Endogenous Imitation and Endogenous Growth In a North-South Model: A Theoretical Analysis.[*]

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[*]This is a part of the research work of the first author under the supervision of the second author leading towards the prospective Ph.D. degree of Indian Statistical Institute. Any remaining errors are ours.

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Abstract

This paper presents a modified North-South product variety model of Grossman-Helpman(1991b) introducing localised knowledge spillover in the Northern R&D sector as opposed to the globalised knowledge spillover assumed in Grossman-Helpman(1991b) model. We show that a tighter IPR in the South leads to an increase in the rate of innovation in the North in this case. We analyse the comparative steady-state effects with respect to change in the labour endowment of the two countries. Results are different from those obtained in Grossman-Helpman(1991b) model. We also analyse some transitional dynamic properties of this modified model; and then derive the welfare effects of the policy of IPR strengthening and of changes in labour endowments. These results are not necessarily identical to those obtained in Helpman(1993) model.

JEL Classification: F23, O31, O34, O40.

Keywords: Product variety, Imitation, Innovation, Steady-state Equilibrium, Transitional Dynamics, North, South, Intellectual Property Rights.

1 Introduction

In an interesting and widely noted paper published in The Economic Journal, Grossman and Helpman(1991b) have developed a North-South endogenous growth model based on product variety framework. The North innovates and the South imitates in this model like that in Helpman(1993). The rate of innovation in the North as well as the rate of imitation in the South is endogenously determined in the steady-state equilibrium of this model. Various comparative steady-state exercises have been performed in that model without testing the stability property of the steady-state equilibrium. The increase in the size of any region measured by the size of the labour endowment raises the rate of innovation in the narrow-gap equilibrium and also raises the relative wage of that region. Rate of imitation always varies positively (negatively) with the size of the South (North). The policy of strengthening Intellectual Property Rights (IPR) protection in the South has no effect on the rate of innovation and on the rate of imitation in the narrow gap case; and this only increases the North-South relative wage in the long-run[1].

The basic model of Grossman-Helpman(1991b) has been extended by various authors in various directions[2]. However, Grossman-Helpman(1991b) and its various extensions share a common assumption which we want to modify in this present paper. In Grossman-Helpman(1991b), the knowledge capital stock in the North is assumed to be proportional to the economy's cumulative research experience measured by the number of product designs already developed. This knowledge capital, treated as the public input into the R&D sector generates positive externalities and thus lowers the cost of developing new blue prints in the R&D sector. Instead of this so-called Marshall-Arrow-Romer type of knowledge spillover, we consider Jacobs(1969) type of localised knowledge spillover. Now the agglomeration of different production unit in one region decreases the cost of doing R&D there. Thus here knowledge spillover originates from the presence of producers of different goods in one region rather than the experience of developing product designs in the past. The researchers might benefit from interactions with producers of other goods. They observe the production process directly and find it easier to invent how new goods can be produced. These Jacobs(1969) type of externalities[3] in the Northern R&D sector have been considered by Dollar(1986, 1987), Martin and Ottaviano (1999), Baldwin et al.(2001) etc. in their North-South models although they have not analysed the problem of imitation and IPR protection in the South.

[1] Grossman-Helpman(1991b) have defined a subsidy (tax) to the Southern imitation as equivalent to a lax (stringent) of IPR protection.

[2] See for example, Lai (1995), Chui. M. et al. (2001), Currie et al. (1999). The analysis of IPR protection in a Grossman-Helpman style model has been greatly advanced by Grossman and Lai(2004). However, their focus is different from that in Helpman(1993).

[3] These types of knowledge spillovers at the level of a city or a region have been documented by Glaeser et al.(1992), Henderson et al.(1995) and also Jacobs(1969). For papers treating the case of a country-specific knowledge stocks (which are Marshall-Arrow-Romer type of spillover) see Feenstra(1996) and Grossman-Helpman(1991c, Ch-9).

This is the only minor change in assumption we introduce in this present paper. However, this gives interesting results. If we introduce this change in an otherwise identical Grossman-Helpman(1991b) model, we find that many of the comparative steady-state results in the narrow gap equilibrium of this modified model differs from the corresponding ones in the original Grossman-Helpman(1991b) model. Firstly, the policy of strengthening IPR in the South raises the rate of innovation in the North[4]. Secondly, an increase in the Southern (Northern) labour force decreases (increases) the rate of innovation in the North and produces positive (ambiguous) effect on the rate of imitation in the South.

We also analyse the comparative transitional dynamic effects with respect to changes in various parameters in this modified model. Grossman and Helpman(1991b, 1991c, Ch-11) did not do any such exercises while Helpman(1993) did the same in his exogenous imitation model. In such a case one can distinguish between the short-run effect and the long-run effect. In this model short-run effects and the long-run effects on the relative wage are not qualitatively similar. As the IPR gets stronger in the South, the Northern relative wage overshoots on impact in the short-run though rises steadily in the long-run. This relative wage of a region varies directly with the size of that region in the long-run but it varies inversely with the size of that region in the short-run. This short-run result is consistent with that in Krugman(1979) while the long-run result is similar to that in Grossman-Helpman(1991b).

We also analyse some welfare effects of the policies. No other works in the existing literature have done this in the endogenous imitation model. However, Helpman(1993) has done this in his exogenous imitation model. We find that a policy of strengthening IPR in the South may lead to welfare gain in both the countries and the marginal welfare gain in the North is higher than that in the South. The increase in the size of the labour endowment in the South may raise the welfare of each of the two regions. These results are different from the corresponding ones obtained in the exogenous imitation model of Helpman (1993) where the South always faces a welfare loss.

In section 2, we describe the model. In section 3, we analyse the various comparative steady-state effects. In section 4 we analyse the various transitional dynamic effects. The effects on welfare are analysed in section 5. Concluding remarks are made in section 6.

[4]We have defined stringent IPR in the South as increasing the labour requirement in imitation. This definition has been taken from Glass and Saggi(2002). According to this definition, a stronger IPR in the South leads to a decrease in the rate of innovation in the Grossman-Helpman(1991b) model.

2 The Model

There are two countries in the world - the North, N, and the South, S; and they are linked by free trade in differentiated products which are invented in the North and imitated in the South. A representative Northern firm incurs an upfront innovation cost to invent a product and then earns a stream of monopoly profits from that product until it gets imitated by a potential Southern firm. Patents are perfectly protected in the North but are imperfectly protected in the South which leads to imitation there. Because of lower labour costs a successful imitator from the South earns an infinite stream of positive profit which it balances against the positive imitation cost. Labour is the only factor of production in each of the two countries. It is used in production as well as in R&D. However, labour is internationally immobile.

2.1 The demand for goods

We consider a world where all the households are identical in terms of preferences irrespective of their origin; and the representative household maximises the intertemporal utility function given by

$$W_i = \int_t^{\infty} e^{-\rho(\tau - t)} log(U_i(\tau)) d\tau$$

subject to the intertemporal budget constraint given by

$$\int_t^{\infty} e^{-r_i(\tau - t)} E_i(\tau) d\tau = \int_t^{\infty} e^{-r_i(\tau - t)} I_i(\tau) d\tau + A_i(\tau) \qquad for\ all\ \ t.$$

Here $E_i(\tau)$, $I_i(\tau)$, $U_i(\tau)$ and $A_i(\tau)$, stand for the instantaneous expenditure, instantaneous income, instantaneous utility and current value of assets at time τ of the representative consumer in the ith region for i= N, S. ρ and r_i stand for the rate of time preference and the nominal interest rate in the ith region respectively. We assume that there is no financial capital mobility between the North and the South. This implies that the trade account of both the economy should balance at every point in time.

The instantaneous utility function is assumed to have the following form:

$$U_i(t) = (\int_0^{n(t)} x_i(z)^{\alpha} dz)^{\frac{1}{\alpha}} \qquad with \quad 0 < \alpha < 1.$$

Here n(t) and $x_i(z)$ stand for the number of products at time t and the amount consumed of the zth variety by the representative consumer in the ith region.

Solving the optimisation problem we obtain the following demand function for zth variety given by

$$x_i(z) = E_i(t) \frac{p(z)^{-\varepsilon}}{\int_0^{n(t)} p(u)^{1-\varepsilon} du} \tag{1}$$

for i=N, S. This is true for all z $\in [0, n(t)]$ and for all t. Here $\varepsilon = \frac{1}{1-\alpha} > 1$, is the constant price elasticity of demand. We will see later that the products produced in the same region are priced equally. Hence the aggregate demand for a product z is given by

$$x(z) = x_N(z) + x_S(z) = (E_N(t) + E_S(t)) \frac{p(z)^{-\varepsilon}}{\int_0^{n(t)} p(u)^{1-\varepsilon} du}.$$

The products $z \in (0, n_N)$ are produced in the North and the products $z \in (0, n_S)$ are produced in the South. So we have the demand function faced by a representative Northern producer given by

$$x_N = (E_N(t) + E_S(t)) \frac{p_N^{-\varepsilon}}{\int_0^{n(t)} p(u)^{1-\varepsilon} du} \qquad for \quad z \in (0, n_N) \tag{2}$$

and that faced by a representative Southern producer given by

$$x_S = (E_N(t) + E_S(t)) \frac{p_S^{-\varepsilon}}{\int_0^{n(t)} p(u)^{1-\varepsilon} du} \qquad for \quad z \in (0, n_S). \tag{3}$$

We also obtain the following optimal time path of expenditure given by

$$\frac{\dot{E}_i}{E_i} = r_i - \rho. \tag{4}$$

Its derivation is omitted because it is similar to that in Grossman-Helpman(1991b).

$$n(t) = n_N(t) + n_S(t) \tag{5}$$

where $n_i(t)$ is the number of products produced in the ith region for i=N,S.

2.2 The North

There are two sectors in the North - the competitive R&D sector and the production sector. In the production sector n_N firms produce n_N differentiated products and each firm is a monopolist on its own product. Labour is the only input used in both the sectors; and there is perfect intersectoral mobility of labour leading to the same wage in equilibrium. In the R&D sector, the blue prints of the new goods are produced.

