Social Capital, Individual Incentives and Loan Repayment

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Preliminary Version: September 2009

Abstract: In the last few years the loan programs of several prominent microfinance institutions have moved away from group liability towards individual liability. The Grameen Bank, for example, has been relying exclusively on individual liability contracts since 2002. We investigate the question of social repayment incentives versus incentives generated through individual liability in a theoretical model. We remove the usual assumption of exogenous social penalties, and consider the interaction between incentives generated through social risk-sharing and those generated through a group-loan program that bootstraps on those incentives. In our model agents are heterogeneous, with differing degrees of risk aversion, and the setting is one of full information implying that strategic default is the only issue. We first consider a loan that depends purely on social enforcement. We show that the threat of exclusion from social risk-sharing fails to deter deviation by types below a cutoff, which could be quite high. Essentially, a loan program using social penalties does not add to penalties, but adds to deviation benefits, creating greater incentive to deviate simultaneously from social risk-sharing as well as the loan program. This reduces the fraction of types covered by the loan program. Further, coverage is decreasing in loan duration. We then show that an individual loan program augmented by a mandatory savings plan can deliver greater coverage, and can even cover types excluded from social risk-sharing (i.e. types for whom social penalties are not available at all). Further, the coverage of an individual loan program increases with loan size as well as loan duration. The results offer an explanation for the observation by Giné and Karlan (2009) that, compared to group lending, individual lending does no worse in procuring repayment, and expands the scope of lending. Finally, we show that stronger social ties (greater underlying social cooperation) enhance the performance of individual loans. Thus fostering social cooperation is beneficial even though it has limited usefulness as a penalty under social enforcement of repayment.

JEL classification: O12

1 Introduction

A large body of literature analyzing collective lending programs in developing countries assumes that compliance is born of the threat of social penalties.(1) The analysis typically assumes that an exogenous social penalty is available, and members of schemes such as group loans can harness this effectively to engender repayment. It follows that group loans can foster repayment incentives even in the absence of useful collateral, which causes individual loan programs to fail.

However, in the last few years many loan programs in developing countries have moved away from group loans towards offering individual loans. The most prominent change has been in the contracts offered by the Grameen Bank, which originated the idea of group loans. Since 2002, the Grameen Bank has moved to "Grameen II" contracts, which are purely individual loans with certain obligatory savings requirements. Other well known micro finance institutions such as Bolivia's BancoSol have similarly moved from group loans to individual loans across several loan categories.

In general, it might be difficult to separate the effect of incentives arising purely from group loans, and other incentives such as monitoring by the micro-lending organization, regular small repayments, and incentives from any reduction in the future creditworthiness. There is some emerging work comparing the performance of group loans to that of individual loans. An interesting paper by Giné and Karlan (2009) study the results from a randomized control trial by a Philippine bank. The bank removed group liability from randomly selected group-screened lending groups. After three years, Gine and Karlan find no increase in default as well as larger group sizes in the converted centers. Thus coverage increased, but performance did not worsen.

The assumption of an exogenous social penalty obviates the need to address the interaction between existing social arrangements and associated incentives. This paper presents a simple model that removes the assumption of exogenous social penalties, and considers the interaction between group loan incentives and existing social incentives. As in Bloch et al. (2007), social capital is assumed to be the result of an underlying risk-sharing arrangement. Further, the agents are assumed to be heterogeneous - with different degrees of risk aversion.

(1) See Ghatak and Guinnane (1999) for a survey.

The paper first analyzes the effect of a loan programme that exploits social repayment incentives, and shows that loan coverage (fraction of types covered by a loan) shrinks with loan duration. Next, the paper shows that an incentive compatible individual loan program augmented by a mandatory savings plan can deliver greater coverage compared to the scheme that uses social penalties. Further, unlike the latter, the coverage of an individual loan program increases with loan size as well as loan duration.

The basic premise is simple: social capital, born of social interactions, need not itself be immune to other incentives arising from a loan programme that is based on social capital.

An agent participates in social interactions and the loan program at the same time - and therefore the ability to penalize is mitigated by the possibility of simultaneous deviation from both arrangements. In other words, ceratin penalties (such as exclusion from social cooperation) sustain social cooperation in a repeated game. When a loan program is introduced that relies on social penalties, it piggybacks on the same penalties to ensure repayment incentives. Therefore a loan program that depends on social penalties increases the deviation payoff, but the penalties remain the same. Agents for whom the incentive constraint in the underlying game binds or is close to binding would, if given a loan, now have an incentive to deviate from paying the underlying transfers as well as repaying the loan. Thus relying on social sanctions for repayment can dilute the effectiveness of the sanctions themselves.

The paper presents a simple model to illustrate this phenomenon. We assume the simple case of complete information, and consider the incentive for strategic default. To model social sanctions, we assume, as in Bloch et al. (2007), that social capital is the result of an underlying risk-sharing arrangement with agents with different degrees of risk aversion. We then show the effect of a loan programme that exploits social repayment incentives. To maximize the power of social enforcement, we assume complete risk-sharing.

We first consider a loan that depends purely on social enforcement. We show that the threat of exclusion from social risk-sharing fails to deter deviation by types below a cut-off, which could be quite high. Essentially, a loan program using social penalties does not add to penalties, but adds to deviation benefits, creating greater incentive to deviate simultaneously from social risk-sharing as well as the loan program. This reduces the fraction of types covered by the loan program. Further, coverage is decreasing in loan duration.

We then show that an individual loan program augmented by a mandatory savings plan can deliver greater coverage, and can even cover types excluded from social risk-sharing (i.e. types for whom social penalties are not available at all). Further, the coverage of an individual loan program increases with loan size as well as loan duration. The results offer an explanation for the observation by Giné and Karlan (2009) that, compared to group lending, individual lending does no worse in procuring repayment, and expands the scope of lending.

Finally, we show that stronger social ties (greater underlying social cooperation) enhance the performance of individual loans. Thus fostering social cooperation is beneficial even though it has limited usefulness as a penalty under social enforcement of repayment.

Related Literature

This section is incomplete.

Ahlin and Townsend (2007): A striking aspect of their result is that "strong social ties - measured by sharing among non-relatives, cooperation, and clustering of relatives, and village-run savings and loan institutions (PCGs) - having seemingly adverse effects on repayment performance."

Our paper assumes full information. Thus all types that cannot be made to repay under social penalties are already excluded from a loan programme. In other words, non-repayment is an entirely out-of-equilibrium phenomenon. However, if peer information is not complete and therefore screening is imperfect, there would be types who secure a loan and then default. Note that this would happen even if social ties are strong (so that the risk-sharing is full and coverage of risk-sharing is high).

Related work: Coate and Ravallion (1993), Besley and Coate (1995), Jain (1996), Cull et al. (2007), Karlan (2007), Armendáriz de Aghion and Morduch (2005), Roy Chowdhury (2007), Conning (2005), Madajewicz (2005), Yunus (2008).

2 The Model

There is a continuum of agents. Each agent draws a type ρ from some distribution F on $[0, \overline{\rho}]$. The utility function of an agent with wealth x and type ρ is

$$u(x\,;\rho) = \frac{1 - \mathrm{e}^{-x\rho}}{\rho} \tag{2.1}$$

The parameter ρ is the degree of absolute risk aversion. Note that as $\rho \to 0$, $u(x\,;\rho) \to x$, the risk neutral utility function.

Investment opportunity for T periods arises for some agents. Agents have no savings technology. For simplicity, we assume that a project needs an indivisible investment of size M each period and the gross return each period is $M(1+R)$ where $R > 0$. Note that we simply study the incentive for strategic default which arises after any return is realized - and therefore assume the simplest possible investment opportunity.

An external lender funds the project with a loan for T periods. We do the accounting as follows. The first loan made at time 0 and the first repayment date is $t = 1$. The second loan is also made at $t = 1$, and $t = 2$ is the second repayment date. Thus in general, the k-th loan is made in period $k-1$ and the repayment is due in period k. Therefore a T period loan consists of T loans (given at dates $0, \ldots, T-1$) and T repayment dates $t = 1, \ldots, T$.

We assume full information on types. Removing this assumption makes defaults occur in equilibrium, but does not otherwise change results.

3 Risk Sharing and Social Capital

Social capital is formed by risk sharing. This forms basis of social incentives to sustain loan repayment - an agent who defaults is punished by exclusion from the underlying social risk sharing.

As shown below, reducing risk sharing simply reduces the ability of social incentives to provide repayment incentives in a loan program. However, we aim to show that individual loans outperform loans based on social incentives even when the latter operate at full power. Therefore, we assume that the risk sharing is full.

Suppose agents share a fraction α of their income. In other words, all agents with an income of 1 pays α into a common pot, which is then equally distributed across agents. In any period, the payoff from conforming to this arrangement is $U(\alpha) = \text{p}u((1-\alpha) + \text{p}\alpha) + (1-\text{p})u(\text{p}\alpha)$. Therefore the payoff from conforming is given by $U(\alpha)/(1-\delta)$. Deviation has an immediate gain only if the agent has an income of 1, and in future the agent loses the social insurance. Therefore the deviation payoff is $U^D = u(1\,;\rho) + \delta/(1-\delta)(\text{p}u(1\,;\rho) + (1-\text{p})u(0\,;\rho))$.

