Narrow identities

Partha Dasgupta^{*}

Sanjeev Goyal[†]

December 13, 2008 First draft: December 2008

Abstract

Personal identity has many facets, as it involves race, language, personal interests, religion, and ethnicity, among other attributes. Yet all over the world we see individuals and groups defining themselves in narrow and exclusive terms. This paper develops a simple model of individual incentives and group interests to explain this puzzle.

^{*}Faculty of Economics, University of Cambridge.

[†]Faculty of Economics, University of Cambridge.

We are grateful to Andrea Galeotti, Rachel Kranton and Bryony Reich for very helpful suggestions. We also thank Ana Babus, Andrea Pattachini, Tanmay Shukla, Bauke Visser, and seminar participants at Cambridge and Groningen for comments.

1 Introduction

It is generally agreed that personal identity involves many features – such as race, religion, profession, personal interests, ethnicity, and language – and people share many of them. Yet all over the world we see individuals and groups defining themselves in narrow and exclusive terms.¹ In this paper, our aim is to provide an explanation for this puzzle using a simple model of individual incentives and collective interests.

We take the view that, in day to day life, the different aspects of personality remain latent. Social and economic context present a background against which individuals choose to retain these different possibilities or to commit to one of these possibilities and to renounce the others. We seek to delineate the contexts in which these different outcomes arise and this analysis leads us naturally to an explanation for the salience of birth based criteria – such as caste, ethnicity and race – in the definition of group identity.

We consider a simple model in which groups are engaged in furthering their objectives. Our notion of groups is quite general. A group may refer to people who share a language or a religion and the objective is greater public funding for specific cultural activities. Alternatively, groups may be communities in conflict over land. Finally, a group may a national research body aiming to promote fundamental research. The labels defining the groups depend upon the context; in some cases they are explicitly defined by the context, such as when some public funds are to be allocated for the development of different languages. In other contexts, as for instance, individuals competing for scarce land or mineral resources, the criteria for groups is at the start quite vague and unclear. The definition of the criteria defining a group is a key element in our approach. Once the criteria exists, individuals choose to commit to one or more groups. Individuals choose join one or more groups and this element of choice is the second key element. We interpret being a member of one group as having a *narrow* identity and being a member of many groups as having a *multiple* identity. Once individual choices are made, group sizes are defined and the groups then interact to generate payoffs for each of the individuals. A group's payoff depends on its own size but also on the size of the other group.

We study the process of group formation and relate incentives of individuals to join group(s) with social and group incentives.

¹Section 2 illustrates these points through examples.

We first study the case where a group's payoffs are increasing in its own size as well as increasing in the size of the other group, i.e., the case of positive spillovers. In such situations, multiple identities are in the interests of each group as well as society at large. Moreover, if costs of joining groups are negligible then they are consistent with individual incentives as well. If costs of joining different groups are significant then there is a systematic divergence between individual incentives and group interests: an individual does not take into account the positive effects of his membership on the payoffs of group members and this leads to narrow identities in excess of what is socially desirable. The group may seek to address this by offering subsidies to its members for joining the other group. We therefore offer a simple explanation for the research fellowships offered by research foundations to individuals to visit other groups. It also explains why these fellowships often come with the obligation that the individual has to return to the sponsoring group for a minimum period of time.

We then turn to the study of situations in which a group's payoff is increasing in its own size but decreasing in the size of the other group, i.e., the case of negative spillovers. Our first insight here is that individual choice to join a second group generates a negative externality: the 'original' group incurs a cost, but the cost is shared with other group members. Building on this insight, we identify a class of economic settings in which individuals, left on their own, have a preference for multiple identities while the group itself wants the individual to commit to a single group.

This tension between individual incentives and group interests leads groups to insist on exclusive membership or, in other words, *narrow identity*. We observe that if all except for one group imposes exclusive membership rules then the choice of the last group is immaterial as it does not alter individual possibilities. This shows that universal narrow identities is a stable social outcome. Our result also provides an explanation for our initial observation: definitions of identities are narrow.

We then turn to the welfare implications of narrow identities. We identify a class of social and economic situations in which exclusive memberships lead to socially inefficient outcomes. This class of situations covers those in which the gain from adding a new member to a group exceeds the loss to the other group (due to negative externality). In all these cases, universal multiple identities are socially efficient, but exclusive memberships and narrow identities are a robust outcome as well. This result supports the widely held view in the literature that narrow identities are undesirable and therefore unfortunate.²

We then turn to the implementation of exclusive membership rules. In the presence of intergroup conflict, communication across groups is problematic. Moreover, since individuals desire multiple identities, they have an incentive to misreport their group memberships . So groups need simple mechanisms for verifying membership. This tension naturally leads groups to construct criteria for membership which are outside the control of the individual, unambiguous and exclusive. This helps explain the salience of birth based criteria – such as caste, color and ethnicity – in the definition of identity.

We briefly relate our analysis to key elements of the problems in Rwanda. Ethnic divisions between Hutus and Tutsi's played a central role in the conflict in Rwanda. The historical origins these ethnics distinctions are uncertain and appear at present to be a subject of considerable controversy. Moreover, there are some well documented cases which concern individuals who were prominent in the ruling Hutu group. These individuals were mistaken for being Tutsi's and in spite of their best efforts, they could not convince the Hutus gangs of their 'identity', as they physically resembled Tutsi's and so they were captured and killed by Hutu gangs. This story illustrates one, the salience of ethnicity in definitions of identity, two, the contested origins of identity, and three, the difficulties involved in implementing narrow identity notions, and finally, the use of simple physical markets in resolving difficult individual cases.

The notion of identity has traditionally been a major theme of research in social psychology. In influential work, Tajfel and Turner (1986) develop three ideas on identity: one, identity is multidimensional, two, it is fluid, and three, it arises out of choice. We take these ideas as our point of departure and develop a model to explain the salience of birth based and exclusive criteria in the definition of identity.

