# Electrification and the Household

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Countries and regions with low electrification rates have high fertility rates and low female employment rates. Can electrification improve these outcomes? This paper develops a cooperative Nash bargaining model to explain how changes in home production technology might impact fertility and women's time use when children represent savings for old age. Data from Guatemala is employed to test the model. As predicted, electrification is shown to have important effects on women's fertility, employment probabilities, and time spent cooking and earning money.

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## 1 Introduction

The potential for electrification to modernise societies has been recognised at least since the Russian Revolution. Speaking to the newly-formed Russian Soviet on November 8th 1917, Lenin stated "Communism is Soviet power plus the electrification of the entire country".[1]

Countries and regions with low electrification rates tend to have high fertility rates, low wages, low educational attainment, low female labour force participation and low household savings levels. According to the World Bank (2007), more than one billion people gained access to electricity between

[1]Lenin's address continued ".... We must show the peasants that the organisation of industry on the basis of modern, advanced technology, on electrification which will provide a link between town and country, will put an end to the division between town and country, will make it possible to raise the level of culture in the countryside and to overcome, even in the most remote corners of land, backwardness, ignorance, poverty, disease, and barbarism."

1975 and 2000. Still, at least 1.6 billion people, mainly in rural areas, do not have access to electricity. Figure 1 shows that, across countries, higher household electrification rates are strongly associated with lower fertility. The correlation coefficient relating country-level household electrification rates and the total fertility rate in the developing countries of the Demographic and Health Surveys (2007) is -.80.[2] Unsurprisingly, the poorest countries also have the lowest electification rates.[3] The correlation coefficient between year-specific GDP per capita at PPP, from the Penn World Tables (Version 6.2, Heston, Summers, and Aten (2006)), and household electrification rates is 0.67 amongst the developing countries of the DHS.

Should improvements in the efficiency of home production increase or decrease fertility and female labour force participation? The answer may depend on several factors, such as whether or not children represent a form of savings for old age, and the price of the new technology. As noted by Pollack and Wachter (1975), the shadow price of home-produced goods depends on the technology by which they are made. Because of this, household electrification might be expected to change savings and investment decisions, and the relative value of time spent working outside of the household. In very poor countries, where home production is inefficient and where children represent old age security, we might expect very different effects of new home production technology than in developed countries. Aside from direct effects on time use and home production decisions, the introduction of electricity to a community may provide a stimulus to local economies. This would further change the value of market production relative to that of home production and further impact fertility and time use decisions.

Existing research suggests that improvements in home production technology should have large impacts on fertility and the working lives of women. Many studies for the US have related increases in female labour force participation to technological improvements, and particularly to technologies that relaxed the time constraints from childbearing.[4] Greenwood, Seshadri, and Yorukoglu (2005) show that the advent of labour-saving household appliances in the US is a major explanation for rising female labour force participation in the 1900s. Greenwood, Seshadri, and Vandenbrouke (2005) present an overlapping generations model which explains how the post-war US Baby Boom could have been caused by the proliferation of household appliances. The explanation is that technology lowered the opportunity cost of having children. Cornwell and Robinson (1988) employ US Census

[2]Table 6 of Data Appendix A lists the most recent household electrification rates amongst countries included in the DHS surveys.

[3]The countries of the Former Soviet Union, particularly those in Central Asia, have almost universal electrification but relatively low GDP. This is clearly related to the Soviet drive for electrification and the subsequent less-than-succesful industrialisation of these lands.

[4]See, for example Angrist and Evans (1998), Goldin and Katz (2002), and Bailey (2006)).

and Agricultural Census data from 1930, 1940, and 1950 and find a large impact of electrification on modes of home production and socioeconomic outcomes in 473 counties. Dinkleman (2008) examines the initial effects of a massive rural electrification program in KwaZulu-Natal, South Africa, on household fuel use and female employment, and finds substantial impacts on both.

A large literature examines the determinants of household electricity demand in developed countries (see for example Dubin and McFadden (1984)) but relatively few studies relate electrification to fertility decisions. However, existing studies from low-income countries generally concur in finding a strong association between fertility and electricity provision.[5] Cornwell and Robinson (1988) find that electrification reduced fertility in the poorer Southern US states but increased it in the richer Northern ones. Greenwood, Seshadri, and Vandenbrouke (2005) show that post-World War II improvements in the efficiency of home production can partially account for the US Baby Boom, and later Baby Bust. To date and to our knowledge, however, no studies have attempted to model the effects of improvements in home production technology on fertility or female labour supply for the case in which children represent a form of saving for old age.[6]

This paper presents a cooperative model of the household in which a change in the efficiency of home production could explain both why fertility and female employment rose with the advent of labour-saving devices in the post-War US, and why fertility might fall but female employment rise with electrification if children are primarily investment goods, as in poor countries. Data from Guatemala provides empirical support for the model. The 2000 Guatemala Living Standards Monitoring Survey (LSMS), a 7276 household sample, contains a community-level survey indicating both the existence and vintage of all community infrastructural improvements. Community-level and individual information from this survey can be combined to examine the effects of electrification on fertility. The detailed time use data in the survey can be used to identify the effects of household electrification on the home production activities in which women specialise.

The paper proceeds as follows. Section 2 develops the theoretical model. Section 3 provides background information on electrification in Guatemala and introduces the data to be used in this analysis, the Guatemala household survey data collected by the Guatemala National Statistical Agency (ENCOVI), as part of the World Bank's LSMS surveys. Section 4 tests the predictions of our

[5] See, for example, Herrin (1979), Sokari and Dimpka-Harry (1991), and Harbison and Robinson (1985).

[6] *A priori*, it is not theoretically obvious that electrification should either reduce fertility or increase female labour force participation. A standard female labour supply model such as Gronau (1977) would predict that women spend *more* time on home production when this production becomes more efficient. This is because the marginal product of work in the home will now only equal the market wage after much more home production has been done. Similarly, if children are considered home-produced goods, their shadow price could be expected to fall under more efficient home production.

model regarding the relationship between electrification and fertility. In Section 5 the causal impact of household electrification on employment probabilities and time spent in key home production activities is identified. Section 6 concludes.

# 2 The model

We describe household decisions regarding labour, consumption and savings using a cooperative Nash bargaining model. The weights of $\alpha$ and $1-\alpha$ are used for the individual Cobb Douglas utility functions, for each member of the couple. Each party has preferences over consumption in each period, number of children and another public good that is produced at home. For each member of the couple we denote the various consumption levels by $x_{f0}$, $x_{f1}$,$x_{m0}$, and $x_{m1}$. The number of children is denoted by $n$, and home production by $y$. The male's utility function and the female's utility function are given as follows:

$$U_m = x_{m_0}^{\theta_m} x_{m_1}^{\gamma_m} n^{\delta_m} y^{\phi_m}$$
$$U_f = x_{f_0}^{\theta_f} x_{f_1}^{\gamma_f} n^{\delta_f} y^{\phi_f}.$$

Putting the two utility functions together to form the generalized Nash product, which we will call $U$, we find

$$U = x_{m_0}^{\alpha\theta_m} x_{m_1}^{\alpha\gamma_m} x_{f_0}^{(1-\alpha)\theta_f} x_{f_1}^{(1-\alpha)\gamma_f} n^{\alpha\delta_m+(1-\alpha)\delta_f} y^{\alpha\phi_m+(1-\alpha)\phi_f}.$$

The two parties will maximise this objective with the usual budget constraints in each period, as well as a time constraint. Although in principle we could have a non-zero disagreement point to reflect each agent's possible alternative payoffs or outside options, for simplicity we take the disagreement points to be zero.[7]

We assume that the male spends all of his available time, which we normalise to be 1, working. On the other hand, the female divides her time, also normalised to 1, between caring for children, home production, and participation in the labour force. The time spent caring for children is an increasing function, $h(n)$, of the number of children.[8] Home production has a production technology

[7]Not only does this choice result in a simpler solution to the model, it also allows us to have a simpler story. To properly include outside options, we would have to provide a model of what determines outside options. This would require a full model of all the alternatives that can be taken if agreement is not reached. Since we already have quite a rich model with four consumption goods for each agent, as well as several other choice variables and two time periods, outside options are not here treated explicitly.

[8]Although, in principle, the time necessary to care for a given number of children could also depend on the level of electricity consumption, our data indicate that controlling for the number of children the time spent on child care is fairly constant. As a function of the number of children, this relationship should be convex for the second order

$y = k(e)\ell$, where $\ell$ denotes the labour the woman devotes to home production and $k(e)$ is a production coefficient that increases with the amount of electricity, $e$. We suppose that, without electricity, home production is very inefficient, so $k(0)$ is quite low. With electricity, home production rapidly becomes more efficient up to a maximum efficiency. Thereafter it is constant. This means that, beyond a point, more electricity does not improve efficiency. Formally, $k(e)$ is increasing concave and eventually constant. Finally, we denote the time for labour force participation by $\ell_f$. This gives us the time constraint for the female

$$\ell_f + \frac{y}{k(e)} + h(n) \leq 1.$$

In addition there are the budget constraints for each time period

$$\begin{aligned} x_{m_0} + x_{f_0} + Pe + cn &\leq w_f \ell_f + w_m - S, \\ x_{m_1} + x_{f_1} &\leq (1+r)S + gn, \end{aligned}$$

where, in the first period, consumption is numeraire, $P$ is the price of electricity, $c$ is the money cost for each child, $w_f$ is the female wage rate, $w_m$ is the male wage rate, $S$ is the amount saved.

In the second period neither party works, consumption is again numeraire, $r$ is the interest rate, and $g$ is the amount of second period monetary support received from each child. Thus the problem reduces to maximizing the generalized Nash product with three constraints

$$\max_{x_{m_0}, x_{m_1} x_{f_0}, x_{f_1}, n, y, e, S, \ell_f} \quad x_{m_0}^{\alpha\theta_m} x_{m_1}^{\alpha\gamma_m} x_{f_0}^{(1-\alpha)\theta_f} x_{f_1}^{(1-\alpha)\gamma_f} n^{\alpha\delta_m + (1-\alpha)\delta_f} y^{\alpha\phi_m + (1-\alpha)\phi_f}$$

subject to the constraints

$$\begin{aligned} \ell_f + \frac{y}{k(e)} + h(n) &\leq 1 \\ x_{m_0} + x_{f_0} + Pe + cn &\leq w_f \ell_f + w_m - S, \\ x_{m_1} + x_{f_1} &\leq (1+r)S + gn. \end{aligned}$$

Our main interest is in observing how the choices relating to time allocation for the female and the number of children change with electrification. In our model, from an initial situation of no electricity ($e = 0$) we associate electrification with a large decrease in the price of electricity. In order to simplify the model we assume that, although the prices for the dated consumption goods can change, they will move together. For this reason it is possible to treat all consumption (for both

conditions to hold. In reality we would think that there could be some initial economies to scale of time required for children, which would eventually disappear. We abstract from this possibility for the purpose of being able to solve using the first order conditions. Our results, then, apply in the portion where there are diseconomies of scale. Although we do find that the number of children in many of our comparative statics are shown to decrease, they would not likely decrease to zero if there were initial economies of scale.

parties, over both periods) as a composite commodity.[9] We let $x$ denote all consumption, where $x = x_{m_0} + \frac{x_{m_1}}{r+1} + x_{f_0} + \frac{x_{f_1}}{r+1}$. We also combine the two budget constraints by eliminating the savings variable.[10] This reduces the problem to

$$\max_{x,n,y,e,\ell_f} x^\theta n^\delta y^\phi$$
$$\text{such that } w_f \ell_f + w_m + \frac{g}{1+r} n - x - Pe - cn \geq 0;$$
$$\text{and } 1 - \ell_f - \frac{y}{k(e)} - h(n) \geq 0,$$

where $\theta$ is the coefficient for $x$, in the reduced problem, $\delta$ is the coefficient for $n$ and $\phi$ is the coefficient for $y$. We define the Lagrangean

$$L = x^\theta n^\delta y^\phi + \lambda\{w_f \ell_f + w_m + \frac{g}{1+r} n - x - Pe - cn\} + \mu\{1 - \ell_f - \frac{y}{k(e)} - h(n)\}.$$

The first order conditions are:

$$\begin{aligned} U_x - \lambda &= 0, \\ U_n + \lambda(\frac{g}{1+r} - c) - \mu h'(n) &= 0, \\ U_y - \frac{\mu}{k(e)} &= 0, \\ \frac{\mu y k'(e)}{k^2(e)} - \lambda P &= 0, \\ \lambda w_f - \mu &= 0, \\ w_f \ell_f + w_m + \frac{g}{1+r} n - x - Pe - cn &= 0, \\ \text{and } 1 - \ell_f - \frac{y}{k(e)} - h(n) &= 0. \end{aligned}$$

The second order conditions are found from the bordered Hessian, which has a negative determi-

[9] There are some conditions necessary for the composite commodity approach to be valid, and these are discussed in the appendix under the section entitled **Composite Commodity.**

[10] Combining the two budget constraints allows the possibility that each one of them may not hold individually, actually obscures the question of whether savings is negative, since we solve for and eliminate savings. However, we can be sure that, if the starting point is one with positive savings, as the price of electricity decreases this will move the solution in the direction of even more savings. This rules out the possibility that savings will ever become negative.

nant[11] for a maximum.

$$\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & 0 & 0 & -1 & 0 \\ U_{nx} & U_{nn}-\mu h''(n) & U_{ny} & 0 & 0 & \frac{g}{1+r}-c & -h'(n) \\ U_{yx} & U_{yn} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & 0 & -1/k(e) \\ 0 & 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)}-\frac{2(k'(e))^2}{k^3(e)} & 0 & -P & \frac{yk'(e)}{k^2(e)} \\ 0 & 0 & 0 & 0 & 0 & w_f & -1 \\ -1 & \frac{g}{1+r}-c & 0 & -P & w_f & 0 & 0 \\ 0 & -h'(n) & -1/k(e) & \frac{yk'(e)}{k^2(e)} & -1 & 0 & 0 \end{bmatrix}$$

To understand the effects of household electrification we look at the comparative statics. We interpret electrification as a large decrease in the price of electricity. Thus, $\frac{\partial n}{\partial P}$ and $\frac{\partial \ell_f}{\partial P}$ are of most interest, since they predict the effect of electrification on fertility and women's labour force participation. We will also examine the effect of men's and women's wages on fertility.

We use Cramer's rule to find $\frac{\partial n}{\partial w_m}$, $\frac{\partial n}{\partial P}$ and $\frac{\partial n}{\partial w_f}$, by replacing the second column with $[0,0,0,0,0,-1,0]$, $[0,0,0,\lambda,0,e,0]$, and $[0,0,0,0,-\lambda,-\ell_f,0]$, respectively, and taking the ratio of the determinant thus obtained with the determinant of the original bordered Hessian matrix. We find $\frac{\partial \ell_f}{\partial P}$ using the same procedure but replacing the fifth column with $[0,0,0,\lambda,0,e,0]$.

The signs of the comparative statics turn on the relative magnitudes of $[\frac{k'(e)}{k^2(e)}]^2 \geq 0$, $[\frac{k'(e)}{k^2(e)}] \geq 0$, and $[\frac{k''(e)}{k^2(e)} - \frac{(k'(e))^2}{k^3(e)}] \leq 0$. For each comparative static, we can write the numerator in the form $-A \cdot [\frac{k'(e)}{k^2(e)}]^2 - B \cdot [\frac{k'(e)}{k^2(e)}] + C \cdot [\frac{k''(e)}{k^2(e)} - \frac{(k'(e))^2}{k^3(e)}]$, where the values of the coefficients $A$, $B$, and $C$ will naturally be different for each comparative static. Notice, that the sign of each component of the effect is the same as the sign of the relevant coefficient, since the determinant in the denominator is negative. We argue that in situations where the current level of electricity consumption is very low, the sign of the effect will be dominated by the sign of the coefficient $A$, since when $e \approx 0$, $k(e)$ is at its minimum value and $k'(e)$ is very large, and so $[\frac{k'(e)}{k^2(e)}]^2$ will be extremely large as compared to the other terms. When electricity consumption is at a higher level all three of $A$, $B$, and $C$ become important in the comparative static. As electricity consumption becomes extremely high, the three coefficients become less and less important since $k'(e)$ and $k''(e)$ are approximately zero; in this case the value of the comparative static tends to zero. Thus, for each comparative static, we will simply focus on the sign of the value of $A$. This gives us the effect when the level of electricity consumption is very low. For higher levels of electricity all three coefficients will have a role in affecting the sign of the comparative statics.

Each comparative static is worked out in detail in the Model Appendix. The general method that

[11] See *Further Mathematics for Economics Analysis* by Sydæter, Hammond, Seierstad, and Strøm [2008], page 126.

we use is to expand the determinant in the numerator and then collect terms to get it in the form $-A \cdot [\frac{k'(e)}{k^2(e)}]^2 - B \cdot [\frac{k'(e)}{k^2(e)}] + C \cdot [\frac{k''(e)}{k^2(e)} - \frac{(k'(e))^2}{k^3(e)}]$. Next, if possible we evaluate the signs of the coefficients. The strategy used in this step is that we try to rearrange the parts of the coefficients into recognizable terms.,[12] One such term is $w_f h'(n) + c - \frac{g}{1+r} > 0$, the effective net price of the last child in terms of the value of the time needed plus the money cost of raising the child less the present value of the monetary support to be expected next period form the child. We also have terms of the form

$$\begin{aligned} U_{xx}U_{ny} - U_{nx}U_{xy} &= \theta(\theta-1)x^{\theta-2}n^{\delta}y^{\phi}\delta\phi x^{\theta}n^{\delta-1}y^{\phi-1} \\ &\quad - \theta\delta x^{\theta-1}n^{\delta-1}y^{\phi}\theta\phi x^{\theta-1}n^{\delta}y^{\phi-1} \\ &= \{\theta(\theta-1)\delta\phi - \theta^2\delta\phi\}x^{2\theta-2}n^{2\delta-1}y^{2\phi-1} \\ &= -\theta\delta\phi x^{2\theta-2}n^{2\delta-1}y^{2\phi-1} < 0; \end{aligned}$$

or terms of the form

$$\begin{aligned} U_{xx}U_{yy} - U_{yx}U_{xy} &= \theta(\theta-1)x^{\theta-2}n^{\delta}y^{\phi}\phi(\phi-1)x^{\theta}n^{\delta}y^{\phi-2} \\ &\quad - \theta\phi x^{\theta-1}n^{\delta}y^{\phi-1}\phi\theta x^{\theta-1}n^{\delta}y^{\phi-1} \\ &= \{(\theta^2-\theta)(\phi^2-\phi) - \theta^2\phi^2\}x^{2\theta-2}n^{2\delta}y^{2\phi-2} \\ &= \theta\phi(1-\theta-\phi)x^{2\theta-2}n^{2\delta}y^{2\phi-2} > 0. \end{aligned}$$

Looking at the effect of the men's wage on fertility, $\frac{\partial n}{\partial w_m}$, we find $A < 0$, $B = 0$, and $C > 0$.[13] Thus, when electricity consumption is low and $A$ dominates, the effect is negative: An increase in men's wages will decrease fertility. Generally with a Cobb-Douglas objective function, we expect all goods to be normal goods, and so this result might initially be puzzling. However, upon a closer look we see that with more income, more electricity can be purchased, and that will make the relative price of home production (which is effectively $\frac{w_f}{k(e)}$) much lower as compared to all the other goods. The substitution effect caused by the relative price reduction outweighs the income effect of an increase in $w_m$.

We expect the effect of men's wages on fertility to vary depending on the level of electricity consumption in the household. At higher levels of electricity consumption, both coefficients will have an effect, and it is plausible that the effect could be positive. In any case it eventually tends to zero. The effect eventually becoming zero may conflict with the usual understanding of a normal good. However, at this high level of electricity consumption, the family already has a high income and so it may be having a high number of children, each of whom is transferring $g$ to their parents in the next

[12] Of course, we also use the basic information that the choice variable $x$, $n$, $y$ are non-negative, as are $e$, $\ell_f$, $k(e)$, $h(n)$, $P$, $r$, $w_m$, $w_f$, $c$, $g$, $S$, $\mu$, and $\lambda$, and the coefficients $\theta$, $\delta$, and $\phi$ are normalized to sum to 1.

[13] The actual expressions for each of these coefficients can be found in the Model Appendix.

period. This may already be more than the amount they would want to transfer across generations, so they would stop having more children.

The effect of electricity prices on fertility, $\frac{\partial n}{\partial P}$, is the main effect of interest. Recall that starting at no electricity consumption we were depicting electrification as a large decrease in the price of electricity. We find that in general the effect is ambiguous, since $A > 0$, $B$ has an ambiguous sign, and $C < 0$. However, at extremely low levels of electricity consumption we have some clear results. We find that electrification should result in lower fertility (since $A > 0$, price of electricity decreases means fertility goes down), which is consistent with the main finding in our data. For intermediate levels of electricity consumption where $B$ and $C$ have a greater role, it is plausible to have an increase in fertility with a decrease in the price of electricity. These latter results are aligned with the increases in fertility observed in the post-war period in the US, which coincided with the development of several labour saving technologies for home production. The post-war economies in home production can be thought of as equivalent to a reduction in the price of electricity.

Next we consider the effect of female wages on fertility, $\frac{\partial n}{\partial w_f}$. This comparative static and the next one that we shall see both have very complex expressions. For $A$ we evaluate only the part of that expression containing the term $[\frac{1}{k(e)}]$, which should be the most significant part of $A$ and so which determines the sign of $A$ for very small levels of $e$.[14] We conclude that for very low levels of electricity consumption, women's wages will have a negative effect on fertility; however, for intermediate levels the effect has ambiguous sign.

Next, we consider the effect of the price of electricity on female labour force participation, $\frac{\partial \ell_f}{\partial P}$. This comparative static, too, has a complex expression for $A$ and we evaluate only the part of that expression containing the term $[\frac{1}{k(e)}]$. When $e$ is very small, we find this to be negative except possibly for one term containing containing $h''(n)$, which has an ambiguous sign that will be opposite to the sign of $h''(n)$.[15] Thus, when $h''(n)$ is non-negative or zero, the comparative static is negative. This

[14] For $C$ there is also such a term, there is an additional term involving $[\frac{1}{k(e)}]^2$. Since this is the most significant part of the coefficient, $C$ will have the most effect at low levels of electrification, and we find this to have a positive sign. The coefficient $B$ is zero. These results are worked out fully in the Model Appendix under the section entitled **Comparative Statics**, at the end of the derivation of $\frac{\partial n}{\partial w_f}$ and where we discuss the signs of the coefficients $A$, $B$, and $C$.

[15] Perhaps initially (i.e. at $n = 0$), $h''(n)$ is negative since there might be economies in the time needed to raise children. But, as long as the effect is small in the range of $n$ that we are actually at, that second derivative is positive or zero, and it will accord with the rest of the term which is negative. Whether it is sensible to think that eventually $h''(n)$ should be positive (that is, with very numerous children there are some sort of confounding externalities that result in actually requiring more and more time per child) or whether it just becomes zero, is equally plausible, but either case is not a problem.

means that, as the price of electricity decreases, we would expect a higher participation rate for women.

Finally, the last comparative static is $\frac{\partial \ell_f}{\partial w_m}$. As with the previous comparative static, in looking at behaviour when electricity consumption is close to zero, $h''(n)$ has a role in the sign. If it is positive it reinforces the rest of the terms, and the effect is positive.[16] That is, with an increase in men's wages we expect greater labour force participation by women. Intuitively, this occurs because, with men earning more income, more electricity is affordable. This makes home production very efficient and frees up women's time for more labour force participation.

# 3 Background and Data

The rapid expansion of the rural electricity network in Guatemala during 1996 to 1999 resulted in poor and indigenous communities being more than twice as likely to receive a new electricity connection as they had been previously (Foster and Araujo (2004)). Still, by 1998, only about 60% of the population of Guatemala had access to electricity (World Bank (2008)). To further promote rapid rural electrification, the Programa de Electrificacíon Rural, PER, was begun in 1998. Under the terms of a privatisation agreement which broke up the state power authority and launched the PER, two private power companies are paid 650 US dollars for the electrification of each household in rural areas. For the company to receive the funds, the connection must be for a residential dwelling and the dwelling must be located more than 200 metres from the existing local electric grid. The new infrastructure becomes an asset of the company.[17] The companies are also required to connect any potential consumer within 200 metres of the existing grid who requests the service. This incentive structure means that communities who obtained electricity under this scheme have nearly-universal household electrification rates. Under the PER the Guatemalan government also allocated 333 million US dollars to fund expansion of the rural electricity distribution network through new substations and lines (see Foster and Araujo (2004) and World Bank (2008)). In the first three years of the PER, about 150 450 rural households obtained electricity. Because the National Commission for Electricity determines the prices that companies can charge, most rural customers receive a subsidy, and electricity bills average about 3 US dollars per month.

