Human Capital Accumulation, Environmental Quality, Taxation and Endogenous Growth *

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Abstract

We consider a Rebelo (1991) type model of endogenous growth in which the environmental quality positively affects the rate of human capital accumulation and the environmental quality itself is positively affected by human capital accumulation and is negatively affected by physical capital accumulation. We analyse the effects of taxation on the steady state equilibrium growth rate in this model. We also analyse the transitional dynamic properties of this model.

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1 Introduction

There exists a substantial theoretical literature focusing on the interaction between the economic growth and the environmental degradation[1]. In these models, the degradation of environmental quality either lowers the utility of the consumer or lowers the productivity of the factors. Most of these models are built in an one sector Ramsey-Solow framework. Environmental degradation is viewed as the social byproduct of the use of modernized machineries in the production sector because the operation of these modenized machines requires the use of pollution enhancing raw materials like oil, coal etc. Some authors like Mohtadi (1996), Bretschger and Smulders (2007), Perez and Ruiz (2007), Hettich (1998) etc. assume a direct relation between the level of environmental pollution and the stock of physical capital when entire physical capital stock is fully utilized[2]. Other authors like Grimaud and Tournemaine (2007), Hettich (1998), Grimaud (1999) etc. assume the level of environmental pollution to be a function of the level of output of the aggregate production sector.

There exists another set of theoretical literature focusing on the role of human capital accumulation on economic growth[3]. The literature starts with the Lucas (1988) model; and this model has been extended and reanalysed by various authors in different directions. The rate of labour augmenting technical progress, i.e., the rate of human capital accumulation is endogenous to the analysis; and the productivity parameter of the human capital accumulation technology is an important determinant of the rate of growth. Rebelo (1991) extends Lucas (1988) model including the contribution of physical capital in human capital accumulation. Some of the works focusing on the interaction between economic growth and environmental pollution are based on the Lucas (1988) framework. In Hettich (1998), environmental pollution negatively affects the welfare of the household; and, in Rosendahl (1996), environmental quality produces a positive effect on the productivity of capital. Ricci (2007) makes a survey of the literature. However, in none of these existing works, except of Gradus and Smulders (1993), environmental quality affects the learning ability of the individuals.

When human capital accumulation is the engine of economic growth, the learning ability of the individual becomes an important determinant of the rate

[1]See, for example, Mohtadi (1996), Dinda (2005), Gradus and Smulders (1993), Hettich (1998), Rosendahl (1996), Perez and Ruiz (2007), Endress, Roumasset and Zhou (2005), Grimaud(1999), Ricci (2007), Grimaud and Tournemaine (2007) etc.

[2]If capital accumulation means replacement of old machines by more eco-friendly machines, then environmental pollution should vary negatively with capital accumulation.

[3]See for example, Lucas (1988), Rebelo (1991), Bengad (2003), Caballe and Santos (1993), Ortigueira (1998), Faig (1995), Mino(1996), Greiner and Semmler (2002), Alonso-Carrera and Freire- Seren (2004), Chamley (1993) etc.

of human capital accumulation. Environmental pollution produces negative effects on the health of the individual; and this lowers the ability to learn. Noise pollution disturbs the academic environment. Margulis (1991) finds significant empirical correlation between lead in air and blood lead levels. Next, he shows that children with higher blood lead levels have a lower cognitive development and requires supplemental education. Kauppi (2006) shows that methyl mercury, whose exposure to human comes from fish consumption, may lower the learning ability of the children. Air pollution also causes problems related to eye sight and functioning of the brain. Gradus and Smulders (1993) consider this negative effect of environmental pollution in an otherwise identical Lucas (1988) model. However, they do not analyse the effects of various fiscal policies and the transitional dynamic properties of that model.

