HEALTH, INFRASTRUCTURE, ENVIRONMENT AND ENDOGENOUS GROWTH

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ABSTRACT

This paper attempts to develop a model of endogenous growth with special focus on the role of health capital, public infrastructure and environmental pollution. It is an extension of the model of Agenor (2008) who does not consider environmental pollution. We analyse properties of optimal fiscal policy in the steady-state growth equilibrium when the level of production of the final good is the source of pollution. Tax revenue of the government is channelized into three expenditure heads-health expenditure, pollution abatement expenditure and public infrastructure expenditure. It is found that the optimum ratio of public infrastructural expenditure to national income in the steady-state equilibrium is less than the competitive output share of the public input; and it varies inversely with the magnitude of the pollution-output coefficient. There is no conflict between the social welfare maximizing solution and the growth rate maximizing solution in the steady-state equilibrium. There may exist indeterminacy in the transitional growth path converging to the unique steady-state equilibrium point that never satisfies saddle-point stability. The market economy growth rate is not necessarily less than the socially efficient growth rate in the steady-state equilibrium.

KEY WORDS: Health capital, Environmental pollution, Public infrastructure, Abatement expenditure, Income tax, Endogenous growth, Steady-state growth.

JEL Classification Number: H4, H51, H54, I18, O4, O13, Q5

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1. INTRODUCTION

There exists a substantial literature on the theory of endogenous economic growth explained in terms of public infrastructural expenditure. Barro (1990) has started this literature and has shown that the optimum ratio of the income tax financed public infrastructural spending to national income is equal to the competitive output share of the public input. However, his model fails to show transitional dynamic properties; and his assumption, that public expenditure is a flow variable, has been questioned by many others[1]. Futagami, Morita and Shibata (hereafter called FMS) (1993) have extended the Barro (1990) model assuming that the productive public expenditure is a stock variable like physical capital. Transitional dynamic properties come back to this extended model; and the Barro (1990) result about the optimal fiscal policy remains valid in the steady-state equilibrium but not in the transitional phase of development. Both the Barro (1990) model and the FMS (1993) model have been extended by various authors in various directions[2]. Agenor (2008) extends the Barro (1990) model introducing productive health expenditure in addition to the infrastructural expenditure, where financing of both types of expenditure is made by the allocation of income tax-revenue. However, none of these models other than Greiner (2005) and Economides and Philippopoulos (2008) deals with the interaction between economic growth and environmental pollution when public infrastructural expenditure is the engine of economic growth.

[1] See, for example, Aschauer (1989), Futagami, Morita and Shibata (1993) etc.

[2] See the works of Dasgupta (1999, 2001), Chen (2006), Tsoukis and Miller (2003), Chang (1999), Turnovsky (1997, 1996), Hu, Ohdoi and Shimomura (2008), Ohdoi (2007), Greiner and Semmler (2000), Kalaitzidakis and Kalyvitis (2004), Baier and Glomm (2001), Yakita (2004), Shioji (2001), Tamai (2007), Burguet and Fernandez-Ruiz (1998), Ghosh and Mourmouras (2004), Raurich-Puigdevall (2000), Cazzavillan (1996), Zhang (2000), Neill (1996), Mourmouras and Lee (1999), Park and Phillippopoulos (2002), Tanaka (2002), Chen and Lee (2007), etc.

On the other hand, existing dynamic models[3] focusing on the interaction between economic growth and environmental pollution are either exogenous growth models or endogenous growth models explaining growth in terms of endogenous technical progress. However, these models do not analyse the role of infrastructural expenditure and health expenditure on economic growth and environmental pollution.

The objective of the present paper is to develop a model of endogenous economic growth and to analyse the properties of optimal fiscal policy in the presence of public infrastructural expenditure, health expenditure and environmental pollution. Greiner (2005) develops an FMS (1993) type of model with environmental pollution affecting the household's utility function. Economides and Philippopoulos (2008) extend the Barro (1990) model in this direction. We follow these authors to assume that the level of production is the source of pollution. However, in our model environmental quality is an accumulable variable; and it affects the productivity of the inputs in the final goods sector. We follow Agenor (2008) to introduce health capital as an input in the production function. However, we assume health capital to be an accumulable input in the production function following the second model of Agenor (2008) while, in his first model, it is in the form of a flow variable. We also consider the negative role of environmental pollution on the depreciation of public health capital.

We obtain interesting results analysing this model. The optimum ratio of combined public expenditure on infrastructure and health to national income is equal to the sum of the competitive shares of the public infrastructural input and the health capital in the unpolluted output of the final good; and hence this optimum ratio varies inversely with the rate of pollution per unit of production. However, in Barro (1990) and in FMS (1993), there is neither any environmental pollution nor any productive health capital; hence this ratio is

[3] See the works of Managi(2006), Dinda(2005), Di Vita(2008), Hartman and Kwon(2005), D. Ayong Le Kama(2001), Elbasha and Roe (1996), Oueslati(2002), Bertinelli, Strobl and Zou(2007), Smulders and Gradus(1996), Byrne (1997), Itaya(2008), Bovenberg and Smulders(1995), Benarroch and Weder(2006), Selden and Song(1995),

always equal to the competitive output share of the public infrastructural input. In Greiner (2005), the optimum share of investment to national income is also independent of the rate of pollution per unit of production because pollution, being a flow variable in his model, enters the utility function. However, neither the environmental quality nor the health infrastructure enters the production function as an accumulable input in his model. Secondly, in our model, optimum income tax rate is higher than that predicted by Barro (1990) and FMS (1993); and this rate varies positively with the pollution-output coefficient. This is so because a part of the income tax revenue is spent as abatement expenditure and health expenditure in this model. However, this is not necessarily so in Greiner (2005) who considers pollution tax as an alternative instrument of financing abatement expenditure. In Agenor (2008), the optimum tax rate is lower than that in our model but is higher than the Barro-FMS optimum tax rate because in Agenor (2008), there is a tax financed health expenditure though there is no abatement expenditure. Thirdly, our model exhibits transitional dynamic properties though it follows Barro (1990) to assume public expenditure to be a flow variable. Introducing environmental quality and health capital as accumulable inputs in the production function, we protect our model from being an AK model and thus get back transitional dynamic properties. The model of Agenor (2008) shows (does not show) transitional dynamic properties when health expenditure is a stock (flow) variable. However, steady-state equilibrium is a saddle-point when health expenditure is a stock variable in his model. In our model, with both health capital and environmental quality being stock variables, steady-state equilibrium never satisfies saddle-point stability but we find a possibility of indeterminacy of the transitional growth path what Agenor (2008) does not find in his model. FMS (1993) and Greiner (2005) also find the saddle-point stability property of the steady-state equilibrium in their models. Fourthly, like Barro (1990) and FMS (1993), we do not find any conflict between the growth rate maximizing solution and the social welfare maximizing solution along the steady-state equilibrium growth path. Agenor (2008) finds a conflict between these two goals because health affects the utility function of the household in his model. Greiner (2005) also finds a similar conflict because environmental pollution affects the utility function in his model. Fifthly, the competitive equilibrium growth rate in this model is not necessarily less than the socially efficient growth rate which is unlike in Barro (1990) or in FMS (1993). This is so because we have two conflicting types of externalities on production-positive externality arising from the gross public expenditure and negative externality arising from capital accumulation and environmental pollution. Market economy growth rate may exceed the socially efficient growth rate when the pollution-output coefficient takes a high value. Barro (1990) and FMS (1993) consider only the positive externality of public expenditure. Agenor (2008) also considers two positive externalities from health expenditure and infrastructural expenditure. So market economy growth rate falls short of the socially efficient growth rate in their models.

The paper is organized as follows. Section 2 describes the basic model of the household economy. Section 3 analyses its dynamic equilibrium properties. Subsection 3.1 shows the existence of a unique steady-state equilibrium growth rate in the market economy and subsection 3.2 analyses the properties of optimal fiscal policy along the steady-state equilibrium path. Section 4 shows transitional dynamic results; and section 5 compares the market economy steady-state equilibrium growth rate to the command economy steady-state equilibrium growth rate. Final remarks are made in section 6.

2. THE MODEL

The single production sector of the economy uses physical capital, labour, public infrastructural input and the health capital as four inputs in production. The production function is of Cobb-Douglas type satisfying increasing returns to scale in these four inputs. However, it satisfies constant returns to scale in physical capital, public infrastructural input and health capital; and there is diminishing marginal productivity to each input.

The product market and private input markets are perfectly competitive; and every producer maximizes profit. The public infrastructural expenditure is treated as a flow variable like that in Barro (1990). However, health capital is a stock variable. The government imposes a proportional tax on income of the representative household who consumes a part of the post-tax income and saves (invests) the other part. Government allocates a part of the tax revenue to build up the additional infrastructure on health and provides it free of charge to the representative household. However, the health capital deteriorates with pollution of environment. The environmental quality is also considered a stock variable; and it deteriorates with pollution and is improved by the abatement activities undertaken by the government. Environmental quality is non-rival and is a free good. The budget of the government is balanced; and the allocation of tax revenue is made among three expenditure heads-public infrastructural expenditure, health expenditure and abatement expenditure.

There is a negative congestion effect on the public infrastructural input; and so the effective benefit of public infrastructural expenditure derived by the representative producer varies inversely with the average private capital stock of the society and positively with the environmental quality.

There is no population growth; and so labour endowment is normalized to unity. Every household maximizes her lifetime utility subject to the budget constraint. The lifetime utility is defined as the infinite integral of the discounted present value of instantaneous utility where instantaneous utility is assumed to be a positive and concave function of the level of consumption and the discounting is made at a constant rate. All variables are measured in terms of the final product.

Following equations describe the model.

$$Y = K^{\alpha}\hat{G}_I^{1-\alpha-\beta}H^{\beta} \text{ } with \text{ } 0 < \alpha, \beta < 1; \qquad \ldots\ldots(1)$$

$$\hat{G}_I = G_I\overline{K}^{-\theta}E^{\theta} \text{ } with \text{ } 0 < \theta < 1; \qquad \ldots\ldots(2)$$

$$\dot{K} = (1-\tau)Y - C; \quad \ldots\ldots(3)$$

$$\dot{E} = TY - \delta Y \; with \; 0 < \delta < 1; \quad \ldots\ldots(4)$$

$$\dot{H} = G_H - \eta\delta Y \; with \; 0 < \eta, \delta < 1; \quad \ldots\ldots(5)$$

$$G = G_I + G_H = (\tau - T)Y \; with \; 0 < T < \tau < 1; \quad \ldots\ldots(6)$$

$$G_i = v_i(\tau - T)Y \; with \; i = I, H; \quad \ldots\ldots(7)$$

and

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma} \; with \; \sigma > 0. \quad \ldots\ldots(8)$$

Equation (1) describes the Cobb-Douglas production function of the final good. Y is the level of output produced, K is the stock of physical capital, and $\hat{G}_I$ is the congestion effect adjusted effective benefit derived from the public infrastructural input. H is the stock of health capital. Since labour endowment is normalized to unity, Y, H and K can be considered as per capita variables with the labour elasticity of output being $(1-\alpha)$. Elasticities of output with respect to physical capital, public infrastructural input, and health capital are denoted by α, $(1-\alpha-\beta)$ and β respectively.