[16] If it is zero, as with the last comparative static, it will have no effect on the sign.

[17] The state power authority was privatised by auction and divided into two companies. A Spanish company bid for both companies, under terms which included investor protection, provided by the Multilateral Investment Guarantee Agency.

### 3.1 The ENCOVI 2000 Survey

The recent large-scale investments in electrification in Guatemala, low household electrification rates, and the availability of comprehensive community-level infrastructure information as part of the Guatemalan ENCOVI survey, combine to make an ideal environment for the study of the effects of electrification on households. As well, the 2000 ENCOVI data constitute one of the largest and most comprehensive of all of the World Bank LSMS surveys. More than 37 700 individuals took part, and answered detailed questions on education, labour supply, time use, agricultural work, health, fertility, and participation in community activities. At the community level, another detailed survey was administered to obtain information on the infrastructure and services provided in the community. This survey, unlike many World Bank surveys containing community-level information, records information on the timing of the introduction of water, sanitation, telephone, and electricity infrastructure. Two-thirds of communities sampled by the LSMS individual-level and household questionnaires also completed the community-level survey, giving a total of 485 communities.[18]

Concurrent with the household survey, another detailed survey was administered at the community level to obtain information on the infrastructure and services provided in the communities of selected households. This survey, unlike many World Bank surveys containing community level data, records information on the timing of the introduction of water, sanitation, telephone, and electricity infrastructure. It is this information which is crucial to identifying the effects of electrification on the labour market outcomes of individuals. Community level surveys were completed by community members assembled in focus groups. Two-thirds of communities sampled by the ENCOVI individual level and household questionnaires also completed this community survey, so that individuals in 485 communities were represented. Respondents in the community survey were described the physical boundaries of their communities principally by showing a map of the geographical limits of the community, as defined by the ENCOVI survey team. These geographical boundaries mostly represent the natural boundaries of villages and towns, but in larger cities such as Guatemala City, represent geographically-distinct neighbourhoods.

There are many notable differences in the access to public infrastructure of individuals residing in communities with and without electricity. For example, electrified communities are much more likely to have paved roads which are open during winter, to have police services, and to have a doctor in the community.[19] More than 84% of communities with electricity have existed for twenty or more

[18]More information on the ENCOVI survey is available on the World Bank Living Standards Monitoring Survey website, *www.worldbank.org/lsms*.

[19]There is considerable variability in both the presence and vintage of basic infrastructure across communities. For example, 86% of the estimation sample members who reside in electrified communities also have piped water

years, versus only 75% of those without.

The sequencing of major infrastructure within communities is such that piped water usually arrives before electricity, and electricity usually arrives before improved sanitation system. In only 37% of communities with both electricity and piped water did electricity arrive first. However, in 68% of communities with both electricity and sanitation (defined as a centralised community sewage or septic tank system), electricity arrived first. About 13% of the communities in which ENCOVI respondents reside had none of these three major infrastructure types at the 2000 ENCOVI interview. In communities with at least one of these three infrastructure types, 87% had electricity.

The survey question determining community electrification status translates roughly as *"Does this community have electricity in the households?"*. This question provides information on whether or not the electric grid extends to a community at the time of the ENCOVI interview. In Guatemala local off-grid energy sources such as solar power make up only a small fraction of residential energy consumption (International Energy Agency (2005)).

Communities are described to respondents of the community-level survey principally by showing a map of the geographical boundaries of the community defined by the ENCOVI survey team. These geographical boundaries sometimes represent the boundaries of municipalities, but in larger cities such as Guatemala City and Huehuetenango, represent neighbourhoods. More information on the ENCOVI survey is available on the World Bank Living Standards Monitoring Surveys website, *www.worldbank.org/lsms*.

Data from earlier household surveys of Guatemala are informative of how electrification has proceeded across the country, and of the correlations between the geography of a region and the extent of electrification at different points in time. The Demographic and Health Surveys collected in Guatemala in 1987 and 1999, can inform us about how electrification rates in counties (*departamentos*) of Guatemala relate to geographic and socioeconomic features of different regions, and how electrification proceeded historically. Table 7 of Data Appendix A gives the mean household electrification rates in Guatemala by county for 1987 and 1999, as well as information on the extent of fighting in the county during the civil war, and the elevation of the county above sea level. To summarise, Guatemalan counties at higher elevations, such as Totonicapán, Sololá, and Quichè, tended to have the lowest rural household electrification rates (10% or less) in the 1987 DHS survey. Those counties where more civil war fighting occurred, as crudely measured by the number of documented

in the community. However, only 44% of people residing electrified communities also have improved sanitation in their community. Amongst those residing in communities with improved sanitation, 99% also have electricity in their community. As with improved sanitation, piped water is also more likely to be available in communities that have been electrified for longer periods.

massacres during the civil war period, also had very low household electrification rates in the 1987 DHS survey. While rural electrification rates generally improved the most in these disadvantaged, rural areas during 1987-1999, they remained under 35% in the most sparsely-populated counties, Petén and Izabal.

In the next section, the causal relationship between electrification and fertility is examined. We test the model's prediction that electrification reduces the incentives of women to bear children in a high-fertility environment in which they children arguably primarily investment goods. We assume that electricity use is low, where present in the household, since the vast majority of electrified households in Guatemala have few appliances and do not use electricity for cooking.

# 4 Impacts of Electrification on Fertility

Our model predicts that household electrification will result in women having fewer children in situations where originally the number of children was large. This is primarily because the old-age savings motive to have children diminishes with this technological innovation. The advent of electricity should also change both the allocation of time of women within and outside the household, since the main home production activities such as cooking (which is done mainly by women) can be executed more efficiently. This section tests the predictions of our model regarding the impact of household electrification on fertility in the context in which children are initially primarily investment goods.

To examine the impact of electrification on fertility, we employ information from the community-level survey on the date at which a community was electrified in conjunction with individual responses from the ENCOVI women's questionnaire. It is not known how old a woman was when she first lived in an electrified household, but it is known when the electric grid first reached a community.

Households in Guatemala receive electricity if the electric grid is extended to their community, so household electrification status can be considered largely exogenous to households and their members. When the grid reaches a community, households obtain electric connections nearly universally and simultaneously. This is particularly true under the Guatemalan government's post-war PER initiative because of the 650 dollars received by electricity companies for each new household brought onto the grid. As a result, about 97% of electrified households in the ENCOVI data are in electrified communities, and more than 85% of households in electrified communities have electricity. Only 7% of households in unelectrified communities report having some electric power. Because household electrification depends mainly on whether the electric grid reaches a community, an individual's employment status or time use preferences of household members are unlikely to be major factors influencing household electrification probabilities.

## 4.1 Estimation

A sample of women aged 12-49 is employed, and multivariate regressions are estimated. The sample is a young one, with a mean age of 30. This reflects the high fertility rates in Guatemala, and the relatively low life expectancy of the population. Both women in currently electrified communities and those in unelectrified communities are included. Because the ENCOVI sample contains large numbers of respondents from each community, it is possible to control in estimation for community fixed effects, current community electrification status, the current age of individuals, and the age of individuals at the time of placement of other infrastructure.

Prior to estimation of causal effects, it is shown that there are substantial differences in the reproductive behaviour of women in households with and without electric connections. Figure 2 compares the number of children ever born to a women by age amongst women in unelectrified and electrified households, using locally-weighted least squares. It is clear from the Figure that women in unelectrified households have more children at any age. It is also apparent from these data that fertility is higher amongst women in unelectrified communities, as our model predicts. For example, the mean number of children born to a woman aged 30-49 in electrified communities is 4.5, as compared to 6.1 amongst women in unelectrified communities.[20] A similar result prevails when examining this statistic by household electrification status, since 97% of those in electrified households reside in communities connected to the electric grid. However, because there are likely many other socioeconomic differences between individuals residing in electrified and unelectrified communities, and because infrastructure placement is likely both economically and politically motivated, it is not clear that these observed differences in fertility are caused by electrification. For this reason, a multivariate strategy which combines individual and community-level information is used to identify causal effects. The identification strategy involves comparing the fertility outcomes of women who were of different ages at the advent of electricity to their community, and so were exposed to this new household technology to differing extents during their reproductive lives.

The identification strategy relies on three main factors. First, if electrification impacts fertility, women who have had access to electricity for greater fractions of their young lives should have less children, conditional on current age. Second, women older than 40 at the advent of electricity to their communities should experience essentially no fertility effects from this new technology, since the ability to bear children is arguably the main constraint on childbearing beyond this age. The data

[20] These differences are statistically significant at the 5% level, with the |t|-statistic in a test of equality of 10.3.

support this assumption: the mean number of children born to a woman whose community obtained electricity after her fortieth birthday is 6.83, statistically the same as the mean number of children ever born to a woman older than 40 in an unelectrified community, 7.17.[21] Third, the age of an individual at the time of the community's electrification should be uncorrelated with unobservables which also impact fertility. Under these assumptions, the effects on fertility of being of different ages at community electrification can be identified relative to a control group containing women aged 40 or older at the advent of electricity to their community, whose fertility is unaffected by this technological innovation. The association between current age and the number of children ever born to a woman is captured by the inclusion of women residing both in electrified and unelectrified communities. The inclusion of women in unelectrified communities in the estimation sample facilitates distinguishing between effects of current age on the total number of children born to a woman from the main effects of interest, those of her the age at the electrification of the community.

Respondents in electrified communities are grouped by their age at the time of electrification of the community. The age at electrification groups are: Born after community electrification (ALLELEC), under age 10 at electrification (ELEC0TO9), age 10 to 19 at electrification (ELEC10TO19), age 20-29 at electrification (ELEC20TO29), age 30-39 at electrification (ELEC30TO39), and older than age 40 at electrification (ELEC40plus). Women in both currently electrified and unelectrified communities are included, and a dummy variable is used to indicate current community electrification status (COMELEC).

Ordinary least squares regressions are estimated to explain the log of the number of children (NOKIDS) woman $i$ in community $j$ has given birth at the time of the survey, as a function of her age when electricity reached her community, her current age, and extensive socio-economic and local controls. Formally, the regression to be estimated takes the form:

$$\begin{aligned} LNNOKIDS_{ij} = \quad & \beta_0 + \beta_1 * ALLELEC + \beta_2 * ELEC0TO9_{ij} + \\ & \beta_3 * ELEC10TO19_{ij} + \beta_4 * ELEC20TO29_{ij} + \\ & \beta_5 * ELEC30TO39_{ij} + \beta_6 * COMELEC + \\ & \gamma * CONTROLS_{ij} + \mu_j + \epsilon_{ij} \end{aligned}$$

The individual level controls included in all specifications are: dummies for the 10-year age group representing current age (age 40 plus is the reference), dummies for the presence of piped water and the age of a woman at the advent of piped water to her community, and dummies for the presence of improved sanitation and the age of a women at the advent of improved sanitation to her community.

[21] A t-test of the equality of these means accepts the null hypothesis at the 5% level, with |t|-value 0.955.

As with the age at electrification dummies, the dummies representing the age of a women at the advent of piped water and improved sanitation are defined in 10 year cohorts.[22] The variable $\mu_j$ represents local fixed effects. Standard errors are clustered at the community level.

Table 1 presents estimation results for the specification described above, with three different controls for local fixed effects. In column (1), twenty-two county (*departamento*) dummies are included as well as controls for whether or not the community has a preschool, an elementary school, or a secondary school, whether or not the community has a paved highway, whether or not the community was affected by Hurricane Mitch, and whether or not the community is rural.[23] Column (2) includes these local controls plus county-rural interaction terms. Column (3), the preferred specification, includes community level fixed effects in place of these other local controls. In all three specifications, standard errors are clustered at the community level.

Results are very similar across the three specifications. Relative to the reference group of women aged 40 plus at the advent of electricity, whose fertility is assumed not to be affected when electricity arrives, women who were younger when electricity arrived currently have significantly fewer children. In the preferred specification, Column (3), women who were under 30 at the time of electricity to their communities are shown to have about 28% fewer children currently than women in their community who were older than this age when electricity arrived.[24] Moreover, those who were aged zero to 9 when electricity arrived have significantly fewer children currently than those who were in their twenties at the advent of electricity.[25] However, individuals who were born into already-electrified communities have statistically the same number of children as those who were age zero to 9 when electricity arrived.[26] These findings are consistent with the hypothesis that women who have had electricity for greater fractions of their lives should have fewer total children *ceteris paribus*, and also

[22]The full set of improved sanitation dummies is as follows: A dummy for the presence of improved sanitation in community, a dummy having had sanitation all one's life, a dummy for being aged 0-9 at the advent of sanitation, a dummy for being aged 10-19 at the advent of sanitation, a dummy for being aged 20-29 at the advent of sanitation, and a dummy for being aged 30-39 at the advent of sanitation. The reference age at the advent of sanitation is above 40. The full set of piped water dummies is as follows: A dummy for the presence of piped water in community, a dummy having had piped water all one's life, a dummy for being aged 0-9 at the advent of piped water, a dummy for being aged 10-19 at the advent of piped water, a dummy for being aged 20-29 at the advent of piped water, and a dummy for being aged 30-39 at the advent of piped water. The reference age at the advent of piped water is above 40.

[23]A community is considered rural if focus group respondents in the community survey so define it.

[24]The calculation is $e^{-0.33}$-1.

[25]A t-test of the equality of coefficients of the dummy variables ELEC0TO9 and ELEC20TO29 just rejects the null hypothesis at the 10% level, with |t|-statistic 1.62.

[26]A t-test of the equality of coefficients of the dummy variables ELEC0TO9 and ALLELEC accepts the null hypothesis at the 10% level, with |t|-statistic 1.29.

with the idea that those who gained access to electricity before reproduction could have begun have similar fertility patterns to those who were born into already-electrified communities.

The preferred specification of Table 1, that reported in column (3), suggests that women under age 30 at electrification have significantly fewer children than those above this age when electrification occurs, conditional on current age and on community fixed effects. However, those who are aged 30 or older when electricity arrives do not have significantly different numbers of children, conditional on current age, than those over 40 at electrification. For this reason, we also present results of a more parsimonious specification of age at electrification effects, in which we distinguish only between women who were born into electrified communities, women who were aged 0 to 29 when electricity arrived, and those who were older than this age when the electric grid arrived. Those who were born into electrified communities have a mean current age of 21, those who were 0 to 29 when electricity arrived a mean age of 24, and those in the reference group (women older than 30 at electrification plus women in unelectrified communities) a mean age of 39.

The equation to be estimated is as follows:

$$\begin{aligned} LNNOKIDS_{ij} = \; & \beta_0 + \beta_1 * ALLELEC + \beta_2 * ELEC0TO29_{ij} + \\ & \beta_3 * COMELEC + \gamma * CONTROLS_{ij} + \mu_j + \epsilon_{ij} \end{aligned}$$

As previously, individual level controls included in all specifications are: dummies for the 10-year age group representing current age (age 40 plus is the reference), dummies for the presence of piped water and age of a woman at the advent of piped water to her community, and dummies for the presence of improved sanitation and the age of a women at the advent of improved sanitation to her community.[27] The variable $\mu_j$ represents local fixed effects. Standard errors are clustered at the community level.

Columns (1) through (3) of Table 2 present the results of this more parsimonious specification. As expected, those who were born into electrified communities, and those whose community of residence was electrified before age 30 have significantly fewer children at the time of the ENCOVI interview, *ceteris paribus*. In column (3), the preferred specification which includes community level fixed effects, it is shown that those who were under age 30 at electrification have about 28 percent less children than do women of the same age who were older when their communities were electrified.[28]

[27]The full set of improved sanitation dummies is as follows: A dummy for the presence of improved sanitation in community, a dummy having had improved sanitation all one's life, and a dummy for being under age 30 at the advent of improved sanitation. The reference age at the advent of improved sanitation is 30 and above. The full set of piped water dummies is as follows: A dummy for the presence of piped water in community, a dummy having had piped water all one's life, and a dummy for being under age 30 at the advent of piped water. The reference age at the advent of piped water is 30 and above.

[28]The calculation is $e^{-0.33} - 1$.

## 4.2 Falsification Exercise

Although all of the specifications of Tables 1 and 2 account for the presence of other major infrastructure in communities, and the age of individuals at the advent of piped water and sanitation, it remains possible that there are unobserved factors correlated with the timing of electricity provisions which also affect women's fertility. For this reason, a falsification exercise is undertaken. In this exercise both the presence and vintage of electricity in communities has been randomised. To do this, we first re-assign electricity to communities on the basis of a random number generator. The fraction of communities with electricity is made the same as in the true data, 76%. Amongst the electrified communities in the falsified data, we assign a vintage of electricity according to this same random number generator, such that the mean and variance of electricity vintage is as in the true data. We then merge this false electrification information to the individual fertility data, and to the true data on the presence and timing of other infrastructure provisions in the community. The specifications of Tables 1 and 2 are then re-estimated.

Results of the falsification exercise are presented in Tables 8 and 9 of Data Appendix A. To summarise, after randomising the presence and vintage of electricity across communities, no statistically significant effect on fertility of being younger versus older at electrification is apparent. While this finding by no means proves that the specifications of Tables 1 and 2 capture pure effects of electrification on fertility, it does support the hypothesis that real events coincident with the timing of electrification cause fertility to decline.

In this section we have shown that electrification reduces fertility in Guatemala by comparing the number of children ever born to women who were different ages at the time of electrification of their communities. While infrastructure placement is likely associated with initial community wealth and with the political say of residents, this possibility is largely accounted for in estimation by employing community-level fixed effects. It is shown that, within communities, women who were relatively young at electrification have fewer children than those who were older at the time of community electrification, conditional on current age and community fixed effects. These results concur with the stylised facts from our sample statistics that women in unelectrified households have more children by a given age.

Nevertheless, the finding that electrification causes substantial reductions in fertility for women does not prove that the mechanism through which fertility reductions occur is as described by the model. While these findings are consistent with the model, it is also potentially the case that electrification substantially improves local wages earned by both men and women. Our model also predicts that higher wages of men and women will decrease fertility when electricity consumption is very low.[29] The findings above are consistent with our model, but the ENCOVI data do not permit further decomposition of the avenues through which electrification reduces fertility.

# 5 Work in and outside the home

Our model assumes a strong sexual division of labour within the household, so that only women's time use is directly impacted by household electrification. The Guatemalan ENCOVI survey contains extensive information on time use within and outside the household, which permits testing of this assumption.

Employment status information from the ENCOVI survey supports the model's assumption that only women's working lives are directly affected by household electrification. In Panel A of Table 3 it is shown that, while women in electrified households are significantly more likely to be employed outside the home than are women in unelectrified households, the same is not true for men. Men's employment probabilities do not appear to be systematically related to household electrification status.[30]

The time use data also suggest strong differences across the sexes in how household electrification impacts time use. Panel B of Table 3 presents a comparison of mean amounts of time spent in three activities in the week prior to the ENCOVI interview: work outside the home in paid employment ('*empresas*' and '*negocios*'), time spent looking after children, and time spent cooking. The latter two activities are the two most time consuming home production activities in which women engage.[31]

The comparison of mean time spent in different activities supports the idea that men's time in paid work, childcare and cooking is not affected by household electrification. Mean time spent in these activities is the same for men across electrified and unelectrified households. However, this is not true for women. Women in electrified households spend significantly less time looking after children and cooking, at the mean, and significantly more time working for money, than do women

[29] As well, parents might readily substitute towards having fewer, better-educated and fed children as their incomes rise, even if the causal effect of income on fertility is positive (Becker (1960), Becker (1991)). Our model does not consider child quality.

[30] In the analysis which follows, the sample is composed of individuals aged 20-55. Since community level data is not essential to the identification strategies to be employed, individuals in communities where no community-level survey was undertaken can also be included in the samples.

[31] ENCOVI interviewers asked respondents to note the amount of time in which they were primarily engaged in each specific activity, since it is possible to combine some activities, such as looking after children and cooking. A separate question regarding combined activities was also posed, but here the focus is on time spent primarily on one specific activity.

in unelectrified households.

One aspect of the results of Panel B which does not, at first, seem to square with our model's assumptions is that mean time spent by men cooking and looking after children is not zero. However, further inspection of the ENCOVI time use data reveals that, as assumed by the model, a majority of men spend virtually no time on either of these two home production activities. Panel C of Table 3 shows this. It describes the probability of spending less than 10 minutes per week on paid work, looking after children, and cooking, respectively, by household electrification status.

As expected, far more women than men spent less than ten minutes per week on paid work activities in the week prior to the ENCOVI interview. About 84% of women did not spend more than 10 minutes earning money, but only 27% of men fell into this category. In Panel A it was found that mean employment probabilities differ amongst women but not amongst men living in electrified versus unelectrified households, and in Panel B it was found that mean time spent earning money differs by household electrification status only for women. The results of Panel C are consistent with this: the differences in the probability of spending less than ten minutes working for money in the past week by household electrification status are significant for women but not for men.

In both electrified and unelectrified households, about 78% of men spent less than ten minutes of the week prior to the ENCOVI interview looking after children. In contrast, a majority of women in both unelectrified and electrified households spent more than ten minutes engaged primarily in looking after children. More importantly, there is no difference in the probability of spending less than ten minutes per week looking after children amongst men, but women in electrified households were significantly more likely to have spent less time than this minimum preforming child care than were women in unelectrified households.

Panel C also demonstrates that cooking is essentially a female activity in Guatemala. More than 96% of men spent less than ten minutes in the week prior to the ENCOVI interview engaged primarily in cooking. In contrast, only 9% of women in unelectrified households, and 18% in electrified households spent less than this threshold amount of time cooking. Significant differences are found by household electrification status amongst women, but not amongst men.

The summary statistics on employment status and time use presented in Table 3 provide empirical support for the assumption that women's time use, but not men's, is affected by the introduction of electricity to the household. The validity of this assumption for the outcomes of interest will next be further demonstrated using multivariate analyses. It will be shown that, in the analysis of the impact of household electrification on women's employment probabilities, time spent earning money, looking after children, and cooking, men can be used as a comparison or 'control' group. In the ENCOVI data, household electrification status is not significantly associated with these outcomes for men. Men are employed outside the home regardless of whether the household has electricity, and obtaining electricity is largely exogenous to the household and its' members. As well, men spend a negligible amount of time on the main home production activities in which women engage, namely caring for children and cooking. This sexual division of labour is not altered by household electrification.

### 5.1 Causal effects of household electrification on women's work

Can the presence of electricity in a household be considered exogenous, conditional on observable socioeconomic and local characteristics? If the effect of household electrification on time use is causal, we should expect to see associations between time use patterns and household electrification status for women, but not for men, in multivariate settings. This is indeed the case in the ENCOVI data.

Time use regressions run separately by sex support the findings of Table 3 that the association between household electrification status and work decisions is important only for women. Four outcomes are considered: current employment status, time spent working for money yesterday, time spent caring for children yesterday, and time spent cooking yesterday. To examine the association between household electrification status and employment probabilities, a probit model is estimated. In the case of time spent earning money, caring for children, and cooking, tobit models which account for left-censoring are implemented, and semi-log specifications are used.

Table 4 presents the results of regressions containing a dummy for household electrification status plus controls for the following: 10-year age group dummies, the highest grade level attained by the respondent, a rural dummy, controls for the number of adults and children under 18 in the household, and regional dummies.[32]

To summarise the results of this exercise, household electrification is associated with major differences in the time use of women but not of men. In no case is the dummy representing household electrification status statistically significant at the 10% level for men, but for all four outcomes this dummy is statistically significant for women. If these significant associations for women were actually due to unobserved hetergeneity in wealth or members' education or ability, we might expect to observe "effects" of household electrification on men too. Thus, the results support the conjecture that household electrification status is essentially exogenous to households, and also unrelated to the time men spend in these activities. It also seems unlikely that the time spent by women cooking, caring for children, or earning money is causing household electrification status since women earn so little relative to men.[33]

[32]The eight regions of Guatemala are: *Metropolitana, Norte, Nororiente, Suroriente, Central, Suroccidente, Noroccidente, and Petén.*

[33]Similar specifications examining the association between household electrification status and leisure time show

The sex-specific time use regressions of Table 4 motivate the use of men as a control group in identifying the causal effects of household electrification on women's time use. If time use does not cause household electrification status, the causal effects of household electrification on women's work outcomes can be identified relative to those on men. Because the effects on men are both theoretically and empirically insignificant, this amounts to identifying the absolute causal impact of household electrification on women's outcomes.