Human capital accumulation also has a positive effect on the upgradation of the environmental quality. Education makes the people aware of the environmental problems and of the importance of protecting environment; and the educated people can protect the environment in a scientific way. This positive effect of human capital accumulation on the environmental quality is ignored not only by Gradus and Smulders(1993) but also in the other theoretical models like of Mohtadi (1996), Dinda (2005), Hettich (1998), Rosendahl (1996), Ricci (2007)[4]etc. However, there are empirical supports in favour of this positive relationship. Torras and Boyce (1998) regress environmental pollution on income, on literacy rate, Gini coefficient of income inequality etc; and find that the literacy rate has a significant negative effect on pollution particularly in low income countries. Petrosillo and Zurlini et.al. (2007) find that the attitudes of the tourists, who visit Marine protected area, are highly dependent on their education level. Clarke and Maantay (2006) find that the participation rate of the people in the recycling program counducted in New York city and its neighbourhood is highly dependent on the education level of the participators.

In this paper, we consider a modified version of Rebelo (1991) model with two special features. (i)Environmental quality positively affects the marginal return to education; and (ii) Environmental quality varies positively with the stock of human capital and negatively with the stock of physical capital whose full utilization is ensured by the perfect flexibility of factor prices. We analyse the effect of taxation on the steady state equilibrium rate of growth of the economy. The interesting results obtained in this paper are as follows. Firstly, the steady state equilibrium rate of growth, in this model, varies positively with the proportional tax rate imposed on on capital income when tax revenue is spent as lumpsum payment. This result holds in both modified Rebelo

[4]Some authors e.g. Grimaud (1999), Goulder and Mathai (2000), Hart (2004) study the issue of environment in R& D driven growth model where innovations help to improve the environment.

(1990) and modified Lucas (1988) model. In Lucas (1988), this rate of growth is independent of the tax rate imposed on labour income or subsidy given on education. However, the growth rate in modified Rebelo (1990) model varies negatively with the tax rate imposed on labour income and may vary in either direction with change in subsidy given on education. Secondly, we have found that there exists a set of optimal policy parameters that can lead decentralized economy solution to command economy solution.

The rest of the paper is organized as follows. Section 2 presents the basic model and contains the analysis of the effect of factor taxation on the steady state equilibrium rate of growth when tax revenue is distributed as lumpsum payment. Section 3 presents the analysis related to the centrally planned economy and derives the optimal policy. Concluding remarks are made in Section 4.

2 The model

The model presented in this paper is an extension of Rebelo (1991) model. The government imposes a proportional tax on output and the tax revenue is distributed among the individuals as lumpsum payment. The dynamic optimization problem of the representative individual is to maximize

$$\int_0^\infty U(C)e^{-\rho t}dt$$

subject to the production function given by

$$Y = A(aH)^\gamma(\phi K)^{1-\gamma} \tag{1}$$

with $A > 0$ and $0 < \gamma < 1$; the dynamic budget constraint given by

$$\dot{K} = (1-\tau_K)r\phi K + (1-\tau_l)waH + sw(1-a)H - C + P \tag{2}$$

with $0 \leq \tau \leq 1$; the human capital accumulation function given by

$$\dot{H} = m(E)^\delta((1-a)H)^\psi((1-\phi)K)^{1-\psi} \tag{3}$$

with $\delta > 0$; and the environmental stock accumulation function given by

$$E = E_0 K^{-\beta} H^\beta \tag{4}$$

with E_0, $\beta > 0$; the utility function is given by

$$U(C) = \frac{C^{1-\sigma}}{(1-\sigma)}.$$

Here A is the technology parameter; K is the stock of physical capital; H is the stock of human capital and τ_K and τ_l are the proportional tax rates imposed on physical capital and labour, s is the rate of subsidy given to the representative individual to compensate foregone earnings for attending school. E is the environmental quality; P is the lumpsum income transfer resulting from the distribution of tax revenue; and C is the level of consumption of the representative household. Y is the Level of output and a is the fraction of labour time allocated to production. m is the productivity parameter in the human capital accumulation function; u(.) is the utility function; ρ is the rate of discount and γ is the elasticity of output with respect to human capital. Equations (3) and (4) make the present model different from Rebelo (1991). Equation (4) with $\beta > 0$ implies that environmental quality varies positively with the stock of human capital and negatively with the stock of physical capital. Equation (3) with $\delta > 0$ implies that the positive external effect of environmental quality is present in the human capital accumulation function. If $\delta = 0$, or $\beta = 0$, then we come back to the original Lucas (1988) model. The representative individual solves this optimization problem with respect to the control variables C, a and ϕ. K and H are two state variables. However the individual can not internalize the externality.

