Heterogeneity and Nonrenewable Resources: When Bad is Good

by

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Abstract

A well known theorem by Herfindahl states that if nonrenewable resources differ by their cost of extraction, then their use must follow the "least cost first" principle. The low cost or the "good" resource must be exploited first. In this paper we consider resources that are differentiated not by cost but by their pollution characteristics. For instance, both coal and natural gas are used to produce electricity, yet coal is more polluting. The regulator imposes a cap on the stock of pollution. We show that if the cap is binding, the cleaner natural gas must be used before coal, as Herfindahl had predicted. However, when the pollution stock is below the cap, regulation may trigger a race to the ceiling by using coal before natural gas is used. A perverse policy implication is that regulating the stock of pollution may accelerate coal burning.

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1. Introduction

Energy markets are often characterized by the use of multiple nonrenewable resources, such as coal and natural gas with different pollution characteristics. The same resource may be used with different qualities, as when high and low sulfur coal is used to generate electricity. Multiple resources are used in providing transportation services, such as cars running on gasoline or hybrids fueled by electricity generated from coal. Sweet and sour varieties of crude oil (oil with low and high sulfur content) are other examples of this phenomenon.

When these nonrenewable resources contribute differentially to pollution, what is the sequence of extraction over time? Hotelling (1931) developed the classical theory of extraction of a nonrenewable resource over time. Herfindahl (1967) extended the Hotelling model by considering many resources with different unit costs of extraction and proposed the "least cost first" principle: extraction must be ordered by cost, with the cheapest resource used first. Others have examined whether Herfindahl's model remains valid in a variety of situations: in a general equilibrium setting (Kemp and Long, 1980; Lewis, 1982) under heterogenous demands (Chakravorty and Krulce, 1994) and when the extraction rate is constrained (Amigues et al, 1998).

In all these studies following Hotelling and Herfindahl, resources were differentiated by cost alone. In this paper, we abstract from cost considerations and focus on how the sequence of extraction may be affected when resources are differentiated only by their pollution characteristics. We consider two resources, one more polluting than the other. Without loss of generality, consider coal to be the dirty resource and natural gas the clean one. Both may be used in electricity generation. Environmental regulation is imposed through a cap on the stock of pollution. This may be a stylized approximation of an international agreement such as the Kyoto Protocol which aims to stabilize the concentration of greenhouse gases in the atmosphere.[2]

[2] In a parallel effort, Smulders and van der Werf (2005) develop a model to examine the extraction of heterogenous resources when the flow of emissions (not the stock as in our case) is constrained. In their model, the resources are imperfect substitutes. Their findings suggest that the economy may use more of

With heterogeneity, the results are non-intuitive and differ sharply from Herfindahl. When the economy is already at its allowable stock of pollution, the clean natural gas is used first and use of the dirty coal is postponed to the future. This is what one would expect, armed with insights from the Herfindahl model. However, when the economy is below the ceiling and accumulating pollution, coal may be used first and use of the clean natural gas is postponed. The optimal strategy is to build the stock of pollution as fast as possible thereby benefiting from natural dilution. This is done by burning coal rather than natural gas. This phenomenon is the reverse of Herfindahl – the "bad" resource is used first. Only when natural gas is abundant is it used before coal.

The pattern of extraction is dependent upon the initial endowments of the two resources. If the stock of coal is relatively low, then the Hotelling rents of the two resources are exactly equal and regulation is never binding. However, if coal is abundant, it has a lower Hotelling rent than natural gas. With abundant resources, extraction paths have a turnpike feature during which both resources are jointly extracted at the maximum allowed level. All paths must pass through this turnpike. With heterogeneity in pollution, we get a complete "preference reversal" over resources, something we do not get with cost heterogeneity. That is coal may be used for a period of time, followed by natural gas, and again coal for another time period.

Stylized facts suggest that coal is much more abundant than natural gas. Our results imply that if the economy is below the regulated stock of emissions, then it is optimal to use the more polluting resource first and get to the ceiling. Once the ceiling is achieved, we must use the cleaner fuel first. From a policy point of view, we thus obtain a perverse result: imposing a cap on the stock of pollution may trigger a race to the ceiling by burning coal.

Section 2 extends the textbook Hotelling model with two costless but polluting resources. Section 3 characterizes the sequence of extraction when the economy is already at the

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the clean resource before the constraint is binding in order to use more of the dirty resource during the period when the constraint is binding. It may be useful at a later stage to develop a more general framework in which the effect of different types of regulation (stock vs flow constraints) on the extraction path is better understood.

ceiling. Section 4 extends the analysis by characterizing the path to the ceiling. Section 5 concludes the paper.

2. The Model

Consider an economy in which energy consumption at any time is given by q and the corresponding gross surplus u is differentiable, strictly increasing and strictly concave over the interval $[0,\bar{q})$ with $\bar{q} > 0$ and constant over the interval $[\bar{q},\infty)$ with $u(q) = \bar{u}$ for $q \geq \bar{q}$. Let $\lim_{q \downarrow 0} u'(q) = +\infty$.[3] This implies that $\lim_{q \uparrow \bar{q}} u'(q) = 0$. Denote the marginal utility of energy consumption by $p(q) \equiv u'(q)$ and by $d(\ p\ )$ the corresponding inverse demand function. Welfare W is the sum of the gross surplus discounted at some constant rate $\rho > 0$ given by

$$W = \int_0^\infty u(\ q\ ) e^{-\rho t} dt\ .$$

Since $u(\ q\ ) \leq \bar{u}$, this integral is well defined. We consider two nonrenewable resources indexed by $i=1,2$ which are perfect substitutes in demand. Each resource is characterized by the vector $\{\theta_i, X_i^{\ 0}\}$ where θ_i is the pollution generated by one unit of the resource and $X_i^{\ 0}$ is its given initial stock. Let $X_i(\ t\ )$ be the residual stock at time t and x_i the extraction rate. Then

$$\dot{X}_i(\ t\ ) = -x_i, i = 1,2\ . \tag{1}$$

Let the cost of extraction of both resources be zero. Without loss of generality, let us assume that resource *1* (say, natural gas) is cleaner than resource *2* (coal), $0 < \theta_1 < \theta_2$. Let $X^0 = X_1^0 + X_2^0$ be the total initial stock, $X \equiv X_1 + X_2$ the total residual stock at time t, and $x \equiv x_1 + x_2$ the aggregate extraction at t.

As in most Hotelling models, we assume an abundant renewable backstop resource (e.g., solar energy) that is non-polluting. Its unit cost is given by $c_r > 0$. Let y be the rate of

[3] To keep the notation simple, we avoid writing the time argument t whenever the context is obvious.

extraction of the backstop resource. Once coal and natural gas are exhausted, the price of the renewable resource is equal to c_r and its consumption is determined by $\tilde{y}$, the solution to the equation $u'(y) = c_r$.

Burning of the fossil fuels increases the aggregate stock of pollution denoted by $Z(t)$. This may be the level of carbon in the atmosphere. We assume that there is a natural decay of the aggregate stock of pollution at the constant rate $\alpha > 0$ so that

$$\dot{Z}(t) = \sum_i \theta_i x_i - \alpha Z, \quad Z(0) = Z^0 \leq \bar{Z} \text{ given,} \tag{2}$$

where $\bar{Z}$ is the regulated limit on the stock of pollution. This may be exogenously imposed by some regulatory authority or the outcome of a negotiated international agreement.[4] It may also approximate a specific form of a damage function that is zero at stock levels below $\bar{Z}$ but imposes prohibitive damages beyond that threshold value. The inequality in (2) suggests that the initial level of pollution is below the ceiling.

Suppose only resource i is being used when the stock of pollution is at the ceiling over a time period. Let $\bar{x}_i$ be this maximum extraction rate. By (2), $\bar{x}_i = \dfrac{\alpha \bar{Z}}{\theta_i}$ and $\bar{p}_i$, the marginal gross surplus is given by $\bar{p}_i = u'(\bar{x}_i)$. Since natural gas is less polluting than coal, more gas would be used at the ceiling, so $\bar{x}_1 > \bar{x}_2$.