Note that deviation payoff does not depend on α, while the payoff from conforming given by $U(\alpha)$ is maximized when risk sharing is full, i.e. $\alpha = 1$. Clearly, full risk sharing maximizes the net benefit of social risk sharing. In what follows, we assume that the underlying social risk sharing game is characterized by full risk sharing. This maximizes the benefit of cooperating, which therefore maximizes the power of social penalties.

Since risk sharing is full, by conforming, any agent receives a constant payoff p per period. Therefore the payoff from conforming is given by $u(\text{p}\,;\rho)/(1-\delta)$. Deviation has an immediate gain only if the agent has an income of 1, and in future the agent loses the social insurance. Therefore the deviation payoff is $u(1\,;\rho) + \delta/(1-\delta)(\text{p}u(1\,;\rho) + (1-\text{p})u(0\,;\rho))$. The following result shows that there is a unique cutoff type that is indifferent between deviating and conforming. Further all types below this cutoff prefer to deviate while all types above conform. Clearly, this cutoff is decreasing in δ.

Proposition 1. *There is a unique type $\rho_{\min} > 0$ such that the subset $[\rho_{\min}, \overline{\rho}]$ of types cooperate and share risk. Types below $\rho_{\min}$ are excluded from the risk sharing scheme. The type $\rho_{\min}$ is decreasing in δ.*

Therefore any loan program based on social sanctions can cover at most types $[\rho_{\min}, \overline{\rho}]$. However, as we show next, coverage might be considerably lower.

4 Loan Program Under Social Sanctions

A loan program (M, T) is said to be sustainable under social sanctions for any type $\rho \in [0, \overline{\rho}]$ if the threat of exclusion form social risk-sharing is sufficient to deter this type from not making the required repayment at all dates $t \in \{1, \dots, T\}$.

Note that to separate the incentive effect of social sanctions it is assumed that even after a default at $t < T$, the loans are given out as scheduled at future dates up to T. This might seem as studying an entirely artificial case since a default is detected with certainty in our simple model. However, a simple extension suffices to rectify this artificiality. Let us suppose that a project succeeds with probability $q \in (0, 1)$ and that peers know the realized state while an external lender must incur some positive cost $c > 0$ to learn the realized state. A loan under social sanction then uses no monitoring by the lender and relies purely on social enforcement. Thus an agent who defaults strategically at period $t < T$ expects to be excluded from social risk-sharing but expects to receive loans in future as normal. An individual loan, in contrast, verifies the state at every date and penalizes strategic defaulters directly (through denial of future loans and transfers). For small c, this setting does not change any result qualitatively, while a large c would obviously reduce the efficacy of individual loans. (The next version of the paper will incorporate these details in the formal results.)

4.1 Preliminaries

Consider a $T > 1$ period loan program. For any $1 \leqslant t \leqslant T$, the payoff in period t from conforming is given by

$$V_t = \sum_{k=0}^{T-t} \delta^k u(\mathrm{p} + MR\,; \rho) + \frac{\delta^{T-t+1}}{1-\delta} u(\mathrm{p}\,; \rho)$$

Note that a type deviating from loan payment in any period $t \leqslant T$ also deviates from loan payment in every subsequent period until T. To see this, note that in any subsequent period, the type is not included in social cooperation - and therefore there is noting to be gained from conforming, but the extra payoff from deviation is lost. Specifically, after deviation at $t > 0$, in any subsequent period the payoff from conforming is either $u(1 + MR\,; \rho)$ (with probability p) or $u(MR\,; \rho)$ (with probability $(1 - \mathrm{p})$). The payoff from deviating in any such period, on the other hand, is $u(1 + M + MR\,; \rho)$ in the first case and $u(M + MR\,; \rho)$ in the second case. Clearly, deviating dominates conforming.

Using this, the payoff from deviating in period t is given by:

$$V_t^D = u(1+M+MR\,;\rho) + \sum_{k=1}^{T-t} \delta^k \Big(\mathrm{p}u(1+M+MR\,;\rho) + (1-\mathrm{p})u(M+MR\,;\rho) \Big) + \frac{\delta^{T-t+1}}{1-\delta} \Big(\mathrm{p}u(1\,;\rho) + (1-\mathrm{p})u(0\,;\rho) \Big)$$

Let

$$\mathbb{G}(X\,;\rho) \equiv u(1+X+XR\,;\rho) - u(\mathrm{p}+XR\,;\rho) \tag{4.1}$$

$$\mathbb{L}(X\,;\rho) \equiv u(\mathrm{p}+XR\,;\rho) - \Big(\mathrm{p}u(1+X+XR\,;\rho) + (1-\mathrm{p})u(X+XR\,;\rho) \Big) \tag{4.2}$$

If an agent of type ρ deviates in any period $t \leqslant T$, the immediate gain is $\mathbb{G}(M\,;\rho)$. In every subsequent period up to period T, the agent incurs a loss of $\mathbb{L}(M\,;\rho)$. After period T, the loss in each period is $u(\mathrm{p}\,;\rho) - (\mathrm{p}u(1\,;\rho) + (1-\mathrm{p})u(0\,;\rho))$, which is simply $\mathbb{L}(0\,;\rho)$. Thus the total loss from deviation in any period $t \leqslant T$ is

$$\begin{aligned} \mathbb{TL}_t(M\,;\rho) &= \sum_{k=1}^{T-t} \delta^k \mathbb{L}(M\,;\rho) + \frac{\delta^{T-t+1}}{1-\delta}\mathbb{L}(0\,;\rho) \\ &= \frac{\delta}{1-\delta}\Big[(1-\delta^{T-t})\mathbb{L}(M\,;\rho) + \delta^{T-t}\mathbb{L}(0\,;\rho)\Big] \end{aligned} \tag{4.3}$$

Note that $\mathbb{TL}_t(M\,;\rho) - \mathbb{G}(M\,;\rho) = V_t - V_t^D$. Let $\widehat{\rho}_t$ be the type that is indifferent between conforming and deviating. It is the type for which gain and loss are exactly equal. Thus $\widehat{\rho}_t$ is given by the solution to $V_t = V_t^D$, which is the same as $\mathbb{G}(M\,;\rho) = \mathbb{TL}_t(M\,;\rho)$. Using equations (4.1) and (4.2), this can be written as follows:

$\widehat{\rho}_t$ is given by the solution to

$$\frac{\delta}{1-\delta}\left[(1-\delta^{T-t})\frac{\mathbb{L}(M\,;\rho)}{\mathbb{G}(M\,;\rho)} + \delta^{T-t}\frac{\mathbb{L}(0\,;\rho)}{\mathbb{G}(M\,;\rho)}\right] = 1 \tag{4.4}$$

We now establish that this cutoff type exists and is unique. First, we need the following result:

Lemma 1. $\frac{\mathbb{L}(0\,;\rho)}{\mathbb{G}(M\,;\rho)}$ *is increasing in ρ.*

The following result now shows that there exists a unique cutoff $\widehat{\rho}_t$ which is indifferent between deviating and conforming in any period t. Only types above this threshold conform, and types below are excluded from the loan program.

Lemma 2. *For any $t \in \{0, \ldots, T\}$, and for any $M, R \geqslant 0$ there is a unique solution $\widehat{\rho}_t$ to equation* (4.4)*. Further, $V_t(T\,;\rho) \gtreqless V_t^D(T\,;\rho)$ according as $\rho \gtreqless \widehat{\rho}_t$.*

Let $\rho^*(T)$ denote the repayment threshold under a T period loan program. This is given by

$$\rho^*(T) \equiv \max\{\widehat{\rho}_1, \ldots, \widehat{\rho}_T\}$$

This cutoff separates those types who deviate in some period $t \leqslant T$, and therefore cannot be included in the loan program, and and the types who conform always who are covered by the loan.

The result below now shows that the relevant threshold is the initial one. In other words, to determine the types to be excluded from the loan programme, we only need to consider types who would deviate in the very first period.

Proposition 2. ***(Participation Threshold)*** *For a loan program lasting T periods, the incentive to deviate is strongest in the initial period of the loan and therefore the participation threshold $\rho^*(T)$ is given by*

$$\rho^*(T) = \widehat{\rho}_1 \tag{4.5}$$

where $\widehat{\rho}_1$ is the solution to equation (4.4) *for any $t = 1$.*

The proof proceeds through the following Lemma which shows that the total loss from deviation is increasing in time. In other words, the loss from deviation is lower for earlier deviations. This then suggests - as explained below - that it is the incentives at date 1 that determines overall incentive compatibility.