We assume that the products produced in the South do not contribute to the knowledge capital in the Northern R&D sector. This is the only point by which this model differs from that of Grossman-Helpman(1991b) in which all the products produced in both the countries contribute to the knowledge capital in the Northern R&D sector at equal rates. Thus the production function in the R&D sector in the North takes the following form:

$$\dot{n} = (\frac{n_N}{a_N}) L_N^R \tag{6}$$

where L_N^R and $\frac{a_N}{n_N}$ stand for the level of employment and the per unit labour requirement in the R &D sector. Here $a_N > 0$ is a technological parameter. We can justify this modification in the case of localised knowledge spillovers. Here the externalities come from the presence of different producers in a locality and not from the number of blue prints developed by the R&D sector. R&D sector derives benefit from the interaction with the producers of different goods. These benefits may come from the direct observation of the production process by which the researchers learn how to invent a new good at cheaper cost.

Assuming that one unit of labour is required to produce one unit of product of any brand produced in the North and, then using equation (2), we can express the labour market clearing equation as follows:

$$L_N = a_N(\frac{\dot{n}}{n_N}) + n_N x_N \tag{7}$$

where L_N stand for the labour endowment in the North[5].

The monopoly price and the monopoly profits of the Northern firm producing each of the n_N varieties are given by the following:

$$p_N = \frac{w_N}{\alpha} \tag{8}$$

and

$$\pi_N = \frac{1-\alpha}{\alpha} w_N x_N. \tag{9}$$

Here the Northern wage rate, w_N, is the marginal cost of production of any of the varieties. Let v_N denotes the value of a typical Northern firm. Then the free-entry condition in the R&D sector in the North is given by

$$v_N = \frac{a_N}{n_N} w_N$$

where $\frac{a_N}{n_N} w_N$ is the cost of developing a new product design in the R&D sector. The Northern no-arbitrage condition is given by

$$\frac{\dot{v_N}}{v_N} + \frac{\pi_N}{v_N} = r_N + m. \tag{10}$$

We define the rate of imitation, m, as

$$m = \frac{\dot{n_S}}{n_N} \tag{11}$$

and the fraction of products staying in the North, ξ, as

$$\xi = \frac{n_N}{n}. \tag{12}$$

[5]All the commodities in the North are produced in equal quantities because the utility function is symmetric and the technologies are identical.

We also have the trade balance equation in the North as

$$E_N = p_N n_N x_N. \tag{13}$$

With the definition of R&D sector production function in the North given by (6), we have

$$L_N^R = a_N \frac{\dot{n}}{n} \frac{n}{n_N} = a_N \frac{g}{\xi}$$

where we denote g as

$$g = \frac{\dot{n}}{n}. \tag{14}$$

We define

$$\frac{g}{\xi} = \theta. \tag{15}$$

Then the labour market clearing condition in the North can be modified as

$$L_N = a_N \theta + n_N x_N. \tag{16}$$

We have from the free entry condition

$$v_N = \frac{a_N}{n_N} w_N.$$

Then

$$p_N = \frac{n_N v_N}{a_N \alpha}$$

and

$$\frac{\pi_N}{v_N} = \frac{1-\alpha}{a_N \alpha} n_N x_N = \frac{1-\alpha}{a_N \alpha}(L_N - a_N \theta).$$

Since the equation (13) is satisfied at each point of time we have

$$\frac{\dot{E_N}}{E_N} = \frac{\dot{p_N}}{p_N} + \frac{(n_N \dot{x}_N)}{n_N x_N}.$$

Now using equations (4) and (16) we have[6]

$$\dot{\theta} = [\rho + \theta - \frac{1-\alpha}{\alpha}(\frac{L_N}{a_N} - \theta)](\frac{L_N}{a_N} - \theta). \tag{17}$$

Also from equations (12) and (15) we have[7]

$$\dot{\xi} = (\theta - m - \theta\xi)\xi. \tag{18}$$

These two equations of motion describe the dynamics of the North.

[6]See Appendix(4) for the derivations.

[7]Note that $\frac{\dot{\xi}}{\xi} = \frac{\dot{n_N}}{n_N} - \frac{\dot{n}}{n} = \frac{\dot{n} - \dot{n_S}}{n_N} - g = \theta - m - \theta\xi$, which we obtain using equation (15)

2.3 The South

The South does not innovate but imitates the Northern products. It has a competitive imitative R&D sector and a production sector producing the imitated products. The production function of the imitative R &D sector takes the following form:

$$\dot{n_S} = \frac{n_S}{a_S} L_S^R. \tag{19}$$

Here L_S^R, $\dot{n_S}$ and (a_S/n_S) stand for the amount of labour used in the imitative R&D, the number of new imitated products and the effective labour output coefficient in this sector respectively. This specification is similar to that in Grossman-Helpman(1991b). We assume that one unit of labour in the South can produce one unit of output of any brand produced in the South. Then using equation (13), we can express the labour market clearing equation as follows:

$$L_S = a_S(\frac{\dot{n_S}}{n_S}) + n_S x_S.$$

Here L_S stands for the labour endowment in the South. From equation (11) and (12) we get

$$\frac{\dot{n_S}}{n_S} = m\frac{n_N}{n_S} = m\frac{\xi}{1-\xi}.$$

Hence the labour market clearing equation can be written as

$$L_S = a_S m \frac{\xi}{1-\xi} + n_S x_S. \tag{20}$$

As like North, the monopoly price and the monopoly profits of a typical Southern producer are

$$p_S = \frac{w_S}{\alpha} \tag{21}$$

and

$$\pi_S = \frac{1-\alpha}{\alpha} w_S x_S. \tag{22}$$

Here the Southern wage rate is denoted by w_S. We always assume that $w_N > w_S$.

The Southern firm can maintain its monopoly position on its imitated product if the price charged by him does not exceed the marginal cost of production (wage-rate) in the North. This implies that

$$p_S = \frac{w_S}{\alpha} \leq w_N.$$

This is the *wide gap* assumption. However in the *narrow gap* case, when $\frac{w_S}{\alpha} > w_N$, the Southern firm charges the limit price as

$$p_S = w_N; \tag{23}$$

and then its profit is

$$\pi_S = (w_N - w_S)x_S. \tag{24}$$

Thus in the *narrow gap* case we have

$$\frac{w_S}{\alpha} > w_N > w_S. \tag{25}$$

In Appendix 1, we have shown that the steady-state growth equilibrium in the wide gap case is unstable; and, in Appendix 2, we show that the equilibrium in the narrow gap case is saddle point stable. Hence we are interested in analysing the comparative steady-state effects in the narrow gap equilibrium[8]. The rest of the paper is concerned with the narrow gap equilibrium only.
Let us denote

$$k = \frac{w_N}{w_S} > 1.$$

Here we have, from equations (2) and (3),

$$\frac{x_N}{x_S} = \alpha^{\varepsilon}. \tag{26}$$

Then using equations (26), (16) and (20) and the fact that $\frac{n_S}{n_N} = \frac{1-\xi}{\xi}$ which we obtain from equation (12), we have

$$\frac{L_N - a_N\theta}{L_S - ma_s\frac{\xi}{1-\xi}}\frac{1-\xi}{\xi} = \alpha^{\varepsilon}.$$

This can be reexpressed as

$$m\frac{\xi}{1-\xi} = \frac{L_S}{a_S} - \frac{1-\xi}{\xi}(\frac{L_N}{a_N} - \theta)\frac{a_N\alpha^{-\varepsilon}}{a_S}. \tag{20.1}$$

The free entry condition implies that the value of a typical Southern firm, v_S, is given by

$$v_S = \frac{a_S}{n_S}\frac{w_N}{k};$$

and using equation (23), this above equation can be written as

$$w_N = p_S = \frac{v_S n_S k}{a_S}. \tag{27}$$

The standard no-arbitrage condition in the Southern asset market is given by

$$\frac{\dot{v_S}}{v_S} + \frac{\pi_S}{v_S} = r_S. \tag{27.1}$$

[8]Various comparative static exercises with respect to policy parameters have been performed in Grossman-Helpman(1991b) model in which the steady-state equilibrium is also saddle point stable. In fact Grossman and Helpman(1991c) did not investigate the stability property of the steady-state growth equilibrium but did the comparative steady-state exercises. It is Mondal(2005) who has proved this saddle point stability property in the Grossman-Helpman(1991c) model.

Using equations (24), (20) and (27) we have

$$\frac{\pi_S}{v_S} = \frac{(w_N - w_S)x_S}{\frac{a_S}{n_S}w_S} = \frac{k-1}{a_S}(L_S - a_S m \frac{\xi}{1-\xi}). \tag{24.1}$$

Now from the trade balance equation of the South given by,

$$E_S = p_S n_S x_S,$$

we have,

$$\frac{\dot{E_S}}{E_S} = \frac{\dot{p_S}}{p_S} + \frac{(\dot{n_S x_S})}{n_S x_S}. \tag{28}$$

Using equations (4) and (20) and (28), we have

$$r_S - \rho = \frac{\dot{v_S}}{v_S} + \frac{\dot{n_S}}{n_S} + \frac{\dot{k}}{k} - a_S \frac{(\dot{m\frac{\xi}{1-\xi}})}{L_S - m a_s \frac{\xi}{1-\xi}};$$

which can be further simplified as[9]

$$\dot{k} = k^2 \frac{a_N}{a_S}\alpha^{-\varepsilon}(\frac{L_N}{a_N} - \theta)\frac{1-\xi}{\xi} - k[\frac{L_S}{a_S} + \rho] + [\frac{k\dot{\theta}}{\frac{L_N}{a_N} - \theta} + \frac{k\dot{\xi}}{\xi(1-\xi)}]. \tag{29}$$

Then replacing the value of m in terms of ξ and θ from equation (20.1) in equation (18) we have

$$\dot{\xi} = (1-\xi)[\xi\theta - \frac{L_S}{a_S} + \frac{1-\xi}{\xi}(\frac{L_N}{a_N} - \theta)\frac{a_N \alpha^{-\varepsilon}}{a_S}]. \tag{30}$$

3 The Steady-State Equilibrium

3.1 Uniqueness of Equilibrium

Equations (17), (29) and (30) are three equations of motion describing the world economy. The steady-state equilibrium point of this system could be obtained by putting $\dot{\theta} = \dot{k} = \dot{\xi} = 0$ and then solving for θ^*, k^* and ξ^* [10]. The steady-state equilibrium system of equations are

$$\theta^* = (1-\alpha)\frac{L_N}{a_N} - \rho\alpha, \tag{17.1}$$

$$k^* a_N \alpha^{-\varepsilon}(\frac{L_N}{a_N} - \theta^*)\frac{1-\xi^*}{\xi^*} = L_S + \rho a_S, \tag{29.1}$$

and

$$a_N \alpha^{-\varepsilon}(\frac{L_N}{a_N} - \theta^*)\frac{1-\xi^*}{\xi^*} = L_S - \theta^* \xi^* a_S. \tag{30.1}$$

[9] See Appendix 3 for the detail derivation.

