Human Capital Accumulation and Endogenous Growth in a Dual Economy

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Abstract

This paper develops an endogenous growth model of a dual economy where human capital accumulation is the source of economic growth. The dualism between the rich individuals and the poor individuals exists in the mechanism of human capital accumulation. Rich individuals allocate labour time not only for their own production and knowledge accumulation but also to train the poor individuals. Steady-state growth paths are studied for Market Economy (Decentralised Economy) and Command Economy. Optimal tax policy that helps to achieve steady-state growth rate of command economy through market economy is also derived.

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1 Introduction

With the emergence of the 'new' growth theory, human capital accumulation and its role on economic growth has become a major area of research in macroeconomics. The literature starts with the seminal paper of Lucas (1988) which shows that growth rate of per capita income depends on the growth rate of human capital which again depends on the time allocation of the individuals for acquiring skill. Rebelo (1991) extends the model introducing physical capital as an additional input in the human capital accumulation function. Both Rebelo and Stokey (1995) and Jones et. al. (1993) study the growth effects of fiscal policy when both physical capital and human capital accumulate over time. Alonso-Carrera and Freire Seren (2004) consider the case where households accumulate human capital with intermediate good and labour owned by them. They show that multiple equilibria can arise in the presence of market imperfection generated by fiscal policy. Caballe and Santos (1993) presents a model similar to Lucas (1988) without the external effect in production; and, in their model, human capital accumulation is a concave function of the time devoted to education. They also analyse the transitional dynamics properties of the model. In Arrow (1962), Chamley (1993), Greiner (1996) etc. human capital accumulation takes place through learning by doing process of the workers. Accumulated stock of human capital is formed as a by-product (spillover effect) of accumulated gross investments to physical capital. Mauro and Carmeci (2003) analyze how unemployment rate adversely influences the human capital accumulation. Gomez (2003) devised optimal fiscal policy in the Lucas (1988) model with externalities. Robertson (2002) shows in Lucas (1988) framework how demographic shocks affect growth of human capital. Albelo (1999) developed an endogenous growth model in which formal education and learning by using new capital goods act as perfect substitutes to each other in human capital accumulation.

However, these endogenous growth models do not provide appropriate framework for analysing the problems of growth of less developed countries. Less developed economies are often characterized by the existence of opulence and poverty side by side. Rich individuals who are capital owners stay in contrast with the poor individuals who have very little income over consumption to save and invest in physical and human capital. This co-existence of rich and poor individuals leads to dualism in the less developed countries.

There exists a substantial theoretical literature dealing with dualism and income inequalities in less developed countries. Different dual economy models e.g Lewis (1954), Ranis and Fei (1961), Sen (1966), Dixit (1968), Todaro (1969), Banerjee and Newman (1998) etc. deal with the problems of co-existence of the advanced sector and the backward sector in less developed countries. Eicher and Penalosa (2001) deal with the complex relationship between growth and inequality due to offsetting supply of and demand for skills in an economy with skilled and unskilled workers. Leach (1996) deals with the regional income disparities due to differences in productivity of advanced and backward regions and also due to differences in training cost and migration cost among the workers of the two regions. Benabou (1994) shows how minor differences in educational technologies or preferences or minor imperfections in capital markets can lead to a persistent income inequality and inefficiently low level of growth. Saint Paul (1996) shows the possibility of multiple equilibria when real wage rates of skilled and unskilled workers are negatively related with their unemployment rate. There are a number of models dealing with the choice of optimal tax for the provision of public education in a society with individuals having different levels of parental income or wealth level e.g Glomm, Ravikumar (1992); Gradstein, Justman (1996); Nordblom (2003). However, none of the existing models [1] focuses on the dualism in the mechanism of human capital formation of two different class of people.

In a less developed economy, the stock of human capital of the poor individuals is far lower than that of rich individuals. Also there exists difference in the mechanism of human capital accumulation of rich and poor individuals. There are rich individuals who spend time and resources for schooling of their children. They spend a lot in purchasing books and other educational equipments and in paying high tuition fees to private educational institutes. Even if they are benefitted by the infrastructural facilities created by the governments, these public goods are mainly financed by their tax payments. On the other hand there are poor individuals who spend a negligible fraction of their income and resources in education and child care. Moreover, the opportunity cost of schooling of their children is very high because they can be alternatively employed as child labour. So unless the education is highly subsidized, they can not send their children to schools and hence their efficiency level remains persistently low. So, the benevolent government wants to improve the efficiency level of the poor workers. Government sets up free public schools and gives scholarships to the meritorious students who come from poor families. The poor individuals send their children to the state aided free schools and accumulate their human capital with the help of the government. The government recruits teachers in the public schools and the salary paid to these teachers and doctors are financed by the tax on labour and capital. So the accumulation of human capital of poor individual depends on help provided by the government while the rich individual can accumulate human capital

[1] Benabou (1996) constructs a model of human capital formation that depends on quality of social interaction and resources devoted per student which differ from one community to other. Significant polarization arises from these differences in the inputs of education technology.

without any help of the government. Thus, efficiency enhancement mechanism for wealthier individuals and poor individuals are different.

In the present paper, we develop a growth model of an economy in which human capital accumulation is viewed as the source of economic growth and in which difference exists in the mechanism of human capital accumulation of the two types of individuals — rich and poor. The government deducts a fraction of the time of the rich individuals for helping the poor individuals accumulating human capital. So, in this model rich individuals not only allocates its labour time between production and his (her) own skill accumulation but also allocates a part of the labour time to the training of the poor people. We assume that there exists aggregate external effect associated with human capital of both rich and poor individuals on production and external effect associated with only human capital of rich individual on the human capital accumulation of the poor individuals.

In this paper we start with market (decentralised) economy in which government imposes tax and gives subsidy to the educational sector of both rich and poor individuals and also gives salary to the teachers recruited to give training to the poor individuals. We compare the steady state values of different variables in market economy with those in economy where human capital and physical capital are allocated by solving a grand optimization exercise. The latter is called a Command Economy, which may be thought of as a fully planned system. The analysis in this paper is restricted to steady-states alone when it comes to a relative evaluation of the steady-state values of the variables. In this paper we find that the rate of growth of human capital of both types of individuals and consequently the rate of growth of income in the market economy may be less or greater than that in the command economy and if there is no externality present in the economy, rates of growth in command economy is same as that in market economy which is again same as that in the Lucas (1988) model. This paper is partly motivated by Lucas's (1988) seminal work and partly by Dasgupta (1999)'s paper in which it is shown that there exists a tax rate for which the market equilibrium rate of growth is same as that of the command economy in a model of nonrival infrastructure.

This paper is organized as follows. Section 2 discusses the assumptions of the model with specified focus on the difference between rich and poor individuals. Section 3 analyses the properties of steady state growth path in the market (competitive) economy and section 4 does the same in the case of a planned economy. In section 5, we derive the optimal fiscal policy that helps to achieve command solutions through market economy. Concluding remarks are made in section 6.