The social planner maximizes the net surplus by choosing extraction rates of natural gas and coal and the backstop resource at each time t as follows:

$$\underset{\{x_i, i=1,2;\, y\}}{Max} \int_0^\infty \{ u(\sum_i x_i + y) - c_r y \} e^{-\rho t} dt \tag{3}$$

subject to (1) and (2), X_i^0, Z^0, and $\bar{Z}$ given with $\bar{Z} - Z(t) \geq 0$. The corresponding current value Lagrangian can be written as

[4] For example, a policy goal of say, 550 parts per million carbon concentration in the atmosphere may correspond to a specific value of $\bar{Z}$.

$$L = u(\sum_i x_i + y) - c_r y - \sum_i \lambda_i x_i + \mu(t)[\sum_i \theta_i x_i - \alpha Z] + \nu(t)[\bar{Z} - Z] + \sum_i \gamma_i x_i + \gamma_r y$$

where ν is the multiplier attached to $\bar{Z} - Z \geq 0$, γ_i and γ_r are the multipliers for the non-negativity constraints $x_i \geq 0$ and $y \geq 0$. The first order conditions are

$$u'(\sum_i x_i + y) = \lambda_i - \theta_i \mu - \gamma_i, i = 1,2 \text{ and} \tag{4}$$

$$u'(\sum_i x_i + y) = c_r - \gamma_r. \tag{5}$$

The dynamics of the costate variables is given by

$$\dot{\lambda}_i(t) = \rho \lambda_i \text{ which gives } \lambda_i = \lambda_i^0 e^{\rho t}, i = 1,2 \text{ and} \tag{6}$$

$$\dot{\mu}(t) = (\rho + \alpha)\mu + \nu, \tag{7}$$

and the complimentary slackness conditions are

$$\gamma_i \geq 0, x_i \geq 0, \gamma_i x_i = 0, i = 1,2 \tag{8}$$

$$\gamma_r \geq 0, y \geq 0, \gamma_r y = 0, \text{ and} \tag{9}$$

$$\nu \geq 0, \bar{Z} - Z \geq 0, \nu(\bar{Z} - Z) = 0. \tag{10}$$

Finally, the transversality conditions are

$$lim_{t \uparrow \infty} e^{-\rho t} \lambda_i(t) X_i(t) = \lambda_i^0 \, lim_{t \uparrow \infty} X_i(t) = 0, i = 1,2, \text{ and} \tag{11}$$

$$lim_{t \uparrow \infty} e^{-\rho t} \mu(t) Z(t) = 0 \tag{12}$$

where we write λ_i^0 for $\lambda_i(0)$. Condition (6) implies that the scarcity rent of oil and gas must rise at the rate of discount. Define the price of resource i as its full marginal cost at time *t*, given by $p_i = \lambda_i - \theta_i \mu$. It has two components: the scarcity rent and the externality cost, the extraction cost being zero. Since μ is the shadow price of the pollution stock, it is negative. If $Z < \overline{Z}$ over some time period, regulation does not constrain the extraction rates of the fossil fuels. Then $\nu = 0$ and $\mu(t) = \mu^0 e^{(\rho+\alpha)t}$ where $\mu(0)$ is written as μ^0. During this period, the shadow price of pollution grows at an exponential rate given by the sum of the discount rate and the dilution rate of the pollution stock. Depending upon whether the ceiling is attained or not, the initial shadow price μ^0 is either strictly negative or zero.

It is convenient to split the analysis into two parts. In section 3 we assume that the initial pollution stock is at the ceiling, $Z^0 = \overline{Z}$. Then in section 4, the initial stock is assumed to be strictly under the ceiling $Z^0 < \overline{Z}$.

3. The Initial Stock of Pollution is at the Ceiling

Let $Z^0 = \overline{Z}$. We plan to show that when the aggregate stock of the two resources is small, both coal and natural gas must have the same scarcity rent and extraction is "pure" Hotelling, as if regulation is a non-issue. When the stock of coal is small but natural gas is abundant, both resources have the same scarcity rent but the price paths are non-Hotelling. When coal is abundant, resource rents are not equal and the price paths are non-Hotelling.

Extraction with Only One Resource

To understand how environmental regulation affects extraction, assume that there is only a single resource *i*, which may be coal or natural gas. Since extraction is costless, the resource price $p_i(t)$ is equal to the scarcity rent and is completely determined by its initial value λ_i^0. The Hotelling price path is given by $p_i(t) \equiv \lambda_i^0 e^{\rho t}$. Define T as the time at which this Hotelling price equals the cost of the backstop c_r. Then $T = [\ln c_r - \ln \lambda_i^0\,] / \rho$. This is the switch point to the backstop.[5] The initial scarcity rent can written as a decreasing function of the initial stock, $\lambda_i^0(X_i^0)$.

[5] The optimal value of λ_i^0 is determined uniquely by the cumulative demand/supply balance equation $\int_0^T d(\lambda_i^0)dt = X^0$.

Since $Z^0 = \overline{Z}$, let the lowest value of the scarcity rent such that regulation is non-binding be given by $\lambda_i^0 = \overline{p}_i$. At resource prices lower than this level, initial resource extraction will lead to a pollution stock higher than the regulated level $\overline{Z}$ and the ceiling constraint will be binding. Since extraction is pure Hotelling, the resource price rises at the rate of discount starting from an initial level $\overline{p}_i$. Define the corresponding extraction period as $\Delta_i^H = [\ln c_r - \ln \overline{p}_i]/\rho$, where the superscript H stands for the Hotelling path. Cumulative extraction is given by $X_i^H = \int_0^{\Delta_i^H} d(\overline{p}_i)dt$. Thus X_i^H is the highest initial stock of resource i that generates a pure Hotelling path. Since coal is more polluting, less of it can be used beginning at the ceiling, so its price at the ceiling must be higher, hence $\overline{p}_1 < \overline{p}_2$. The aggregate stock of coal that could be used up over this Hotelling path must be lower, i.e., $X_1^H > X_2^H$.

If the initial stock is higher than this maximal Hotelling stock, $X_i^0 > X_i^H$, then at the beginning, extraction must be limited to $\overline{x}_i$, otherwise the stock will exceed the ceiling (see Fig. 1). When the stock declines to X_i^H, the Hotelling path begins. The duration of the first phase is then given by $\Delta_i = \dfrac{X_i^0 - X_i^H}{\overline{x}_i}$. Until time Δ_i, the price is $\overline{p}_i$. Since regulation is binding, the shadow cost of pollution $\mu(t)$ is strictly negative and (4) yields $\overline{p}_i = \lambda_i^0 e^{\rho t} - \mu\theta_i$.[6] The gap between the resource price $\overline{p}_i$ and the shadow price λ_i is the externality cost per unit of the resource *i*, given by $-\mu\theta_i$. Beyond Δ_i, the ceiling is no longer binding hence $\mu \equiv 0$ and both resource price and scarcity rents are equal.

[Fig. 1 here]

Extraction with Both Resources

Consider that both resources are available, $X_i^0 > 0, i = 1,2$. If $X^0 \leq X_2^H$, i.e., the aggregate stock is lower than the minimum of the two critical stocks, the solution must still be the

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[6] Hence $\mu(t) = \dfrac{\lambda_i^0 e^{\rho t} - \overline{p}_i}{\theta_i}$.

Hotelling path with initial extraction lower than the maximum allowed at the ceiling, $\bar{x}_2$. The scarcity rents of the two resources are equal, and their extraction rates a matter of indifference.[7] Regulation is never active except possibly at the initial instant.

This is shown in $[X_1^0, X_2^0]$ space in Fig.2 where each point represents an initial endowment of the two resources. Points *A* and *B* denote the stock X_2^H on each axis. In zone I, which is the triangle bounded by the axes and the straight line *AB* with slope -1, the aggregate stock is always lower than X_2^H and endowments in this zone yield a Hotelling solution in which only the aggregate extraction is determinate but not its composition.

[Fig. 2 here]

Now let $X_2^H < X^0 < X_1^H$. The aggregate stock is higher than the maximum Hotelling-induced stock of coal, but lower than that of natural gas. As X^0 approaches X_1^H we search for the maximum endowment of coal that generates a Hotelling price path, i.e., one unconstrained by regulation. In the limit, the pollution constraint can only bind at discrete points in time but not over an interval of non-zero duration. Consider the extraction sequence in which gas is consumed first for some time period followed by coal over another period. In the first interval, as price increases because of scarcity, extraction of gas falls below the critical $\bar{x}_1$ so that the pollution stock declines from the initial level of $\bar{Z}$. When we switch from gas to coal, the stock is strictly lower than $\bar{Z}$. Therefore we could extract an aggregate stock of coal strictly larger than X_2^H and still not violate the ceiling. As shown in Fig.3, natural gas is consumed first to bring down the stock of pollution, then coal is extracted at rates higher than $\bar{x}_2$ to restore the stock of pollution to $\bar{Z}$. Exactly at that instant, the residual coal stock must equal X_2^H and the Hotelling path follows as before. Aggregate coal use in this case is strictly larger than X_2^H.

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[7] However, they must satisfy $x(t) = d(\lambda^0)$ and $\int_0^{\Delta(X^0)} x_i dt = X_i^0, i = 1,2$, where λ^0 is the common value of the initial scarcity rent of the two resources and $\Delta(X^0)$ the duration of this extraction period.

[Fig. 3 here]

There may be other such Hotelling paths along which both resources are consumed simultaneously while maintaining the stock of pollution at less than or equal to $\overline{Z}$ and again letting it rise to the ceiling $\overline{Z}$, followed by a phase during which only coal is consumed. A polar case of such a path is one in which both resources are used while the stock of pollution is maintained exactly at its maximal level $\overline{Z}$ in the first phase, followed by exclusive use of coal. Appendix A shows that this path maximizes the consumption of coal for any given stock of natural gas such that the aggregate stock satisfies $X^0 \in (X_2^H , X_1^H]$.[8]

Consider the curve AA' in Fig.2. The point $A = (0, X_2^H)$ corresponds to an initial resource price $\overline{p}_2$. As we travel on AA' from A towards A' we increase the stock of natural gas and the corresponding maximal stock of coal that can be used to remain in the Hotelling path. The point A' denotes the endowment pair $(\overline{X}_1^0, \overline{X}_2^0)$ such that $\overline{X}_1^0 + \overline{X}_2^0 = X_1^H$. The path AA' can not cross the 45°line through X_1^H because the aggregate stock would exceed the maximum stock of natural gas compatible with the Hotelling path. Because the aggregate stock increases along AA', the common initial scarcity rent declines to $\overline{p}_1$ at A'.