Lemma 3. *For any $t \leqslant T$, $\mathbb{TL}_{t-1}(M\,;\rho) < \mathbb{TL}_t(M\,;\rho)$.*

Proof: Using the value of $\mathbb{TL}_t(M\,;\rho)$ from equation (4.3),

$$\begin{aligned}
\mathbb{TL}_t(M\,;\rho) - \mathbb{TL}_{t-1}(M\,;\rho) &= \frac{\delta}{1-\delta}\left((\delta^{T-t+1} - \delta^{T-t})\mathbb{L}(M\,;\rho) + \left(\delta^{T-t} - \delta^{T-t+1}\right)\mathbb{L}(0\,;\rho)\right) \\
&= \delta^{T-t+1}\left(\mathbb{L}(0\,;\rho) - \mathbb{L}(M\,;\rho)\right)
\end{aligned}$$

We know that $\mathbb{L}(0\,;\rho) > 0$. If $\mathbb{L}(M\,;\rho) \leqslant 0$, the right hand side is strictly positive. Next, suppose $\mathbb{L}(M\,;\rho) > 0$. Using the expression for $u(\cdot\,;\rho)$,

$$\mathbb{L}'(M) = -\mathrm{e}^{-(1+M+MR)\rho}\left((1-\mathrm{p})\mathrm{e}^{\rho} + \mathrm{p}\right) - \rho R\mathbb{L}(M\,;\rho)$$

Since $\mathbb{L}(M\,;\rho) > 0$, $\mathbb{L}'(M) < 0$. Therefore $\mathbb{L}(0\,;\rho) - \mathbb{L}(M\,;\rho) > 0$. Therefore, the right hand side is again strictly positive. This completes the proof.$\|$

Let us now complete the proof of the proposition.

Proof of Proposition 2 (completed) Consider the incentive of type $\widehat{\rho}_t$ in period $t-1$. The gain from deviation for this type remains the same as that in period t. But from Lemma 3 the loss at $t-1$ is lower. Therefore it is clear that type $\widehat{\rho}_t$ is among the types that strictly prefer to deviate in period $t-1$. In other words, $\widehat{\rho}_{t-1} > \widehat{\rho}_t$ for any $t \leqslant T$. This proves that $\widehat{\rho}_1 = \max\{\widehat{\rho}_1, \ldots, \widehat{\rho}_T\}$.‖

4.2 Project Size, Loan Duration and Loan Coverage

We now show that under a loan program supported by social sanctions, coverage (i.e. fraction of types covered) decreases in loan duration, and is non-monotonic in loan size. The example below then shows that a project lasting even a few periods causes the coverage to shrink significantly compared to the coverage of the underlying social risk-sharing arrangement.

Proposition 3. (Loan Duration and Coverage) $\rho^*(T+1) > \rho^*(T)$, *which implies that loan coverage decreases in the duration of the loan program.*

The next result shows that the effect of loan size on coverage is non-monotonic.

Proposition 4. (Loan Size and Coverage) *Given any loan program* $(M, R, T) > 0$, *there exists* $\underline{\delta}(T) < 1$ *and* $\underline{M} > 0$ *such that for* $\delta > \underline{\delta}(T)$, *loan coverage decreases in loan size for* $M < \underline{M}$, *and increases in loan size for* $M > \underline{M}$. *Further,* $\underline{\delta}(T)$ *decreases in* T.

5 Individual Loan Program

This section analyzes incentives created by an individual loan program augmented by a compulsory savings program. We show that such a program improves on social sanctions in several ways. Social sanctions obviously cannot cover types not covered by social risk-sharing. Further, as shown in the previous section, the coverage of a loan program supported by social sanctions decreases with l loan duration - implying that for longer duration projects coverage can be significantly reduced.

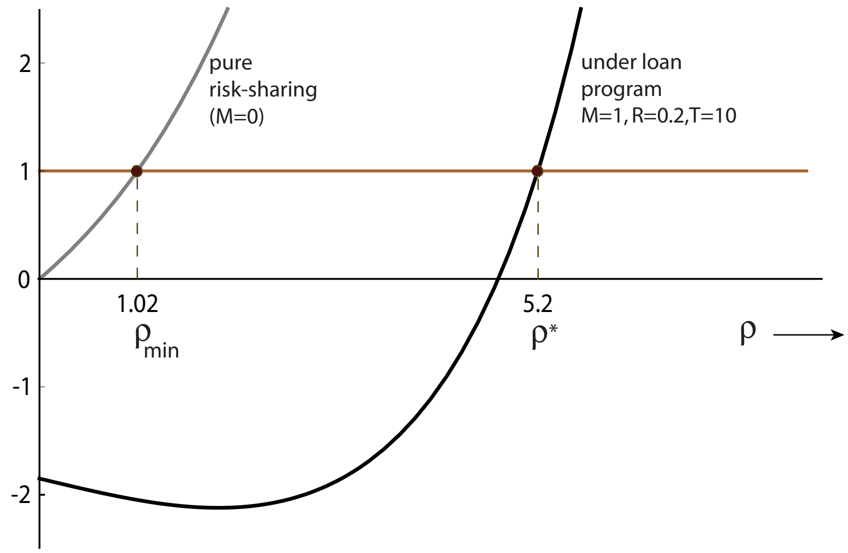

Figure 1: This shows the coverage of social risk-sharing as well as that of a loan program that uses social enforcement. In this case, the loan program specification is $M = 1$, $R = 0.2$, and $T = 10$. Further, p $= 0.5$ and $\delta = 0.75$. Type above the cutoff $\rho_{\min} = 1.02$ participate in social risk-sharing. However, the loan program is incentive compatible only for types above $\rho^* = 5.2$.

An individual loan program can, in contrast, cover *all* types - irrespective of whether they are covered by social risk sharing or not. Further, even when an individual loan program has less than full coverage, we show that coverage *increases* unambiguously with loan size as well as loan duration.

Finally, we show that even though social sanctions are not made use of by an individual loan program, such a program can indirectly benefit from stronger underlying risk-sharing. The idea is that the types who are excluded from social risk-sharing have a weakly greater incentive to deviate from a loan program compared to types who participate in social risk-sharing. We show that an implication of this is that in some cases the minimum discount factor required to induce all types to conform to repayment rises as $\rho_{\min}$ rises. In other words, the minimum discount factor required to cover all types by an individual loan program can fall as the coverage of social risk sharing rises. Thus the presence of social capital extends the scope of an individual loan program even though it plays no direct role in generating repayment incentives.

5.1 The Loan Mechanism

Mimicking the Grameen Bank loans (as noted before, since 2002 the Grameen Bank has abandoned the idea of joint liability and offers purely individual loans), we define an individual loan as a finite sequence of loans with a compulsory savings program.

We show that a finite sequence of loans plus a savings program is budget balanced and can generate full repayment incentives for all types.

As before, loans are made across T periods $t = 0, \dots, T - 1$. A loan given in period t requires repayment in period $t + 1$. This implies that there are T corresponding repayment dates $t = 1, \dots, T$. We assume that the loan program is designed so that the lender makes a zero profit. Therefore for a loan of size M made at date t, the required repayment at $t + 1$ is also M.

Finally, the loan program also includes a compulsory savings clause. This requires a borrowing agent to deposit an amount rM each period, $r \in (0, 1)$, into a savings account held with the lender. The borrower has no withdrawal privileges while the loan program is ongoing. If the borrower does not repay in any period $t \leqslant T$, the lender can seize the amount available in the account. At the end of the loan program, when all repayments are made, the borrower is given full access to the accumulated account.

The purpose of the compulsory savings account is obvious. This serves as collateral. In the absence of some such provision, borrowers would definitely default in the last period. With the savings account clause, they stand to lose the accumulated amount in the savings account if they default. Therefore, incentive to repay can be sustained by accumulating the right amount into this fund. However, the fact that such deposits must be made itself affects repayment incentives in any period before the last period. We show below that the mechanism succeeds in sustaining repayments for all types of agents so long as the discount factor is not too low.

5.2 Repayment Incentives for Types with Social Capital

The repayment condition is as follows. The expected utility from repaying the loan in each period starting from any period $t \leqslant T$ is given by

$$\begin{aligned} W_t &= u(M(R-r)+\mathrm{p}\,;\rho)+\ldots+\delta^{T-t-1}u(M(R-r)+\mathrm{p}\,;\rho) \\ &\quad +\delta^{T-t}u(M(R-r)+\mathrm{p}+S\,;\rho)+\frac{\delta^{T-t+1}}{1-\delta}u(\mathrm{p}\,;\rho) \\ &= \frac{1-\delta^{T-t}}{1-\delta}u(M(R-r)+\mathrm{p}\,;\rho)+\delta^{T-t}u(M(R-r)+\mathrm{p}+S\,;\rho)+\frac{\delta^{T-t+1}}{1-\delta}u(\mathrm{p}\,;\rho) \end{aligned}$$

The expected payoff from defaulting in period $t \leqslant T$ is

$$W_t^D = u(M+MR+\mathrm{p}\,;\rho)+\frac{\delta}{1-\delta}u(\mathrm{p}\,;\rho)$$

Now, for the last period,

$$W_T - W_T^D = u(M(R-r)+\mathrm{p}+S\,;\rho)-u(M+MR+\mathrm{p}\,;\rho)$$

Since $u(\cdot;\cdot)$ is increasing in the first argument, incentive compatibility requires $M(R-r)+\mathrm{p}+S \geqslant M+MR+\mathrm{p}$, implying $S \geqslant M+Mr$. The budget balance condition is

$$S = TMr \tag{5.1}$$

Therefore incentive compatibility requires that $(T-1)Mr \geqslant M$, which says that the the net transfer in the last period must be at least M. Since $\delta < 1$, it is better for the agents to receive income earlier rather than later. Therefore the optimal arrangement is to set r so that $(T-1)Mr = M$, i.e.

$$S^* = M + Mr \tag{5.2}$$

Now, using the value of $u(\cdot;\cdot)$ from equation (2.1), and S^* from above,

$$W_t - W_t^D = \frac{\mathrm{e}^{\,-(\mathrm{p}+M(1+R))\rho}}{\rho}\frac{1-\delta^{T-t}}{1-\delta}\left(1-\delta+\delta\mathrm{e}^{\,M(1+R)\rho}-\mathrm{e}^{\,M(1+r)\rho}\right)$$

Therefore the incentive compatibility condition for any ρ is independent of t and is given by

$$1-\delta+\delta\mathrm{e}^{\,M(1+R)\rho}-\mathrm{e}^{\,M(1+r)\rho} \geqslant 0 \tag{5.3}$$

This proves the following result.