The current value Hamiltonian is given by

$$Z = \frac{C^{1-\sigma}}{(1-\sigma)} + \lambda_K[(1-\tau_K)r\phi K + (1-\tau_l)waH + sw(1-a)H - C + P]$$
$$+\lambda_H[m(E)^{\delta}((1-a)H)^{\psi}((1-\phi)K)^{1-\psi}].$$

Here λ_K and λ_H are the co state variables of K and H.

The first order optimality conditions are given by the following.

$$\frac{\partial Z}{\partial C} = C^{-\sigma} - \lambda_K = 0; \tag{5}$$

$$\frac{\partial Z}{\partial a} = \lambda_K(1-\tau_l-s)wH - \lambda_H m\psi(1-a)^{\psi-1}H^{\psi}((1-\phi)K)^{1-\psi}(E)^{\delta} = 0 \tag{6}$$

and

$$\frac{\partial Z}{\partial \phi} = \lambda_K(1-\tau_K)rK - \lambda_H m(1-\psi)((1-a)H)^{\psi}((1-\phi))^{-\psi}K^{1-\psi}(E)^{\delta} = 0. \tag{7}$$

Time behaviour of the co state variables along the optimum growth path should satisfy the following.

$$\dot{\lambda_K} = \rho\lambda_K - \lambda_K(1-\tau_K)r\phi - \lambda_H m(1-\psi)((1-a)H)^{\psi}((1-\phi))^{1-\psi}K^{-\psi}(E)^{\delta}, \tag{8}$$

and

$$\dot{\lambda_H} = \rho\lambda_H - \lambda_K(1-\tau_l)wa - \lambda_K sw(1-a) - \lambda_H m\psi(1-a)^{\psi}H^{\psi-1}((1-\phi)K)^{1-\psi}(E)^{\delta}. \tag{9}$$

Transversality conditions are given by the followings.

$$\lim_{t\to\infty} e^{-\rho t}\lambda_K(t)K(t) = \lim_{t\to\infty} e^{-\rho t}\lambda_H(t)H(t) = 0.$$

We assume that the budget of the government is balanced at every instance ; and hence

$$P = \tau_K r\phi K + \tau_l waH - sw(1-a)H.$$

Hence, at the aggregate level, equation (2) is modified as follows:

$$\dot{K} = r\phi K + waH - C. \tag{10}$$

Using equations (13) and (14) we have

$$\frac{\psi(1-\phi)}{(1-\tau_l-s)wH} = \frac{(1-a)(1-\psi)}{(1-\tau_K)rK} \tag{11}$$

The equilibrium factor prices are given by

$$w = \frac{\partial Y}{\partial(aH)} = A\gamma(\frac{\phi K}{aH})^{1-\gamma}$$

$$r = \frac{\partial Y}{\partial(\phi K)} = A(1-\gamma)(\frac{\phi K}{aH})^{-\gamma}$$

Using equations (5) and (8), we have

$$\frac{\dot{C}}{C} = \frac{(1-\tau_K)r-\rho}{\sigma}. \tag{12}$$

Now we define three new variables x, y and ω such that $x = \frac{C}{H}$, $y = \frac{K}{H}$ and $\omega = \frac{w}{r} = \frac{\gamma}{(1-\gamma)}\frac{\phi K}{aH}$

Using equation (11) and definitions of ω and y we have

$$a = \frac{1 - \frac{(1-\tau_l-s)(1-\psi)}{(1-\tau_K)\psi}\frac{\omega}{y}}{[\frac{(1-\gamma)}{\gamma} - \frac{(1-\tau_l-s)(1-\psi)}{(1-\tau_K)\psi}]\frac{\omega}{y}} \tag{13}$$

and

$$\phi = \frac{(1-\gamma)}{\gamma}\frac{[1 - \frac{(1-\tau_l-s)(1-\psi)}{(1-\tau_K)\psi}\frac{\omega}{y}]}{[\frac{(1-\gamma)}{\gamma} - \frac{(1-\tau_l-s)(1-\psi)}{(1-\tau_K)\psi}]} \tag{14}$$

