A non-cooperative theory of quantity-rationing international transfrontier pollution

Sudhir A. Shah*

September 13, 2007

Abstract

We study a remedy for the problem caused by international transfrontier pollution. Our results are derived from the analysis of a non-cooperative game model of the determination of emissions in a quantity-rationing setting. We model the emission capping negotiations using the best response dynamic process and provide natural conditions under which the process has a unique and globally asymptotically stable stationary point. We then analyze the link between type profiles and the stationary points of the negotiation process to derive various comparative statics results and the type-contingent ordering of emission allocations. These results are used to study the investment strategies that nations can use prior to the negotiations in order to manipulate the equilibrium emission caps. A policy implication of our model is that a cap-and-trade arrangement is inferior to a cap-and-hold arrangement if the policy aim is to reduce equilibrium total emission. We also point out some implications of our results regarding the political economy of emission capping.

JEL classification: D74, H41, Q21, Q25, R11

Key phrases: Emission capping, non-cooperative game, negotiations, incomplete information, manipulation

1 Introduction

Consider n nations whose firms emit some pollutant into a shared medium. The aggregate emission hurts consumers in all these nations. The pre-protocol situation is that each firm chooses its production plan, and therefore emission, to maximize its own profit subject to pre-protocol domestic regulations.[1] The resulting outcome is inefficient for standard specifications of

*All correspondence should be addressed to: Department of Economics, Delhi School of Economics, University of Delhi, Delhi 110007, India. Telephone: (+91)(11) 2766-7005. Fax: (+91)(11) 2766-7159. E-mail: sudhir@econdse.org

[1]The resulting outcome is also called "the *status quo*" or "business-as-usual" in the literature. However, the modeling of this choice does vary in substance and interpretation.

national welfare that take consumer welfare into account. Suppose these nations attempt to improve upon this situation by means of a protocol that caps each nation's emission to some country-specific level.[2]

The contributions of this paper are threefold. First, we present a tractable general model of protocol formation that generates a self-enforcing emission capping protocol with various attractive features. Secondly, we use the model to study two issues of political-economic significance. One application is to understand the effect of historically given asymmetries among nations on protocol outcomes. The other application is to use the model to predict the nature of pre-protocol strategic manoeuvering that can be expected from various parties, specifically the use of investment as a commitment device to manipulate protocol outcomes. Thirdly, we derive the following policy implication of our analysis: if the negotiating nations are sufficiently concerned about the total world emission and the damages caused by it, then they should opt for a cap-and-hold arrangement rather than a cap-and-trade arrangement because the equilibrium total emission under the autarkic arrangement is lower than the equilibrium total emission under the trading arrangement.

Model

Each nation consists of two classes of entities: (a) pollution emitters (henceforth, labeled as "firms"), and (b) non-emitters who are hurt by the aggregate international emission (henceforth, labeled as "consumers").[3] While the production and emission decisions are made by the firms to maximize their profits, the emission caps under the protocol are negotiated by the national governments, who are concerned not only about the profits of domestic firms but also about the damage suffered by domestic consumers.

A nation's emission cap is its endowment of emission rights, which is an upper bound on the pollution that can be emitted by that nation. We assume that each nation has a mechanism for distributing its endowment of emission rights among domestic firms and ensuring their compliance with the implied firm-level caps.[4] As the equilibrium national caps generated by our protocol are self-enforcing, each nation has the incentive to ensure that domestic

[2]Our aim is to work out the consequences of the regime modeled in this paper, not to rationalize any actual protocol. In any case, most actual protocols cannot be so rationalized as they are merely exhortative. Improving on this situation requires us to understand the consequences of different ways of constructing international institutions and protocols. In this paper, we analyze the non-cooperative quantity-rationing strategy.

[3]Clearly, these classes are not necessarily congruent with the usual economic classes of firms and consumers. For instance, if the pollutant is a greenhouse gas, then there may be firms in the usual economic sense who are non-emitters (e.g., farms) and are hurt by the effects of greenhouse gas emissions, and therefore categorized as "consumers" by us.

[4]The domestic regulator must monitor emissions in order to enforce compliance, but this is essential for other regulatory schemes too, such as emission taxation.

firms respect the caps imposed on them. We do not model the domestic capping mechanism because, given its assumed effectiveness, its exact nature is irrelevant in the context of international capping negotiations: foreign nations care only about the total emission from a country, not its domestic distribution.

The domestic mechanism maps the national cap into a profile of caps for domestic firms and cap-constrained profit maximization by these firms maps each domestic cap profile into a profile of profits and emissions. We assume that each government cares about total domestic profit and total emission but not their distribution. Therefore, we model the domestic implementation of a cap by postulating an aggregate national firm that maximizes profit subject to the national emission cap.[5] For the sake of convenience, we also aggregate each nation's consumers into a single national consumer.

A nation is described by its type, which consists of two parameters. The first is private capital, which consists of the fixed inputs that determine the nation's (equivalently, the national firm's) production technology. The second is adaptation capital, which determines the relationship between aggregate emission and the damage suffered by that nation (equivalently, by the national consumer). Adaptation capital consists of assets that are used to mitigate the damage caused by emissions, e.g., water and forest management systems, meteorological facilities, knowledge of the ways to cope with the effects of pollution, research facilities that generate such technologies, etc. We assume that types are common knowledge.[6] National types are a device for modeling the heterogeneity of nations and studying the consequences of historical asymmetries among nations. In addition, our second application allows the study of another asymmetry among nations. As this application involves nations investing prior to protocol formation, a forward-looking asymmetry among nations is their differing ability to invest.

Given this data about the players, their preferences and their types, our model of protocol formation and implementation has two stages. In Stage 1, the national governments use the best response dynamic (henceforth, BRD) procedure to negotiate emission caps: given a proposed profile of caps, the new profile of proposed caps is the profile of best responses to the given profile. As a cap cannot be imposed on a sovereign nation, the negotiations continue until a stationary profile is reached; we identify "reached" with asymptotic convergence. The stationary caps are self-enforcing as, by definition, no country can unilaterally improve upon a stationary profile. In

[5]Given the assumed nature of government preferences, it is possible to show formally, using the usual apparatus of production sets, that our postulate involves no loss of generality. We omit this demonstration and simply black-box the domestic implementation issue as it is tangential to our main objectives.

[6]The results extend, with appropriate qualification, to the incomplete information case (Shah [20]). However, as this complicates the notation and does not generate new phenomena of substantive interest, we restrict attention to the complete information model.

Stage 2, each national firm chooses a production and emission plan subject to that nation's emission cap determined in Stage 1. Naturally, nations will take into account the Stage 2 choices of firms when negotiating in Stage 1.

While governments negotiate the caps in Stage 1, they do not themselves use the emission rights in Stage 2. Therefore, the endowments of emission rights negotiated in Stage 1 necessarily devolve to the national firms. However, as nations and their national firms have different preferences, it is possible that a stationary profile of caps from the perspective of the nations may still imply gains-from-trade from the firms' perspective if they can trade rights among themselves. Our model does not permit such trade. The cap-and-trade variant of our model is analyzed and compared with our cap-and-hold model in Section 8.

Results

In Stage 1, for each profile of national types, the equilibrium profile of caps is the stationary point of the BRD procedure. We characterize the set of stationary points of the dynamic process as the set of Nash equilibria of an artificial non-cooperative game (Proposition 4.5). Given this characterization, the existence of stationary points follows from an application of Nash's existence theorem (Proposition 4.6). Assuming the Hahn condition implies that the artificial game is dominance-solvable, which has very strong implications (Proposition 4.7). First, this guarantees the existence of a unique Nash equilibrium of the artificial game, and therefore, a unique stationary point of the capping negotiations. Secondly, the steady-state solution of the BRD system corresponding to the unique stationary point is globally asymptotically stable. Thirdly, the implied mapping from type profiles to equilibrium cap profiles is simply characterized and sufficiently smooth given standard regularity assumptions regarding the primitives of the model (Proposition 5.2). As suggested by the correspondence principle, the assumptions implying the asymptotic stability of the equilibrium also enable us to derive the effects of parametric variations on the equilibrium cap profile. These general n-player comparative statics results (Proposition 5.3) are the key ingredients for both our applications.

The first application uses these results to show how historical asymmetries among nations affect equilibrium caps. We show that, *ceteris paribus*, (a) nations with more adaptation capital get larger caps (Proposition 6.1), and (b) among nations with clean technology, a nation with more private capital gets a smaller cap, while among nations with dirty technology, a nation with more private capital gets a larger cap (Proposition 6.2).

The second application analyzes the means and nature of strategic manipulation of the protocol. This is an innovation in the formal literature on emission protocols, especially the analysis of the differing uses of the two natural instruments in this context.[7] We show how pre-negotiation investment choices can be used to manipulate the equilibrium emission caps (Propositions 7.2, 7.4, 7.6, 7.8). We show that nations will overinvest in adaptation capital, underinvest in domestic private capital if domestic technology is clean, overinvest in domestic private capital if domestic technology is dirty, not invest in foreign adaptation capital and invest in foreign private capital only if the recipient nation has clean technology. While the directional aspects of these results are invariant with respect to the identity of the investor, there are noteworthy differences between the nature of investment choices made by a nation and the firm within it (Propositions 7.1, 7.4(B), 7.5, and Remarks 7.3 and 7.7).

In our model, we endogenize each nation's participation decision with respect to the protocol by identifying "non-participation" with an infinite cap in equilibrium. While we allow this possibility, in equilibrium every nation will accept a finite emission cap that is lower than its pre-protocol emission level, i.e., every nation will choose to "participate". As the caps are incentive compatible (by definition) and domestically implementable by the national governments (by assumption), the equilibrium aggregate emission will be lower than the pre-protocol level. Indeed, *from the perspective of the nations*, the equilibrium outcome will be Pareto superior to the pre-protocol outcome without relying on international transfers (Proposition 4.9). For conditions that generate maximal improvements, i.e., reaching the Pareto frontier, see the cooperative approach discussed in Section 9.

A final result that has strong policy implications is our demonstration (Proposition 8.1) that a cap-and-trade institutional arrangement, i.e., capping is followed by trade in emission rights before the re-allocated rights are implemented, leads to higher total emission in equilibrium than the autarkic cap-and-hold scheme we have studied in this paper.

Modeling choices

The problem of modeling environmental protocols has generated a varied literature that can be classified broadly in terms of three modeling choices: (a) the modeling of the pre-protocol situation, (b) the method used to regulate emissions, and (c) the degree of collusion among nations that is implicitly assumed to be institutionally feasible.

The treatment of (a) depends on the identity of the entity choosing the pre-protocol emission. Actual emission choices are typically made by firms and not by governments. This distinction would be immaterial if pre-protocol regulations succeed in identifying the interests of the firm and the government. However, it seems unlikely that, in the absence of an international capping protocol, governments unilaterally align the domestic firms'

[7] Of course, there are numerous models with similar results in the dynamic oligopoly literature; for a survey, see Fudenberg and Tirole [11].

incentives with those of the state so as to force firms to take into account the damage suffered by domestic consumers on account of the *international* emission externality. Certainly, regulatory practices do not suggest such a global vision. The disaggregation of a nation into a firm and a consumer allows us to model the pre-protocol emission choice as being made by the firm. Other models (see Section 9) treat nations as unitary entities that choose emissions and the emission caps, in effect conflating the two variables and identifying the interests of the entities choosing these variables.

With respect to (b), our model features pure quantity-rationing instead of the standard fiscal remedies for dealing with externalities, or quantity-rationing supplemented by fiscal transfers. With respect to (c), our purely non-cooperative model embodies pessimism regarding the existence of institutions that can exercise supranational fiscal authority over sovereign nations or sustain collusive agreements among coalitions of nations. Whether this pessimism is justified is, of course, an empirical matter.

A final remark is in order before we proceed to the formalism. Typically, the pollutants (e.g., greenhouse gases and CFCs) that cause the global externalities that we seek to address are stock pollutants. For such pollutants, the problem of accounting for past emissions at a point of time is conceptually distinct from that of regulating future flows. While the former is the problem of apportioning responsibility for a noxious stock of exogenously given size, the latter is the problem of regulating the marginal externalities generated by future emissions. As these problems require different theoretical approaches, it is desirable to treat the two problems separately. Our model addresses the latter problem, taking history as given.