The equations identifying the causal effect of household electrification on women's outcomes take the general form:

$$\begin{aligned} WKOUTCOME_{ihj} = \; & \beta_0 + \beta_1 * ELECHH_{hj} + \beta_2 * ELECFEM_{ihj} + \\ & \beta_3 * FEMALE_{ihj} + \gamma * CONTROLS_{ihj} + \mu_j + \epsilon_{ihj} \end{aligned}$$

where WKOUTCOME $\in$ {employed, log(time spent working for money), log(time spent caring for children), and log(time spent cooking)}. The variable ELECHH refers to whether or not a household has an electricity connection, and ELECFEM to the interaction term between household electrification status and whether or not the respondent is female. Here $i$ refers to an individual in household $h$ in region $j$. Control variables include 10 year age cohort dummies (age 50 plus is the reference), the highest grade attained in school, the number of adults in the household, the number of children (under 18) in the household, a rural dummy, and regional fixed effects. Also included as controls are gender interactions terms for each of these socioeconomic and regional variables.

The above specification provides what is essentially a difference-in-differences estimate of the average effect of household electrification on time use. Women are the treatment group, and men the control group. Pre-treatment households are those in communities where the grid has not yet arrived, while treated households are those with electricity currently. Given that household electrification status is essentially driven by community electrification status, and men's time use should not be impacted by the treatment, this specification yields causal impacts of household electrification on women's time use.

Table 5 presents results identifying the causal impact of household electrification on key activities in women's working lives. The interaction term between household electrification status and female identifies these effects in the pooled sample of women and men.

that, conditional on these same control variables, there is no statistically significant relationship between either men's leisure time and having electricity. For women, having electricity is associated with a 3% increase in leisure time, statistically significant at the 5% level. Leisure is defined to include sleep and all other home activities which are not directly productive. These specifications are not presented here but are available on request from the authors.

Household electrification causes an increase of more than 9% in the probability that a woman is employed outside the household, as evaluated at the mean value of the $X$s in these data (column (1)). Consistent with this, the electrification of households is found to cause a 188 log point, or more than 5-fold increase, on average, in time spent by women working for money in the day prior to the ENCOVI interview (column (2)).[34] This large increase is attributable to the dual facts that more than 84% of women in unelectrified households spend no time working for money, and that mean time spent by women on this activity is about 1 1/2 hours per week.

Results for the main home production activities of caring for children and cooking are mixed. While the relationship between time spent caring for children and household electrification status is negative, it is found that the causal effect of household electrification on this outcome for women is not statistically significant at the 10% level (column (3)). However, women spend, on average, about 34 percent less time cooking as a result of household electrification (column (4) of Table 5).[35]

Food preparation may take less time if household electrification causes a switch to fuels which cook food more quickly, but this is clearly not the only potential way in which this new technology might impact cooking time. It may also be that electric light facilitates aspects of food preparation other than warming food, or the combining of other activities with food preparation. Since we here examine time spent *primarily* in "cooking or preparing breakfast, lunch or dinner", an increase in combining cooking with other activities could result in a reduction in reported time spent cooking. The data do not show that the reduction in cooking time caused by household electrification is attributable to households switching to using electricity for cooking when they are electrified. Only 5% of electrified households report that they used some electricity for cooking in the month prior to the ENCOVI survey.[36]

Could improvements in men's wages caused by community electrification lead to the observed changes in women's time use? In Appendix B, we examine the association in the data between men's and women's earnings and the electrification status of households. To summarise the results of this analysis, it appears that there are increases in men's earnings associated with household electrification. These earnings increases cannot be explained by men spending more hours in paid work when their household has electricity. Given the fact that the community generally has electricity

[34]The calculation is $e^{1.88} - 1$.

[35]The calculation is $e^{-0.41} - 1$.

[36]Rather, it appears that household electrification is associated with households changing their cooking fuel to propane. Nearly 55% of electrified household use some propane for cooking, compared to only 7% of unelectrified households. The switching of cooking fuels from solid (wood, charcoal) to modern fuel sources (propane, kerosene) when households are electrified is common across countries (see, for example Heltzburg (2003), who examines household energy use in Brazil, Ghana, Guatemala, India, Nepal, South Africa and Vietnam.).

if a household does, higher male wage offers caused by community electrification are a potential cause. In contrast, it appears that there are no significant increases in women's earnings associated with household electrification, after accounting for the impact of household electrification on the participation margin.

It remains possible that women in electrified households participate more because community electrification causes increases household wealth, via higher men's wages. Our model predicts that increases in men's wages will cause their wives to participate more because the household can purchase more labour-saving electricity. In estimation we have considered a binary outcome: Households have electricity or they do not, and whether or not they do is determined almost entirely by whether or not their community has electricity. However, the time use results reported here are also robust to the inclusion of controls for household wealth, as proxied by features of the dwelling and non-electric consumer durables possessed by the household.[37]This suggests that any increases in men's wages and household wealth caused by community electrification are not likely the primary causes of the observed changes in women's time use.

In this section it has been shown, using difference-in-difference type estimators, that household electrification causes women to spend substantially less time cooking, and more time working outside the home. It should be noted that these results are also robust to the exclusion of individuals for whom reverse causality from time use to household electricity status may be an issue: individuals living in electrified households in non-electrified communities, and those who live in unelectrified households in electrified communities.[38] The results confirm the predictions of our theoretical model that household electrification should increase women's labour force participation and reduce time spent in home production.

# 6 Conclusions

This paper develop a model which explains the implications of labour-saving technological innovations such as electrification for intrahousehold resource allocation. The model shows that, in high-fertility environments in which the initial price of technology is very high, electrification reduces the number of children, increases the amount of time women spend in the labour market, and reduces time spent in home production. Data from Guatemala provides strong support for the model. It is shown that electrification causes substantial reductions in women's fertility. As well, employment probabilities, time spent earning money, and time spent cooking change significantly for women because of

[37]These results are not presented here but are available on request from the authors.

[38]These results are not presented here but are available on request from the authors.

household electrification.

This paper is, to our knowledge, the first to model the effects of improvements in home production technology for the case in which children might represent a form of saving for old age. The quarter of the world's population which does not yet have access to electricity reside in developing countries with high fertility rates and scarce access to health care or old age pensions, so allowing for the possibility of children as investment goods seems particularly relevant.

This paper suggests that the important effects of electrification on household resource allocation and labour market prospects should be taken into account when calculating the potential benefits of electricity provision to communities. The large fertility reductions and female employment improvements from electrification suggest that this infrastructure provision might have large poverty reducing impacts. Whereas the economics literature has not focused to date on the potential gender implications of infrastructural improvements, it appears that infrastructure which affects home production technology might also substantially increase the earnings potential of women. Together with fertility reductions in high-fertility environments, this suggests that certain infrastructure types may be important vectors of improvements in the status of women both within the household and society.

# References


Angrist, J. and W. Evans (1998). Children and their parents' labour supply: Evidence from exogenous variation in family size. *American Economic Review 88*(1), 450–477.

Bailey, M. (2006). More power to the pill: The impact of contraceptive freedom on women's lifecycle labor supply. *Quarterly Journal of Economics 121*(1), 289–320.

Becker, G. (1960). An economic analysis of fertility. *Demographic and Economic Change in Developed Countries. NBER: New York*.

Becker, G. (1991). *A Treatise on the Family*. Harvard University Press, Cambridge Mass.

Cornwell, G. and W. Robinson (1988). Fertility of U.S. farm women during the electrification era 1930-1950. *Journal of Population Research and Policy Review (online edition) 7*(3).

Demographic and Health Surveys (2007). ORC Macro International. *www.measureddhs.org*.

Dinkleman, T. (2008). The effects of rural electrification on employment: New evidence from South Africa. *University of Michigan (mimeo)*.

Dubin, J. and D. McFadden (1984). An econometric analysis of residential electric appliance holdings and consumption. *Econometrica 52*(2), 345–362.

Foster, V. and C. Araujo (2004). Does infrastructure reform work for the poor? A case study from Guatemala. *World Bank Policy Research Working Paper No. 3185*.

Goldin, C. and L. Katz (2002). The power of the pill: Oral contraceptives and women's career and marriage decisions. *Journal of Political Economy 110*(1), 730–770.

Greenwood, J., A. Seshadri, and M. Yorukoglu (2005). Engines of liberation. *Review of Economic Studies 72*(1), 109–133.

Greenwood, J., S. Seshadri, and G. Vandenbrouke (2005). The Baby Boom and Baby Bust. *American Economic Review 95*(1), 183–207.

Gronau, R. (1977). Leisure, home production and work: The theory of the allocation of time revisited. *Journal of Political Economy 85*(6), 1099–1123.

Harbison, S. and W. Robinson (1985). Rural electrification and fertility change. *Population Research and Policy Review 4*(2), 149–171.

Heltzburg, R. (2003). Household fuel and energy use in developing countries: A multi-country study. *World Bank*.

Herrin, A. (1979). Rural electrification and fertility change in the Southern Philippines. *Population and Development Review 5*(1), 61–86.

Heston, A., R. Summers, and B. Aten (2006). Penn world tables version 6.2. *Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania*.

International Energy Agency (2005). Electricity/heat in Guatemala in 2005. *www.iea.org (last accessed July 16th 2008)*.

Pollack, R. and M. Wachter (1975). The relevance of the household production function and its' implications for the allocation of time. *Journal of Political Economy 83*(2), 255–278.

Sokari, G. and S. Dimpka-Harry (1991). Rural electrification: A study of socio-economic and fertility change in Rivers State, Nigeria. *African Study Monographs 12*(4), 167–183.

World Bank (2007). Global data monitoring. Results measurement system. *http://ddp-ext.worldbank.org/ext/GMIS/gdmis.do?siteId=1targetId=9menuId=LNAV01LST3SUB4 (accessed Nov. 26th 2007)*.

World Bank (2008). The welfare impact of rural electrification: A reassessment of the costs and benefits. *World Bank Independent Evaluation Group Report*.

Table 1: Community electrification and fertility

| OLS regressions. Dependent variable is log(no. children born to a woman) | | | |
|---|---|---|---|
| Specification: | (1) | (2) | (3) |
| electricity all life | -0.2631** | -0.2959** | -0.4897** |
| | (0.106) | (0.106) | (0.156) |
| community elec. >0, ≤9 | -0.7784** | -0.8046** | -0.9704** |
| | (0.181) | (0.180) | (0.337) |
| community elec. >10, ≤19 | -0.6168** | -0.6292** | -0.7425** |
| | (0.151) | (0.151) | (0.262) |
| community elec. >20, ≤29 | -0.2487* | -0.2617* | -0.3297* |
| | (0.139) | (0.139) | (0.207) |
| community elec. > 30, ≤39 | -0.1472 | -0.1512 | -0.1885 |
| | (0.122) | (0.122) | (0.150) |
| aged 12-19 | -3.0637** | -3.0565** | -2.3384** |
| | (0.128) | (0.129) | (0.157) |
| aged 20-29 | -0.5606** | -0.5487** | -0.2163* |
| | (0.109) | (0.110) | (0.122) |
| aged 30-39 | 0.4634 | 0.4553 | 0.4354 |
| | (0.396) | (0.395) | (0.300) |
| community has no preschool | 0.0602 | 0.0599 | |
| | (0.063) | (0.063) | |
| community has no elementary school | -0.1209* | -0.1329* | |
| | (0.068) | (0.070) | |
| community has no secondary school | -0.1533* | -0.1350 | |
| | (0.082) | (0.083) | |
| community affected by Mitch | 0.0155 | 0.0174 | |
| | (0.057) | (0.056) | |
| community has paved highway | -0.0448 | -0.0439 | |
| | (0.061) | (0.061) | |
| community elec. status dummy | yes | yes | yes |
| community sanitation dummies [a] | yes | yes | yes |
| community piped water dummies[b] | yes | yes | yes |
| dept | yes | yes | no |
| dept X rural interactions | no | yes | no |
| rural | yes | yes | no |
| community dummies | no | no | yes |
| $R^2$ | 0.58 | 0.58 | 0.63 |
| adj-$R^2$ | 0.58 | 0.58 | 0.60 |
| no. obs. | 6817 | 6817 | 6817 |

Source: Guatemala ENCOVI data 2000. Standard errors are clustered at the community level.

[a] The full set of improved sanitation dummies is as follows: A dummy for the presence of improved sanitation in community, a dummy having had sanitation all one's life, a dummy for being aged 0-9 at the advent of sanitation, a dummy for being aged 10-19 at the advent of sanitation, a dummy for being aged 20-29 at the advent of sanitation, and a dummy for being aged 30-39 at the advent of sanitation. The reference age at the advent of sanitation is above 40.

[b] The full set of piped water dummies is as follows: A dummy for the presence of piped water in community, a dummy having had piped water all one's life, a dummy for being aged 0-9 at the advent of piped water, a dummy for being aged 10-19 at the advent of piped water, a dummy for being aged 20-29 at the advent of piped water, and a dummy for being aged 30-39 at the advent of piped water. The reference age at the advent of piped water is above 40.

Table 2: Community electrification and fertility

| **OLS regressions. Dependent variable is log (no. children born to a woman)** | | | |
|---|---|---|---|
| Specification: | (1) | (2) | (3) |
| electricity all life | -0.5082** | -0.5526** | -0.5215** |
| | (0.121) | (0.124) | (0.230) |
| electricity before age 30 | -0.3568** | -0.3665** | -0.3277** |
| | (0.077) | (0.077) | (0.120) |
| aged 12-19 | -4.2716** | -4.2661** | -3.5080** |
| | (0.085) | (0.086) | (0.132) |
| aged 20-29 | -0.8815** | -0.8749** | -0.2308* |
| | (0.098) | (0.098) | (0.131) |
| aged 30-39 | 0.3422 | 0.3385 | 0.2585 |
| | (0.269) | (0.269) | (0.276) |
| community has no preschool | 0.0107 | 0.0110 | |
| | (0.063) | (0.062) | |
| community has no elementary school | -0.1141* | -0.1319* | |
| | (0.069) | (0.069) | |
| community has no secondary school | -0.0697 | -0.0651 | |
| | (0.083) | (0.083) | |
| community affected by Mitch | 0.0122 | 0.0153 | |
| | (0.057) | (0.055) | |
| community has paved highway | -0.0663 | -0.0654 | |
| | (0.059) | (0.060) | |
| community elec. status dummy | yes | yes | yes |
| community sanitation dummies [a] | yes | yes | yes |
| community piped water dummies[b] | yes | yes | yes |
| dept | yes | yes | no |
| dept X rural interactions | no | yes | no |
| rural | yes | yes | no |
| community dummies | no | no | yes |
| $R^2$ | 0.56 | 0.56 | 0.61 |
| adj-$R^2$ | 0.55 | 0.55 | 0.58 |
| no. obs. | 6817 | 6817 | 6817 |

Source: Guatemala ENCOVI data 2000. Standard errors are clustered at the community level.

[a] The full set of improved sanitation dummies is as follows: A dummy for the presence of improved sanitation in community, a dummy having had improved sanitation all one's life, and a dummy for being under age 30 at the advent of improved sanitation. The reference age at the advent of improved sanitation is 30 and above.

[b] The full set of piped water dummies is as follows: A dummy for the presence of piped water in community, a dummy having had piped water all one's life, and a dummy for being under age 30 at the advent of piped water. The reference age at the advent of piped water is 30 and above.

Table 3: Summary statistics (means): Time use in past day by household electrification status and sex

| | Hhld Electricity? | Women | Men |
|---|---|---|---|
| **Panel A: Employment status** | | | |
| **Employed outside home (=1 if yes)** | no hhld elec. | 0.2560 | 0.9155 |
| | | (0.010) | (0.007) |
| | hhld elec. | 0.4772 | 0.9073 |
| | | (0.007) | (0.006) |
| | T-stat (equality) | -17.035 | 0.897 |
| **Panel B: Time use in day prior to interview (means)** | | | |
| **Work for money (mean minutes)** | no hhld elec. | 98.1200 | 434.7556 |
| | | (14.910) | (23.861) |
| | hhld elec. | 168.7997 | 442.7787 |
| | | (6.913) | (12.595) |
| | T-stat (equality) | -4.803 | -0.318 |
| **Looking after children (mean minutes)** | no hhld elec. | 224.5679 | 65.9315 |
| | | (8.930) | (11.966) |
| | hhld elec. | 178.6148 | 60.0449 |
| | | (4.388) | (6.252) |
| | T-stat (equality) | 5.016 | 0.468 |
| **Cooking (mean minutes)** | no hhld elec. | 151.1465 | 43.9087 |
| | | (7.337) | (11.905) |
| | hhld elec. | 117.0854 | 33.5801 |
| | | (3.249) | (6.148) |
| | T-stat (equality) | 4.844 | 0.832 |
| **Panel C: Probability of spending less than 10 minutes per day on activity** | | | |
| **Work for money** | no hhld elec. | 0.8352 | 0.2734 |
| | | (0.009) | (0.011) |
| | hhld elec. | 0.6592 | 0.2802 |
| | | (0.006) | (0.007) |
| | T-stat (equality) | 14.590 | -0.546 |
| **Looking after children** | no hhld elec. | 0.3408 | 0.7814 |
| | | (0.011) | (0.010) |
| | hhld elec. | 0.4656 | 0.7867 |
| | | (0.007) | (0.006) |
| | T-stat (equality) | -9.445 | -0.465 |
| **Cooking** | no hhld elec. | 0.0875 | 0.9715 |
| | | (0.007) | (0.004) |
| | hhld elec. | 0.1817 | 0.9641 |
| | | (0.005) | (0.003) |
| | T-stat (equality) | -9.697 | 1.460 |

Source: Guatemala ENCOVI data 2000. Individuals aged 20-55 included in sample.

Table 4: Household electrification status and time use of men and women

| Outcome: | **Employed** | | **ln(time working for money)** | | **ln(time caring for children)** | | **ln(time cooking)** | |
|---|---|---|---|---|---|---|---|---|
| Estimator: | **Probit** | | **Tobit** | | **Tobit** | | **Tobit** | |
| | women (1) | men (2) | women (3) | men (4) | women (5) | men (6) | women (7) | men (8) |
| hhld has elec. | 0.1049 ** | 0.0002 | 2.6259** | 0.0705 | -0.7987** | -0.3979 | -0.3802** | 0.7217 |
| | (0.016) | (0.010) | (0.527) | (0.220) | (0.241) | (0.566) | (0.121) | (1.205) |
| age 20-29 | -0.0611 ** | -0.0372 ** | -1.0701* | -0.9276** | 4.6489** | 3.5825** | -0.8363** | -1.3358 |
| | (0.017) | (0.011) | (0.554) | (0.249) | (0.286) | (0.671) | (0.138) | (1.227) |
| age 30-39 | 0.0094 | 0.0054 | 1.1587* | -0.1201 | 2.1734** | 3.0987** | -0.3521** | -0.0997 |
| | (0.020) | (0.012) | (0.595) | (0.266) | (0.307) | (0.696) | (0.149) | (1.309) |
| age 40-49 | 0.0195 | 0.0025 | 0.8567 | 0.3977 | -0.4608 | -1.5597** | 0.1401 | -0.8821 |
| | (0.020) | (0.012) | (0.608) | (0.273) | (0.325) | (0.756) | (0.153) | (1.381) |
| highest grade | 0.0130 ** | 0.0020 | 0.3783** | 0.0195 | 0.1809** | 0.2891** | -0.0334 | 0.0097 |
| | (0.003) | (0.002) | (0.085) | (0.040) | (0.044) | (0.102) | (0.022) | (0.197) |
| rural | -0.1370 ** | 0.0181 | -4.8368** | 0.0952 | -0.1256 | -1.0831** | 0.7730** | -2.5771** |
| | (0.014) | (0.009) | (0.434) | (0.196) | (0.218) | (0.508) | (0.108) | (0.999) |
| no. adults in hhld | 0.0052 | -0.0211 ** | 0.2713** | -0.4328** | -1.2746** | -1.9450** | -0.5564** | -0.9096** |
| | (0.005) | (0.002) | (0.130) | (0.061) | (0.069) | (0.174) | (0.033) | (0.320) |
| no. kids under 18 in hhld | -0.0157 ** | 0.9022 | -0.5723** | 0.1345** | 1.3998** | 0.9853** | 0.0976** | -2.0167** |
| | (0.003) | (0.912) | (0.099) | (0.042) | (0.049) | (0.114) | (0.024) | (0.256) |
| obs. P | 0.4213 | 0.9022 | | | | | | |
| pred. P at $\bar{X}$ | 0.4140 | 0.9115 | | | | | | |
| no. obs. | 7379 | 6455 | 7379 | 6455 | 7379 | 6455 | 7379 | 6455 |
| Prob > $\chi^2$ | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Pseudo $R^2$ | 0.0668 | 0.0700 | 0.0257 | 0.0045 | 0.0482 | 0.0206 | 0.0175 | 0.0373 |

Source: Guatemala ENCOVI data 2000. Columns (1) and (2) present probit marginal effects evaluated at $\bar{X}$. Standard errors are clustered at the household level. Columns (3) through (8) present the results of tobit estimation which accounts for left censoring (ie. respondents spending no time in an activity in the week prior to the ENCOVI interview.). Time spent working for money includes both time spent working for businesses ('empresas') and in individual economic activities ('negocios'). Time use refers to the day prior to the ENCOVI interview, and to time which was *primarily* spent engaging in each of the three time use outcomes. Individuals aged 20-55 are included in the analysis.

Table 5: The causal effect of household electrification status on the time use of women

| Outcome: | Employed | ln(time working for money) | ln(time caring for children) | ln(time cooking) |
|---|---|---|---|---|
| Estimator: | Probit | Tobit | Tobit | Tobit |
| | (1) | (2) | (3) | (4) |
| hhld has elec. X female | 0.0940 ** | 1.8760** | -0.5689 | -0.4118* |
| | (0.025) | (0.451) | (0.472) | (0.254) |
| hhld has elec. | 0.0004 | 0.0738 | -0.2967 | 0.2124 |
| | (0.022) | (0.290) | (0.374) | (0.299) |
| female | -0.5562 ** | -10.3080** | 4.8717** | 12.4172** |
| | (0.034) | (0.828) | (0.922) | (0.608) |
| age 20-29 | -0.0789 ** | -1.0487** | 2.6648** | -0.4906 |
| | (0.024) | (0.328) | (0.441) | (0.307) |
| age 30-39 | 0.0119 | -0.1357 | 2.3088** | -0.0796 |
| | (0.026) | (0.349) | (0.459) | (0.329) |
| age 40-49 | 0.0055 | 0.4447 | -1.1415** | -0.2824 |
| | (0.027) | (0.358) | (0.498) | (0.345) |
| highest grade | 0.0044 | 0.0218 | 0.2179** | 0.0103 |
| | (0.004) | (0.052) | (0.068) | (0.050) |
| rural | 0.0226 | 0.1096 | -0.8142** | -0.8132** |
| | (0.027) | (0.258) | (0.336) | (0.248) |
| no. adults in hhld | -0.0035 | -0.4879** | -1.4329** | -0.3030** |
| | (0.032) | (0.080) | (0.113) | (0.079) |
| no. adults in hhld X female | 0.0507 ** | 0.7009** | 0.0347 | -0.2850** |
| | (0.006) | (0.118) | (0.139) | (0.089) |
| no. kids under 18 in hhld | -0.0330 ** | 0.1505** | 0.7226** | -0.6874** |
| | (0.005) | (0.056) | (0.075) | (0.061) |
| no. kids under 18 in hhld X female | 0.0189 ** | -0.5852** | 0.8175** | 0.7896** |
| | (0.005) | (0.086) | (0.095) | (0.068) |
| female X age 20-29 | -0.1696 ** | 0.2467 | 2.4412** | -0.3904 |
| | (0.024) | (0.491) | (0.558) | (0.350) |
| female X age 30-39 | -0.0460 ** | 1.0249* | 0.0873 | -0.2907 |
| | (0.005) | (0.526) | (0.588) | (0.376) |
| female X age 40-49 | 0.0119 | 0.2157 | 0.6426 | 0.4303 |
| | (0.032) | (0.538) | (0.633) | (0.393) |
| female X highest grade | 0.0073 | 0.2674** | -0.0195 | -0.0454 |
| | (0.005) | (0.077) | (0.085) | (0.056) |
| female X rural | 0.0394 ** | -3.7870** | 0.6745 | 1.6242** |
| | (0.019) | (0.384) | (0.425) | (0.281) |
| obs. P | 0.6457 | | | |
| pred. P at $\bar{X}$ | 0.6964 | | | |
| no. obs. | 13834 | 13834 | 13834 | 13834 |
| Prob > $\chi^2$ | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Pseudo $R^2$ | 0.26 | 0.0589 | 0.0736 | 0.2299 |

Source: Guatemala ENCOVI data 2000. Column (1) presents probit marginal effects evaluated at $\bar{X}$. Standard errors are clustered at the household level. Columns (2) through (4) present the results of tobit estimation which accounts for left censoring (ie. respondents spending no time in an activity in the week prior to the ENCOVI interview.). Time spent working for money includes both time spent working for businesses ('empresas') and in individual economic activities ('negocios'). Time use refers to the day prior to the ENCOVI interview, and to time which was primarily spent engaging in each of the three time use outcomes. Individuals aged 20-55 are included in the analysis.