Using equations (3), (4) and (12) we derive the equation of motion of x.

$$\frac{\dot{x}}{x} = M_3\frac{(1-\tau_K)}{\sigma}\omega^{-\gamma} - \frac{\rho}{\sigma} - \frac{m}{M_2}{M_1}^{1-\psi}\omega^{-\psi}y^{1-\beta\delta}{E_0}^{\delta}[\frac{(1-\gamma)\omega}{\gamma y} - 1]. \quad (15)$$

Using equations (1), (3), (4) and (10) we derive the equation of motion of y.

$$\frac{\dot{y}}{y} = \frac{M_3}{\gamma M_2}\omega^{-\gamma}(1 - M_1\frac{\omega}{y}) - \frac{x}{y} - \frac{m}{M_2}{M_1}^{1-\psi}\omega^{-\psi}y^{1-\beta\delta}{E_0}^{\delta}[\frac{(1-\gamma)\omega}{\gamma y} - 1]. \quad (16)$$

and using equations (3), (4), (10), (13) and (14) we derive the equation of motion of ω.

$$\frac{\dot{\omega}}{\omega} = \frac{m}{(\gamma-\psi)}{M_1}^{1-\psi}\omega^{-\psi}y^{1-\beta\delta}{E_0}^{\delta}[\frac{\psi(1-\tau_l)\omega}{(1-\tau_l-s)y} - \frac{\beta\delta}{M_2}\{\frac{(1-\gamma)\omega}{\gamma y} - 1\}]$$

$$+\frac{M_3\omega^{-\gamma}}{(\gamma-\psi)}[\frac{\beta\delta}{\gamma M_2}(1 - M_1\frac{\omega}{y}) - (1-\tau_K)] - \frac{\beta\delta x}{(\gamma-\psi)y} \quad (17)$$

where

$$M_1 = \frac{(1-\tau_l-s)(1-\psi)}{(1-\tau_K)\psi}$$

$$M_2 = \frac{(1-\gamma)}{\gamma} - \frac{(1-\psi)(1-\tau_l-s)}{\psi(1-\tau_K)}$$

and

$$M_3 = A\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}$$

2.1 Steady State Equilibrium

Along the steady state equilibrium growth path, $\frac{\dot{x}}{x} = \frac{\dot{y}}{y} = \frac{\dot{\omega}}{\omega} = 0$. Their steady state equilibrium values are denoted by x^*, y^* and ω^*. Alternatively $\frac{\dot{K}}{K} = \frac{\dot{H}}{H} = \frac{\dot{C}}{C} = g$, where g is constant and $\frac{\dot{a}}{a} = \frac{\dot{\phi}}{\phi} = 0$ along the steady state equilibrium growth path.

The steady state equilibrium growth rate is given by the equation (12). Hence using the expression of rate of interest, r, the balanced growth rate of the economy, denoted by g, is obtained as

$$g = \frac{(1-\tau_K)}{\sigma}A(1-\gamma)^{(1-\gamma)}\gamma^{\gamma}\omega^{-\gamma} - \frac{\rho}{\sigma}. \quad (18)$$

Equating $\frac{\dot{y}}{y} = \frac{\dot{\omega}}{\omega} = 0$ from equations (16) and (17) we have

$$y^* = M_4\omega^{\frac{(1-\psi+\gamma)}{\beta\delta}}$$

where

$$M_4 = [\frac{m\psi(1-\tau_l){E_0}^{\delta}{M_1}^{1-\psi}}{(1-\tau_K)M_3(1-\tau_l-s)}]^{\frac{1}{\beta\delta}}$$