Outline of paper

In Section 2, we formally state our model. As is standard, the model is solved backwards, with Stages 2 and 1 of the model studied in Sections 3 and 4 respectively. In Section 5, we derive the comparative statics results that link the equilibrium points of the model to the underlying data. We use these results to study our two applications in Sections 6 and 7. Section 8 is devoted to the comparison of the total equilibrium emissions resulting from the cap-and-trade and the cap-and-hold institutional arrangements. Section 9 contains a brief overview of the literature. We conclude in Section 10 by interpreting our results and suggesting extensions. The technical proofs are collected in the Appendix. The assumptions underlying our results are stated in the nested sequence of Assumptions 3.1, 4.1 and 5.1, with Assumption 3.1 being the weakest set of assumptions and Assumption 5.1 being the strongest set of assumptions. This is done to clarify the precise assumptions underlying the results.

2 The model in extensive form

$N = \{1, \ldots, n\}$ is the set of nations. Nation i's type space is $\Theta = \Re^2_{++}$ and the space of type profiles is Θ^n. Nation i consists of Firm i and Consumer i. Nation i's type is $(t_i, k_i) = \theta_i \in \Theta$, where t_i is Firm i's private capital and k_i is Consumer i's adaptation capital.

The model has two stages: (1) protocol formation, and (2) implementation. Suppose $\theta \in \Theta^n$ is the type profile with $\theta_i = (t_i, k_i)$. In Stage 1, the nations negotiate using the BRD procedure and arrive at the stationary profile of caps $e = (e_1, \ldots, e_n) \in \bar{\Re}^n$. In Stage 2, Firm i chooses variable input v_i, which determines Firm (and Nation) i's profit $g(t_i, v_i)$ and emission $h(t_i, v_i)$. The choice of v_i is subject to the emission constraint $h(t_i, v_i) \leq e_i$. The total world emission $\sum_{j \in N} h(t_j, v_j)$ is consumed by the consumer of every nation and this causes damage $\delta(k_i, \sum_{j \in N} h(t_j, v_j))$ to Consumer (and Nation) i.

We interpret g and h as incorporating the effects of non-quantitative domestic emission regulations, e.g., emission taxes and subsidies for adoption of clean technologies. Thus, our welfare results should be interpreted as reflecting improvements compared to the pre-protocol situation with its full complement of domestic emission regulations.

3 Analysis of the second stage

Suppose the profile of types is θ and the profile of emission caps is e. Given this data, we analyze Firm i's decision-making assuming that it maximizes its profit.

Assumption 3.1 *$g : \Re^2_+ \rightarrow \Re_+$ and $h : \Re^2_+ \rightarrow \Re_+$ are continuous. For every $t \in \Re_+$,*

(a) $g(t, .)$ has a unique maximum at $V(t)$ where $V : \Re_+ \rightarrow \Re_{++}$ is continuous; moreover, $g(t, .)$ is strictly increasing on $[0, V(t)]$ and strictly concave on $\Re_+$, and

(b) $h(t, 0) = 0$ and $h(t, .)$ is strictly increasing and strictly convex.

(a) implies that profit increases with variable input until the unconstrained maximum is attained and use of the variable input faces diminishing returns in value terms. (b) implies that emission increases at an increasing rate with the variable input.

Consider a firm with private capital $t \in \Re_+$ and emission cap $e \in \bar{\Re}_+$. The firm's problem is to choose variable input v to maximize profit $g(t, v)$ subject to the constraint $h(t, v) \leq e$. Since $V(t)$ is an unconstrained optimal choice for the firm and $h(t, .)$ is increasing, the firm's constraint can be written as $v \in \Gamma(t, e) \equiv \{v \in \Re_+ \mid h(t, v) \leq e\} \cap [0, V(t)]$.

Let $v : \Re_+ \times \bar{\Re}_+ \to \Re_+$ be such that, for every $(t, e) \in \Re_+ \times \bar{\Re}_+$, $v(t, e)$ solves the firm's problem; consequently, for every $(t, e) \in \Re_+ \times \bar{\Re}_+$, we have $h(t, v(t, e)) \leq e$. Define $f : \Re_+ \times \bar{\Re}_+ \to \Re$ by $f(t, e) = g(t, v(t, e))$. Consequently, Firm i's choice of variable input is $v(t_i, e_i)$, its profit is $f(t_i, e_i)$ and its emission is $h(t_i, v(t_i, e_i))$.

Proposition 3.2 *Given Assumption 3.1,*

(A) for every $(t, e) \in \Re_+ \times \bar{\Re}_+$ there exists a unique $v(t, e) \in \Gamma(t, e)$ such that $g(t, v(t, e)) \geq g(t, v)$ for every $v \in \Gamma(t, e)$, and

(B) $v : \Re_+ \times \bar{\Re}_+ \to \Re_+$ and $f : \Re_+ \times \bar{\Re}_+ \to \Re$ are continuous functions. Moreover, for every $(t, e) \in \Re_+ \times \bar{\Re}_+$,

(C) $e \geq h(t, V(t))$ if and only if $v(t, e) = V(t)$, and

(D) $e \leq h(t, V(t))$ if and only if $h(t, v(t, e)) = e$.

Furthermore, for every $t \in \Re_+$,

(E) $f(t, .)$ is strictly increasing on $[0, h(t, V(t))]$,

(F) $f(t, .)$ is strictly concave on $[0, h(t, V(t))]$, and

(G) $f(t, e) = g(t, V(t))$ for $e \geq h(t, V(t))$.

$V(t)$ is to be interpreted as the pre-protocol level of variable input use. Therefore, $g(t, V(t))$ and $h(t, V(t))$ are the pre-protocol profit and emission. Thus, a cap e is a binding constraint on the firm if and only if $e \leq h(t, V(t))$. Given these interpretations, Proposition 3.2 is easy to interpret.

4 Analysis of the first stage

Suppose the profile of types is $\theta \in \Theta^n$ with $\theta_j = (t_j, k_j)$. Suppose the profile of emission caps $e = (e_j)_{j \in N} \in \bar{\Re}^n_+$ is proposed by a mediator; $e_j = \infty$ is interpreted as "non-participation". The nations inform the mediator about their best responses, which are used as the proposals for the next round. A stationary point of this iterative procedure is the implemented profile of caps in Stage 2.

To ease analysis, we enrich Assumption 3.1 as follows. *Henceforth, a function being $\mathcal{C}^p$ means it is p times continuously differentiable on the specified set.*

Assumption 4.1 *In addition to the requirements of Assumption 3.1, suppose that, for every $t \in \Re_+$ and every $k \in \Re_+$,*

(a) $g(t, .)$ and $h(t, .)$ are $\mathcal{C}^2$ on $\Re_{++}$,

(b) $\delta(k, .)$ is continuous, $\mathcal{C}^2$ on $\Re_{++}$, strictly increasing and strictly convex.

Moreover, for every $\theta = (t_j, k_j)_{j \in N} \in \Theta^n$ and $i \in N$,

(c) $g(t_i, V(t_i)) - \delta(k_i, \sum_{j \in N} h(t_j, V(t_j))) > 0$.

The smoothness assumptions enable simple characterizations of optima. The substantive part of (b) assumes that damage increases at an increasing rate with respect to total emission. (c) is an innocuous assumption that Nation i's pre-protocol payoff is positive. Its only role is to ensure that no nation will accept a zero emission cap in equilibrium. This fact is not crucial for our results but does help to shorten some technical arguments. In any case, such a condition can be ensured by changing the weights of Firm i and Consumer i in Nation i's welfare function.

Proposition 4.2 *Given Assumption 4.1 and* $t \in \Re_+$,
(A) $D_v g(t, V(t)) = 0$,
(B) $D_e v(t, e) = 0$ *and* $D_v g(t, v(t, e)) = 0$ *for* $e > h(t, V(t))$,
(C) $v(t, .)$ *is* $\mathcal{C}^1$ *on* $(0, h(t, V(t)))$, *and*
(D) $g(t, v(t, .))$ *is* $\mathcal{C}^1$ *on* $(0, h(t, V(t))) \cup (h(t, V(t)), \infty)$.

Suppose a profile of caps $e \in \bar{\Re}^n_+$ is proposed. Nation i's payoff is

$$u_i(\theta, e) = f(t_i, e_i) - \delta\left(k_i, h(t_i, v(t_i, e_i)) + \sum_{j \in N - \{i\}} h(t_j, v(t_j, e_j))\right) \tag{1}$$

if e is implemented.[8] If $e_i \in [0, h(t_i, V(t_i))]$, then Proposition 3.2(D) implies

$$u_i(\theta, e) = f(t_i, e_i) - \delta\left(k_i, e_i + \sum_{j \in N - \{i\}} h(t_j, v(t_j, e_j))\right) \tag{2}$$

Thus, Proposition 3.2(F) and Assumption 4.1(b) imply that $u_i(\theta, ., e_{-i})$ is strictly concave on $[0, h(t_i, V(t_i))]$. If $e \in \prod_{j \in N}[0, h(t_j, V(t_j))]$, then (1) further simplifies to

$$u_i(\theta, e) = f(t_i, e_i) - \delta(k_i, e_+) \tag{3}$$

where $e_+ = \sum_{j \in N} e_j$. The set of Nation i's best responses to a proposal e is

$$\beta_i(e_{-i}; \theta) = \bigcap_{x \in \bar{\Re}_+} \{b \in \bar{\Re}_+ \mid u_i(\theta, b, e_{-i}) \geq u_i(\theta, x, e_{-i})\}$$

Note that we allow the response to be an infinite cap, which amounts to non-participation. It seems possible that $h(t_i, v(t_i, b)) < b$ for some $b \in \beta_i(e_{-i}; \theta)$, i.e., Nation i's emission given a best response cap may be less than the cap. This can complicate the analysis of emission capping as the costs and benefits of capping are generated by the actual emissions, rather than the caps *per se*. The following result eliminates this potential problem.

[8]The equal weights given to the national firm and the national consumer in the national welfare function are just a matter of convenience; specifying other weights will not affect the nature of our results.

Proposition 4.3 *Let $\theta \in \Theta^n$, $\theta_i = (t_i, k_i)$ and $e \in \bar{\Re}^n_+$. Given Assumption 4.1, if $b \in \beta_i(e_{-i};\theta)$, then*

(A) $0 \leq h(t_i, v(t_i, b)) = b < h(t_i, V(t_i))$, and

(B) $\{b\} = \beta_i(e_{-i};\theta)$, i.e., i's best response is unique.

Given this uniqueness property, proposals are generated using the BRD procedure:

$$e_i(\tau + 1) = \beta_i(e_{-i}(\tau);\theta) \tag{4}$$

for $i \in N$ and $\tau \in \mathcal{N}$, where τ represents the τ-th round of negotiations. We say that $e \in \bar{\Re}^n_+$ is a stationary point of (4), given $\theta \in \Theta^n$, if $e_i = \beta_i(e_{-i};\theta)$ for every $i \in N$.

Proposition 4.4 *Given $\theta \in \Theta^n$, if e is a stationary point of (4), then for every Nation i, $e_i = \beta_i(e_{-i};\theta) \in (0, h(t_i, V(t_i)))$ and the payoff is given by (3).*

The above results reduce the number of variables involved in the analysis of an equilibrium by identifying equilibrium caps with equilibrium emissions, thereby simplifying (1) to (3). Secondly, Nation i's payoff in the relevant range $[0, h(t_i, V(t_i))]$ is *not* monotonically increasing with respect to e_i, for a larger cap in this range induces greater emission by Firm i, thereby increasing Firm i's profit, but also increasing Consumer i's damage. Thus, by attaching an endogenously generated shadow value to emission rights in the form of damages, our model forces nations to trade-off profits against damages, thereby preventing them from pursuing arbitrarily large caps. Consequently, results in Section 4 asserting that a nation manipulates its type to increase its emission cap do not reflect a trivial desire to have a larger amount of a free positive-valued option, but a desire to have a specific larger cap for strategic reasons.

Given $\theta \in \Theta^n$ with $\theta_i = (t_i, k_i)$, define the non-cooperative game $G(\theta) = \{N, ([0, h(t_i, V(t_i))], u_i(\theta, .))_{i \in N}\}$; N is the set of players, $[0, h(t_i, V(t_i))]$ is player i's strategy space and $u_i(\theta, .) : \prod_{j \in N}[0, h(t_j, V(t_j))] \to \Re$ is player i's payoff function.