2 The dual economy model

We consider an economy with two types of individuals – rich and poor individuals. All workers are employed in a single aggregative sector that produces a single good. Both rich and poor individuals invest in physical capital employed in the production sector. By human capital as Lucas we mean the set of specialized skills or efficiency level of the workers that accumulate over time. The skill formed by rich individuals and poor individuals are imperfectly substitute to each other in production. The mechanism of human capital accumulation is different for the two types of individuals. Population size of either type of individual is normalised to unity. All individuals belonging to each group are assumed to be identical to each other. There is full employment of labour and capital. The labour and capital market are perfectly competitive.

2.1 Production technology

The single production sector behaves competitively and employs the labour and capital by profit maximizing rule. The government deducts $(1-x)$ fraction of total time of rich individual for helping poor individuals in accumulating human capital. A rich individual allocates 'a' fraction of the total remaining (x fraction of) non-leisure time in production. A poor individual allocates $(1-u)$ fraction of total time in production. Let H_R and H_P be the skill level of the representative rich and poor individual (worker) respectively.

The production function takes the form:

$$Y = A(axH_R)^{\alpha}((1-u)H_P)^{\beta}K^{1-\alpha-\beta}{H_R}^{\epsilon_R}{H_P}^{\epsilon_P} \tag{1}$$

where $0 < \alpha < 1$, $0 < \beta < 1$. Here $\epsilon_R > 0$, $\epsilon_P > 0$ are the parameters representing the magnitude of the external effect of human capital of rich individual and poor individual on production. Production function satisfies CRS in terms of inputs but shows IRS if external effect is taken into consideration. Y stands for the level of output. Here $K = K_R + K_P$ i.e. the aggregate physical capital constitutes of the physical capital owned by the rich individuals (K_R) and the physical capital owned by the poor individuals (K_P).

2.2 Difference in the mechanism of human capital accumulation

Education is the channel through which a person acquires skill. Mechanism of human capital accumulation for the rich individuals is assumed to be similar to that in Lucas (1988). The rate at which human capital is formed is proportional to the time or effort devoted to acquire skill by the rich individual. Hence

$$\dot{H}_R = m(1-a)xH_R \tag{2}$$

where $(1-a)$ is the fraction of the non-leisure time devoted to acquiring the own skill level. Here $0 < a < 1$ and m is a positive constant, representing the productivity parameter of human capital formation of rich individuals.

However the mechanism of human capital formation for the two classes of individuals are different. The poor individuals can not acquire human capital on their own. The skill formation of a poor person takes place through training programme conducted by the rich individuals. The government recruits (deducts $(1-x)$ fraction of the time of) the rich individuals to give training to the poor individuals so that they can improve their skill and can take part in the production process more efficiently. So the poor individual can not accumulate human capital without the help of the government. Each rich individual spends $(1-x)$ fraction of its time in this training. Poor individuals devote u fraction of non-leisure time for learning. The additional skill acquired by the representative poor worker is assumed to be a linear homogeneous function in terms of the effort level put by the poor individual and time spent in training by rich individuals including the external effect associated (γ) with H_R. So the human capital accumulation function of poor individual follows social CRS.

Hence we have

$$\dot{H_P} = \{(1-x)H_R\}^{\delta}(uH_P)^{1-\delta-\gamma}{H_R}^{\gamma} \tag{3}$$

Here $0 < \delta < 1$ and $\gamma > 0$. If $x = 1$, $\dot{H_P} = 0$, so H_P will get stuck to a very low value.

3 The Decentralized Economy

In decentralized economy the government taxes physical capital at a rate τ and labour income at a rate t and finances the salary of the rich individuals for devoting $(1-x)$ fraction of their time in training of the poor individuals, subsidizes investment in education by rich individuals at a rate s_R and investment in education by poor individuals at a rate s_P. For rich individuals sole cost of education is foregone earnings that is borne by rich individuals themselves. For poor individuals cost of education is partly foregone earning that is borne by the poor individuals themselves and the salary paid to the rich individuals who are recruited by the government for the training of the poor individuals. So the cost of education of poor individuals is shared between poor agents and the government. So the budget constraint of the government is as follows:

$$\begin{aligned} w_R(1-x)H_R = \tau rK + tw_R axH_R + tw_P(1-u)H_P + tw_R(1-x)H_R \\ -s_R w_R(1-a)xH_R - s_P w_P uH_P \end{aligned} \tag{4}$$

We assume that perfectly competitive firms producing the final good maximize their profit. Profit maximization implies that labours and capital are used up to the point at which marginal product equates marginal cost:

$$r = (1 - \alpha - \beta)\frac{Y}{K} \tag{5}$$

$$w_R = \frac{\alpha Y}{axH_R} \tag{6}$$

$$w_P = \frac{\beta Y}{(1-u)H_P} \tag{7}$$

Both the representative rich individual (worker) and poor individual consume a part of their income and invests the remaining part in physical capital. So we have

$$\dot{K_R} = (1-t)w_R axH_R + (1-\tau)rK_R + (1-t)w_R(1-x)H_R + s_R w_R(1-a)xH_R - C_R \tag{8}$$

and

$$\dot{K_P} = (1-\tau)rK_P + (1-t)w_P(1-u)H_P + s_P w_P uH_P - C_P \tag{9}$$

where C_P and C_R are the level of consumption of the representative poor worker and the rich worker respectively, K_R and K_P are the stock of physical capital owned by the rich and poor individual, τ is the tax rate imposed on physical capital and t is the tax rate imposed on wage income, w_R is the wage rate of rich individual and w_P is the wage rate of poor individuals. $w_R(1-x)$ is the salary paid to the rich individuals who are employed for giving training to the poor individuals. Both the representative rich individual (worker) and poor individual maximize their respective discounted present value of utility over the infinite time horizon with respect to some control variables. The instantaneous utility function from consumption is given by

$$U(C_i) = \frac{C_i^{1-\sigma}}{1-\sigma}, \sigma > 0, \tag{10}$$

$i = R$ for rich individuals and $i = P$ for poor individuals. Here σ is the constant elasticity of instantaneous marginal utility.

3.1 The optimization problem

3.1.1 Optimization by rich household

The objective of the representative rich individual is to maximize the discounted present value of utility over the infinite time horizon. The objective function is given by:

$$J_{H_R} = \int_0^\infty U(C_R)e^{-\rho t}dt$$

This is to be maximized with respect to C_R and a subject to the equations of motion given by

$$\begin{aligned}\dot{H_R} &= m(1-a)xH_R;\\ \dot{K_R} &= (1-t)w_R axH_R + (1-\tau)rK_R + (1-t)w_R(1-x)H_R + s_R w_R(1-a)xH_R - C_R\end{aligned}$$

and given the initial values of H_R and H_P. Here $U(C_R)$ is given by equation (10) and Y is given by equation (1). Here ρ is the constant positive discount rate. Here $0 \leq a \leq 1$, $0 \leq x \leq 1$, $0 \leq \tau \leq 1$ and $0 < t < 1$. The state variables are H_R and H_P. The current value Hamiltonian is given by

$$H^c = \frac{C_R^{1-\sigma}}{1-\sigma} + \lambda_{K_R}[(1-t)w_R axH_R + (1-\tau)rK_R + (1-t)w_R(1-x)H_R + s_R w_R(1-a)xH_R - C_R] + \lambda_R m(1-a)xH_R$$

where λ_R, λ_{K_R} are co-state variables of H_R, K_R respectively representing the shadow prices of the human capital of rich individuals and of the physical capital owned by rich individuals. While maximizing their own present discounted value of utility, the rich individuals consider x, τ, t, s_R to be given.