From $\frac{\dot{x}}{x} = 0$ we have

$$[M_3\frac{(1-\tau_K)}{\sigma} - m{E_0}^{\delta}\frac{{M_1}^{1-\psi}}{M_2}\frac{(1-\gamma)}{\gamma M_4}]\omega^{-\gamma} = \frac{\rho}{\sigma} - m{E_0}^{\delta}\frac{{M_1}^{1-\psi}}{M_2}\omega^{\frac{(1+\gamma)(1-\beta\delta)-\psi}{\beta\delta}}] \quad (19)$$

Let

$$M_5 = [M_3\frac{(1-\tau_K)}{\sigma} - m{E_0}^{\delta}\frac{{M_1}^{1-\psi}}{M_2}\frac{(1-\gamma)}{\gamma M_4}]$$

and

$$M_6 = m{E_0}^{\delta}\frac{{M_1}^{1-\psi}}{M_2}$$

(i) When $\psi < Min\{(1+\gamma)(1-\beta\delta), \frac{\gamma(1-\tau_l-s)}{(1-\gamma)(1-\tau_K)+\gamma(1-\tau_l-s)}\}$ M_2 is negative and hence LHS of equation (19) is negatively related to ω and RHS of equation (19) is positively related to ω. So unique ω^* exists. No restriction on σ is required for the existence of ω. (ii) When $\psi > Max\{(1+\gamma)(1-\beta\delta), \frac{\gamma(1-\tau_l-s)}{(1-\gamma)(1-\tau_K)+\gamma(1-\tau_l-s)}\}$ RHS of equation (19) is positively related to ω. If σ is very low (so that $M_5 > 0$), LHS of equation (19) is negatively related to ω. If $\omega \to 0$; LHS $\to \infty$ and RHS $\to -\infty$ and if $\omega \to \infty$, LHS $\to 0$ and RHS $\to \frac{\rho}{\sigma}$ Hence unique ω^* exists.

Proposition 1 *Under certain parametric condition, unique growth rate exists.*

When $\psi = 1$ and $\beta = 0$, i.e. in Lucas (1988) model M_1 turns out to be zero and hence from equation (15) we obtain a finite and unique ω^*, that is given by

$$\omega^* = \frac{\rho}{M_3(1-\tau_K)}$$

Hence unique growth rate exists.

Numerical Example

Consider the following numerical specifications. For instance, let $A = 1; E = 0.2; \rho = 0.5; \gamma = 0.5; \delta = 1; \psi = 0.2; \sigma = 2; \beta = 0.5; m = 0.5; \tau_K = 0.4; \tau_l = 0.3; s = 0.5$; The solutions of x, y and ω turn out to be $\omega = 4.375; y = 0.209; x = 8.602$. So there exists a unique real solution of x, y and ω.

2.2 Comparative Static Results

In this section, we study the comparative static effect of change in tax rates and subsidy rate on the growth rate. From the equation (18) we see that growth rate is negatively related to ω. In this section we are considering case (i).

$$\frac{\partial M_5}{\partial \tau_K} = -\frac{M_3}{\sigma} - \frac{mE_0{}^{\delta}(1-\gamma)}{\gamma M_2{}^2 M_4{}^2}[M_1{}^{-\psi} M_2 M_4 \frac{(1-\tau_l-s)(1-\psi)^2}{\psi(1-\tau_k)^2} - M_1{}^{1-\psi}\{-\frac{(1-\psi)(1-\tau_l-s)}{\psi(1-\tau_k)^2} M_4$$

$$-M_2[\frac{m\psi(1-\tau_l)E_o{}^{\delta}}{(1-\tau_l-s)M_3}\{\frac{(1-\tau_l-s)(1-\psi)}{\psi}\}^{1-\psi}]^{\frac{1}{\beta\delta}}\frac{(\psi-2)}{\beta\delta}(1-\tau_k)^{\frac{\psi-2-\beta\delta}{\beta\delta}}\}] < 0$$

$$\frac{\partial M_6}{\partial \tau_K} = \frac{mE_0{}^{\delta} M_1{}^{-\psi}}{M_2{}^2}(1-\psi)^2\frac{(1-\tau_l-s)}{\psi(1-\tau_k)^2}\{\frac{(1-\gamma)}{\gamma} + \frac{(1-\tau_l-s)}{(1-\tau_k)}\} > 0$$

In figure 1, as a result of increase in τ_K LHS1 curve shifts in downward direction to LHS2 curve and RHS1 curve shifts in leftward direction to RHS2. So with the increase in τ_K, ω^* decreases to ω_R.