Zone II in Fig.2 denotes the set of initial stocks that generate this Hotelling solution. If the vector of stocks is on the curve AA', the stock of coal is the maximum allowed on the Hotelling path, given the stock of natural gas. Each point on AA' corresponds to a unique extraction path, as shown in Appendix A. Extraction proceeds along the locus $A'A$

[8] During this period, aggregate supply must equal demand, $x_1 + x_2 = d(\lambda^0)$. From (2) the extraction rate of the fossil fuels satisfies $x_i = \overline{x}_i - \frac{\theta_j}{\theta_i} x_j,\ \ i, j = 1,2; i \neq j,$ so that $\theta_1 x_1 + \theta_2 x_2 = \theta_1 \overline{x}_1$.

Differentiating over time gives $\dot{x}_1(t) = -\frac{\theta_2}{\theta_1} \dot{x}_2(t) < 0.$ That is, the extraction rates must move in opposite directions when both resources are used jointly.

towards the point A.[9] Once A is reached, all natural gas is exhausted and the stock of coal equals X_2^H. In the next phase only coal is extracted.

If the set of initial stocks is in zone II but strictly below the curve AA', the amount of coal available is less than the maximal amount required and there may be some flexibility in sequencing the use of the two resources, since resource prices are still equal. For instance, optimal extraction beginning at point C may involve use of both resources at a maximal rate while staying at the ceiling followed by the exclusive use of coal, and finally the residual stock of natural gas. This is shown by the path $CC'C''O$ where the segment CC' is the translation of AA' through point C. However, since we are in the strict interior of zone II, regulation does not bind, hence some natural gas extraction at the beginning may be substituted by an equal compensating amount of coal later in time.[10]

The union of zones I and II is the area $OAA'B'$, the set of all solutions that are pure Hotelling, i.e., unaffected by environmental regulation.

Only Natural Gas is Abundant

This case is depicted by zone III in Fig. 2, where $X^0 > X_1^H$ and $X_2^0 < \overline{X}_2^0$. The pattern of resource use is a variation of the one described in Fig.1. Consider an initial vector of stocks at point D. Since natural gas is abundant, it is used first at the maximal level $\overline{x}_1$ until point D' is reached. The resource price is constant at $\overline{p}_1$. The aggregate stock at D' equals X_1^H. From here the extraction sequence is similar to the pure Hotelling paths originating in zone II. The scarcity rents of the two resources is equal and its initial value is given by $\lambda^0 = \overline{p}_1 e^{-\rho\Delta_1} = c_r e^{-\rho(\Delta_1 + \Delta_1^H)}$ where Δ_1 and Δ_1^H are the durations of the first and second phases respectively. The shadow cost of pollution $\mu(t)$ is non-zero until D' and zero beyond when regulation does not bind. As we see below this is the only non-Hotelling path with equal scarcity rents for the two resources.

[9] Let Δ_{12} denote the duration when the two resources are consumed jointly. From A to A', Δ_{12} increases from zero at A to $\dfrac{\ln \overline{p}_2 - \ln \overline{p}_1}{\rho}$ at A'.

[10] See Appendix B for further characterization of paths starting from zone II.

When Coal is Abundant

If coal, the more polluting resource is abundant, extraction may be constrained because of environmental regulation. The scarcity rents of the two resources may no longer be equal. For purposes of exposition it is easier to first discuss the case when both resources are abundant, before considering abundant coal but small stocks of natural gas, the clean resource.

This is depicted by endowments in zone IV, defined by $X_1^0 > \overline{X}_1^0$ and $X_2^0 > \overline{X}_2^0$, such as point *E* in Fig.2. As in zone III, natural gas is used at the maximal rate $\overline{x}_1$ until its stock is reduced to $\overline{X}_1^0$, shown as point F'. The duration of this phase equals $\Delta_1 = \dfrac{X_1^0 - \overline{X}_1^0}{\overline{x}_1}$. The corresponding price paths are shown in Fig. 4. The price of gas $\overline{p}_1$, shown by the solid line is lower than that of coal given by $p_2 = \lambda_2^0 e^{\rho t} - \mu\theta_2$, marked by the dashed line. Let this period of gas use be denoted by Δ_1. The price of energy is constant at $\overline{p}_1$ until F' is reached. At this instant, both resource prices are equal and a period of joint use begins. The price of energy increases to $\overline{p}_2$ at location *F* as extraction proceeds along a path $F'F$ that is a vertical translation of the curve $A'A$. Denote this period of joint use by Δ_{12}.

[Fig. 4 here]

Natural gas is exhausted at location *F*. When both resources are consumed, $x_i > 0$ and $\gamma_i = 0, i = 1,2$. From (4), we have $\lambda_1 - \mu\theta_1 = \lambda_2 - \mu\theta_2$ which implies that $\lambda_1 - \lambda_2 = -\mu(\theta_2 - \theta_1)$ so that $\mu = \dfrac{-(\lambda_1^0 - \lambda_2^0)e^{\rho t}}{\theta_2 - \theta_1}$. Thus $\lambda_1^0 > \lambda_2^0$. The cleaner resource has a higher scarcity rent, which is to be expected given that they have the same (zero) extraction cost. Beyond this time, the price of gas exceeds the price of coal. Coal is used at the maximum level (see Fig.4). Its price $\overline{p}_2$ is constant until time $\Delta_1 + \Delta_{12} + \Delta_2$. This is followed by the terminal phase when regulation is non-binding and the price of coal follows a Hotelling path $p_2(t) = \lambda_2 = \lambda_2^0 e^{\rho t}$ until the resource is exhausted and the backstop is used. The stock of coal at the starting location *E* (in Fig.2) determines the corresponding path $F'F$ each of which is a vertical translation of $A'A$.

However, now consider smaller endowments of natural gas, specifically $X_1^0 \leq \overline{X}_1^0$. If it is a strict equality, the first period of exclusive natural gas use in zone IV disappears completely. If the inequality is strict, consider a starting vector of stocks shown as point *G* in Fig.2. Extraction begins with both resources and a common price in the interval $[\,\overline{p}_1, \overline{p}_2\,]$ that depends upon the location of *G* on the curve FF'. The higher the endowment of natural gas, the closer is point *G* to point F' and the closer is the initial price to $\overline{p}_1$ (see also Fig.4). Extraction proceeds along the curve GF until all gas is exhausted at *F*. The resource price at this instant is $\overline{p}_2$. The remaining phases are as in zone IV.

The demarcation of the different zones is dependent upon $\overline{Z}$. If $\overline{Z}$ is higher, point A moves up in Fig.2 and B' moves to the right, enlarging the set of endowments that result in a Hotelling path (zones I and II). On the other hand, if $\overline{Z}$ approaches zero, the Hotelling set shrinks towards the origin until in the limit, none of the resources may be used. The pollution content of the resources also determines the set of Hotelling paths. If coal was more polluting and gas cleaner, point *A* will move down and B' will shift to the right, leading to a flat and elongated Hotelling set.

The main insight when regulation is binding is that excess natural gas must be used first. When both resources are used, natural gas extraction declines and that of coal increases. This turnpike feature allows for a smooth transition from the clean to the polluting resource until only coal is used. Loosely speaking, the order of extraction is according to pollution content. The more polluting resource is used latter. This trend is driven by time preference. Using the cleaner resource allows for higher extraction rates and higher profits earlier in time. However, as we see below, this tendency does not hold when the initial pollution stock is below the regulated level.

4. The Initial Stock of Pollution is Below the Ceiling

For the stock to build up to the ceiling, there must be a period when emissions are strictly greater than the maximum allowed at the ceiling. We adopt the same approach as before, differentiating between resource endowments that lead to an unconstrained Hotelling path and those that do not. However, the analysis in this section has an added dimension, the initial stock of pollution Z^0. This pollution stock together with the endowment of resources determines the approach to the ceiling. Now all variables are also a function of Z^0.

Hotelling Paths with a Single Resource

We can write $\lambda_i^0(Z^0)$ as the initial scarcity rent so that when $\lambda_i^0 e^{\rho t} = \overline{p}_i$ then $Z_i = \overline{Z}$. When the scarcity rent equals $\overline{p}_i$, the residual stock is exactly at the ceiling. In a Hotelling path, the stock must be off the ceiling both before and after this instant. It is the lowest possible scarcity rent and depends upon the initial stock Z^0. A lower rent will mean higher emissions, so the path will be constrained by regulation and will no longer be Hotelling. A higher rent will imply that the stock will not reach the ceiling. This scarcity rent $\lambda_i^0(Z^0)$ must increase with the initial stock of pollution Z^0. The cumulative extraction until the ceiling is reached is given by $X_i^H(Z^0) = \int_0^{T^0} d(\lambda_i^0(Z^0))dt$ where we use T^0 to denote the time it takes to get to the ceiling. Then $X_1^H(Z^0) > X_2^H(Z^0)$ and $\lambda_1^0(Z^0) < \lambda_2^0(Z^0)$ since more of the clean fuel can be used on the way to the ceiling.[11] Let $\Delta_i^H(Z^0)$ be the time duration during which resource i is extracted along a Hotelling path. We have $\Delta_1^H(Z^0) > \Delta_2^H(Z^0)$. For an initial stock of resource i lower than the critical level $X_i^H(Z^0)$, the stock does not reach the ceiling at all. For a higher initial stock, the initial scarcity rent is lower and regulation binds over a non-zero time period.