[10] Superscript * of a variable denotes its steady-state value. This notation is followed everywhere in the rest of this paper.

The steady-state equilibrium value of m can be determined from the following equation

$$m^* = \frac{1-\xi^*}{\xi^*}\{L_S - \frac{1-\xi^*}{\xi^*}(\frac{L_N}{a_N} - \theta^*)a_N\alpha^{-\varepsilon}\}\frac{1}{a_S} = \theta^*(1-\xi^*). \tag{31}$$

where the last equality follows from equation (30.1). Also, using equation (29.1) and (30.1) we get

$$(k^* - 1)L_S = \rho a_S + k^*\theta^*\xi^* a_S. \tag{32}$$

We first show that a unique steady-state equilibrium point exists. From equation (17.1), θ^* is determined uniquely. From equation (29.1) a unique k^* is determined given θ^* and ξ^*; and, from equation (31), a unique m^* is determined. So the existence of a unique solution is ensured if equation (30.1) can solve for ξ^* uniquely in terms of θ^* satisfying $0 \le \xi^* \le 1$. In fact, it is ensured and we show it using the Figure 1. The Left Hand Side (LHS) of equation (30.1) is shown by the AA curve which slopes negatively being asymptotic to the vertical axis and meeting the horizontal axis at $\xi = 1$. The Right Hand Side (RHS) of this equation is shown by the negatively sloped BB curve. It meets the vertical axis because L_S is finite and also meets the horizontal axis at $\xi = \frac{L_S}{\theta^* a_S} > 1$[11]. So the two curves must intersect at only one point satisfying $0 < \xi^* < 1$.

—insert Figure 1 here—

3.2 The Comparative Steady-State Effects

3.2.1 Strengthening IPR

We now perform some of the comparative static exercises around this steady-state equilibrium. First we want to study the effects of a tighter IPR protection by the South. We consider

$$a_S = a_m + \lambda$$

where a_m is the technology parameter and λ is a policy parameter representing the degree of strengthening the IPR protection in the South. The stronger the IPR, the greater is the value of λ ; and hence the greater is the effective labour requirement[12] in the imitative R &D sector. Thus a tighter IPR protection in the South increases the value of a_S.

[11] A sufficient assumption for this to happen is $\frac{L_S}{a_S} > \frac{L_N}{a_N}$. In Appendix 2 we have shown that this is also sufficient to ensure the local saddle point stability of the steady-state equilibrium in the narrow gap case.

[12] The increase in the labour requirement means the increase in the cost of imitation and strengthening IPR means the increase in the cost of imitation. We follow Glass and Saggi(2002) for this kind of definition of IPR tightening in the South.

Equation (17.1) shows that θ^* is independent of a_S. From figure 1, an increase in a_S causes the BB curve to shift downward. However, the AA curve remains unchanged. Thus ξ^* increases in the new equilibrium. As ξ^* increases, the LHS of equation (29.1) decreases for given k^*. The RHS of this equation increases due to the increase in a_S. Thus k^* is to increase in the new equilibrium. Equation (31) implies that m^* decreases due to an increase in a_S. Also, g^* $(= \theta^*\xi^*)$ will increase in the new steady-state equilibrium. Hence we can establish the following proposition:

Proposition 1 *A policy of strengthening IPR protection in the South raises the rate of innovation and the proportion of unimitated products in the North and lowers the rate of imitation in the South and the South-North relative wage in the new steady-state growth equilibrium.*

The result is important because it is not fully similar to that obtained in Grossman-Helpman(1991b). The effect on the rate of imitation in the South and on the North-South wage gap are qualitatively similar in both the cases. However, the effect on the rate of innovation is negative in Grossman-Helpman(1991b) and is positive in ours.

The intuition of the results are as follows. The strengthening of IPR implies a rise in a_S. An increase in a_S makes imitation costly in the South. So the rate of imitation is decreased which implies that smaller number of products are getting imitated at each point in time, given n. So the fraction of the imitated products produced in the South falls. As North now produces a higher fraction of products its demand for labour in the production sector is increased and the opposite happens in the South. This leads to an increase in the North-South relative wage because labour is internationally immobile. Also, as the rate of imitation falls, the cost of capital of a Northern firm is reduced. However the profit-rate remains unchanged because both the instantaneous profit and the cost of blue print fall at equal rates. So the incentive to innovate in the North increases; and this leads to an increase in the rate of innovation there. In Grossman-Helpman(1991b), a decrease in the imitation rate lowers the profit rate at a higher proportion than the reduction in the cost of capital of a typical Northen firm. This generates negative incentive to innovate in the North and hence the rate of innovation falls.

3.2.2 Change in labour endowment

The increase in the Southern labour force, L_S, has no impact on θ^*. In Figure 1, BB curve shifts upward and AA curve remains unchanged when L_S rises. This decreases the value of ξ^* in the new equilibrium. Now equation (32) shows that k^* falls in the new equilibrium. Also, $g^* = \theta^*\xi^*$ is decreased. Equation (31) shows that m^* rises.

Equation (17.1) shows that an increase in L_N raises θ^* as well as $(\frac{L_N}{a_N} - \theta^*)$ because $0 < \alpha < 1$. This causes the AA curve to shift upward while the BB curve remains the same. This raises ξ^* in the new equilibrium. k^* is increased while the effect on m^* is not unambiguous. $g^* = \theta^* \xi^*$ must go up. We can now summarize these effects in the following proposition:

Proposition 2 *The increase in the size of the labour endowment in the South (North) lowers (raises) the rate of innovation and the proportion of unimitated products and the North-South relative wage and produces positive (indeterminate) effect on the rate of imitation in the South in the new steady-state growth equilibrium.*

The intuition of the results are as follows. An increase in the Southern labour endowment raises its availability to both the production sector and the imitative R&D sector there. This leads to an increase in the rate of imitation and in the fraction of products being manufactured in the South. As the rate of imitation is increased the incentive to innovate in the North falls leading to a decrease in the rate of innovation. Also, the relative wage of the South over North rises because of the higher demand for labour from the Southern production sector caused by the increase in the share of products produced in the South.

Similarly, an increase in the Northern labour endowment makes more labour available to both of its R&D sector and the production sector. This increases the share of products produced in the North and the size of its R&D sector (measured by $a_N \theta^*$ in the steady-state). Increased R&D sector's size leads to an increase in the rate of innovation. Since the share of products produced in the North is increased, an increase in the demand for labour takes place in the Northern labour market. So the relative wage of North over South rises. However, the effect on the rate of imitation is ambiguous[13].

The results regarding the relationship between the region's size of its labour endowment and its relative wage are consistent with that of Grossman-Helpman(1991b). However, the effect of the increase in the Southern labour endowment on the rate of innovation is just the opposite here to that obtained in Grossman-Helpman(1991b). This is so because there is a negative relationship between m and g in this model; and the intuition of this has already been explained in the discussion of proposion 1. Also in our model the effect of the increase in the Northern labour endowment on the rate of imitation is ambiguous which is not so in the Grossman-Helpman(1991b) model.

4 Transitional Dynamics

We now analyse how the variables behave in transition from one steady-state to another with respect to the once for all change in the parameters. Though Helpman(1993) analysed these comparative transitional effects in his exogenous imitation

[13]Since in the steady-state $m^* = \theta^*(1 - \xi^*)$ and both θ^* and ξ^* increases, effect on m^* is not clear.

model, Grossman-Helpman (1991c) and no other works on dynamic endogenous imitation models have done similar exercises. In Appendix 2, we have derived the general solution of the linearised version of the differential equations (17), (29) and (30) along the saddle path. These are given by

$$\theta(t) = \theta^*, \tag{17.2}$$

$$\xi(t) = \xi^* - [\xi^* - \xi(0)]e^{a_{22}t}, \tag{29.2}$$

and

$$k(t) = k^* + [\xi^* - \xi(0)]e^{a_{22}t}(\frac{a_{32}}{a_{33} - a_{22}}). \tag{30.2}$$

Here

$$a_{22} = \frac{\partial \dot{\xi}}{\partial \xi}]_{(\theta^*,\xi^*,k^*)} = [-(\frac{L_S}{a_S} - \theta^*) - \frac{1}{k^*}(\frac{L_S}{a_S} + \rho)(\frac{1-\xi^*}{\xi^*})] < 0,$$

$$a_{33} = \frac{\partial \dot{k}}{\partial k}]_{(\theta^*,\xi^*,k^*)} = [\rho + \frac{L_S}{a_S}] > 0,$$

and

$$a_{32} = \frac{\partial \dot{k}}{\partial \xi}]_{(\theta^*,\xi^*,k^*)} = \frac{k^*}{\xi^*(1-\xi^*)}[-k^*\frac{a_N}{a_S}\alpha^{-\varepsilon}(\frac{L_N}{a_N} - \theta^*)\frac{1-\xi^*}{\xi^*} + \frac{\partial \dot{\xi}}{\partial \xi}] < 0.$$

4.1 Strengthening IPR

We assume that the economies are initially in the steady-state; and then analyse the effects of parametric change on its transitional behaviour. Note that if $\xi(0) = \xi^*$, then the entire system is in steady-state initially. The first order response of tightening of IPR can be evaluated at $\xi(0) = \xi^*$ as

$$\frac{d\theta(t)}{da_S} = 0, \tag{33}$$

$$\frac{d\xi(t)}{da_S} = (1 - e^{a_{22}t})\frac{d\xi^*}{da_S}, \tag{34}$$

and,

$$\frac{dk(t)}{da_S} = \frac{dk^*}{da_S} + \frac{d\xi^*}{da_S}e^{a_{22}t}(\frac{a_{32}}{a_{33} - a_{22}}). \tag{35}$$

Here $\frac{d\xi^*}{da_S} > 0$; and hence

$$\frac{d\xi(0)}{da_S} = 0,$$

and

$$\frac{d\xi(t)}{da_S} > 0$$

for any $t > 0$. Also, using equation (29.1), it can be shown that

$$(\frac{a_{32}}{a_{33} - a_{22}}) = \frac{\frac{k^*}{\xi^*(1-\xi^*)}[-k^*\frac{a_N}{a_S}\alpha^{-\varepsilon}(\frac{L_N}{a_N} - \theta^*)\frac{1-\xi^*}{\xi^*} + \frac{\partial \dot{\xi}}{\partial \xi}]}{[\rho + \frac{L_S}{a_S}] - [\frac{\partial \dot{\xi}}{\partial \xi}]} = -\frac{k^*}{\xi^*(1-\xi^*)}; \tag{36}$$

and, differentiating (29.1) with respect to a_S, we have

$$\frac{1-\xi^*}{\xi^*}[\frac{dk^*}{da_S} - \frac{k^*}{\xi^*(1-\xi^*)}\frac{d\xi^*}{da_S}]a_N\alpha^{-\varepsilon}(\frac{L_N}{a_N} - \theta^*) = \rho. \tag{36.1}$$