When $\psi = 1$; the model reduces to Lucas (1988) model. In this case, $M_2 = \frac{(1-\gamma)}{\gamma}$, $M_1 = 0$, $\frac{\partial M_5}{\partial \tau_K} = -\frac{M_3}{\sigma}$ and $\frac{\partial M_6}{\partial \tau_K} = 0$ As a result of increase in τ_K LHS curve shifts in downward direction while the RHS curve remains unchanged. So with the increase in τ_K, ω^* decreases to ω_L.

Hence the effect of increase in τ_K on growth rate is uncertain in modified Rebelo (1990) as well as modified Lucas (1988) model. Growth rate is being pulled down because of direct effect of tau_K on it and the growth rate increased via decrease in ω. But the increase in growth rate is lower in modified Lucas (1988) model compared to modified Rebelo (1990) model.

$$\frac{\partial M_5}{\partial \tau_l} = -\frac{mE_0{}^{\delta}(1-\gamma)M_1{}^{-\psi}}{\gamma M_2{}^2 M_4{}^2}[-\frac{M_4(1-\psi)^2}{\psi(1-\tau_k)}\{\frac{(1-\gamma)}{\gamma} + \frac{(1-\tau_l-s)}{(1-\tau_k)}\}$$

$$-M_1 M_2\{\frac{m\psi E_0{}^{\delta}}{(1-\tau_k)M_3}\}^{\frac{1}{\beta\delta}}\frac{(1-\psi)M_1{}^{\frac{1-\psi-\beta\delta}{\beta\delta}}}{(1-\tau_l-s)\psi(1-\tau_k)}\{s-(1-\tau_l)\frac{(1-\psi)}{\beta\delta}\}] > 0$$

if $\frac{(1-\tau_l)(1-\psi)}{\beta\delta} > s$

$$\frac{\partial M_6}{\partial \tau_l} = -\frac{mE_0{}^{\delta} M_1{}^{-\psi}}{M_2{}^2}\frac{(1-\psi)^2}{\psi(1-\tau_k)}\{\frac{(1-\gamma)}{\gamma} + \frac{(1-\tau_l-s)}{(1-\tau_k)}\} < 0$$

As a result of increase in τ_l, in figure 2, LHS1 curve shifts in upward direction to LHS2 and RHS1 curve shifts in downward direction to RHS2. So as τ_l increases ω^* is increased to ω_R and growth rate decreases. When $\psi = 1$,

$\frac{\partial M_5}{\partial \tau_l} = 0$ and $\frac{\partial M_6}{\partial \tau_l} = 0$. So τ_l does not have any effect on growth rate in modified Lucas(1988) model.

$$\frac{\partial M_5}{\partial s} = \frac{(1-\psi)}{\psi(1-\tau_k)(1-\tau_l-s)}[\frac{\{(1-\tau_k)\psi+1-\beta\delta\psi\}}{\beta\delta}\{\frac{(1-\psi)(1-\tau_l-s)}{\psi(1-\tau_k)}-\frac{(1-\gamma)}{\gamma}\}-\frac{(1-\gamma)}{\gamma}]$$

may be positive or may be negative.

$$\frac{\partial M_6}{\partial s} = [\frac{m\psi(1-\tau_l){E_0}^{\delta}}{(1-\tau_k)M_3}]^{\frac{1}{\beta\delta}}\frac{{M_1}^{-\psi}}{\beta\delta(1-\tau_l-s)^2}[\frac{{M_1}^{1-\psi}}{(1-\tau_l-s)}]^{\frac{1}{\beta\delta}-1}[(1-\psi)(1-\tau_l-s)+M_1] > 0$$

So the effect of increase in s on ω and its effect on growth rate is uncertain in modified Rebelo (1990) model. When $\psi = 1$, $\frac{\partial M_5}{\partial s} = 0$ and $\frac{\partial M_6}{\partial s} = 0$. So rate of subsidy given on education does not have any influence on economic growth rate in modified Lucas (1988) model.