The optimality conditions

(A) The first order conditions necessary for this optimization problem with respect to the control variables C_R, a are given by the following:

$$C_R^{-\sigma} - \lambda_{K_R} = 0; \tag{11}$$

$$\lambda_{K_R}[(1-t)w_R xH_R - s_R w_R xH_R] - \lambda_R mxH_R = 0; \tag{12}$$

(B) Time derivatives of the co-state variables satisfying the optimum growth path are given by the following:

$$\dot{\lambda_{K_R}} = \rho\lambda_{K_R} - \lambda_{K_R}(1-\tau)r; \tag{13}$$

$$\dot{\lambda_R} = \rho\lambda_R - \lambda_{K_R}[(1-t)w_R ax + (1-t)w_R(1-x) + s_R w_R(1-a)x] - \lambda_R m(1-a)x; \tag{14}$$

(C)Solving the system there will be family of time paths of state and costate variables satisfying the given initial condition. The member of this family that satisfies the transversality conditions given by

$$\lim_{t\to\infty} e^{-\rho t}\lambda_{K_R}(t)K_R(t) = \lim_{t\to\infty} e^{-\rho t}\lambda_R(t)H_R(t)$$

is the optimal path.

Equation (11) states that in equilibrium the marginal utility of consumption of rich individual is equal to the shadow price of physical capital. Equation (12) states that the marginal contribution of time allocation of a rich individual to own skill accumulation evaluated at shadow price of H_R must be equal to the marginal contribution of time allocation in producing commodity evaluated at the shadow price of physical capital. The equations (13), (14) depict the rates of change of the shadow price of physical capital and the shadow price of human capital of the representative rich individual.

3.1.2 Optimization by poor household

The objective of the representative poor individual is to maximize the discounted present value of utility over the infinite time horizon. The objective functional is given by:

$$J_{H_P} = \int_0^\infty U(C_P)e^{-\rho t}dt$$

This is to be maximized with respect to C_P and u subject to the equations of motion given by

$$\begin{aligned} \dot{H_P} &= (1-x)^\delta u^{1-\delta-\gamma} {H_R}^{\delta+\gamma} {H_P}^{1-\delta-\gamma}; \\ \dot{K_P} &= (1-t)w_P(1-u)H_P + (1-\tau)rK_P + s_P w_P u H_P - C_P \end{aligned}$$

and given the initial values of H_R and H_P. Here $U(C_P)$ is given by equation (10) and Y is given by equation (1). Here ρ is the constant positive discount rate. We assume that the discount rate for the rich household and poor household are same. Here $0 \leq u \leq 1$, $0 \leq x \leq 1$. The state variables are H_R and H_P. The current value Hamiltonian is given by

$$H^c = \frac{C_P^{1-\sigma}}{1-\sigma} + \lambda_{K_P}[(1-t)w_P(1-u)H_P + (1-\tau)rK_P + s_P w_P u H_P - C_P] \\ + \lambda_P(1-x)^\delta u^{(1-\delta-\gamma)} {H_R}^{(\delta+\gamma)} {H_P}^{1-\delta-\gamma}$$

where λ_P, λ_{K_P} are co-state variables of H_P, K_P respectively representing the shadow prices of the human capital of poor individuals and of the physical capital of poor individuals. While maximizing their present discounted value of utility the poor individuals consider H_R, x, τ, t, s_P to be given.

3.2 The optimality conditions

(A) The first order conditions necessary for this optimization problem with respect to the control variables C_P, u are given by the following:

$$C_P^{-\sigma} - \lambda_{K_P} = 0; \tag{15}$$

$$-\lambda_{K_P}(1-t)w_P H_P+\lambda_{K_P}s_P w_P H_P+\lambda_P(1-\delta-\gamma)(1-x)^{\delta}u^{-\delta-\gamma}{H_R}^{\delta+\gamma}{H_P}^{(1-\delta-\gamma)}=0; \tag{16}$$

(B) Time derivatives of the co-state variables satisfying the optimum growth path are given by the following:

$$\dot{\lambda_{K_P}}=\rho\lambda_{K_P}-\lambda_{K_P}(1-\tau)r; \tag{17}$$

$$\dot{\lambda_P}=\rho\lambda_P-\lambda_{K_P}(1-t)w_P(1-u)-\lambda_{K_P}s_P w_P u-\lambda_P(1-\delta-\gamma)(1-x)^{\delta}u^{(1-\delta-\gamma)}(\frac{H_R}{H_P})^{(\delta+\gamma)}; \tag{18}$$

(C)Solving the system there will be family of time paths of state and costate variables satisfying the given initial condition. The member of this family that satisfies the transversality conditions given by

$$\lim_{t\to\infty}e^{-\rho t}\lambda_{K_P}(t)K_P(t)=\lim_{t\to\infty}e^{-\rho t}\lambda_P(t)H_P(t)$$

is the optimal path.

Equation (15) states that in equilibrium the marginal utility of consumption of poor individual is equal to the shadow price of physical capital. Equation (16) states that the marginal contribution of time allocation of a poor individual to own skill accumulation evaluated at shadow price of H_P must be equal to the marginal contribution of time allocation in producing commodity evaluated at the shadow price of physical capital. The equations (17), (18) depict the rates of change of the shadow price of physical capital and the shadow price of human capital of the representative poor individual.

3.3 Steady state growth path

Now, we analyze the steady state growth properties of the system. Along the steady state growth path (SGP) C_R, C_P, K, Y grow at constant and same rates and H_R and H_P grow at constant and same rates and a, u and x are time independent.

3.3.1 Steady state growth rate

Let us define the follwing variables $y={H_P}^{\alpha+\beta+\epsilon_P+\epsilon_R}K^{-(\alpha+\beta)}$; $z=\frac{H_R}{H_P}$ and $q=\frac{(C_R+C_P)}{K}$

From equation (2) the growth rate of the human capital of a representative rich individual is given by

$$\frac{\dot{H_R}}{H_R}=m(1-a)x \tag{19}$$

Now, from equation (3) the growth rate of human capital of a representative poor individual denoted by r is given by

$$r = (1-x)^{\delta} u^{(1-\delta-\gamma)} (\frac{H_R}{H_P})^{(\delta+\gamma)} \tag{20}$$

Since on BGP x, u and r are constant, the growth rate of H_R and H_P is equal, i.e. z is constant. So along BGP

$$m(1-a)x = (1-x)^{\delta} u^{(1-\delta-\gamma)} z^{(\delta+\gamma)} \tag{21}$$

This will hold only if $0 < x < 1$ and $0 < a < 1$. From equation (4) and using equations (5), (6), (7) we have

$$\frac{\hat{x}}{(1-\hat{x})} = \frac{(1-t)}{[\{\tau\frac{(1-\alpha-\beta)}{\alpha} + t + s_R + \frac{\beta t}{\alpha} - \frac{\beta s_P \hat{u}}{\alpha(1-\hat{u})}\}\hat{a} - s_R]} \tag{22}$$