In recent years, identity has been extensively studied in economics; see e.g., Akerlof and Kranton (2000), Alessina and La Ferarra (2000) and Alesina, Baqir, and Easterley (1999) and Bisin and Verdier (2000). Most of this work takes a standard form of identity – ethnicity, race or religion – as given and examines its implications for economic outcomes. For example, in Akerlof and Kranton (2000) identity is taken as an exogenously specified argument of the utility function, and plays the role of a socially constructed point of reference. By contrast,

²See for example the discussions in Appiah (2004) and Sen (2006).

our interest is in the origins of these reference points, in particular, in the form which these reference points take. In a recent book, Sen (2006) argues that individuals have multiple identities and that claims for special and narrow identities are somehow unnatural. However, a casual examination of much of recent history suggests that individuals and societies often do choose to define themselves rather narrowly. This observation placed in the context of the rich literature in social psychology leads us to examine the economic circumstances under which narrow notions of identities emerge and dominate our perceptions of who we are.

We now discuss some recent work in economics which takes a group formation approach to understanding the importance of ethnicity and religion. Esteban and Ray (2007) study a model of group formation where groups which are formed compete for share of public money. They shows that it is more attractive for the rich and the poor of a community to form a group rather than the rich of the two communities to form a group. They interpret this result as saying that ethnicity is salient over class. Our paper allows for individuals to choose between a narrow and a broad identity; by contrast, Esteban and Ray assume that individuals have ex-ante multiple identities – an individual has an ethnic identity as well as a class identity – but is *obliged* to choose between two exclusive notions of identity. Their concerns and main findings are thus orthogonal to ours.³

The paper is organized as follows. Section 2 discusses a number of examples to illustrate the salience of birth based exclusive criteria in the definition of identity. Section 3 sets out our basic model while sections 4–?? analyze this model. Section 7 concludes.

2 Examples

This section discusses a wide range of examples which bring out three general points. One, individuals sharing a number of social and cultural characteristics often separate themselves into exclusive hostile groups. Two, the narrow characteristics which are chosen are easily observable and exclusive. We conclude by discussing the social construction of identity.

Language as identity: In many contexts, individuals which share a number of different characteristics such as religion, ethnicity, food habits, come into conflict over the role of language.

³Similarly, in a recent paper Dasgupta and Kanbur (2007) study a model of communities and class, in which members of a community have access to a privately provided public good, which non-members are excluded from. In such a setting, the impact of changes in the income distribution depends very much on whether the income changes within or across communities.

This conflict takes place against a background of allocation of public funds or competition for cultural space. The following examples illustrate:

- Linguistic reorganization of state boundaries in India: This involved a redrawing of the boundaries of states which existed at the time of independence from Great Britian in 1947. It was a major change in policy and led, over the years, to the creation of a number of new states. This change can be traced back to the large scale agitation for redrawing of the state boundary of the state of Madras, initiated by Telugu speakers in the early 1950's. Telugu speakers are similar to Tamil speakers in religion – most of them being Hindus – and share a variety of other cultural attributes.
- 2. *Federal Belgium:* Flemish and French speakers share a common religion and ethnicity but have been in conflict over public funds and state power for several decades. This conflict has led to a profound transformation of the nature and powers of central Belgian government.
- 3. *Independence movement in Ireland:* Attempts were made to popularize Gaelic as a strategy to bridge the divide between Catholics and Protestants in the struggle for Irish independence. This is an instance of creating a linguistic identity which overarches other differences.

Religion and sects: Religion has served as a key organizing principle in social and economic conflict throughout history. It also appears to be a major factor in the modern age. The following examples illustrate.

- 1. *Religious divide in Iraq:* Most Iraqis are Muslims, but within this religion there is a division into two groups, the Shias and the Sunnis. The division between Shias and Sunnis among people who share a common ethnicity (they are Arabs), religion (they are Muslims) and a range of cultural traits, in a key element which lies at the center of the political turmoil in Iraq.
- 2. Partition of India: The partition of India into two countries India and Pakistan in 1947 (which subsequently became three countries as the Eastern part of Pakistan seceded and became an independent state Bangladesh, in 1972), is a dramatic instance of large scale classification and consequent separation of people along narrow and exclusive lines. In this partition, at the local level, communities of people sharing very similar

characteristics were carved out into separate states which were assigned to India and Pakistan. For example, the erstwhile state of Bengal was carved into two parts, an Eastern part which was allotted to Pakistan, while the western part was allotted to India. In the east part, a majority of people were Muslim, while in the western part a majority of people were Hindu. But people in the state shared a common Bengali culture along many dimensions, which included ethnicity, language and food habits.

Ethnicity: Ethnicity – reflected in color, height and other physical distinctions between people – has played a major role in shaping social attitudes and is also a key dimension along which groups define themselves in contexts of acute conflict.

- 1. *The Rwanda Conflict:* The Hutus and Tutsi's share religion and language, and a variety of other cultural traits. The gruesome conflict between them between highlights the salience of ethnicity as well as its contested nature.
- 2. Secession and partition in Yugoslavia: Conflict between Serbs, Croats, and Bosnian's, over territory and political power.

The social construction of identity and its dependence on contextual factors has been studied extensively in social psychology. The Robbers Cave Experiment carried out by Muzafer Sharif and his associates in 1954 is very well known in this literature and we discuss it briefly. *The social construction of identity:* We conclude with a discussion of a well known experiment on the social construction of group and personal identity, the *Robbers Cave experiment (1954)*. In this experiment, 22 teenage boys from a similar background (lower middle class, Protestant, same year of schooling, well settled families) were brought together in the Robbers Cave State Park in Okhlahoma. The boys were assigned to two groups of 11.

For the first few weeks, these groups were kept separate. In this period, the boys were asked to pursue activities which involved common goals and required cooperation at the group level. It was observed that very soon each group evolved a hierarchy and behavioral norms. Gradually, these groups were made aware of each other. They entered into competitive activities with a trophy as a prize for the group which accumulated the most points. A key finding of the study was that these otherwise homogeneous individuals quickly formed strong and aggressive group identities. Each group then took on aggressive position vis-a-vis the other group: this was reflected in burning the flag of the other group, refusing to eat together, singing derogatory songs about the other group etc. In the last stage of the experiment, the two groups were brought together to pursue superordinate goals, i.e., goals which required cooperation across groups. Over time, participation in these activities led to an abatement in group aggression.

3 A simple model

Suppose that there are $N = \{1, 2, ...n\}, n \ge 2$ individuals and they each choose to belong to any subset of groups $M = \{A, B\}$. The payoff to a group depends on its own size as well as the size of the other group. We will also assume that, within a group, the group payoff is equally divided among all the members.