Proposition 4.5 *Given $\theta \in \Theta^n$, e^* is a stationary point of (4) if and only if e^* is a Nash equilibrium of $G(\theta)$.*

Needless to say, this result neither assumes nor implies that the nations are playing $G(\theta)$ in Stage 1. However, the characterization is very useful as it allows the application of many standard results; e.g., an application of Nash's existence theorem yields the following.

Proposition 4.6 *Given Assumption 4.1 and $\theta \in \Theta^n$, (4) has a stationary point.*

Theorem 3 in Moulin [19] implies the following much stronger result.

Proposition 4.7 *Suppose Assumption 4.1 is satisfied. Also suppose that, for every $i \in N$, for every $\theta \in \Theta^n$ with $\theta_j = (t_j, k_j)$, and for every $e \in \prod_{j \in N}[0, h(t_j, V(t_j))]$, we have*

$$-D_{ee}f(t_i, e_i) > (n-2)D_{e_+e_+}\delta(k_i, e_+) \tag{5}$$

Then, for every $\theta \in \Theta^n$, $G(\theta)$ is dominance-solvable and has a unique Nash equilibrium that is globally asymptotically stable with respect to (4).

(5) is the Hahn condition. Combining this result with Proposition 4.5, we have

Corollary 4.8 *Given the assumptions of Proposition 4.7, (4) has a unique stationary point that is globally asymptotically stable.*

Thus, given appropriate assumptions, the BRD procedure converges to the same stationary point, independent of the initial proposed profile. If $n = 2$, then (5) is automatically satisfied. (5) becomes easier to satisfy as (a) n decreases, (b) the curvature of f increases, and (c) the curvature of δ decreases. For example, the closer the damage function δ is to being linear in total emission e_+, the more easily is (5) satisfied.

We now assess the welfare properties of stationary points. Proposition 4.4 implies that $e \ll (h(t_j, V(t_j)))_{j \in N}$ for every stationary point e of (4). As every firm is constrained to emit less than its pre-protocol optimal choice, every firm is worse off compared to the pre-protocol outcome. As the total emission is less than the pre-protocol level, every consumer is better off. *From the perspective of the nations*, we show that the stationary point represents a Pareto improvement over the pre-protocol outcome. Indeed, the stationary profile is Pareto superior to every intermediate profile also.

Proposition 4.9 *Suppose Assumption 4.1 is satisfied. If e is a stationary point of (4) and $z \in \prod_{j \in N}[e_j, h(t_j, V(t_j))] - \{e\}$, then $u_i(\theta, z) < u_i(\theta, e)$ for every $i \in N$.*

Finally, we define an equilibrium of our model.

Definition 4.10 *$\{e, v\}$ is an equilibrium if*

(a) $v : \Re_+ \times \bar{\Re}_+ \to \Re_+$ is such that, for every $(t, e) \in \Re_+ \times \bar{\Re}_+$, $v(t, e)$ maximizes $g(t, .)$ subject to the constraint $h(t, .) \leq e$, and

(b) $e : \Theta^n \to \bar{\Re}^n_+$ is such that $e(\theta)$ is a stationary point of (4) for every $\theta \in \Theta^n$.

Given Assumption 4.1, Propositions 3.2(A) and 4.6 ensure the existence of such an equilibrium. (a) requires optimal production choices by all firms in Stage 2 given *any* profile of emission caps chosen in Stage 1. Given v, (b) requires the selection of a stationary profile of emission caps in Stage 1.

Remark 4.11 *Consider the following artificial two-stage game. In the first stage, for every $\theta \in \Theta^n$ with $\theta_i = (t_i, k_i)$, the game $G(\theta)$ is played. Let e be an outcome of $G(\theta)$. In the second stage, for every $i \in N$, Firm i selects $v \in \Gamma(t_i, e_i)$ to maximize $g(t_i, v)$. Consider an equilibrium $\{e, v\}$ of our model. By Proposition 3.2(A), for every $t_i \in \Re_{++}$, the restriction of $v(t_i, .)$ to $[0, h(t_i, V(t_i))]$ generates Firm i's unique optimal Stage 2 action. By Proposition 4.7, $e(\theta)$ is the unique Nash equilibrium of $G(\theta)$. Thus, $\{e, v\}$ generates the unique subgame perfect Nash equilibrium of the artificial two-stage game.*

Given Assumption 4.1, Propositions 3.2(A) and 4.6 guarantee the existence of such an equilibrium. Indeed, existence is guaranteed even if the differentiability assumptions and part (c) of Assumption 4.1 are dropped. The differentiability assumptions are used to derive the uniqueness and stability properties of the stationary point in Proposition 4.7. Part (c) of Assumption 4.1 is used only in Proposition 4.4 to ensure that a stationary point of (4) cannot involve a zero emission cap for some nation.

5 Comparative statics for the first stage

Suppose Assumption 4.1 and (5) hold. By Proposition 4.4 and Corollary 4.8, there is a unique function $e : \Theta^n \to \prod_{j \in N}(0, h(t_j, V(t_j)))$ that describes the stationary cap profiles of (4) contingent on the type profile. In this section, we analyze the variational properties of this mapping. By Propositions 4.3 and 4.4, $e(\theta)$ is identical to the profile of emissions for every profile of types θ. Henceforth, "Nation i's equilibrium cap" is interchangeable with "Nation i's emission".

Our analysis so far has taken the type profile θ as given. Consequently, we have only made assumptions about the dependence of g, h and δ on the variable input and emissions, with types taken as parameters. Our analysis in this section will require information on the effects of type variations on emission, profit and damage.

Assumption 5.1 *In addition to Assumption 4.1 and (5), assume that*

(a) for every $v \in \Re_+$, $g(., v)$ is strictly increasing and $h(., v)$ is strictly decreasing,

(b) for every $e \in \Re_+$, $f(., e)$ is strictly concave,

(c) g, h and δ are $\mathcal{C}^2$ on $\Re^2_{++}$.

(d) $D_{ke_+}\delta < 0$, and

(e) for every $e_+ \in \Re_+$, $\delta(., e_+)$ is strictly decreasing and strictly convex.

(a) implies that profit is increasing and emission is decreasing with respect to private capital. (b) implies that private capital faces diminishing returns. (d) implies that greater adaptation capital reduces a nation's vulnerability to damage. (e) means that a nation's damage is a decreasing function of adaptation capital but this beneficial effect is subject to diminishing returns. We note the following consequences of these assumptions.

Proposition 5.2 *Given Assumption 5.1,*
(A) v and f are $\mathcal{C}^2$ on $\{(t,e) \in \Re^2_{++} \mid e < h(t, V(t))\}$,
(B) given $e, t, t' \in \Re_+$, if $t < t'$, $e \leq h(t, V(t))$ and $e \leq h(t', V(t'))$, then $f(t,e) < f(t',e)$, and
(C) for $i \in N$, u_i is $\mathcal{C}^2$ on $\Theta^n \times \prod_{j \in N}(0, h(t_j, V(t_j)))$.

Consider $j \in N$ and $\theta \in \Theta^n$ with $\theta_j = (t_j, k_j)$. By Proposition 4.4, u_j is given by (3). By definition, $u_j(\theta, ., e_{-j}(\theta))$ is maximized at $e_j(\theta)$. Proposition 4.4 implies $e_j(\theta) \in (0, h(t_j, V(t_j)))$. Therefore, Proposition 5.2(C) implies that $u_j(\theta, ., e_{-j}(\theta))$ is $\mathcal{C}^2$ on $(0, h(t_j, V(t_j)))$. Proposition 4.4 implies that

$$D_{e_j} u_j(\theta, e(\theta)) = 0 \tag{6}$$

for every $j \in N$. $D_{e_j} u_j(\theta, e(\theta))$ is the shadow value of emission rights to Nation j when the emission cap profile is $e(\theta)$. Thus, (6) means that, in equilibrium, the shadow value of emission rights for every nation is equal to the cost of acquiring the marginal right, which is zero. Analogously, $D_e f(t_j, e_j)$ is the shadow value of emission rights to Firm j with private capital t_j when Nation j's emission cap is e_j. It follows from Proposition 3.2 that the firm's shadow value of emission rights is positive at $e_j(\theta)$. Thus, the national firm will prefer a larger cap than the equilibrium one negotiated by the national government.

Combining Propositions 3.2(F) and 5.2(A) with Assumptions 5.1(c) and 4.1(b) implies the second order condition

$$D_{e_j e_j} u_j(\theta, e(\theta)) < 0 \tag{7}$$

for every $j \in N$. The effects on $e(\theta)$ of varying either component of θ_1 are as follows; with appropriate notational adjustments, the same result holds for all nations.

Proposition 5.3 *Given Assumption 5.1, and interpreting x as either t_1 or k_1,*
(A) $e : \Theta^n \to \Re^n_+$ is $\mathcal{C}^1$ on Θ^n,
(B) Sign $D_x e_j = -$Sign $D_{e_1 x} u_1$ for $j \in N - \{1\}$,
(C) Sign $D_x e_1 =$ Sign $D_{e_1 x} u_1$, and
(D) Sign $D_x \sum_{j \in N} e_j =$ Sign $D_{e_1 x} u_1$.

Evidently, the signs of all the variational formulae depend on how the type variations affect the shadow values of emission rights for the nation whose type is being perturbed. We classify technology as locally clean (resp. dirty) if the firm's shadow value of emission rights decreases (resp. increases) with increases in private capital.

Definition 5.4 *Technology f is dirty (resp. clean) at (t', e') if $D_{te}f(t', e') > 0$ (resp. $D_{te}f(t', e') < 0$).*

Using this definition, we have the following corollary of Proposition 5.3.

Corollary 5.5 *Let $\theta \in \Theta^n$ with $\theta_i = (t_i, k_i)$. Let $j \neq 1$. Given Assumption 5.1,*

(A) $D_{t_1}e_1(\theta) < 0$, $D_{t_1}e_j(\theta) > 0$ and $D_{t_1}e_+(\theta) < 0$ if $D_{te}f(t_1, e_1(\theta)) < 0$,

(B) $D_{t_1}e_1(\theta) > 0$, $D_{t_1}e_j(\theta) < 0$ and $D_{t_1}e_+(\theta) > 0$ if $D_{te}f(t_1, e_1(\theta)) > 0$, and

(C) $D_{k_1}e_1(\theta) > 0$, $D_{k_1}e_j(\theta) < 0$ and $D_{k_1}e_+(\theta) > 0$.

(A) (resp. (B)) means that the growth of private capital in a clean (resp. dirty) nation implies lower (resp. higher) domestic emission, higher (resp. lower) foreign emissions and lower (resp. higher) aggregate emission. (C) means that the growth of adaptation capital implies higher domestic emission, lower foreign emissions and higher aggregate emission. The directional effects on aggregate emission are identical to the directional effects on domestic emission and opposite to the directional effects on foreign emissions.

6 Application: effects of historical asymmetries

Suppose the Stage 1 stationary emission caps are generated by a function $e : \Theta^n \to \prod_{j \in N}(0, h(t_j, V(t_j)))$. We first derive the ordering of emission caps implied by the ordering of adaptation capital, *ceteris paribus*.

Proposition 6.1 *Let $\theta \in \Theta^n$ with $\theta_i = (t_i, k_i)$ for $i \in N$. Given Assumption 5.1, if $t_1 = t_2$ and $k_1 > k_2$, then $e_1(\theta) > e_2(\theta)$.*

Proof. By (6) and Assumption 5.1(d), $k_1 > k_2$ implies

$$D_e f(t_1, e_1(\theta)) = D_{e_+}\delta(k_1, e_+(\theta)) < D_{e_+}\delta(k_2, e_+(\theta)) = D_e f(t_2, e_2(\theta))$$

As $t_1 = t_2$, Proposition 3.2(F) implies $e_1(\theta) > e_2(\theta)$. ■

Ceteris paribus, nations with more adaptation capital have larger emissions. In the case of private capital, the analogous result is more complicated as the nature of technology affects the directions in which the emissions change as private capital varies.

Proposition 6.2 *Let $\theta \in \Theta^n$ with $\theta_i = (t_i, k_i)$ for $i \in N$. Given Assumption 5.1, if*

(a) $k_1 = k_2$ and $t_1 > t_2$,

(b) $D_{te}f(t_1, e_1(\theta)) > 0$ and $D_{te}f(t_2, e_2(\theta)) > 0$ (resp. $D_{te}f(t_1, e_1(\theta)) < 0$ and $D_{te}f(t_2, e_2(\theta)) < 0$), and

(c) $D_{te}f(., e_1(\theta))$ is decreasing,

then $e_1(\theta) > e_2(\theta)$ (resp. $e_1(\theta) < e_2(\theta)$).