Then, using equations (35) and (36), we have

$$\frac{dk(t)}{da_S} = \frac{dk^*}{da_S} - \frac{k^*}{\xi^*(1-\xi^*)}\frac{d\xi^*}{da_S}e^{a_{22}t};$$

and, from the above equation, using equation (36.1), we obtain

$$\frac{dk(0)}{da_S} = [\frac{dk^*}{da_S} - \frac{k^*}{\xi^*(1-\xi^*)}\frac{d\xi^*}{da_S}] = \frac{\rho}{a_N\alpha^{-\varepsilon}(\frac{L_N}{a_N} - \theta^*)\frac{1-\xi^*}{\xi^*}} > 0.$$

Also as $t \to \infty$, $\frac{dk(t)}{da_S} \to \frac{dk^*}{da_S}$. Now we can describe how ξ and k will behave during transition from one steady state to another due to an increase in a_S. As a_S increases, k jumps from point A to the new saddle path and then increases over time to reach the new steady-state equilibrium at point B. ξ increases steadily from A to B. Since θ^* is constant for all t, g= $\theta^*\xi$ increases proportionately with ξ and $m = \theta(1-\xi)$ decreases with ξ. Since we are in the narrow gap case, we need to assume that, $1 < k < \frac{1}{\alpha}$, is satisfied at the new equilibrium point B, otherwise we shall violate the narrow gap condition. However, we can establish the following proposition from this comparative dynamic analysis.

Proposition 3 *As IPR gets stronger once for all, Northern relative wage initially overshoots on impact and then increases steadily to reach the new steady state while the proportion of unimitated products and the rate of innovation increases steadily over time along the transition path.*

—insert Figure 2 here—

4.2 Change in Labour Endowment

Differentiating the equations (17.2), (29.2) and (30.2) with respect to L_S and using the initial condition $\xi(0) = \xi^*$, we get

$$\frac{d\theta(t)}{dL_S} = 0, \tag{37}$$

$$\frac{d\xi(t)}{dL_S} = (1 - e^{a_{22}t})\frac{d\xi^*}{dL_S}, \tag{38}$$

and

$$\frac{dk(t)}{dL_S} = \frac{dk^*}{dL_S} - \frac{d\xi^*}{dL_S}e^{a_{22}t}(\frac{k^*}{\xi^*(1-\xi^*)}). \tag{39}$$

From the comparative steady-state exercises we have

$$\frac{d\xi^*}{dL_S} < 0 \ \ and \ \ \frac{dk^*}{dL_S} < 0.$$

Differentiating equation (29.1) with respect to L_S we have

$$\frac{1-\xi^*}{\xi^*} a_N \alpha^{-\varepsilon} (\frac{L_N}{a_N} - \theta^*)[\frac{dk^*}{dL_S} - \frac{k^*}{\xi^*(1-\xi^*)} \frac{d\xi^*}{dL_S}] = 1.$$

So the Right Hand Side of equation (39) is positive at $t = 0$ which implies that $\frac{dk(0)}{dL_S} > 0$. Equation (38) clearly shows that $\frac{d\xi(0)}{dL_S} = 0$. However, $a_{22} < 0$ and this means that $\frac{dk(t)}{dL_S} < 0$ for all $t > T_1 > 0$; and $\frac{d\xi(t)}{dL_S} < 0$ for all $t > 0$. In transition from A to B, $k(t)$ first jumps to reach the new saddle path given $\xi(0) = \xi^*$; and then decreases to B along the saddle path. However, $\xi(t)$ falls steadily over time; and so $g(t)$ also falls proportionately. This leads to the following proposition

Proposition 4 *As the size of the Southern labour endowment is increased once for all, the Northern relative wage rises initially and then falls to reach the new steady-state while the proportion of unimitated products and the rate of innovation falls steadily over time along the transitional path.*

—insert Figure 3 here—

Differentiating equations (17.2), (29.2) and (30.2) with respect to L_N and using the initial condition $\xi(0) = \xi^*$, we have

$$\frac{d\theta(t)}{dL_N} = \frac{1-\alpha}{a_N} > 0\,, \tag{40}$$

$$\frac{d\xi(t)}{dL_N} = (1 - e^{a_{22}t})\frac{d\xi^*}{dL_N}\,, \tag{41}$$

and

$$\frac{dk(t)}{dL_N} = \frac{dk^*}{dL_N} - \frac{d\xi^*}{dL_N} e^{a_{22}t} (\frac{k^*}{\xi^*(1-\xi^*)}). \tag{42}$$

From the comparative steady-state exercises we have

$$\frac{d\xi^*}{dL_N} > 0, \ \ \frac{dk^*}{dL_N} > 0 \ \ and \ \ \frac{dg^*}{dL_N} > 0.$$

Differentiating equation (29.1) with respect to L_N we have

$$\frac{1-\xi^*}{\xi^*} a_N \alpha^{-\varepsilon} (\frac{L_N}{a_N} - \theta^*)[\frac{dk^*}{dL_N} - \frac{k^*}{\xi^*(1-\xi^*)} \frac{d\xi^*}{dL_N}] + k^* \frac{1-\xi^*}{\xi^*} \alpha^{1-\varepsilon} = 0.$$

Using this equation and equations (41) and (42) we have

$$\frac{dk(0)}{dL_N} < 0 \ \ and \ \ \frac{d\xi(0)}{dL_S} = 0.$$

However, $a_{22} < 0$; and this means that $\frac{dk(t)}{dL_N} > 0$ for all $t > T_2 > 0$ and $\frac{d\xi(t)}{dL_N} > 0$ for all $t > 0$. Evaluating $g(t) = \theta(t)\xi(t)$ at $t = 0$ and then differentiating this with respect to L_N and using the initial condition, $\xi(0) = \xi^*$, we have

$$\frac{dg(0)}{dL_N} = \frac{d\theta^*}{dL_N}\xi^* = \xi^* \frac{1-\alpha}{a_N} > 0 \quad and \quad \frac{dg(t)}{dL_N} > 0\,,\; for\, all\; t.$$

We can now explain how $g(t)$, $k(t)$ and $\xi(t)$ behave in the transitional phase following an increase in L_N. $g(t)$ rises on impact and then increases steadily over time to attain the new steady-state equilibrium. $k(t)$ decreases on impact and then increases steadily to reach the new equilibrium. $\xi(t)$ increases steadily. This leads us to the following proposition:

Proposition 5 *As the size of the Northern labour endowment is increased once for all, (i) the Northern relative wage decreases initially and then rises to reach the new steady-state, (ii) the proportion of unimitated products increases steadily over time and (iii) the rate of innovation over-shoots on impact and then increases steadily over time.*

From propositions 4 and 5 it now follows that, in the short-run, the relative wage of a region varies inversely with the size of the labour endowment of that region and directly with that of the other region. However, in the long-run, it varies directly with its size and inversely with that of the other. Grossman and Helpman(1991c) have found that the relative wage of a region varies directly with the size of that region and inversely with the size of the other region. In their own words, "This result may be surprising to readers versed in the neo-classical growth model, and it stands in sharp contrast to the findings reported by Krugman(1979)" (Grossman-Helpman(1991c), page 304, last paragraph). However, Grossman and Helpman(1991c) have dealt with the comparative steady-state properties only; and did not analyse the transitional dynamic properties. We do the transitional dynamic analysis in the narrow gap equilibrium in this modified Grossman-Helpman(1991c) model and show that it is possible to reconcile both Krugman(1979) result and Grossman-Helpman(1991c) result in terms of the difference between short-run and long-run effects obtained from this model. Thus the short-run impact on the relative wage is consistent with the result of Krugman(1979); and the long-run impact is in the line with that of Grossman-Helpman(1991c)[14].

Our result regarding the impact of a change in the region's size of labour endowment on its relative wage is consistent with that of Dollar(1986). He uses a dynamic general equilibrium model of North-South trade and has shown that the short-run effect

[14] It is worthwhile to report one result of Lai(1995) in this context. Lai(1995), using a product-variety endogenous growth model like Grossman-Helpman(1991c), finds that an increase in the supply of unskilled labor in a country lowers its steady-state equilibrium relative wage while an increase in supply of skilled labor in a country raises its steady-state equilibrium relative wage when the elasticity of substitution between the goods is sufficiently high. However, Lai(1995) deals with the comparative steady-state effects only.

of an increase in the Southern labour force is to raise the relative wage of the North by increasing the demand for Northern products. He mentioned this as the "classical result". However, in the long-run, relative wage of the North is decreased in his model by accelerating the transfer of technology and capital flow from the North to the South. We do not have capital in our model as another factor of production. However, it is the faster imitation rate in the South that gradually increases the share of products produced by the South; and hence this leads to a higher Southern relative wage in the long run.