Equation (2) describes the nature of the combined effect of congestion and of environment on the effectiveness of public infrastructure.
It shows that the effective production benefit obtained from the public infrastructural input varies inversely with the average physical capital stock of all private producers, $\bar{K}$, and positively with the environmental quality, E.

The justification of this assumption of the negative external effect of average physical capital stock is available in the existing literature[4]. With the increase in the number of factories and housing colonies, limited roads cannot provide effective transportation service. The increase in power consumption disturbs the supply of power. Parks and footpaths are occupied by informal sector traders. This effective benefit of public infrastructure, $\hat{G}$, is treated as a composite input in the production function. We assume that the negative congestion effect of the average physical capital stock of the society is not

[4] See the works of Ott and Soretz (2008), Van Tuijl *et. al* (1997), Raurich-Puigdevall (2000), Turnovsky (1996, 1997), Eicher and Turnovsky, etc.

strong enough to outweigh the positive private technological contribution of physical capital of the representative producer. Hence, we assume $(\alpha - \theta\overline{1-\alpha-\beta}) > 0$. Here $(\alpha - \theta\overline{1-\alpha-\beta})$ represents the social elasticity of output with respect to physical capital.

Degradation of environmental quality reduces the effective benefit of the public infrastructural expenditure and health expenditure in various ways. For example, deforestation reduces rainfall and thus lowers the efficiency of the public irrigation programme by reducing the supply of canals' water flow and by reducing the recharging of groundwater. Poor qualities of natural resources (coal) and the lack of current in the flowing water of streams and rivers negatively affect the generation of electricity. Global warming leads to natural disasters like floods, earthquakes, cyclones, etc., which, in turn, cause a heavy loss of infrastructural capital damaging roads, electricity lines, power plants, building, industrial plants, etc. Water pollution and air pollution create a disease-friendly environment; and hence the public health expenditure programme cannot provide the maximum benefit to the workers. This, in turn, lowers the efficiency of the workers.

Equation (3) describes the private budget constraint of the household who allocates its post tax disposable income between consumption, C, and savings (investment); and there is no depreciation of physical capital.

Equation (4) shows how environmental quality changes over time depending upon the magnitudes of pollution and abatement activity. Abatement activities bring improvements in environmental quality; and there exists a substantial theoretical and empirical literature dealing with the role of abatement activities and abatement policies of the government[5]. TY is the abatement expenditure made by the government. We call T as abatement expenditure rate. Here environmental pollution is assumed to be a flow variable and is proportional to the level of production of the final good; and δ is the

[5] See the works of Liddle (2001), Managi (2006), Dinda (2005), Di Vita (2008), Smulders and Gradus (1996), Byrne (1997), etc.

constant pollution-output coefficient. Many models of environmental pollution assume pollution to be a positive function of the level of production[6] of the final good. This is consistent with only one segment of the Environmental Kuznets curve[7], according to which, there exists an inverted U-shaped relationship between the pollution level and the income level.

The accumulation of the stock of health capital is given by equation (5). The government spends an amount G_H on the health infrastructure. Pollution causes depreciation of this stock; and this relationship is assumed to be proportional for the sake of simplicity. η is the resulting depreciation per unit of pollution.

Equation (6) describes the government budget constraint. The government finances the public infrastructural expenditure and health expenditure with its tax revenue after meeting the abatement expenditure. T is the ratio of abatement expenditure to income; and τ is the income tax rate. A fraction v_I of the tax revenue net of the abatement expenditure is used to finance the infrastructural expenditure, G_I; $v_H$is the fraction allocated to health expenditure. Public expenditure allocation ratios are given by equation (7).

Equation (8) describes the instantaneous utility function of the household. The instantaneous utility is assumed to be a positive and concave function of the level of consumption. σ represents the constant elasticity of marginal utility with respect to consumption. Many models assume utility to be a positive function of the environmental quality[8]. Agenor (2008) introduces health as an argument in the utility function and Greiner (2005) introduces pollution as an argument in the utility function. We ignore these complications in this model for the sake of simplicity.

[6] For example, see the works of Liddle (2001), Oueslati (2002), Hartwick (1991), Smulders and Gradus (1996), Byrne (1997), Gruver (1976), Dinda (2005), etc.
[7] Analysis on this curve is available in Managi (2006), Dinda (2005), Di Vita (2008), Hartman and Kwon (2005), Seldon and Song (1995), etc.
[8] See the works of Howarth (1996), Tahvonen and Kuuluvainen (1991), Smulders and Gradus (1996), D. Ayong Le Kama (2001), Greiner (2005), Gruver (1976), Itaya (2008), etc.

Stocks of E, H and K are exogenous at a particular point of time. E and H are non rival stocks and G is a non rival flow. Given the stocks of physical capital, health capital and environmental quality, and given the fiscal instrument rates, equations (1), (2) and (6) together determine Y and G at each point of time. Thus equations (4) and (5) determine absolute rates of improvement in the health capital and the environmental quality, denoted by $\dot{H}$ and $\dot{E}$ respectively. The household then chooses C and this determines the absolute rate of physical capital accumulation, $\dot{K}$.

3. DYNAMIC EQUILIBRIUM

The representative household maximizes $\int_0^\infty u(C)\, e^{-\rho t}\, dt$ with respect to C subject to equations (3) and (8). The demand rate of growth[9] of consumption is derived from this maximizing problem as follows.

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}\left[\alpha(1-\tau)\{v_I(\tau - T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}}\left(\frac{E}{K}\right)^{\frac{\beta+\theta(1-\alpha-\beta)}{\alpha+\beta}}\left(\frac{H}{E}\right)^{\frac{\beta}{\alpha+\beta}} - \rho\right]. \qquad \ldots\ldots(9)$$

We consider a steady-state growth equilibrium where all macroeconomic variables grow at the same rate, g_m. Hence, we have

$$\frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{E}}{E} = \frac{\dot{H}}{H} = g_m. \qquad \ldots\ldots(10)$$

3.1 EXISTENCE OF STEADY STATE GROWTH EQUILIBRIUM

Using equations (1) to (7), (9) and (10), we obtain the following equations.

$$\frac{1}{\sigma}\left[\alpha(1-\tau)\{v_I(\tau - T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}}\left(\frac{E}{K}\right)^{\frac{\beta+\theta(1-\alpha-\beta)}{\alpha+\beta}}\left(\frac{H}{E}\right)^{\frac{\beta}{\alpha+\beta}} - \rho\right] = g_m; \qquad \ldots\ldots(11)$$

[9] The demand rate of growth of consumption is derived in appendix (A).

$$(1-\tau)\{v_I(\tau-T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}}\left(\frac{E}{K}\right)^{\frac{\beta+\theta(1-\alpha-\beta)}{\alpha+\beta}}\left(\frac{H}{E}\right)^{\frac{\beta}{\alpha+\beta}}-\frac{C}{K}=g_m; \qquad \dots\dots(12)$$

$$(T-\delta)\{v_I(\tau-T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}}\left(\frac{E}{K}\right)^{\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}}\left(\frac{H}{E}\right)^{\frac{\beta}{\alpha+\beta}}=g_m; \qquad \dots\dots(13)$$

and

$$\{v_H(\tau-T)-\eta\delta\}\{v_I(\tau-T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}}\left(\frac{E}{K}\right)^{\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}}\left(\frac{H}{E}\right)^{-\frac{\alpha}{\alpha+\beta}}=g_m. \qquad \dots\dots(14)$$

Then we use equations (11), (12), (13) and (14) to obtain the following equation[10] that solves for g_m.

$$g_m{}^{\beta+\theta(1-\alpha-\beta)}(\sigma g_m+\rho)^{\alpha-\theta(1-\alpha-\beta)}=\alpha^{\alpha-\theta(1-\alpha-\beta)}(1-\tau)^{\alpha-\theta(1-\alpha-\beta)}$$
$$\{v_H(\tau-T)-\eta\delta\}^{\beta}\{v_I(\tau-T)\}^{1-\alpha-\beta}(T-\delta)^{\theta(1-\alpha-\beta)} \qquad \dots\dots(15)$$

The L.H.S. of equation (15) is an increasing function of g_m and its R.H.S. is a constant term, given τ, T and v. Figure 1 shows the existence of a unique value of g_m.

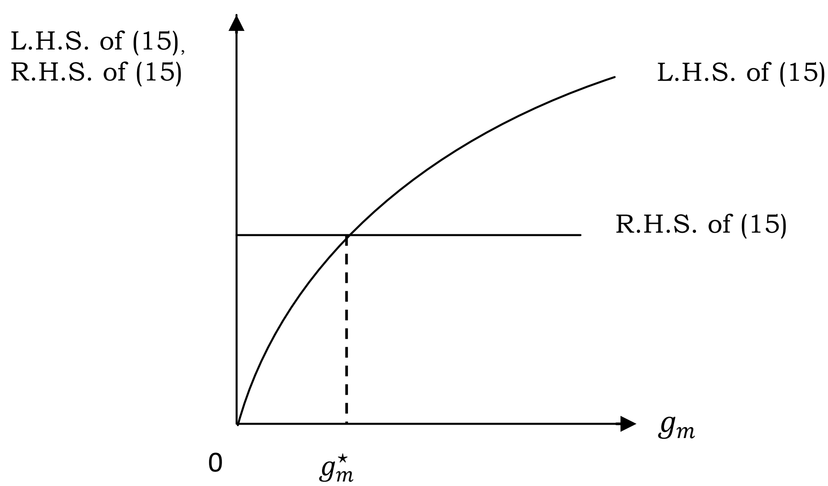

FIGURE 1

[10] The derivation of equation (15) is worked out in appendix (B).

We have the following proposition.

Proposition 1: There exists a unique steady-state equilibrium growth rate in the market economy given the income tax rate, the abatement expenditure rate and the public expenditure allocation ratio.

Equations (B7), (B8) and (B9) in Appendix (B) show that $\frac{E}{K}$, $\frac{H}{E}$ and $\frac{C}{K}$ in the steady-state equilibrium are functions of g_m. This proves that the steady-state equilibrium is also unique.

3.2 OPTIMAL TAXATION

We first assume that the government maximizes the steady-state equilibrium growth rate with respect to fiscal instruments-τ, T and v_I. The L.H.S. of equation (15) is a monotonically increasing function of g_m, because, by assumption, $\alpha > \theta(1-\alpha-\beta)$. Since the L.H.S. is always equal to the R.H.S. in the steady-state growth equilibrium, maximization of g_m implies maximization of the R.H.S. of equation (15).