The group membership game is as follows: all players simultaneously choose their membership strategy. For player *i* let her strategy $s_i \in S = \{\{A\}, \{B\}, \{A, B\}\}$. Let $s = \{s_1, s_2, ..., s_n\}$ be the strategy profile and let $S = \prod_{i \in N} S$, be the set of all strategy profiles. Define $K_A(s)$ as the number of players who choose A and $K_B(s)$ as the number of players who choose B in profile s. The membership of each group ranges from 0 to n. For any allocation of individuals between the two groups $\{x, y\}$, R(x, y) is the aggregate surplus of a group with x members when the other group has y members. We turn next to a player's payoffs. Given a strategy profile s a player i's payoffs are given by:

$$\Pi_i(s_i, s_{-i}) = \frac{\mathbf{1}_{s_i(A)}}{F(K_A(s))} R(K_A(s), K_B(s)) + \frac{\mathbf{1}_{s_i(B)}}{F(K_B(s))} R(K_B(s), K_A(s))$$
(1)

where $\mathbf{1}_{s_i(A)}$ is the indicator function for membership in group A under strategy s_i , and $\mathbf{1}_{s_i(B)}$ is the indicator function for membership in group B. The function F(.) reflects the rules of payoff division within a group. We will focus on two polar cases: one, equal division within the group, $F(K_i(s)) = K_i(s)$, and two, pure public good, $F(K_i(s)) = 1$.

A strategy profile s^* is a Nash equilibrium if for each player i, $\Pi_i(s_i^*, s_{-i}^*) \ge \Pi_i(s_i, s_{-i}^*)$, for all $s_i \in S$.

Assume that R(.,.) is (weakly) increasing in the first argument. Also assume that R(0, y) = 0, for all $y \in \mathbb{Z}$. Define the total social surplus as S(x, y) = R(x, y) + R(y, x). A strategy profile s yields a configuration (x, y) which yields the highest social surplus, $S(x, y) \ge S(x', y')$, for all (x, y).

3.1 Examples

The following examples illustrate the range of situations which are covered by our model.

Example 1 Independent groups

When R(x, y) = R(x) the payoffs to a group are independent of the membership in the other group. This constitutes a special case of our model.

Example 2 Positive spillovers: Promotion of Fundamental Research

A group is a research organization promoting fundamental research, e.g., a scientific laboratory or a science foundation. Each member who joins the group brings a special skill to the group and the combination of the skills gives rise to inventions. Suppose that there is knowledge spillover across groups. The payoffs to a group A faced with memberships (K_A, K_B) are given by:

$$R(K_A, K_B) = \begin{cases} \frac{[D+K_A(s)+\beta K_B(s)]^2}{4} & \text{if } K_A > 0\\ 0 & \text{if } K_A = 0. \end{cases}$$

where D > 0 is a positive parameter and $\beta > 0$ reflects the magnitude of spillover. Clearly a group's payoff is increasing in own as well as other group membership. Moreover, marginal payoffs increase in own membership are also increasing in own group as well as other group size.

In this setting, an individual gains by joining a second group, his current groups gains by dual membership due to positive spillovers and so social, group and individual interests all point toward universal multiple membership. Notice that in this example, there is no cost to joining a group. If joining a group entails personal costs while the returns are shared with the group then we face a familiar situation of positive externalities and individuals will typically provide too little of membership.

Example 3 Negative spillovers: conflict over land, minerals and public money

There is a fixed resource available, and groups compete for a share of this resource. The resource may be land or minerals and the groups are tribes, ethnicities or religions. Alternatively the resource may be public funds for cultural activities, while the groups are language groups. The resource may be market share and the group may be an alliance of firms seeking an innovation.

Individuals provide ideas and labor; so a larger group is at an advantage, and larger opponent group is a disadvantage. The payoffs to group A are:

$$R(K_A, K_B) = \begin{cases} \frac{[D+2K_A(s)-K_B(s)]^2}{9} & \text{if } K_A, K_B > 0\\ \frac{[D+K_A(s)]^2}{4} & \text{if } K_A > 0, K_B = 0\\ 0 & \text{if } K_A = 0. \end{cases}$$

The payoffs to group B are analogous. The payoffs are increasing in own members and decreasing in membership of other group. The payoff to player i under a strategy profile $s = (s_i, s_{-i})$ are given by:

$$\Pi_i(s_i, s_{-i}) = \frac{\mathbf{1}_{s_i(A)}}{K_A(s)} R(K_A, K_b) + \frac{\mathbf{1}_{s_i(B)}}{K_B(s)} R(K_B, K_A)$$
(2)

A player only earns payoffs from a group if he joins the group. What are the incentives of an individual player? Let us start with the case where all players join group A, and we now ask if player i would gain by also joining group B. The payoff to player i from being in a group with n players, while the other group is empty is:

$$\Pi_i(s_i, s_{-i}) = \frac{1}{n} \frac{[D+n]^2}{4}$$
(3)

In this situation, the payoff to player i if he joins group B in addition to A is:

$$\Pi_i(s_i, s_{-i}) = \frac{1}{n} \frac{[D+2n-1]^2}{9} + \frac{[D+2-n]^2}{9}$$
(4)

It can be checked that if n is relatively small compared to D then a player strictly gains by becoming a member of both groups.

Next consider the case where all players have joined both groups. Is this an equilibrium? The payoffs to a player are:

$$\Pi_i(s_i, s_{-i}) = \frac{2}{n} \frac{[D+n]^2}{9}$$
(5)

Exiting from one group leads to the following payoff:

$$\Pi_i(s_i, s_{-i}) = \frac{1}{n} \frac{[D+1+n]^2}{9} \tag{6}$$

It is now easily verified that retaining membership of both groups is optimal so long as $n \ge 3$.

However, social welfare has the following ranking: S(n,0) = R(n,0) > 2R(n,n) = S(n,n). Thus, there is a conflict between social and individual incentives.

We now turn to the issue of whether optimal group formations can be sustained in equilibrium? To fix ideas consider the case n = 2. The social welfare levels of the different configurations are given by:

$$S(2,0) = \frac{(D+2)^2}{4} \quad S(1,1) = 2\frac{(D+1)^2}{9}$$
$$S(2,1) = \frac{(D+3)^2}{9} + \frac{D^2}{9} \quad S(2,2,) = 2\frac{(D+2)^2}{9} \tag{7}$$

Thus (2,0) is socially optimal. Can this be sustained in equilibrium? Simple calculations reveal that if D > 4 then the *unique* equilibrium is (2,2). The key to understanding why (2,0) is not an equilibrium is the following: when a player becomes a member of a second group the loss to the current group is shared with the other member, while the gain to joining a new group accrues fully to this player. So this player underestimates the costs of dual membership. This gives rise to excessive membership in equilibrium.