When the stock exceeds $X_i^H(Z^0)$, the excess stock is extracted at the maximum level until the Hotelling stock X_i^H remains. Now we must append an initial period when the pollution stock rises from some level Z^0 to the regulated level $\overline{Z}$. With a single resource *i*, the price at the ceiling is a constant $\overline{p}_i$. Suppose it takes time T_i^0 to go to the ceiling with resource *i* and the second phase at the ceiling is of duration Δ_i.[12] In the first phase the

[11] A special case is $\lambda_i^0(\overline{Z})$ for which T^0 equals zero. The initial stock is at the ceiling as in section 3.

[12] These variables must solve $Z(\overline{p}_i e^{-\rho T_i^0}) = \overline{Z}$ and $\int_0^{T_i^0} d(\overline{p}_i e^{-\rho(T_i^0 - t)})dt + \overline{x}_i \Delta_i = X_i^0 - X_i^H$.

price of the resource must rise from a lower level to $\overline{p}_i$ and the shadow price of pollution μ must be non-zero since regulation is binding. Beyond time $T_i^0 + \Delta_i$ regulation does not matter so the value of μ is zero and extraction is pure Hotelling.

Extraction with Both Resources

As in section 3, if Hotelling paths were to hold, for initial aggregate stocks lower than $X_2^H(Z^0)$, regulation will not bind, so which resource is extracted is a matter of indifference. This solution is given by initial stocks in zone I in Fig.5, which is a generalization of Fig.2. For a vector of stocks $X^0 \in [X_2^H(Z^0), X_1^H(Z^0)]$ we may obtain pure Hotelling paths as shown in section 3, provided the stock of coal is not too large. The Hotelling set corresponding to zone II in section 3 is given by figure $KLL'MN$ where $K = (X_2^H(Z^0), 0)$ $L = (0, X_2^H(Z^0))$, L' is located vertically above A', $M = (X_1^H(Z^0) - \overline{X}_2^0, \overline{X}_2^0)$ and $N = (0, X_1^H(Z^0))$. These vectors are all functions of the initial stock of pollution Z^0. As Z^0 approaches $\overline{Z}$ the curve LL' approaches AA' and $L'M$ approaches the vertex A'.

[Fig. 5 here]

Consider an initial endowment $F = (F_1, F_2)$ located on the AA' curve. We already know the resource use profile beginning from location F where by construction, the pollution stock is at the ceiling. With some abuse of notation, let the common resource price at F be given by $p(F)$ which lies in the interval $(\overline{p}_2, \overline{p}_1)$ as shown in Fig.4. As before, let T^0 denote the time taken from an initial endowment to reach the ceiling at F. Suppose only coal is used to go to the ceiling. Then the price path beginning from F is shown in Fig. 6.[13]

[Fig. 6 here]

[13] This path solves $Z(T^0) = \overline{Z}$ and $F' = (F_1, F_2 + \int_0^{T^0} d(p(F)e^{-\rho(T^0 - t)}dt)$.

The curve LL' is the locus of all such points F' that map to points F lying on the curve AA'. However LL' is not a vertical translation of AA'. Since $p(L') < \overline{p}_1 < \overline{p}_2 < p(L)$ resource prices are lower starting from L' relative to L. This will lead to higher coal use and higher emissions starting from L' given the same starting initial pollution stock. The distance $F'F$ is smaller, the closer F is to A'. From the ceiling at F, extraction follows the sequence described in section 3.[14]

Points on the segment $L'M$ (in Fig.5) can be explained similarly. At vertex M with aggregate endowment $X_1^H(Z^0)$ only natural gas is used during the transition to the ceiling at point A'. Since this is again a Hotelling path, both resource rents are equal. Consider an intermediate point J on the path $A'M$, where by construction, the stock of pollution is below the ceiling. Starting from M only gas is extracted to arrive at J. But starting from J' only coal is extracted. Since both of these paths must start from pollution stock Z^0, follow Hotelling and arrive at the same stock of resources and pollution, more natural gas needs to be used than coal. Thus traveling to J will take longer from L than from J'. This explains why point J' is located to the left of the extension of the line NM which is the locus of points $X_1^0 + X_2^0 = X_1^H(Z^0)$.

In the area of zone II above the line $AA'M$, coal is used first. For example, from F' only coal is used until the ceiling is reached at F followed by simultaneous extraction. For points located on the line $J'J$, coal is used first to point J, followed by natural gas until the ceiling is attained at A'.

Why will coal be used *before* natural gas on the way to the ceiling? Consider two instants of time t_1 and t_2 with $t_1 < t_2$. Since the two resources are perfect substitutes and their scarcity rents are equal in zone II, it is more profitable to burn a unit of coal at t_1 and gas later at t_2. Then at some time $t > t_2$ the increment to the pollution stock will be $[\theta_2 e^{-\alpha(t_2-t_1)} + \theta_1]e^{-\alpha(t-t_2)}$. Now consider the alternative – burning a unit of gas at

[14] To show that the extraction path $F'FAO$ is optimal, because this is a Hotelling path, $\lambda^0 = \lambda_1^0 = \lambda_2^0 = p(F)e^{-\rho T^0}$ and $\mu = 0, \upsilon = 0$ for $t \in [0, T^0]$. It is easy to check that all the necessary conditions are satisfied. In the interval $[T^0, T]$ the analysis from section 3 applies.

t_1 and coal later at t_2. The addition to pollution is $[\theta_1 e^{-\alpha(t_2-t_1)}+\theta_2]e^{-\alpha(t-t_2)}$. Subtracting the latter from the former gives $(\theta_2-\theta_1)(e^{-\alpha(t-t_2)}-1)e^{-\alpha(t-t_2)}<0$. Burning coal rather than natural gas will imply a lower pollution stock in the future because dilution is an increasing function of *Z*.[15]

Only Gas is Abundant

Here the initial coal stock is less than $\overline{X}_2^0(Z^0)$ while the aggregate stock is larger than $X_1^H(Z^0)$ as shown as zone III in Fig. 5. Consider a vector of initial stocks such as *G*. Natural gas is used until the stock reaches the ceiling. If the ceiling is achieved before arriving at the frontier $A'B'$ then gas is extracted at the maximum level $\overline{x}_1$ until the stocks are at a point such as G' which is on the boundary of zone II from section 3. The remaining path is Hotelling. Shadow prices of the two resources are still equal.

When Coal is Abundant

There are three alternative paths to the ceiling with abundant coal, given by a stock located higher than the line $LL'MQ$ shown in Fig.7. These are demarcated by zones IV, V and VI. It is easier to discuss them in reverse order starting with zone VI. In this zone coal is abundant with only a small stock of natural gas lower than $\overline{X}_1^0$. We know from section 3 that once the ceiling is attained both resources are used until natural gas is exhausted. Moreover, because regulation binds, the scarcity rent of coal is lower than that of natural gas. Consider the point *R* in zone 6 where the stock of pollution is exactly at the ceiling. Then by section 3, this point must be on some path HH' that is parallel to AA' and the sequence of extraction from this point is well-defined.

[Fig.7 here]

Let the price of energy at R be given by $p(R)$. Then T^0 is the time at which the ceiling is achieved exactly at location *R*. Since both resources are used from this location, $p(R)=p_1(t)=p_2(t)=\lambda_i-\mu\theta_i, i=1,2$ with $p(R)\in[\overline{p}_1,\overline{p}_2]$. Before T^0 the ceiling is

[15] Instead of the linear specification αZ in (2), any monotonically increasing function of Z will suffice.

non-binding, hence $\nu = 0$ so that $\mu(t) = \mu^0 e^{(\rho+\alpha)t}$. Differentiating $p_i(t)$ with respect to time yields $\dot{p}_i(t) = \frac{d}{dt}[\lambda_i(t) - \mu(t)\theta_i] = \dot{\lambda}_i(t) - \dot{\mu}(t)\theta_i = \rho\lambda_i(t) - (\rho+\alpha)\mu(t)\theta_i$ $= \rho[\lambda_i(t) - \mu(t)\theta_i)] - \alpha\mu(t)\theta_i = \rho p_i(t) - \alpha\mu(t)\theta_i$. As $t \to T^0$, $p_1(T^0) = p_2(T^0)$ and since $\theta_1 < \theta_2$, we get $\dot{p}_1(t) < \dot{p}_2(t)$. That is, just before location *R*, the price of coal rises at a faster rate than that of gas and both are equal at *R*, hence it must be lower than the price of gas. This in turn implies that in the period before the ceiling is hit, only coal is used.

There exists a vector of stocks R' shown in Fig.7 with the initial stock of pollution Z^0 such that coal is used until the ceiling is attained at R. Beyond this point, both resources are used jointly along the curve $H'H$ as in section 3. There is a unique locus of points through R' such that beginning with initial vectors on this curve, coal is used first until the ceiling is reached at some point on HH'. Conversely any starting vector R' in zone VI will imply an exclusive use of coal until the ceiling is reached. Because natural gas is not abundant, it is relatively expensive, hence coal is used on the fast track to the ceiling for the same reason as in zone III.