Maximizing the R.H.S. of equation (15) with respect to τ, T and v_I respectively, we obtain following expressions of their optimum values[11].

$$\tau^\star = 1-(1-\delta-\eta\delta)\{\alpha-\theta(1-\alpha-\beta)\}; \qquad \ldots\ldots(16)$$

$$T^\star = \delta+(1-\delta-\eta\delta)\theta(1-\alpha-\beta); \qquad \ldots\ldots(17)$$

and

$$v_I^\star = \frac{(1-\delta-\eta\delta)(1-\alpha-\beta)}{\eta\delta+(1-\delta-\eta\delta)(1-\alpha)}. \qquad \ldots\ldots(18)$$

Using equations (16) and (17), we have

$$\tau^\star - T^\star = \eta\delta+(1-\delta-\eta\delta)(1-\alpha). \qquad \ldots\ldots(19)$$

[11] The derivation of equations (16), (17) and (18) is worked out in appendix (C).

To ensure that the growth rate is non-negative deterioration of the two accumulable inputs-environmental quality and health infrastructure-due to pollution is neutralized by allocating δ and $\eta\delta$ fractions of the total output to abatement expenditure, TY and aggregate productive public expenditure, $(\tau - T)Y$, respectively. The optimum net abatement expenditure rate is then $(T^\star - \delta)$ and $(1 - \delta - \eta\delta)\theta(1 - \alpha - \beta)$ is the competitive unpolluted output share of environmental quality in the output of the final good. So the net optimum ratio is equal to the competitive share of environmental input in the unpolluted output. Similarly $(\tau^\star - T^\star - \eta\delta)$ is the optimum ratio of net aggregate public expenditure on the intermediate public good and health infrastructure to the national income; and $(1 - \delta - \eta\delta)(1 - \alpha)$ is the net competitive unpolluted output share of the two inputs taken together which are financed by government's tax revenue. So the net optimum ratio is equal to the competitive share of the public intermediate good in the unpolluted output. In Barro (1990) and in FMS (1993), entire output is pollution free and this ratio is equal to the competitive share of the public input in the total output.

We now examine whether the growth rate maximizing solution is consistent with the social welfare maximizing solution in the steady-state equilibrium. The social welfare function is given by

$$W = \int_0^\infty e^{-\rho t} \frac{C^{1-\sigma}}{1-\sigma} dt. \qquad \ldots\ldots(20)$$

Using equations (11) and (12) and assuming that the economy is on the steady-state equilibrium growth path, it can be shown that

$$C = \frac{1}{\alpha}[\rho - (\alpha - \sigma)g_m]K(0)e^{g_m t}. \qquad \ldots\ldots(21)$$

Using equations (20) and (21) we have

$$W = \alpha^{\sigma-1}\frac{K(0)^{1-\sigma}}{1-\sigma}\left[\frac{\rho-(\alpha-\sigma)g_m}{\rho-(1-\sigma)g_m}\right][\rho - (\alpha - \sigma)g_m]^{-\sigma}. \qquad \ldots\ldots(22)$$

Equation (22) shows that W varies positively with g_m.

Thus the level of social welfare in the steady-state equilibrium is maximized when the steady-state equilibrium growth rate is maximized[12]. We now can state the following proposition.

PROPOSITION 2: (i) The optimum income tax rate, the optimum abatement expenditure rate and the optimum public infrastructural expenditure allocation ratio in the steady-state growth equilibrium are given by

$$\tau^\star = 1 - (1 - \delta - \eta\delta)\{\alpha - \theta(1 - \alpha - \beta)\},$$

$$T^\star = \delta + (1 - \delta)\theta(1 - \alpha),$$

and

$$v_I^\star = \frac{(1-\delta-\eta\delta)(1-\alpha-\beta)}{\eta\delta+(1-\delta-\eta\delta)(1-\alpha)}.$$

(ii) The net optimum ratio of combined public expenditure on infrastruture and health to national income in the steady-state equilibrium is equal to the combined competitive share of these two inputs in the unpolluted output of the final good; and hence this optimum ratio varies inversely with the magnitude of the pollution-output coefficient.

The presence of three different effects makes our result different from those available in the existing literature. These are (i) congestion effect on public expenditure that makes $\theta > 0$, (ii) the environmental pollution effect causing $\delta > 0$ and (iii) the effect of pollution on health capital causing $\eta > 0$. If we assume $\theta = \delta = 0$, we obtain $\tau^\star = 1 - \alpha$ and $T^\star = 0$; and these results are identical to those of Barro (1990) and FMS (1993). The net optimum ratio of combined public expenditure on infrastructure and health to national income in this model, with $0 < \delta, \eta < 1$ and $\theta > 0$, appears to be lower than that obtained in Barro (1990) and in FMS (1993). This is obvious because production of the final good generates environmental pollution. This, in turn,

[12] We do not analyse social welfare maximization including transitional dynamics. FMS (1993) does that.

lowers the rate of accumulation of environmental quality and of health capital. Thus the effective producer's benefit derived from the public expenditure is reduced. So it is optimal for the government to allocate a smaller fraction of tax revenue to meet this expenditure. However,

$$\tau^{\star} = 1 - \alpha + \alpha\delta + \alpha\eta\delta + (1 - \delta - \eta\delta)\theta(1 - \alpha - \beta).$$

Here, $\tau^{\star} > 1 - \alpha$ because $0 < \delta, \eta < 1$, $0 < \alpha < 1$, and $\theta > 0$. So the optimum income tax rate in the present model is higher than the corresponding rate obtained in the models like Barro (1990), FMS (1993), Agenor (2008). This is so because income tax is the only source of public revenue in this model and a part of that revenue is used to meet the abatement expenditure. This is not so in the models of Barro (1990), FMS (1993), Agenor (2008), etc., because there is no environmental pollution in those models.

In this model, not only the aggregate of productive public expenditure, i.e., the excess of tax revenue over the abatement expenditure, but also the level of environmental pollution is proportional to the level of income. So $(\tau^{\star} - T^{\star})$ varies inversely with the pollution-output coefficient, δ. If the level of pollution is independent of the level of income, then $\delta = 0$; and the Barro (1990)-FMS (1993)-Agenor (2008) result comes back in this model in this special case.

4. TRANSITIONAL DYNAMICS

We now turn to investigate the stability properties of the unique steady-state equilibrium point in the market economy. Equations of motion of the dynamic system are given by (3), (4), (5) and (9). We define the following ratio variables.

$x = \frac{C}{K}$;

$y = \frac{E}{K}$;

and

$z = \frac{H}{E}$.

Using equations (3), (4), (5) and (9), we have

$$\frac{\dot{x}}{x} = \left(\frac{\alpha}{\sigma} - 1\right)(1-\tau)\{v_I(\tau - T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}} y^{\frac{\theta(1-\alpha-\beta)+\beta}{\alpha+\beta}} z^{\frac{\beta}{\alpha+\beta}} + x - \frac{\rho}{\sigma}; \qquad \ldots\ldots(23)$$

$$\frac{\dot{y}}{y} = (T-\delta)\{v_I(\tau - T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}} y^{\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}} z^{\frac{\beta}{\alpha+\beta}}$$
$$-(1-\tau)\{v_I(\tau - T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}} y^{\frac{\theta(1-\alpha-\beta)+\beta}{\alpha+\beta}} z^{\frac{\beta}{\alpha+\beta}} + x; \qquad \ldots\ldots(24)$$

and

$$\frac{\dot{z}}{z} = \{v_H(\tau - T) - \eta\delta\}\{v_I(\tau - T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}} y^{\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}} z^{-\frac{\alpha}{\alpha+\beta}}$$
$$-(T-\delta)\{v_I(\tau - T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}} y^{\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}} z^{\frac{\beta}{\alpha+\beta}}. \qquad \ldots\ldots(25)$$

The determinant of the Jacobian matrix[13] corresponding to the differential equations (23), (24) and (25) is given by

$$|J| = \frac{\alpha-\theta(1-\alpha-\beta)}{\alpha+\beta}(T-\delta)\{v_I(\tau - T)\}^{2\frac{1-\alpha-\beta}{\alpha+\beta}}\{v_H(\tau - T) - \eta\delta\} y^{2\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}-1} z^{-2\frac{\alpha}{\alpha+\beta}}$$
$$+\frac{\alpha}{\sigma}\frac{\theta(1-\alpha-\beta)}{\alpha+\beta}(1-\tau)\{v_I(\tau - T)\}^{2\frac{1-\alpha-\beta}{\alpha+\beta}}\{v_H(\tau - T) - \eta\delta\} y^{2\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}} z^{-2\frac{\alpha}{\alpha+\beta}}$$
$$+\frac{\alpha}{\sigma}\frac{\beta}{\alpha+\beta}(1-\tau)(T-\delta)\{v_I(\tau - T)\}^{2\frac{1-\alpha-\beta}{\alpha+\beta}} y^{2\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}} z^{2\frac{\beta}{\alpha+\beta}-1}.$$

Here $\alpha - \theta(1-\alpha-\beta) > 0$. Also $1 > \tau > T > \delta$ when τ and T are optimally chosen and when $\theta > 0$. Also $v_H(\tau - T) - \eta\delta > 0$. So $|J| > 0$ in this case, when it is evaluated at the steady-state equilibrium point. So either all the three latent roots of J matrix are positive or two of them are negative with the third one being positive. Hence the steady-state equilibrium cannot be a saddle point. Either it is unstable with all the latent roots being positive or there exists

[13] The derivation of the determinant is worked out in appendix (D).

indeterminacy in the transitional growth path converging to the equilibrium point.

The trace of the Jacobian matrix is given by

$$Tr\,J = 1 + \frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}(T-\delta)\{v_I(\tau-T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}} y^{\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}-1} z^{\frac{\beta}{\alpha+\beta}}$$
$$-\frac{\theta(1-\alpha-\beta)+\beta}{\alpha+\beta}(1-\tau)\{v_I(\tau-T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}} y^{\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}} z^{\frac{\beta}{\alpha+\beta}}$$
$$-\frac{\alpha}{\alpha+\beta}\{v_H(\tau-T)-\eta\delta\}\{v_I(\tau-T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}} y^{\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}} z^{-\frac{\alpha}{\alpha+\beta}-1}$$
$$-\frac{\beta}{\alpha+\beta}(T-\delta)\{v_I(\tau-T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}} y^{\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}} z^{-\frac{\alpha}{\alpha+\beta}}.$$

Using equations (16), (17), (18) and using the expression of the steady-state equilibrium values of x, y and z in terms g_m from equations (B7), (B8) and (B9) in Appendix (B), we find that the trace of the Jacobian matrix is negative[14] if

$$1 + \frac{\rho}{\sigma}\frac{\{\alpha-\theta(1-\alpha-\beta)\}^2}{\alpha+\beta}\frac{\alpha}{\theta(1-\alpha-\beta)}\frac{g_m}{(\sigma g_m+\rho)} + \frac{\rho}{\sigma}\frac{\theta(1-\alpha-\beta)}{\beta} < \Big[\frac{\alpha}{\sigma}\frac{\{\alpha-\theta(1-\alpha-\beta)\}}{\alpha+\beta} + \frac{\theta(1-\alpha-\beta)+\beta}{\alpha+\beta}$$
$$\frac{1}{\sigma}\frac{\{\theta(1-\alpha-\beta)\}^2}{\beta\{\alpha-\theta(1-\alpha-\beta)\}}\frac{(\sigma g_m+\rho)}{g_m}\Big](1-\delta-\eta\delta)^{\frac{1}{\alpha+\beta}}(1-\alpha-\beta)^{\frac{(1-\alpha-\beta)}{\alpha+\beta}}$$
$$\left\{\frac{\beta}{\theta(1-\alpha-\beta)}\right\}^{\frac{\beta}{\alpha+\beta}}\{\alpha-\theta(1-\alpha-\beta)\}^{1+\frac{\{\alpha-\theta(1-\alpha-\beta)\}}{\alpha+\beta}}\left\{\frac{\alpha}{\theta(1-\alpha-\beta)}\right\}^{\frac{\{\alpha-\theta(1-\alpha-\beta)\}}{\alpha+\beta}}$$
$$\left\{\frac{g_m}{(\sigma g_m+\rho)}\right\}^{\frac{\{\alpha-\theta(1-\alpha-\beta)\}}{\alpha+\beta}}.$$

If the determinant of the Jacobian matrix takes a positive sign and its trace takes a negative sign, then there are one positive and two negative latent roots of this matrix[15]. It means that there exists indeterminacy in the transitional growth path converging to the unique equilibrium point. So we have the following proposition.