Example 4 Indirect negative spillovers: private provision of local public goods

A group consists of individuals who provide time and effort for group specific local public good. Individuals have a fixed budget and allocate allocate resources equally the groups they join. Let n_A be the number of individuals who join solely group A, n_B be the number of individuals who join group B solely, and n_{AB} be the number of individuals who join both groups. The payoff to group A are:

$$\hat{R}(s) = f(n_A + \frac{1}{2}n_{AB})$$
(8)

where f(.) is increasing. The payoff to player *i* under a strategy profile $s = (s_i, s_{-i})$ is:

$$\Pi_i(s_i, s_{-i}) = \hat{R}_A(s) \mathbf{1}_{s_i(A)} + \hat{R}_B(s) \mathbf{1}_{s_i(B)}.$$
(9)

So when a person switches to the other group the group loses 1 unit of contribution, while if a person moves from sole membership to dual membership then the group loses 1/2 contribution. Thus the payoff of a group is negatively affected by the size of the other group. The allocation of effort across groups is a reduced form of the model in which individuals choose groups and then choose effort level in different groups. This model shares some features with the model of religious sects developed in Iannaconi (1992).

Finally observe that this example corresponds to the case where $F(K_i(s)) = 1$, and the payoff takes the form of a pure local public good, i.e., there is no congestion.

4 Identity: multiple vs exclusive

This section studies the relation between socially efficient and Nash equilibrium construction of social identity. An outcome is said to exhibit *narrow* identities if all individuals join a single group, while it is said to exhibit *multiple* identities if some individuals join both groups. We focus on the case where $F(K_i(s)) = K_i(s)$.⁴

It is useful to start with the benchmark case in which one group's payoffs are independent of the size of the other group. In this case R(x, y) = R(x), for all y; recall that payoffs are increasing in own membership $R(x + 1) \ge R(x)$, for all $x \in \{0, 1, ..., n\}$. The following result is then immediate.

Proposition 1 Suppose R(x, y) = R(x) and $R(x + 1) \ge R(x)$. Then universal multiple identities is socially optimal. This group formation is also an equilibrium. If R(x) is strictly increasing in x then universal multiple identities is uniquely efficient and also the unique equilibrium.

While group reward is increasing in own membership we do not have any restrictions on the per capita payoffs; they may be rising or falling or indeed the maximum may be attained at

⁴The case of pure public goods, $F(K_i(s)) = 1$, for all $K_i(s)$ is of independent interest and is taken up in section ??.

an intermediate point. Thus while there may be no conflict between social and individual incentives, there could still be a conflict between the incentives of insiders in a group and players outside the group who want to join it.

We now turn to the interesting case where payoffs of a group are related to the size of both groups. Consider examples 2 and 3. There are two aspects of the examples we want to bring out: one, the relation between aggregate payoffs and groups sizes and two, the relation between marginal payoffs and group sizes. With regard to the former relation we note that in example 2, group payoffs are increasing in own size and in the size of the other group, while in example 3 group payoffs are increasing in own group size, they are falling in other group size. With regard to marginal returns, in example 2 we note that marginal payoff to a new member is increasing in the size of the other group, while in example 3, the marginal gains from a new member are falling with an increase in other group size. The concepts of positive and negative spillovers and strategic complements and substitutes capture these different effects.

Definition 1 A game exhibits positive (negative) spillover if the payoffs of a group are increasing (decreasing) in the size of the other group. Formally, positive spillover obtains if for all $(x, y) \in \mathbb{Z}^2_+$, $R(x, y + 1) \ge R(x, y)$, while negative spillover obtains if for all $(x, y) \in \mathbb{Z}^2_+$, $R(x, y + 1) \ge R(x, y)$.

The following result summarizes our analysis of games with positive spillovers.

Proposition 2 If group payoffs are increasing in own group size and exhibit positive spillover then universal multiple identities is socially efficient as well as an equilibrium.

Proof: Consider social efficiency. Starting at any (x, y) < (n, n) the payoffs of both groups can be raised by increasing identities of any group. It follows that universal multiple identities is socially optimal. Similarly, starting at any (x, y) < (n, n), a player *i* has an incentive to join both groups, and so universal identities is an equilibrium. If the payoff gains to own group membership are strict then universal multiple membership is uniquely optimal as well as the unique equilibrium.

XYZ.

We now turn to the case where the spillover across groups is negative. We will present two results. The first result covers the case when the marginal returns to increase in own group size exceed the fall in payoff of the other group. **Proposition 3** Suppose group payoff is increasing in own group size and exhibits negative spillover. If $R(x+1,y) - R(x,y) \ge R(y,x) - R(y,x+1)$, for all $x < n, y \le n$, then universal multiple identities is socially efficient and also an equilibrium. If this inequality is strict then universal multiple memberships is the unique equilibrium and socially efficient outcome.

Proof: The proof for social optimality is straightforward: suppose that some $(x, y) \neq (n, n)$ is socially optimal. Social welfare is given by S(x, y). Suppose, without loss of generality, that x < n and consider the configuration (x + 1, y).

$$S(x+1,y) = R(x+1,y) + R(y,x+1)$$
(10)

The hypothesis that $R(x+1, y) - R(x, y) \ge R(y, x) - R(y, x+1)$ implies that $S(x+1, y) \ge S(x, y)$. Iterating on the argument yields the first part of the result.

Consider next the equilibrium statement. Start with the configuration (n, n); Individual payoff is given by 2R(n, n)/n. Next consider deviations by an individual; suppose that a player withdraws from one of the two groups. Then his payoff is given by R(n, n - 1)/n. Note that $S(n, n) = 2R(n, n) \ge S(x, y)$, for all $(x, y) \le (n, n)$. In particular then S(n, n) = $2R(n, n) \ge R(n, n - 1)$. No individual has an incentive to deviate from the configuration (n, n).

We now take up the uniqueness result. First observe that unique efficient outcome result follows from the argument above plus the strictness of the inequality. Next consider the equilibrium result.