Suppose that there is a higher stock of natural gas where $X_1^0 \geq \overline{X}_1^0$ but gas is not abundant, as shown by endowments in zone V (see Fig.7). Then as from zone VI, we use coal at the beginning, but because there is a higher endowment of natural gas, some of it is used to get to the ceiling. From an initial stock of resources *S*, coal is used until location S' then natural gas is used exclusively until S'' when the ceiling is reached. Beyond this location, natural gas continues to be used at its maximum level $\overline{x}_1$ and extraction proceeds as in zone IV in section 3. When the vector of stocks reaches the boundary of zone VI, both resources are used until gas is exhausted and there is exclusive coal use.

Why not only coal to the ceiling as in zone IV? Intuitively with increased gas resources, there is competition between the two resources. The benefit of coal use on the access to the ceiling is that it allows for increased pollution and therefore, increased dilution which is costless. However, with higher reserves of gas, its scarcity rent is lower and it is less polluting, so it makes sense to use some gas to ensure continuity of the price path as the pollution stock approaches the ceiling.[16] This is shown in Fig.8. Coal is cheaper at first followed by natural gas, until the ceiling is reached at time T^0.[17]

[Fig.8 here]

Finally consider zone IV where both resources are abundant (Fig.7). Intuitively, as the endowment of natural gas increases, its scarcity rent decreases. Even though $\lambda_1^0 > \lambda_2^0$, large stocks of gas will imply $\lambda_1^0 - \mu^0\theta_1 < \lambda_2^0 - \mu^0\theta_2$. That is during the initial phase, gas is used exclusively all the way to the ceiling.[18] This is followed by maximal use of gas until joint use from the boundary of zone 4. The extraction path is shown starting from an initial endowment V in Fig. 7.

5. Concluding Remarks

We extend Hotelling theory to resources when they are differentiated by their pollution characteristics. Herfindahl showed that when there is cost heterogeneity among resources, extraction follows the "least cost first" principle. We show that when resources are differentiated by pollution there is no such ordering of extraction. Even if there is a "clean" and a "dirty" resource, the order of extraction suggested by Herfindahl breaks down. When the economy starts with the regulated level of pollution, the clean resource is used first, analogous to the Herfindahl principle. However, when the economy starts

[16] It may be useful in empirical work to check the relative duration of these phases by using plausible economic data.

[17] To prove that this is the only possible sequence, we can show that the price of coal is strictly higher than that of natural gas and the differential increases as we move recursively from $T^0 + \Delta_1$ to T^0 (see Fig.8). Then coal can not be used in the preceeding interval. Thus natural gas must be used. Consider the price differential $p_2(t) - p_1(t)$ at any time $\tau \in [0, \Delta_1]$ so that $T^0 + \Delta_1 - \tau \in [T^0, T^0 + \Delta_1]$. That is, τ is measured from $T^0 + \Delta_1$. In this interval by (5), $\bar{p}_1 = \lambda_1 - \mu\theta_1$ which gives $\mu(t) = \dfrac{\lambda_1 - \bar{p}_1}{\theta_1}$ We have

$$p_2(T^0 + \Delta_1 - \tau) - \bar{p}_1 = \lambda_2(T^0 + \Delta_1 - \tau) - \frac{\lambda_1((T^0 + \Delta_1 - \tau) - \bar{p}_1)\theta_2}{\theta_1} - \bar{p}_1$$

$$= \lambda_2^0 e^{\rho(T^0 + \Delta_1 - \tau)} + \frac{\theta_2[\bar{p}_1 - \lambda_1^0 e^{\rho(T^0 + \Delta_1 - \tau)}}{\theta_1} - \bar{p}_1$$ which after some manipulation yields

$$p_2 - \bar{p}_1 = \frac{(\theta_2 - \theta_1)\bar{p}_1}{\theta_1}[1 - e^{-\rho\tau}]$$ so that $$\frac{d}{d\tau}(p_2 - \bar{p}_1) = \frac{(\theta_2 - \theta_1)\bar{p}_1}{\theta_1}[\rho e^{-\rho\tau}] > 0.$$

[18] The case of natural gas use followed by coal to the ceiling can be eliminated because natural gas use at the ceiling can not be preceded by coal use, as discussed earlier for endowments in zone 5.

from a lower level of pollution, it tends to use the dirty resource first and build the pollution stock as quickly as possible. In this manner, it benefits from natural dilution of the pollution stock, which is "free." However, if the stock of the clean resource is large, it is used even during the approach to the ceiling.

When resources are differentiated by cost, under some conditions there may be joint use of two resources, although a complete reversal of comparative advantage over resources is impossible.[19] This paper shows that when resources are differentiated by pollution characteristics, a resource may be used over two disjoint intervals. Coal may be used exclusively at the beginning, followed by exclusive use of gas, then again later in time, the exclusive use of coal, as in zone V. This sort of complete "preference reversal" over resources does not emerge in models with cost differentiation among resources.

Unlike in the general literature following Hotelling, the sequence of extraction in this pollution model depends strongly upon the initial endowment of the resources. A common feature of extraction when both resources are abundant is a turnpike property, in which the two resources are used jointly at their maximal levels at the ceiling, until the clean resource is completely depleted. Which resources will be used to get to the ceiling depends on their relative abundance.

These results have implications for the order of extraction when an economy has to meet environmental goals. For instance, a stated aim of the Kyoto Treaty is to stabilize the atmospheric concentration of carbon at approximately 550 parts per million. Currently it is about 370 parts per million. Since estimated reserves of clean fuels is limited while polluting fuels such as coal are abundant, our results suggest that coal should be used exclusively to get to the ceiling. This seems counter-intuitive, but by getting to the ceiling as quickly as possible, we also dilute a larger part of the pollution stock. However, once the ceiling is achieved, we may use both resources jointly for a period until all gas reserves are exhausted. Beyond this time, only coal will be used until exhaustion and transition to the backstop.

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[19] See Kemp and Long (1980) and Amigues et al (2001) for models in which a low and a high cost resource may be jointly used.

An important assumption in the model is that both resources are homogenous with respect to cost. In future work, it may be useful to generalize Herfindahl's framework to examine how cost heterogeneity interacts with pollution heterogeneity. To keep the analysis tractable, we also use a very specific form of environmental regulation which is perfectly inelastic at some exogenously fixed level. However, more realistic damage functions may be used in later work to explore how the sequence of extraction may be sensitive to the specification of damages. For instance if damages were strictly positive at all levels of the pollution stock, we may not get these sharp transitions between resources. The present paper may thus be thought of as a first step towards increased understanding of how environmental regulation affects the use of nonrenewable resources.

Appendix A

Maximizing Cumulative Extraction of Coal along a Hotelling Path that satisfies the Ceiling Constraint:

For $X^0 \in (X_2^H, X_1^H)$, we have a common scarcity rent $\lambda(X^0) \in (\bar{p}_1, \bar{p}_2)$. Consider the Hotelling path $d(\lambda^0(X^0))$ over the time interval $[0, \Delta_{12})$ during which $p(t) = \lambda^0(X^0)e^{\rho t}$ increases from $\lambda^0(X^0)$ to $\bar{p}_2$, i.e., $\Delta_{12} = \dfrac{\ln \bar{p}_2 - \ln \lambda^0(X^0)}{\rho}$ and the set of paths $\{x_1, x_2\}$ such that $x_1 + x_2 = d(t)$. We show that among these paths starting from $Z^0 = \bar{Z}$ and satisfying the constraint $Z(t) \le \bar{Z}$, $Z(\Delta_{12}) = \bar{Z}$, the path that maximizes the extraction of coal is one for which the ceiling constraint is always binding. That is, we must have $Z(t) = \bar{Z}$ over the entire interval $[0, \Delta_{12})$. Define $\hat{x}_1(t)$ and $\hat{x}_2(t)$ to be the extraction rates of gas and coal when the pollution stock is at the ceiling. The maximization problem can be written as:

$$\underset{x_2}{Maximize} \int_0^{\Delta_{12}} x_2(t)dt$$

subject to

$$\dot{Z}(t) = \theta_1(d(t) - x_2) + \theta_2 x_2 - \alpha Z(t),$$

$$Z^0 = \bar{Z}, \bar{Z} - Z(t) \ge 0, t \in [0, \Delta_{12}), Z(\Delta_{12}) = \bar{Z}, \text{ and}$$

$$d(t) - x_2 \ge 0, \quad x_2 \ge 0.$$

The Lagrangian can be written as

$$L = x_2 + \pi[\theta_1(d - x_2) + \theta_2 x_2 - \alpha Z_t] + \nu[\bar{Z} - Z] + \bar{\gamma}_2[d - x_2] + \underline{\gamma}_2 x_2.$$

The first order conditions are

$$\frac{\partial L}{\partial x_2} = 0 \Leftrightarrow 1 + \pi(\theta_2 - \theta_1) = \bar{\gamma}_2 - \underline{\gamma}_2, \tag{A1}$$

$$\dot{\pi}(t) = -\frac{\partial L}{\partial Z} \Leftrightarrow \dot{\pi}(t) = \alpha\pi + \nu, \tag{A2}$$

$$\nu \geq 0, \ \ \nu[\bar{Z} - Z] = 0 \tag{A3}$$

$$\bar{\gamma}_2 \geq 0, \ \ \bar{\gamma}_2(d - x_2) = 0, \text{ and} \tag{A4}$$

$$\underline{\gamma}_2 \geq 0, \ \ \underline{\gamma}_2 x_2 = 0. \tag{A5}$$

The shadow price of pollution $\pi(t)$ must be non-positive. First we show that $\hat{x}_1(t)$ and $\hat{x}_2(t), t \in (0, \Delta_{12})$ satisfy the above first order conditions (A1-A5). Since $\hat{x}_2(t) \in (0, d(t))$, both $\bar{\gamma}_2(t), \underline{\gamma}_2(t)$ equal zero. Then (A1) implies $\pi(t) = -\frac{1}{\theta_2 - \theta_1} < 0$.