**Data Appendix A**

Table 6: Household Electrification Rates Around the World

| Country | Year | Hhld Electrification rate (%) | | |
|---|---|---|---|---|
| | | Urban | Rural | Total |
| **Latin America and the Caribbean** | | | | |
| Bolivia | 2003 | 94 | 35.7 | 72.3 |
| Brazil | 1996 | 98.8 | 72.4 | 93.6 |
| Colombia | 2005 | 99.3 | 89.2 | 96.8 |
| Dominican Republic | 2002 | 98.7 | 80.6 | 92.3 |
| Guatemala | 1998/99 | 91.3 | 54 | 70.9 |
| Haiti | 2005 | 68.9 | 11.7 | 33.9 |
| Nicaragua | 2001 | 94.5 | 40.1 | 72.6 |
| Peru | 2000 | 92.5 | 28.9 | 69.3 |
| **Sub-Saharan Africa** | | | | |
| Benin | 2001 | 49.6 | 5.5 | 21.9 |
| Burkina Faso | 2003 | 52.4 | 0.8 | 11.4 |
| Cameroon | 2004 | 77.1 | 15.5 | 47.1 |
| Central African Republic | 1994/95 | 8 | 0.3 | 3 |
| Chad | 2004 | 16.4 | 0.3 | 3.5 |
| Comoros | 1996 | 51.8 | 19.6 | 28.9 |
| Congo (Brazzaville) | 2005 | 50.8 | 14.8 | 33.8 |
| Côte d'Ivoire | 1998/99 | 85.9 | 22.5 | 48.2 |
| Eritrea | 2002 | 78.3 | 3 | 32.2 |
| Ethiopia | 2005 | 85.7 | 1.9 | 14 |
| Gabon | 2000 | 90.2 | 29.7 | 73.6 |
| Ghana | 2003 | 76.9 | 24.1 | 48.3 |
| Guinea | 2005 | 63.8 | 2.8 | 20.2 |
| Kenya | 2003 | 50.2 | 4.6 | 16 |
| Lesotho | 2004 | 26.2 | 0.8 | 6.8 |
| Madagascar | 1997 | 38.1 | 2.1 | 10.9 |
| Malawi | 2004 | 30.2 | 2.2 | 6.9 |
| Mali | 2001 | 37 | 2.2 | 10.8 |
| Mauritania | 2000/01 | 49.7 | 2.5 | 22.2 |
| Mozambique | 2003 | 25 | 1.1 | 8.1 |
| Namibia | 2000 | 73.2 | 13.2 | 36.5 |
| Niger | 2006 | 47.2 | 1.5 | 9.3 |
| Nigeria | 2003 | 84.9 | 33.8 | 52.2 |
| Rwanda | 2005 | 25.1 | 1.3 | 4.8 |
| Senegal | 2005 | 80.4 | 15.8 | 47.1 |
| South Africa | 1998 | 84.2 | 37.1 | 64.9 |
| Tanzania | 2004 | 38.9 | 1.6 | 11.4 |
| Togo | 1998 | 41.2 | 2.4 | 15.3 |
| Uganda | 2006 | 41.8 | 2.9 | 9 |
| Zimbabwe | 2005/06 | 91.4 | 8.7 | 37.2 |
| **North Africa/West Asia/Europe** | | | | |
| Armenia | 2005 | 99.9 | 99.7 | 99.8 |
| Egypt | 2005 | 99.8 | 99.1 | 99.4 |
| Jordan | 2002 | 99.7 | 98.7 | 99.5 |
| Moldova | 2005 | 99.4 | 98 | 98.6 |
| Yemen | 1997 | 89.8 | 27.1 | 42.6 |
| **Central Asia** | | | | |
| Kazakhstan | 1999 | 99.4 | 93.9 | 97 |
| Kyrgyz Republic | 1997 | 100 | 99.6 | 99.8 |
| Turkmenistan | 2000 | 99.7 | 99.6 | 99.6 |
| Uzbekistan | 1996 | 100 | 99.3 | 99.6 |
| **South and Southeast Asia** | | | | |
| Cambodia | 2005 | 66.8 | 12.6 | 20.5 |
| India | 1998/99 | 91.3 | 48.1 | 60.1 |
| Indonesia | 2002/2003 | 98.1 | 84.5 | 90.7 |
| Nepal | 2006 | 90.1 | 43.2 | 51.2 |
| Pakistan | 1990/91 | 95.2 | 44.7 | 59.6 |
| Philippines | 2003 | 92 | 59.8 | 76.6 |
| Vietnam | 2002 | 99.4 | 86.6 | 89.1 |

Source: Demographic and Health Surveys. For countries in which more than one DHS survey has been undertaken, data from the latest year are presented. These data are used in Figures 1a and 1b. Only DHS surveys for which data was collected on the full population are included in these figures. Note that GDP per capita and PPP (from the Penn World Tables), and fertility data (from the DHS) were not available for all countries in the year of the DHS survey.

Table 7: Electrification rates and characteristics of *departamentos* (counties) in Guatemala

| Departamento (county) | Capital | elevation (metres) ENCOVI 2000 | no. massacres during civil war | population density in 1981/km$^2$ | population density in 2002/km$^2$ | Hhld elec. rate DHS 1987 | | Hhld elec. rate DHS 1999 | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | urban | rural | urban | rural |
| Guatemala | Guatemala | 1493 | 3 | 616.74 | 1195.47 | 0.778 | 0.506 | 0.913 | 0.854 |
| Alta Verapáz | Cobán | 1 316.91 | 61 | 37.07 | 88.23 | 0.750 | 0.007 | 0.953 | 0.089 |
| Baja Verapáz | Salamá | 940.48 | 28 | 31.00 | 69.11 | 0.500 | 0.000 | 0.765 | 0.500 |
| Chimaltenango | Chimaltenango | 1 800.17 | 70 | 116.25 | 225.42 | 0.685 | 0.260 | 0.932 | 0.756 |
| Chiquimula | Chiquimula | 423.82 | 8 | 71.07 | 127.31 | 0.500 | 0.000 | 0.997 | 0.243 |
| El Progreso | Guastatoya | 561 | 0 | 42.24 | 72.58 | 0.500 | 0.679 | 0.801 | 0.655 |
| Escuintla | Escuintla | 346.91 | 3 | 76.34 | 122.89 | 0.792 | 0.102 | 0.981 | 0.462 |
| Huehuetenango | Huehuetenango | 1 901.64 | 38 | 58.29 | 114.40 | 0.402 | 0.037 | 0.936 | 0.351 |
| Izabal | Puerto Barrios | 0 | 3 | 21.53 | 34.78 | 0.639 | 0.270 | 0.846 | 0.192 |
| Jutiapa | Jutiapa | 905.96 | 0 | 78.00 | 120.87 | 0.648 | 0.301 | 0.841 | 0.639 |
| Petén | Flores | 115 | 13 | 3.68 | 10.23 | – | – | 0.775 | 0.232 |
| Quetzaltenango | Quetzaltenango | 2 371 | 6 | 188.08 | 320.20 | 0.628 | 0.438 | 0.924 | 0.660 |
| Sacatepquez | Antigua Guatemala | 1 533 | 0 | 260.49 | 533.37 | 0.711 | 0.140 | 0.988 | 0.509 |
| Quiché | Santa Cruz del Quiché | 2021 | 344 | 39.17 | 78.24 | 0.121 | 0.118 | 0.789 | 0.391 |
| San Marcos | San Marcos | 2397 | 15 | 124.59 | 209.69 | 0.714 | 0.111 | 1.000 | 0.514 |
| Retalhuleu | Retalhuleu | 239.39 | 1 | 81.32 | 130.07 | 0.885 | 0.163 | 1.000 | 0.693 |
| Sololá | Sololá | 2 113.50 | 16 | 145.38 | 289.97 | 0.452 | 0.109 | 0.850 | 0.834 |
| Santa Rosa | Cuilapa | 889 | 0 | 65.71 | 101.99 | 0.774 | 0.380 | 0.979 | 0.823 |
| Totonicapán | Totonicapán | 2 495.30 | 1 | 192.67 | 319.75 | 0.596 | 0.000 | 0.986 | 0.817 |
| Suchitépeqez | Mazatenango | 371.13 | 0 | 94.64 | 160.93 | 0.711 | 0.140 | 0.988 | 0.509 |
| Zacapa | Zacapa | 312 | 1 | 43.02 | 74.41 | 0.786 | 0.648 | 1.000 | 1.000 |
| Jalapa | Jalapa | 1700 | 0 | 65.97 | 117.75 | 0.667 | 0.085 | 0.923 | 0.459 |

Note: Elevations as reported in *http://www.aquiguatemala.net/*, except for the following: Santa Rosa, Guatemala City, Baja Verapáz, El Progreso, Izabal, Peten, Sacatepquez, and Quetzaltenango. These were obtained principally from *http://www.geonames.org*. Fraction HHELEC is the fraction of the estimation sample whose household has electricity. Information on civil war massacres was obtained from the Commission for Historical Clarification - "Guatemala: Memory of Silence". February 1999. Information on population density was obtained by combining 1981 and 2002 Census data with information on the geographical area of the *departamento*, as given by the Guatemala National Institute of Statistics.

Table 8: Falsification Exercise: Community electrification and fertility

| **OLS regressions. Dependent variable is log(no. children born to a woman)** | | | |
|---|---|---|---|
| Specification: | (1) | (2) | (3) |
| false: electricity all life | -1.1505 | -1.2010 | -1.1666 |
| | (1.163) | (1.163) | (1.255) |
| false: community elec. >0, ≤9 | -1.2340 | -1.2677 | -0.8461 |
| | (1.164) | (1.166) | (1.213) |
| false: community elec. >10, ≤19 | -1.0923 | -1.1238 | -0.5813 |
| | (1.153) | (1.154) | (1.182) |
| false: community elec. >20, ≤29 | -0.7426 | -0.7607 | -0.3366 |
| | (1.153) | (1.155) | (1.176) |
| false: community elec. > 30, ≤39 | -0.4207 | -0.4440 | -0.1579 |
| | (1.125) | (1.126) | (1.150) |
| aged 12-19 | -4.5266** | -4.5223** | -2.3129** |
| | (0.082) | (0.081) | (0.154) |
| aged 20-29 | -1.1225** | -1.1226** | -0.1850 |
| | (0.089) | (0.089) | (0.119) |
| aged 30-39 | 0.4032 | 0.4055 | 0.4127 |
| | (0.374) | (0.374) | (0.300) |
| no preprimary schools in community | 0.0884 | 0.0912 | |
| | (0.066) | (0.065) | |
| no public schools in area | -0.1740** | -0.1949** | |
| | (0.075) | (0.076) | |
| no secondary school, public or private | 0.0969 | 0.0800 | |
| | (0.082) | (0.083) | |
| community was affected by Mitch | 0.0614 | 0.0578 | |
| | (0.059) | (0.058) | |
| community has paved highway inspring | -0.1321** | -0.1384** | |
| | (0.063) | (0.064) | |
| community elec. status dummy | yes | yes | yes |
| community sanitation dummies [a] | yes | yes | yes |
| community piped water dummies[b] | yes | yes | yes |
| dept | yes | yes | no |
| dept X rural interactions | no | yes | no |
| rural | yes | yes | no |
| community dummies | no | no | yes |
| $R^2$ | 0.55 | 0.55 | 0.63 |
| adj-$R^2$ | 0.55 | 0.55 | 0.60 |
| no. obs. | 6817 | 6817 | 6817 |

Source: Guatemala ENCOVI data 2000. The falsification exercise is described in detail in the main text. Standard errors are clustered at the community level.

[a] The full set of improved sanitation dummies is as follows: A dummy for the presence of improved sanitation in community, a dummy having had sanitation all one's life, a dummy for being aged 0-9 at the advent of sanitation, a dummy for being aged 10-19 at the advent of sanitation, a dummy for being aged 20-29 at the advent of sanitation, and a dummy for being aged 30-39 at the advent of sanitation. The reference age at the advent of sanitation is above 40.

[b] The full set of piped water dummies is as follows: A dummy for the presence of piped water in community, a dummy having had piped water all one's life, a dummy for being aged 0-9 at the advent of piped water, a dummy for being aged 10-19 at the advent of piped water, a dummy for being aged 20-29 at the advent of piped water, and a dummy for being aged 30-39 at the advent of piped water. The reference age at the advent of piped water is above 40.

Table 9: Falsification Exercise: Community electrification and fertility

| **OLS regressions. Dependent variable is log(no. children born to a woman)** | | | |
|---|---|---|---|
| Specification: | (1) | (2) | (3) |
| false: electricity all life | -0.7729 | -0.8008 | -1.0763 |
| | (0.817) | (0.816) | (0.882) |
| false: electricity before 30 | -0.6896 | -0.6973 | -0.5305 |
| | (0.797) | (0.796) | (0.713) |
| aged 12-19 | -4.6295** | -4.6297** | -3.4752** |
| | (0.075) | (0.075) | (0.132) |
| aged 20-29 | -1.1631** | -1.1653** | -0.1914 |
| | (0.087) | (0.087) | (0.130) |
| aged 30-39 | 0.3743 | 0.3743 | 0.2934 |
| | (0.373) | (0.373) | (0.377) |
| no preprimary schools in community | 0.0877 | 0.0905 | |
| | (0.065) | (0.065) | |
| no public schools in area | -0.1675** | -0.1867** | |
| | (0.074) | (0.074) | |
| no secondary school, public or private | 0.0951 | 0.0787 | |
| | (0.080) | (0.082) | |
| community was affected by Mitch | 0.0587 | 0.0566 | |
| | (0.059) | (0.057) | |
| community has paved highway inspring | -0.1382** | -0.1446** | |
| | (0.062) | (0.062) | |
| community elec. status dummy | yes | yes | yes |
| community sanitation dummies [a] | yes | yes | yes |
| community piped water dummies[b] | yes | yes | yes |
| dept | yes | yes | no |
| dept X rural interactions | no | yes | no |
| rural | yes | yes | no |
| community dummies | no | no | yes |
| $R^2$ | 0.55 | 0.55 | 0.61 |
| adj-$R^2$ | 0.54 | 0.54 | 0.58 |
| no. obs. | 6817 | 6817 | 6817 |

Source: Guatemala ENCOVI data 2000. The falsification exercise is described in detail in the main text. Standard errors are clustered at the community level.

[a] The full set of improved sanitation dummies is as follows: A dummy for the presence of improved sanitation in community, a dummy having had improved sanitation all one's life, and a dummy for being under age 30 at the advent of improved sanitation. The reference age at the advent of improved sanitation is 30 and above.

[b] The full set of piped water dummies is as follows: A dummy for the presence of piped water in community, a dummy having had piped water all one's life, and a dummy for being under age 30 at the advent of piped water. The reference age at the advent of piped water is 30 and above.

**Appendix B: Wages**

How do we know that these time use results are not attributable to higher wages offered as a result of community electrification? It is potentially the case that the advent of the electric grid improves substantially the productivity of labour outside of the home, and generally increases labour demand. Higher wage offers could lead to changes in women's time allocation, even in the absence of efficiency gains in the household.

To show that higher female wages cannot be causing the changes in female time use observed, we run similar similar regressions to the above, but with the log of monthly earnings as the dependent variable. We are interested in both whether the household electrification variable, is statistically significant, and also whether the electrification times female interaction is so. Table 6 presents the results. In Column (1) the full sample is included. Both the coefficients on the household electrification dummy, and its' interaction with the female dummy are large and statistically significant. However, conditioning on labour force participation changes the story substantially. In column (2) results are presented for a tobit regression including only labour force participants. While the household electrification dummy is still large and statistically significant, the interaction between female and this variable is no longer so. Also the size of the electrification dummy coefficient is the statistically the same whether or not we condition on labour force participation. Together these results suggest that women's earnings do not increase because of wage increases associated with community electrification, but rather because of participation increases. In contrast, men's earnings likely increase with electrification because of higher wages. These findings are consistent with the causal effect of household electrification on women's time use running from improvements in home production to greater incentives for market work.

Table 10: Household electrification status and male and female earnings

| **Dependent var: ln(earnings in month prior to interview)** | | |
|---|---|---|
| Estimator | **Tobit** | **Tobit** |
| | (1) Full sample | (2) Labour force participants only |
| Hhld has elec. X female | 0.4958** | -0.2726 |
| | (0.216) | (0.178) |
| Hhld has elec. | 0.8861** | 0.8007** |
| | (0.145) | (0.094) |
| Female | -4.4429** | -0.1475 |
| | (0.398) | (0.310) |
| aged 20-29 | -0.0686 | 0.1267 |
| | (0.163) | (0.106) |
| aged 30-39 | 0.2845 | 0.2597** |
| | (0.174) | (0.112) |
| aged 40-49 | 0.3675** | 0.2843** |
| | (0.179) | (0.116) |
| female X aged 20-29 | -0.6877** | -0.2261 |
| | (0.237) | (0.181) |
| female X aged 30-39 | 0.1700 | -0.0731 |
| | (0.254) | (0.192) |
| female X aged 40-49 | 0.1401 | 0.1152 |
| | (0.260) | (0.196) |
| Highest grade | 0.1984** | 0.1600** |
| | (0.026) | (0.017) |
| female X edgrade | 0.0297 | -0.0587** |
| | (0.037) | (0.028) |
| rural | -0.9226** | -0.9907** |
| | (0.128) | (0.083) |
| female X rural | -1.3319** | -0.2002 |
| | (0.185) | (0.142) |
| no. adults in hhld | -0.3472** | -0.1788** |
| | (0.040) | (0.026) |
| female X no. adults in hhld | 0.4058** | 0.1700** |
| | (0.057) | (0.043) |
| no. kids in hhld | 0.0492* | -0.0052 |
| | (0.028) | (0.018) |
| female X no. kids under 18 in hhld | -0.2884** | -0.0880** |
| | (0.041) | (0.032) |
| Prob > $\chi^2$ | 0.0000 | 0.0000 |
| adj-$R^2$ | 0.0780 | 0.0309 |
| no. obs. | 13834 | 9050 |

Source: Guatemala ENCOVI data 2000. Individuals aged 20-55 included in sample.

# 7 Model Appendix

**NOT INTENDED FOR PUBLICATION**

## 7.1 Composite commodity

We start with nine choice variables and three constraints, giving a total of twelve variables; with the composite commodity the problem reduces to five choice variables and two constraints. However, for this composite commodity approach to be valid, we need that savings be non-zero throughout the time during the price change for electricity. The reason is that with zero savings it would not be valid to use a price of $\frac{1}{1+r}$ for second period consumption, since the implicit price would be $\frac{w_f h'(n)+c}{g} > \frac{1}{1+r}$. The way this can happen is if the agents actually prefer that savings become negative: the parties want borrow now, if possible, and raise a large number of children who would then be able to support them as well as pay back the loan. However, this not very realistic since we do not expect credit markets for such loans to exist. We therefore want to rule out this possibility. We can be assured that in our model it will not happen if we suppose that $h(n)$ is sufficiently convex. Alternatively, another plausible way to rule it out is to assume that $g$, the amount of support we can expect from each child, decreases with the number of children.

Nevertheless, if savings are zero, then the single composite commodity cannot be used for the reasons discussed, and we can only reduce the problem somewhat by forming two composite commodities, one for all consumption in each time period. Such composite commodities will reduce the problem to six choice variables and three constraints; nine variable in total. This case, although very implausible, we expect will give qualitatively similar results.

## 7.2 The Comparative Statics

We assume that savings $S$ are never zero and so the simplification of the single composite commodity for all consumption is valid. In each case, the signs of the comparative statics turn on the relative magnitudes of $[\frac{k'(e)}{k^2(e)}]^2 \geq 0$,

$[\frac{k'(e)}{k^2(e)}] \geq 0$, and $[\frac{k''(e)}{k^2(e)} - \frac{(k'(e))^2}{k^3(e)}] \leq 0$. We can express the comparative statics as linear combinations all three of these terms, in the form $-A \cdot [\frac{k'(e)}{k^2(e)}]^2 - B \cdot [\frac{k'(e)}{k^2(e)}] + C \cdot [\frac{k''(e)}{k^2(e)} - \frac{(k'(e))^2}{k^3(e)}]$. Notice, then, that the sign of the effect is the same as the sign of the relevant coefficient, since the determinant in the denominator is negative. We argue that in situations where the current level of electricity consumption is very low, the sign of the effect will be dominated by the sign of the coefficient $A$, since when $e \approx 0$, $k'(e)$ is very large and so $[\frac{k'(e)}{k^2(e)}]^2$ will be extremely large as compared to the other terms. For higher levels of electricity consumption, some of the gains in home production have already been realized all three of $A$, $B$, and $C$ become important. Finally, at extremely high levels of electricity consumption, when all the gains of home production have been realized (and thus the price of electricity is close to zero), all effects should also be almost zero, since $k'(e)$ and $k''(e)$ are approximately zero. In signing the coefficients we will use results such as $w_f h'(n) - \frac{g}{1+r} + c > 0$ and $w_f y \frac{k'(e)}{k^2(e)} = P$ which are obtained from the first order conditions.