From equations (11); (13) and (15); (17) we have,

$$g = \frac{\dot{C_R}}{C_R} = \frac{\dot{C_P}}{C_P} = -\frac{1}{\sigma}\frac{\dot{\lambda_{K_R}}}{\lambda_{K_R}} = -\frac{1}{\sigma}\frac{\dot{\lambda_{K_P}}}{\lambda_{K_P}} = \frac{r(1-\tau)-\rho}{\sigma} \tag{23}$$

Using equations (23) and (5) we have

$$\rho + \sigma g = (1-\tau)(1-\alpha-\beta)A(ax)^{\alpha}(1-u)^{\beta} {H_R}^{(\alpha+\epsilon_R)} {H_P}^{(\beta+\epsilon_P)} K^{(1-\alpha-\beta)} \tag{24}$$

Since the common growth rate of C_R and C_P is constant $\frac{Y}{K}$ is also constant. and growth rate of physical capital is

$$\frac{\dot{K}}{K} = \frac{\dot{K_R} + \dot{K_P}}{K} \tag{25}$$

Using the equations (8), (9) we have,

$$\frac{\dot{K}}{K} = (1-\tau)r + [(1-t)ax + (1-t)(1-x) + s_R(1-a)x]\frac{w_R H_R}{K} + [(1-t)(1-u) + s_P u]\frac{w_P H_P}{K} - \frac{(C_R + C_P)}{K}$$

Using the equation (5), (6), (7), (4) we have,

$$\frac{\dot{K}}{K} = \frac{Y}{K} - \frac{(C_R + C_P)}{K} \tag{26}$$

Since $\frac{\dot{K}}{K}$ and $\frac{Y}{K}$ are constants and $\frac{\dot{C_R}}{C_R}$ and $\frac{\dot{C_P}}{C_P}$ are equal, on BGP

$$\frac{\dot{K}}{K} = \frac{\dot{C_R}}{C_R} = \frac{\dot{C_P}}{C_P} = \frac{\dot{Y}}{Y} = g(say) \tag{27}$$

So, along BGP

$$g = A(ax)^{\alpha}(1-u)^{\beta} z^{\alpha+\epsilon_R} y - q \tag{28}$$

where

Differentiating the equation (24) and using (19), (20), (21) and (27) we have the common growth rate of C_R, C_P, Y, K as given by

$$g = \frac{(\alpha+\beta+\epsilon_R+\epsilon_P)}{(\alpha+\beta)} m(1-a)x \tag{29}$$

Note that if $\epsilon_P = \epsilon_R = 0$, then $g = m(1-a)x$.

Proposition 1 *If there is no external effect in production sector ($\epsilon_P = \epsilon_R = 0$) in production sector, on BGP all variables would grow at the same common rate as the growth rate of human capital of rich individual. External effect in human capital accumulation does not play any role in determining the growth rate of the economy.*

From equation (14) and using equation (12) we have

$$\frac{\dot{\lambda_R}}{\lambda_R} = \rho - \frac{m(1-t)}{(1-t-s_R)} \tag{30}$$

Log-differentiating the equation (12) and using equation (6) with respect to t we have,

$$\frac{\dot{\lambda_{K_R}}}{\lambda_{K_R}} - \frac{\dot{\lambda_R}}{\lambda_R} = -\left(\frac{\dot{Y}}{Y} - \frac{\dot{H_R}}{H_R}\right)$$

Using equations (23), (30), (19), (27) and (29) we have,

$$m(1-a)x = \frac{(m-\rho)(1-t)+\rho s_R}{(1-t-s_R)\{1-\frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}\}} \tag{31}$$

This is the rate of growth of human capital of rich individual in the market economy. Note that, if $s_R = 0$ then this rate of growth of human capital of rich individual does not depend on tax rate t. γ does not have any role in determining this growth rate of human capital of rich individual. If $\sigma = 1$, $m(1-a)x$ is independent of external effects. The condition that ensures a to lie between 0 and 1 is

$$\sigma > \frac{(\epsilon_P+\epsilon_R)}{(\alpha+\beta+\epsilon_P+\epsilon_R)} + \frac{(\alpha+\beta)\{(m-\rho)(1-t)+\rho s_R\}}{mx(1-t-s_R)(\alpha+\beta+\epsilon_P+\epsilon_R)}$$

From the above equation we obtain the equilibrium value of a denoted by $\hat{a}$ as follows:

$$\hat{a} = 1 - \frac{\{(m-\rho)(1-t)+\rho s_R\}}{m\hat{x}(1-t-s_R)\{1-\frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}\}} \tag{32}$$

From equation (18)

$$\frac{\dot{\lambda_P}}{\lambda_P} = \rho - (1-\delta-\gamma)(1-x)^{\delta}u^{-(\delta+\gamma)}z^{(\delta+\gamma)}[\frac{(1-t)}{(1-t-s_P)}(1-u)+u] \qquad (33)$$

Log-differentiating (16)we have,

$$\frac{\dot{\lambda_{K_P}}}{\lambda_{K_P}} - \frac{\dot{\lambda_P}}{\lambda_P} = -(\frac{\dot{Y}}{Y} - \frac{\dot{H_P}}{H_P})$$

Using equations (23), (33), (20), (21), (27) we obtain the ratio of equilibrium time allocation to production and education by the poor individual.

$$\frac{(1-\hat{u})}{\hat{u}} = \frac{(1-t-s_P)}{(1-\delta-\gamma)(1-t)}[\frac{\rho(1-t-s_R)}{(m-\rho)(1-t)+\rho s_R}\{1-\frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}\} + (\delta+\gamma) - \frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}] \qquad (34)$$

The above ratio is derived under the assumption that $0 < \hat{x} < 1$. If $\hat{x} = 1$ then from equation (16) optimal $\hat{u} = 0$ under the assumption that $(1-t-s_P) > 0$. Note that $\frac{(1-\hat{u})}{\hat{u}}$ is positively related with γ and negatively related with production external effects if $(1-\sigma) > 0$. If $\sigma = 1$, $\frac{(1-\hat{u})}{\hat{u}}$ does not depend on production externalities. Substituting the values of $\hat{a}$ and $\hat{u}$ from equations (32), (34) in equation (22) we obtain the equilibrum value of x denoted by $\hat{x}$.

$$\hat{x} = \frac{D_1}{D_2} \qquad (35)$$

where

$$D_1 = [(1-t)(1-t-s_R)m\{1-\frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}\} + \{(m-\rho)(1-t)+\rho s_R\} \{\frac{\tau(1-\alpha-\beta)}{\alpha} + t + \frac{\beta t}{\alpha} - \frac{\beta s_P \hat{u}}{\alpha(1-\hat{u})} + s_R\}]$$

$$D_2 = [m\{1-\frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}\}(1-t-s_R)\{\frac{\tau(1-\alpha-\beta)}{\alpha} + \frac{\beta t}{\alpha} - \frac{\beta s_P \hat{u}}{\alpha(1-\hat{u})} + 1\}]$$

Note that if $t = s_R = s_P = \tau = 0$ then $\hat{x} = 1$. This is quite obvious in the sense that if there is no government intervention no tax is imposed then wage bill of the rich workers employed in the educational sector of the poor individuals can not be paid and hence they can not be recruited in that sector.

So $\hat{u}$, $\hat{x}$, $\hat{a}$ are derived in terms of tax and subsidy rates.