—insert Figure 4 here—

5 Welfare

Our analysis in this section is similar to that in Helpman(1993). The instantaneous utility function of the representative individual of the ith region is given by

$$U_i(t) = E_i[n_N p_N^{1-\varepsilon} + n_S p_S^{1-\varepsilon}]^{\frac{1}{\varepsilon-1}} = E_i[p_N^{-1} n^{\frac{1}{\varepsilon-1}} \{\xi + (1-\xi)\alpha^{1-\varepsilon}\}^{\frac{1}{\varepsilon-1}}]$$

for $i = N, S$. Here E_i represents the per capita income in the ith region with

$$E_N = \frac{p_N n_N x_N}{L_N} = p_N(1 - \frac{a_N \theta}{L_N})$$

and

$$E_S = \frac{p_S n_S x_S}{L_S} = p_S(1 - \frac{a_S m^{\frac{\xi}{1-\xi}}}{L_S}) = p_S\{\frac{1-\xi}{\xi}(\frac{L_N}{a_N} - \theta)\frac{a_N}{L_S}\alpha^{-\varepsilon}\}.$$

Using the above set of equations we can write

$$log(U_N(t)) = log(1 - \frac{a_N\theta}{L_N}) + \frac{1}{\varepsilon-1}log(n(t)) + \frac{1}{\varepsilon-1}log\{\xi + (1-\xi)\alpha^{1-\varepsilon}\} \quad (43)$$

and

$$log(U_S(t)) = log\{\frac{1-\xi}{\xi}(\frac{L_N}{a_N} - \theta)\frac{a_N}{L_S}\alpha^{-\varepsilon}\} + \frac{1}{\varepsilon-1}log(n(t)) + \frac{1}{\varepsilon-1}log\{\xi + (1-\xi)\alpha^{1-\varepsilon}\} + log(\alpha). \quad (44)$$

From equations (43) and (44) we have

$$log(U_S(t)) - log(U_N(t)) = log(\frac{1-\xi}{\xi}\frac{L_N}{L_S}) - (\varepsilon - 1)log(\alpha). \quad (45)$$

Equation (45) implies that the relative instantaneous utility of a representative individual in any region depends on the relative size of its labour endowment, interregional allocation of manufacturing goods and on the taste parameter which in turn determines the monopoly power of a Northern producer. Various parametric changes affect the relative instantaneous utility through the endogenous variable, ξ.

The welfare, discounted at period 0, of a representative individual in the ith region is given by

$$W_i(0) = \int_0^\infty e^{-\rho t} log(U_i(t))dt$$

for i= N, S. Using equation (45) we have

$$W_N(0) - W_S(0) = \int_0^\infty e^{-\rho t}[(\varepsilon - 1)log(\alpha) - log(\frac{1-\xi}{\xi}\frac{L_N}{L_S})]dt. \quad (46)$$

5.1 Strengthening IPR

We assume that the economies are initially at the steady-state equilibrium. Then differentiating equation (46) with respect to a_S and evaluating it at the steady state we have

$$\frac{dW_N(0)}{da_S} - \frac{dW_S(0)}{da_S} = \frac{1}{\xi^*(1-\xi^*)}\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho(\rho - a_{22})} > 0. \quad (47)$$

Here $\frac{dW_i(0)}{da_S}$ is the marginal welfare change in the ith region (i=N, S) due to strengthening of IPR in the South. So North has a higher marginal welfare gain than South in this case.

In our present model with endogeous imitation and localised knowledge spillover, the absolute welfare effects on each of the two regions are ambigous. From equation (44), it can be shown that[15]

$$\frac{dW_S(0)}{da_S} = -\frac{1}{\xi^*(1-\xi^*)}\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho(\rho - a_{22})} + \frac{1}{\varepsilon - 1}\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho(\rho - a_{22})}[\frac{\theta^*}{\rho} + \frac{1-\alpha^{1-\varepsilon}}{\xi^* + (1-\xi^*)\alpha^{1-\varepsilon}}]$$

where the first term in the RHS is negative and represents the marginal welfare loss through the endogenous reallocation of intertemporal R&D expenditure, the second term is positive and represents the marginal welfare gain through the availability of greater variety of products and the third term is negative and it represents the marginal welfare loss through the inter-regional allocation of production. Similarly, from equation (43), it can be shown that

$$\frac{dW_N(0)}{da_S} = \frac{1}{\varepsilon - 1}\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho(\rho - a_{22})}[\frac{\theta^*}{\rho} + \frac{1-\alpha^{1-\varepsilon}}{\xi^* + (1-\xi^*)\alpha^{1-\varepsilon}}]$$

where the first term in the RHS is positive and represents the marginal welfare gain through the availability of greater variety of products and the third term is negative and it represents the marginal welfare loss through the inter-regional allocation of production. Both of these expressions may have positive signs. So both the countries may have welfare gain in this case. Now using this result and the results summarised in equation (47) we have the following proposition:

[15]Derivations are done in Appendix 5

Proposition 6 *Both South and North may gain in welfare due to the policy of strengthening IPR and the North always has a higher marginal welfare gain than the South in this case.*

This result is interesting because it is different from that in Helpman(1993). In an exogenous imitation model with globalised knowledge spillover Helpman(1993) has shown that the South always loses due to stronger IPR protection there. However, North may or may not gain in his model. This difference arises because, in this present model, the policy of strengthening IPR raises the steady-state equilibrium rate of growth in both the countries while, in Helpman(1993), this rate of growth is reduced. Hence, in this modified model, both North and South gain in welfare due to increased variety in consumption. If this positive effect outweighs the negative effect of inter-regional allocation of production and intertemporal reallocation of R&D expenditure, then there is net welfare gain of each of the two countries. There is no terms of trade effect in this narrow gap equilibrium case. North derives higher marginal welfare gain because there is no intertemporal reallocation of R&D expenditure there; and hence the welfare of the North is not affected through this channel which causes welfare loss to the South.

5.2 Change in labour endowment

The welfare effect in the ith region with respect to the change in labour endowment of the jth region can be derived as

$$\frac{dW_i(0)}{dL_j} = \int_0^{\infty} e^{-\rho t}[\frac{dlog(U_i(t))}{dL_j}]dt = \frac{1}{\varepsilon - 1}(\Delta_N^{L_j} + \Delta_e^{L_j}) + \Delta_s^{iL_j} \tag{48}$$

for i, j=N, S. Here $\Delta_N^{L_j}$ and $\Delta_e^{L_j}$ capture the welfare effect through a change in the variety in consumption and the welfare effect through a change in the inter-regional allocation of production respectively due to change in the jth region's labour endowment. $\Delta_s^{iL_j}$ represents the welfare effect through a change in the ith region's worker's savings rate due to change in the jth region's labour endowment. In Appendix (6) we have shown that

$$\begin{aligned}
\Delta_N^{L_j} = \theta^* \frac{-a_{22}}{\rho^2(\rho - a_{22})} \frac{d\xi^*}{dL_j} &> 0 \text{ and } < 0 \quad \text{for j=N and j=S respectively,} \\
\Delta_e^{L_j} = \frac{1-\alpha^{1-\varepsilon}}{\xi^* + (1-\xi^*)\alpha^{1-\varepsilon}} \frac{1}{\rho} \frac{d\xi^*}{dL_j} &< 0 \text{ and } > 0 \quad \text{for j=N and j=S respectively,} \\
\Delta_s^{NL_j} &= 0 \text{ and } < 0 \quad \text{for j= S and j=N respectively} \\
\textit{and} & \\
\Delta_s^{SL_j} &= \text{is ambiguous in sign for j= S and j= N}
\end{aligned}$$

From the above mentioned expressions we can say that both the direct marginal effect and the cross marginal effect on welfare with respect to change in labour endowment in either region may take any sign. So both North and South may either gain or lose in welfare.

Helpman(1993) did not analyse the welfare effect of changes in factor endowments in his model. However, one can show that, in the Helpman(1993) model, an increase in the size of the labour endowment in the North increases the rate of innovation, North-South relative wage, the fraction of unimitated products and the savings rate in the steady-state equilibrium. Out of these four effects the first two causes welfare gain and the last two causes welfare loss in the case of North; and the South gains in welfare only due to the first effect and faces welfare loss due to the second and the third effect. The fourth effect does not apply to the South because imitation is costless and exogenous in the South in the Helpman(1993) model. Hence the net welfare effects on both the North and the South are ambiguous in his model.

However, an increase in the Southern labour endowment only increases the North-South relative wage and does not affect any other variable in the Helpman(1993) model. This clearly increases the Northern welare and decreases the Southern welfare. Thus the welfare of the South (North) varies inversely (directly) with the size of the Southern labour endowment in the Helpman(1993) model. Our analysis is important because we have shown that this inverse relationship between the size of the labour endowment of the South and its welfare and the direct relationship between the size of the labour endowment of the South and Northern welfare are not necessarily valid once we endogenise the rate of imitation and introduce localised knowledge spillover in the Helpman(1993) model.

6 Conclusion

This paper modifies the Grossman-Helpman(1991b) model assuming that the stock of knowledge capital in the North is measured by the number of firms currently producing there. This is the case of localised knowledge spillover whose empirical supports are provided by Glaeser et al.(1992), Henderson et al.(1995) and Jacobs(1969) and which has been introduced in the theoretical models of Dollar(1986, 1987), Martin and Ottaviano (1999), Baldwin et. al.,(2001) etc.

This modified model is technically less complicated than the original Grossman-Helpman (1991b) model. So we have been able to derive the stability properties of the steady-state equilibrium of this modified model. For example, we have shown that the steady-state growth equilibrium is locally unstable in the wide gap case and is saddle-point stable in the narrow gap case with a unique saddle path converging to the steady-state equilibrium point. Grossman and Helpman (1991b) have not derived any stability property of the long-run equilibrium in their model. Helpman(1993) has derived similar stability property in his model. However, he assumed exogenous imitation rate.

We have analysed the various transitional dynamic properties of the model which Grossman and Helpman (1991b) did not do. We have shown how the behaviour of the transitional path from one steady-state equilibrium to the other is sensitive to the once for all parametric changes. Helpman(1993) performed similar exercises in his exogenous imitation model. However, this is the pioneering attempt to derive the transitional dynamic effects of the parametric changes in an endogenous imitation model. Our ability to derive the transitional dynamic effects helps us to derive the welfare effects in both the regions with respect to the once for all parametric change.

Our results have significantly different implications in the context of the relevance of the policy of strengthening IPR in the South. Such a policy should be justified if it leads to welfare gain in both the countries. In Helpman(1993) this policy leads to a welfare loss in the South. Grossman and Helpman (1991b) did not analyse the welfare effect. However, in the present model, this policy may lead to a welfare gain in both the countries because it has a positive comparative steady-state effect on the rate of growth, which in turn may cause a positive welfare effect through the availability of greater variety of products if it outweighs the combined negative effect of inter-regional allocation of production and of the intertemporal R&D expenditure[16]. In Helpman(1993) model as well as in the Grossman-Helpman(1991b) model, we find a negative comparative steady-state effect on the rate of growth.

However, our analysis is subject to all other limitations common to Grossman-Helpman(1991b) model. For example it is subject to Jones'(1995a) critique of scale effect[17]. We have not considered the case of innovation in the South[18]. We have not considered imitation through multinationalisation of the Northern firm[19]. Also we have not introduced the North-South labour mobility[20] in this model. Simultaneous consideration of all these issues will make the model highly complicated.

[16]The effect throuth the intertemporal R&D expenditure is valid only in the case of South.