Proposition 5. *Consider a type $\rho \in [\rho_{\min}, \overline{\rho}]$, and an individual loan program (M, R, T). The following condition is necessary and sufficient for the loan program to be incentive compatible for type ρ: condition* (5.3) *holds for some $r \geqslant \dfrac{1}{T-1}$.*

Note that the incentive compatibility constraint (5.3) implies a maximum value of r for any given values of other parameters. So long as this maximum value exceeds $1/(T-1)$, we can find r such that the conditions stated in the result above are satisfied. Therefore we can write the incentive compatibility condition more compactly as the following:

Corollary 1. *A project (M, R, T) can be supported by an incentive compatible individual lending program for a type $\rho \geqslant \rho_{\min}$ if and only if $\overline{r}(\rho) > \dfrac{1}{T-1}$, where*

$$\overline{r}(\rho) = \frac{\ln\left[1 - \delta + \delta \mathrm{e}^{\,M(1+R)\rho}\right]}{\rho M} - 1 \tag{5.4}$$

The next result shows that $\overline{r}(\rho)$ is increasing in ρ and M. This then gives us a sufficient condition for all types under social risk-sharing to be covered by an individual loan program.

Lemma 4. *$\overline{r}(\rho)$ is strictly increasing in ρ and M.*

The result above proves that if $\overline{r}(\rho) > 1/(T-1)$ for $\rho = \widehat{\rho}$, it is also met for all $\rho > \widehat{\rho}$. Therefore an individual loan program can cover *all* types $\rho \geqslant \rho_{\min}$ so long as the condition holds for $\rho = \rho_{\min}$. A sufficient condition for this is that the condition holds at $\rho = 0$. Now, $\lim_{\rho \to 0} \overline{r}(\rho) = \delta(1+R) - 1$. This proves the following result.

Proposition 6. *Consider any project (M, R) with $M > 0$ and $R > 0$. The condition $\delta(1+R) - 1 \geqslant \dfrac{1}{T-1}$ is sufficient for this project to be supported for all types $\rho \in [\rho_{\min}, \overline{\rho}]$ by an incentive compatible individual loan program.*

Finally, let us consider the case of $\overline{r}(\rho_{\min}) < 1/(T-1)$. In this case, $\overline{r}(\rho) > 1/(T-1)$ only for types exceeding some cutoff $\widehat{\rho} > \rho_{\min}$, where the cutoff $\widehat{\rho}$ is the solution for ρ to[(2)]

$$\overline{r}(\rho) = \frac{1}{T-1} \tag{5.5}$$

Whenever $\widehat{\rho} < \overline{\rho}$, we can support a loan program for types $(\widehat{\rho}, \overline{\rho}]$.

[(2)] Formally, for any $\delta < 1/(1+R)$, $\exists\, \widehat{\rho} > 0$ such that $\forall \rho > \widehat{\rho}$, $\overline{r}(\rho) > 1/(T-1)$. This is because there is a unique solution $\widehat{\rho}$ to $\overline{r}(\rho) = \frac{1}{T-1}$. Since $\overline{r}(\rho)$ is strictly increasing in ρ (Lemma 4), the result follows.

Proposition 7. *$\widehat{\rho}$ is decreasing in both M and T.*

Proof: We have

$$\frac{\partial \widehat{\rho}}{\partial M} = - \left. \frac{\frac{\partial \overline{r}(\rho,\delta,M)}{\partial M}}{\frac{\partial \overline{r}(\rho,\delta,M)}{\partial \rho}} \right|_{\rho=\widehat{\rho}}$$

From Lemma 4, the derivatives in the numerator and the denominator are both positive, and therefore $\frac{\partial \widehat{\rho}}{\partial M} < 0$. Finally, since $\overline{r}(\rho)$ is increasing in ρ, and right hand side of equation (5.5) is decreasing in T, $\widehat{\rho}$ is also decreasing in T.$\|$

The previous section showed that under social-incentive-based lending, the loan coverage *decreases* with T and is non-monotonic in M. The result above shows that in contrast, under individual lending, a higher project size M *increases* coverage unambiguously, and coverage also *Increases* in the length T of the program.

5.3 Repayment Incentives for Types without Social Capital

Social incentives are of course unavailable for types who do not participate in social risk-sharing. Here we show that individual loans can cover these types as well, but repayment requires satisfying a stricter condition. This proves that social capital is helpful in supporting individual loans.

Proposition 8. *Consider a type $\rho \in (0, \rho_{\min})$, and an individual loan program (M, R, T). The following condition is necessary and sufficient for the loan program to be incentive compatible for type ρ:*

$$\delta \left(\mathrm{e}^{M(1+R)\rho} - 1 \right) (p + (1-p)\mathrm{e}^{\rho}) - \mathrm{e}^{\rho} \left(\mathrm{e}^{M(1+r)\rho} - 1 \right) \geqslant 0 \tag{5.6}$$

for some $r \geqslant \dfrac{1}{T-1}$.

Before proving this result, we prove the following lemma which is useful for the proof as well as for other results.

Lemma 5. *If condition* (5.6) *holds, this implies condition* (5.3) *holds as well, but the reverse is not true. In other words, the incentive compatibility condition for an individual loan program is stricter for types without social capital.*

Proof: Let $\phi(\mathrm{p}) \equiv \delta\big(\mathrm{e}^{M(1+R)\rho}-1\big)\left(\mathrm{p}+(1-\mathrm{p})\mathrm{e}^{\rho}\right)-\mathrm{e}^{\rho}\big(\mathrm{e}^{M(1+r)\rho}-1\big)$. This is simply the expression on the left hand side of condition (5.6). Note that

$$\phi(0)=\mathrm{e}^{\rho}\big(1-\delta+\delta\mathrm{e}^{\,M(1+R)\rho}-\mathrm{e}^{\,M(1+r)\rho}\big)$$

Thus $\phi(0)\geqslant 0$ is the same as condition (5.3). Now, $\phi'(\mathrm{p})=-\delta(\mathrm{e}^{\rho}-1)(\mathrm{e}^{M(1+R)\rho}-1)<0$. Therefore $\phi(\mathrm{p})\geqslant 0$ implies $\phi(0)>0$, but $\phi(0)\geqslant 0$ does not imply $\phi(\mathrm{p})\geqslant 0$. This proves that the incentive compatibility condition for an individual loan program is stricter for types without social capital.$\|$

Let us now outline the proof of Proposition 8. The formal proof is in the appendix. First, note that for an agent who is not socially insured, income is either 0 or 1. It is then easy to see that the deviation gain is highest if the income is 0 - the gain in utility from retaining the entire return of $M(1+R)$ (rather than repaying $M(1+r)$ and retaining $M(R-r)$) is the highest in this case. Incentive compatibility then requires this gain to be lower than the future loss at every $t<T$. As before, the transfers at T ensure that repayment is incentive compatible at T.

Now, if we consider the incentive compatibility condition at $t=T-1$, we get condition (5.6). This establishes the necessity of the condition. This condition would also be sufficient if the loss from deviation is higher for earlier deviations (i.e. for deviations at $t<T-1$). In this case, clearly, if incentive compatibility holds at $T-1$, it also holds for all earlier periods. It turns out that loss from earlier deviations is higher if and only if condition (5.3) holds. But Lemma 5 above shows that condition (5.6) $\Longrightarrow$ condition (5.3). Therefore condition (5.6) is also sufficient for incentive compatibility.

As in the case of types with social capital, the incentive compatibility condition can be written more compactly as follows. A project (M,R,T) can be supported by an incentive compatible individual lending program for a type $\rho\in(0,\rho_{\min}]$ if and only if $\widetilde{r}(\rho)\geqslant\dfrac{1}{T-1}$, where

$$\widetilde{r}(\rho)=\frac{1}{M\rho}\ln\Big[(1-\mathrm{p})\delta\mathrm{e}^{\,(1+M(1+R))\rho}+\big(1-(1-\mathrm{p})\delta\big)\mathrm{e}^{\rho}+\mathrm{p}\delta\big(\mathrm{e}^{\,M(1+R)\rho}-1\big)\Big]-\frac{M+1}{M}$$

Note that the constraint obviously becomes more lax as T increases (so that the right hand side decreases). The next Lemma shows that $\widetilde{r}(\rho)$ is increasing in M.