Proof. Let $t_1 > t_2$, $D_{te}f(t_1, e_1(\theta)) > 0$ and $D_{te}f(t_2, e_2(\theta)) > 0$. It follows from (c) that $D_{te}f(x, e_1(\theta)) > D_{te}f(t_1, e_1(\theta)) > 0$ for every $x \in [t_2, t_1]$. As $k_1 = k_2$, (6) implies $D_e f(t_2, e_2(\theta)) - D_e f(t_2, e_1(\theta)) = D_e f(t_1, e_1(\theta)) - D_e f(t_2, e_1(\theta)) = \int_{t_2}^{t_1} dx\, D_{te}f(x, e_1(\theta)) > 0$. From Proposition 3.2(F), we have $e_1(\theta) > e_2(\theta)$. The other case follows analogously. ■

If (a) both nations have the same adaptation capital stock, (b) both nations have clean (resp. dirty) technology, and (c) technology becomes cleaner as private capital grows, then the nation with the greater private capital stock has lower (resp. higher) emission.

7 Application: manipulation of protocol outcomes

Suppose that, prior to negotiating the protocol in Stage 1 of the model studied in Sections 2 to 4, there is Stage 0, in which nations can invest in private and adaptation capital. Let $\bar{\theta} \in \Theta^n$ be the initial profile of types for Stage 0, with $\bar{\theta}_i = (\bar{t}_i, \bar{k}_i)$. Let $\theta \in \Theta^n$, with $\theta_i = (t_i, k_i)$, be the modified profile of types after the nations have invested in Stage 0. θ serves as data for Stage 1 of the model studied in Sections 2 to 4. Nation i's investment in Stage 0 is $I_i = \theta_i - \bar{\theta}_i = (t_i - \bar{t}_i, k_i - \bar{k}_i)$; the cost of this investment is $C(I_i) = t_i - \bar{t}_i + k_i - \bar{k}_i$. Negative investment is interpreted as disinvestment, e.g., the depletion of adaptation capital by cutting down trees. Naturally, if there is disinvestment, the "cost of investment" is negative, i.e., it is the revenue from disinvestment. Also, the amount of feasible disinvestment is bounded because the modified type must be non-negative. Suppose the Stage 1 stationary emission caps are generated by a function $e : \Theta^n \to \prod_{j \in N}(0, h(t_j, V(t_j)))$. The payoff function relevant for Nation i in Stage 0 is $u_i(\theta, e(\theta)) - C(I_i)$, where the first term is given by (1). As e is the equilibrium mapping, Proposition 4.4 implies that u_i is given by (3) and $u_i(\theta, e(\theta)) = f(t_i, e_i(\theta)) - \delta(k_i, e_+(\theta))$.

We shall consider a number of hypotheses about the determination of I_i, depending on the identity of the decision-maker. Given the investment profile I_{-i} of the other nations, $t_i - \bar{t}_i$ (resp. $k_i - \bar{k}_i$) may be chosen by Nation i or Firm i. In addition, there is the possibility that Nation j or Firm j select $t_i - \bar{t}_i$ and $k_i - \bar{k}_i$. Thus, there are eight possible combinations of decision-makers with respect to Nation i's type. In Stage 0, Nation i's investment choices seek to maximize

$$(x, y) \mapsto u_i(\bar{t}_i + x, \bar{k}_i + y, \bar{\theta}_{-i} + I_{-i}, e(\bar{t}_i + x, \bar{k}_i + y, \bar{\theta}_{-i} + I_{-i})) - C(x, y) \quad (8)$$

while Firm i's investment choices seek to maximize

$$(x, y) \mapsto f(\bar{t}_i + x, e_i(\bar{t}_i + x, \bar{k}_i + y, \bar{\theta}_{-i} + I_{-i})) - C(x, y) \quad (9)$$

Nation j's investment choices in Nation i seek to maximize

$$(x, y) \mapsto u_j(\bar{t}_i + x, \bar{k}_i + y, \bar{\theta}_{-i} + I_{-i}, e(\bar{t}_i + x, \bar{k}_i + y, \bar{\theta}_{-i} + I_{-i})) - C(x, y)$$

while Firm j's investment choices in Nation i seek to maximize

$$(x, y) \mapsto f(t_j, e_j(\bar{t}_i + x, \bar{k}_i + y, \bar{\theta}_{-i} + I_{-i})) - C(x, y)$$

In the following two sections we analyze the determination of I_1; the same arguments apply to every nation. As the cost of investments is linear and additive, investment can move without friction between private and adaptation capital. This allows us to study the investment choices in a piecemeal manner without loss of generality. Moreover, it allows us to focus on the purely strategic role of these investments.

Investment in private capital

If t_1 is chosen by Nation 1 (resp. Firm 1), then denote t_1 by t^* (resp. t^{**}). (8) and (9) imply that $t_1 = t^*$ and $t_1 = t^{**}$ maximize

$$u_1(t_1, k_1, \theta_{-1}, e(t_1, k_1, \theta_{-1})) - t_1 \qquad \text{and} \qquad f(t_1, e_1(t_1, k_1, \theta_{-1})) - t_1$$

respectively, taking k_1 and θ_{-1} as given. Using (6), t^* and t^{**} are characterized by

$$D_t f(t^*, e_1(t^*, k_1, \theta_{-1})) - D_{e_+}\delta(k_1, e_+(t^*, k_1, \theta_{-1})) \sum_{j \neq 1} D_t e_j(t^*, k_1, \theta_{-1}) = 1 \quad (10)$$

and

$$D_t f(t^{**}, e_1(t^{**}, k_1, \theta_{-1})) + D_e f(t^{**}, e_1(t^{**}, k_1, \theta_{-1})) D_t e_1(t^{**}, k_1, \theta_{-1}) = 1 \quad (11)$$

respectively. We first ask: in what direction would Nation 1 like to perturb Firm 1's choice t^{**}? Let $G(t_1, k_1) = u_1(t_1, k_1, \theta_{-1}, e(t_1, k_1, \theta_{-1})) - t_1$. (11) implies that

$$D_t G(t^{**}, k_1) = -D_{e_+}\delta(k_1, e_+(t^{**}, k_1, \theta_{-1})) D_t e_+(t^{**}, k_1, \theta_{-1})$$

Therefore, Assumption 4.1(b) and Corollary 5.5 imply

Proposition 7.1 *Given that the Stage 1 stationary emission caps are generated by a function e, Nation 1 prefers a higher (resp. lower) level of private capital than Firm 1 if Nation 1's technology is clean (resp. dirty) at* $(t^{**}, e_1(t^{**}, k_1, \theta_{-1}))$.

Let $t_1 = t^0$ and $t_1 = t^{00}$ maximize

$$u_1(t_1, k_1, \theta_{-1}, e(t^*, k_1, \theta_{-1})) - t_1 \qquad \text{and} \qquad f(t_1, e_1(t^{**}, k_1, \theta_{-1})) - t_1$$

respectively. Given the caps $e(t^*, k_1, \theta_{-1})$, Nation 1 prefers t^0 to t^*. Similarly, given the cap $e_1(t^{**}, k_1, \theta_{-1})$, Firm 1 prefers t^{00} to t^{**}. The difference between t^0 (resp. t^{00}) and t^* (resp. t^{**}) represents Nation (resp. Firm) 1's ability to manipulate the Stage 1 equilibrium caps *via* its Stage 0 choice of private investment. $t^* > t^0$ (resp. $t^* < t^0$) is interpreted as strategic overinvestment (resp. underinvestment) by Nation 1 in domestic private capital. Similarly, $t^{**} > t^{00}$ (resp. $t^{**} < t^{00}$) is interpreted as strategic overinvestment (resp. underinvestment) by Firm 1 in domestic private capital. t^0 and t^{00} are characterized by

$$D_t f(t^0, e_1(t^*, k_1, \theta_{-1})) = 1 \tag{12}$$

and

$$D_t f(t^{00}, e_1(t^{**}, k_1, \theta_{-1})) = 1 \tag{13}$$

respectively.

Proposition 7.2 *Suppose the Stage 1 stationary emission caps are generated by a function e.*

(A) If Nation 1's technology is clean (resp. dirty) at $(t^*, e_1(t^*, k_1, \theta_{-1}))$, *then Nation 1 underinvests (resp. overinvests) in domestic private capital.*

(B) If Nation 1's technology is clean (resp. dirty) at $(t^*, e_1(t^*, k_1, \theta_{-1}))$, *then Firm 1 underinvests (resp. overinvests) in domestic private capital.*

(C) In all the cases considered in (A) and (B), the effect of the strategic manipulation is to raise e_1, *lower* e_{-1} *and raise* e_+.

Proof. Suppose $D_{te} f(t^*, e_1(t^*, k_1, \theta_{-1})) < 0$. The proofs for the other case are analogous.

(A) (10), (12), Assumption 4.1(b) and Corollary 5.5(A) imply that

$$\begin{aligned} D_t f(t^0, e_1(t^*, k_1, \theta_{-1})) &= D_t f(t^*, e_1(t^*, k_1, \theta_{-1})) \\ &\quad - D_{e_+} \delta(k_1, e_+(t^*, k_1, \theta_{-1})) \sum_{j \neq 1} D_t e_j(t^*, k_1, \theta_{-1}) \\ &< D_t f(t^*, e_1(t^*, k_1, \theta_{-1})) \end{aligned}$$

Assumption 5.1(b) implies $t^0 > t^*$.

(B) (11), (13), Proposition 3.2(E) and Corollary 5.5(A) imply that

$$D_t f(t^{00}, e_1(t^{**}, k_1, \theta_{-1})) < D_t f(t^{**}, e_1(t^{**}, k_1, \theta_{-1}))$$

Assumption 5.1(b) implies $t^{00} > t^{**}$.

(C) follows from Corollary 5.5(A). ■

Although the directions of the manipulations carried out by Nation 1 and Firm 1 are identical, their targets for manipulation are different. Since, in equilibrium, e_1 is a best response to e_2 in terms of Nation 1's preference, we have the following observation.

Remark 7.3 *(10) shows that Nation 1 strategically uses investment in private capital to manipulate downwards the equilibrium emission of other nations. (11) shows that Firm 1 strategically manipulates upwards Nation 1's equilibrium emission.*

Finally, we consider the possibility of Nation 2 investing in Nation 1's private capital. By Corollary 5.5(B), if Nation 1's technology is dirty, then an increase in Nation 1's private capital hurts Firm 2 by reducing its emission, and therefore its profit, while it hurts Consumer 2 by increasing the total emission. Thus, in this case, Nation (resp. Firm) 2 will not invest in Nation 1's private capital.

Proposition 7.4 *(A) Nation 2 (resp. Firm 2) invests in Nation 1's private capital only if Nation 1's technology after investment is clean.*

(B) If Firm 2 invests in Nation 1's private capital, then Nation 2 has an even stronger incentive to make such an investment.

(B) follows as Nation 2 stands to gain from the fall in total emission in addition to the rise in Firm 2's emission.

Investment in adaptation capital

The analysis of investment in adaptation capital mimics the analysis of the previous Section. If k_1 is chosen by Nation 1 (resp. Firm 1), then denote k_1 by k^* (resp. k^{**}). (8) and (9) imply that $k_1 = k^*$ and $k_1 = k^{**}$ maximize

$$u_1(t_1, k_1, \theta_{-1}, e(t_1, k_1, \theta_{-1})) - k_1 \qquad \text{and} \qquad f(t_1, e_1(t_1, k_1, \theta_{-1})) - k_1$$

respectively, taking t_1 and θ_{-1} as given. Using (6), k^* and k^{**} are characterized by

$$D_k\delta(k^*, e_+(t_1, k^*, \theta_{-1})) + D_{e_+}\delta(k^*, e_+(t_1, k^*, \theta_{-1})) \sum_{j\neq 1} D_k e_j(t_1, k^*, \theta_{-1}) = -1 \tag{14}$$

and

$$D_e f(t_1, e_1(t_1, k^{**}, \theta_{-1})) D_k e_1(t_1, k^{**}, \theta_{-1}) = 1 \tag{15}$$

respectively. Consider the choice of k^*. Combining Assumption 5.1, and Propositions 3.2 and 5.3, the benefits of investment in adaptation capital are: (a) an increase in domestic profit caused by higher domestic emission, (b) lower domestic vulnerability to damage, and (c) a decrease in domestic damage on account of lower foreign emission. The costs are: (d) an increase in domestic damage caused by higher domestic emission, and (e) the opportunity cost of investment. In equilibrium, each nation's emission (cap) is chosen to balance benefit (a) and cost (d) at the margin. Thus, (14) ensures that marginal benefits (b) and (c) are balanced by the marginal cost (e). (b) is the direct benefit of investment in adaptation capital, while (c) is the indirect or strategic benefit.