Substituting in (A2) and using $\dot{\pi}(t) = 0$ yields $\nu(t) = \frac{\alpha}{\theta_2 - \theta_1} > 0, t \in (0, \Delta_{12})$.

Suppose that $Z < \bar{Z}$ over some time interval $[0, \delta) \in [0, \Delta_{12}]$. We show that this leads leads to a contradiction or an upward jump in the costate variable $\pi(t)$ which is not feasible. Then $\nu = 0$ and $\dot{\pi}(t) = \alpha\pi$ by (A2) which gives $\pi(t) = \pi^0 e^{\alpha t}$. Since $\pi(t) \leq 0$ suppose $\pi^0 \in (-\frac{1}{\theta_2 - \theta_1}, 0]$. Then $1 + \pi^0 e^{\rho t}(\theta_2 - \theta_1) > 0$ over some interval $(0, \delta), \delta > 0$. From (A1) and (A4), we get $x_2(t) = d(t), t \in [0, \delta)$. But this contradicts $\hat{x}_2(t) < d(t)$. Now let $\pi^0 \leq -\frac{1}{\theta_2 - \theta_1}$. Then $1 + \pi^0 e^{\rho t}(\theta_2 - \theta_1) < 0$ and by (A1) this yields $\bar{\gamma}_2 < \underline{\gamma}_2$ which implies by (A4) and (A5) that $x_2(t) = 0, t \in [0, \delta)$. Since the cumulative extraction of $x_2(t)$ is being maximized, it must be strictly positive over some subinterval of $[0, \Delta_{12})$. Let $\bar{t}$ be the beginning of the first such subinterval.
Then $\bar{t} = \inf\{t : \exists \varepsilon > 0, x_2(\tau) > 0, \tau \in (\bar{t}, \bar{t} + \varepsilon)\}$. If $\delta > \bar{t}$ then $x_2(t) > 0, t \in (\bar{t}, \min\{\bar{t} + \varepsilon, \delta\})$, a contradiction. If $\delta = \bar{t}$, then since $x_2(t) > 0$, we have $\underline{\gamma}_2(t) = 0$ hence $1 + \pi(\theta_2 - \theta_1) = \bar{\gamma}_2 \geq 0, t \in (\bar{t}, \bar{t} + \varepsilon)$. Thus $\lim_{t \uparrow \bar{t}} \pi(t) = \pi^0 e^{\alpha \bar{t}} < -\frac{1}{\theta_2 - \theta_1}$

and $\liminf_{t \downarrow \bar{t}} \pi(t) \geq -\dfrac{1}{\theta_2 - \theta_1}$, so that $\pi(t)$ would have to jump upwards at $t = \bar{t} = \delta$.

Finally if $\delta < \bar{t}$ then again $x_2(t) = 0, t \in (0, \bar{t})$ and $x_2(t) > 0, t \in (\bar{t}, \bar{t} + \varepsilon)$ resulting in the same type of jump at $t = \bar{t}$.

If $Z(t) = \overline{Z}, t \in (0, \delta)$ by the same argument, we can not have $Z(t) < \overline{Z}, t \in (\delta, \delta + \delta')$, $\delta' \in (0, \Delta_{12} - \delta)$. Thus $Z(t) = \overline{Z}, t \in (0, \Delta_{12})$.

Appendix B

Characterization of Optimal Hotelling Paths at the Ceiling starting from zone II:

Consider point *C* in Fig. B1. It is chosen so that the stock of coal at *C* is higher than at *A*. Let $A_C A'_C$ be the translation of the AA' curve in the direction (+1,-1) through *C*. One possible path from *C* is to extract both resources while keeping the stock of pollution at the ceiling, $Z(t) = \overline{Z}$. This is a Hotelling path in which the common scarcity rent λ^0 corresponds to the initial aggregate stock at *C*. This rent must equal the one starting from point D on the AA' curve, since the global aggregate stock is equal for both and the resources are perfect substitutes and the paths are Hotelling. From *C*, extraction proceeds along the $A'_C A_C$ curve. At any point on this curve, extraction rates are exactly equal to the corresponding points on the AA' curve obtained by drawing a 45° line as shown for point *C*. This program ends at point A_C which has the same aggregate stock X_2^H as in point *A*. The price of energy is $\overline{p}_2$, although the residual stock at A_C is lower than X_2^H. In general, any path from A_C to the origin may be followed provided the ceiling constraint is not violated and $x_1 + x_2 = d(\lambda^0 e^{\rho t})$.

[Fig. B1 here]

Since the vector of endowments at *C* is under the AA' curve, there are alternative extraction sequences that will not violate the ceiling. One such path may only use natural gas at first $\left(x_1 = d(\lambda^0 e^{\rho t}), x_2 = 0\right)$. Since the aggregate endowment at *C* is strictly lower than X_1^H (lies left of the 45° line through B'), scarcity rent will be higher at *C* and the allowable extraction rate of natural gas $x_1(t) = d(\lambda^0(X^0))$ lower than $\bar{x}_1$. This path may cross the *AA*′ curve until some point *E*. As the price of the resource increases, extraction of gas decreases, and the stock of pollution also decreases. At *E* the stock of pollution is lower than the ceiling and thus larger than on the path $A'_C A_C$. Coal can be used at rates beginning from *E* to go back to the *AA*′curve at point *F*. Since the aggregate stock is higher at *E* relative to *A*, the common shadow price is lower, hence coal consumption at *E* will be higher than $\bar{x}_2$. The stock of pollution will rise from *E* towards point *F*. The lengths of these two periods can be so chosen that the stock at *F* is exactly $\bar{Z}$. The remaining path may follow the curve *AA*′ until *A* and then use coal until exhaustion.

Yet another alternative may be to use gas until point *G* on the *AA*′curve where $Z(t) < \bar{Z}$ then use coal until some location *H* where $Z(t) = \bar{Z}$. From there extraction can follow the translation of the *AA*′curve through *H*. Many alternative sequences are possible including single or joint use of the two resources such that $\bar{Z}$ is not exceeded. Once the vector of stocks achieves the boundary *AB* of zone I, the proportion of each resource that can be used in response to the common scarcity rent is no longer restrained. For instance, from location A_C, only coal can be used until exhaustion, and the ceiling will not be violated. This is not possible for initial coal stocks larger than X_2^H such as from point *C*.

An important feature of extraction from any location *C* in zone II is that the residual vector of stocks must stay either on or above the $A'_C A_C$ curve for some initial period. Paths such as *CJK* are not allowed since they imply extraction of the polluting resource at rates higher than $\bar{x}_2$ and violation of the ceiling constraint.

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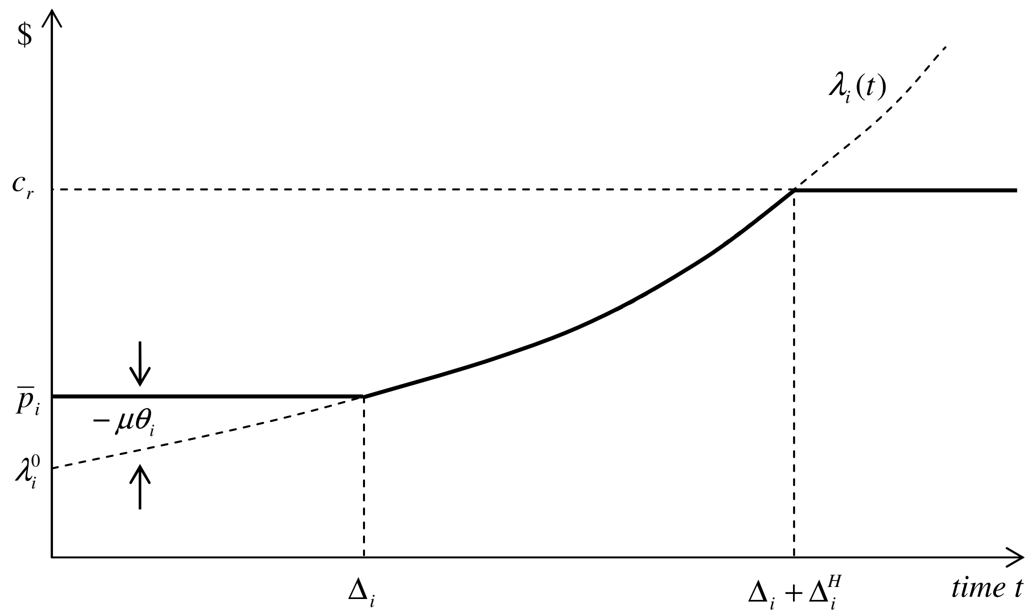