Proposition 2 *For unique specification of fiscal policy (τ, t, s_R, s_P), there exist unique equilibrium values of a, u, x denoted by $\hat{a}, \hat{u}, \hat{x}$.*

4 Command Economy

In the command economy the social planner maximizes the discounted present value of instantaneous social welfare function over the infinite time horizon. Instantaneous social welfare is assumed to be a positive function of the level of consumption of the representative rich individual as well as that of poor individual. Instantaneous social welfare function of the economy is defined as

$$W = \frac{({C_R}^{\gamma}{C_P}^{1-\gamma})^{1-\sigma}}{1-\sigma}, 0 \leq \theta \leq 1 \tag{36}$$

where γ is the weight given to the consumption of the representative rich individual and $(1-\gamma)$ is the weight given to the consumption of the representative poor individual. If $\gamma = 1$, it is same as the utility function of the representative rich individual which we have considered in section 3.

4.1 The optimisation problem

The objective of the social planner is to maximize

$$J_C = \int_0^{\infty} W e^{-\rho t} dt$$

with respect to C_R, a and b subject to the constraints

$$\begin{aligned} \dot{K} &= Y - C_R - C_P, \\ \dot{H}_R &= m(1-a)xH_R, \\ and & \\ \dot{H}_P &= \{(1-x)H_R\}^{\delta}(uH_P)^{1-\delta-\gamma}{H_R}^{\gamma} \end{aligned}$$

Here Y is given by equations (1); and W is given by (36). Here the control variables are C_R, C_P, a, u, x. There are three important points by which the optimization problem in the planned economy is different from that in the household economy. First, the social planner can internalise the externalities what the household in the competitive economy can not do. Secondly, the objective function in command economy and household economy are different. Thirdly, in command economy x is a control variable of government. We form the appropriate current value Hamiltonian and maximize it at each time point with respect to the control variables. λ_K, λ_R, λ_P are co-state variables of K, H_R, H_P respectively representing their shadow prices.

4.2 The optimality conditions

(A) The first order conditions of maximization with respect to C_R, a, b are as follows:

$$({C_R}^{\gamma}{C_P}^{1-\gamma})^{-\sigma}\gamma{C_R}^{\gamma-1}{C_P}^{1-\gamma} - \lambda_K = 0; \tag{37}$$

$$(C_R{}^{\gamma}C_P{}^{1-\gamma})^{-\sigma}(1-\gamma)C_R{}^{\gamma}C_P{}^{-\gamma} - \lambda_K = 0; \tag{38}$$

$$\lambda_K \frac{\alpha Y}{a} - \lambda_R m x H_R = 0; \tag{39}$$

$$\lambda_K \frac{Y(-\beta)}{(1-u)} + \lambda_P(1-\delta-\gamma)\frac{\dot{H}_P}{u} = 0 \tag{40}$$

$$\lambda_K \frac{\alpha Y}{x} + \lambda_R m(1-a)H_R - \lambda_P \delta (1-x)^{(\delta-1)} H_R{}^{\delta}(uH_P)^{1-\delta-\gamma} H_R{}^{\gamma} = 0 \tag{41}$$

(B) Time derivative of co-state variables which satisfy the optimum growth path are given by the followings:

$$\dot{\lambda_K} = \rho\lambda_K - \lambda_K(1-\alpha-\beta)\frac{Y}{K}; \tag{42}$$

$$\dot{\lambda_R} = \rho\lambda_R - [\lambda_K(\alpha+\epsilon_R)\frac{Y}{H_R} + \lambda_R m(1-a)x + \lambda_P(\delta+\gamma)(1-x)^{\delta}(\frac{H_R}{H_P})^{\delta+\gamma-1}u^{1-\delta-\gamma}]; \tag{43}$$

and

$$\dot{\lambda_P} = \rho\lambda_P - \lambda_K \frac{(\beta+\epsilon_P)Y}{H_P} - \lambda_P(1-\delta-\gamma)(1-x)^{\delta}u^{1-\delta-\gamma}(\frac{H_R}{H_P})^{(\delta+\gamma)}. \tag{44}$$

(C) Transversality Conditions: Optimum time path of $K, H_R, H_P, \lambda_K, \lambda_R$ and λ_P should satisfy the followings:

$$\lim_{t\to\infty} e^{-\rho t}\lambda_K K = \lim_{t\to\infty} e^{-\rho t}\lambda_R H_R = \lim_{t\to\infty} e^{-\rho t}\lambda_P H_P = 0.$$

4.3 Steady state growth path

We define steady state growth path (SGP) following the same way as in section 3.3.

From equations (37) and (38)

$$\frac{C_R}{C_P} = \frac{\gamma}{1-\gamma} \tag{45}$$

We consider the case where $0 < \gamma < 1$. So, the equilibrium growth rate of C_R and C_P are same. From equations (37), (38) and (42) we have,

$$-\sigma\frac{\dot{C}_R}{C_R} = -\sigma\frac{\dot{C}_P}{C_P} = \frac{\dot{\lambda_K}}{\lambda_K} = -\sigma g = \rho - (1-\alpha-\beta)\frac{Y}{K} \tag{46}$$

From equation (41) and using equations (39) and (40) we have

$$\frac{u^*}{1-u^*} = \frac{(1-\delta-\gamma)\alpha(1-x^*)}{\beta\delta a^* x^*} \tag{47}$$

Note that if $(1-\delta-\gamma) = 0$ i.e. if poor individuals do not have any role in their own skill accumulation then optimal effort that poor individuals should put in education is zero i.e. $u^* = 0$ From (44) and using equations (40), (47) and (21) we have,

$$\frac{\dot{\lambda_P}}{\lambda_P} = \rho - [\frac{\delta(\beta+\epsilon_P)ax}{\alpha(1-x)} + (1-\delta-\gamma)]m(1-a)x \tag{48}$$

From (43) and using equations (39) and (40) we have

$$\frac{\dot{\lambda_R}}{\lambda_R} = \rho - max(\frac{\alpha+\epsilon_R}{\alpha}) - m(1-a)x - \frac{\beta(\delta+\gamma)umax}{(1-\delta-\gamma)(1-u)\alpha} \tag{49}$$

Now the equations (19), (20), (21) and (29) of household economy also hold in planned economy. Since the above mentioned equations are identical, the common growth rate of Y, K, C_R and C_P in the planned economy is exactly same as the common growth rate obtained in the household economy provided that $m(1-a)x$ takes the same value in both the systems.

Log-differentiating equation (39) and using equations (46), (49), (19) and using the fact that in steady state Y also grows at the same rate (g) as C_R, C_P, K we have,

$$(1-\sigma)g + max[\frac{(\alpha+\epsilon_R)}{\alpha} + \frac{u\beta(\delta+\gamma)}{(1-\delta-\gamma)(1-u)\alpha}] = \rho$$

Now using the equations (29), (47) we obtain the optimal value of a in terms of x.

$$a^* = \frac{\rho - \frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}mx^* - \frac{m(\delta+\gamma)(1-x^*)}{\delta}}{mx^*[\frac{(\alpha+\epsilon_R)}{\alpha} - \frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}]} \tag{50}$$

From the above equation we find that a^* is negatively related with x^* i.e. more the time government deducts from the rich individual's total time endowment for the training purpose of poor individuals, it is optimal for rich individuals to allocate more time to production. From equation (50) we have,

$$(1-a^*)mx^* = \frac{\frac{(\alpha+\epsilon_R)}{\alpha}mx^* - \rho + \frac{m(\delta+\gamma)(1-x^*)}{\delta}}{\frac{(\alpha+\epsilon_R)}{\alpha} - \frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}} \tag{51}$$

This is the growth rate of human capital of rich individual in command economy. Note that in this case the growth rate of human capital of rich individuals positively related with the external effect present in the human capital accumulation of poor individuals (γ) unlike the case of market economy where growth rate of human capital is not related to γ.