We ask: in what direction would Nation 1 like to perturb Firm 1's choice k^{**}? Let $G(t_1, k_1) = u_1(t_1, k_1, \theta_{-1}, e(t_1, k_1, \theta_{-1})) - k_1$. Then,

$$\begin{aligned} D_k G(t_1, k^{**}) &= -D_k \delta(k^{**}, e_+(t_1, k^{**}, \theta_{-1})) \\ &\qquad - D_{e_+} \delta(k^{**}, e_+(t_1, k^{**}, \theta_{-1})) D_k e_+(t_1, k^{**}, \theta_{-1}) \end{aligned}$$

Assumption 4.1(b), Assumption 5.1(e) and Corollary 5.5(C), imply that this marginal incentive cannot be signed unambiguously as an increase in Nation 1's adaptation capital has two opposing effects on Nation 1's damage. On the one hand, it directly decreases domestic damage (the direct effect), but on the other hand, it increases domestic damage by inducing higher total emission (the indirect effect).

Proposition 7.5 *If the direct effect is larger (resp. smaller) than the indirect effect, then Nation 1 prefers a higher (resp. lower) level of adaptation capital than Firm 1.*

Let $k_1 = k^0$ and $k_1 = k^{00}$ maximize

$$u_1(t_1, k_1, \theta_{-1}, e(t_1, k^*, \theta_{-1})) - k_1 \qquad \text{and} \qquad f(t_1, e_1(t_1, k^{**}, \theta_{-1})) - k_1$$

respectively. Given the caps $e(t_1, k^*, \theta_{-1})$, Nation 1 prefers k^0 to k^*. Similarly, given the cap $e_1(t_1, k^{**}, \theta_{-1})$, Firm 1 prefers k^{00} to k^{**}. The difference between k^0 (resp. k^{00}) and k^* (resp. k^{**}) represents Nation (resp. Firm) 1's ability to manipulate the Stage 1 equilibrium caps *via* its Stage 0 choice of adaptation investment. $k^* > k^0$ (resp. $k^* < k^0$) is interpreted as strategic overinvestment (resp. underinvestment) by Nation 1 in domestic adaptation capital. Similarly, $k^{**} > k^{00}$ (resp. $k^{**} < k^{00}$) is interpreted as strategic overinvestment (resp. underinvestment) by Firm 1 in domestic adaptation capital. k^0 and k^{00} are characterized by

$$D_k \delta(k^0, e_+(t_1, k^*, \theta_{-1})) = -1 \qquad \text{and} \qquad k^{00} = 0 \tag{16}$$

respectively.

Proposition 7.6 *Nation 1 (resp. Firm 1) overinvests in domestic adaptation capital, thereby strategically raising e_1, lowering e_2 and raising e_+.*

Proof. (16), (14), Assumption 4.1(b) and Corollary 5.5(C) imply that

$$\begin{aligned} D_k\delta(k^0, e_+(t_1, k^*, \theta_{-1})) &= D_k\delta(k^*, e_+(t_1, k^*, \theta_{-1})) \\ &\quad + D_{e_+}\delta(k^*, e_+(t_1, k^*, \theta_{-1})) \sum_{j \neq 1} D_k e_j(t_1, k^*, \theta_{-1}) \\ &< D_k\delta(k^*, e_+(t_1, k^*, \theta_{-1})) \end{aligned}$$

Assumption 5.1(e) implies $k^0 < k^*$. (15) and (16) imply that $k^{**} > k^{00}$. The strategic effects follow from Proposition 10(C). ■

We also record an observation analogous to Remark 7.3.

Remark 7.7 *(14) shows that Nation 1 uses investment in adaptation capital to manipulate downwards the equilibrium emissions of other nations. (15) shows that Firm 1 strategically manipulates upwards Nation 1's equilibrium emission.*

We finally consider the possibility of Nation (resp. Firm) 2 investing in Nation 1's adaptation capital. By Corollary 5.5(C), an increase in Nation 1's adaptation capital hurts Firm 2 by reducing its emission, and therefore its profit, while it hurts Consumer 2 by increasing the total emission.

Proposition 7.8 *Nation (resp. Firm) 2 will not invest in Nation 1's adaptation capital.*

8 The model with post-capping trade

Consider the model of Section 2 with an intermediate stage between Stages 1 and 2, say Stage 1*, in which the endowments of emission rights negotiated in Stage 1 are traded on a competitive market by the firms. In Stage 2, firms make production decisions that respect the new distribution of emission rights. As the analysis of Stage 2 of this modified model is identical to that contained in Section 3, we begin the analysis of the modified model by studying the Stage 1* emission rights market.

Suppose $\theta \in \Theta^n$ is the type profile and $e \in \prod_{i \in N}(0, h(t_i, V(t_i)))$ is the profile of endowments available for trading in Stage 1*. Since e_i constrains Firm i to emit less than its pre-protocol level, it has an incentive to acquire more emission rights than provided by e_i. As this holds for every firm, the equilibrium price of emission rights in Stage 1* will be positive. Let $p(t, e) > 0$ be this equilibrium price given e and the profile of private capitals $t = (t_i)_{i \in N}$. The damage functions and adaptation capitals do not affect the price of emission rights as firms are the only traders in the rights market.

Let $z_i(t,e)$ be the equilibrium quantum of rights traded by Firm i, interpreting $z_i(t,e) > 0$ (resp. $z_i(t,e) < 0$) as a purchase (resp. sale). Given t, e, $p(t,e)$ and $i \in N$, $z_i = z_i(t,e)$ maximizes $f(t_i, e_i + z_i) - p(t,e)z_i$. $f(t_i, e_i + z_i)$ is Firm i's profit from using emission rights $e_i + z_i$ in Stage 2. $p(t,e)z_i$ is the cost of buying rights (resp. revenue from selling rights) in Stage 1* if $z_i > 0$ (resp. $z_i < 0$). The equilibrium trades $(z_i(t,e))_{i \in N}$ are characterized by the conditions

$$\sum_{i \in N} z_i(t,e) = 0 \qquad \text{and} \qquad D_e f(t_i, e_i + z_i(t,e)) = p(t,e) \tag{17}$$

for every $i \in N$. The first equation represents market-clearing in the rights market, while the second equation represents the optimality of Firm i's trade; given the first order condition, Proposition 3.2(F) ensures optimality. $N(t,e) = \{i \in N \mid z_i(t,e) > 0\}$ is the set of buyers in Stage 1*. Using (6), (17) and Proposition 3.2(F), we have $i \in N(t,e)$ if and only if $D_e f(t_i, e_i) > D_e f(t_i, e_i + z_i(t,e)) = p(t,e)$. Equivalently, $i \in N - N(t,e)$ if and only if $D_e f(t_i, e_i) \leq D_e f(t_i, e_i + z_i(t,e)) = p(t,e)$. Thus, firms in $N(t,e)$ profit from trade by buying emission rights at a price less than their own shadow value of the rights and firms in $N - N(t,e)$ profit by selling emission rights at a price higher than their own shadow value of the rights. Given e, trading in Stage 1* re-allocates emission rights to make all firms better-off but does not change the total emission e_+ and the resulting damages suffered by nations. So, *given* e, all firms and all nations prefer the trading arrangement to the autarkic arrangement.

Now consider the capping negotiations in Stage 1. If firms can trade emission rights in Stage 1*, then the equilibrium implications of this trading will be anticipated when caps are negotiated in Stage 1. Given a cap profile e, the marginal cost to Nation i entailed by an additional emission right is $D_{e_+}\delta(k_i, e_+)$, while the marginal benefit is either $D_e f(t_i, e_i + z_i(t,e))$ if Firm i uses this right to offset an extra unit of domestic emission or $p(t,e)$ if Firm i sells the marginal right in Stage 1*. The Stage 1* equilibrium condition (17) equates the two benefits at the margin. Thus, a profile e will be accepted by all nations in Stage 1 only if the marginal cost of an extra emission right exceeds the marginal benefit for all nations, i.e.,

$$p(t,e) \leq \min\{D_{e_+}\delta(k_i, e_+) \mid i \in N\} \tag{18}$$

Consider a profile of emission caps e such that $e_+ \leq e_+(\theta)$ and $e + z(t,e) \neq e(\theta)$, where $e(\theta)$ is the equilibrium profile of caps for the model of Section 2. Then, $e_i + z_i(t,e) < e_i(\theta)$ for some $i \in N$. It follows from (17), Proposition 3.2(F), (6) and Assumption 4.1 that

$$\begin{aligned} p(t,e) = D_e f(t_i, e_i + z_i(t,e)) &> D_e f(t_i, e_i(\theta)) \\ &= D_{e_+}\delta(k_i, e_+(\theta)) \geq D_{e_+}\delta(k_i, e_+) \end{aligned}$$

which violates (18). So, e cannot be an equilibrium profile of caps.

Proposition 8.1 *Consider the model with trading in Stage 1*. In an equilibrium for this model, the profile of caps e negotiated in Stage 1 is either* $e(\theta)$ *(i.e., the autarkic profile) or is such that* $e_+ > e_+(\theta)$.

9 The literature

With the description and analysis of our model complete, we can now juxtapose it with other models in the same area.

The cooperative game approach (Chander and Tulkens [7], [8], [9]) uses the core as the solution concept to generate a striking result: for emission allocation games with many nations, they characterize zero-sum transfers that implement efficient emission allocations and satisfy individual and group rationality constraints. The use of international transfers and the ability of members of a coalition of nations to correlate decisions so as to maximize the sum of their utilities generate large welfare improvements from the pre-protocol situation. Our non-cooperative model does not permit transfers and makes relatively conservative assumptions about international institutions, resulting in smaller improvement from the pre-protocol situation.

The stable coalition approach (Barrett [2], Black et al. [4] and Carraro and Siniscalco [5]) adopts the methodology of d'Aspremont et al. [1] to derive the likely size of a "stable coalition" of participating nations. This methodology, which sits between the cooperative and the non-cooperative game categories, seems expressly designed to model the participation decision and cannot readily be adapted for other purposes such as the concerns of this paper.

The non-cooperative multi-period approach (Cesar [6], Dockner and van Long [10], Hoel [13], Martin et al. [18], van der Ploeg and de Zeeuw [23]) explores strategies other than emission capping for dealing with the global emission externality. They study the possibility of sustaining emission agreements among nations in repeated interaction settings *via* (1) implicit penal codes that provide appropriate incentives, or (2) explicit international fiscal incentives.

10 Conclusions

The results of Sections 3 to 8 show that our model of protocol formation and implementation is tractable and has attractive properties. For every profile of types, there is a unique profile of equilibrium caps. The profile of resulting emissions is identical to the profile of caps. Moreover, this profile is globally asymptotically stable. The assumptions on the primitives of the model that imply these strong properties also facilitate the comparative statics results. Finally, from the perspective of the nations, for every profile of types, the equilibrium profile of caps is a Pareto improvement over the pre-protocol situation.

With respect to our first application, we find that, *ceteris paribus*, nations with greater adaptation capital have larger caps in equilibrium. The result in terms of private capital depends on the assumption that larger private capital implies cleaner technology. Given this assumption, *ceteris paribus*, the emission caps of nations with dirty (resp. clean) technology are positively (resp. negatively) related to the size of their private capital. While such comparisons are empirically problematic as "*ceteris paribus*" rarely holds when comparing actual nations, the theoretical results do identify the principles governing the ordering of caps.