[14] The derivation is worked out in Appendix (D).
[15] It is a sufficient condition but not a necessary one. There may be one positive and two negative roots even if the trace takes a positive sign. However, all the roots may also be positive in that case implying that no trajectory converges to the equilibrium point. See Benhabib and Perili (1994).

Proposition 3: the unique steady-state equilibrium point never satisfies saddle-point stability; but there exists indeterminacy in the transitional growth path converging to the steady-state equilibrium point if the steady-state equlibrium growth rate satisfies the following condition:

$$1+\frac{\rho}{\sigma}\frac{\{\alpha-\theta(1-\alpha-\beta)\}^2}{\alpha+\beta}\frac{\alpha}{\theta(1-\alpha-\beta)}\frac{g_m}{(\sigma g_m+\rho)}+\frac{\rho}{\sigma}\frac{\theta(1-\alpha-\beta)}{\beta}<\Big[\frac{\alpha}{\sigma}\frac{\{\alpha-\theta(1-\alpha-\beta)\}}{\alpha+\beta}+\frac{\theta(1-\alpha-\beta)+\beta}{\alpha+\beta}$$
$$\frac{1}{\sigma}\frac{\{\theta(1-\alpha-\beta)\}^2}{\beta\{\alpha-\theta(1-\alpha-\beta)\}}\frac{(\sigma g_m+\rho)}{g_m}\Big](1-\delta-\eta\delta)^{\frac{1}{\alpha+\beta}}(1-\alpha-\beta)^{\frac{(1-\alpha-\beta)}{\alpha+\beta}}$$
$$\left\{\frac{\beta}{\theta(1-\alpha-\beta)}\right\}^{\frac{\beta}{\alpha+\beta}}\{\alpha-\theta(1-\alpha-\beta)\}^{1+\frac{\{\alpha-\theta(1-\alpha-\beta)\}}{\alpha+\beta}}\left\{\frac{\alpha}{\theta(1-\alpha-\beta)}\right\}^{\frac{\{\alpha-\theta(1-\alpha-\beta)\}}{\alpha+\beta}}$$
$$\left\{\frac{g_m}{(\sigma g_m+\rho)}\right\}^{\frac{\{\alpha-\theta(1-\alpha-\beta)\}}{\alpha+\beta}}.$$

This sufficient condition is always satisfied for low values of $\frac{g_m}{(\sigma g_m+\rho)}$; and the value of g_m is determined by the exogenous values of the parameters. Here very low values of δ and η will ensure that $(1-\delta-\eta\delta)$ is positive; and this is necessary for the inequality to be satisfied. Note that $\frac{g_m}{(\sigma g_m+\rho)}$ is low when g_m is high; and figure 3 shows that g_m is high when δ and η take very low values[16].

This is an important result. Barro (1990) model, with a flow public expenditure, does not exhibit any transitional dynamic properties. FMS (1993) brings back transitional dynamic properties in Barro (1990) model introducing durable public input but shows the saddle-point stability property of the unique steady-state equilibrium. The model of Agenor (2008) shows saddle-point stability property of the steady-state equilibrium when health expenditure is a stock variable but does not exhibit transitional dynamic properties when health expenditure is a flow variable. Greiner (2005), Dasgupta (1999), etc., also prove the saddle-point stability property of the long-run equilibrium in their models. However, we show that saddle-point stability property of the steady-state equilibrium is never satisfied in our model. On the

[16] Since it is a sufficient condition and not a necessary one, a low value of g_m does not rule out the possibility of indeterminacy.

contrary, we find a possibility of indeterminacy of the transitional growth path without introducing physical capital stock on public expenditure into the utility function[17]. This is so because both the environmental quality and health infrastructure are stock variables in our model generating externalities in the productivity of the system. Also the physical capital stock generates a negative externality through congestion effect. That the externality of physical capital may generate indeterminacy in the transitional growth path has been explained by Benhabib and Farmer (1993), Chen and Lee (2006), Mino (2001), Benhabib, Meng and Nishimura (2000). These externalities cannot be internalized by the private agents; and the interaction among conflicting type of externalities may generate indeterminacy in the transitional growth path.

5. COMMAND ECONOMY

The market economy solution may not coincide with the socially efficient solution in the steady-state equilibrium due to the distortion caused by the proportional income tax and by the failure of private individuals to internalize externalities. The presence of three non rival inputs in the production function - public infrastructure, health capital and environmental quality - causes positive externalities. Also, physical capital generates negative externalities through congestion effects. Therefore, we next turn to solve the planner's problem in order to obtain the first best solution. The planner, who also maximizes a social welfare function identical to that of the representative household's lifetime utility function, can internalize the externalities. Equations (1), (2), (5) and (8) remain unchanged; equations (3), (4), (6) and (7) are modified as follows.

$$\dot{K} = Y - \Pi - C; \qquad \ldots\ldots(3.1)$$

[17] Cazzavillan (1996), Chang (1999), Chen (2006), Zhang (2000), Raurich-Puigdevall (2000), etc. explain indeterminacy when public expenditure enters as an argument in the utility function.

$$\dot{E} = \Omega - \delta Y; \quad \ldots\ldots (4.1)$$

$$G = G_I + G_H = \Pi - \Omega; \quad \ldots\ldots (6.1)$$

and

$$G_i = v_i(\Pi - \Omega) with\; i = I, H. \quad \ldots\ldots (7.1)$$

Here Π denotes planner's combined lump sum expenditure on public intermediate input, health infrastructure and abatement activities; and the abatement expenditure is denoted by Ω.

The planner's problem is to maximize $\int_0^\infty e^{-\rho t} \frac{C^{1-\sigma}}{1-\sigma} dt$ with respect to C, Π, Ω and v_i subject to equations (3.1), (4.1), (5), (6.1) and (7.1). We consider a steady-state growth equilibrium where the growth rate is denoted by g_c; and the following equation solves for the steady-state equilibrium growth rate[18] in the command (planned) economy.

$$(\rho + \sigma g_c)^{\alpha+\beta} = (1 - \delta - \eta\delta)\beta^\beta (1 - \alpha - \beta)^{1-\alpha-\beta} \{\theta(1 - \alpha - \beta)\}^{\theta(1-\alpha-\beta)} \{\alpha - \theta(1 - \alpha - \beta)\}^{\alpha-\theta(1-\alpha-\beta)} \quad \ldots\ldots (26)$$

The L.H.S. of equation (26) is an increasing function of g_c and the R.H.S. is a parametric constant. Figure 2 shows the determination of the unique value of g_c, when $\theta > 0$.

[18] Equation (26) is derived in the appendix (E).

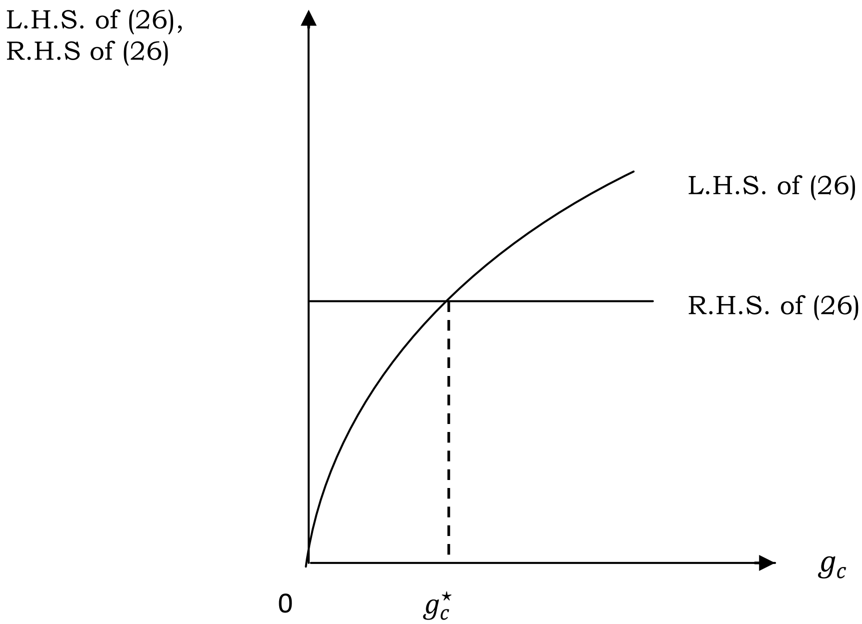

FIGURE 2

We compare the market economy solution to the socially efficient solution by comparing equation (26) to equation (15) when $\tau = \tau^\star$, $T = T^\star$ and $v_I = v_I^\star$. We modify equation (15) with $\tau = \tau^\star$, $T = T^\star$ and $v_I = v_I^\star$ as follows.

$$\alpha^{-\alpha-\beta}\left[\frac{\alpha g_m}{(\sigma g_m+\rho)}\right]^{\beta+\theta(1-\alpha-\beta)}(\sigma g_m+\rho)^{\alpha+\beta} = (1-\delta-\eta\delta)\beta^{\beta}(1-\alpha-\beta)^{1-\alpha-\beta}$$
$$\{\theta(1-\alpha-\beta)\}^{\theta(1-\alpha-\beta)}\{\alpha-\theta(1-\alpha-\beta)\}^{\alpha-\theta(1-\alpha-\beta)}. \quad \ldots\ldots (15.1)$$

The R.H.S. of equations (26) and (15.1) are identical. However, the L.H.S. of equation (15.1) is greater than that of equation (26) for all values of $g_m = g_c > \frac{\rho}{\alpha^{\frac{\theta(1-\alpha-\beta)-\alpha}{\theta(1-\alpha-\beta)+\beta}}-\sigma}$. Hence comparing equation (15.1) to equation (26) we find that g_m exceeds (falls short of) g_c when the parametric term $(1-\delta-\eta\delta)$ takes a low (high) value. This is shown in figure 3. The L.H.S. of equations (15.1) and (26) are plotted as positively sloped curves and the R.H.S. is depicted by horizontal straight lines for exogenous values of the parameters. The L.H.S.

curve obtained from equation (15.1) starts from the origin but the L.H.S. curve obtained from equation (26) starts from a point on the vertical axis. The intersection point of the two L.H.S. curves shows that $g_m = g_c = \frac{\rho}{\alpha^{\frac{\theta(1-\alpha-\beta)-\alpha}{\theta(1-\alpha-\beta)+\beta}} - \sigma}$.