Lemma 6. *$\widetilde{r}(\rho)$ is increasing in M.*

If the inequality $\tilde{r}(\rho) \geqslant \frac{1}{T-1}$ is satisfied for all $\rho \in (0, \rho_{\min}]$, clearly further increase in M or T has no impact. Also, if the inequality is satisfied for no $\rho \in (0, \rho_{\min}]$, an increase in M or T would reduce the extent by which the right hand side exceeds the left hand side,but the constraint still might not hold for any $\rho \in (0, \rho_{\min}]$. However, suppose the inequality is satisfied for at least some values of $\rho \in (0, \rho_{\min}]$. As T increases, or M increases, the inequality is now satisfied for further values of $\rho \in (0, \rho_{\min}]$.

5.4 Some Examples

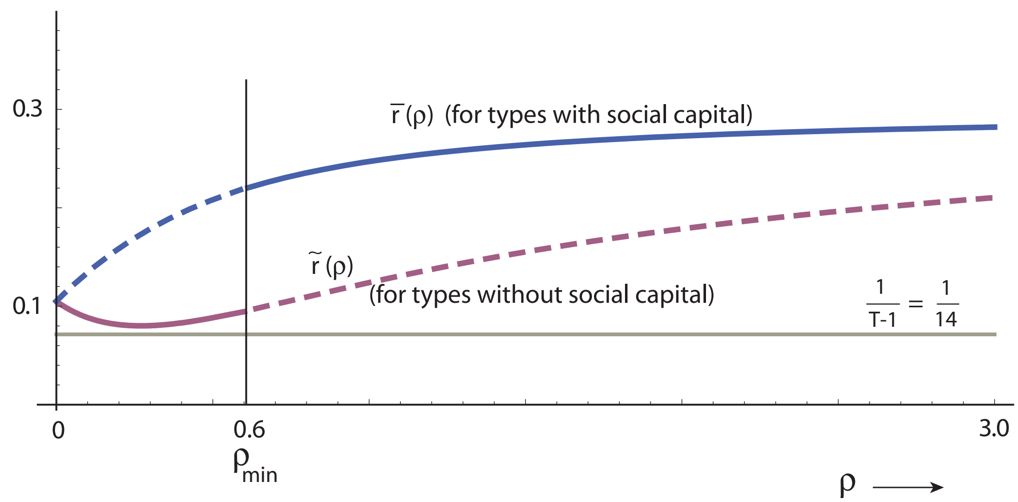

Figure 2: Here the loan program is $M = 3, R = 0.3, T = 15$, and p $= 0.5, \delta = 0.85$. Note that all types - both those with social capital and those without - are covered by the individual loan program.

Now the loan size is reduced to $M = 2$ - with all other parameters the same as above. We see that all types with social capital are still covered, but only types $(0, 0.125]$ of the types out o those that do not have social capital are covered.

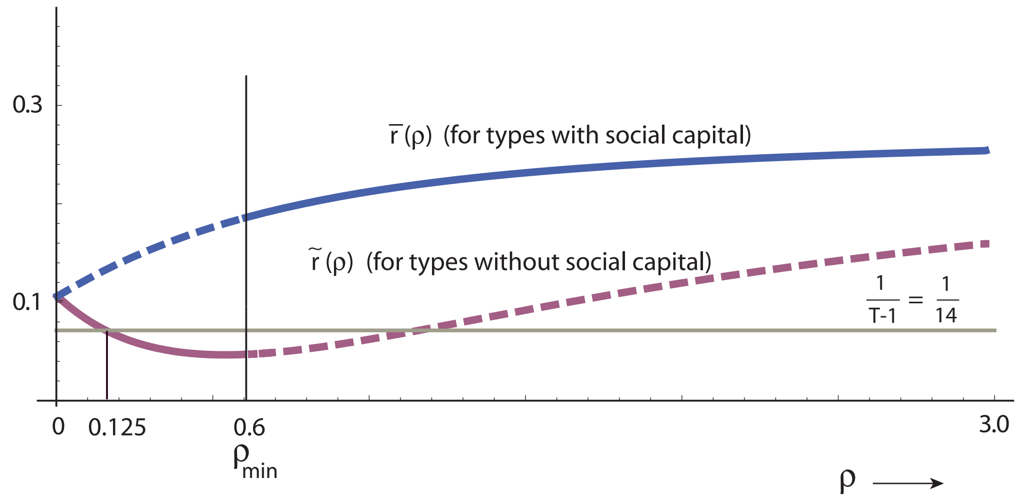

Figure 3: Here the loan program is $M = 2, R = 0.3, T = 15$, and p $= 0.5, \delta = 0.85$. Note that all types with social capital are covered by the individual loan program, and out of the types without social capital (types in the interval $(0, 0.6)$) only types $(0, 0.125)$ are covered.

Appendix: Proofs

A.5 Proof of Proposition 1

The immediate gain from deviation is $G = u(1\,;\rho) - u(\mathrm{p}\,;\rho)$, and starting next period, the loss per period is $L = u(\mathrm{p}\,;\rho) - (\mathrm{p}u(1\,;\rho) + (1-\mathrm{p})u(0\,;\rho))$. Now, ρ_0 is the solution to

$$G = \frac{\delta}{(1-\delta)} L \tag{A.7}$$

Now, $\dfrac{L}{G} = \dfrac{(1-\mathrm{p})e^{(1+\mathrm{p})\rho} + \mathrm{p}e^{\mathrm{p}\rho} - e^{\rho}}{e^{\rho} - e^{\mathrm{p}\rho}}$. Therefore

$$\frac{\partial L/G}{\partial \rho} = \frac{(1-\mathrm{p})e^{(1+\mathrm{p})\rho}((1-\mathrm{p}) + \mathrm{p}e^{\rho} - e^{\mathrm{p}\rho})}{(e^{\rho} - e^{\mathrm{p}\rho})^2} > 0$$

where the last inequality follows from the fact that $(1-\mathrm{p}) + \mathrm{p}e^{\rho} - e^{\mathrm{p}\rho} > 0$, which follows from Jensen's inequality. Finally, L/G goes to 0 as $\rho \to 0$, and increases without bound as ρ increases. Therefore ρ_0 is positive and unique, and $L \gtreqless G$ according as $\rho \gtreqless \rho_0$. This completes the proof.‖

A.6 Proof of Lemma 1

Let $\widehat{\mathbb{G}}(M\,;\rho) \equiv \dfrac{\mathbb{G}(M\,;\rho)}{u(\mathrm{p}\,;\rho)}$ and $\widehat{\mathbb{L}}(0\,;\rho) \equiv \dfrac{\mathbb{L}(0\,;\rho)}{u(\mathrm{p}\,;\rho)}$. Clearly, $\dfrac{\mathbb{L}(0\,;\rho)}{\mathbb{G}(M\,;\rho)} = \dfrac{\widehat{\mathbb{L}}(0\,;\rho)}{\widehat{\mathbb{G}}(M\,;\rho)}$. We now show that $\widehat{\mathbb{G}}(M\,;\rho)$ is strictly decreasing in ρ and $\widehat{\mathbb{L}}(0\,;\rho)$ is strictly increasing in ρ.

Let us first note a case of Jensen's inequality that is used repeatedly later. Let $f(x) = \mathrm{e}^{\rho x}$. Since $f(\cdot)$ is convex in x, Jensen's inequality implies $\mathrm{p}f(1) + (1-\mathrm{p})f(0) > f(\mathrm{p})$. This is stated below.

$$\mathrm{p}\mathrm{e}^{\rho} + (1-\mathrm{p}) - \mathrm{e}^{\mathrm{p}\rho} > 0 \qquad \textbf{(Jensen's Inequality)} \tag{A.8}$$

Next consider the derivatives of $\widehat{\mathbb{L}}(0\,;\rho)$ and $\widehat{\mathbb{G}}(M\,;\rho)$ with respect to ρ.

$$\frac{\partial \widehat{\mathbb{L}}(0\,;\rho)}{\partial \rho} = \left(\frac{\delta}{1-\delta}\right)\left(\frac{\mathrm{p}\mathrm{e}^{(\mathrm{p}-1)\rho}}{(\mathrm{e}^{\mathrm{p}\rho}-1)^2}\right)\left(\mathrm{p}\mathrm{e}^{\rho} + (1-\mathrm{p}) - \mathrm{e}^{\mathrm{p}\rho}\right)$$

The first two terms are obviously strictly positive. The last term is positive from equation (A.8). Therefore $\frac{\partial \widehat{\mathbb{L}}(0\,;\rho)}{\partial \rho} > 0$. Next, we show $\frac{\partial \widehat{\mathbb{G}}(M\,;\rho)}{\partial \rho} < 0$.

Lemma 7. $\frac{\partial \widehat{\mathbb{G}}(M\,;\rho)}{\partial \rho} < 0$ *for any* $M > 0$ *and* $R \geqslant 0$.

Proof:

$$\frac{\partial \widehat{\mathbb{G}}(M\,;\rho)}{\partial \rho} = -\frac{\mathrm{e}^{-(1+M+LR)\rho}}{(\mathrm{e}^{\mathrm{p}\rho}-1)^2} Z(M)$$

where

$$Z(M,\rho) = -MR\mathrm{e}^{(M+1)\rho} - (1+M+LR)\mathrm{e}^{2\mathrm{p}\rho} + (\mathrm{p}+MR)\mathrm{e}^{(1+\mathrm{p}+M)\rho} + ((1-\mathrm{p})+M+LR)\mathrm{e}^{\mathrm{p}\rho}$$

It follows that if we can show $Z(M,\rho) > 0$ for any $M > 0$ and any $\rho > 0$, that would establish $\frac{\partial \mathbb{G}(M\,;\rho)}{\partial \rho} < 0$.