Looking first at $\frac{\partial n}{\partial w_m}$, the determinant in the numerator is

$$\det\begin{bmatrix} U_{xx} & 0 & U_{xy} & 0 & 0 & -1 & 0 \\ U_{nx} & 0 & U_{ny} & 0 & 0 & \frac{g}{1+r}-c & -h'(n) \\ U_{yx} & 0 & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & 0 & -1/k(e) \\ 0 & 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)} & 0 & -P & \frac{yk'(e)}{k^2(e)} \\ 0 & 0 & 0 & 0 & 0 & w_f & -1 \\ -1 & -1 & 0 & -P & w_f & 0 & 0 \\ 0 & 0 & -1/k(e) & \frac{yk'(e)}{k^2(e)} & -1 & 0 & 0 \end{bmatrix}$$

$$= (-1)\det\begin{bmatrix} U_{xx} & U_{xy} & 0 & 0 & -1 & 0 \\ U_{nx} & U_{ny} & 0 & 0 & \frac{g}{1+r}-c & -h'(n) \\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & 0 & -1/k(e) \\ 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)} & 0 & -P & \frac{yk'(e)}{k^2(e)} \\ 0 & 0 & 0 & 0 & w_f & -1 \\ 0 & -1/k(e) & \frac{yk'(e)}{k^2(e)} & -1 & 0 & 0 \end{bmatrix}$$

$$= \det\begin{bmatrix} U_{xx} & U_{xy} & 0 & -1 & 0 \\ U_{nx} & U_{ny} & 0 & \frac{g}{1+r}-c & -h'(n) \\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & -1/k(e) \\ 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)} & -P & \frac{yk'(e)}{k^2(e)} \\ 0 & 0 & 0 & w_f & -1 \end{bmatrix}$$

$$= (\frac{\mu k'(e)}{k^2(e)})\det\begin{bmatrix} U_{xx} & U_{xy} & -1 & 0 \\ U_{nx} & U_{ny} & \frac{g}{1+r}-c & -h'(n) \\ 0 & \frac{\mu k'(e)}{k^2(e)} & -P & \frac{yk'(e)}{k^2(e)} \\ 0 & 0 & w_f & -1 \end{bmatrix}$$

$$-\Big(\frac{k''(e)}{k^2(e)}-\frac{2(k'(e))^2}{k^3(e)}\Big)\det\begin{bmatrix} U_{xx} & U_{xy} & -1 & 0\\ U_{nx} & U_{ny} & \frac{g}{1+r}-c & -h'(n)\\ U_{yx} & U_{yy} & 0 & -1/k(e)\\ 0 & 0 & w_f & -1\end{bmatrix}$$

$$= -\Big(\frac{w_f\mu k'(e)}{k^2(e)}\Big)\det\begin{bmatrix} U_{xx} & U_{xy} & 0\\ U_{nx} & U_{ny} & -h'(n)\\ 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{yk'(e)}{k^2(e)}\end{bmatrix} - \Big(\frac{\mu k'(e)}{k^2(e)}\Big)\det\begin{bmatrix} U_{xx} & U_{xy} & -1\\ U_{nx} & U_{ny} & \frac{g}{1+r}-c\\ 0 & \frac{\mu k'(e)}{k^2(e)} & -P\end{bmatrix}$$

$$+\Big(\frac{k''(e)}{k^2(e)}-\frac{2(k'(e))^2}{k^3(e)}\Big)\Big\{w_f\det\begin{bmatrix} U_{xx} & U_{xy} & 0\\ U_{nx} & U_{ny} & -h'(n)\\ U_{yx} & U_{yy} & -1/k(e)\end{bmatrix} + \det\begin{bmatrix} U_{xx} & U_{xy} & -1\\ U_{nx} & U_{ny} & \frac{g}{1+r}-c\\ U_{yx} & U_{yy} & 0\end{bmatrix}\Big\}$$

$$= -h'(n)\Big(\frac{w_f\mu k'(e)}{k^2(e)}\Big)[U_{xx}\frac{\mu k'(e)}{k^2(e)}] - \Big(\frac{w_f\mu k'(e)}{k^2(e)}\Big)\frac{yk'(e)}{k^2(e)}[U_{xx}U_{ny}-U_{nx}U_{xy}]$$

$$+\Big(\frac{\mu k'(e)}{k^2(e)}\Big)\frac{\mu k'(e)}{k^2(e)}[U_{xx}\Big(\frac{g}{1+r}-c\Big)+U_{nx}] + P\Big(\frac{\mu k'(e)}{k^2(e)}\Big)[U_{xx}U_{ny}-U_{nx}U_{xy}]$$

$$+\Big(\frac{k''(e)}{k^2(e)}-\frac{2(k'(e))^2}{k^3(e)}\Big)\Big\{w_f h'(n)[U_{xx}U_{yy}-U_{yx}U_{xy}] - \frac{w_f}{k(e)}[U_{xx}U_{ny}-U_{nx}U_{xy}]$$

$$-[U_{nx}U_{yy}-U_{xy}U_{ny}] - \Big(\frac{g}{1+r}-c\Big)[U_{xx}U_{yy}-U_{yx}U_{xy}]\Big\}$$

$$= \Big[\frac{k'(e)}{k^2(e)}^2\Big]\{-w_f h'(n)\mu^2 U_{xx} + \mu^2[U_{xx}\Big(\frac{g}{1+r}-c\Big)+U_{nx}]\}$$

$$+\Big[\frac{k''(e)}{k^2(e)}-\frac{2(k'(e))^2}{k^3(e)}\Big]\Bigg\{\Big(w_f h'(n)-\frac{g}{1+r}+c\Big)[U_{xx}U_{yy}-U_{yx}U_{xy}]$$

$$-\frac{w_f}{k(e)}[U_{xx}U_{ny}-U_{nx}U_{xy}] - [U_{nx}U_{yy}-U_{xy}U_{ny}]\Bigg\}$$

$$= \Big[\frac{k'(e)}{k^2(e)}\Big]^2\{\mu^2\Big(w_f h'(n)-\frac{g}{1+r}+c\Big)\theta(1-\theta)x^{\theta-2}n^{\delta}y^{\phi}\}$$

$$+\Big[\frac{k''(e)}{k^2(e)}-\frac{2(k'(e))^2}{k^3(e)}\Big]\Big\{\Big(w_f h'(n)-\frac{g}{1+r}+c\Big)\theta\phi(1-\theta-\phi)x^{\theta-2}n^{\delta}y^{\phi-2}$$

$$+\frac{w_f}{k(e)}\theta\delta\phi x^{\theta-2}n^{\delta-1}y^{\phi-1} + \theta\delta\phi x^{\theta-1}n^{\delta-2}y^{\phi-1}\Big\}$$

We notice that $A<0$, $B=0$, and $C>0$. Thus, for low levels of electricity consumption, we find that the $A$ term dominates, and we conclude that the effect on fertility of an increase in men's wages is negative. For intermediate levels of electricity consumption $C$ does have a role, and it is plausible that the overall outcome could be positive.

Next, looking at $\frac{\partial n}{\partial P}$, we find

$$\frac{\det\begin{bmatrix} U_{xx} & 0 & U_{xy} & 0 & 0 & -1 & 0 \\ U_{nx} & 0 & U_{ny} & 0 & 0 & \frac{g}{1+r}-c & -h'(n) \\ U_{yx} & 0 & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & 0 & -1/k(e) \\ 0 & \lambda & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)} & 0 & -P & \frac{yk'(e)}{k^2(e)} \\ 0 & 0 & 0 & 0 & 0 & w_f & -1 \\ -1 & e & 0 & -P & w_f & 0 & 0 \\ 0 & 0 & -1/k(e) & \frac{yk'(e)}{k^2(e)} & -1 & 0 & 0 \end{bmatrix}}{\det\{\text{Bordered Hessian}\}}$$

$$= -e\frac{\partial n}{\partial w_m} + \frac{\lambda\det\begin{bmatrix} U_{xx} & U_{xy} & 0 & 0 & -1 & 0 \\ U_{nx} & U_{ny} & 0 & 0 & \frac{g}{1+r}-c & -h'(n) \\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & 0 & -1/k(e) \\ 0 & 0 & 0 & 0 & w_f & -1 \\ -1 & 0 & -P & w_f & 0 & 0 \\ 0 & -1/k(e) & \frac{yk'(e)}{k^2(e)} & -1 & 0 & 0 \end{bmatrix}}{\det\{\text{Bordered Hessian}\}}$$

Looking at the numerator of the second term,

$$= -\lambda w_f \det\begin{bmatrix} U_{xx} & U_{xy} & 0 & -1 & 0 \\ U_{nx} & U_{ny} & 0 & \frac{g}{1+r}-c & -h'(n) \\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & -1/k(e) \\ 0 & 0 & 0 & w_f & -1 \\ 0 & -1/k(e) & \frac{yk'(e)}{k^2(e)} & 0 & 0 \end{bmatrix}$$

$$-\lambda\det\begin{bmatrix} U_{xx} & U_{xy} & 0 & -1 & 0 \\ U_{nx} & U_{ny} & 0 & \frac{g}{1+r}-c & -h'(n) \\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & -1/k(e) \\ 0 & 0 & 0 & w_f & -1 \\ -1 & 0 & -P & 0 & 0 \end{bmatrix}$$

$$
\begin{aligned}
&= -\lambda w_f^2 \det \begin{bmatrix} U_{xx} & U_{xy} & 0 & 0 \\ U_{nx} & U_{ny} & 0 & -h'(n) \\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & -1/k(e) \\ 0 & -1/k(e) & \frac{yk'(e)}{k^2(e)} & 0 \end{bmatrix} \\
&\quad - \lambda w_f \det \begin{bmatrix} U_{xx} & U_{xy} & 0 & -1 \\ U_{nx} & U_{ny} & 0 & \frac{g}{1+r} - c \\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 \\ 0 & -1/k(e) & \frac{yk'(e)}{k^2(e)} & 0 \end{bmatrix} \\
&\quad - \lambda w_f \det \begin{bmatrix} U_{xx} & U_{xy} & 0 & 0 \\ U_{nx} & U_{ny} & 0 & -h'(n) \\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & -1/k(e) \\ -1 & 0 & -P & 0 \end{bmatrix} \\
&\quad - \lambda \det \begin{bmatrix} U_{xx} & U_{xy} & 0 & -1 \\ U_{nx} & U_{ny} & 0 & \frac{g}{1+r} - c \\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 \\ -1 & 0 & -P & 0 \end{bmatrix} \\
&= \lambda w_f^2 h'(n) \det \begin{bmatrix} U_{xx} & U_{xy} & 0 \\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} \\ 0 & -1/k(e) & \frac{yk'(e)}{k^2(e)} \end{bmatrix} \\
&\quad - \frac{\lambda w_f^2}{k(e)} \det \begin{bmatrix} U_{xx} & U_{xy} & 0 \\ U_{nx} & U_{ny} & 0 \\ 0 & -1/k(e) & \frac{yk'(e)}{k^2(e)} \end{bmatrix} \\
&\quad - \lambda w_f \det \begin{bmatrix} U_{nx} & U_{ny} & 0 \\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} \\ 0 & -1/k(e) & \frac{yk'(e)}{k^2(e)} \end{bmatrix} \\
&\quad - \Big(\frac{g}{1+r} - c\Big)\lambda w_f \det \begin{bmatrix} U_{xx} & U_{xy} & 0 \\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} \\ 0 & -1/k(e) & \frac{yk'(e)}{k^2(e)} \end{bmatrix} \\
&\quad + \lambda w_f h'(n) \det \begin{bmatrix} U_{xx} & U_{xy} & 0 \\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} \\ -1 & 0 & -P \end{bmatrix} \\
&\quad - \frac{\lambda w_f}{k(e)} \det \begin{bmatrix} U_{xx} & U_{xy} & 0 \\ U_{nx} & U_{ny} & 0 \\ -1 & 0 & -P \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&- \lambda \det \begin{bmatrix} U_{xy} & 0 & -1 \\ U_{ny} & 0 & \frac{g}{1+r} - c \\ U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 \end{bmatrix} \\
&- P\lambda \det \begin{bmatrix} U_{xx} & U_{xy} & -1 \\ U_{nx} & U_{ny} & \frac{g}{1+r} - c \\ U_{yx} & U_{yy} & 0 \end{bmatrix} \\
= {} & \lambda w_f^2 h'(n) \frac{\mu k'(e) U_{xx}}{k^3(e)} + \lambda w_f^2 h'(n) \frac{y k'(e)}{k^2(e)} [U_{xx}U_{yy} - U_{yx}U_{xy}] \\
&\qquad - \frac{\lambda w_f^2 y k'(e)}{k^3(e)} [U_{xx}U_{ny} - U_{nx}U_{xy}] \\
&- \frac{\mu k'(e)}{k^3(e)} \lambda w_f \{U_{nx} + \Big(\frac{g}{1+r} - c\Big) U_{xx}\} \\
&\qquad - \frac{y k'(e)}{k^2(e)} \lambda w_f \{[U_{nx}U_{yy} - U_{yx}U_{ny}] \\
&\quad + \Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yy} - U_{yx}U_{xy}]\} \\
&- \lambda w_f h'(n) U_{xy} \frac{\mu k'(e)}{k^2(e)} - P\lambda w_f h'(n)[U_{xx}U_{yy} - U_{yx}U_{xy}] \\
&\quad + P \frac{\lambda w_f}{k(e)} [U_{xx}U_{ny} - U_{nx}U_{xy}] \\
&+ \lambda \frac{\mu k'(e)}{k^2(e)} [U_{xy}\Big(\frac{g}{1+r} - c\Big) + U_{ny}] \\
&\quad + P\lambda[U_{nx}U_{yy} - U_{yx}U_{ny}] + P\lambda\Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yy} - U_{yx}U_{xy}] \\
= {} & \Big[\frac{k'(e)}{k^2(e)}\Big]\Big\{\frac{\mu\lambda w_f^2 h'(n) U_{xx}}{k(e)} + \lambda w_f^2 h'(n) y [U_{xx}U_{yy} - U_{yx}U_{xy}] \\
&\quad - \frac{\lambda w_f^2 y}{k(e)} [U_{xx}U_{ny} - U_{nx}U_{xy}] \\
&\quad - \frac{\mu}{k(e)} \lambda w_f \{U_{nx} + \Big(\frac{g}{1+r} - c\Big) U_{xx}\} - y\lambda w_f\{[U_{nx}U_{yy} - U_{yx}U_{ny}] \\
&\quad + \Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yy} - U_{yx}U_{xy}]\} \\
&\quad - \lambda\mu[w_f h'(n) U_{xy} - U_{xy}\Big(\frac{g}{1+r} - c\Big) - U_{ny}]\Big\} \\
&- P\lambda w_f h'(n)[U_{xx}U_{yy} - U_{yx}U_{xy}] + P\frac{\lambda w_f}{k(e)}[U_{xx}U_{ny} - U_{nx}U_{xy}] \\
&\quad + P\lambda[U_{nx}U_{yy} - U_{yx}U_{ny}] + P\lambda\Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yy} - U_{yx}U_{xy}]
\end{aligned}
$$

$$
\begin{aligned}
&= \Big[\frac{k'(e)}{k^2(e)}\Big]\Big\{\Big(h'(n)w_f + c - \frac{g}{1+r}\Big)\Big[\frac{\mu\lambda w_f}{k(e)}\theta(1-\theta)x^{\theta-2}n^{\delta}y^{\phi} \\
&\qquad\qquad + \theta\phi(1-\theta-\phi)\lambda w_f x^{2\theta-2}n^{2\delta}y^{2\phi-2} - \lambda\mu\theta\phi x^{\theta-1}n^{\delta}y^{\phi-1}\Big] \\
&\quad - \frac{\mu\lambda w_f}{k(e)}\theta\delta x^{\theta-1}n^{\delta-1}y^{\phi} + \lambda\mu\delta\phi x^{\theta}n^{\delta-1}y^{\phi-1}\Big\} \\
&\quad - P\lambda\Big(w_f h'(n) + c - \frac{g}{1+r}\Big)\theta\phi(1-\theta-\phi)x^{2\theta-2}n^{2\delta}y^{2\phi-2}.
\end{aligned}
$$

Since $P = [\frac{k'(e)}{k^2(e)}$ we can rewrite this expression as

$$
\begin{aligned}
&= \Big[\frac{k'(e)}{k^2(e)}\Big]\Big\{\Big(h'(n)w_f + c - \frac{g}{1+r}\Big)\Big[\frac{\mu\lambda w_f}{k(e)}\theta(1-\theta)x^{\theta-2}n^{\delta}y^{\phi} \\
&\qquad\qquad + \theta\phi(1-\theta-\phi)\lambda w_f x^{2\theta-2}n^{2\delta}y^{2\phi-2} - \lambda\mu\theta\phi x^{\theta-1}n^{\delta}y^{\phi-1}\Big] \\
&\quad - \frac{\mu\lambda w_f}{k(e)}\theta\delta x^{\theta-1}n^{\delta-1}y^{\phi} + \lambda\mu\delta\phi x^{\theta}n^{\delta-1}y^{\phi-1} \\
&\quad - w_f y\lambda\Big(w_f h'(n) + c - \frac{g}{1+r}\Big)\theta\phi(1-\theta-\phi)x^{2\theta-2}n^{2\delta}y^{2\phi-2}\Big\} \\
&= \Big[\frac{k'(e)}{k^2(e)}\Big]\Big\{\Big(h'(n)w_f + c - \frac{g}{1+r}\Big)\Big[\frac{\mu\lambda w_f}{k(e)}\theta(1-\theta)x^{\theta-2}n^{\delta}y^{\phi} \\
&\qquad\qquad - \lambda\mu\theta\phi x^{\theta-1}n^{\delta}y^{\phi-1}\Big] \\
&\quad - \frac{\mu\lambda w_f}{k(e)}\theta\delta x^{\theta-1}n^{\delta-1}y^{\phi} + \lambda\mu\delta\phi x^{\theta}n^{\delta-1}y^{\phi-1}\Big\}.
\end{aligned}
$$

Putting the two terms together, and looking at the numerator only (the denominator continues to be the determinant of the bordered Hessian) we find

$$
\det\begin{bmatrix}
U_{xx} & 0 & U_{xy} & 0 & 0 & -1 & 0 \\
U_{nx} & 0 & U_{ny} & 0 & 0 & \frac{g}{1+r} - c & -h'(n) \\
U_{yx} & 0 & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & 0 & -1/k(e) \\
0 & \lambda & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)} & 0 & -P & \frac{yk'(e)}{k^2(e)} \\
0 & 0 & 0 & 0 & 0 & w_f & -1 \\
-1 & e & 0 & -P & w_f & 0 & 0 \\
0 & 0 & -1/k(e) & \frac{yk'(e)}{k^2(e)} & -1 & 0 & 0
\end{bmatrix}
$$

$$
\begin{aligned}
&= -e\Big\{\Big[\frac{k'(e)}{k^2(e)}\Big]^2\{\mu^2\Big(w_f h'(n) - \frac{g}{1+r} + c\Big)\theta(1-\theta)x^{\theta-2}n^{\delta}y^{\phi}\} \\
&\quad + \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]\Big\{\Big(w_f h'(n) - \frac{g}{1+r} + c\Big)\theta\phi(1-\theta-\phi)x^{\theta-2}n^{\delta}y^{\phi-2} \\
&\qquad + \frac{w_f}{k(e)}\theta\delta\phi x^{\theta-2}n^{\delta-1}y^{\phi-1} + \theta\delta\phi x^{\theta-1}n^{\delta-2}y^{\phi-1}\Big\}\Big\} \\
&\quad + \Big\{\Big[\frac{k'(e)}{k^2(e)}\Big]\Big\{\Big(h'(n)w_f + c - \frac{g}{1+r}\Big)\Big[\frac{\mu\lambda w_f}{k(e)}\theta(1-\theta)x^{\theta-2}n^{\delta}y^{\phi} \\
&\qquad - \lambda\mu\theta\phi x^{\theta-1}n^{\delta}y^{\phi-1}\Big] \\
&\quad - \frac{\mu\lambda w_f}{k(e)}\theta\delta x^{\theta-1}n^{\delta-1}y^{\phi} + \lambda\mu\delta\phi x^{\theta}n^{\delta-1}y^{\phi-1}\Big\}\Big\} \\
&= -\Big[\frac{k'(e)}{k^2(e)}\Big]^2 e\{\mu^2\Big(w_f h'(n) - \frac{g}{1+r} + c\Big)\theta(1-\theta)x^{\theta-2}n^{\delta}y^{\phi}\} \\
&\quad + \Big[\frac{k'(e)}{k^2(e)}\Big]\Big\{\Big(h'(n)w_f + c - \frac{g}{1+r}\Big)\Big[\frac{\mu\lambda w_f}{k(e)}\theta(1-\theta)x^{\theta-2}n^{\delta}y^{\phi} \\
&\qquad - \lambda\mu\theta\phi x^{\theta-1}n^{\delta}y^{\phi-1}\Big] \\
&\qquad - \frac{\mu\lambda w_f}{k(e)}\theta\delta x^{\theta-1}n^{\delta-1}y^{\phi} + \lambda\mu\delta\phi x^{\theta}n^{\delta-1}y^{\phi-1}\Big\} \\
&\quad - \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]e\{\Big(w_f h'(n) - \frac{g}{1+r} + c\Big)\theta\phi(1-\theta-\phi)x^{\theta-2}n^{\delta}y^{\phi-2} \\
&\qquad + \frac{w_f}{k(e)}\theta\delta\phi x^{\theta-2}n^{\delta-1}y^{\phi-1} + \theta\delta\phi x^{\theta-1}n^{\delta-2}y^{\phi-1}\}
\end{aligned}
$$

We see, then, that $A > 0$, $B$ is of ambiguous sign, and $C < 0$ , indicating that at low levels of electricity consumption the effect of a price change for electricity has a positive effect on fertility, or in other words as the price of electricity becomes lower the number of children will also be lower. As electricity consumption increases eventually fertility and price of electricity move in opposite directions. Intuitively, initially a drop in the price of electricity will result in families reducing the number of children, as they get the home production done in very little time, and then increase their second period income by savings from increased women's labour force participation. Eventually the wealth effect only remains where further reductions in the price of electricity means more children.

Next we consider the effect of women's wages on fertility, $\frac{\partial n}{\partial w_f}$ is found by evaluating

$$\frac{\det\begin{bmatrix} U_{xx} & 0 & U_{xy} & 0 & 0 & -1 & 0 \\ U_{nx} & 0 & U_{ny} & 0 & 0 & \frac{g}{1+r}-c & -h'(n) \\ U_{yx} & 0 & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & 0 & -1/k(e) \\ 0 & 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)} & 0 & -P & \frac{yk'(e)}{k^2(e)} \\ 0 & -\lambda & 0 & 0 & 0 & w_f & -1 \\ -1 & -\ell_f & 0 & -P & w_f & 0 & 0 \\ 0 & 0 & -1/k(e) & \frac{yk'(e)}{k^2(e)} & -1 & 0 & 0 \end{bmatrix}}{\det\{\text{Bordered Hessian}\}}$$

$$= -\ell_f \frac{\partial n}{\partial w_m} + \frac{\lambda \det\begin{bmatrix} U_{xx} & U_{xy} & 0 & 0 & -1 & 0 \\ U_{nx} & U_{ny} & 0 & 0 & \frac{g}{1+r}-c & -h'(n) \\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & 0 & -1/k(e) \\ 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)} & 0 & -P & \frac{yk'(e)}{k^2(e)} \\ -1 & 0 & -P & w_f & 0 & 0 \\ 0 & -1/k(e) & \frac{yk'(e)}{k^2(e)} & -1 & 0 & 0 \end{bmatrix}}{\det\{\text{Bordered Hessian}\}}.$$