When $(1-\delta-\eta\delta)$ takes a very low value the points of intersection of the two L.H.S. curves with the lower horizontal line in figure 3 show that $g_c^{\star}$ falls short of $g_m^{\star}$. When $(1-\delta-\eta\delta)$ takes a high value we find that $\widehat{g_c} > \widehat{g_m}$.

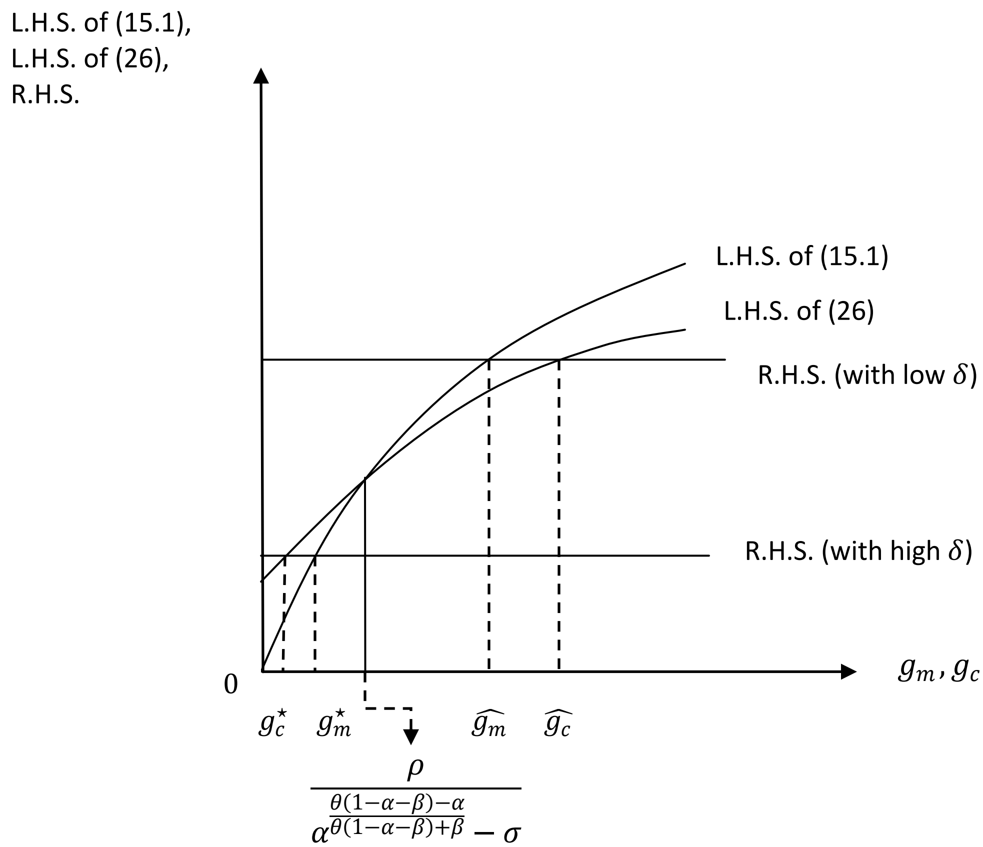

FIGURE 3

We can state the following proposition.

Proposition 4: If $\theta > 0$, then $(g_c - g_m)$ takes a positive (negative) sign when $(1 - \delta - \eta\delta)$ takes a high (low) value[19].

Barro (1990) and FMS (1993) show that the market economy growth rate in the steady-state equilibrium falls short of the socially efficient growth rate. Agenor (2008) does not find out the socially efficient solution but the implication should be same as those of Barro (1990) and FMS (1993). Each of them considers the role of a positive externality. The result obtained from the present model may be different from theirs'. Here the planner internalizes two conflicting types of externalities-the negative externality arising due to pollution of the environment as well as due to the congestion effect of capital accumulation, and the positive externalities caused by the presence of the public infrastructure, the health capital and the environmental quality. So the net benefit of internalization of externalities is ambiguous. Socially efficient growth rate should exceed (fall short of) the competitive equilibrium growth rate when a positive (negative) externality is internalized.

The relationship between the market economy equilibrium growth rate and the socially efficient growth rate in our model depends on the value $(1 - \delta - \eta\delta)$. This term takes a high (low) value if the pollution-output coefficient, δ,takes a low (high) value or if the pollution produces a weak (strong) negative effect on the depreciation of capital. When $(1 - \delta - \eta\delta)$ takes a low value, the negative externality of environmental pollution dominates all other positive externalities; and the opposite happens when $(1 - \delta - \eta\delta)$ takes a high value.

[19] We assume the existence of a unique point of intersection of two L.H.S. curves in the figure 3. If they never intersect, g_m is always greater than g_c. If they intersect twice, $(g_c - g_m)$ takes a positive sign for very low and very high values of $(1 - \delta - \eta\delta)$ but is negative for its intermediate values.

6. CONCLUSION

The paper develops an endogenous growth model with a special focus on the role of public infrastructural expenditure, health expenditure and environmental pollution. Economic growth leads to environmental pollution which lowers the rate of improvement in environmental quality as well as the rate of accumulation of health capital; and the improvement in environmental quality and the accumulation of the health capital lead to an increase in the social marginal productivity of physical capital. This model is different from those of Greiner (2005) and of Economides and Philippopoulos (2008) where environmental pollution affects the utility of the representative household.

We derive following interesting results from this model. The optimal rate of allocation of the income tax revenue to the productive public expenditure (combined health and infrastructural expenditure) is less than the combined competitive output share of the two public inputs-health and infrastructure. This result differs from what Barro (1990), FMS (1993), Agenor (2008), etc. obtain. Secondly, the model exhibits transitional dynamic properties and we find the possibility of indeterminacy in the transitional growth path converging to the steady-state equilibrium point. There is no transitional dynamics property in Barro (1990) model; and the steady state equilibrium is saddle-point stable in Greiner (2005) model, FMS (1993) model and also in the stock health version of Agenor (2008) model. Thirdly, the competitive equilibrium growth rate in this model is not necessarily less than the socially efficient growth rate due to the presence of conflicting types of externalities on production; and thus the result is different from what Barro (1990), FMS (1993), Agenor (2008), etc., obtain.

However, our model is abstract and fails to consider many aspects of reality. We rule out the possibility of skill accumulation and technical progress and hence do not consider the allocation of tax revenue to education and R & D sectors. We do not consider durable public capital and hence ignore the problem of depreciation of public capital and the role of maintenance expenditure. We ignore external effects of public infrastructural expenditure, health infrastructure and environmental quality on the utility function of the representative household. We consider the level of production as the only source of environmental pollution. Ignoring labour as a variable factor of production, we cannot analyse the problem of unemployment. One sector aggregative framework fails to highlight the inter-relationship among different sectors in the context of environmental pollution and revenue generation; and thus we cannot analyse the differences in the properties of sector-specific optimal tax rates. We plan to do further research in future attempting to remove these problems.

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APPENDIX (A)

DERIVATION OF EQUATION (9) IN SECTION 3

The dynamic optimization problem of the representative household is to maximize $\int_0^\infty e^{-\rho t}\frac{C^{1-\sigma}}{1-\sigma}dt$ with respect to C subject to equation (3) and given $K(0)$. Here C is the control variable satisfying $0 \le C \le (1-\tau)Y$; and K is the state variable.

The Hamiltonian to be maximized at each point of time is given by

$H = e^{-\rho t}\frac{C^{1-\sigma}}{1-\sigma} + e^{-\rho t}\lambda_K[(1-\tau)Y - C]$.

Here λ_K is the co-state variable representing the shadow price of investment. Maximizing the Hamiltonian with respect to C and assuming an interior solution, we obtain

$$C^{-\sigma} = \lambda_K. \qquad \ldots\ldots(A1)$$

Also the optimum time path of λ_K satisfies the following.

$$\frac{\dot{\lambda_K}}{\lambda_K} = \rho - (1-\tau)\alpha K^{\alpha-1}\hat{G}^{1-\alpha-\beta}H^{\beta}. \qquad \ldots\ldots(A2)$$

Using equations (1), (2), (6), (7) and (A2) we have

$$\frac{\dot{\lambda_K}}{\lambda_K} = \rho - \alpha(1-\tau)\{v_I(\tau - T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}}\left(\frac{E}{K}\right)^{\frac{\beta+\theta(1-\alpha-\beta)}{\alpha+\beta}}\left(\frac{H}{E}\right)^{\frac{\beta}{\alpha+\beta}}. \qquad \ldots\ldots(A3)$$

Using the two optimality conditions (A1) and (A3), we have

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}\left[\alpha(1-\tau)\{v_I(\tau - T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}}\left(\frac{E}{K}\right)^{\frac{\beta+\theta(1-\alpha-\beta)}{\alpha+\beta}}\left(\frac{H}{E}\right)^{\frac{\beta}{\alpha+\beta}} - \rho\right], \qquad \ldots\ldots(A4)$$

which is same as equation (9) in the body of the paper.