We have already shown that (n, n) is an equilibrium. We now establish uniqueness. Suppose there is an equilibrium configuration $(x, y) \neq (n, n)$. First suppose that x = n while y < n. The payoff to a player who is a member of one group only is R(n, y)/n. A deviation by a player which involves joining both groups yields this player:

$$\frac{1}{n}R(n,y+1) + \frac{1}{y+1}R(y+1,n)$$
(11)

A deviation is strictly profitable if

$$\frac{1}{n}R(n,y+1) + \frac{1}{y+1}R(y+1,n) > \frac{1}{n}R(n,y)$$
(12)

which is true if:

$$\frac{1}{y+1}R(y+1,n) > \frac{1}{n}[R(n,y) - R(n,y+1)].$$
(13)

Since $y + 1 \le n$, this inequality is satisfied if:

$$\frac{1}{n}R(y+1,n) > \frac{1}{n}[R(n,y) - R(n,y+1)]$$
(14)

This last inequality is satisfied since R(y+1,n)+R(n,y+1) > R(n,y)+R(y,n) and $R(x,y) \ge 0$, for all $0 \le x, y \le n$.

We now take up the case where x, y < n. Without loss of generality suppose $x \ge y$. The case where $x \ge y + 1$ can be proved using a variation of the argument above. We turn to the case x = y.

A deviation in which a player in group A also joins group B is profitable if:

$$\frac{1}{y+1}R(y+1,x) > \frac{1}{x}[R(x,y) - R(x,y+1)].$$
(15)

Noting that x = y and rewriting, we get

$$xR(y+1,x) > (x+1)[R(x,y) - R(x,y+1)].$$
(16)

Rearranging terms we get:

$$x[R(y+1,x) + R(x,y+1)] > xR(x,y) + [R(x,y) - R(x,y+1)].$$
(17)

which is satisfied since x[R(y+1,x)+R(x,y+1)] > x[R(x,y)+R(y,x)] under our hypothesis, R(y,x) = R(x,y), and $R(x,y+1) \ge 0$.

This result suggests that if negative spillovers are modest relative to the positive effects of own group size effects then social efficiency dictates multiple identities and this is also the unique equilibrium. While there is congruence between aggregate social and individual incentives, there may well be a tension between group incentives and individual incentives. This is because a group always wants the other group to be smaller due to negative spillovers. We discuss this tension further in section 5 below.

We now turn our attention to games in which own group size effects may be weaker than negative effects on other group. In some contexts, it is reasonable to suppose that competition among groups grows in intensity and wastefulness as groups get more equal. This situation is reflected in the following assumption: positive own size effects dominate negative spillover on other group if the growing group is larger. The following result considers socially efficient networks.

Proposition 4 Suppose group payoff is increasing in own group size and exhibits negative spillover. Suppose that for $0 \le x, y \le n$, $R(x + 1, y) - R(x, y) \ge R(y, x) - R(y, x + 1)$ if and only if $x \ge y$, then narrow identities with all players joining one group is socially efficient. If the inequality in payoffs is strict then it is the unique socially optimal outcome.

Proof: Start with any profile (x, y) and set $x \ge y$, without loss of generality. Suppose to start that x < n. Then it follows from the hypothesis $R(x + 1, y) - R(x, y) \ge R(y, x) - R(y, x + 1)$ for $x \ge y$ that $S(x + 1, y) \ge S(x, y)$. Iterate on this argument and we arrive at S(n, y), where at each step social welfare increases weakly. Note that by hypothesis $n \ge y$. Next lower y, and note by the hypothesis $R(x+1, y) - R(x, y) \ge R(y, x) - R(y, x + 1)$ if and only if $x \ge y$; so R(y, x) - R(y - 1, x) < R(n, y - 1) - R(n, y) which implies that S(n, y - 1) > S(n, y). Iterate on this and we arrive at S(n, 0), where at each step we have increased the payoff weakly. This completes the argument.

We now turn to individual incentives. Recall, that payoffs to a player i given profile s are:

$$\Pi_i(s_i, s_{-i}) = \frac{\mathbf{1}_{s_i(A)}}{K_A(s)} R(K_A, K_B) + \frac{\mathbf{1}_{s_i(B)}}{K_B(s)} R(K_B, K_A).$$
(18)

Start with the case of (n, 0). In this configuration a player earns R(n, 0)/n. If she deviates and joins both groups then she will earn,

$$\frac{1}{n}R(n,1) + R(1,n).$$
(19)

Thus (n, 0) is an equilibrium if and only if

$$\frac{1}{n}R(n,1) + R(1,n) \le \frac{R(n,0)}{n}.$$
(20)

This can be rewritten as

$$\frac{1}{n}[R(n,0) - R(n,1)] \ge R(1,n)$$
(21)

Similarly, we can check that (n, n) is an equilibrium if and only if,

$$R(n,n) \ge \frac{1}{2}R(n,n-1).$$
 (22)

A comparison of Proposition 3 and 4 is worthwhile as it brings out the role of negative spillovers nicely. The hypothesis in Proposition 3 says that the gain from an extra member is greater than the negative externality on the other group imposed by this move. In this case, social surplus grows as individuals subscribe to both groups. However, in Proposition 4 this inequality only holds if and only if the group gaining new members is the larger one: so to raise social surplus, we make a large group still larger by taking away individuals from the smaller group.

Let us now compare aggregate returns and individual incentives in games with negative spillovers and in cases where the hypothesis of Proposition 4 hold. In this case, (n, 0) is socially optimal and so $R(1, n) \leq [R(n, 0) - R(n, 1)]$. The divergence between social and private incentives is clear: when a player joins a second group, she shares the loss with the existing group but gets the full share of the gain from the new group. This creates incentives for multiple membership in excess of what is socially desirable. Next we ask if the converse is possible? Suppose universal multiple identities is socially optimal; do players always have an incentive to join both groups? The payoff to a player in the (n, n) configuration is 2R(n, n)/n. The only deviation involves withdrawing from one group and remaining a member of only one group and the payoff from this deviation is R(n, n - 1)/n. For this to be optimal it must be the case that 2R(n, n) < R(n, n - 1). This however contradicts the social optimality of (n, n). Thus (n, n) is an equilibrium outcome as well. We have thus shown that if (n, n) is socially optimal then it is also an equilibrium. The following result summarizes this discussion.