Looking at the numerator of the second term we find

$$\lambda \det\begin{bmatrix} U_{xx} & U_{xy} & 0 & 0 & -1 & 0 \\ U_{nx} & U_{ny} & 0 & 0 & \frac{g}{1+r}-c & -h'(n) \\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & 0 & -1/k(e) \\ 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)} & 0 & -P & \frac{yk'(e)}{k^2(e)} \\ -1 & 0 & -P & w_f & 0 & 0 \\ 0 & -1/k(e) & \frac{yk'(e)}{k^2(e)} & -1 & 0 & 0 \end{bmatrix}$$

$$= -w_f \lambda \det\begin{bmatrix} U_{xx} & U_{xy} & 0 & -1 & 0 \\ U_{nx} & U_{ny} & 0 & \frac{g}{1+r}-c & -h'(n) \\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & -1/k(e) \\ 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)} & -P & \frac{yk'(e)}{k^2(e)} \\ 0 & -1/k(e) & \frac{yk'(e)}{k^2(e)} & 0 & 0 \end{bmatrix}$$

$$
+ -\lambda \det \begin{bmatrix} U_{xx} & U_{xy} & 0 & -1 & 0 \\ U_{nx} & U_{ny} & 0 & \frac{g}{1+r} - c & -h'(n) \\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & -1/k(e) \\ 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)} & -P & \frac{yk'(e)}{k^2(e)} \\ -1 & 0 & -P & 0 & 0 \end{bmatrix}
$$

$$
\begin{aligned}
&= \frac{-\lambda w_f}{k(e)} \det \begin{bmatrix} U_{xx} & 0 & -1 & 0 \\ U_{nx} & 0 & \frac{g}{1+r} - c & -h'(n) \\ U_{yx} & \frac{\mu k'(e)}{k^2(e)} & 0 & -1/k(e) \\ 0 & \frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)} & -P & \frac{yk'(e)}{k^2(e)} \end{bmatrix} \\
&\quad - \frac{\lambda w_f y k'(e)}{k^2(e)} \det \begin{bmatrix} U_{xx} & U_{xy} & -1 & 0 \\ U_{nx} & U_{ny} & \frac{g}{1+r} - c & -h'(n) \\ U_{yx} & U_{yy} & 0 & -1/k(e) \\ 0 & \frac{\mu k'(e)}{k^2(e)} & -P & \frac{yk'(e)}{k^2(e)} \end{bmatrix} \\
&\quad + \lambda \det \begin{bmatrix} U_{xy} & 0 & -1 & 0 \\ U_{ny} & 0 & \frac{g}{1+r} - c & -h'(n) \\ U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & -1/k(e) \\ \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)} & -P & \frac{yk'(e)}{k^2(e)} \end{bmatrix} \\
&\quad + P\lambda \det \begin{bmatrix} U_{xx} & U_{xy} & -1 & 0 \\ U_{nx} & U_{ny} & \frac{g}{1+r} - c & -h'(n) \\ U_{yx} & U_{yy} & 0 & -1/k(e) \\ 0 & \frac{\mu k'(e)}{k^2(e)} & -P & \frac{yk'(e)}{k^2(e)} \end{bmatrix} \\
&= \frac{-\lambda w_f}{k(e)} \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big] \det \begin{bmatrix} U_{xx} & -1 & 0 \\ U_{nx} & \frac{g}{1+r} - c & -h'(n) \\ U_{yx} & 0 & -1/k(e) \end{bmatrix} \\
&\quad + \frac{\lambda w_f}{k(e)} \frac{\mu k'(e)}{k^2(e)} \det \begin{bmatrix} U_{xx} & -1 & 0 \\ U_{nx} & \frac{g}{1+r} - c & -h'(n) \\ 0 & -P & \frac{yk'(e)}{k^2(e)} \end{bmatrix} \\
&\quad + \lambda w_f \frac{yk'(e)}{k^2(e)} \det \begin{bmatrix} U_{nx} & U_{ny} & -h'(n) \\ U_{yx} & U_{yy} & -1/k(e) \\ 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{yk'(e)}{k^2(e)} \end{bmatrix} \\
&\quad + \lambda w_f \frac{yk'(e)}{k^2(e)} \Big(\frac{g}{1+r} - c\Big) \det \begin{bmatrix} U_{xx} & U_{xy} & 0 \\ U_{yx} & U_{yy} & -1/k(e) \\ 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{yk'(e)}{k^2(e)} \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&\quad - \lambda w_f \frac{yk'(e)}{k^2(e)} P \det \begin{bmatrix} U_{xx} & U_{xy} & 0 \\ U_{nx} & U_{ny} & -h'(n) \\ U_{yx} & U_{yy} & -1/k(e) \end{bmatrix} \\
&+ \lambda \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big] \det \begin{bmatrix} U_{xy} & -1 & 0 \\ U_{ny} & \frac{g}{1+r} - c & -h'(n) \\ U_{yy} & 0 & -1/k(e) \end{bmatrix} \\
&\quad - \lambda \frac{\mu k'(e)}{k^2(e)} \det \begin{bmatrix} U_{xy} & -1 & 0 \\ U_{ny} & \frac{g}{1+r} - c & -h'(n) \\ \frac{\mu k'(e)}{k^2(e)} & -P & \frac{yk'(e)}{k^2(e)} \end{bmatrix} \\
&- P\lambda \det \begin{bmatrix} U_{nx} & U_{ny} & -h'(n) \\ U_{yx} & U_{yy} & -1/k(e) \\ 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{yk'(e)}{k^2(e)} \end{bmatrix} \\
&\quad - P\lambda \Big(\frac{g}{1+r} - c\Big) \det \begin{bmatrix} U_{xx} & U_{xy} & 0 \\ U_{yx} & U_{yy} & -1/k(e) \\ 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{yk'(e)}{k^2(e)} \end{bmatrix} \\
&\quad + P^2 \lambda \det \begin{bmatrix} U_{xx} & U_{xy} & 0 \\ U_{nx} & U_{ny} & -h'(n) \\ U_{yx} & U_{yy} & -1/k(e) \end{bmatrix} \\
= &\Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big] \Big\{\frac{\lambda w_f}{k(e)} h'(n) U_{yx} + \frac{\lambda w_f}{k^2(e)} [U_{xx}\Big(\frac{g}{1+r} - c\Big) + U_{nx}]\Big\} \\
&- \Big[\frac{k'(e)}{k^2(e)}\Big] \frac{\lambda \mu w_f}{k(e)} h'(n) U_{xx} P + \Big[\frac{k'(e)}{k^2(e)}\Big]^2 \frac{\lambda \mu y w_f}{k(e)} [U_{xx}\Big(\frac{g}{1+r} - c\Big) + U_{nx}] \\
&+ \Big[\frac{k'(e)}{k^2(e)}\Big]^2 \Big\{\lambda w_f y \mu [\frac{U_{nx}}{k(e)} - U_{xy} h'(n)] + \lambda w_f y^2 [U_{nx} U_{yy} - U_{yx} U_{ny}]\Big\} \\
&+ \Big[\frac{k'(e)}{k^2(e)}\Big]^2 \Big\{\lambda w_f \mu y \Big(\frac{g}{1+r} - c\Big) \frac{U_{xx}}{k(e)} + \lambda w_f y^2 \Big(\frac{g}{1+r} - c\Big) [U_{xx} U_{yy} - U_{xy} U_{yx}]\Big\} \\
&- \Big[\frac{k'(e)}{k^2(e)}\Big] \Big\{\lambda w_f h'(n) y P [U_{xx} U_{yy} - U_{xy} U_{yx}] - \frac{\lambda w_f y P}{k(e)} [U_{xx} U_{ny} - U_{xy} U_{nx}]\Big\} \\
&+ \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big] \Big\{\lambda h'(n) U_{yy} - \frac{\lambda}{k(e)} [U_{xy}\Big(\frac{g}{1+r} - c\Big) + U_{ny}]\Big\} \\
&- \Big[\frac{k'(e)}{k^2(e)}\Big]^2 \Big\{\lambda \mu^2 h'(n) + \lambda \mu y [U_{xy}\Big(\frac{g}{1+r} - c\Big) + U_{ny}]\Big\} + \Big[\frac{k'(e)}{k^2(e)}\Big] \lambda \mu P h'(n) U_{xy} \\
&- \Big[\frac{k'(e)}{k^2(e)}\Big] \Big\{P\lambda \mu [\frac{U_{nx}}{k(e)} - U_{yx} h'(n)] + P\lambda y [U_{nx} U_{yy} - U_{ny} U_{yx}]\Big\} \\
&- \Big[\frac{k'(e)}{k^2(e)}\Big] \Big\{P\lambda \mu \Big(\frac{g}{1+r} - c\Big) \frac{U_{xx}}{k(e)} + P\lambda y \Big(\frac{g}{1+r} - c\Big) [U_{xx} U_{yy} - U_{yx} U_{xy}]\Big\}
\end{aligned}
$$

$$+ P^2\lambda h'(n)[U_{xx}U_{yy} - U_{yx}U_{xy}] - \frac{P^2\lambda}{k(e)}[U_{xx}U_{ny} - U_{nx}U_{xy}]$$

Since $P = w_f y \frac{k'(e)}{k^2(e)}$, we find

$$\begin{aligned}
= \Big[\frac{k'(e)}{k^2(e)}\Big]^2\Big\{ &- \frac{\lambda\mu w_f^2 y}{k(e)}h'(n)U_{xx} + \frac{\lambda\mu y w_f}{k(e)}[U_{xx}\Big(\frac{g}{1+r} - c\Big) + U_{nx}] \\
&+ \lambda w_f y\mu[\frac{U_{nx}}{k(e)} - U_{xy}h'(n)] + \lambda w_f y^2[U_{nx}U_{yy} - U_{yx}U_{ny}] \\
&+ \lambda w_f \mu y\Big(\frac{g}{1+r} - c\Big)\frac{U_{xx}}{k(e)} + \lambda w_f y^2\Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yy} - U_{xy}U_{yx}] \\
&- \lambda w_f^2 h'(n)y^2[U_{xx}U_{yy} - U_{xy}U_{yx}] + \frac{\lambda w_f^2 y^2}{k(e)}[U_{xx}U_{ny} - U_{xy}U_{nx}] \\
&- \lambda\mu^2 h'(n) - \lambda\mu y[U_{xy}\Big(\frac{g}{1+r} - c\Big) + U_{ny}] + \lambda\mu w_f y h'(n)U_{xy} \\
&- w_f y\lambda\mu[\frac{U_{nx}}{k(e)} - U_{yx}h'(n)] - w_f\lambda y^2[U_{nx}U_{yy} - U_{ny}U_{yx}] \\
&- w_f y\lambda\mu\Big(\frac{g}{1+r} - c\Big)\frac{U_{xx}}{k(e)} - w_f\lambda y^2\Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yy} - U_{yx}U_{xy}] \\
&+ w_f^2 y^2\lambda h'(n)[U_{xx}U_{yy} - U_{yx}U_{xy}] - \frac{w_f^2 y^2\lambda}{k(e)}[U_{xx}U_{ny} - U_{nx}U_{xy}]\Big\} \\
+ \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]\Big\{ &\frac{\lambda w_f}{k(e)}h'(n)U_{yx} + \frac{\lambda w_f}{k^2(e)}[U_{xx}\Big(\frac{g}{1+r} - c\Big) + U_{nx}] \\
&+ \lambda h'(n)U_{yy} - \frac{\lambda}{k(e)}[U_{xy}\Big(\frac{g}{1+r} - c\Big) + U_{ny}]\Big\} \\
= \Big[\frac{k'(e)}{k^2(e)}\Big]^2\Big\{ &\Big(\frac{g}{1+r}\Big)\Big[\frac{\lambda\mu y w_f}{k(e)} - \lambda\mu y U_{xy}\Big] + U_{nx}\frac{\lambda\mu y w_f}{k(e)} \\
&- \lambda\mu^2 h'(n) - \lambda\mu y U_{ny}\Big\} \\
+ \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]\Big\{ &\frac{\lambda}{k^2(e)}[U_{xx}\Big(\frac{g}{1+r} - c\Big) + U_{nx}] \\
&+ \lambda h'(n)U_{yy} - \frac{\lambda}{k(e)}[U_{xy}\Big(\frac{g}{1+r} - c - w_f h'(n)\Big) + U_{ny}]\Big\}
\end{aligned}$$

$$
\begin{aligned}
&= \Big[\frac{k'(e)}{k^2(e)}\Big]^2 \Big\{\Big(\frac{g}{1+r}\Big)\Big[\frac{\lambda\mu y w_f}{k(e)} - \lambda\mu y\theta\phi x^{\theta-1}n^{\delta}y^{\phi-1}\Big] \\
&\qquad + \theta\delta x^{\theta-1}n^{\delta-1}y^{\phi}\frac{\lambda\mu y w_f}{k(e)} - \lambda\mu^2 h'(n) \\
&\qquad - \lambda\mu y\delta\phi x^{\theta}n^{\delta-1}y^{\phi-1}\Big\} \\
&\quad + \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]\Big\{\frac{\lambda}{k^2(e)}[-\theta(1-\theta)x^{\theta-2}n^{\delta}y^{\phi}\Big(\frac{g}{1+r} - c\Big) \\
&\qquad\quad + \theta\delta x^{\theta-1}n^{\delta-1}y^{\phi}] \\
&\qquad - \lambda h'(n)\phi^2 x^{\theta}n^{\delta-}y^{\phi-2} - \frac{\lambda}{k(e)}[\theta\phi x^{\theta-1}n^{\delta}y^{\phi-1}\Big(\frac{g}{1+r} - c - w_f h'(n)\Big) \\
&\qquad\quad + \delta\phi x^{\theta}n^{\delta-1}y^{\phi-1}]\Big\}.
\end{aligned}
$$

Putting the two terms together we find that the numerator for $\frac{\partial n}{\partial w_f}$ is

$$
\begin{aligned}
&- \ell_f\Bigg\{\Big[\frac{k'(e)}{k^2(e)}\Big]^2 \{\mu^2\Big(w_f h'(n) - \frac{g}{1+r} + c\Big)\theta(1-\theta)x^{\theta-2}n^{\delta}y^{\phi}\} \\
&\quad + \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]\Big\{\Big(w_f h'(n) - \frac{g}{1+r} + c\Big)\theta\phi(1-\theta-\phi)x^{\theta-2}n^{\delta}y^{\phi-2} \\
&\qquad + \frac{w_f}{k(e)}\theta\delta\phi x^{\theta-2}n^{\delta-1}y^{\phi-1} + \theta\delta\phi x^{\theta-1}n^{\delta-2}y^{\phi-1}\Big\}\Bigg\} \\
&+ \Bigg\{\Big[\frac{k'(e)}{k^2(e)}\Big]^2 \Big\{\Big(\frac{g}{1+r}\Big)\Big[\frac{\lambda\mu y w_f}{k(e)} - \lambda\mu y\theta\phi x^{\theta-1}n^{\delta}y^{\phi-1}\Big] \\
&\qquad + \theta\delta x^{\theta-1}n^{\delta-1}y^{\phi}\frac{\lambda\mu y w_f}{k(e)} - \lambda\mu^2 h'(n) \\
&\qquad - \lambda\mu y\delta\phi x^{\theta}n^{\delta-1}y^{\phi-1}\Big\} \\
&\quad + \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]\Big\{\frac{\lambda}{k^2(e)}[-\theta(1-\theta)x^{\theta-2}n^{\delta}y^{\phi}\Big(\frac{g}{1+r} - c\Big) \\
&\qquad\quad + \theta\delta x^{\theta-1}n^{\delta-1}y^{\phi}] \\
&\qquad - \lambda h'(n)\phi^2 x^{\theta}n^{\delta-}y^{\phi-2} - \frac{\lambda}{k(e)}[\theta\phi x^{\theta-1}n^{\delta}y^{\phi-1}\Big(\frac{g}{1+r} - c - w_f h'(n)\Big) \\
&\qquad\quad + \delta\phi x^{\theta}n^{\delta-1}y^{\phi-1}]\Big\}\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
&= \Big[\frac{k'(e)}{k^2(e)}\Big]^2\Big\{ - \ell_f\{\mu^2\Big(w_f h'(n) - \frac{g}{1+r} + c\Big)\theta(1-\theta)x^{\theta-2}n^{\delta}y^{\phi}\} \\
&\quad + \Big(\frac{g}{1+r}\Big)\Big[\frac{\lambda\mu y w_f}{k(e)} - \lambda\mu y\theta\phi x^{\theta-1}n^{\delta}y^{\phi-1}\Big] + \theta\delta x^{\theta-1}n^{\delta-1}y^{\phi}\frac{\lambda\mu y w_f}{k(e)} \\
&\qquad - \lambda\mu^2 h'(n) - \lambda\mu y\delta\phi x^{\theta}n^{\delta-1}y^{\phi-1}\Big\} \\
&\quad + \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]\Big\{ - \ell_f\{\Big(w_f h'(n) - \frac{g}{1+r} + c\Big)\theta\phi(1-\theta-\phi)x^{\theta-2}n^{\delta}y^{\phi-2} \\
&\qquad + \frac{w_f}{k(e)}\theta\delta\phi x^{\theta-2}n^{\delta-1}y^{\phi-1} + \theta\delta\phi x^{\theta-1}n^{\delta-2}y^{\phi-1}\} \\
&\qquad + \{\frac{\lambda}{k^2(e)}[-\theta(1-\theta)x^{\theta-2}n^{\delta}y^{\phi}\Big(\frac{g}{1+r} - c\Big) \\
&\qquad\quad + \theta\delta x^{\theta-1}n^{\delta-1}y^{\phi}] \\
&\qquad - \lambda h'(n)\phi^2 x^{\theta}n^{\delta-}y^{\phi-2} - \frac{\lambda}{k(e)}[\theta\phi x^{\theta-1}n^{\delta}y^{\phi-1}\Big(\frac{g}{1+r} - c - w_f h'(n)\Big) \\
&\qquad\quad + \delta\phi x^{\theta}n^{\delta-1}y^{\phi-1}]\}\Big\} \\
&= -\Big[\frac{k'(e)}{k^2(e)}\Big]^2\Bigg\{ - \Big[\frac{1}{k(e)}\Big]\Big\{\lambda\mu y w_f\Big(\frac{g}{1+r}\Big) + \lambda\mu y w_f\theta\delta x^{\theta-1}n^{\delta-1}y^{\phi}\Big\} \\
&\qquad + \Big\{\Big(\frac{g}{1+r}\Big)\lambda\mu y\theta\phi x^{\theta-1}n^{\delta}y^{\phi-1} \\
&\qquad\quad + \ell_f\{\mu^2\Big(w_f h'(n) - \frac{g}{1+r} + c\Big)\theta(1-\theta)x^{\theta-2}n^{\delta}y^{\phi} \\
&\qquad\quad + \lambda\mu^2 h'(n) + \lambda\mu y\delta\phi x^{\theta}n^{\delta-1}y^{\phi-1}\Big\}\Bigg\} \\
&\quad + \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]\Bigg\{\Big[\frac{1}{k^2(e)}\Big]\Big\{\lambda\theta\delta x^{\theta-1}n^{\delta-1}y^{\phi} \\
&\qquad\quad - \lambda\theta(1-\theta)x^{\theta-2}n^{\delta}y^{\phi}\Big(\frac{g}{1+r} - c\Big)\Big\} \\
&\qquad - \Big[\frac{1}{k(e)}\Big]\Big\{\ell_f w_f\theta\delta\phi x^{\theta-2}n^{\delta-1}y^{\phi-1} + \lambda\delta\phi x^{\theta}n^{\delta-1}y^{\phi-1} \\
&\qquad\quad + \lambda\theta\phi x^{\theta-1}n^{\delta}y^{\phi-1}\Big(\frac{g}{1+r} - c - w_f h'(n)\Big)\Big\} \\
&\qquad + \Big\{ - \ell_f\Big(w_f h'(n) - \frac{g}{1+r} + c\Big)\theta\phi(1-\theta-\phi)x^{\theta-2}n^{\delta}y^{\phi-2} \\
&\qquad\quad - \ell_f\theta\delta\phi x^{\theta-1}n^{\delta-2}y^{\phi-1}\} + \delta\phi x^{\theta}n^{\delta-1}y^{\phi-1}]\Big\}
\end{aligned}
$$

From the above equation, we see that $A$ and $C$ are of ambiguous sign, and $B = 0$. However, if we look at these two ambiguous coefficients more closely, we see that each has a component involving $[\frac{1}{k(e)}]$ and in the case of $C$ we have $[\frac{1}{k(e)}]^2$. Since for $e$ very small these components will be large, it is enough to be able to sign these components. We see that for $A$ the sign is negative and for $C$ it is positive. Thus, since the sign of $A$ is most important when $e$ is small, we see that at low levels of electricity an increase in women's wages will have a negative effect on fertility. As more electricity is consumed the coefficient $C$ will become relevant, and all the components of both coefficients will also have a role resulting in an ambiguous effect on fertility.

The next comparative static is $\frac{\partial \ell_f}{\partial P}$, and here the determinant in the numerator is

$$\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & 0 & 0 & -1 & 0 \\ U_{nx} & U_{nn}-\mu h''(n) & U_{ny} & 0 & 0 & \frac{g}{1+r}-c & -h'(n) \\ U_{yx} & U_{yn} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & 0 & -1/k(e) \\ 0 & 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)}-\frac{2(k'(e))^2}{k^3(e)} & \lambda & -P & \frac{yk'(e)}{k^2(e)} \\ 0 & 0 & 0 & 0 & 0 & w_f & -1 \\ -1 & \frac{g}{1+r}-c & 0 & -P & e & 0 & 0 \\ 0 & -h'(n) & -1/k(e) & \frac{yk'(e)}{k^2(e)} & 0 & 0 & 0 \end{bmatrix}$$

$$= -\lambda\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & 0 & -1 & 0 \\ U_{nx} & U_{nn}-\mu h''(n) & U_{ny} & 0 & \frac{g}{1+r}-c & -h'(n) \\ U_{yx} & U_{yn} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & -1/k(e) \\ 0 & 0 & 0 & 0 & w_f & -1 \\ -1 & \frac{g}{1+r}-c & 0 & -P & 0 & 0 \\ 0 & -h'(n) & -1/k(e) & \frac{yk'(e)}{k^2(e)} & 0 & 0 \end{bmatrix}$$

$$- e\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & 0 & -1 & 0 \\ U_{nx} & U_{nn}-\mu h''(n) & U_{ny} & 0 & \frac{g}{1+r}-c & -h'(n) \\ U_{yx} & U_{yn} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & -1/k(e) \\ 0 & 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)}-\frac{2(k'(e))^2}{k^3(e)} & -P & \frac{yk'(e)}{k^2(e)} \\ 0 & 0 & 0 & 0 & w_f & -1 \\ 0 & -h'(n) & -1/k(e) & \frac{yk'(e)}{k^2(e)} & 0 & 0 \end{bmatrix}$$

$$= w_f \lambda \det \begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & 0 & 0 \\ U_{nx} & U_{nn} - \mu h''(n) & U_{ny} & 0 & -h'(n) \\ U_{yx} & U_{yn} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & -1/k(e) \\ -1 & \frac{g}{1+r} - c & 0 & -P & 0 \\ 0 & -h'(n) & -1/k(e) & \frac{yk'(e)}{k^2(e)} & 0 \end{bmatrix}$$

$$+ \lambda \det \begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & 0 & -1 \\ U_{nx} & U_{nn} - \mu h''(n) & U_{ny} & 0 & \frac{g}{1+r} - c \\ U_{yx} & U_{yn} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 \\ -1 & \frac{g}{1+r} - c & 0 & -P & 0 \\ 0 & -h'(n) & -1/k(e) & \frac{yk'(e)}{k^2(e)} & 0 \end{bmatrix}$$

$$- w_f e \det \begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & 0 & 0 \\ U_{nx} & U_{nn} - \mu h''(n) & U_{ny} & 0 & -h'(n) \\ U_{yx} & U_{yn} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & -1/k(e) \\ 0 & 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)} & \frac{yk'(e)}{k^2(e)} \\ 0 & -h'(n) & -1/k(e) & \frac{yk'(e)}{k^2(e)} & 0 \end{bmatrix}$$

$$- e \det \begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & 0 & -1 \\ U_{nx} & U_{nn} - \mu h''(n) & U_{ny} & 0 & \frac{g}{1+r} - c \\ U_{yx} & U_{yn} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 \\ 0 & 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)} & -P \\ 0 & -h'(n) & -1/k(e) & \frac{yk'(e)}{k^2(e)} & 0 \end{bmatrix}$$

$$= h'(n) w_f \lambda \det \begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & 0 \\ U_{yx} & U_{yn} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} \\ -1 & \frac{g}{1+r} - c & 0 & -P \\ 0 & -h'(n) & -1/k(e) & \frac{yk'(e)}{k^2(e)} \end{bmatrix}$$

$$- \lambda \frac{w_f}{k(e)} \det \begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & 0 \\ U_{nx} & U_{nn} - \mu h''(n) & U_{ny} & 0 \\ -1 & \frac{g}{1+r} - c & 0 & -P \\ 0 & -h'(n) & -1/k(e) & \frac{yk'(e)}{k^2(e)} \end{bmatrix}$$

$$-\lambda\det\begin{bmatrix} U_{nx} & U_{nn}-\mu h''(n) & U_{ny} & 0 \\ U_{yx} & U_{yn} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} \\ -1 & \frac{g}{1+r}-c & 0 & -P \\ 0 & -h'(n) & -1/k(e) & \frac{yk'(e)}{k^2(e)} \end{bmatrix}$$

$$-\lambda\Big(\frac{g}{1+r}-c\Big)\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & 0 \\ U_{yx} & U_{yn} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} \\ -1 & \frac{g}{1+r}-c & 0 & -P \\ 0 & -h'(n) & -1/k(e) & \frac{yk'(e)}{k^2(e)} \end{bmatrix}$$

$$-w_f h'(n)e\det\begin{bmatrix} U_{xx} & U_{xy} & 0 & 0 \\ U_{nx} & U_{ny} & 0 & -h'(n) \\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & -1/k(e) \\ 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)}-\frac{2(k'(e))^2}{k^3(e)} & \frac{yk'(e)}{k^2(e)} \end{bmatrix}$$

$$+\frac{ew_f}{k(e)}\det\begin{bmatrix} U_{xx} & U_{xn} & 0 & 0 \\ U_{nx} & U_{nn}-\mu h''(n) & 0 & -h'(n) \\ U_{yx} & U_{yn} & \frac{\mu k'(e)}{k^2(e)} & -1/k(e) \\ 0 & 0 & \frac{k''(e)}{k^2(e)}-\frac{2(k'(e))^2}{k^3(e)} & \frac{yk'(e)}{k^2(e)} \end{bmatrix}$$

$$+w_f e\frac{yk'(e)}{k^2(e)}\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & 0 \\ U_{nx} & U_{nn}-\mu h''(n) & U_{ny} & -h'(n) \\ U_{yx} & U_{yn} & U_{yy} & -1/k(e) \\ 0 & 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{yk'(e)}{k^2(e)} \end{bmatrix}$$