APPENDIX (B)

DERIVATION OF EQUATION (15) IN SECTION 3.1

Using equations (1) to (7), (9) and (10) we have the following equations.

$$g_m = \frac{\dot{C}}{C} = \frac{1}{\sigma}\left[\alpha(1-\tau)\{v_I(\tau-T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}}\left(\frac{E}{K}\right)^{\frac{\beta+\theta(1-\alpha-\beta)}{\alpha+\beta}}\left(\frac{H}{E}\right)^{\frac{\beta}{\alpha+\beta}}-\rho\right]; \quad \ldots\ldots(B1)$$

$$g_m = \frac{\dot{K}}{K} = (1-\tau)\{v_I(\tau-T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}}\left(\frac{E}{K}\right)^{\frac{\beta+\theta(1-\alpha-\beta)}{\alpha+\beta}}\left(\frac{H}{E}\right)^{\frac{\beta}{\alpha+\beta}}-\frac{C}{K}; \quad \ldots\ldots(B2)$$

$$g_m = \frac{\dot{E}}{E} = (T-\delta)\{v_I(\tau-T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}}\left(\frac{E}{K}\right)^{\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}}\left(\frac{H}{E}\right)^{\frac{\beta}{\alpha+\beta}}; \quad \ldots\ldots(B3)$$

and

$$g_m = \frac{\dot{H}}{H} = \{v_H(\tau-T)-\eta\delta\}\{v_I(\tau-T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}}\left(\frac{E}{K}\right)^{\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}}\left(\frac{H}{E}\right)^{-\frac{\alpha}{\alpha+\beta}}; \quad \ldots\ldots(B4)$$

From equation (B1) we have,

$$\frac{E}{K} = \left[\{v_I(\tau-T)\}^{1-\alpha-\beta}\left\{\frac{(\sigma g_m+\rho)}{\alpha(1-\tau)}\right\}^{-\alpha-\beta}\left(\frac{H}{E}\right)^{\beta}\right]^{-\frac{1}{\theta(1-\alpha-\beta)+\beta}}. \quad \ldots\ldots(B5)$$

Again, from equation (B3) we have,

$$\frac{E}{K} = \left[\{v_I(\tau-T)\}^{1-\alpha-\beta}\left(\frac{g_m}{T-\delta}\right)^{-\alpha-\beta}\left(\frac{H}{E}\right)^{\beta}\right]^{\frac{1}{\alpha-\theta(1-\alpha-\beta)}}. \quad \ldots\ldots(B6)$$

Using equations (B5) and (B6) we derive the following equation.

$$\frac{H}{E} = \left[\{v_I(\tau-T)\}^{1-\alpha-\beta}\left(\frac{T-\delta}{g_m}\right)^{\beta+\theta(1-\alpha-\beta)}\left\{\frac{\alpha(1-\tau)}{(\sigma g_m+\rho)}\right\}^{\alpha-\theta(1-\alpha-\beta)}\right]^{-\frac{1}{\beta}}. \quad \ldots\ldots(B7)$$

Using equations (B6) and (B7) we obtain the following equation.

$$\frac{E}{K} = \frac{(\sigma g_m+\rho)}{g_m}\frac{(T-\delta)}{\alpha(1-\tau)}. \quad \ldots\ldots(B8)$$

Similarly using equations (B1) and (B2) we can show that

$$\frac{C}{K} = \frac{(\sigma-\alpha)g_m+\rho}{\alpha}. \qquad \dots\dots(B9)$$

Now, using equations (B4), (B7) and (B8) we derive the following equation.

$$g_m = \{v_H(\tau-T)-\eta\delta\}\{v_I(\tau-T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}}\left[\frac{(\sigma g_m+\rho)}{g_m}\frac{(T-\delta)}{\alpha(1-\tau)}\right]^{\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}}$$

$$\left[\{v_I(\tau-T)\}^{1-\alpha-\beta}\left(\frac{T-\delta}{g_m}\right)^{\beta+\theta(1-\alpha-\beta)}\left\{\frac{\alpha(1-\tau)}{(\sigma g_m+\rho)}\right\}^{\alpha-\theta(1-\alpha-\beta)}\right]^{\frac{\alpha}{\beta(\alpha+\beta)}},$$

or,

$$g_m{}^{\beta+\theta(1-\alpha-\beta)}(\sigma g_m+\rho)^{\alpha-\theta(1-\alpha-\beta)} = \alpha^{\alpha-\theta(1-\alpha-\beta)}(1-\tau)^{\alpha-\theta(1-\alpha-\beta)}$$

$$\{v_H(\tau-T)-\eta\delta\}^{\beta}\{v_I(\tau-T)\}^{1-\alpha-\beta}(T-\delta)^{\theta(1-\alpha-\beta)}. \qquad \dots\dots(B10)$$

This is same as equation (15) in the body of the paper.

APPENDIX (C)

DERIVATION OF EQUATIONS (16), (17) AND (18) AND THE SECOND ORDER CONDITIONS IN SECTION 3.2

Maximizing the R.H.S. of equation (15) with respect to τ, we obtain the following first order condition.

$$\alpha^{\alpha-\theta(1-\alpha-\beta)}(1-\tau)^{\alpha-\theta(1-\alpha-\beta)}\{v_H(\tau-T)-\eta\delta\}^{\beta}\{v_I(\tau-T)\}^{1-\alpha-\beta}(T-\delta)^{\theta(1-\alpha-\beta)}$$
$$\begin{bmatrix}\beta(1-v_I)\{v_H(\tau-T)-\eta\delta\}^{-1}-\{\alpha-\theta(1-\alpha-\beta)\}(1-\tau)^{-1}\\ +(1-\alpha-\beta)(\tau-T)^{-1}\end{bmatrix}=0;$$

or,

$$\beta(1-v_I)\{v_H(\tau-T)-\eta\delta\}^{-1}-\{\alpha-\theta(1-\alpha-\beta)\}(1-\tau)^{-1}$$
$$+(1-\alpha-\beta)(\tau-T)^{-1}=0. \qquad \dots\dots(C1)$$

Maximizing the R.H.S. of equation (15) with respect to T, we obtain the following first order condition.

$$\alpha^{\alpha-\theta(1-\alpha-\beta)}(1-\tau)^{\alpha-\theta(1-\alpha-\beta)}\{v_H(\tau-T)-\eta\delta\}^{\beta}\{v_I(\tau-T)\}^{1-\alpha-\beta}(T-\delta)^{\theta(1-\alpha-\beta)}$$
$$\begin{bmatrix}-\beta(1-v_I)\{v_H(\tau-T)-\eta\delta\}^{-1}+\theta(1-\alpha-\beta)(T-\delta)^{-1}\\ -(1-\alpha-\beta)(\tau-T)^{-1}\end{bmatrix}=0;$$

or,

$$-\beta(1-v_I)\{v_H(\tau-T)-\eta\delta\}^{-1}+\theta(1-\alpha-\beta)(T-\delta)^{-1}$$
$$-(1-\alpha-\beta)(\tau-T)^{-1}=0. \qquad \dots\dots(C2)$$

Maximizing the R.H.S. of equation (15) with respect to v_I, we obtain the following first order condition.

$$\alpha^{\alpha-\theta(1-\alpha-\beta)}(1-\tau)^{\alpha-\theta(1-\alpha-\beta)}\{v_H(\tau-T)-\eta\delta\}^{\beta}\{v_I(\tau-T)\}^{1-\alpha-\beta}(T-\delta)^{\theta(1-\alpha-\beta)}$$
$$[-\beta(\tau-T)\{v_H(\tau-T)-\eta\delta\}^{-1}+(1-\alpha-\beta){v_I}^{-1}]=0. \qquad \dots(C3)$$

Using equations (C1), (C2) and (C3) we arrive at the following expressions for the optimal tax rates and the optimal public expenditure allocation ratio.

$$\tau^\star = 1 - (1-\delta-\eta\delta)\{\alpha - \theta(1-\alpha-\beta)\};$$

$$T^\star = \delta + (1-\delta-\eta\delta)\theta(1-\alpha-\beta);$$

and

$$v_I^\star = \frac{(1-\delta-\eta\delta)(1-\alpha-\beta)}{\eta\delta+(1-\delta-\eta\delta)(1-\alpha)}.$$

These are the same as equations (16), (17) and (18) in the body of the paper.

To check the second order conditions for optimality we twice differentiate equation (15), with respect to τ, T and v_I respectively and arrive at the following three second order conditions.

$$-\left[\{\beta+\theta(1-\alpha-\beta)\}{g_m}^{-2} + \sigma^2\{\alpha-\theta(1-\alpha-\beta)\}(\sigma g_m+\rho)^{-2}\right]\left(\frac{\partial g_m}{\partial \tau}\right)^2$$
$$+[\{\beta+\theta(1-\alpha-\beta)\}{g_m}^{-1} + \sigma\{\alpha-\theta(1-\alpha-\beta)\}(\sigma g_m+\rho)^{-1}]\frac{\partial^2 g_m}{\partial \tau^2}$$
$$= -[(1-\alpha-\beta)(\tau-\mathrm{T})^{-2} + \{\alpha-\theta(1-\alpha-\beta)\}(1-\tau)^{-2}$$
$$+\beta\{v_H(\tau-T)-\eta\delta\}^{-2}v_H^2]; \qquad \ldots\ldots(C4)$$

$$-\left[\{\beta+\theta(1-\alpha-\beta)\}{g_m}^{-2} + \sigma^2\{\alpha-\theta(1-\alpha-\beta)\}(\sigma g_m+\rho)^{-2}\right]\left(\frac{\partial g_m}{\partial T}\right)^2$$
$$+[\{\beta+\theta(1-\alpha-\beta)\}{g_m}^{-1} + \sigma\{\alpha-\theta(1-\alpha-\beta)\}(\sigma g_m+\rho)^{-1}]\frac{\partial^2 g_m}{\partial T^2}$$
$$= -[(1-\alpha-\beta)(\tau-\mathrm{T})^{-2} + \theta(1-\alpha-\beta)(T-\delta)^{-2}$$
$$+\beta\{v_H(\tau-T)-\eta\delta\}^{-2}v_H^2]; \qquad \ldots\ldots(C5)$$

and

$$-\left[\{\beta+\theta(1-\alpha-\beta)\}{g_m}^{-2} + \sigma^2\{\alpha-\theta(1-\alpha-\beta)\}(\sigma g_m+\rho)^{-2}\right]\left(\frac{\partial g_m}{\partial v_I}\right)^2$$
$$+[\{\beta+\theta(1-\alpha-\beta)\}{g_m}^{-1} + \sigma\{\alpha-\theta(1-\alpha-\beta)\}(\sigma g_m+\rho)^{-1}]\frac{\partial^2 g_m}{\partial v_I^2}$$
$$= -[\beta(\tau-\mathrm{T})^2\{v_H(\tau-T)-\eta\delta\}^{-2} + (1-\alpha-\beta){v_I}^{-2}]; \qquad \ldots\ldots(C6)$$

Now we evaluate the above three second order conditions at $\tau = \tau^\star$, $T = T^\star$ and $v_I = v_I^\star$ where $\frac{\partial g_m}{\partial \tau} = \frac{\partial g_m}{\partial T} = \frac{\partial g_m}{\partial v_I} = 0$. Hence we obtain the followings.

$$\frac{\partial^2 g_m}{\partial \tau^2} = -\frac{(1-\alpha-\beta)(\tau^\star-T^\star)^{-2} + \{\alpha-\theta(1-\alpha-\beta)\}(1-\tau^\star)^{-2} + \beta\{v_H^\star(\tau^\star-T^\star)-\eta\delta\}^{-2}{v_H^\star}^2}{[\{\beta+\theta(1-\alpha-\beta)\}{g_m}^{-1} + \sigma\{\alpha-\theta(1-\alpha-\beta)\}(\sigma g_m+\rho)^{-1}]};$$

$$\frac{\partial^2 g_m}{\partial T^2} = -\frac{(1-\alpha-\beta)(\tau^\star-T^\star)^{-2}+\theta(1-\alpha-\beta)(T^\star-\delta)^{-2}+\beta\{v_H^\star(\tau^\star-T^\star)-\eta\delta\}^{-2}{v_H^\star}^2}{[\{\beta+\theta(1-\alpha-\beta)\}{g_m}^{-1}+\sigma\{\alpha-\theta(1-\alpha-\beta)\}(\sigma g_m+\rho)^{-1}]};$$

and

$$\frac{\partial^2 g_m}{\partial v_I^2} = -\frac{\left[\beta(\tau^\star-T^\star)^2\{v_H(\tau^\star-T^\star)-\eta\delta\}^{-2}+(1-\alpha-\beta){v_I^\star}^{-2}\right]}{[\{\beta+\theta(1-\alpha-\beta)\}{g_m}^{-1}+\sigma\{\alpha-\theta(1-\alpha-\beta)\}(\sigma g_m+\rho)^{-1}]}.$$

Hence the R.H.S. of each of these three equations is negative. Thus the second order conditions are also satisfied.