Figure 1. Price path with only one resource

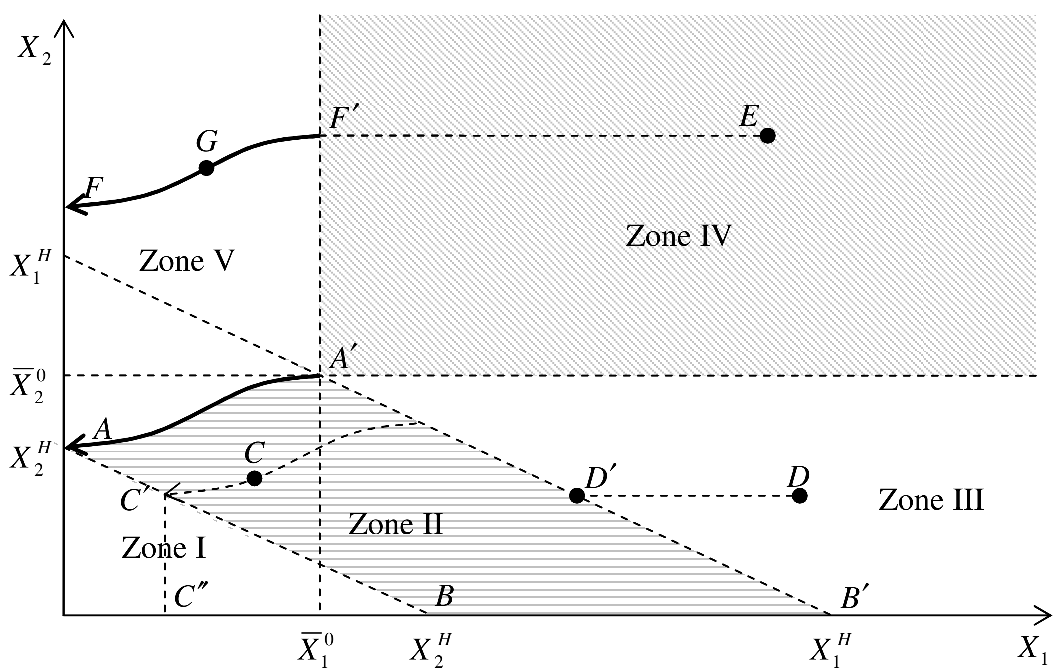

Figure 2. The sequence of extraction depends on initial endowments $(A' : \overline{X}_1^0 + \overline{X}_2^0 = X_1^H)$