$$-eh'(n)\det\begin{bmatrix} U_{xx} & U_{xy} & 0 & -1 \\ U_{nx} & U_{ny} & 0 & \frac{g}{1+r}-c \\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 \\ 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)}-\frac{2(k'(e))^2}{k^3(e)} & -P \end{bmatrix}$$

$$+\frac{e}{k(e)}\det\begin{bmatrix} U_{xx} & U_{xn} & 0 & -1 \\ U_{nx} & U_{nn}-\mu h''(n) & 0 & \frac{g}{1+r}-c \\ U_{yx} & U_{yn} & \frac{\mu k'(e)}{k^2(e)} & 0 \\ 0 & 0 & \frac{k''(e)}{k^2(e)}-\frac{2(k'(e))^2}{k^3(e)} & -P \end{bmatrix}$$

$$+e\frac{yk'(e)}{k^2(e)}\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & -1 \\ U_{nx} & U_{nn}-\mu h''(n) & U_{ny} & \frac{g}{1+r}-c \\ U_{yx} & U_{yn} & U_{yy} & 0 \\ 0 & 0 & \frac{\mu k'(e)}{k^2(e)} & -P \end{bmatrix}$$

$$=h'(n)w_f\lambda\frac{\mu k'(e)}{k^2(e)}\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} \\ -1 & \frac{g}{1+r}-c & 0 \\ 0 & -h'(n) & -1/k(e) \end{bmatrix}$$

$$+h'(n)w_f\lambda P\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} \\ U_{yx} & U_{yn} & U_{yy} \\ 0 & -h'(n) & -1/k(e) \end{bmatrix}$$

$$+h'(n)w_f\lambda\frac{yk'(e)}{k^2(e)}\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} \\ U_{yx} & U_{yn} & U_{yy} \\ -1 & \frac{g}{1+r}-c & 0 \end{bmatrix}$$

$$-\lambda P\frac{w_f}{k(e)}\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} \\ U_{nx} & U_{nn}-\mu h''(n) & U_{ny} \\ 0 & -h'(n) & -1/k(e) \end{bmatrix}$$

$$-\lambda\frac{yk'(e)}{k^2(e)}\frac{w_f}{k(e)}\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} \\ U_{nx} & U_{nn}-\mu h''(n) & U_{ny} \\ -1 & \frac{g}{1+r}-c & 0 \end{bmatrix}$$

$$-\lambda\frac{\mu k'(e)}{k^2(e)}\det\begin{bmatrix} U_{nx} & U_{nn}-\mu h''(n) & U_{ny} \\ -1 & \frac{g}{1+r}-c & 0 \\ 0 & -h'(n) & -1/k(e) \end{bmatrix}$$

$$-\lambda P\det\begin{bmatrix} U_{nx} & U_{nn}-\mu h''(n) & U_{ny}\\ U_{yx} & U_{yn} & U_{yy}\\ 0 & -h'(n) & -1/k(e)\end{bmatrix}$$

$$-\lambda\frac{yk'(e)}{k^2(e)}\det\begin{bmatrix} U_{nx} & U_{nn}-\mu h''(n) & U_{ny}\\ U_{yx} & U_{yn} & U_{yy}\\ -1 & \frac{g}{1+r}-c & 0\end{bmatrix}$$

$$-\lambda\frac{\mu k'(e)}{k^2(e)}\Big(\frac{g}{1+r}-c\Big)\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy}\\ -1 & \frac{g}{1+r}-c & 0\\ 0 & -h'(n) & -1/k(e)\end{bmatrix}$$

$$-\lambda P\Big(\frac{g}{1+r}-c\Big)\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy}\\ U_{yx} & U_{yn} & U_{yy}\\ 0 & -h'(n) & -1/k(e)\end{bmatrix}$$

$$-\lambda\frac{yk'(e)}{k^2(e)}\Big(\frac{g}{1+r}-c\Big)\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy}\\ U_{yx} & U_{yn} & U_{yy}\\ -1 & \frac{g}{1+r}-c & 0\end{bmatrix}$$

$$+w_f h'(n)e\frac{\mu k'(e)}{k^2(e)}\det\begin{bmatrix} U_{xx} & 0 & 0\\ U_{nx} & 0 & -h'(n)\\ U_{yx} & \frac{\mu k'(e)}{k^2(e)} & -1/k(e)\end{bmatrix}$$

$$+w_f h'(n)e\Big[\frac{k''(e)}{k^2(e)}-\frac{2(k'(e))^2}{k^3(e)}\Big]\det\begin{bmatrix} U_{xx} & U_{xy} & 0\\ U_{nx} & U_{ny} & -h'(n)\\ U_{yx} & U_{yy} & -1/k(e)\end{bmatrix}$$

$$-w_f h'(n)e\frac{yk'(e)}{k^2(e)}\det\begin{bmatrix} U_{xx} & U_{xy} & 0\\ U_{nx} & U_{ny} & 0\\ U_{yx} & U_{yy} & \frac{\mu k'(e)}{k^2(e)}\end{bmatrix}$$

$$+\frac{ew_f}{k(e)}\frac{\mu k'(e)}{k^2(e)}\det\begin{bmatrix} U_{xx} & U_{xn} & 0\\ U_{nx} & U_{nn}-\mu h''(n) & -h'(n)\\ 0 & 0 & \frac{yk'(e)}{k^2(e)}\end{bmatrix}$$

$$-\frac{ew_f}{k(e)}\Big[\frac{k''(e)}{k^2(e)}-\frac{2(k'(e))^2}{k^3(e)}\Big]\det\begin{bmatrix}U_{xx} & U_{xn} & 0\\ U_{nx} & U_{nn}-\mu h''(n) & -h'(n)\\ U_{yx} & U_{yn} & -1/k(e)\end{bmatrix}$$

$$-w_f e\frac{yk'(e)}{k^2(e)}\frac{\mu k'(e)}{k^2(e)}\det\begin{bmatrix}U_{xx} & U_{xn} & 0\\ U_{nx} & U_{nn}-\mu h''(n) & -h'(n)\\ U_{yx} & U_{yn} & -1/k(e)\end{bmatrix}$$

$$+w_f e\frac{yk'(e)}{k^2(e)}\frac{yk'(e)}{k^2(e)}\det\begin{bmatrix}U_{xx} & U_{xn} & U_{xy}\\ U_{nx} & U_{nn}-\mu h''(n) & U_{ny}\\ U_{yx} & U_{yn} & U_{yy}\end{bmatrix}$$

$$-eh'(n)\frac{\mu k'(e)}{k^2(e)}\det\begin{bmatrix}U_{xx} & U_{xy} & -1\\ U_{nx} & U_{ny} & \frac{g}{1+r}-c\\ 0 & \frac{\mu k'(e)}{k^2(e)} & -P\end{bmatrix}$$

$$+eh'(n)\Big[\frac{k''(e)}{k^2(e)}-\frac{2(k'(e))^2}{k^3(e)}\Big]\det\begin{bmatrix}U_{xx} & U_{xy} & -1\\ U_{nx} & U_{ny} & \frac{g}{1+r}-c\\ U_{yx} & U_{yy} & 0\end{bmatrix}$$

$$+\frac{e}{k(e)}\frac{\mu k'(e)}{k^2(e)}\det\begin{bmatrix}U_{xx} & U_{xn} & -1\\ U_{nx} & U_{nn}-\mu h''(n) & \frac{g}{1+r}-c\\ 0 & 0 & -P\end{bmatrix}$$

$$-\frac{e}{k(e)}\Big[\frac{k''(e)}{k^2(e)}-\frac{2(k'(e))^2}{k^3(e)}\Big]\det\begin{bmatrix}U_{xx} & U_{xn} & -1\\ U_{nx} & U_{nn}-\mu h''(n) & \frac{g}{1+r}-c\\ U_{yx} & U_{yn} & 0\end{bmatrix}$$

$$-e\frac{yk'(e)}{k^2(e)}\frac{\mu k'(e)}{k^2(e)}\det\begin{bmatrix}U_{xx} & U_{xn} & -1\\ U_{nx} & U_{nn}-\mu h''(n) & \frac{g}{1+r}-c\\ U_{yx} & U_{yn} & 0\end{bmatrix}$$

$$-eP\frac{yk'(e)}{k^2(e)}\det\begin{bmatrix}U_{xx} & U_{xn} & U_{xy}\\ U_{nx} & U_{nn}-\mu h''(n) & U_{ny}\\ U_{yx} & U_{yn} & U_{yy}\end{bmatrix}$$

$$
\begin{aligned}
&= [h'(n)]^2 w_f \lambda \frac{\mu k'(e)}{k^2(e)} U_{xy} - h'(n) w_f \lambda \frac{\mu k'(e)}{k^3(e)} [U_{xx}\Big(\frac{g}{1+r} - c\Big) + U_{xn}] \\
&\quad + [h'(n)]^2 w_f \lambda P [U_{xx}U_{yy} - U_{yx}U_{xy}] - \frac{h'(n) w_f \lambda P}{k(e)} [U_{xx}U_{yn} - U_{yx}U_{xn}] \\
&\quad - h'(n) w_f \lambda \frac{y k'(e)}{k^2(e)} [U_{xn}U_{yy} - U_{yn}U_{xy}] \\
&\quad - h'(n) w_f \lambda \frac{y k'(e)}{k^2(e)} \Big(\frac{g}{1+r} - c\Big) [U_{xx}U_{yy} - U_{yx}U_{xy}] \\
&\quad - \lambda P \frac{w_f}{k(e)} h'(n) [U_{xx}U_{ny} - U_{nx}U_{xy}] + \lambda P \frac{w_f}{k^2(e)} [U_{xx}\Big(U_{nn} - \mu h''(n)\Big) - U_{nx}U_{xn}] \\
&\quad + \lambda \frac{y k'(e)}{k^2(e)} \frac{w_f}{k(e)} [U_{xn}U_{yy} - \Big(U_{nn} - \mu h''(n)\Big) U_{xy}] \\
&\quad + \lambda \frac{y k'(e)}{k^2(e)} \frac{w_f}{k(e)} \Big(\frac{g}{1+r} - c\Big) [U_{xx}\Big(U_{nn} - \mu h''(n)\Big) - U_{nx}U_{xn}] \\
&\quad - \lambda h'(n) \frac{\mu k'(e)}{k^2(e)} U_{ny} + \lambda \frac{\mu k'(e)}{k^3(e)} [U_{nx}\Big(\frac{g}{1+r} - c\Big) + \Big(U_{nn} - \mu h''(n)\Big)] \\
&\quad - \lambda P h'(n) [U_{nx}U_{yy} - U_{yx}U_{ny}] + \frac{\lambda P}{k(e)} [U_{nx}U_{yn} - U_{yx}\Big(U_{nn} - \mu h''(n)\Big)] \\
&\quad + \lambda \frac{y k'(e)}{k^2(e)} [\Big(U_{nn} - \mu h''(n)\Big) U_{yy} - U_{yn}U_{ny}] \\
&\quad + \lambda \frac{y k'(e)}{k^2(e)} \Big(\frac{g}{1+r} - c\Big) [U_{nx}U_{yy} - U_{yx}U_{ny}] \\
&\quad - \lambda \frac{\mu k'(e)}{k^2(e)} \Big(\frac{g}{1+r} - c\Big) h'(n) U_{xy} \\
&\qquad\quad + \lambda \frac{\mu k'(e)}{k^3(e)} \Big(\frac{g}{1+r} - c\Big) [U_{xx}\Big(\frac{g}{1+r} - c\Big) + U_{xy}] \\
&\quad - \lambda P \Big(\frac{g}{1+r} - c\Big) h'(n) [U_{xx}U_{yy} - U_{yx}U_{xy}] \\
&\qquad\quad + \frac{\lambda P}{k(e)} \Big(\frac{g}{1+r} - c\Big) [U_{xx}U_{ny} - U_{nx}U_{xy}] \\
&\quad + \lambda \frac{y k'(e)}{k^2(e)} \Big(\frac{g}{1+r} - c\Big) [U_{xn}U_{yy} - U_{yn}U_{xy}] \\
&\qquad\quad + \lambda \frac{y k'(e)}{k^2(e)} \Big(\frac{g}{1+r} - c\Big)^2 [U_{xx}U_{yy} - U_{yx}U_{xy}] \\
&\quad + w_f [h'(n)]^2 e \frac{\mu k'(e)}{k^2(e)} \frac{\mu k'(e)}{k^2(e)} U_{xx}
\end{aligned}
$$

$$
\begin{aligned}
&+ w_f[h'(n)]^2 e\Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big][U_{xx}U_{yy} - U_{yx}U_{xy}] \\
&- \frac{w_f h'(n) e}{k(e)}\Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big][U_{xx}U_{ny} - U_{nx}U_{xy}] \\
&- w_f h'(n) e \frac{yk'(e)}{k^2(e)}\frac{\mu k'(e)}{k^2(e)}[U_{xx}U_{ny} - U_{nx}U_{xy}] \\
&+ \frac{ew_f}{k(e)}\frac{\mu k'(e)}{k^2(e)}\frac{yk'(e)}{k^2(e)}[U_{xx}\Big(U_{nn} - \mu h''(n)\Big) - U_{nx}U_{xn}] \\
&- \frac{ew_f}{k(e)}\Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big][U_{xx}U_{yn} - U_{yx}U_{xn}] \\
&+ \frac{ew_f}{k^2(e)}\Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big][U_{xx}\Big(U_{nn} - \mu h''(n)\Big) - U_{nx}U_{xn}] \\
&- w_f e h'(n)\frac{yk'(e)}{k^2(e)}\frac{\mu k'(e)}{k^2(e)}[U_{xx}U_{yn} - U_{yx}U_{xn}] \\
&+ w_f e \frac{yk'(e)}{k^3(e)}\frac{\mu k'(e)}{k^2(e)}[U_{xx}\Big(U_{nn} - h''(n)\Big) - U_{nx}U_{xn}] \\
&+ w_f e \frac{yk'(e)}{k^2(e)}\frac{yk'(e)}{k^2(e)}\Big[U_{xx}[\Big(U_{nn} - \mu h''(n)\Big)U_{yy} - U_{yn}U_{ny}] \\
&\qquad - U_{nx}[U_{xn}U_{yy} - U_{yn}U_{xy}] \\
&\qquad + U_{yx}[U_{xn}U_{ny} - \Big(U_{nn} - \mu h''(n)\Big)U_{xy}]\Big] \\
&+ eh'(n)\frac{\mu k'(e)}{k^2(e)}\frac{\mu k'(e)}{k^2(e)}[U_{xx}\Big(\frac{g}{1+r} - c\Big) + U_{nx}] \\
&+ eh'(n)\frac{\mu k'(e)}{k^2(e)}P[U_{xx}U_{ny} - U_{xy}U_{nx}] \\
&- eh'(n)\Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big][U_{nx}U_{yy} - U_{yx}U_{ny}] \\
&- eh'(n)\Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]\Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yy} - U_{xy}U_{yx}] \\
&- \frac{e}{k(e)}\frac{\mu k'(e)}{k^2(e)}P[U_{xx}\Big(U_{nn} - \mu h''(n)\Big) - U_{xn}U_{nx}] \\
&+ \frac{e}{k(e)}\Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]\Big[[U_{nx}U_{yn} - U_{yx}\Big(U_{nn} - \mu h''(n)\Big)] \\
&\qquad + \Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yn} - U_{yx}U_{xn}]\Big]
\end{aligned}
$$

$$
\begin{aligned}
&+ e\frac{yk'(e)}{k^2(e)}\frac{\mu k'(e)}{k^2(e)}[U_{nx}U_{yn} - U_{yx}\Big(U_{nn} - \mu h''(n)\Big)] \\
&+ e\frac{yk'(e)}{k^2(e)}\frac{\mu k'(e)}{k^2(e)}\Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yn} - U_{yx}U_{xn}] \\
&- eP\frac{yk'(e)}{k^2(e)}\Big[U_{xx}[\Big(U_{nn} - \mu h''(n)\Big)U_{yy} - U_{yn}U_{ny}] - U_{nx}[U_{xn}U_{yy} - U_{yn}U_{xy}] \\
&\qquad - U_{yx}[\Big(U_{nn} - \mu h''(n)\Big)U_{xy} - U_{xn}U_{ny}]\Big] \\
= &\Big[\frac{k'(e)}{k^2(e)}\Big]^2\Bigg\{w_f[h'(n)]^2 e\mu^2 U_{xx} - w_f h'(n)ey\mu[U_{xx}U_{ny} - U_{nx}U_{xy}] \\
&\qquad + \frac{ew_f\mu y}{k(e)}[U_{xx}(U_{nn} - \mu h''(n)) - U_{nx}U_{xn}] - w_f eh'(n)y\mu[U_{xx}U_{ny} - U_{nx}U_{xy}] \\
&\qquad + \frac{w_f ey\mu}{k(e)}[U_{xx}(U_{nn} - \mu h''(n)) - U_{nx}U_{xn}] + w_f ey^2 U_{xx}[(U_{nn} - \mu h''(n))U_{yy} - U_{yn}U_{ny}] \\
&\qquad - w_f ey^2 U_{nx}[U_{xn}U_{yy} - U_{yn}U_{xy}] + w_f ey^2 U_{yx}[U_{xn}U_{ny} - (U_{nn} - \mu h''(n))U_{xy}] \\
&\qquad + eh'(n)\mu^2[U_{xx}(\frac{g}{1+r}) + U_{nx}] + eh'(n)\mu y w_f[U_{xx}U_{ny} - U_{xy}U_{nx}] \\
&\qquad - \frac{e\mu y w_f}{k(e)}[U_{xx}(U_{nn} - \mu h''(n)) - U_{nx}U_{xn}] + ey\mu big(\frac{g}{1+r})[U_{xx}U_{yn} - U_{yx}U_{xn}] \\
&\qquad - ey^2 w_f U_{xx}[(U_{nn} - \mu h''(n))U_{yy} - U_{yn}U_{ny}] - ey^2 w_f U_{xn}[U_{yy}U_{ny} - U_{yn}U_{xy}] \\
&\qquad - ey^2 w_f[(U_{nn} - \mu h''(n))U_{xy} - U_{xn}U_{ny}]\Bigg\} \\
&+ \Big[\frac{k'(e)}{k^2(e)}\Big]\Bigg\{[h'(n)]^2 w_f\lambda\mu U_{xy} - \frac{h'(n)w_f\lambda\mu}{k(e)}[U_{xx}\Big(\frac{g}{1+r} - c\Big) + U_{xn}] \\
&\qquad + [h'(n)]^2 w_f^2\lambda y[U_{xx}U_{yy} - U_{yx}U_{xy}] - \frac{h'(n)w_f^2\lambda y}{k(e)}[U_{xx}U_{yn} - U_{yx}U_{xn}] \\
&\qquad - h'(n)w_f\lambda y[U_{xn}U_{yy} - U_{yn}U_{xy}] \\
&\qquad - h'(n)w_f\lambda y\Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yy} - U_{yx}U_{xy}] \\
&\qquad - \lambda\frac{w_f^2 y}{k(e)}h'(n)[U_{xx}U_{ny} - U_{nx}U_{xy}] + \lambda\frac{w_f^2 y}{k^2(e)}[U_{xx}\Big(U_{nn} - \mu h''(n)\Big) - U_{nx}U_{xn}] \\
&\qquad + \frac{\lambda y w_f}{k(e)}[U_{xn}U_{yy} - \Big(U_{nn} - \mu h''(n)\Big)U_{xy}] \\
&\qquad + \frac{\lambda w_f y}{k(e)}\Big(\frac{g}{1+r} - c\Big)[U_{xx}\Big(U_{nn} - \mu h''(n)\Big) - U_{nx}U_{xn}]
\end{aligned}
$$

$$
\begin{aligned}
&- \lambda h'(n)\mu U_{ny} + \frac{\lambda\mu}{k(e)}[U_{nx}\Big(\frac{g}{1+r} - c\Big) + \Big(U_{nn} - \mu h''(n)\Big)] \\
&- \lambda y w_f h'(n)[U_{nx}U_{yy} - U_{yx}U_{ny}] \\
&+ \frac{\lambda y w_f}{k(e)}[U_{nx}U_{yn} - U_{yx}\Big(U_{nn} - \mu h''(n)\Big)] \\
&+ \lambda y[\Big(U_{nn} - \mu h''(n)\Big)U_{yy} - U_{yn}U_{ny}] \\
&+ \lambda y\Big(\frac{g}{1+r} - c\Big)[U_{nx}U_{yy} - U_{yx}U_{ny}] \\
&- \lambda\mu\Big(\frac{g}{1+r} - c\Big)h'(n)U_{xy} \\
&+ \lambda\mu\Big(\frac{g}{1+r} - c\Big)[U_{xx}\Big(\frac{g}{1+r} - c\Big) + U_{xy}] \\
&- \lambda y w_f\Big(\frac{g}{1+r} - c\Big)h'(n)[U_{xx}U_{yy} - U_{yx}U_{xy}] \\
&+ \frac{\lambda y w_f}{k(e)}\Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{ny} - U_{nx}U_{xy}] \\
&+ \lambda y\Big(\frac{g}{1+r} - c\Big)[U_{xn}U_{yy} - U_{yn}U_{xy}] \\
&+ \lambda y\Big(\frac{g}{1+r} - c\Big)^2[U_{xx}U_{yy} - U_{yx}U_{xy}]\Big\} \\
+ \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]\Big\{&w_f[h'(n)]^2[U_{xx}U_{yy} - U_{yx}U_{xy}] - \frac{w_f h'(n)e}{k(e)}[U_{xx}U_{ny} - U_{nx}U_{xy}] \\
&- \frac{w_f e}{k(e)}[U_{xx}U_{yn} - U_{yx}U_{xn}] + \frac{w_f e}{k^2(e)}[U_{xx}(U_{nn} - \mu h''(n)) - U_{nx}U_{xn}] \\
&- eh'(n)[U_{nx}U_{yy} - U_{yx}U_{ny}] - eh'(n)\Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yy} - U_{yx}U_{xy}] \\
&+ \frac{e}{k(e)}[U_{nx}U_{yn} - U_{yx}(U_{nn} - \mu h''(n))]\Big\}
\end{aligned}
$$

Since this final comparative static is substantially more complex, we will only identify the coefficient $A$ since it indicates the effect at extremely low or zero levels of electricity consumption. We notice that, part of the coefficient includes terms with $\Big[\frac{1}{k(e)}\Big]$ and this will be the part that we will identify since for $e$ very small this part will determine the sign. We find that the coefficient for $-\left[\frac{k'(e)}{k^2(e)}\right]^2\left[\frac{1}{k(e)}\right]$ is

$$- ew_f\mu y\theta\delta[1-\theta-\delta]x^{2\theta-2}n^{2\delta-2}y^{2\phi} - w_f e y\mu^2 h''(n)\theta(1-\theta)x^{\theta-2}n^{\delta}y^{\phi}$$

which is negative except possibly for the last term containing $h''(n)$, which will have the opposite sign as $h''(n)$. Even when $h''(n)$ is negative, as long as this term is small[1] in absolute terms, the whole effect is negative. That is for low levels of electricity, women's participation in the labour market will increase with a decrease in the price of electricity. As electricity consumption increases all three terms will become important and the effect has ambiguous sign.