APPENDIX (D)

DERIVATION OF THE DETERMINANT AND THE TRACE OF THE JACOBIAN MATRIX IN SECTION 4

We define the following variables.

$$M = (1-\tau)\{v_I(\tau - T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}} y^{\frac{\theta(1-\alpha-\beta)+\beta}{\alpha+\beta}} z^{\frac{\beta}{\alpha+\beta}}; \qquad \ldots\ldots(D1)$$

$$N = (T-\delta)\{v_I(\tau - T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}} y^{\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}} z^{\frac{\beta}{\alpha+\beta}}; \qquad \ldots\ldots(D2)$$

and

$$Q = \{v_H(\tau - T) - \eta\delta\}\{v_I(\tau - T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}} y^{\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}} z^{-\frac{\alpha}{\alpha+\beta}}. \qquad \ldots\ldots(D3)$$

Now we consider following equations from the body of the paper.

$$\frac{\dot{x}}{x} = \left(\frac{\alpha}{\sigma} - 1\right)(1-\tau)\{v_I(\tau - T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}} y^{\frac{\theta(1-\alpha-\beta)+\beta}{\alpha+\beta}} z^{\frac{\beta}{\alpha+\beta}} + x - \frac{\rho}{\sigma}; \qquad \ldots\ldots(23)$$

$$\frac{\dot{y}}{y} = (T-\delta)\{v_I(\tau - T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}} y^{\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}} z^{\frac{\beta}{\alpha+\beta}}$$
$$-(1-\tau)\{v_I(\tau - T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}} y^{\frac{\theta(1-\alpha-\beta)+\beta}{\alpha+\beta}} z^{\frac{\beta}{\alpha+\beta}} + x; \qquad \ldots\ldots(24)$$

and

$$\frac{\dot{z}}{z} = \{v_H(\tau - T) - \eta\delta\}\{v_I(\tau - T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}} y^{\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}} z^{-\frac{\alpha}{\alpha+\beta}}$$
$$-(T-\delta)\{v_I(\tau - T)\}^{\frac{1-\alpha-\beta}{\alpha+\beta}} y^{\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}} z^{\frac{\beta}{\alpha+\beta}}. \qquad \ldots\ldots(25)$$

Thus using equations (D1), (D2) and (D3) we modify equations (23), (24) and (25) as follows.

$$\frac{\dot{x}}{x} = \left(\frac{\alpha}{\sigma} - 1\right) M + x - \frac{\rho}{\sigma}; \qquad \ldots\ldots(D4)$$

$$\frac{\dot{y}}{y} = N - M + x; \qquad \ldots\ldots(D5)$$

and

$$\frac{\dot{z}}{z} = Q - N. \qquad \ldots\ldots(D6)$$

We obtain the following partial derivatives corresponding to three modified differential equations.

$$\frac{\partial\left(\frac{\dot{x}}{x}\right)}{\partial x} = 1;$$

$$\frac{\partial\left(\frac{\dot{x}}{x}\right)}{\partial y} = \frac{\theta(1-\alpha-\beta)+\beta}{\alpha+\beta}\left(\frac{\alpha}{\sigma}-1\right)\frac{M}{y};$$

$$\frac{\partial\left(\frac{\dot{x}}{x}\right)}{\partial z} = \frac{\beta}{\alpha+\beta}\left(\frac{\alpha}{\sigma}-1\right)\frac{M}{z};$$

$$\frac{\partial\left(\frac{\dot{y}}{y}\right)}{\partial x} = 1;$$

$$\frac{\partial\left(\frac{\dot{y}}{y}\right)}{\partial y} = \frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}\frac{N}{y} - \frac{\theta(1-\alpha-\beta)+\beta}{\alpha+\beta}\frac{M}{y};$$

$$\frac{\partial\left(\frac{\dot{y}}{y}\right)}{\partial z} = \frac{\beta}{\alpha+\beta}\frac{N}{z} - \frac{\beta}{\alpha+\beta}\frac{M}{z};$$

$$\frac{\partial\left(\frac{\dot{z}}{z}\right)}{\partial x} = 0;$$

$$\frac{\partial\left(\frac{\dot{z}}{z}\right)}{\partial y} = \frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}\frac{Q}{y} - \frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}\frac{N}{y};$$

and

$$\frac{\partial\left(\frac{\dot{z}}{z}\right)}{\partial z} = -\frac{\alpha}{\alpha+\beta}\frac{Q}{z} - \frac{\beta}{\alpha+\beta}\frac{N}{z}.$$

So the determinant of the Jacobian matrix can be written as follows.

$$\begin{aligned}
|J| = &\left[\frac{\{\theta(1-\alpha-\beta)-\alpha\}}{(\alpha+\beta)}\frac{N}{y} - \frac{\{\theta(1-\alpha-\beta)+\beta\}}{(\alpha+\beta)}\frac{M}{y}\right]\left[-\frac{\alpha}{(\alpha+\beta)}\frac{Q}{z} - \frac{\beta}{(\alpha+\beta)}\frac{N}{z}\right] \\
&- \left[\frac{\{\theta(1-\alpha-\beta)-\alpha\}}{(\alpha+\beta)}\frac{Q}{y} - \frac{\{\theta(1-\alpha-\beta)-\alpha\}}{(\alpha+\beta)}\frac{N}{y}\right]\left[\frac{\beta}{(\alpha+\beta)}\frac{N}{z} - \frac{\beta}{(\alpha+\beta)}\frac{M}{z}\right] \\
&- \frac{\{\theta(1-\alpha-\beta)+\beta\}}{(\alpha+\beta)}\left(\frac{\alpha}{\sigma}-1\right)\frac{M}{y}\left[-\frac{\alpha}{(\alpha+\beta)}\frac{Q}{z} - \frac{\beta}{(\alpha+\beta)}\frac{N}{z}\right] \\
&+ \frac{\beta}{(\alpha+\beta)}\left(\frac{\alpha}{\sigma}-1\right)\frac{M}{z}\left[\frac{\{\theta(1-\alpha-\beta)-\alpha\}}{(\alpha+\beta)}\frac{Q}{y} - \frac{\{\theta(1-\alpha-\beta)-\alpha\}}{(\alpha+\beta)}\frac{N}{y}\right];
\end{aligned}$$

or,

$$|J| = \frac{\{\alpha-\theta(1-\alpha-\beta)\}}{(\alpha+\beta)}\frac{N}{y}\frac{Q}{z} + \frac{\alpha}{\sigma}\frac{\theta(1-\alpha-\beta)}{(\alpha+\beta)}\frac{M}{y}\frac{Q}{z} + \frac{\alpha}{\sigma}\frac{\beta}{(\alpha+\beta)}\frac{M}{y}\frac{N}{z};$$

or,

$$|J| = \frac{\alpha-\theta(1-\alpha-\beta)}{\alpha+\beta}(T-\delta)\{v_I(\tau-T)\}^{2\frac{1-\alpha-\beta}{\alpha+\beta}}\{v_H(\tau-T)-\eta\delta\}y^{2\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}-1}z^{-2\frac{\alpha}{\alpha+\beta}}$$

$$+\frac{\alpha}{\sigma}\frac{\theta(1-\alpha-\beta)}{\alpha+\beta}(1-\tau)\{v_I(\tau-T)\}^{2\frac{1-\alpha-\beta}{\alpha+\beta}}\{v_H(\tau-T)-\eta\delta\}y^{2\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}}z^{-2\frac{\alpha}{\alpha+\beta}}$$

$$+\frac{\alpha}{\sigma}\frac{\beta}{\alpha+\beta}(1-\tau)(T-\delta)\{v_I(\tau-T)\}^{2\frac{1-\alpha-\beta}{\alpha+\beta}}y^{2\frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}}z^{2\frac{\beta}{\alpha+\beta}-1}.$$

Here $\alpha > \theta(1-\alpha-\beta)$, $1 > \tau > T > \delta$ and $v_H(\tau-T)-\eta\delta > 0$. Thus the determinant is positive in sign.

The trace of the Jacobian matrix is given by,

$$Tr\,J = 1 + \frac{\theta(1-\alpha-\beta)-\alpha}{\alpha+\beta}\frac{N}{y} - \frac{\theta(1-\alpha-\beta)+\beta}{\alpha+\beta}\frac{M}{y} - \frac{\alpha}{\alpha+\beta}\frac{Q}{z} - \frac{\beta}{\alpha+\beta}\frac{N}{z}.$$

Using equations (D4), (D5) and (D6) the trace can be written as follows.

$$Tr\,J = 1 + \left[\frac{\{\theta(1-\alpha-\beta)-\alpha\}}{(\alpha+\beta)}\frac{\alpha}{\sigma}\frac{1}{y} - \frac{\{\theta(1-\alpha-\beta)+\beta\}}{(\alpha+\beta)}\frac{1}{y} - \frac{\alpha}{\sigma}\frac{1}{z}\right]M - \frac{\{\theta(1-\alpha-\beta)-\alpha\}}{(\alpha+\beta)}\frac{\rho}{\sigma}\frac{1}{y} + \frac{\rho}{\sigma}\frac{1}{z}.$$

Now, $Tr\,J < 0$ if

$$1 + \frac{\{\alpha-\theta(1-\alpha-\beta)\}}{(\alpha+\beta)}\frac{\rho}{\sigma}\frac{1}{y} + \frac{\rho}{\sigma}\frac{1}{z} < \left[\frac{\{\alpha-\theta(1-\alpha-\beta)\}}{(\alpha+\beta)}\frac{\alpha}{\sigma}\frac{1}{y} + \frac{\{\theta(1-\alpha-\beta)+\beta\}}{(\alpha+\beta)}\frac{1}{y} + \frac{\alpha}{\sigma}\frac{1}{z}\right]M$$

At the steady-state equilibrium, $\frac{\dot{x}}{x} = \frac{\dot{y}}{y} = \frac{\dot{z}}{z} = 0$. Using $\frac{\dot{z}}{z} = 0$ and the optimal values of the policy variables given by equations (16), (17) and (18) we have,

$$z = \frac{v_H(\tau-T)-\eta\delta}{T-\delta} = \frac{\beta}{\theta(1-\alpha-\beta)}. \qquad \dots\dots(D7)$$