First, fix any $R \geqslant 0$.

Next, note that $Z(0,\rho) = \mathrm{e}^{\mathrm{p}\rho}(-\mathrm{e}^{\mathrm{p}\rho} + \mathrm{p}\mathrm{e}^{\rho} + (1-\mathrm{p})) > 0$, where the inequality follows from equation (A.8).

Next, consider the derivative of Z with respect to M, denoted $Z_M(M,\rho)$.

$$Z_M(M,\rho) = R(\rho M+1)(\mathrm{e}^{\mathrm{p}\rho}-1)\mathrm{e}^{(1+M)\rho} + \mathrm{p}\rho\mathrm{e}^{(1+M+\mathrm{p})\rho} - (R+1)\mathrm{e}^{\mathrm{p}\rho}(\mathrm{e}^{\mathrm{p}\rho}-1)$$

Claim: $Z_M(M,\rho) > 0$ **for any** $M > 0$ **and any** $\rho > 0$**.**

Proof: M enters the first two terms which are both positive and strictly increasing in M. Therefore $Z_{LL}(M,\rho) > 0$. Now, if we can show that Z_M is positive at $M = 0$, the proof would be complete.

$$Z_M(0,\rho) = R(\mathrm{e}^{\mathrm{p}\rho}-1)\mathrm{e}^{\rho} + \mathrm{p}\rho\mathrm{e}^{(1+\mathrm{p})\rho} - (R+1)\mathrm{e}^{\mathrm{p}\rho}(\mathrm{e}^{\mathrm{p}\rho}-1)$$

Consider the value of $Z_M(0,\rho)$ at $\rho = 0$. Clearly, $Z_M(0,0) = 0$. Also, the derivative of $Z_M(0,\rho)$ with respect to ρ is given by

$$Z_{M\rho}(0,\rho) = (\mathrm{e}^{\rho}-\mathrm{e}^{\mathrm{p}\rho})(\mathrm{e}^{\mathrm{p}\rho}-1)\,R + \mathrm{e}^{\mathrm{p}\rho}(\mathrm{p}\rho\mathrm{e}^{\rho}-\mathrm{e}^{\mathrm{p}\rho}+1)$$

The first term is obviously positive. In the second term, the coefficient of $\mathrm{e}^{\mathrm{p}\rho}$ is $Y(\rho) = \mathrm{p}\rho\mathrm{e}^{\rho} - \mathrm{e}^{\mathrm{p}\rho} + 1$. This is 0 at $\rho = 0$, and $Y'(\rho) = \mathrm{p}(\mathrm{e}^{\rho} + \rho\mathrm{e}^{\rho} - \mathrm{e}^{\mathrm{p}\rho}) > 0$ for any $\mathrm{p} \in [0,1]$. Therefore $Y(\rho) > 0$ for $\rho > 0$. It follows that $Z_{M\rho}(0,\rho) > 0$ for $\rho > 0$. Coupled with the fact that $Z_M(0,0) = 0$, this implies that $Z_M(0,\rho) > 0$ for any $\rho > 0$. In turn, this, coupled with the fact that $Z_{LL}(M,\rho) > 0$, implies that $Z_M(M,\rho) > 0$ for any $M > 0$ and any $\rho > 0$. This completes the proof of the claim.

To continue with the proof of the Lemma, we now know that given any $R \geqslant 0$, the function Z is positive at $M = 0$ and strictly increasing in M. It follows that $Z(M,\rho) > 0$ for all strictly positive values of M and ρ. Note also that beyond assuming $R \geqslant 0$, we have not restricted the value of R any further. This completes the proof.$\|$

To continue with the proof of Lemma 1, we have now established that $\widehat{\mathbb{G}}(M\,;\rho)$ is strictly decreasing in ρ and $\widehat{\mathbb{L}}(0\,;\rho)$ is strictly increasing in ρ. Therefore the stated ratio is strictly increasing in ρ.$\|$

A.7 Proof of Lemma 2

Step 1 Using the form of the utility function (from equation (2.1)),

$$\frac{\mathbb{L}(M\,;\rho)}{\mathbb{G}(M\,;\rho)} = (1-\mathrm{p})\frac{(\mathrm{e}^{\rho}-1)}{\mathrm{e}^{(1-\mathrm{p}+M)\rho}-1} - 1 \tag{A.9}$$

ρ appears only in the coefficient of $(1-\mathrm{p})$, which is of the form $\dfrac{\mathrm{e}^{a\rho}-1}{\mathrm{e}^{b\rho}-1}$ where a, b are positive real numbers with $b \gtreqless a$ according as $M \gtreqless \mathrm{p}$. The following Lemma establishes some useful properties of such a ratio.

Lemma 8. *Let $Z_1(x) \equiv \mathrm{e}^{x\rho}-1$. Let a, b be positive real numbers. Then $\dfrac{\partial}{\partial\rho}\left(\dfrac{Z_1(a)}{Z_1(b)}\right) \gtreqless 0$ as $b \gtreqless a$. Further, if $b > a$, $\dfrac{\partial^2}{\partial\rho^2}\left(\dfrac{Z_1(a)}{Z_1(b)}\right) > 0$.*

Proof: Let $Z_2(x) \equiv \dfrac{x\mathrm{e}^{x\rho}}{\mathrm{e}^{x\rho}-1}$ and $Z_3(x) \equiv \dfrac{x^2\mathrm{e}^{x\rho}}{(\mathrm{e}^{x\rho}-1)^2}$. Then

$$\frac{\partial}{\partial\rho}\left(\frac{Z_1(a)}{Z_1(b)}\right) = \frac{Z_1(a)}{Z_1(b)}\Big(Z_2(a)-Z_2(b)\Big) \tag{A.10}$$

$$\frac{\partial^2}{\partial\rho^2}\left(\frac{Z_1(a)}{Z_1(b)}\right) = \frac{Z_1(a)}{Z_1(b)}\Big(Z_2(a)-Z_2(b)\Big)^2 + \frac{Z_1(a)}{Z_1(b)}\Big(Z_3(b)-Z_3(a)\Big) \tag{A.11}$$

Now,

$$\frac{\partial}{\partial x}Z_2(x) = \frac{\mathrm{e}^{x\rho}}{(\mathrm{e}^{x\rho}-1)^2}\Big(\mathrm{e}^{x\rho}-(1+x\rho)\Big) \;>\; 0$$

and

$$\frac{\partial}{\partial x}Z_3(x) = \frac{x\mathrm{e}^{x\rho}}{(\mathrm{e}^{x\rho}-1)^2}\left(x\rho + 2\left(1-\frac{x\rho}{\mathrm{e}^{x\rho}-1}\right)\right) \;>\; 0$$

where both inequalities follow from the fact that $\mathrm{e}^{x\rho} > 1 + x\rho$. Therefore $Z_2(a)-Z_2(b) \lesseqgtr 0$ as $b \gtreqless a$ and for $b > a$, and $Z_3(b)-Z_3(a) > 0$. This completes the proof.$\|$

The result above shows that $\mathbb{L}(M\,;\rho)/\mathbb{G}(M\,;\rho)$ increases in ρ for $M < \mathrm{p}$ and decreases in ρ for $M > \mathrm{p}$, and in the latter case the derivative with respect to ρ is increasing - i.e. the derivative becomes less negative as ρ increases. Further, for $b > a$, $\lim_{\rho\to\infty}\frac{Z_1(a)}{Z_1(b)} = 0$. It follows that $\frac{\partial}{\partial\rho}\left(\frac{Z_1(a)}{Z_1(b)}\right) \to 0$ as ρ increases. Therefore for $M > \mathrm{p}$, the derivative of $\mathbb{L}(M\,;\rho)/\mathbb{G}(M\,;\rho)$ with respect to ρ falls to zero as ρ increases.

Step 2 The left hand side of equation (4.4) is $\frac{\delta}{1-\delta}$ times a convex combination of the two ratios $\mathbb{L}(0\,;\rho)/\mathbb{G}(M\,;\rho)$ and $\mathbb{L}(M\,;\rho)/\mathbb{G}(M\,;\rho)$. From Lemma 1, $\mathbb{L}(0\,;\rho)/\mathbb{G}(M\,;\rho)$ is increasing in ρ. Further, from step 1, for $M < \mathrm{p}$, $\mathbb{L}(M\,;\rho)/\mathbb{G}(M\,;\rho)$ is increasing in ρ and for $M = \mathrm{p}$, $\mathbb{L}(M\,;\rho)/\mathbb{G}(M\,;\rho)$ is a constant function of ρ. Therefore for $M \leqslant \mathrm{p}$, the left hand side increases in ρ.