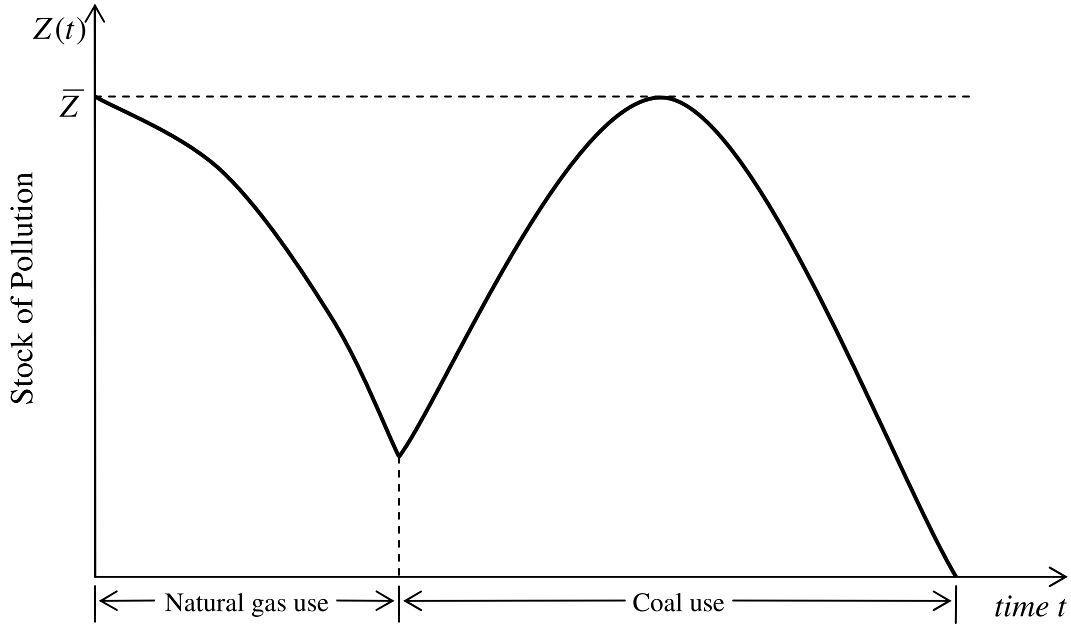

Figure 3. Natural gas is used, then coal. $X^0 \in [X_2^H, X_1^H]$

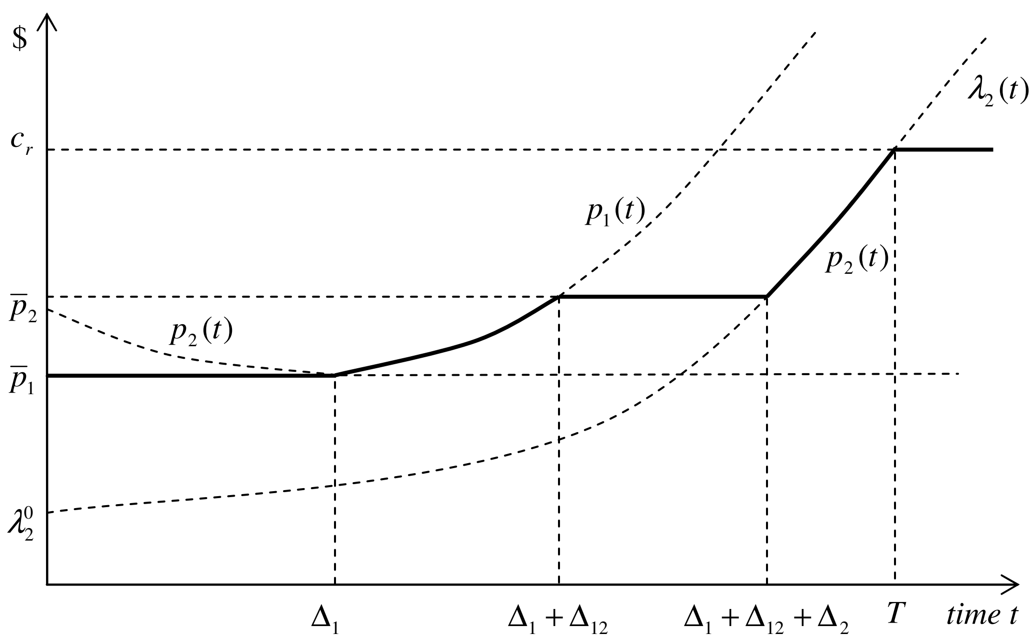

Figure 4. Price paths when both resources are abundant

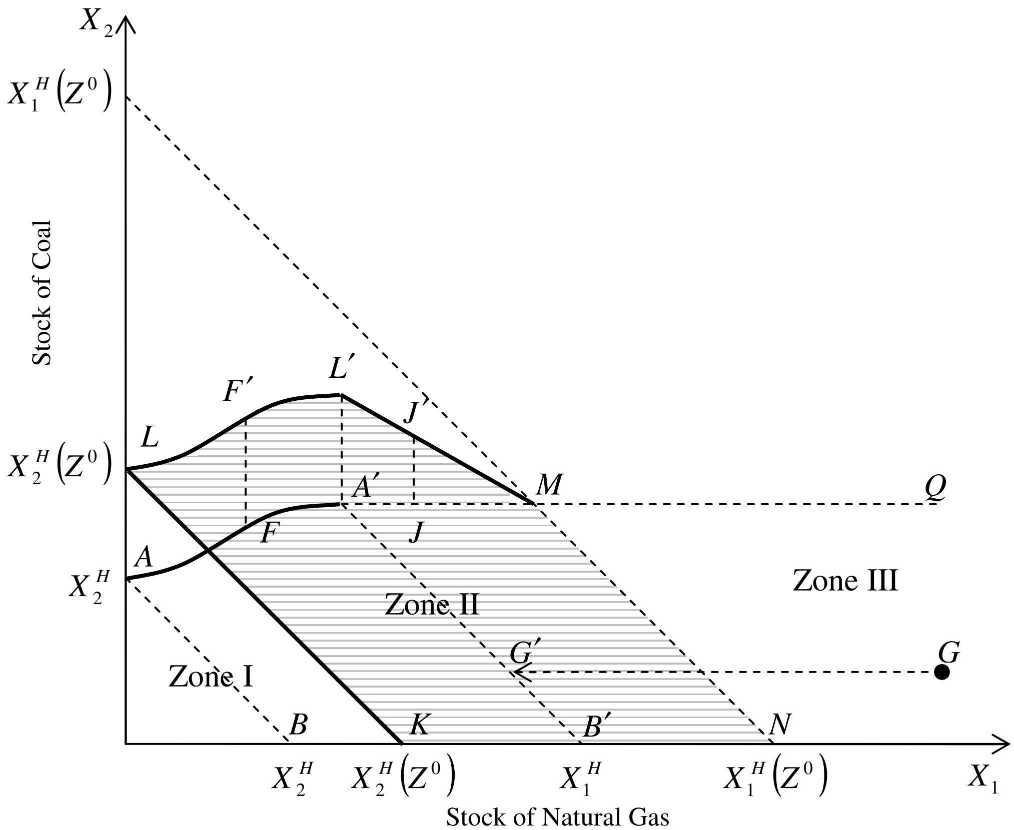

Figure 5. Hotelling paths from below the ceiling

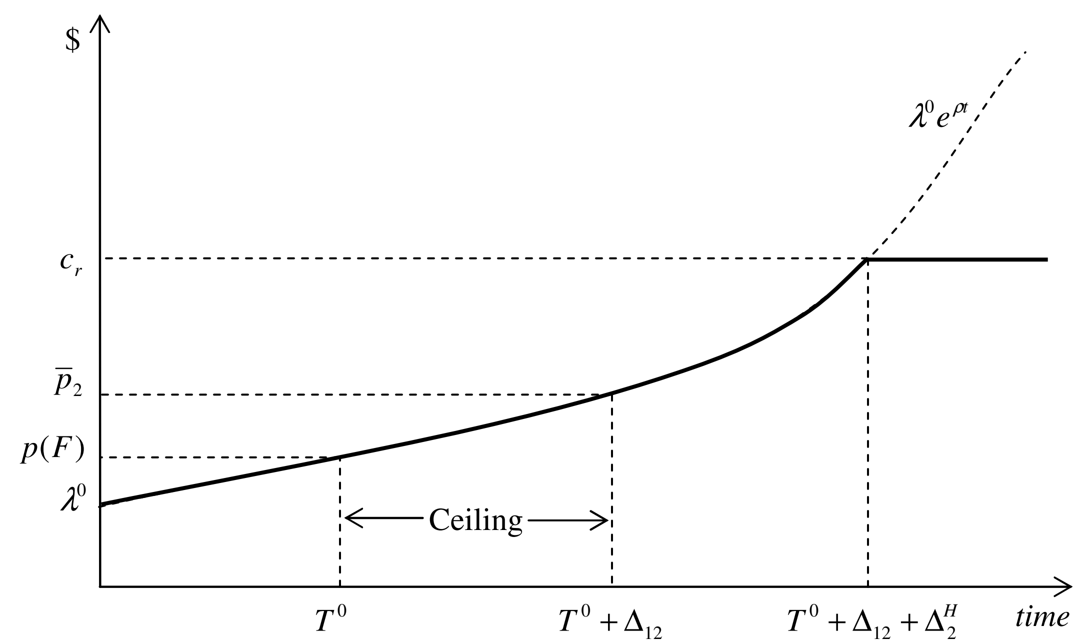

Figure 6. Approach to the ceiling when natural gas is abundant

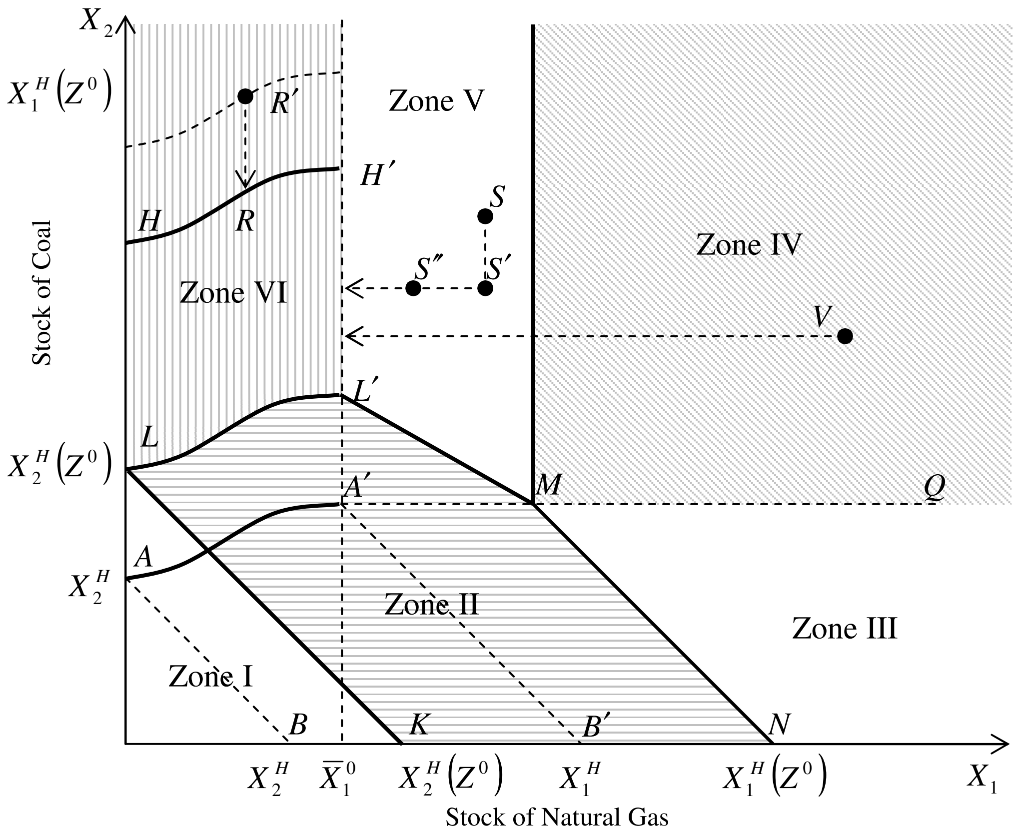

Figure 7. Approach to the ceiling with abundant resources

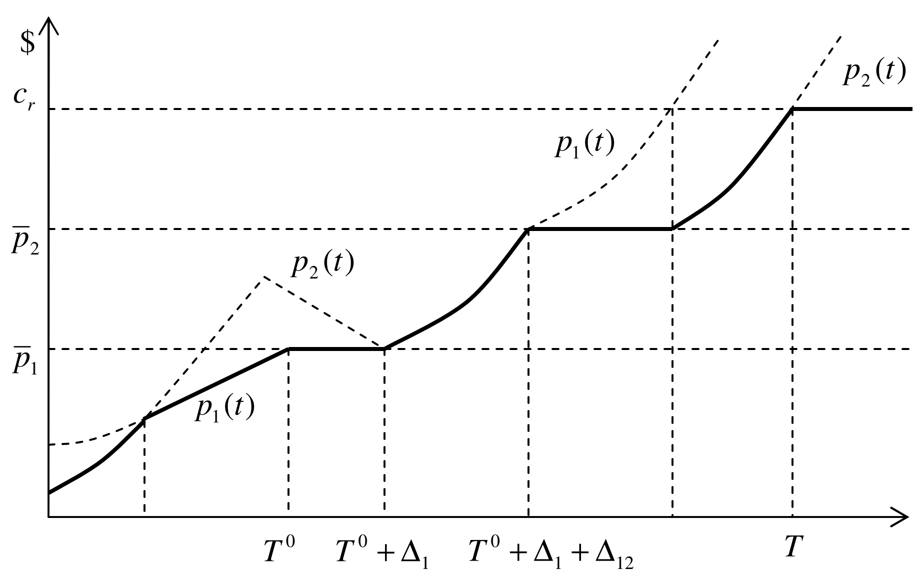

Figure 8. Price paths with abundant coal and limited gas reserves

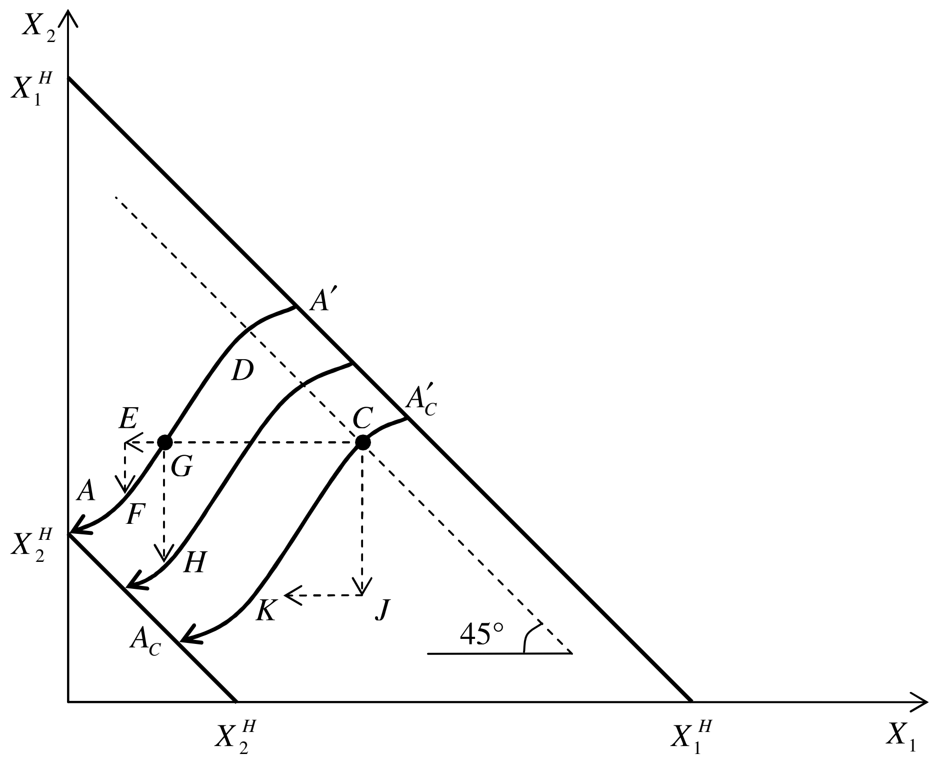

Figure B1. Hotelling paths starting from Zone II