[17]For models with R&D technologies and without scale effects, see Jones (1995b), Segerstrom (1998), and Arnold (1998).

[18]This has been considered by Arnold (2003)

[19]This has been considered by Lai(1998) in an exogenous imitation model.

[20]This has been considered by Mondal and Gupta(2005).

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Appendix

Appendix(1)

In this appendix, we shall prove that the steady-state equilibrium in the wide gap case is unstable. The full derivation of the dynamic equations is not shown. This is available from the authors on request. Dynamic equations are

$$\dot{\theta} = [\rho + \theta - \frac{1-\alpha}{\alpha}(\frac{L_N}{a_N} - \theta)](\frac{L_N}{a_N} - \theta), \qquad (A1)$$

$$\dot{m}\frac{\xi}{1-\xi} = (\rho + m\frac{\xi}{1-\xi})(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}) - \frac{1-\alpha}{\alpha}(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})^2 - \frac{m}{(1-\xi)^2}\{\xi\theta - (\xi\theta + m)\xi\}, \qquad (A2)$$

and

$$\dot{\xi} = \xi\theta - (\xi\theta + m)\xi. \qquad (A3)$$

Steady state system of equations are

$$\rho + \theta - \frac{1-\alpha}{\alpha}(\frac{L_N}{a_N} - \theta) = 0, \qquad (A1.1)$$

$$\rho + m\frac{\xi}{1-\xi} - \frac{1-\alpha}{\alpha}(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}) = 0, \qquad (A2.1)$$

and

$$\theta = \frac{m}{1-\xi}. \qquad (A3.1)$$

Linearising the above system of equations (A1), (A2) and (A3) around their steady-state equilibrium point we obtain

$$\begin{bmatrix} \dot{m} \\ \dot{\xi} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{m}}{\partial m}]_{(m^*,\xi^*,\theta^*)} & \frac{\partial \dot{m}}{\partial \xi}]_{(m^*,\xi^*,\theta^*)} & \frac{\partial \dot{m}}{\partial \theta}]_{(m^*,\xi^*,\theta^*)} \\ \frac{\partial \dot{\xi}}{\partial m}]_{(m^*,\xi^*,\theta^*)} & \frac{\partial \dot{\xi}}{\partial \xi}]_{(m^*,\xi^*,\theta^*)} & \frac{\partial \dot{\xi}}{\partial \theta}]_{(m^*,\xi^*,\theta^*)} \\ \frac{\partial \dot{\theta}}{\partial m}]_{(m^*,\xi^*,\theta^*)} & \frac{\partial \dot{\theta}}{\partial \xi}]_{(m^*,\xi^*,\theta^*)} & \frac{\partial \dot{\theta}}{\partial \theta}]_{(m^*,\xi^*,\theta^*)} \end{bmatrix} \cdot \begin{bmatrix} m(t) - m^* \\ \xi(t) - \xi^* \\ \theta(t) - \theta^* \end{bmatrix} \qquad (A4)$$

All the following derivatives are evaluated at the steady-state equilibrium values given by θ^*, m^* and ξ^*.

$$\frac{\partial \dot{m}}{\partial m}]_{(m^*,\xi^*,g^*)} = (\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})\frac{1}{\alpha} + \frac{m}{1-\xi}$$

$$\frac{\partial \dot{m}}{\partial \xi}]_{(m^*,\xi^*,g^*)} = (\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})\frac{m}{\alpha}\frac{1}{\xi(1-\xi)} + \frac{\theta m}{1-\xi}$$

$$\frac{\partial \dot{\xi}}{\partial \xi}]_{(m^*,\xi^*,\theta^*)} = -\theta\xi \quad ; \qquad \frac{\partial \dot{\xi}}{\partial m}]_{(m^*,\xi^*,\theta^*)} = -\xi$$

$$\frac{\partial\dot{\theta}}{\partial\theta}]_{(m^*,\xi^*,\theta^*)} = (\frac{L_N}{a_N} - \theta)\frac{1}{\alpha} \quad ; \qquad \frac{\partial\dot{\theta}}{\partial\xi}]_{(m^*,\xi^*,\theta^*)} = \frac{\partial\dot{\theta}}{\partial m}]_{(m^*,\xi^*,\theta^*)} = 0$$

Let us denote the matrix of the right hand side of (A4) as C. Then we have

$$C = \begin{bmatrix} (\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})\frac{1}{\alpha} + \frac{m}{1-\xi} & (\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})\frac{m}{\alpha}\frac{1}{\xi(1-\xi)} + \frac{\theta m}{1-\xi} & ? \\ -\xi & -\theta\xi & ? \\ 0 & 0 & (\frac{L_N}{a_N} - \theta)\frac{1}{\alpha} \end{bmatrix}_{(m^*,\xi^*,\theta^*)}$$

Trace of C is:

$$Tr(C) = (\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})\frac{1}{\alpha} + (\frac{m}{1-\xi} - \theta\xi) + (\frac{L_N}{a_N} - \theta)\frac{1}{\alpha} > 0.$$

This is positive because (A3.1) implies that

$$\theta\xi = \frac{m\xi}{1-\xi}$$

; and this makes the bracketed second term of the expression of Tr(C) positive. Now the determinant of C is:

$$Det(C) = \frac{1}{\alpha}(\frac{L_N}{a_N} - \theta)\frac{m}{\alpha}(\frac{L_S}{a_S} - m\frac{\xi}{1-\xi}) > 0.$$

Thus both the trace and the determinant of C are positive. In order to determine the sign of the roots of the C matrix we apply the Routh-Hurwitz Theorem. The characteristic equation associated with C is

$$-q^3 + Tr(C)q^2 - M(C)q + Det(C) = 0 \qquad (A4.1)$$

where

$$M(C) = \begin{vmatrix} (\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})\frac{1}{\alpha} + \frac{m}{1-\xi} & (\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})\frac{m}{\alpha}\frac{1}{\xi(1-\xi)} + \frac{\theta m}{1-\xi} \\ -\xi & -\theta\xi \end{vmatrix}_{(m^*,\xi^*,\theta^*)} +$$

$$\begin{vmatrix} -\xi\theta & ? \\ 0 & (\frac{L_N}{a_N} - \theta)\frac{1}{\alpha} \end{vmatrix}_{(m^*,\xi^*,\theta^*)} + \begin{vmatrix} (\frac{L_S}{a_S} - m\frac{\xi}{1-\xi})\frac{1}{\alpha} + \frac{m}{1-\xi} & ? \\ 0 & (\frac{L_N}{a_N} - \theta)\frac{1}{\alpha} \end{vmatrix}_{(m^*,\xi^*,\theta^*)}$$

Routh-Hurwitz Theorem states that the number of positive roots of the characteristic equation (A4.1) is equal to the number of variations of signs in the scheme

$$\{\, -1, \;\; Tr(C), \;\; -M(C) + \frac{Det(C)}{Tr(C)}, \;\; Det(C) \,\}. \qquad (A5)$$

Now,

$$-M(C)+\frac{Det(C)}{Tr(C)}=-\frac{m}{\alpha}(\frac{L_S}{a_S}-m\frac{\xi}{1-\xi})-(\frac{L_N}{a_N}-\theta)\frac{1}{\alpha}[(\frac{L_S}{a_S}-m\frac{\xi}{1-\xi})\frac{1}{\alpha}+m]$$

$$+\frac{\frac{1}{\alpha}(\frac{L_N}{a_N}-\theta)\frac{m}{\alpha}(\frac{L_S}{a_S}-m\frac{\xi}{1-\xi})}{(\frac{L_S}{a_S}-m\frac{\xi}{1-\xi})\frac{1}{\alpha}+m+(\frac{L_N}{a_N}-\theta)\frac{1}{\alpha}}$$

$$=-\frac{m}{\alpha}(\frac{L_S}{a_S}-m\frac{\xi}{1-\xi})-(\frac{L_N}{a_N}-\theta)\frac{m}{\alpha}-\frac{1}{\alpha}(\frac{L_N}{a_N}-\theta)\frac{1}{\alpha}(\frac{L_S}{a_S}-m\frac{\xi}{1-\xi})$$

$$[1-\frac{m}{(\frac{L_S}{a_S}-m\frac{\xi}{1-\xi})\frac{1}{\alpha}+m+(\frac{L_N}{a_N}-\theta)\frac{1}{\alpha}}].$$

The last term in the third braket of the above expression is positive and hence

$$[-M(C)+\frac{Det(C)}{Tr(C)}]<0$$

So the number of variations in sign in (A5) is equal to three, which means that all the three roots of the equation (A4.1) are positive. This implies that the steady-state equilibrium of the system is locally unstable.

Appendix(2)

Dynamic equations are

$$\dot{\theta}=[\rho+\theta-\frac{1-\alpha}{\alpha}(\frac{L_N}{a_N}-\theta)](\frac{L_N}{a_N}-\theta), \tag{B1}$$

$$\dot{k}=k^2\frac{a_N}{a_S}\alpha^{-\varepsilon}(\frac{L_N}{a_N}-\theta)\frac{1-\xi}{\xi}-k[\frac{L_S}{a_S}+\rho]+[\frac{k\dot{\theta}}{\frac{L_N}{a_N}-\theta}+\frac{k\dot{\xi}}{\xi(1-\xi)}], \tag{B2}$$

and

$$\dot{\xi}=(1-\xi)[\xi\theta-\frac{L_S}{a_S}+\frac{1-\xi}{\xi}(\frac{L_N}{a_N}-\theta)\frac{a_N\alpha^{-\varepsilon}}{a_S}]. \tag{B3}$$

The steady state system of equations are

$$\rho+\theta-\frac{1-\alpha}{\alpha}(\frac{L_N}{a_N}-\theta)=0, \tag{B1.1}$$

$$k\frac{a_N}{a_S}\alpha^{-\varepsilon}(\frac{L_N}{a_N}-\theta)\frac{1-\xi}{\xi}=\frac{L_S}{a_S}+\rho, \tag{B2.1}$$

and

$$\frac{a_N}{a_S}\alpha^{-\varepsilon}(\frac{L_N}{a_N}-\theta)\frac{1-\xi}{\xi}=\frac{L_S}{a_S}-\theta\xi. \tag{B3.1}$$

Using equations (B2.1) and (B3.1) we get

$$\frac{L_S}{a_S} - \theta\xi = (\frac{L_S}{a_S} + \rho)\frac{1}{k}$$

$$\Rightarrow \theta\xi = \frac{L_S}{a_S} - (\frac{L_S}{a_S} + \rho)\frac{1}{k} \qquad (B4)$$