Now, as $\rho \to 0$, $\frac{\mathbb{L}(M\,;\rho)}{\mathbb{G}(M\,;\rho)} \to \frac{-M}{1-\mathrm{p}+M} < 0$, and $\frac{\mathbb{L}(0\,;\rho)}{\mathbb{G}(M\,;\rho)} \to 0$. Therefore, for ρ close to 0, the left hand side of equation (4.4) is below 1. Further, the left hand side is continuous in ρ and, as shown above, rises without bound as ρ increases. Therefore there is a unique $\widehat{\rho}_t$ that solves equation (4.4).

Step 3 Next, consider $M > \mathrm{p}$. As ρ increases, $\mathbb{L}(0\,;\rho)/\mathbb{G}(M\,;\rho)$ increases without bound as before, and $\mathbb{L}(M\,;\rho)/\mathbb{G}(M\,;\rho)$ is negative, decreases at a decreasing rate, and is bounded below by -1. Further, the derivative of this ratio is the most negative at $\rho = 0$ at which point is equals $-\frac{1}{2}\frac{(M-\mathrm{p})(1-\mathrm{p})}{M-\mathrm{p}+1} > -\frac{1}{2}$. Therefore the left hand side of equation (4.4), if it decreases at all, is decreasing for values of ρ close to zero, but there exists $\rho_* \geqslant 0$ such that the left hand side of equation (4.4) is increasing in ρ for $\rho > \rho_*$. Since the left hand side is negative at $\rho = 0$, it is also negative at $\rho = \rho_*$. By the same argument as in step 2, it follows that there is a unique $\widehat{\rho}_t > \rho_*$ that solves equation (4.4).$\|$

A.8 Proof of Proposition 4

1. As shown in the proof of Lemma 2, the left hand side of equation (4.4) is (eventually) increasing in ρ, and it is increasing at the solution $\widehat{\rho}_t$. If we can show that the left hand side of equation (4.4) is decreasing in M, this will establish that as M rises, the solution $\widehat{\rho}_t$ rises.

2. The left hand side of equation (4.4) is a convex combination of the two ratios $\frac{\mathbb{L}(M\,;\rho)}{\mathbb{G}(M\,;\rho)}$ and $\frac{\mathbb{L}(0\,;\rho)}{\mathbb{G}(M\,;\rho)}$. As T rises, the weight attached to the first ratio increases.

3. $\frac{\mathbb{L}(M\,;\rho)}{\mathbb{G}(M\,;\rho)}$ is given by equation (A.9), from which it is clear that the ratio is decreasing in M.

4. Now consider the ratio $\frac{\mathbb{L}(0;\rho)}{\mathbb{G}(M;\rho)}$.

$$\frac{\partial}{\partial M}G(M;\rho) = \mathrm{e}^{-MR\rho}\big((1+R)\mathrm{e}^{-(1+M)\rho} - R\mathrm{e}^{-\mathrm{p}\rho}\big)$$

The above derivative is positive for $M \leqslant \underline{M}$ where

$$\underline{M} \equiv \frac{1}{\rho}\ln\left(\frac{1+R}{R}\right) - (1-\mathrm{p})$$

$\underline{M} > 0$ if $\rho < \dfrac{1}{1-\mathrm{p}}\ln\left(\dfrac{1+R}{R}\right)$.

This proves that for $\rho < \dfrac{1}{1-\mathrm{p}}\ln\left(\dfrac{1+R}{R}\right)$ there exists $\underline{M} > 0$ such that $\frac{\mathbb{L}(0;\rho)}{\mathbb{G}(M;\rho)}$ is decreasing in M for $M < \underline{M}$.

5. From steps 1-4 above, we can conclude that there exists $\lambda(T)$ which is positive and increasing in T such that for $\rho < \dfrac{1}{1-\mathrm{p}}\ln\left(\dfrac{1+R}{R}\right) + \lambda(T)$, there exists $\underline{M} > 0$ such that he left hand side of equation (4.4) is decreasing in M for $M < \underline{M}$ and increasing thereafter.

6. Finally, as δ increases to 1, $\rho_{\min}$ decreases to 0. Thus there exists $\underline{\delta}(T)$ such that for $\delta > \underline{\delta}(T)$, $\rho_{\min} < \frac{1}{1-\mathrm{p}}\ln\left(\frac{1+R}{R}\right) + \lambda(T)$. It then follows that coverage shrinks initially as M rises. It is also clear that since $\lambda(T)$ rises in T, $\underline{\delta}(T)$ decreases in T.

A.9 Proof of Proposition 3

For ease of exposition, let us change the notation slightly and write the total program duration as an explicit argument of total loss - i.e. the total loss starting period t associated with a loan program of duration T is now written as $\mathbb{TL}_t(T, M;\rho)$. Further, let $\widehat{\rho}_t(T)$ denote the solution to equation 4.4 given duration T.

From Lemma 3, we know that given any program of duration T, and any $t \leqslant T$,

$$\mathbb{TL}_t(T, M;\rho) > \mathbb{TL}_{t-1}(T, M;\rho)$$

Further, it is clear that $\mathbb{TL}_t(T, M;\rho) = \mathbb{TL}_{t+1}(T+1, M;\rho)$. Therefore

$$\mathbb{TL}_1(T, M;\rho) = \mathbb{TL}_2(T+1, M;\rho) > \mathbb{TL}_1(T+1, M;\rho)$$

From Lemma 2, $\mathbb{TL}_1(T, M;\rho) = \mathbb{G}(M)$ at $\rho = \widehat{\rho}_1(T)$. It follows from above that at $\rho = \widehat{\rho}_1(T)$, $\mathbb{TL}_1(T+1, M;\rho) < \mathbb{G}(M)$. Now, the proof of Lemma 2 shows that for any given T, $\mathbb{TL}_1(T, M; \rho) \lesseqqgtr \mathbb{G}(M)$ as $\rho \lesseqqgtr \widehat{\rho}_1(T)$. It follows that $\widehat{\rho}_1(T+1) > \widehat{\rho}_1(T)$. Finally, using Proposition 2, this implies that $\rho^*(T+1) > \rho^*(T)$.‖

A.10 Proof of Lemma 4

Let $M(1+R)\rho \equiv A$. Then $\bar{r}(\rho)$ can be written as $\phi(A)$, where

$$\phi(A) = (1+R)\frac{\ln[1-\delta+\delta \mathrm{e}^{A}]}{A} - 1$$

Note that

$$\mathrm{sign}\frac{\partial \phi(A)}{\partial A} = \mathrm{sign}\frac{\partial \bar{r}(\rho)}{\partial \rho} \tag{A.12}$$

Now, $\frac{\partial \phi(A)}{\partial A} = \frac{1+R}{A^2 B} H(A)$ where $H(A) \equiv A(B-(1-\delta)) - B\ln[B]$ and $B \equiv 1-\delta+\delta \mathrm{e}^{A}$. Thus $\mathrm{sign}\frac{\partial \phi(A)}{\partial A} = \mathrm{sign}\frac{\partial H(A)}{\partial A}$. At $A=0$, we have $B=1$. Therefore $H(0)=0$. Next, $\frac{\partial H(A)}{\partial A} = B-(1-\delta)+(A-\ln[B]-1)\frac{dB}{dA}$. Using $\frac{dB}{dA} = \delta \mathrm{e}^{A}$ and simplifying,

$$\frac{\partial H(A)}{\partial A} = \delta \mathrm{e}^{A}(A - \ln[B]) > 0$$

Since $H(0)=0$ and $\frac{\partial H(A)}{\partial A} > 0$, it follows that $H(A) > 0$ for $A > 0$. Thus $\frac{\partial \phi(A)}{\partial A} > 0$ for $A > 0$. But $A > 0$ for $\rho > 0$. Using equation (A.12), this implies that $\frac{\partial \bar{r}(\rho)}{\partial \rho} > 0$ for $\rho > 0$.‖

A.11 Proof of Lemma 9

Proof that types with social insurance do not deviate from social insurance under an incentive compatible individual loan program.

Lemma 9. *An incentive compatible individual loan program does not alter social cooperation.*

Define the following functions:

$$\begin{aligned}
\widehat{\mathbb{G}}(X;\rho) &\equiv u(1+X;\rho) - u(\mathrm{p}+X;\rho) \\
\widehat{\mathbb{L}}(X;\rho) &\equiv u(\mathrm{p}+X;\rho) - \Big(\mathrm{p}u(1+X;\rho) + (1-\mathrm{p})u(X;\rho)\Big)
\end{aligned}$$

Suppose a type participating in social insurance who receives and repays an individual loan contemplates deviation from social insurance in any period $t \leqslant T$. The immediate gain is $\widehat{\mathbb{G}}(M(R-r)\,;\rho)$ and the total future loss is given by

$$\widehat{\mathbb{TL}}_t(M\,;\rho) = \sum_{k=1}^{T-t-1} \delta^k \widehat{\mathbb{L}}(M(R-r)\,;\rho) + \delta^{T-t}\widehat{\mathbb{L}}(M(1+R)\,;\rho) + \frac{\delta^{T-t+1}}{1-\delta}\widehat{\mathbb{L}}(0\,;\rho)$$

Now, types $\rho \geqslant \rho_0$ participate in social insurance where ρ_0 is given by equation (A.7). Using current notation, this can be rewritten as

$$\widehat{\mathbb{G}}(0\,;\rho_0) = \frac{\delta}{1-\delta}\widehat{\mathbb{L}}(0\,;\rho_0) \tag{A.13}$$