Log differentiating the equation (40) and using equations (46), (48), (20), (21) and (29) we have

$$[(1-\sigma)\frac{(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}-(\delta+\gamma)+\frac{\delta(\beta+\epsilon_P)ax}{\alpha(1-x)}]m(1-a)x=\rho \tag{52}$$

From equations (51) and (52) we have

$$B_1x^2+B_2x+B_3=0 \tag{53}$$

where

$$B_1=\frac{m^2\alpha}{\delta(\beta+\epsilon_P)}\{(\delta+\gamma)-(1-\sigma)\frac{(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}\}\{\frac{(\alpha+\epsilon_R)}{\alpha}-\frac{(\delta+\gamma)}{\delta}\}$$

$$B_2=\frac{(\delta+\gamma)}{\delta}+\frac{\alpha\rho}{\delta(\beta+\epsilon_P)}\{\frac{(\alpha+\epsilon_R)}{\alpha}-(\delta+\gamma)\}+\frac{m\alpha}{\delta(\beta+\epsilon_P)}\{(\delta+\gamma)-(1-\sigma)\frac{(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}\}$$
$$\{2\frac{(\delta+\gamma)}{\delta}-\frac{(\alpha+\epsilon_R)}{\alpha}\}-(1-\sigma)\frac{(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}$$

$$B_3=[\rho-m\frac{(\delta+\gamma)}{\delta}]-\frac{m\alpha}{\delta}[\frac{\rho}{(\beta+\epsilon_P)}\{\frac{(\alpha+\epsilon_R)}{\alpha}-(\delta+\gamma)\}+\{(\delta+\gamma)-(1-\sigma)\frac{(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}\frac{m(\delta+\gamma)}{\delta(\beta+\epsilon_P)}]$$

Note that if $x=1$ LHS is

$$\rho-\frac{(1-\sigma)}{(\alpha+\beta)}(\alpha+\beta+\epsilon_P+\epsilon_R)m$$

The above term is positive since a^* is positive. If $x=0$ then LHS is as follows

$$[\rho-m\frac{(\delta+\gamma)}{\delta}]-\frac{m\alpha}{\delta}[\frac{\rho}{(\beta+\epsilon_P)}\{\frac{(\alpha+\epsilon_R)}{\alpha}-(\delta+\gamma)\}+\{(\delta+\gamma)-(1-\sigma)\frac{(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}\frac{m(\delta+\gamma)}{\delta(\beta+\epsilon_P)}]$$

Since without any external effect $m(1-a)x>0$, $\frac{m(\delta+\gamma)}{\delta}-\rho>0$. Moreover we are assuming

$$\frac{(\alpha+\epsilon_R)}{\alpha}>\frac{(\delta+\gamma)}{\delta} \tag{54}$$

and

$$(\delta+\gamma)>\frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)} \tag{55}$$

So when the conditions (54) and (55) are satisfied if $x=0$ LHS is negative. Here, $\frac{B_3}{B_1}<0$. So between the two roots of the equation one root is positive and the other one is negative and the positive root lies between 0 and 1.

Proposition 3 *There exists a unique, positive optimal x^* for which aggregate welfare is maximized in command economy provided the external effect associated with the human capital of rich individual is stronger in production than in the human capital accumulation of poor individual and the role of human capital of rich individual is sufficiently important in the human capital accumulation mechanism of poor individual.*

Once x^* is determined, from equation (50) a^* and hence from equation (47) $\frac{u^*}{1-u^*}$ can be determined. From equation (21) the ratio of human capital of rich individual and that of poor individual in command economy (z^*) is determined.

$$z^* = [\frac{m(1-a^*)x^*}{(1-x^*)^{\delta}u^{*1-\delta-\gamma}}]^{\frac{1}{(\delta+\gamma)}} \tag{56}$$

5 Optimal Fiscal Policy

Inefficiency arises in market economy because of two reasons. Firstly, in market economy individual agents can not internalise the externalities associated with the human capital. Secondly, in decentralized economy the effort put by rich individual for augmenting the skill of poor individuals is determined by the government's budget constraint and not by the marginal benefit of time allocation for this purpose multiplied by the shadow price of human capital of poor individual. So intuitively it is obvious that command economy dominates market economy in welfare.

Since from equation (29) the growth rates of C_R, C_P, Y, K, H_R, H_P in command economy and market economy are same if $m(1-a)x$ takes the same value. So using equations (31), (51) we compare the values of $m(1-a)x$ in command economy with that in market economy and find that the command economy growth rate is higher than competitive economy growth rate if

$$\alpha m\{x^*(\frac{\epsilon_R}{\alpha} - \frac{\gamma}{\delta}) + \frac{\gamma}{\delta}\} > \frac{(m-\rho)\epsilon_R}{\{1 - \frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}\}}$$

So if there is no external effect command economy growth rate is same as market economy growth rate. If $\epsilon_R = 0$ and $\gamma > 0$ then command economy growth rate is always higher than market economy growth rate. But if $\gamma = 0$ and $\epsilon_R > 0$ then command economy growth rate is higher than market economy growth rate if

$$x^* > \frac{(m-\rho)}{m\{1 - \frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}\}}$$

Castrillo, Sanso (2000), Gomez(2003) devise optimal fiscal poly that is capable in making decentralized economy to move along optimal transitional path in Lucas (1988) type model. They compare the entire dynamic system in decentralized economy with that in the command economy. Here just to avoid complications we are comparing steady state values of growth rates and steady-state values of time allocation variables in market economy with those in the command economy and it is possible for decentralized economy to grow at the same rate as command economy and allocating optimal amount of time between production and education in steady-state of decentralized economy by choosing appropriate fiscal policy.

The growth rate of consumption in command economy is given by (from equation (46)) as follows:

$$\frac{\dot{C}_R}{C_R} = \frac{\dot{C}_P}{C_P} = \frac{(1-\alpha-\beta)\frac{Y}{K} - \rho}{\sigma}$$

From equations (23) and (5) the growth rate of consumption in market economy is given by,

$$\frac{\dot{C}_R}{C_R} = \frac{\dot{C}_P}{C_P} = \frac{(1-\alpha-\beta)(1-\tau)\frac{Y}{K} - \rho}{\sigma}$$

Comparing these two equations it can be said that the optimal capital tax to be imposed in market economy is zero.