The last comparative static that we compute is $\frac{\partial \ell_f}{\partial w_m}$. We find the numerator to be

[1]We have argued earlier that it is reasonable to think that $h''(n)$ is negative, or at best zero. For it to be positive, we would need that having more children causes externalities of such a magnitude that they overwhelm any economies of scale, and result in requiring more time for each additional child. For example, although there may be economies of scale for making meals for the children, the time required to sort out all the various fights that they get into grows exponentially, say, and it outweighs the saving in meal preparation, so at the margin an extra child needs more extra time than the last one needed.

$$
\det \begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & 0 & 0 & -1 & 0 \\ U_{nx} & U_{nn} - \mu h''(n) & U_{ny} & 0 & 0 & \frac{g}{1+r} - c & -h'(n) \\ U_{yx} & U_{yn} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & 0 & -1/k(e) \\ 0 & 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)} & 0 & -P & \frac{yk'(e)}{k^2(e)} \\ 0 & 0 & 0 & 0 & 0 & w_f & -1 \\ -1 & \frac{g}{1+r} - c & 0 & -P & -1 & 0 & 0 \\ 0 & -h'(n) & -1/k(e) & \frac{yk'(e)}{k^2(e)} & 0 & 0 & 0 \end{bmatrix}
$$

$$
= \det \begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & 0 & -1 & 0 \\ U_{nx} & U_{nn} - \mu h''(n) & U_{ny} & 0 & \frac{g}{1+r} - c & -h'(n) \\ U_{yx} & U_{yn} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 & -1/k(e) \\ 0 & 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)} & -P & \frac{yk'(e)}{k^2(e)} \\ 0 & 0 & 0 & 0 & w_f & -1 \\ 0 & -h'(n) & -1/k(e) & \frac{yk'(e)}{k^2(e)} & 0 & 0 \end{bmatrix}
$$

$$
= w_f \det \begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & 0 & 0 \\ U_{nx} & U_{nn} - \mu h''(n) & U_{ny} & 0 & -h'(n) \\ U_{yx} & U_{yn} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & -1/k(e) \\ 0 & 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)} & \frac{yk'(e)}{k^2(e)} \\ 0 & -h'(n) & -1/k(e) & \frac{yk'(e)}{k^2(e)} & 0 \end{bmatrix}
$$

$$
+\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & 0 & -1 \\ U_{nx} & U_{nn}-\mu h''(n) & U_{ny} & 0 & \frac{g}{1+r}-c \\ U_{yx} & U_{yn} & U_{yy} & \frac{\mu k'(e)}{k^2(e)} & 0 \\ 0 & 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{k''(e)}{k^2(e)}-\frac{2(k'(e))^2}{k^3(e)} & -P \\ 0 & -h'(n) & -1/k(e) & \frac{yk'(e)}{k^2(e)} & 0 \end{bmatrix}
$$

$$
= -w_f\frac{\mu k'(e)}{k^2(e)}\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & 0 \\ U_{nx} & U_{nn}-\mu h''(n) & U_{ny} & -h'(n) \\ 0 & 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{yk'(e)}{k^2(e)} \\ 0 & -h'(n) & -1/k(e) & 0 \end{bmatrix}
$$

$$
+ w_f\Big[\frac{k''(e)}{k^2(e)}-\frac{2(k'(e))^2}{k^3(e)}\Big]\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & 0 \\ U_{nx} & U_{nn}-\mu h''(n) & U_{ny} & -h'(n) \\ U_{yx} & U_{yn} & U_{yy} & -1/k(e) \\ 0 & -h'(n) & -1/k(e) & 0 \end{bmatrix}
$$

$$
- w_f\frac{yk'(e)}{k^2(e)}\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & 0 \\ U_{nx} & U_{nn}-\mu h''(n) & U_{ny} & -h'(n) \\ U_{yx} & U_{yn} & U_{yy} & -1/k(e) \\ 0 & 0 & \frac{\mu k'(e)}{k^2(e)} & \frac{yk'(e)}{k^2(e)} \end{bmatrix}
$$

$$
- \frac{\mu k'(e)}{k^2(e)}\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & -1 \\ U_{nx} & U_{nn}-\mu h''(n) & U_{ny} & \frac{g}{1+r}-c \\ 0 & 0 & \frac{\mu k'(e)}{k^2(e)} & -P \\ 0 & -h'(n) & -1/k(e) & 0 \end{bmatrix}
$$

$$
+ \Big[\frac{k''(e)}{k^2(e)}-\frac{2(k'(e))^2}{k^3(e)}\Big]\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & -1 \\ U_{nx} & U_{nn}-\mu h''(n) & U_{ny} & \frac{g}{1+r}-c \\ U_{yx} & U_{yn} & U_{yy} & 0 \\ 0 & -h'(n) & -1/k(e) & 0 \end{bmatrix}
$$

$$
- \frac{yk'(e)}{k^2(e)}\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} & -1 \\ U_{nx} & U_{nn}-\mu h''(n) & U_{ny} & \frac{g}{1+r}-c \\ U_{yx} & U_{yn} & U_{yy} & 0 \\ 0 & 0 & \frac{\mu k'(e)}{k^2(e)} & -P \end{bmatrix}
$$

$$
= w_f\frac{\mu k'(e)}{k^2(e)}h'(n)\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} \\ 0 & 0 & \frac{\mu k'(e)}{k^2(e)} \\ 0 & -h'(n) & -1/k(e) \end{bmatrix}
$$

$$
+ w_f\frac{\mu k'(e)}{k^2(e)}\frac{yk'(e)}{k^2(e)}\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} \\ U_{nx} & U_{nn}-\mu h''(n) & U_{ny} \\ 0 & -h'(n) & -1/k(e) \end{bmatrix}
$$

$$- w_f\Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]h'(n)\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} \\ U_{yx} & U_{yn} & U_{yy} \\ 0 & -h'(n) & -1/k(e) \end{bmatrix}$$

$$+ w_f\Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big](1/k(e))\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} \\ U_{nx} & U_{nn} - \mu h''(n) & U_{ny} \\ 0 & -h'(n) & -1/k(e) \end{bmatrix}$$

$$+ w_f\frac{yk'(e)}{k^2(e)}\frac{\mu k'(e)}{k^2(e)}\det\begin{bmatrix} U_{xx} & U_{xn} & 0 \\ U_{nx} & U_{nn} - \mu h''(n) & -h'(n) \\ U_{yx} & U_{yn} & -1/k(e) \end{bmatrix}$$

$$- w_f\frac{yk'(e)}{k^2(e)}\frac{yk'(e)}{k^2(e)}\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} \\ U_{nx} & U_{nn} - \mu h''(n) & U_{ny} \\ U_{yx} & U_{yn} & U_{yy} \end{bmatrix}$$

$$+ \frac{\mu k'(e)}{k^2(e)}h'(n)\det\begin{bmatrix} U_{xx} & U_{xy} & -1 \\ U_{nx} & U_{ny} & \frac{g}{1+r} - c \\ 0 & \frac{\mu k'(e)}{k^2(e)} & -P \end{bmatrix}$$

$$- \frac{\mu k'(e)}{k^2(e)}(1/k(e))\det\begin{bmatrix} U_{xx} & U_{xn} & -1 \\ U_{nx} & U_{nn} - \mu h''(n) & \frac{g}{1+r} - c \\ 0 & 0 & -P \end{bmatrix}$$

$$- \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]h'(n)\det\begin{bmatrix} U_{xx} & U_{xy} & -1 \\ U_{nx} & U_{ny} & \frac{g}{1+r} - c \\ U_{yx} & U_{yy} & 0 \end{bmatrix}$$

$$+ \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big](1/k(e))\det\begin{bmatrix} U_{xx} & U_{xn} & -1 \\ U_{nx} & U_{nn} - \mu h''(n) & \frac{g}{1+r} - c \\ U_{yx} & U_{yn} & 0 \end{bmatrix}$$

$$+ \frac{yk'(e)}{k^2(e)}\frac{\mu k'(e)}{k^2(e)}\det\begin{bmatrix} U_{xx} & U_{xn} & -1 \\ U_{nx} & U_{nn} - \mu h''(n) & \frac{g}{1+r} - c \\ U_{yx} & U_{yn} & 0 \end{bmatrix}$$

$$+ \frac{yk'(e)}{k^2(e)}P\det\begin{bmatrix} U_{xx} & U_{xn} & U_{xy} \\ U_{nx} & U_{nn} - \mu h''(n) & U_{ny} \\ U_{yx} & U_{yn} & U_{yy} \end{bmatrix}$$

$$= w_f\frac{\mu k'(e)}{k^2(e)}h'(n)\frac{\mu k'(e)}{k^2(e)}h'(n)U_{xx} + w_f\frac{\mu k'(e)}{k^2(e)}\frac{yk'(e)}{k^2(e)}h'(n)[U_{xx}U_{ny} - U_{nx}U_{xy}]$$

$$- w_f\frac{\mu k'(e)}{k^2(e)}\frac{yk'(e)}{k^2(e)}(1/k(e))[U_{xx}(U_{nn} - \mu h''(n)) - U_{nx}U_{xn}]$$

$$- w_f\Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big](h'(n))^2[U_{xx}U_{yy} - U_{yx}U_{xy}]$$

$$+ w_f\Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]\frac{h'(n)}{k(e)}[U_{xx}U_{yn} - U_{yx}U_{xn}]$$

$$
\begin{aligned}
&+ w_f\Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big](1/k(e))h'(n)[U_{xx}U_{ny} - U_{nx}U_{xy}] \\
&\quad - w_f\Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]\frac{1}{k^2(e)}[U_{xx}(U_{nn} - \mu h''(n)) - U_{nx}U_{xn}] \\
&+ w_f\frac{yk'(e)}{k^2(e)}\frac{\mu k'(e)}{k^2(e)}h'(n)[U_{xx}U_{yn} - U_{yx}U_{xn}] \\
&\quad - w_f\frac{yk'(e)}{k^2(e)}\frac{\mu k'(e)}{k^2(e)}(1/k(e))[U_{xx}(U_{nn} - \mu h''(n)) - U_{nx}U_{xn}] \\
&- w_f\frac{yk'(e)}{k^2(e)}\frac{yk'(e)}{k^2(e)}U_{xx}[(U_{nn} - \mu h''(n))U_{yy} - U_{yn}U_{ny}] \\
&\quad + w_f\frac{yk'(e)}{k^2(e)}\frac{yk'(e)}{k^2(e)}U_{nx}[U_{xn}U_{yy} - U_{yn}U_{xy}] \\
&\quad - w_f\frac{yk'(e)}{k^2(e)}\frac{yk'(e)}{k^2(e)}U_{yx}[U_{xn}U_{ny} - (U_{nn} - \mu h''(n))U_{xy}] \\
- \frac{\mu k'(e)}{k^2(e)}h'(n)\frac{\mu k'(e)}{k^2(e)}&[U_{xx}\Big(\frac{g}{1+r} - c\Big) + U_{nx}] \\
&\quad - \frac{\mu k'(e)}{k^2(e)}h'(n)P[U_{xx}U_{ny} - U_{nx}U_{xy}] \\
&+ \frac{\mu k'(e)}{k^2(e)}(P/k(e))[U_{xx}(U_{nn} - \mu h''(n)) - U_{nx}U_{xn}] \\
&+ \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]h'(n)[U_{nx}U_{yy} - U_{yx}U_{ny}] \\
&\quad + \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]h'(n)\Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yy} - U_{yx}U_{xy}] \\
&- \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big](1/k(e))[U_{nx}U_{yn} - U_{yx}(U_{nn} - \mu h''(n))] \\
&\quad - \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big](1/k(e))\Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yn} - U_{yx}U_{xn}] \\
&- \frac{yk'(e)}{k^2(e)}\frac{\mu k'(e)}{k^2(e)}[U_{nx}U_{yn} - U_{yx}(U_{nn} - \mu h''(n)\big)] \\
&\quad - \frac{yk'(e)}{k^2(e)}\frac{\mu k'(e)}{k^2(e)}\Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yn} - U_{yx}U_{xn}] \\
&+ \frac{yk'(e)}{k^2(e)}PU_{xx}[(U_{nn} - \mu h''(n))U_{yy} - U_{yn}U_{ny}] \\
&\quad - \frac{yk'(e)}{k^2(e)}PU_{nx}[U_{xn}U_{yy} - U_{yn}U_{xy}]
\end{aligned}
$$

$$
\begin{aligned}
&\quad + \frac{yk'(e)}{k^2(e)} P U_{yx}[U_{xn}U_{ny} - (U_{nn} - \mu h''(n))U_{xy}] \\
&= \Big[\frac{k'(e)}{k^2(e)}\Big]^2 \ \Big[\mu^2 w_f (h'(n))^2 U_{xx} + w_f \mu y h'(n)[U_{xx}U_{ny} - U_{nx}U_{xy}] \\
&\qquad - \frac{w_f \mu y}{k(e)}[U_{xx}(U_{nn} - \mu h''(n)) - U_{nx}U_{xn}] + w_f \mu y h'(n)[U_{xx}U_{yn} - U_{yx}U_{xn}] \\
&\qquad - \frac{w_f \mu y}{k(e)}[U_{xx}(U_{nn} - \mu h''(n)) - U_{nx}U_{xn}] \\
&\qquad - w_f y^2 U_{xx}[(U_{nn} - \mu h''(n))U_{yy} - U_{yn}U_{ny}] + w_f y^2 U_{nx}[U_{xn}U_{yy} - U_{yn}U_{xy}] \\
&\qquad - w_f y^2 U_{yx}[U_{xn}U_{ny} - (U_{nn} - \mu h''(n))U_{xy}] - \mu^2 h'(n)[U_{xx}(\frac{g}{1+r} - c) + U_{nx}] \\
&\qquad - \mu y[U_{nx}U_{yn} - U_{yx}(U_{nn} - \mu h''(n))] - \mu y\Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yn} - U_{yx}U_{xn}]\Big] \\
&\quad + \Big[\frac{\mu k'(e)}{k^2(e)}\Big]\Big[ - \mu h'(n) P[U_{xx}U_{ny} - U_{nx}U_{xy}] \\
&\qquad + \frac{\mu P}{k(e)}[U_{xx}(U_{nn} - \mu h''(n)) - U_{nx}U_{xn}] \\
&\qquad + yPU_{xx}[(U_{nn} - \mu h''(n))U_{yy} - U_{yn}U_{ny}] - yPU_{nx}[U_{xn}U_{yy} - U_{yn}U_{xy}] \\
&\qquad + yPU_{yx}[U_{xn}U_{ny} - (U_{nn} - \mu h''(n))U_{xy}]\Big] \\
&\quad + \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]\Big[ - w_f (h'(n))^2 [U_{xx}U_{yy} - U_{yx}U_{xy}] + \frac{w_f h'(n)}{k(e)}[U_{xx}U_{yn} - U_{yx}U_{xn}] \\
&\qquad + \frac{w_f}{k(e)} h'(n)[U_{xx}U_{ny} - U_{nx}U_{xy}] - \frac{w_f}{k^2(e)}[U_{xx}(U_{nn} - \mu h''(n)) - U_{nx}U_{xn}] \\
&\qquad + h'(n)[U_{nx}U_{yy} - U_{yx}U_{ny}] + h'(n)\Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yy} - U_{yx}U_{xy}] \\
&\qquad - \frac{1}{k(e)}[U_{nx}U_{yn} - U_{yx}(U_{nn} - \mu h''(n))] - \frac{1}{k(e)}\Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yn} - U_{yx}U_{xn}]\Big] \\
&= \Big[\frac{k'(e)}{k^2(e)}\Big]^2 \ \Big[\mu^2 w_f (h'(n))^2 U_{xx} + w_f \mu y h'(n)[U_{xx}U_{ny} - U_{nx}U_{xy}] \\
&\qquad - \frac{w_f \mu y}{k(e)}[U_{xx}(U_{nn} - \mu h''(n)) - U_{nx}U_{xn}] + w_f \mu y h'(n)[U_{xx}U_{yn} - U_{yx}U_{xn}] \\
&\qquad - \frac{w_f \mu y}{k(e)}[U_{xx}(U_{nn} - \mu h''(n)) - U_{nx}U_{xn}] \\
&\qquad - w_f y^2 U_{xx}[(U_{nn} - \mu h''(n))U_{yy} - U_{yn}U_{ny}] + w_f y^2 U_{nx}[U_{xn}U_{yy} - U_{yn}U_{xy}] \\
&\qquad - w_f y^2 U_{yx}[U_{xn}U_{ny} - (U_{nn} - \mu h''(n))U_{xy}] - \mu^2 h'(n)[U_{xx}(\frac{g}{1+r} - c) + U_{nx}]
\end{aligned}
$$

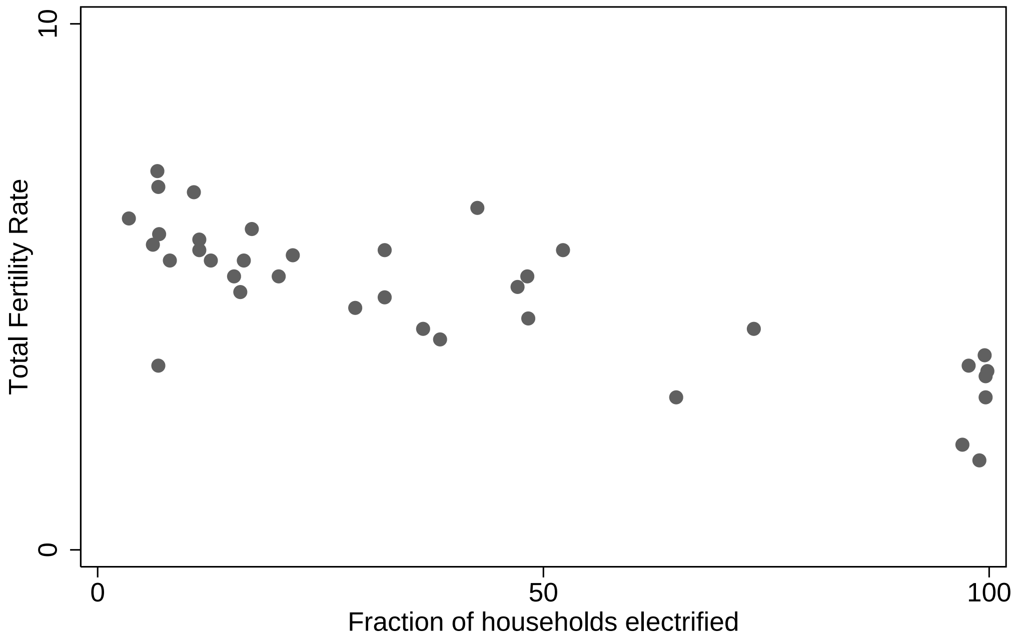


$$
\qquad - \mu y[U_{nx}U_{yn} - U_{yx}(U_{nn} - \mu h''(n))] - \mu y\Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yn} - U_{yx}U_{xn}]
$$

$$
\begin{aligned}
&- \mu h'(n) w_f y [U_{xx}U_{ny} - U_{nx}U_{xy}] \\
&+ \frac{\mu w_f y}{k(e)}[U_{xx}(U_{nn} - \mu h''(n)) - U_{nx}U_{xn}] \\
&+ y^2 w_f U_{xx}[(U_{nn} - \mu h''(n))U_{yy} - U_{yn}U_{ny}] - y^2 w_f U_{nx}[U_{xn}U_{yy} - U_{yn}U_{xy}] \\
&+ y^2 w_f U_{yx}[U_{xn}U_{ny} - (U_{nn} - \mu h''(n))U_{xy}]\Big] \\
+ \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]&\Big[ - w_f (h'(n))^2 [U_{xx}U_{yy} - U_{yx}U_{xy}] + \frac{w_f h'(n)}{k(e)}[U_{xx}U_{yn} - U_{yx}U_{xn}] \\
&+ \frac{w_f}{k(e)} h'(n)[U_{xx}U_{ny} - U_{nx}U_{xy}] - \frac{w_f}{k^2(e)}[U_{xx}(U_{nn} - \mu h''(n)) - U_{nx}U_{xn}] \\
&+ h'(n)[U_{nx}U_{yy} - U_{yx}U_{ny}] + h'(n)\Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yy} - U_{yx}U_{xy}] \\
&- \frac{1}{k(e)}[U_{nx}U_{yn} - U_{yx}(U_{nn} - \mu h''(n))] - \frac{1}{k(e)}\Big(\frac{g}{1+r} - c\Big)[U_{xx}U_{yn} - U_{yx}U_{xn}]\Big] \\
= -\Big[\frac{k'(e)}{k^2(e)}\Big]^2 &\Big\{\Big[\frac{1}{k(e)}\Big]\Big[\theta(1-\theta)\delta(1-\delta)x^{2\theta-2}n^{2\delta-2}y^{2\phi} + \mu h''(n)\theta(1-\theta)x^{\theta-2}n^{\delta}y^{\phi}\Big] \\
&+ [\mu y \theta\delta\phi x^{2\theta-2}n^{2\delta-1}y^{2\phi-1} - \mu^2 h'(n)\theta(1-\theta)x^{\theta-2}n^{\delta}y^{\phi}](\frac{g}{1+r} - c - w_f h'(n)) \\
&\quad + \mu^2\theta\delta x^{\theta-1}n^{\delta-1}y^{\phi}h'(n) + \mu y\theta\delta\phi x^{2\theta-1}n^{2\delta-2}y^{2\phi-1} - \theta\phi)x^{\theta-1}n^{\delta}y^{\phi-1}\mu h''(n)\Big\} \\
+ \Big[\frac{k''(e)}{k^2(e)} - \frac{2(k'(e))^2}{k^3(e)}\Big]&\Big\{ - \Big[\frac{1}{k^2(e)}\Big] w_f [1 - \theta - \delta] x^{2\theta-2}n^{2\delta-2}y^{2\phi} \\
&- \Big[\frac{1}{k(e)}\Big]\Big[\theta\delta\phi x^{2\theta-1}n^{2\delta-2}y^{2\phi-1} + \theta\delta x^{\theta-1}n^{\delta-1}y^{\phi}\mu h''(n) \\
&\quad + \Big(\frac{g}{1+r} - c - w_f h'(n)\Big)\theta\delta\phi x^{2\theta-1}n^{2\delta-2}y^{2\phi-1}\Big] \\
&+ h'(n)\Big(\frac{g}{1+r} - c - w_f h'(n)\Big)[1 - \theta - \phi]x^{2\theta-2}n^{2\delta}y^{2\phi-2} \\
&\quad - h'(n)\theta\delta\phi x^{2\theta-1}n^{2\delta-1}y^{2\phi-2} \\
&\quad + h'(n)\Big(\frac{g}{1+r} - c - w_f h'(n)\Big)[1 - \theta - \phi]x^{2\theta-2}n^{2\delta}y^{2\phi-2}\Big\}
\end{aligned}
$$

The coefficient, $A$, is then positive as long as $h''(n)$ is positive, or negative and small, which should be the case when electricity consumption is low. This means that we find that this effect (of men's wages on women's labour force participation) is positive. The effect seems to be very similar to the first comparative static we found: the effect of men's wages on the number of children. The higher the wages the more electricity is bought, which frees up the women so much, to work in the labour force, that it is better to work more and have fewer children. The coefficient $B = 0$ and $C$ is of ambiguous sign. For greater levels of electricity consumption, the coefficient $C$ will also have an effect, which could possibly result in this comparative static being negative for some range.

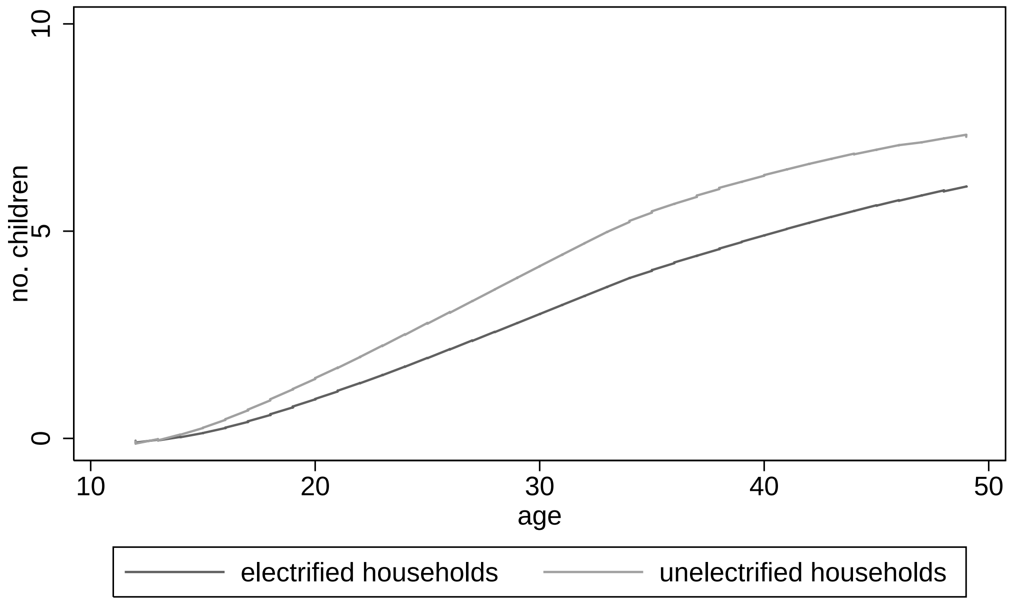
Figure 1: Household electrification rates and fertility




Source: DHS surveys and Penn World Tables version 6.2

Figure 2: Household electrification and children born by age

Women aged 12−49




Source: 2000 ENCOVI Survey. Locally−weighted least squares regressions