All the following derivatives are evaluated at θ^*, k^* and ξ^*.

$$\frac{\partial\dot{\theta}}{\partial\theta} = (\frac{L_N}{a_N} - \theta)[1 + \frac{1-\alpha}{\alpha}] = (\frac{L_N}{a_N} - \theta)\frac{1}{\alpha}$$

$$\frac{\partial\dot{\theta}}{\partial k} = \frac{\partial\dot{\theta}}{\partial\xi} = \frac{\partial\dot{\xi}}{\partial k} = 0$$

$$\frac{\partial\dot{k}}{\partial k} = 2k\frac{a_N}{a_S}\alpha^{-\varepsilon}(\frac{L_N}{a_N} - \theta)\frac{1-\xi}{\xi} - [\frac{L_S}{a_S} + \rho] = \frac{L_S}{a_S} + \rho \qquad [using\ (B2.1)]$$

$$\frac{\partial\dot{\xi}}{\partial\xi} = (1-\xi)[\theta - \frac{1}{\xi^2}(\frac{L_N}{a_N} - \theta)\frac{a_N\alpha^{-\varepsilon}}{a_S}] = \theta - \xi\theta - \frac{1}{\xi}\frac{1}{k}(\frac{L_S}{a_S} + \rho) \qquad [using\ (B2.1)]$$

$$= \theta - \frac{L_S}{a_S} + (\frac{L_S}{a_S} + \rho)\frac{1}{k} - \frac{1}{\xi}\frac{1}{k}(\frac{L_S}{a_S} + \rho) = -(\frac{L_S}{a_S} - \theta) - \frac{1}{k}(\frac{L_S}{a_S} + \rho)(\frac{1-\xi}{\xi}) \quad [using\ (B4)]$$

The linearised system is given by:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\xi} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} \frac{1}{\alpha}(\frac{L_N}{a_N} - \theta) & 0 & 0 \\ ? & -(\frac{L_S}{a_S} - \theta) - \frac{1}{k}(\frac{L_S}{a_S} + \rho)(\frac{1-\xi}{\xi}) & 0 \\ ? & ? & \rho + \frac{L_S}{a_S} \end{bmatrix}_{(\theta^*,\xi^*,k^*)} \cdot \begin{bmatrix} \theta(t) - \theta^* \\ \xi(t) - \xi^* \\ k(t) - k^* \end{bmatrix}$$

We use the following notations a_{11}, a_{22}, a_{33} and a_{32} such that

$$a_{11} = \frac{\partial\dot{\theta}}{\partial\theta}]_{(\theta^*,\xi^*,k^*)} = \frac{1}{\alpha}(\frac{L_N}{a_N} - \theta) > 0,$$

$$a_{22} = \frac{\partial\dot{\xi}}{\partial\xi}]_{(\theta^*,\xi^*,k^*)} = [-(\frac{L_S}{a_S} - \theta) - \frac{1}{k}(\frac{L_S}{a_S} + \rho)(\frac{1-\xi}{\xi})]_{(\theta^*,\xi^*,k^*)} < 0,$$

$$a_{33} = \frac{\partial\dot{k}}{\partial k}]_{(\theta^*,\xi^*,k^*)} = [\rho + \frac{L_S}{a_S}]_{(\theta^*,\xi^*,k^*)} > 0,$$

and

$$a_{32} = \frac{\partial\dot{k}}{\partial\xi}]_{(\theta^*,\xi^*,k^*)} = [k^2\frac{a_N}{a_S}\alpha^{-\varepsilon}(\frac{L_N}{a_N} - \theta)(-\frac{1}{\xi^2}) - (\rho + \frac{L_S}{a_S})]_{(\theta^*,\xi^*,k^*)} < 0.$$

The roots of the characteristic equation of the matrix D are a_{11}, a_{22} and a_{33}. Since exactly one root is negative the system is saddle point stable with a unique saddle path converging to the steady-state equilibrium point. To determine the general solution of the variables along the unique saddle path we choose the eigenvectors corresponding to the two positive roots as zero. This procedures leads to the following solutions of the variables

$$\theta(t) = \theta^*,$$

$$\xi(t) = \xi^* - [\xi^* - \xi(0)]e^{a_{22}t},$$

and

$$k(t) = k^* + [\xi^* - \xi(0)]e^{a_{22}t}(\frac{a_{32}}{a_{33} - a_{22}}).$$

Appendix(3)

We have

$$\frac{\dot{E}_S}{E_S} = \frac{\dot{p}_S}{p_S} + \frac{(\dot{n_S x_S})}{n_S x_S}.$$

Now using equations (4), (27) and (27.1), the last equation imply

$$r_S - \rho = r_S - \frac{\pi_S}{v_S} + \frac{\dot{n}_S}{n_S} + \frac{\dot{k}}{k} - a_S \frac{(\dot{m\frac{\xi}{1-\xi}})}{L_S - ma_s\frac{\xi}{1-\xi}}$$

Differentiating equation (20.1) with respect to time we obtain

$$a_S \frac{(\dot{m\frac{\xi}{1-\xi}})}{L_S - ma_s\frac{\xi}{1-\xi}} = a_N \frac{\dot{\theta}}{L_N - a_N\theta} - \frac{\dot{\xi}}{\xi(1-\xi)}.$$

Then using this in the above mentioned equation we get,

$$r_S - \rho = r_S - \frac{\pi_S}{v_S} + m\frac{\xi}{1-\xi} + \frac{\dot{k}}{k} - a_N \frac{\dot{\theta}}{L_N - a_N\theta} - \frac{\dot{\xi}}{\xi(1-\xi)} \Longrightarrow$$

$$-\rho = -[\frac{k-1}{a_S}\alpha^{-\varepsilon}(L_N - a_N\theta)\frac{1-\xi}{\xi}] + [\frac{L_S}{a_S} - \frac{1-\xi}{\xi}(\frac{L_N}{a_N} - \theta)\frac{a_N\alpha^{-\varepsilon}}{a_S}] + \frac{\dot{k}}{k} - [a_N\frac{\dot{\theta}}{L_N - a_N\theta} + \frac{\dot{\xi}}{\xi(1-\xi)}] \Longrightarrow$$

$$\dot{k} = k^2\frac{a_N}{a_S}\alpha^{-\varepsilon}(\frac{L_N}{a_N} - \theta)\frac{1-\xi}{\xi} - k[\frac{L_S}{a_S} + \rho] + [\frac{k\dot{\theta}}{\frac{L_N}{a_N} - \theta} + \frac{k\dot{\xi}}{\xi(1-\xi)}].$$

Appendix(4)

We have

$$\frac{\dot{v_N}}{v_N} = \frac{\dot{p_N}}{p_N} - \frac{\dot{n_N}}{n_N} \Longrightarrow \frac{\dot{v_N}}{v_N} = \frac{\dot{p_N}}{p_N} - (\theta - m).$$

This we get from the fact that $n_N = n\xi$ and footnote (6). The no-arbitrage condition (10) then implies that

$$\frac{\dot{p_N}}{p_N} = r_N + \theta - \frac{1-\alpha}{\alpha a_N}(L_N - a_N\theta). \qquad (C1)$$

Again from

$$\frac{\dot{E_N}}{E_N} = \frac{\dot{p_N}}{p_N} + \frac{(n_N \dot{x}_N)}{n_N x_N}$$

and using equations (16) and (4), we have

$$r_N - \rho = \frac{\dot{p_N}}{p_N} - a_N \frac{\dot{\theta}}{L_N - a_N\theta},$$

$$\Longrightarrow \frac{\dot{p_N}}{p_N} = r_N - \rho + a_N \frac{\dot{\theta}}{L_N - a_N\theta}. \qquad (C2)$$

Equations (C1) and (C2) together imply that

$$a_N \frac{\dot{\theta}}{L_N - a_N\theta} = \rho + \theta - \frac{1-\alpha}{\alpha a_N}(L_N - a_N\theta),$$

$$\Longrightarrow \dot{\theta} = [\rho + \theta - \frac{1-\alpha}{\alpha a_N}(\frac{L_N}{a_N} - \theta)](\frac{L_N}{a_N} - \theta).$$

Appendix(5)

Here,

$$W_N(0) = \int_0^\infty e^{-\rho t} log(U_N(t))dt = \int_0^\infty e^{-\rho t}[log(1 - \frac{a_N\theta}{L_N}) + \frac{1}{\varepsilon - 1} log(n(t)) + \frac{1}{\varepsilon - 1} log\{\xi + (1-\xi)\alpha^{1-\varepsilon}\}]dt$$

Then differentiating $W_N(0)$ with respect to a_S and evaluating the derivative at the steady-state, we have

$$\frac{dW_N(0)}{da_S} = \int_0^\infty e^{-\rho t}[\frac{dlog(1 - \frac{a_N\theta}{L_N})}{da_S}]dt \;+\; \frac{1}{\varepsilon - 1}\int_0^\infty e^{-\rho t}[\frac{dlog(n(t))}{da_S}]dt$$
$$+\; \frac{1}{\varepsilon - 1}\int_0^\infty e^{-\rho t}[\frac{dlog\{\xi + (1-\xi)\alpha^{1-\varepsilon}\}}{da_S}]dt. \qquad (D1)$$

At steady-state we have $\xi = \xi^*$ and $\theta = \theta^*$, and hence

$$\frac{dlog(1 - \frac{a_N\theta}{L_N})}{da_S}|_{\theta=\theta^*} = 0 \Rightarrow \int_0^\infty e^{-\rho t}[\frac{dlog(1 - \frac{a_N\theta}{L_N})}{da_S}]dt = 0.$$

Note that $log(n(t)) = log(n(0)) + \int_0^t g(\tau)d\tau$. Differentiating this with respect to a_S we have

$$\frac{dlog(n(t))}{da_S} = \int_0^t \frac{dg(\tau)}{da_S}d\tau = \theta^* \frac{d\xi^*}{da_S}\int_0^t (1-e^{a_{22}\tau})d\tau = \theta^* \frac{d\xi^*}{da_S}[\int_0^t d\tau - \int_0^t e^{a_{22}\tau}d\tau]$$

$$= \theta^* \frac{d\xi^*}{da_S}[t - [\frac{1}{a_{22}}e^{a_{22}\tau}]_0^t] = \theta^* \frac{d\xi^*}{da_S}[t + \frac{1}{a_{22}} - \frac{1}{a_{22}}e^{a_{22}t}]$$