Proposition 4 *Optimal capital tax in market economy is zero.*

The growth rate in command economy and that in market economy would be equal if the growth rate of H_R in command economy is equal to that in the market economy i.e. $m(1-\hat{a})\hat{x} = m(1-a^*)x^*$. From equations (31) and (51) these two would be equal when

$$\frac{(1-t)}{s_R} = \frac{M_1}{M_2}$$

where

$$M_1 = [\rho\{(\frac{\alpha+\epsilon_R}{\alpha}) - \frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}\} + \{1 - \frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}\}$$
$$\{(\frac{\alpha+\epsilon_R}{\alpha})mx^* - \rho + m\frac{(\delta+\gamma)}{\delta}(1-x^*)\}]$$

$$M_2 = [\{(\frac{\alpha+\epsilon_R}{\alpha})mx^* - \rho + m\frac{(\delta+\gamma)}{\delta}(1-x^*)\}\{1 - \frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}\}$$
$$-(m-\rho)\{(\frac{\alpha+\epsilon_R}{\alpha}) - \frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}\}]$$

In case of $\sigma = 1$ the above ratio is

$$(\frac{1-t}{s_R}) = \frac{(\frac{\alpha+\epsilon_R}{\alpha})mx^* - \rho + m\frac{(\delta+\gamma)}{\delta}(1-x^*) + \rho(\frac{\alpha+\epsilon_R}{\alpha})}{(\frac{\alpha+\epsilon_R}{\alpha})mx^* - \rho + m\frac{(\delta+\gamma)}{\delta}(1-x^*) - (m-\rho)(\frac{\alpha+\epsilon_R}{\alpha})}$$

It is evident from the above equation that the fraction $\frac{(1-t)}{s_R}$ is greater than unity. Since $0 < t < 1$, subsidy given to the education of rich individual (s_R) is positive if

$$(\frac{\alpha+\epsilon_R}{\alpha})mx^* - \rho + m\frac{(\delta+\gamma)}{\delta}(1-x^*) > (m-\rho)(\frac{\alpha+\epsilon_R}{\alpha})$$

This is the same condition of command economy growth rate to be higher than market economy growth rate without government intervention in case of $\sigma = 1$. As x^* is uniquely determined in the command economy, growth rate equalizing $\frac{(1-t)}{s_R}$ is also unique.

Now we equate $\frac{u}{(1-u)}$ in market economy to that in command economy from equations (47) and (34) and obtain the optimal ratio of subsidy to be given to poor individuals (s_P) and subsidy to be given to rich individuals (s_R).

$$\frac{s_P}{s_R} = \frac{(1-t)}{s_R}[1-[(1-u^*)(1-\delta-\gamma)]/[u^*[\rho(\frac{1-t}{s_R}-1)\{(m-\rho)(\frac{(1-t)}{s_R}+\rho\}$$
$$\{1-\frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}\} + (\delta+\gamma) - \frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}]]$$

In case of $\sigma = 1$ the above ratio is

$$\frac{s_P}{s_R} = \frac{(1-t)[\rho(\frac{(1-t)}{s_R}-1)(1-\delta-\gamma) + m(\frac{1-t}{s_R})\{u^* - (1-\delta-\gamma)\}]}{s_R u^*[\rho(\frac{(1-t)}{s_R}-1)(1-\delta-\gamma) + m(\delta+\gamma)(\frac{1-t}{s_R})]}$$

The sufficient condition for $\frac{s_P}{s_R}$ to be positive is $u^* > (1-\delta-\gamma)$ Using equations (50) and (47) the above condition reduces to $(1-x^*) > \frac{\alpha\beta\rho\delta}{m(\delta+\gamma)(\alpha+\epsilon_R+\beta\alpha)}$

Now we equate $\hat{x}$ of equation (35) with x^* of command economy. Since we have already equated $\hat{u}$ and u^* we can replace $\hat{u}$ by u^* and for simplicity we are assuming $\sigma = 1$. So from equation (35) we have,

$$x^* = \frac{(1-t)(1-t-s_R)m + \{(m-\rho)(1-t) + \rho s_R\}\{t + \frac{\beta t}{\alpha} - \frac{\beta s_P u^*}{\alpha(1-u^*)} + s_R\}}{(1-t-s_R)\{\frac{\beta t}{\alpha} - \frac{\beta s_P u^*}{\alpha(1-u^*)} + 1\}m}$$

Simplifying the above equation we obtain the optimal tax rate to be imposed on wage income.

$$t = \frac{N - mx^*(\frac{1-t}{s_R}-1)}{N + mx^*\frac{\beta}{\alpha}(\frac{1-t}{s_R}-1) - \{(m-\rho)\frac{1-t}{s_R}+\rho\}(1+\frac{\beta}{\alpha})} \tag{57}$$

where

$$N=\frac{s_R}{(1-t)}[\{(m-\rho)(\frac{1-t}{s_R})+\rho\}\{1-\frac{\beta u^*}{\alpha(1-u^*)}+m(\frac{1-t}{s_R}-1)\{\frac{1-t}{s_R}+\frac{\beta u^* s_P}{\alpha(1-u^*)s_R}x^*\}]$$

Now,

$$N-mx^*(\frac{1-t}{s_R}-1)=\frac{s_R}{(1-t)}\{(m-\rho)\frac{1-t}{s_R}+\rho\}\{1-\frac{\beta u^*}{\alpha(1-u^*)}\}$$
$$+m(\frac{1-t}{s_R}-1)[1-x^*\{1-\frac{\beta u^* s_P}{\alpha(1-u^*)(1-t)}\}]$$

Since $(1-t-s_P)>0$, $\frac{s_P}{(1-t)}<1$ Now using equations (47) and (50) we find that the condition of $\frac{\beta u}{\alpha(1-u)}<1$ is $(1-x^*)<\frac{\alpha\delta\rho}{m\epsilon_R\{\frac{(\alpha+\epsilon_R)}{\alpha}-(\delta+\gamma)\}}$ If the above condition is satisfied then $0<[1-\frac{\beta u^* s_P}{\alpha(1-u^*)(1-t)}]<1$ and hence numerator of the expression t is positive.If the numerator is positive then it implies that N is also positive.

$$mx^*\frac{\beta}{\alpha}(\frac{1-t}{s_R}-1)-\{(m-\rho)\frac{1-t}{s_R}+\rho\}(1+\frac{\beta}{\alpha})>0$$

when

$$x^*>\frac{[(m-\rho)(\frac{1-t}{s_R})+\rho](\alpha+\beta)}{m\beta(\frac{1-t}{s_R}-1)}$$

So the condition that t lies between 0 and 1 is that

$$x^*>max\{1-\frac{\alpha\delta\rho}{m\epsilon_R\{\frac{(\alpha+\epsilon_R)}{\alpha}-(\delta+\gamma)\}},\frac{[(m-\rho)(\frac{1-t}{s_R})+\rho](\alpha+\beta)}{m\beta(\frac{1-t}{s_R}-1)}\}$$

From equation (21) we see that

$$z=[\frac{m(1-a)x}{(1-x)^\delta u^{1-\delta-\gamma}}]^{\frac{1}{(\delta+\gamma)}}$$

So if $m(1-a)x$, u and x in market economy and command economy are set to be equal then z in both the economies will also be equal. Since z is the ratio of human capital of rich individual and that of poor individual so we can think z as a measure of equity. So through choosing optimal tax policy if growth rates and other important control variables (x, u) are set to be equal to those in the command economy then automatically improvement will be brought about in terms of equity. Hence we can establish the follwing proposition.