Now using equations (B5), (D7) and the optimal values of the policy variables, the condition for the trace of the Jacobian matrix to be negative can be written as

$$1 + \frac{\alpha\{\alpha-\theta(1-\alpha-\beta)\}^2}{\theta(1-\alpha-\beta)(\alpha+\beta)}\frac{\rho}{\sigma}\frac{g_m}{(\sigma g_m+\rho)} + \frac{\rho}{\sigma}\frac{\theta(1-\alpha-\beta)}{\beta}$$

$$< \left[\frac{\{\alpha-\theta(1-\alpha-\beta)\}}{(\alpha+\beta)}\frac{\alpha}{\sigma} + \frac{\{\theta(1-\alpha-\beta)+\beta\}}{(\alpha+\beta)} + \frac{\alpha}{\sigma}\frac{\{\theta(1-\alpha-\beta)\}^2}{\alpha\beta\{\alpha-\theta(1-\alpha-\beta)\}}\frac{(\sigma g_m+\rho)}{g_m}\right]$$

$$(1-\delta-\eta\delta)^{\frac{1}{\alpha+\beta}}\beta^{\frac{\beta}{\alpha+\beta}}\alpha^{\frac{\alpha-\theta(1-\alpha-\beta)}{\alpha+\beta}}\{1-\alpha-\beta\}^{\frac{1-\alpha-\beta}{\alpha+\beta}}\{\theta(1-\alpha-\beta)\}^{\frac{\alpha-\theta(1-\alpha-\beta)+\beta}{\alpha+\beta}}$$

$$\{\alpha-\theta(1-\alpha-\beta)\}^{\frac{\beta+\theta(1-\alpha-\beta)}{\alpha+\beta}}\left\{\frac{g_m}{(\sigma g_m+\rho)}\right\}^{\frac{\alpha-\theta(1-\alpha-\beta)}{\alpha+\beta}}.$$

APPENDIX (E)

DERIVATION OF EQUATION (26) IN SECTION 5

The relevant Hamiltonian to be maximized by the planner at each point of time is given by

$$L = e^{-\rho t}\frac{C^{1-\sigma}}{1-\sigma} + e^{-\rho t}\lambda_K\left[\{v_I(\Pi-\Omega)\}^{1-\alpha-\beta}K^{\alpha-\theta(1-\alpha-\beta)}E^{\theta(1-\alpha-\beta)}H^{\beta} - \Pi - C\right]$$
$$+e^{-\rho t}\lambda_E\left[\Omega - \delta\{v_I(\Pi-\Omega)\}^{1-\alpha-\beta}K^{\alpha-\theta(1-\alpha-\beta)}E^{\theta(1-\alpha-\beta)}H^{\beta}\right]$$
$$+e^{-\rho t}\lambda_H\left[(1-v_I)(\Pi-\Omega) - \eta\delta\{v_I(\Pi-\Omega)\}^{1-\alpha-\beta}K^{\alpha-\theta(1-\alpha-\beta)}E^{\theta(1-\alpha-\beta)}H^{\beta}\right].$$

The state variables are K, H and E. The control variables are $C, \Pi,\ \Omega$ and v_I. λ_K, λ_H, and λ_E are three co-state variables.

Maximising L with respect to $C, \Pi,\ \Omega$ and v_I we have

$$C^{-\sigma} = \lambda_K; \qquad \dots\dots(E1)$$

$$\left(\frac{\lambda_K}{\lambda_E} - \delta\right)(1-\alpha-\beta)\frac{Y}{\Pi-\Omega} + \frac{\lambda_H}{\lambda_E}\left[(1-v_I) - \eta\delta(1-\alpha-\beta)\frac{Y}{\Pi-\Omega}\right] = \frac{\lambda_K}{\lambda_E}; \qquad \dots\dots(E2)$$

$$\left(\frac{\lambda_K}{\lambda_E} - \delta\right)(1-\alpha-\beta)\frac{Y}{\Pi-\Omega} + \frac{\lambda_H}{\lambda_E}\left[(1-v_I) - \eta\delta(1-\alpha-\beta)\frac{Y}{\Pi-\Omega}\right] = 1; \qquad \dots\dots(E3)$$

and

$$\left(\frac{\lambda_K}{\lambda_E} - \delta\right)(1-\alpha-\beta)\frac{Y}{v_I} = \frac{\lambda_H}{\lambda_E}\left[(\Pi-\Omega) + \eta\delta(1-\alpha-\beta)\frac{Y}{v_I}\right]. \qquad \dots\dots(E4)$$

Using equations (E2) and (E3) we find that

$$\frac{\lambda_K}{\lambda_E} = 1. \qquad \dots\dots(E5)$$

Using equations (E3) and (E5) we obtain the following.

$$\frac{\lambda_H}{\lambda_E} = \frac{1-(1-\delta)(1-\alpha-\beta)\frac{Y}{v_I}}{(1-v_I)-\eta\delta(1-\alpha-\beta)\frac{Y}{\Pi-\Omega}}. \qquad \dots\dots(E6)$$

Using equations (E4), (E5) and (E6) we obtain the following equation.

$$(1-\delta-\eta\delta)(1-\alpha-\beta)\frac{Y}{v_I} = \Pi - \Omega. \qquad \dots\dots\text{(E7)}$$

Now, using equations (E6) and (E7) we find that,

$$\frac{\lambda_H}{\lambda_E} = 1. \qquad \dots\dots(E9)$$

Also, along the optimum path, time behaviour of the co-state variables satisfies the followings.

$$\left(1-\delta\frac{\lambda_E}{\lambda_K}\right)\{\alpha-\theta(1-\alpha-\beta)\}\frac{Y}{K}-\frac{\lambda_H}{\lambda_K}\eta\delta\{\alpha-\theta(1-\alpha-\beta)\}\frac{Y}{K}=\rho-\frac{\dot{\lambda_K}}{\lambda_K}; \qquad \dots\dots(E10)$$

$$\left(\frac{\lambda_K}{\lambda_E}-\delta\right)\theta(1-\alpha-\beta)\frac{Y}{E}-\frac{\lambda_H}{\lambda_E}\eta\delta\theta(1-\alpha-\beta)\frac{Y}{E}=\rho-\frac{\dot{\lambda_E}}{\lambda_E}; \qquad \dots\dots(E11)$$

and

$$\frac{\lambda_K}{\lambda_H}\beta\frac{Y}{H}-\delta\left(\frac{\lambda_E}{\lambda_H}+\eta\right)\beta\frac{Y}{H}=\rho-\frac{\dot{\lambda_H}}{\lambda_H}. \qquad \dots\dots(E12)$$

Equations (E5) and (E9) imply that $\frac{\dot{\lambda_K}}{\lambda_K}=\frac{\dot{\lambda_H}}{\lambda_H}=\frac{\dot{\lambda_E}}{\lambda_E}$. Thus using equations (E5), (E9), (E10) and (E11) we obtain the following equation.

$$\frac{E}{K}=\frac{\theta(1-\alpha-\beta)}{\alpha-\theta(1-\alpha-\beta)}. \qquad \dots\dots(E13)$$

Again using, equations (E5), (E9), (E11) and (E12) we obtain,

$$\frac{H}{E}=\frac{\beta}{\theta(1-\alpha-\beta)}. \qquad \dots\dots(E14)$$

Using equations (1), (2) and (7.1) we have

$$\frac{Y}{v_I(\Pi-\Omega)}\{v_I(\Pi-\Omega)\}^{\alpha+\beta}=K^{\alpha-\theta(1-\alpha-\beta)}E^{\theta(1-\alpha-\beta)}H^{\beta}. \qquad \dots\dots(E15)$$

Now, using equations (E7) and (E15) we derive the following.

$$\{v_I(\Pi-\Omega)\}=\left[(1-\delta-\eta\delta)(1-\alpha-\beta)K^{\alpha-\theta(1-\alpha-\beta)}E^{\theta(1-\alpha-\beta)}H^{\beta}\right]^{\frac{1}{\alpha+\beta}}. \qquad \dots\dots(E16)$$

From equation (E1), we have

$$-\sigma\frac{\dot{C}}{C}=\frac{\dot{\lambda_K}}{\lambda_K}. \qquad \dots\dots(E17)$$

Using equations (E5), (E9), (E10), (E16) and (E17) we obtain the following

$$\frac{\dot{C}}{C}=\frac{1}{\sigma}\left[\{\alpha-\theta(1-\alpha-\beta)\}(1-\delta-\eta\delta)^{\frac{1}{\alpha+\beta}}\right.$$
$$\left.(1-\alpha-\beta)^{\frac{1-\alpha-\beta}{\alpha+\beta}}\left(\frac{E}{K}\right)^{\frac{\theta(1-\alpha-\beta)+\beta}{\alpha+\beta}}\left(\frac{H}{E}\right)^{\frac{\beta}{\alpha+\beta}}-\rho\right]. \qquad \dots\dots(E19)$$

In the steady state growth equilibrium,

$$\frac{\dot{C}}{C}=\frac{1}{\sigma}\left[\{\alpha-\theta(1-\alpha-\beta)\}(1-\delta-\eta\delta)^{\frac{1}{\alpha+\beta}}\right.$$
$$\left.(1-\alpha-\beta)^{\frac{1-\alpha-\beta}{\alpha+\beta}}\left(\frac{E}{K}\right)^{\frac{\theta(1-\alpha-\beta)+\beta}{\alpha+\beta}}\left(\frac{H}{E}\right)^{\frac{\beta}{\alpha+\beta}}-\rho\right]=g_c; \qquad \dots\dots(E20)$$

$$\frac{\dot{K}}{K} = \left(\frac{K}{\Pi - \Omega}\right)^{\alpha-1} \left(\frac{E}{K}\right)^{\theta(1-\alpha)} - \frac{\Pi}{K} - \frac{C}{K} = g_c; \qquad \ldots\ldots(E21)$$

$$\frac{\dot{H}}{H} = v_H \left(\frac{\Pi - \Omega}{\mathrm{H}}\right) - \eta\delta \left\{v_I \left(\frac{\Pi - \Omega}{\mathrm{H}}\right)\right\}^{1-\alpha-\beta} \left(\frac{E}{K}\right)^{\theta(1-\alpha-\beta)-\alpha} \left(\frac{H}{E}\right)^{-\alpha} = g_c; \qquad \ldots\ldots(E22)$$

and

$$\frac{\dot{E}}{E} = \frac{\Omega}{E} - \delta \left(\frac{K}{\Pi - \Omega}\right)^{\alpha-1} \left(\frac{E}{K}\right)^{\theta(1-\alpha)-1} = g_c. \qquad \ldots\ldots(E23)$$

Therefore, using equations (E13), (E14) and (E20) we obtain the following equation.

$$(\rho + \sigma g_c)^{\alpha} = (1 - \delta - \eta\delta)\beta^{\beta}(1 - \alpha - \beta)^{1-\alpha-\beta}\{\theta(1 - \alpha - \beta)\}^{\theta(1-\alpha-\beta)}$$
$$\{\alpha - \theta(1 - \alpha - \beta)\}^{\alpha-\theta(1-\alpha-\beta)}. \qquad \ldots\ldots(E24)$$

This is same as equation (26) in section 5 in the body of the paper.