Proposition 5 If universal multiple identities is socially efficient then it is also an equilibrium. Suppose hypotheses of Proposition 4 hold. There are excessive incentives for multiple identities relative to what is socially optimal if $\frac{1}{n}[R(n,0) - R(n,1)] < R(1,n) \leq [R(n,0) - R(n,1)]$.

Let us briefly consider example 3 again. In this example if $D^2 + 2D > 1$ and D < 1/2 then the hypotheses of Proposition 4 are satisfied but (2, 2) is an equilibrium. The intuition turns on the negative externality generated by dual identity for existing group members.

5 The incentives of groups

This section studies the relation between individual incentives and the interests of groups. The first issue is what do groups care about? There are different perspectives a group can take: for instance, a group may wish to maximize the size of the cake it gets or it may wish to maximize the average payoff of its members. To fix ideas, we will also suppose that groups wish to maximize aggregate group payoff. This appears to be a reasonable assumption in a context of conflict: groups would like to maximize their share of resources or of public funds.

In a game with positive spillover (and with positive size effect), a group gains from having more members and also from a member joining the other group. Proposition 2 suggests that this is also in the interests of the individual. Thus, in games with positive spillovers multiple identities are appealing to the group as well as the individual.

We turn next to games with negative spillover. To get a first impression of the tension between individual and social incentives here, we revisit example 3. In this example, with n = 2 and D > 4, the unique equilibrium is (2, 2); however, in this equilibrium, the payoff is lower than the payoff in the configuration (2, 0). Recall, a group realizes that its members have an incentive to join the other group as the negative effect on the group is shared among the current members, while the benefits are not. A natural response of the group would be to impose an *exclusive membership* rule. What is the equilibrium if both groups use this rule? There are two equilibria: (2,0) and (0,2). In these equilibria, individual players as well the active group fare better than in the non-exclusive rules equilibrium. Moreover, if the equilibria are equally likely then a group gains in an ex-ante sense, as well. This illustrates how exclusive membership rules can lead to a "better" outcomes.

We now examine the scope of such exclusive membership rules, more generally. Our first observation is that if a group imposes an exclusive membership rule then the choice of rule for the other group – whether it is exclusive or not – is immaterial; this suggests that exclusive membership rules observed by both groups constitutes an 'equilibrium' in membership rules. This observation points to a more general point: if all groups except one choose exclusive membership rules then the choice of this last group makes no difference to the options available to the individuals and hence all groups imposing a exclusive membership rule is *always* a Nash equilibrium in a game of membership rules.

What are the welfare and equilibrium implications of exclusive membership rules? In games

which satisfy the hypotheses of Proposition 3, universal multiple membership is attractive in the aggregate as well as incentive compatible. How about the interests of the group? Due to negative spillovers, a group would like larger own size and smaller size for the other group; in particular, a group would like all players to be its members exclusively. What happens in equilibrium? The following result covers the case where returns to own group size are convex.

Proposition 6 Suppose R(x, y) is increasing and convex in x, for all $y \in \{0, ..., n\}$ and exhibits negative spillover. Then there exist two equilibria under exclusive membership rules corresponding to the configurations, (0, n) and (n, 0).

Proof: We first show that (n, 0) is an equilibrium. The payoff to a player in the (n, 0) configuration is R(n, 0)/n. The payoff from a deviation, which entails switching to the other group, is R(1, n - 1). From negative spillovers, we know that R(1, n - 1) < R(1, 0). Since R(0, 0) = 0, from convexity of R(x, y) in x we know that nR(1, 0) < R(n, 0). Thus (n, 0) [and (0, n)] are equilibria of the membership game under the exclusive membership rule.

We next want to show that they are the only equilibria in this game. Consider a configuration (x, y) and suppose without loss of generality that $x \ge y$. We will show that $(x, y) \ne (n, 0)$ is not an equilibrium. First consider the case x = y. Payoffs to a player in group A are R(x, x)/x, while payoffs to deviating to other group are R(x + 1, x - 1)/x + 1. Note that

$$\frac{1}{x}R(x,x) < \frac{1}{x+1}R(x+1,x) < \frac{1}{x+1}R(x+1,x-1),$$
(23)

where the first inequality is due to increasing and convex returns in x, while the second inequality is due to negative spillover. Next consider the case x = y + 1. Note that

$$\frac{1}{x+1}R(x+1,y-1) > \frac{1}{x}R(x,y-1) > \frac{1}{x}R(x,y) > \frac{1}{y}R(y,y) > \frac{1}{y}R(y,x).$$
(24)

where the first inequality follows from hypotheses that payoffs are increasing and convex in own group size, the second inequality follows from negative spillover, the third inequality follows from convexity in own group size, while the last inequality follows from negative spillover. Next, consider the case x > y + 1. Again, a variant of the above argument, applying the hypothesis that payoffs are increasing and convex in own group size, and negative spillover leads to the conclusion that players find it profitable to deviate from smaller group with y players to the group with x players. Thus the only equilibrium with $x \ge y$ is (n, 0). Analogous arguments show that (0, n) is the only other equilibrium with $x \le y$.

Proposition 6 covers the case where payoffs are convex in own group size. We turn next to games in which payoffs are concave in own group size. An examination of the proof of Proposition 6 reveals that under exclusive membership rules, convexity of returns in own group size implies that an individual always prefers to join the larger group. The negative spillover effect goes in the same direction as well: a smaller opponent group is better news as an individual switches to a larger group. If the payoffs to a group are concave in own group size then matters are more complicated as the concavity of returns and negative spillovers press in opposite directions. If diminishing returns dominate then two equal groups, (n/2, n/2) arise. By contrast, if negative spillovers of opponent group size dominates then the outcome will involve a single active group, (n, 0) or (0, n).

Propositions 3, 6, and the above discussion on concave payoffs in own group size, taken together identify a class of games in which free membership rules lead to universal multiple identities, while exclusive membership rules lead to a single group outcome, i.e., either (n, 0)and (0, n). It is important to emphasize that under the hypotheses of Proposition 3, social efficiency dictates universal multiple identities. Taken together with our earlier observation, that exclusive memberships is an equilibrium in a game of rules among groups, this suggests that societies may function with exclusive membership rules even when it is individually and collectively inefficient.