With respect to our second application, we find that all nations will overinvest in domestic adaptation capital and nations with clean (resp. dirty) technologies will underinvest (resp. overinvest) in domestic private capital. The effects of these manipulations are to raise domestic emission, lower foreign emissions and raise the total emission. The results regarding the domestic investment choices by firms' are directionally similar, but there are significant differences. For instance, the variables targeted for manipulation by a nation are the foreign caps, while the variable targeted by a firm is the domestic cap. Nor is the extent of manipulation the same. With respect to domestic private capital, a nation with clean (resp. dirty) technology underinvests (resp. overinvests) less severely than its firm, while the comparison is ambiguous in the case of adaptation capital. Neither nations, nor their firms, will invest in foreign adaptation capital. Nor will they invest in foreign private capital if the investment recipient has dirty technology.

Consider the following thought experiment. Suppose the adaptation capitals of all nations are fixed and nations with clean technology (the "North") emit more than nations with dirty technology (the "South"). In this scenario, Corollary 5.5 suggests that the exogenous growth of either Northern or Southern private capital raises Southern emission and lowers Northern emission, i.e., their emissions "converge".

Our model generates an unambiguous policy conclusion: if one is sufficiently concerned about total global emission and the resulting damages, or in the extreme case, wish to minimize total emission subject to the usual incentive constraints, then an international cap-and-trade arrangement is inferior to the cap-and-hold arrangement modeled by us. This result (Proposition 8.1) cautions against the facile application of the proposition that, given endowments, voluntary trade must be welfare improving. However, in the context of capping protocols, the endowments of emission rights are not given but are generated endogenously. As we show, the expectation of equilibrium trade creates the incentives to raise the total endowment of emission rights, which translates into higher emission. The practical implication of this finding is that pre-protocol deliberations to determine the architecture of a capping regime should not view post-capping trade in rights as an unalloyed blessing, and therefore an automatic choice, but as a choice involving significant welfare trade-offs since it makes all consumers unambiguously worse-off relative to the autarkic arrangement.

Finally, we note three political-economic implications of our model.

First, a nation's equilibrium emission cap is an active constraint on the domestic firm. Growth of a nation's adaptation capital is a means for relaxing this constraint. Thus, reduction of damage to the consumer is not the only motive for investing in domestic adaptation capital; such investment is also a strategic way of gaining head-room for higher emission by the domestic firm, resulting in higher profit and national welfare. The strategic motive is particularly strong for nations with clean technology as investment in their private capital makes an already active emission constraint still tighter; thus, over-investment in adaptation capital is a strategic way of loosening this constraint or counteracting the effects of private capital growth.

Secondly, the growth of Southern private capital raises total emission, while the growth of Northern private capital lowers total emission. If a "Green" is someone who wishes to minimize total emission, then a "Green" will favor private investment (i.e., growth) in the North and not in the South. This points to a conflict between the objectives of Green lobbies and the growth ambitions of the South.

Thirdly, investment is a tool for manipulation in our model. Consequently, affluent nations with the wherewithal for substantial investments can manipulate the emission caps in ways that poorer nations cannot do.

An extension of our model is to embed it in a multi-period setting. It is certain that national types will evolve over time on account of technical change and economic growth, and knowledge of the interaction between the economic process and the environment (modeled by g, h and δ) will also evolve over time. Thus, emission rights will have to be re-allocated periodically. Therefore, learning about the relevant processes is important for every nation so that it can position itself better for future renegotiations. Such concerns have motivated a literature (e.g., Kelly and Kolstad [14], Kolstad [15], Ulph and Maddison [21], Ulph and Ulph [22]) on learning in this context. However, a general model is awaited.

Appendix

Proof of Proposition 3.2 (A) Fix $(t, e) \in \Re_+ \times \bar{\Re}_+$. As $h(t, 0) = 0 \leq e$, we have $\Gamma(t, e) \neq \emptyset$. As h is continuous, $\{v \in \Re_+ \mid h(t, v) \leq e\}$ is closed in $\Re_+$. As $V(t) \in \Re_+$, $[0, V(t)]$ is compact. Therefore, $\Gamma(t, e)$ is compact. As g is continuous, Weierstrass' theorem implies the existence of $v \in \Gamma(t, e)$ such that $g(t, v) = \sup g(t, \Gamma(t, e))$.

We now show that there is a unique solution. Suppose $v, v' \in \Gamma(t, e)$

such that $g(t,v) = g(t,v') = \sup g(t,\Gamma(t,e))$ and $v \neq v'$. Let $\lambda \in (0,1)$. As $v, v' \in \Gamma(t,e)$, we have $h(t,v) \leq e$, $h(t,v') \leq e$ and $v, v' \in [0, V(t)]$. It follows that $\lambda v + (1-\lambda)v' \in [0, V(t)]$, and as $h(t,.)$ is strictly convex, we have $h(t, \lambda v + (1-\lambda)v') < \lambda h(t,v) + (1-\lambda)h(t,v') \leq e$. Thus, $\lambda v + (1-\lambda)v' \in \Gamma(t,e)$, and as $g(t,.)$ is strictly concave, we have $g(t, \lambda v + (1-\lambda)v') > \lambda g(t,v) + (1-\lambda)g(t,v') = \sup g(t,\Gamma(t,e))$, a contradiction.

(B) Define mappings Γ_1, Γ_2 and Γ, each from $\Re_+ \times \bar{\Re}_+$ to $\Re_+$, by $\Gamma_1(t,e) = \{v \in \Re_+ \mid h(t,v) \leq e\}$, $\Gamma_2(t,e) = [0, V(t)]$ and $\Gamma(t,e) = \Gamma_1(t,e) \cap \Gamma_2(t,e)$ respectively. Suppose Γ is continuous and has nonempty and compact values. Using this fact, (A) and the continuity of g, the result follows from the Theorem of the Maximum (Berge [3], Theorem VI.3). We now confirm the hypothesized properties of Γ.

Assumptions 3.1 implies that $0 \in \Gamma(t,e)$ for every $(t,e) \in \Re_+ \times \bar{\Re}_+$. Thus, Γ has nonempty values. As h is continuous, $\operatorname{Gr}\Gamma_1$ is closed in $\Re_+ \times \bar{\Re}_+ \times \Re_+$. It follows immediately that Γ_1 has closed values. As $V(t) \in \Re_{++}$, Γ_2 has compact values. Thus, Γ has compact values.

If Γ_2 is upper hemicontinuous, then so is Γ (Berge [3], Theorem VI.1.7). We show that Γ_2 is upper hemicontinuous. Consider $(t,e) \in \Re_+ \times \bar{\Re}_+$ and $\epsilon > 0$. As V is continuous, there exists an open neighborhood of t in $\Re_+$, say U, such that $V(U) \subset [0, V(t)+\epsilon)$. It follows that $[0, V(t')] \subset [0, V(t)+\epsilon)$ for every $t' \in U$. Thus, $\Gamma_2(U \times \bar{\Re}_+) = \cup_{t' \in U}[0, V(t')] \subset [0, V(t)+\epsilon)$. Suppose E is open in $\Re_+$ and $\Gamma_2(t,e) = [0, V(t)] \subset E$. It follows that there exists $\epsilon > 0$ such that $[0, V(t)+\epsilon)) \subset E$. By the above argument, there exists an open neighborhood of t in $\Re_+$, say U, such that $\Gamma_2(U \times \bar{\Re}_+) \subset [0, V(t)+\epsilon) \subset E$, as required.

We now show that Γ is lower hemicontinuous. Consider $(t,e) \in \Re_+ \times \bar{\Re}_+$ and E open in $\Re_+$ such that $\Gamma(t,e) \cap E \neq \emptyset$.

Suppose $e = 0$. Then $\Gamma(t,e) = \{0\} \subset E$. As Γ is upper hemicontinuous, there exists an open neighborhood of (t,e) in $\Re_+ \times \bar{\Re}_+$, say U, such that $\Gamma(U) \subset E$. Thus, $\Gamma(t',e') \cap E \neq \emptyset$ for every $(t',e') \in U$.

Suppose $e > 0$. Define mappings $\hat{\Gamma}_1$, $\hat{\Gamma}_2$ and $\hat{\Gamma}$, each from $\Re_+ \times \bar{\Re}_+$ to $\Re_+$, by $\hat{\Gamma}_1(t,e) = \{v \in \Re_+ \mid h(t,v) < e\}$, $\hat{\Gamma}_2(t,e) = [0, V(t))$ and $\hat{\Gamma}(t,e) = \hat{\Gamma}_1(t,e) \cap \hat{\Gamma}_2(t,e)$ respectively. As $\Gamma(t,e) \cap E \neq \emptyset$, there exists $v \in E$ such that $v \in [0, V(t)]$ and $h(t,v) \leq e$. If $v = 0$, then $h(t,v) = 0 < e$ and $v = 0 \in [0, V(t))$. Thus, $v \in \hat{\Gamma}(t,e) \cap E$. If $v > 0$, then $\emptyset \neq [0,v) \subset \hat{\Gamma}(t,e)$ as $h(t,.)$ is strictly increasing. As E is open in $\Re_+$ and $v \in E$, $\emptyset \neq [0,v) \cap E \subset \hat{\Gamma}(t,e) \cap E$. We conclude that $\hat{\Gamma}(t,e) \cap E \neq \emptyset$.

If $\hat{\Gamma}$ is lower hemicontinuous at (t,e), then there exists an open neighborhood of (t,e) in $\Re_+ \times \bar{\Re}_+$, say U, such that $\emptyset \neq \hat{\Gamma}(t',e') \cap E \subset \Gamma(t',e') \cap E$ for every $(t',e') \in U$. Thus, Γ is lower hemicontinuous at (t,e), as required.

It remains to show that $\hat{\Gamma}$ is lower hemicontinuous. Suppose E is open in $\Re_+$. As h and V are continuous, $\operatorname{Gr}\hat{\Gamma}_1 = \{(t,e,v) \in \Re_+ \times \bar{\Re}_+ \times \Re_+ \mid h(t,v) - e < 0\}$ and $\operatorname{Gr}\hat{\Gamma}_2 = \{(t,e,v) \in \Re_+ \times \bar{\Re}_+ \times \Re_+ \mid v - V(t) < 0\}$ are open in $\Re_+ \times \bar{\Re}_+ \times \Re_+$. Consequently, $\operatorname{Gr}\hat{\Gamma} = \operatorname{Gr}\hat{\Gamma}_1 \cap \operatorname{Gr}\hat{\Gamma}_2$ is open in

$\Re_+ \times \bar{\Re}_+ \times \Re_+$. Note that

$$\{(t, e) \in \Re_+ \times \bar{\Re}_+ \mid \hat{\Gamma}(t, e) \cap E \neq \emptyset\} = \pi(\mathrm{Gr}\,\hat{\Gamma} \cap \Re_+ \times \bar{\Re}_+ \times E)$$

where $\pi : \Re_+ \times \bar{\Re}_+ \times \Re_+ \to \Re_+ \times \bar{\Re}_+$ is the projection mapping $\pi(t, e, v) = (t, e)$. As π is an open mapping and $\mathrm{Gr}\,\hat{\Gamma}$ and $\Re_+ \times \bar{\Re}_+ \times E$ are open in $\Re_+ \times \bar{\Re}_+ \times \Re_+$, we have $\{(t, e) \in \Re_+ \times \bar{\Re}_+ \mid \hat{\Gamma}(t, e) \cap E \neq \emptyset\}$ open in $\Re_+ \times \bar{\Re}_+$. Thus, $\hat{\Gamma}$ is lower hemicontinuous.

(C) and (D) follow immediately from Assumption 3.1.

(E) Let $e, e' \in [0, h(t, V(t))]$ and $e < e'$. (D) implies $h(t, v(t, e)) = e < e' = h(t, v(t, e'))$. Assumption 3.1 implies $v(t, e) < v(t, e') \leq V(t)$, and therefore, $f(t, e) = g(t, v(t, e)) < g(t, v(t, e')) = f(t, e')$.

(F) Let $e, e' \in [0, h(t, V(t))]$ and $\lambda \in (0, 1)$, with $e \neq e'$. Let $v(t, e) = v$ and $v(t, e') = v'$. By definition, $h(t, v) \leq e$ and $h(t, v') \leq e'$. Assumption 3.1 implies $h(t, \lambda v + (1 - \lambda)v') \leq \lambda h(t, v) + (1 - \lambda)h(t, v') \leq \lambda e + (1 - \lambda)e'$. Therefore, by Assumption 3.1, $f(t, \lambda e + (1 - \lambda)e') \geq g(t, \lambda v + (1 - \lambda)v') > \lambda g(t, v) + (1 - \lambda)g(t, v') = \lambda f(t, e) + (1 - \lambda)f(t, e')$.