Then,

$$\int_0^\infty e^{-\rho t}\frac{dlog(n(t))}{da_S}dt = \theta^* \frac{d\xi^*}{da_S}[\int_0^\infty e^{-\rho t}(t+\frac{1}{a_{22}}-\frac{1}{a_{22}}e^{a_{22}t})dt] = \theta^* \frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho^2(\rho - a_{22})} > 0.$$

Also it can be shown that

$$\frac{dlog\{\xi+(1-\xi)\alpha^{1-\varepsilon}\}}{da_S}|_{\xi=\xi^*} = \frac{1-\alpha^{1-\varepsilon}}{\xi^*+(1-\xi^*)\alpha^{1-\varepsilon}}[1-e^{a_{22}t}]\frac{d\xi^*}{da_S}.$$

So we have

$$\int_0^\infty e^{-\rho t}[\frac{dlog\{\xi+(1-\xi)\alpha^{1-\varepsilon}\}}{da_S}]dt = \frac{1-\alpha^{1-\varepsilon}}{\xi^*+(1-\xi^*)\alpha^{1-\varepsilon}}\frac{d\xi^*}{da_S}\int_0^\infty e^{-\rho t}[1-e^{a_{22}t}]dt$$

$$= \frac{1-\alpha^{1-\varepsilon}}{\xi^*+(1-\xi^*)\alpha^{1-\varepsilon}}\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho(\rho-a_{22})}.$$

Then from equation (D1) we have

$$\frac{dW_N(0)}{da_S} = \frac{1}{\varepsilon-1}[\theta^*\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho^2(\rho-a_{22})} + \frac{1-\alpha^{1-\varepsilon}}{\xi^*+(1-\xi^*)\alpha^{1-\varepsilon}}\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho(\rho-a_{22})}].$$

Now using equation (46) we obtain

$$\frac{dW_S(0)}{da_S} = \frac{dW_N(0)}{da_S} + \int_0^\infty e^{-\rho t}[\frac{dlog(\frac{1-\xi}{\xi}\frac{L_N}{L_S})}{da_S}]dt$$

$$= \frac{dW_N(0)}{da_S} - \frac{1}{\xi^*(1-\xi^*)}\frac{d\xi^*}{da_S}\int_0^\infty e^{-\rho t}[1-e^{a_{22}t}]dt$$

$$= \frac{dW_N(0)}{da_S} - \frac{1}{\xi^*(1-\xi^*)}\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho(\rho-a_{22})}.$$

So we have

$$\frac{dW_S(0)}{da_S} = -\frac{1}{\xi^*(1-\xi^*)}\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho(\rho-a_{22})} + \frac{1}{\varepsilon-1}[\theta^*\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho^2(\rho-a_{22})} + \frac{1-\alpha^{1-\varepsilon}}{\xi^*+(1-\xi^*)\alpha^{1-\varepsilon}}\frac{d\xi^*}{da_S}\frac{-a_{22}}{\rho(\rho-a_{22})}].$$

Appendix(6)

All the following derivatives are evaluated at the steady-state equilibrium values of ξ and θ.

$$\Delta_N^{L_j} = \int_0^\infty e^{-\rho t}[\frac{dlog(n(t))}{dL_j}]dt = \theta^* \frac{d\xi^*}{dL_j}[\int_0^\infty e^{-\rho t}(t+\frac{1}{a_{22}}-\frac{1}{a_{22}}e^{a_{22}t})dt] = \theta^* \frac{d\xi^*}{dL_j}\frac{-a_{22}}{\rho^2(\rho - a_{22})}.$$

Since $\frac{d\xi^*}{dL_N} > 0$ and $\frac{d\xi^*}{dL_S} < 0$, we have $\Delta_N^{L_j} > 0$ for j=N and $\Delta_N^{L_j} < 0$ for j=S.

$$\Delta_e^{L_j} = \int_0^\infty e^{-\rho t}[\frac{dlog\{\xi + (1-\xi)\alpha^{1-\varepsilon}\}}{dL_j}]dt = \frac{1-\alpha^{1-\varepsilon}}{\xi^* + (1-\xi^*)\alpha^{1-\varepsilon}}\frac{d\xi^*}{dL_j}\frac{-a_{22}}{\rho(\rho - a_{22})}.$$

Since $\alpha < 1$ and $\varepsilon > 1$ we have $\alpha^{1-\varepsilon} > 1$. Hence $\Delta_e^{L_j} < 0$ for j=N and $\Delta_e^{L_j} > 0$ for j=S.

At the steady-state equilibrium we have

$$\theta = \theta^* = (1-\alpha)\frac{L_N}{a_N} - \rho\alpha$$

and this implies that

$$\frac{d\theta}{dL_S} = 0 \text{ and } \frac{d\theta}{dL_N} = \frac{1-\alpha}{a_N}$$

Then,

$$\begin{aligned} \Delta_s^{NL_j} &= \int_0^\infty e^{-\rho t}[\frac{dlog(1-\frac{a_N\theta}{L_N})}{dL_j}]dt = 0 \qquad \text{for j=S; and} \\ &= \int_0^\infty e^{-\rho t}[\frac{dlog(\alpha + \frac{\rho\alpha a_N}{L_N})}{dL_j}]dt = \frac{1}{\rho}\frac{1}{\alpha + \frac{\rho\alpha a_N}{L_N}}(-\frac{\rho\alpha a_N}{L_N^2}) < 0 \qquad \text{for j=N} \end{aligned}$$

Again

$$\begin{aligned} \Delta_s^{SL_j} &= \int_0^\infty e^{-\rho t}[\frac{dlog\{\frac{1-\xi}{\xi}(\frac{L_N}{a_N}-\theta)\frac{a_N}{L_S}\alpha^{-\varepsilon}\}}{dL_j}]dt = \int_0^\infty e^{-\rho t}[\frac{dlog\{\frac{1-\xi}{\xi}(\frac{L_N}{L_S}+\rho\frac{a_N}{L_S})\alpha^{1-\varepsilon}\}}{dL_j}]dt \\ &= \int_0^\infty e^{-\rho t}[\frac{dlog(\frac{1-\xi}{\xi})}{dL_j}]dt + \int_0^\infty e^{-\rho t}[\frac{dlog(\frac{L_N}{L_S}+\rho\frac{a_N}{L_S})}{dL_j}]dt. \end{aligned}$$

Here the first term is negative and the second term is positive for j=N. So the net effect is ambiguous. Similarly, for j=S, the first term is positive and the second term is negative. So the net effect is ambiguous again.

Figure

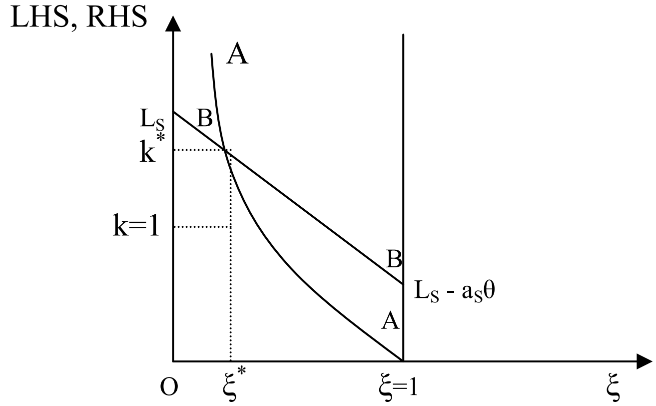

Figure-1: Existence of equilibrium.

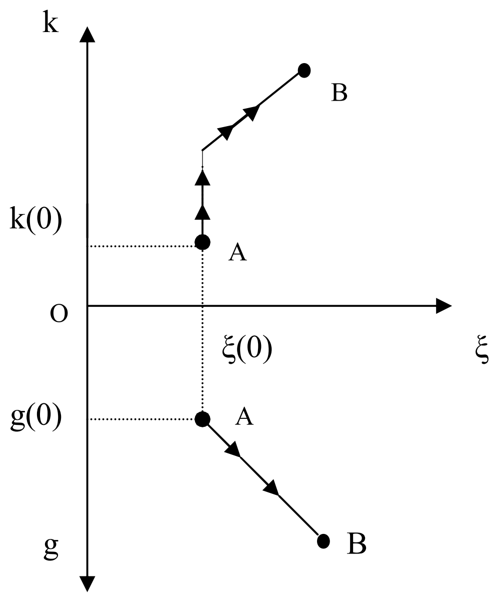

Figure – 2: Transitional movement due to stronger IPR protection

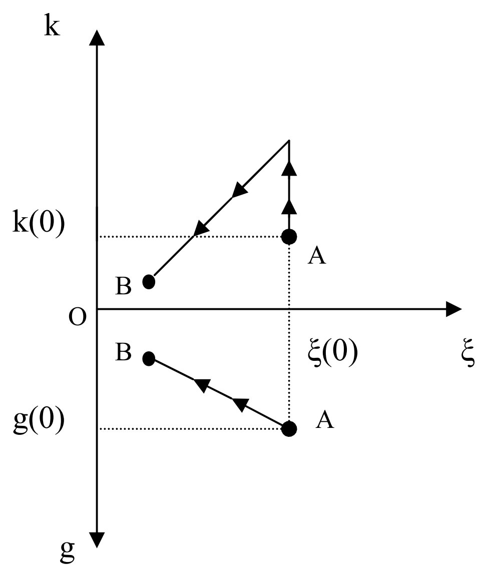

Figure – 3: Transitional movement due to change in L_S

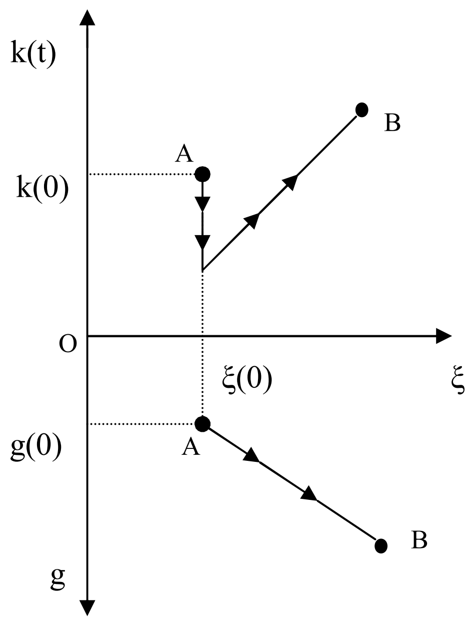

Figure – 4: Transitional movement due to change in L_N