We finally turn to the games covered by Proposition 4. Recall that in such games, we know from Proposition 5 that if $\frac{1}{n}[R(n,0)-R(n,1)] \leq R(1,n) \leq [R(n,0)-R(n,1)]$, then the socially optimal arrangement is not sustainable in equilibrium. We next note that if R(n,0)/n >R(1,n-1) then, under the *exclusive membership* rule, the configurations (n,0) and (0,n)are sustainable in equilibrium. Since these configurations are not sustainable in the free membership scenario, this implies that a policy of exclusive membership can *facilitate* the attainment of higher payoffs in some environments. This is summarized in the following result.

Proposition 7 Suppose the hypotheses of Proposition 4 hold. If R(1, n - 1) < R(n, 0)/n < R(n, 1)/n + R(1, n), then the outcomes (n, 0) and (0, n) are socially efficient, sustainable under exclusive membership rules, but **not** under free membership rules.

We note that in example 3, with n = 2, the game satisfies the hypotheses of Proposition 4 as well as the parametric restrictions developed in Proposition 7 (so long as D < 1/2).

5.1 Implementing narrow identities

Propositions 6-7 together identify a class of games in which individuals prefer multiple identities while groups prefer exclusive memberships. In a context of inter-group conflict, communication across groups is likely to be difficult, and individuals have an incentive to misreport their membership of other groups.⁵ So groups need mechanisms for verifying identity which overcome these incentives.

In some contexts, such as allocation of public funds for language development or land for religious activities (such as mosques, temples and churches) the identity of the groups is implicitly defined within the context of the conflict. In other important contexts, such as population pressure on land or mineral resources, the definition of the group is not given *a priori*. Historical experience is important and limits the range of possible categories; our point is simply that history does not tie down a unique classification of people.

So consider such a context of pressure on resources: individuals realize that on their own they cannot hope to ensure themselves a share of the resource. So they start looking for groups to form which can better protect their interests. In this context, it seems to us natural there is an advantage to being a larger group and to have smaller opponent groups. Given the incentive problem identified earlier it is appealing for the group to define itself on criteria which exclusive and easily verifiable. Religion, race and caste are natural candidates given these criteria: a person cannot be Black and White or Muslim and Christian or Brahmin and Shudra, at the same time.

6 Extension: pure public good

This section considers a model of pure public goods. Formally, $F(K_i(s)) = 1$, for all $K_i(s)$. We will use example 4 from section 3 to study pure public goods. The analysis highlights the tension between individual incentives and group incentives and illustrates the role of exclusive membership rules as a response to these tensions. We also identify the key role of public good returns function: if it is increasing and convex (concave) in group size then exclusive membership rules facilitates (prevents) socially optimal outcomes.

 $^{{}^{5}}$ In the world of espionage, this problem arises in the particularly well known form of the "double agent": an individual who joins both sides of a conflict!

We first analyze the case of increasing and convex returns from group size.

Proposition 8 Suppose the group produces a pure public good and returns function is increasing and convex. Then a single active group is socially optimal. Single active groups constitute an equilibrium, but universal multiple memberships is also an equilibrium.

The proof is provided in the appendix. If returns to group size are convex, and two groups are active then it is better to move a person from the smaller group to the larger group. This raises returns from the larger group more than the decline in the returns in the smaller group; moreover, the pure public good assumption reinforces this effect as the larger group has more people enjoying the larger public good. The proof extends this simple intuition to cover multiple memberships. If everyone is in one group then individual has a strict incentive to also be a member of this group, due to increasing and convex returns from group size. However, if everyone is a member of two groups, then multiple memberships yields a payoff 2f(n/2)while single membership of large group yields a payoff of f(n/2 + 1/2); under reasonable conditions the former is larger than the latter (for instance of f(.) is a quadratic function of group size). In other words, universal multiple memberships constitutes an equilibrium. We note that under under exclusive membership rules, an individual has an incentive to join the larger group, and so there exist two equilibria, both of which involve only one active group.

Proposition 9 Suppose the group produces a pure public good and returns function is increasing and convex. Then exclusive membership rules facilitate the attainment of socially optimal outcomes.

We next examine the case where returns are increasing and concave in group size. Here the efficient outcomes are harder to characterize as growth in group size and concavity push in opposite directions. The following result points to the benefits of multiple memberships.

Proposition 10 Suppose the group produces a pure public good and returns function is increasing and concave. Then universal multiple memberships are socially better than single active group outcomes and also constitute an equilibrium.

The proof is provided in the appendix. In a situation with increasing returns, it follows that exclusive memberships will lead individuals to move to the larger group, and so only single active groups are possible in equilibrium. From Proposition 10 these outcomes are inefficient, and so we have shown: with increasing and concave returns, exclusive membership rules lead to single active group outcomes which are socially inefficient.

7 Concluding remarks

Personal identity has many facets, as it involves race, language, personal interests, religion, and ethnicity, among other attributes. Yet all over the world we see individuals and groups defining themselves in narrow and exclusive terms. In this paper, our aim is to provide an explanation for this puzzle using a simple model of individual incentives and collective interests.

We take the view that, in day to day life, the different aspects of an individual personality remain latent and retain the feature of "perpetual possibilities". Social and economic context present a background against which individuals choose to retain these different possibilities or to commit to one of these possibilities and to renounce the others.

In a context of inter-group conflict, a larger opponent group lowers the payoffs. So a group gains strictly by prohibiting its current members from joining the other group. On the other hand, individual choice creates a negative externality: when a player joins a second group his 'original' group incurs a cost in terms of lost competitiveness, but the individual shares this cost with other group members. Thus individuals prefer to have rich/multiple identities in excess of what groups desire. This conflict between individual incentives and group interests motivate *narrow* membership rules. In a situation of group conflict, communication across groups is poor and unreliable. To implement narrow rules groups therefore seek criteria which are exclusive and generally outside individual control: this helps explain the salience of birth based criteria such as such as caste, race, and ethnicity.

8 Appendix

Proof of Proposition 8: The first step in the proof is to observe that S(n,0) = nf(n) > 2nf(n/2) = S(n/2, n/2), since f(.) is increasing and convex. We next observe that any partition of population (a, n-a), where a < n, is socially dominated by the (n, 0) configuration. Observe that

$$S(a, n - a) = af(a) + (n - a)f(n - a)$$
(25)

Suppose without loss generality that $a \ge n - a$, then it follows from convexity of f that S(a+1, n-a-1) > S(a, n-a), and the proof follows by iterating on this step until we reach

a = n.