(G) By (C), if $e \geq h(t, V(t))$, then $f(t, e) = g(t, v(t, e)) = g(t, V(t))$. ■

Proof of Proposition 4.2 (A) follows immediately from the definition of $V(t)$.

(B) Consider $e > h(t, V(t))$. By Proposition 3.2(C), $v(t, e') = V(t)$ for every $e' > h(t, V(t))$. Thus, $D_e v(t, e) = 0$, and using (A), $D_v g(t, v(t, e)) = D_v g(t, V(t)) = 0$.

(C) By Proposition 3.2(D), if $e \in (0, h(t, V(t)))$, then $h(t, v(t, e)) = e$. In this case, we can combine Assumptions 3.1(c) and 4.1(a), and use the implicit function theorem (Lang [16], XIV, Theorem 2.1) to conclude that $v(t, .)$ is $\mathcal{C}^1$ on $(0, h(t, V(t)))$.

(D) Combining (B), (C) and Assumption 4.1(a), it follows that $f(t, .) = g(t, v(t, .))$ is $\mathcal{C}^1$ on $(0, h(t, V(t))) \cup (h(t, V(t)), \infty)$. ■

Proof of Proposition 4.3 (A) If $e_i \geq h(t_i, V(t_i))$, then Proposition 3.2(C) implies $v(t_i, e_i) = V(t_i)$. It follows that $u_i(\theta, e) = u_i(\theta, h(t_i, V(t_i)), e_{-i})$ is independent of e_i for $e_i \geq h(t_i, V(t_i))$.

Let $e_i \in (0, h(t_i, V(t_i)))$. Proposition 3.2(D) implies that $h(t_i, v(t_i, e_i)) = e_i$ and $u_i(\theta, e)$ is given by (2). Proposition 4.2(C) and Assumption 4.1(a) yield $D_{e_i} v(t_i, e_i) = 1/D_{v_i} h(t_i, v(t_i, e_i))$. Using Proposition 4.2(D), the derivative of the first term of (2) with respect to e_i is

$$D_{v_i} g(t_i, v(t_i, e_i)) D_{e_i} v(t_i, e_i) = \frac{D_{v_i} g(t_i, v(t_i, e_i))}{D_{v_i} h(t_i, v(t_i, e_i))} \tag{19}$$

As v is continuous by Proposition 3.2(B),

$$\lim_{e_i \uparrow h(t_i, V(t_i))} v(t_i, e_i) = v(t_i, h(t_i, V(t_i))) = V(t_i)$$

Using (19), Assumptions 3.1(c) and 4.1(a), and Proposition 4.2(A), we have

$$\lim_{e_i \uparrow h(t_i, V(t_i))} D_{v_i} g(t_i, v(t_i, e_i)) D_{e_i} v(t_i, e_i) = \frac{D_{v_i} g(t_i, V(t_i))}{D_{v_i} h(t_i, V(t_i))} = 0$$

The derivative of the second term of (2) with respect to e_i, evaluated at $h(t_i, V(t_i))$, is

$$-D_{e_+} \delta \left(k_i, h(t_i, V(t_i)) + \sum_{j \in N - \{i\}} h(t_j, v(t_j, e_j)) \right) < 0$$

Thus, $e_i = h(t_i, V(t_i))$ cannot maximize (2). As Nation i's expected payoff is invariant with respect to e_i for $e_i \geq h(t_i, V(t_i))$, we have $b < h(t_i, V(t_i))$. It follows from Proposition 3.2(D) that $h(t_i, v(t_i, b)) = b$.

(B) follows as $u_i(\theta, ., e_{-i})$ is strictly concave on $[0, h(t_i, V(t_i))]$. ■

Proof of Proposition 4.4 Let $\theta \in \Theta^n$ and let e be a stationary point of (4). Proposition 4.3 implies $e_j = \beta_j(e_{-j}; \theta) \in [0, h(t_j, V(t_j)))$ for every $j \in N$. Therefore, $u_j(\theta, e)$ is given by (3). Suppose $e_i = 0$. Consequently, $v(t_i, e_i) = 0$, $f(t_i, e_i) = 0$ and $u_i(\theta, e) = -\delta(k_i, \sum_{j \in N - \{i\}} e_j) \leq 0$. Set $e'_i = h(t_i, V(t_i))$. By Proposition 3.2(G), $f(t_i, e'_i) = g(t_i, V(t_i))$. By Assumption 4.1(b) and the fact that $e_j < h(t_j, V(t_j))$ for every $j \in N$, we have

$$\begin{aligned} u_i(\theta, e'_i, e_{-i}) &= f(t_i, e'_i) - \delta \left(k_i, e'_i + \sum_{j \in N - \{i\}} e_j \right) \\ &> g(t_i, V(t_i)) - \delta \left(k_i, \sum_{j \in N} h(t_j, V(t_j)) \right) \end{aligned}$$

which is positive by Assumption 4.1(c). Thus, $u_i(\theta, e'_i, e_{-i}) > 0 \geq u_i(\theta, e)$, a contradiction. ■

Proof of Proposition 4.5 Let e^* be a stationary point of (4). Consider $j \in N$. By Proposition 4.3, $0 \leq e^*_j = \beta_j(e^*_{-j}; \theta) < h(t_j, V(t_j))$. As $e^*_j = \beta_j(e^*_{-j}; \theta)$, we have $u_j(\theta, e^*) \geq u_j(\theta, b, e^*_{-j})$ for every $b \in \bar{\Re}_+$. Therefore, $u_j(\theta, e^*) \geq u_j(\theta, b, e^*_{-j})$ for every $b \in [0, h(t_j, V(t_j))]$. So, e^* is a Nash equilibrium of $G(\theta)$.

Conversely, suppose e^* is a Nash equilibrium of $G(\theta)$. By definition, for every $j \in N$, $e^*_j \in [0, h(t_j, V(t_j))]$ and $u_j(\theta, e^*) \geq u_j(\theta, b, e^*_{-j})$ for every $b \in [0, h(t_j, V(t_j))]$. As $u_j(\theta, ., e^*_{-j})$ is strictly concave on $[0, h(t_j, V(t_j))]$, $u_j(\theta, e^*) > u_j(\theta, b, e^*_{-j})$ for every $b \in [0, h(t_j, V(t_j))] - \{e^*_j\}$. By Proposition 4.3, $\beta_j(e^*_{-j}; \theta) \subset [0, h(t_j, V(t_j))]$. Thus, if $b' \in \bar{\Re}_+ - [0, h(t_j, V(t_j))]$, then there exists $b \in [0, h(t_j, V(t_j))]$, such that $u_j(\theta, b, e^*_{-j}) > u_j(\theta, b', e^*_{-j})$. Consequently, $u_j(\theta, e^*) > u_j(\theta, b', e^*_{-j})$. Therefore, $e^*_j = \beta_j(e^*_{-j}; \theta)$ for every $j \in N$. Thus, e^* is a stationary point of (4). ■

Proof of Proposition 4.6 Consider $G(\theta)$. By Assumption 3.1(a), each player's strategy set is a nonempty, compact and convex subset of $\Re$. Proposition 3.2(B) and Assumption 4.1(b) imply that each player's payoff function is continuous. Proposition 3.2(F) and Assumption 4.1(b) imply that each player's payoff is concave in his own strategy. Thus, by Nash's existence theorem, $G(\theta)$ has a Nash equilibrium, which is a stationary point of (4) by Proposition 4.5. ■

Proof of Proposition 4.9 Fix $i \in N$ and a stationary point e of (4). Let $R_j = [e_j, h(t_j, V(t_j))]$ for $j \in N$ and let $R = \prod_{j \in N} R_j$. Proposition 4.4 ensures that $\operatorname{Int} R \neq \emptyset$. Given $x \in R - \{e\}$, define $e^j = (e_1, \ldots, e_j)$ and $x^j = (x_j, \ldots, x_n)$ for $j = 1, \ldots, n$; formally define $(e^0, x^1) = x$ and $(e^n, x^{n+1}) = e$.

Consider $x \in R$ such that $x \neq (h(t_j, V(t_j)))_{j \in N}$ and $x \neq e$. As $x_j \in [0, h(t_j, V(t_j))]$ for every $j \in N$, $u_i(\theta, x)$ is given by (3). Assumption 4.1(b) implies $D_{e_k} u_i(\theta, x) = -D_{e_+} \delta(k_i, x_+) < 0$ for every $k \in N - \{i\}$. Proposition 4.4 and Assumption 4.1(b) imply

$$\begin{aligned}
D_{e_i} u_i(\theta, x) &= D_{e_i} u_i(\theta, e) + \sum_{k=1}^{n} \left[D_{e_i} u_i(\theta, e^{k-1}, x^k) - D_{e_i} u_i(\theta, e^k, x^{k+1}) \right] \\
&= D_{e_i} u_i(\theta, e) + \sum_{k=1}^{n} \int_{e_k}^{x_k} dy \, D_{e_i e_k} u_i(\theta, e^{k-1}, y, x^{k+1}) \\
&< D_{e_i} u_i(\theta, e) \\
&= 0
\end{aligned}$$

Given $z \in R - \{e\}$, it follows that

$$\begin{aligned}
u_i(\theta, z) - u_i(\theta, e) &= \sum_{k=1}^{n} \left[u_i(\theta, e^{k-1}, z^k) - u_i(\theta, e^k, z^{k+1}) \right] \\
&= \sum_{k=1}^{n} \int_{e_k}^{z_k} dy \, D_{e_k} u_i(\theta, e^{k-1}, y, z^{k+1}) \\
&< 0
\end{aligned}$$

as required. ■

Proof of Proposition 5.2 (A) Consider $(t, e) \in \Re^2_{++}$ such that $e < h(t, V(t))$. By Proposition 3.2(D), $h(t, v(t, e)) = e$. As $e > 0$, Assumption 3.1(c) implies $v(t, e) > 0$. By Assumption 3.1, h and V are continuous. Consequently, there exist an open neighborhood $T \times E \subset \Re^2_{++}$ of (t, e) such that $h(\tau, V(\tau)) - \eta > 0$ for every $(\tau, \eta) \in T \times E$. Consider the function $F : T \times E \times \Re_{++} \to \Re$ defined by $F(t, e, v) = h(t, v) - e$. By definition, $F(t, e, v(t, e)) = 0$. By Assumption 5.1(c), F is $\mathcal{C}^2$. Combining Assumptions 3.1(c) and 5.1(c), we have $D_v F(t, e, v(t, e)) = D_v h(t, v(t, e)) > 0$. By the implicit function theorem (Lang [16], XIV, Theorem 2.1), there exists an open neighborhood $T' \times E' \subset T \times E$ of (t, e) and a $\mathcal{C}^2$ function

$v' : T' \times E' \to \Re_{++}$ such that $F(\tau, \eta, v'(\tau, \eta)) = 0$ for every $(\tau, \eta) \in T' \times E'$, i.e., $h(\tau, v'(\tau, \eta)) = \eta$ for every $(\tau, \eta) \in T' \times E'$. Therefore, $h(\tau, V(\tau)) > \eta$ for every $(\tau, \eta) \in T' \times E'$. By Proposition 3.2(D), $h(\tau, v(\tau, \eta)) = \eta$ for every $(\tau, \eta) \in T' \times E'$. Thus, $h(\tau, v(\tau, \eta)) = \eta = h(\tau, v'(\tau, \eta))$ for every $(\tau, \eta) \in T' \times E'$. Assumption 3.1(c) implies $v(\tau, \eta) = v'(\tau, \eta)$. Thus, v is $\mathcal{C}^2$ on $\{(t, e) \in \Re^2_{++} \mid e < h(t, V(t))\}$. Assumption 5.1(c) implies that f is $\mathcal{C}^2$ on $\{(t, e) \in \Re^2_{++} \mid e < h(t, V(t))\}$.

(B) Suppose $e, t, t' \in \Re_+$, $t < t'$, $e \leq h(t, V(t))$ and $e \leq h(t', V(t'))$. Proposition 3.2(D) and Assumption 5.1(a) imply that $h(t, v(t, e)) = e = h(t', v(t', e)) < h(t, v(t', e))$. Assumption 3.1(c) implies $v(t, e) < v(t', e)$ and the definition of v implies $v(t', e) \leq V(t')$. Therefore, Assumptions 3.1(b) and 5.1(a) imply $f(t, e) = g(t, v(t, e)) < g(t', v(t, e)) < g(t', v(t', e)) = f(t', e)$.