If we can show that $\widehat{\mathbb{G}}(M(R-r)\,;\rho_0) \leqslant \widehat{\mathbb{TL}}_t(M\,;\rho_0)$, that would imply that none of the types who participate in social insurance have an incentive to deviate from social insurance when the loan is introduced. Now,

$$\begin{aligned}\frac{\widehat{\mathbb{TL}}_t(M\,;\rho_0)}{\widehat{\mathbb{G}}(M(R-r)\,;\rho_0)} = \delta\left(\frac{1-\delta^{T-t-1}}{1-\delta}\right)\frac{\widehat{\mathbb{L}}(M(R-r)\,;\rho_0)}{\widehat{\mathbb{G}}(M(R-r)\,;\rho_0)} \\ + \delta^{T-t}\frac{\widehat{\mathbb{L}}(M(1+R)\,;\rho_0)}{\widehat{\mathbb{G}}(M(R-r)\,;\rho_0)} + \frac{\delta^{T-t+1}}{1-\delta}\frac{\widehat{\mathbb{L}}(0\,;\rho_0)}{\widehat{\mathbb{G}}(M(R-r)\,;\rho_0)}\end{aligned}$$

Using the form of the utility function from equation (2.1), it can be easily verified that the following equalities hold:

$$\begin{aligned}\frac{\widehat{\mathbb{L}}(M(R-r)\,;\rho)}{\widehat{\mathbb{G}}(M(R-r)\,;\rho)} &= \frac{\widehat{\mathbb{L}}(0\,;\rho)}{\widehat{\mathbb{G}}(0\,;\rho)} \\ \frac{\widehat{\mathbb{L}}(M(1+R)\,;\rho)}{\widehat{\mathbb{G}}(M(R-r)\,;\rho)} &= \mathrm{e}^{\,-M(1+r)\rho}\frac{\widehat{\mathbb{L}}(0\,;\rho)}{\widehat{\mathbb{G}}(0\,;\rho)} \\ \frac{\widehat{\mathbb{L}}(0\,;\rho)}{\widehat{\mathbb{G}}(M(R-r)\,;\rho)} &= \mathrm{e}^{\,M(R-r)\rho}\frac{\widehat{\mathbb{L}}(0\,;\rho)}{\widehat{\mathbb{G}}(0\,;\rho)}\end{aligned}$$

Using these, and using equation (A.13),

$$\begin{aligned}\frac{\widehat{\mathbb{TL}}_t(M\,;\rho_0)}{\widehat{\mathbb{G}}(M(R-r)\,;\rho_0)} &= \frac{1-\delta}{\delta}\left(\delta\left(\frac{1-\delta^{T-t-1}}{1-\delta}\right) + \delta^{T-t}\mathrm{e}^{\,-M(1+r)\rho} + \frac{\delta^{T-t+1}}{1-\delta}\mathrm{e}^{\,M(R-r)\rho}\right) \\ &= 1 + \delta^{T-t-1}\mathrm{e}^{\,-M(1+r)\rho}\left(1-\delta+\delta\mathrm{e}^{\,M(1+R)\rho} - \mathrm{e}^{\,M(1+r)\rho}\right)\end{aligned}$$

Therefore $\dfrac{\widehat{\mathbb{TL}}_t(M\,;\rho_0)}{\widehat{\mathbb{G}}(M(R-r)\,;\rho_0)} \geqslant 1$ if and only if $1-\delta+\delta\mathrm{e}^{\,M(1+R)\rho} - \mathrm{e}^{\,M(1+r)\rho} \geqslant 0$, which is the same condition as (5.3), which implies the rest. This completes the proof.‖

A.12 Proof of Proposition 8

Step 1: Loss and gain from deviation In each period an agent is supposed to repay the loan amount M and make a deposit of rM. Therefore the income gain from deviating in any period $t \leqslant T$ is $M(1+r)$. The gain in utility is therefore $\Delta u \equiv u(I + M(1+r)) - u(I)$ where $I = L(R-r)$ in the low income state and $I = 1 + L(R-r)$ in the high income state. Since $u(\cdot)$ is concave, Δu decreases in I: the same income gain leads to a higher utility gain in the low income state. It follows that the deviation gain is highest in the low income state. If the incentive to repay holds in this state, it also holds in the other (high income state).

Therefore the immediate gain from deviation we must consider is

$$\widetilde{\mathbb{G}} = u(L(1+R)\,;\rho) - u(L(R-r)\,;\rho)$$

Next, consider the loss from such deviation in future periods. If the deviation takes place in period $t < T$, then the agent does not get any further loans or lump sum payoffs. Therefore, at each of the next $T-t-1$ periods (i.e. for periods $t+1$ until period $T-1$) he loses $M(R-r)$ which is the amount he would have earned by conforming. Finally, the loss in period T is the final period payoff from conforming, given by $M(1+R)$.

Let

$$\widetilde{\mathbb{L}}(X) \equiv \mathrm{p}(u(1+X\,;\rho) - u(1\,;\rho)) + (1-\mathrm{p})(u(X\,;\rho) - u(0\,;\rho))$$

As noted above, after deviation at t, the loss lasts for $T-t$ periods and the total loss is given by

$$\widetilde{\mathbb{TL}}_t = \delta\frac{1-\delta^{T-t-1}}{1-\delta}\widetilde{\mathbb{L}}(M(R-r)) + \delta^{T-t}\widetilde{\mathbb{L}}(M(1+R))$$

As before, no deviation in the last period requires $(T-1)Mr \geqslant M$, implying that for any given r, we must have $r \geqslant 1/(T-1)$.

Step 2: Necessity Consider the incentive to deviate at period $T-1$ when there is only 1 period left. Clearly, a necessary condition for a loan to be incentive compatible is that the agent does not deviate at $T-1$. This condition is:

$$\widetilde{\mathbb{TL}}_{T-1} \geqslant \widetilde{\mathbb{G}}$$

Using the form of the utility function, this is exactly the condition (5.6). This proves the necessity of condition (5.6).

Step 3: Sufficiency Next, we prove sufficiency.

$$\widetilde{\mathbb{TL}}_{t-1}-\widetilde{\mathbb{TL}}_{t}=\frac{\delta^{T-t}\,\mathrm{e}^{-(1+M+MR)\rho}\Big((1-\mathrm{p})\mathrm{e}^{\rho}+\mathrm{p}\Big)}{\rho}\Big(1-\delta+\delta\mathrm{e}^{\,M(1+R)\rho}-\mathrm{e}^{\,M(1+r)\rho}\Big)$$

Note that the above is positive if the term in the parenthesis at the end is positive. But this is the same as condition (5.3). From lemma 5, this condition holds automatically when condition (5.6) holds. Therefore if condition (5.6) holds, the total loss is increasing for earlier deviations. Therefore if incentive compatibility holds for deviation at $T-1$, it also holds for all earlier deviations. This proves that condition (5.6) is sufficient for the individual loan program to be incentive compatible. This completes the proof.‖

A.13 Proof of Lemma 6

Let $\psi(M)\equiv(1-\mathrm{p})\delta\mathrm{e}^{\,(1+M(1+R))\rho}+\big(1-(1-\mathrm{p})\delta\big)\mathrm{e}^{\rho}+\mathrm{p}\delta\big(\mathrm{e}^{\,M(1+R)\rho}-1\big)$. Note that

$$\psi'(M)=\delta\,\rho\,(1+R)\,\mathrm{e}^{\,M(1+R)\rho}(\mathrm{p}+(1-\mathrm{p})\mathrm{e}^{\rho})>0\quad\text{and}\quad\psi''(M)=\rho\,(1+R)\,\psi'(M)$$

Now, $\widetilde{r}(\rho)=\dfrac{\ln[\psi(M)]}{M\,\rho}-\dfrac{M+1}{M}$. Therefore $\dfrac{\partial\widetilde{r}(\rho)}{\partial M}=\frac{1}{M^2}\xi(M)$ where

$$\xi(M)=1+\frac{M}{\rho}\frac{\psi'(M)}{\psi(M)}-\frac{\ln[\psi(M)]}{\rho}$$

Clearly, $\operatorname{sign}\dfrac{\partial\widetilde{r}(\rho)}{\partial M}=\operatorname{sign}\xi(M)$.

Now, it can be checked easily that $\xi(0)=0$. Further,

$$\begin{aligned}\xi'(M)&=\frac{M}{\rho\,\psi(M)^2}\Big(\psi''(M)\psi(M)-(\psi'(M))^2\Big)\\&=\frac{M\,\psi'(M)}{\rho\,\psi(M)^2}\Big(\rho\,(1+R)\psi(M)-\psi'(M)\Big)\\&=\frac{M\,\psi'(M)}{\rho\,\psi(M)^2}\rho\,(1+R)\Big((1-\delta)\mathrm{e}^{\rho}+\mathrm{p}\,\delta(\mathrm{e}^{\rho}-1)\Big)\;>\;0\end{aligned}$$

Since $\xi(0)=0$ and $\xi'(\cdot)>0$, it follows that $\xi(M)>0$ for any $M>0$. Therefore $\dfrac{\partial\widetilde{r}(\rho)}{\partial M}>0$.‖

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