Now examine a multiple membership configuration (n_A, n_B, n_{AB}) and suppose that $0 < n_{AB}$. We wish to show that any $n_{AB} > 0$ is dominated by $n_A = n$ $(n_B = n)$. Start from (n_A, n_B, n_{AB}) and suppose without loss of generality that $n_A \ge n_B$. We have already shown that $n_{AB} = n$ is socially dominated by $n_{AB} = 0$. So we consider $n_{AB} < n$. So there is at least one person who belongs only to group A. Consider the effects of moving a person from multiple membership to single membership. There are two possibilities: one, it increases social payoffs and two, it lowers payoffs. In the former case:

$$(n_A + n_{AB})f(n_A + \frac{n_{AB}}{2} + \frac{1}{2}) + (n_B + n_{AB} - 1)f(n_B + \frac{n_{AB}}{2} - \frac{1}{2})$$

$$\geq (n_A + n_{AB})f(n_A + \frac{n_{AB}}{2}) + (n_B + n_{AB})f(n_B + \frac{n_{AB}}{2})$$
(26)

This can be re-written as follows:

$$(n_A + n_{AB})[f(n_A + \frac{n_{AB}}{2} + \frac{1}{2}) - f(n_A + \frac{n_{AB}}{2})] \ge (n_B + n_{AB})f(n_B + \frac{n_{AB}}{2}) - (n_B + n_{AB} - 1)f(n_B + \frac{n_{AB}}{2} - \frac{1}{2})$$
(27)

Consider moving the next person from n_{AB} to single membership n_A . The social payoff is given by:

$$(n_A + n_{AB})f(n_A + \frac{n_{AB}}{2} + 1) + (n_B + n_{AB} - 2)f(n_B + \frac{n_{AB}}{2} - 1)$$
(28)

This social payoff is higher than the configuration $S(n_A + 1, n_B, n_{AB} - 1)$ if

$$(n_A + n_{AB})f(n_A + \frac{n_{AB}}{2} + 1) + (n_B + n_{AB} - 2)f(n_B + \frac{n_{AB}}{2} - 1)$$

> $(n_A + n_{AB})f(n_A + \frac{n_{AB}}{2} + \frac{1}{2}) + (n_B + n_{AB} - 1)f(n_B + \frac{n_{AB}}{2} - \frac{1}{2})$ (29)

Rearranging terms and noting that f(.) is increasing and convex and using equation (??) leads us to infer that (29) holds. So, iterating on this step we eventually arrive at a exclusive membership configuration $(n_A + n_{AB}, n_B, 0)$, and at each step we are raising social payoffs. However, we have already shown that any such exclusive membership configuration is dominated by (n, 0) and (0, n).

We next consider latter case, where moving a person from exclusive A membership to multiple membership raises social payoffs. Then we exploit convexity and show that it is socially strictly better to move to universal multiple memberships. But we have already shown that $n_{AB} = n$ is socially dominated by (n, 0) and (0, n). This proves that (n, 0) and (0, n) are socially efficient.

Single active groups are an equilibrium outcome: the payoff to player *i* from a group of size *n* is f(n). The payoff from multiple membership is f(n - 1/2) + f(1/2); f(.) = 0 and convexity of f(.) implies that such a deviation is not profitable. Similarly, moving to a new group is not profitable. Thus (n, 0) is an equilibrium. Analogous argument applies in the case (0, n).

Finally, consider the universal multiple membership outcome (n/2, n/2). Individual payoff is given by 2f(n/2). The payoff from a single group membership is f(n/2+1/2). So universal multiple memberships is an equilibrium so long as $2f(n/2) \ge f(n/2+1/2)$. This condition is satisfied for example if $f(x) = x^2$ and $n \ge 3$.

Proof of Proposition 10: Universal multiple memberships are an equilibrium outcome: the payoff to a player in such a configuration is 2f(n/2). The payoff from a deviation to a single group membership is f(n/2 + 1/2). Observe that:

$$f(\frac{n}{2} + \frac{1}{2}) \le f(\frac{n}{2}) + f(\frac{1}{2}) < f(\frac{n}{2}) + f(\frac{n}{2}) = 2f(\frac{n}{2}), \tag{30}$$

where we have used concavity of f(.) to derive the first inequality and strictly increasing fand $n \ge 2$ to derive second inequality. Thus universal multiple memberships is an equilibrium.

9 References

- 1. References incomplete!!
- Akerlof, G. and Kranton, R. (2000), Economics and identity, *Quarterly Journal of Economics*, 715-753.
- 3. Appiah, K. A. (2004), The ethics of identity.
- 4. Appiah, K. A. (200x), Cosmopolitanism.

- Alesina, A., R. Baqir, and W. Easterley (1999), Public Goods and Ethnic Divisions, Quarterly Journal of Economics, 114, 4, 1243-1284.
- Alesina, A. and E. La Ferrara (2000), Participation in heterogeneous communities, *Quarterly Journal of Economics*, 115, 3, 847-904.
- 7. Dasgupta, I. and Kanbur, R. (2007), Communities and Class Antagonisms. *Journal of Public Economics*, to appear.
- 8. Esteban, J. and Ray, D. (2007), The salience of ethnic conflict, American Economic Review, to appear.
- 9. Goyal, S. and M. Janssen (1997), Non-exclusive conventions and social coordination, Journal of Economic Theory.
- Iannaccone, L. (1992), Sacrifice and Stigma: Reducing Free-Riding in Cults, Communes, and Other Collectives, *Journal of Political Economy*, 100, 271-291.
- Bisin, A. and Verdier, T. (2000), Beyond the melting pot: cultural transmission, marriage and the evolution of ethnic and minority traits, *Quarterly Journal of Economics*, 115, 955-988
- 12. Sen, A. (2006), Identity and violence: the illusion of destiny. Norton Books. New York.
- Tajfel, H. and J.C. Turner (1986), The social identity theory of inter-group behavior, in S. Worchel and L.W. Austin (eds), *Psychology of intergroup relations*, Chicago. Nelson-Hall.
- 14. Weinstein, B (1997), Language strategists: redefining political frontiers on the basis of linguistic choices, *World Politics*, 345-364.