(C) Our assumptions imply that $u_i(\theta, e)$ is given by (3). The result follows from Proposition 5.2(A) and Assumption 5.1(c). ■

Proof of Proposition 5.3 Fix $\theta \in \Theta^n$ with $\theta_i = (t_i, k_i)$. Denote $f(t_i, .)$ and $\delta(k_i, .)$ by f_i and δ_i respectively. In the following arguments, u_i, f_i, δ_i and their derivatives are evaluated at $(\theta, e(\theta))$ for every $i \in N$. Let

$$A = \begin{bmatrix} D_{e_1e_1}u_1 & \dots & D_{e_1e_n}u_1 \\ \vdots & \ddots & \vdots \\ D_{e_ne_1}u_n & \dots & D_{e_ne_n}u_n \end{bmatrix} \qquad \text{and} \qquad b = \begin{bmatrix} -D_{e_1x}u_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

where

$$D_{e_ie_j}u_i = \begin{cases} D_{e_ie_i}f_i - D_{e_+e_+}\delta_i, & \text{if } j = i \\ -D_{e_+e_+}\delta_i, & \text{if } j \in N - \{i\} \end{cases}$$

for $i \in N$. Let A_j be the $(n-1) \times (n-1)$ matrix derived from A by eliminating the first row and the j-th column.

By the fundamental theorem of algebra (Markushevich [17], Theorem 17.7), A has n roots. As A is real, its characteristic polynomial has real coefficients. Therefore, the conjugate of every complex root of A with multiplicity m is also a root of A with multiplicity m. Therefore, the product of all complex roots is positive. As A is similar to its Jordan canonical form (Gantmacher [12], Section VI.6.3), $\det A$ equals the product of its roots. (5) implies that A is a row dominant diagonal matrix. (7) implies that A has a negative diagonal.

Suppose λ is a root of A with a non-negative real part. Let $|c|$ denote the modulus of a complex number c. As $a_{ii} < 0$ for every $i \in N$, $|a_{ii} - \lambda| \geq |a_{ii}| > \sum_{j \in N - \{i\}} |a_{ij}|$ for every $i \in N$. This implies $H = A - \lambda I$ is an $n \times n$ complex matrix with a row dominant diagonal. H is singular as λ is a root of A. Thus, there exists a complex n-tuple $x \neq 0$ such that $Hx = 0$, which implies $h_{ii}x_i + \sum_{j \in N - \{i\}} h_{ij}x_j = 0$ for every $i \in N$. The triangle inequality implies $|h_{ii}||x_i| = |h_{ii}x_i| = |\sum_{j \in N - \{i\}} h_{ij}x_j| \leq \sum_{j \in N - \{i\}} |h_{ij}x_j| =$

$\sum_{j\in N-\{i\}} |h_{ij}||x_j|$. Let $k \in N$ be such that $|x_k| \geq |x_j|$ for every $j \in N$. It follows that $|h_{kk}||x_k| \leq \sum_{j\in N-\{k\}} |h_{kj}||x_k| = |x_k| \sum_{j\in N-\{k\}} |h_{kj}|$. As $x \neq 0$, $|x_k| > 0$. Therefore, $|h_{kk}| \leq \sum_{j\neq k} |h_{kj}|$, a contradiction. Thus, all the roots of A have negative real parts.

It follows that, if n is even (resp. odd), then $\det A > 0$ (resp. $\det A < 0$). By copying this argument, if n is even (resp. odd), then $\det A_1 < 0$ (resp. $\det A_1 > 0$).

(A) Consider the function $F : \Theta^n \times \prod_{j\in N}(0, h(t_j, V(t_j))) \to \Re^n$ such that $F_i = D_{e_i} u_i : \Theta^n \times \prod_{j\in N}(0, h(t_j, V(t_j))) \to \Re$. As (6) implies that $D_{e_i} u_i(\theta, e(\theta)) = 0$ for every $i \in N$, we have $F(\theta, e(\theta)) = 0$. By Proposition 5.2(C), F is $\mathcal{C}^1$. As $D_e F(\theta, e(\theta)) = A$ and A is non-singular, the implicit function theorem (Lang [16], XIV, Theorem 2.1) implies that there exists an open neighborhood $T \subset \Theta^n$ of θ and a unique $\mathcal{C}^1$ function $e' : T \to \Re^n_{++}$ such that $F(\theta', e'(\theta')) = 0$ for every $\theta' \in T$, i.e., $D_{e_j} u_j(\theta', e'(\theta')) = 0$ for every $\theta' \in T$ and every $j \in N$. But (6) implies $D_{e_j} u_j(\theta', e(\theta')) = 0$ for every $\theta' \in T$ and every $j \in N$. Strict concavity of $u_j(\theta', .)$ implies that $e' = e$ on T. The result follows as the same argument holds at every $\theta \in \Theta^n$.

(B) Differentiating (6) with respect to x yields the equation $AD_x e = b$. Using Cramer's rule, we have

$$D_x e_j = \frac{(-1)^j D_{e_1 x} u_1 \det A_j}{\det A} \tag{20}$$

We first evaluate $\det A_j$ for $j \in N - \{1\}$. Subtracting the first column of A_j from every other column yields $\det A_j = \det B_j$, where

$$B_j = \begin{bmatrix} -D_{e_+e_+}\delta_2 & D_{e_2e_2}f_2 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ -D_{e_+e_+}\delta_{j-1} & 0 & \dots & D_{e_{j-1}e_{j-1}}f_{j-1} & 0 & \dots & 0 \\ -D_{e_+e_+}\delta_j & 0 & \dots & 0 & 0 & \dots & 0 \\ -D_{e_+e_+}\delta_{j+1} & 0 & \dots & 0 & D_{e_{j+1}e_{j+1}}f_{j+1} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ -D_{e_+e_+}\delta_n & 0 & \dots & 0 & 0 & \dots & D_{e_ne_n}f_n \end{bmatrix}$$

By a sequence of $j-2$ adjacent column interchanges, we transform B_j into

$$C_j = \begin{bmatrix} D_{e_2e_2}f_2 & \dots & 0 & -D_{e_+e_+}\delta_2 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & D_{e_{j-1}e_{j-1}}f_{j-1} & -D_{e_+e_+}\delta_{j-1} & 0 & \dots & 0 \\ 0 & \dots & 0 & -D_{e_+e_+}\delta_j & 0 & \dots & 0 \\ 0 & \dots & 0 & -D_{e_+e_+}\delta_{j+1} & D_{e_{j+1}e_{j+1}}f_{j+1} & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & -D_{e_+e_+}\delta_n & 0 & \dots & D_{e_ne_n}f_n \end{bmatrix}$$

As interchanging adjacent columns changes the sign of a determinant, we have

$$\det A_j = \det B_j = (-1)^{j-2} \det C_j = (-1)^{j-1} \frac{D_{e_+ e_+} \delta_j}{D_{e_1 e_1} f_1 D_{e_j e_j} f_j} \prod_{k=1}^{n} D_{e_k e_k} f_k$$

Thus,

$$D_x e_j = \left[\frac{(-1)^{2j-1} \prod_{k=1}^{n} D_{e_k e_k} f_k}{\det A} \right] \frac{D_{e_1 x} u_1 D_{e_+ e_+} \delta_j}{D_{e_1 e_1} f_1 D_{e_j e_j} f_j} \tag{21}$$

Proposition 3.2(F) implies $D_{e_k e_k} f_k < 0$ for every $k \in N$. Therefore,

$$\frac{(-1)^{2j-1} \prod_{k=1}^{n} D_{e_k e_k} f_k}{\det A} < 0$$

for every n and $j \in N - \{1\}$. The result follows from (21) and Assumption 4.1(b).

(C) follows from (20) and the fact that $\det A$ and $\det A_1$ have opposite signs.

(D) It follows from (20) that

$$D_x \sum_{j \in N} e_j = \frac{D_{e_1 x} u_1}{\det A} \sum_{j \in N} (-1)^j \det A_j = \frac{D_{e_1 x} u_1}{\det A} \det K$$

where

$$K = \begin{bmatrix} -1 & \dots & -1 \\ D_{e_2 e_1} u_2 & \dots & D_{e_2 e_n} u_2 \\ \vdots & \ddots & \vdots \\ D_{e_n e_1} u_n & \dots & D_{e_n e_n} u_n \end{bmatrix}$$

Subtracting the first column of K from every other column yields $\det K = \det L$, where

$$L = \begin{bmatrix} -1 & 0 & \dots & 0 \\ -D_{e_+ e_+} \delta_2 & D_{e_2 e_2} f_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -D_{e_+ e_+} \delta_n & 0 & \dots & D_{e_n e_n} f_n \end{bmatrix}$$

Thus, $D_x \sum_{j \in N} e_j = D_{e_1 x} u_1 \det L / \det A = -D_{e_1 x} u_1 \prod_{k=2}^{n} D_{e_k e_k} f_k / \det A$. Note that $\prod_{k=2}^{n} D_{e_k e_k} f_k < 0$ if and only if n is even. ■

References

[1] C. d'Aspremont, J. Jacquemin, J. Gabszewicz, J. Weymark: On the stability of collusive price leadership, *Canadian Journal of Economics* 16, 17-25 (1983)

[2] S. Barrett: Self-enforcing international environmental agreements, *Oxford Economic Papers* 46, 878-894 (1994)

[3] C. Berge: *Topological Spaces*, Macmillan (1963)

[4] J. Black, M. Levi, D. De Meza: Creating a good climate: minimum participation for tackling the greenhouse effect, *Economica* 60, 281-294 (1993)

[5] C. Carraro, D. Siniscalco: Strategies for the international protection of the environment, *Journal of Public Economics* 52, 309-328 (1993)

[6] H. Cesar: *Control and Game Models of the Greenhouse Effect* Lecture Notes in Economics and Mathematical Systems 146, Springer-Verlag (1994)

[7] P. Chander, H. Tulkens: Theoretical foundations of negotiations and cost sharing in transfrontier pollution problems, *European Economic Review* 36, 288-299 (1992)

[8] P. Chander, H. Tulkens: A core-theoretic solution for the design of cooperative agreements on transfrontier pollution, *International Tax and Public Finance* 2, 279-294 (1995)

[9] P. Chander, H. Tulkens: The core of an economy with multilateral environmental externalities, *International Journal of Game Theory* 26, 379-401 (1997)

[10] E. Dockner, N. van Long: International pollution control: cooperative versus non-cooperative strategies, *Journal of Environmental Economics and Management* 25, 13-29 (1993)

[11] D. Fudenberg, J. Tirole: *Dynamic Models of Oligopoly*, Harwood Academic Publishers (1986)

[12] F. Gantmacher: *The Theory of Matrices, Vol. I*, Chelsea Publishing Co (1990)

[13] M. Hoel: Intertemporal properties of an international carbon tax, *Resource and Energy Economics* 15, 51-70 (1993)

[14] D. Kelly, C. Kolstad: Bayesian learning, growth and pollution, *Journal of Economic Dynamics and Control* 23, 491-518 (1999)

[15] C. Kolstad: Fundamental irreversibilities in stock externalities, *Journal of Public Economics* 60, 221-233 (1996)

[16] S. Lang: *Real and Functional Analysis, 3rd ed.*, Springer-Verlag (1993)

[17] A. Markushevich: *Theory of Functions of a Complex Variable, Vol. I*, Prentice-Hall (1965)

[18] W. Martin, R. Patrick, B. Tolwinski: A dynamic game of a transboundary externality with asymmetric players, *Journal of Environmental Economics and Management* 25, 1-12 (1993)

[19] H. Moulin: *Game Theory for the Social Sciences*, New York University Press (1986)

[20] S. Shah: A non-cooperative theory of quantity-rationing international transfrontier pollution, CDE Working Paper No. 143 (2006)

[21] A. Ulph, D. Maddison: Uncertainty, learning and international environmental policy coordination, *Environmental and Resource Economics* 9, 451-466 (1997)

[22] A. Ulph, D. Ulph: Global warming, irreversibility and learning, *The Economic Journal* 107, 636-650 (1997)

[23] F. van der Ploeg, A. de Zeeuw: International aspects of pollution control, *Environmental and Resource Economics* 3, 117-139 (1992)