Proposition 5 *There exist unique wage tax and subsidy rates that can equate the market economy growth rate and equity to be equal to those of the command economy.*

6 Conclusion

Existing endogeneous growth models dealing with human capital accumulation have not considered dualism in human capital formation among different classes of people. On the other hand the old growth models which have considered dualism between industry and agriculture in less developed countries do not consider the aspect of human capital accumulation and endogenous growth. This paper tries to bridge the gap.

This paper attempts to develop a theoretical model of growth involving redistributive service to build up human capital of less privileged section of the community. Here we have analyzed the model of an economy with two different class of individuals in which growth stems from human capital accumulation and the dualism exists in the nature of human capital accumulation of two types of individuals. We compare the steady state values of the variables obtained in market economy with those obtained in command economy. We also derive the optimal fiscal policy that can lead to command economy solution through market economy.

The model, in this paper, does not consider many important features of less developed countries. The present model does not consider many problems of dual economy e.g. unemployment of labour, credit-market imperfections etc. Our purpose is to focus on dualism in the human capital accumulation in a less developed economy. In order to keep the analysis otherwise simple, we do all kinds of abstraction—a standard practice often followed in the theoretical literature.

References

Albelo C.D.A, 1999. Complementarity between physical and human capital and speed of convergence, Economics Letters,64, 357-361.

Benabou, R, 1996. Heterogeneity, Stratification and Growth: Macroeconomic Implications of Community Structure and School Finance, American Economic Review, 86 , 584-609.

Banerjee A.V. and A.F. Newman, 1997. Information, the Dual Economy and Development. Review of Economic Studies, 65 , 631-653.

Caballe, J. and M. Santos, 1993. On Endogenous Growth with Physical and Human Capital, Journal of Political Economy, 101, 1042-1067.

Carrera J. A. and M. J. F. Seren, 2004. Multiple equilibria, fiscal policy and human capital accumulation. Journal of Economic Dynamics and Control, 28, 841-856.

Chamley, C, 1993. Externalities and dynamics in models of learning or doing, International Economic Review, 34, 583-610.

Dasgupta, D, 1999. Growth versus welfare in a model of nonrival infrastructure, Journal of Development Economics, 58, 359-385.

Dixit A.K, 1969. Marketable Surplus and Dual Development, Journal of Economic Theory, 1,203-219.

Eicher, T.S. and C.G. Penalosa, 2001. Inequality and growth: The dual role of human capital in development, Journal of Development Economics, 66, 173-197.

Garcia-Castrillo, P and Sanso, M. 2000. Human capital and optimal policy in a Lucas-type model, Review of Economic Dynamics, 3, 757-770.

Glomm, G. and B. Ravikumar, 1992. Public versus Private Investment in human capital: Endogenous growth and income inequality, Journal of Political Economy. 100(4), 818-834.

Gomez M. A. 2003. Equilibrium dynamics in the one sector endogenous growth model with physical and human capital. Journal of Economic Dynamics and Control, 28, 367-375.

Gomez M. A. 2003. Optimal fiscal policy in the Uzawa-Lucas model with externalities. Economic Theory, 22, 917-925.

Gradstein, J, 1996. The political economy of mixed public and private schooling: A dynamic analysis, International Tax and Public Finance, 3, 297-310.

Greiner, Alfred, 1996. Fiscal policy in a model of endogenous growth with learning by doing, Public Finance Quarterly, 24, 371-390.

Greiner, A. and W. Semmler. 2002. Externalities of Investment, Education and Economic Growth, Economic Modelling, 19, 709-724.

Guaitoli D, 2000. Human Capital Distribution, Growth and Convergence, Unpublished, 1-24.

Harris J.R and M. Todaro, 1970. Migration, Unemployment and Development: A two sector analysis, American Economic Review, 60, 126-142.

Jones, L.E., R.E. Manuelli and P.E. Rossi, 1993. Optimal Taxation in Models of Endogenous Growth, Journal of Political Economy, 101, 485-517.

Jorgenson D. W, 1961. The development of a dual economy, The Economic Journal, 71, 309-334.

Krueger,A. B. and M. Lindahl, 2001. Education for Growth: Why and For Whom? Journal of Economic Literature, Vol 39, 1101-1136.

Leach J, 1996. Training,Migration and Regional Income Disparities, Journal of Public Economics, 61, 429-443.

Levhari, D . 1966. Further Implications of Learning by Doing, Review of Economic Studies, 33, 31-38.

Lewis, W.A, 1954. Economic Development with Unlimited Supplies of Labour. Manchester School of Economics and Social Studies, 28, 139-191.

Lucas, R. E, 1988. On the Mechanics of Economic Development, Journal of Monetary Economics, 22, 3-42.

Mankiw, G. N., D. Romer and D.N. Weil, 1992. A Contribution to the Empirics of Economic Growth, The Quarterly Journal of Economics, 407-437.

Mauro L. and G. Carmeci, 2003. Long run growth and investment in education: Does unemployment matter?" Journal of Macroeconomics, 25, 123-137.

Nelson, R. R. and E.S. Phelps, 1996. Investment in Humans, Technological Diffusion and Economic Growth, American Economic Review, Papers and Proceedings, 56, 69-75.

Nordblom K, 2003. Is increased public schooling really a policy for equality? The role within the family education, Journal of Public Economics, 87, 1943-1965.

Ortigueira, S. 1998. Fiscal Policy in an Endogenous Growth Model with Human Capital Accumulation, Journal of Monetary Economics, 42, 323-355.

Ranis, G and C.H. Fei, 1961. A Theory of Economic Development, American Economic Review, 51, 533-556.

Rebelo, S, 1991. Long-run Policy Analysis and Long run Growth, Journal of Political Economy. 99, 500-521.

Robertson, P.E., 2002. Demographic shocks and human capital accumulation in the Uzawa-Lucas model. Economics Letter. 74, 151-156.

Romer, P. M, 1986. Increasing Returns and Long run Growth. Journal of Political Economy, 94, 1002-37.

Romer, P. M, 1990. Endogenous Technological Change." Journal of Political Economy, 98, S71-S102.

Romer, P. M, 1989. Human Capital and Growth, Theory and Evidence. NBER working paper series, working paper no. 3173

Ryder, H. E and G.M. Heal, 1973. Optimum Growth with Intertemporally Dependent Preferences, Review of Economic Studies, 40, 1-33.

Saint-Paul, G, 1996. Unemployment and increasing private returns to human capital, Journal of Public Economics, 61, 1-20.

Schultz, T.W, 1961. Investment in Human Capital, The American Economic Review.

Sen, A.K. 1966. Peasants and Dualism with or without Surplus Labour, The Journal of Political Economy, 74, 425-50.

Solow, R.M, 1956. A Contribution to the Theory of Economic Growth, Quarterly Journal of Economics, 65-94.

Solow, 2000. R.M.Growth Theory—An Exposition,Oxford University Press,2nd Edition.

Stokey, N. L. and S. Rebelo, 1995. Growth Effects of Flat-Rate Taxes, Journal of Political Economy, Vol. 103, 519-50.

Todaro,M, 1969. A model of labour migration and urban unemplyment in less developed countries". American Economic Review, 59, 138-148.

Uzawa, H, 1965. Optimum Technical Change in an Aggregative Model of Economic Growth, International Economic Review, 6, 18-31