Proposition 2 *When human capital accumulation function in the Lucas (1988) and Rebelo (1991) model receives the negative external effect from environmental pollution, the effect of tax rate on physical capital on the steady state equilibrium rate of growth of the economy is uncertain. The growth rate in modified Lucas (1988) model is invariant to the tax rate imposed on labour income and subsidy rate where as growth rate in modified Rebelo (1990) model varies negatively with the tax rate imposed on labour income. The effect of rate of subsidy given on education on economic growth rate is uncertain in modified Rebelo (1991) model.*

The works of Mino (1996), Bond, Wang, Yip (1996),Ortiueira (1998) and Gomez (2000) deal with the policy implications in Rebelo (1991) model in details. Mino (1996) has analysed the effects of capital income taxation on growth in Rebelo (1991) model and has shown that this effect depends on the sectoral capital intensity ranking. Bond, Wang, Yip (1996) have shown that imposition of taxes on factors reduce the growth rate while the subsidization policy raises it. Ortiueira (1998) has pointed out that welfare cost of taxation in the Rebelo (1991) model is higher than that in the Lucas (1988) model. Gomez (2000) has shown that optimal tax on capital income in Rebelo model is significantly different from zero and optimal tax on human capital is higher than that on physical capital. So the results of our model is in contradiction with the standard results. In this paper we obtain the optimal tax on human capital or labour income to be zero. There is also a possibility of obtaining growth maximizing unique tax on physical capital and subsidy rate on education.

Gomez (2003), Garcia-Castrillo and Sanso (2000), Alonso Carrera (2000) and Lucas (1990) analyse the effects of fiscal policy in Lucas (1988) model. According to Garcia Castrillo and Sanso (2000) physical capital must be free from taxes but the tax on labour income should finance educational subsidy. Gomez (2003) also finds that capital income taxes are not optimal and subsidy financed by labour tax is optimal. Lucas (1990) shows that heavy initial capital taxation followed by lower and ultimately zero taxation is optimal and labour income taxation does not affect the growth rate in Lucas (1988) model. But we find that the tax on physical capital may be positive.

The intuition underlying this finding in this model is that in this model, physical capital has negative influence and human capital has positive influence on environment and environment has positive effect on human capital accumulation technology. Hence the optimal tax on physical capital may be positive and optimal tax on human capital is zero.

3 Centrally Planned Economy

In centrally planned economy the government allocates the human capital and physical capital endowment across sectors. In this economy, the aggregate production over consumption is accumulated as physical capital and the government can internalize the externality related to environment. Therefore, the physical capital and human capital accumulation function turns out to be

$$\dot{K} = A(aH)^{\gamma}(\phi K)^{1-\gamma} - C \tag{20}$$

and

$$\dot{H} = m(E_0 K^{-\beta} H^{\beta})^{\delta}((1-a)H)^{\psi}((1-\phi)K)^{1-\psi} \tag{21}$$

The government maximizes the social welfare function

$$\int_0^{\infty} U(C)e^{-\rho t}dt$$

subject to the equations (20) and (21) with respect to the control variables a and ϕ. To make this centrally planned economy model comparable to competitive economy we derive the equations of motion of x, y and ω where ω is defined as

$$\omega = \frac{\gamma\phi K}{(1-\gamma)aH}.$$

In this case a and ϕ are given by

$$a = \frac{1 - \frac{(1-\psi)}{\psi}\frac{\omega}{y}}{[\frac{(1-\gamma)}{\gamma} - \frac{(1-\psi)}{\psi}]\frac{\omega}{y}}$$

and

$$\phi = \frac{(1-\gamma)}{\gamma}\frac{[1-\frac{(1-\psi)}{\psi}\frac{\omega}{y}]}{[\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)}{\psi}]}$$

From the optimality conditions, we derive the equations of motion of x, y and ω

$$\frac{\dot{x}}{x} = \frac{A}{\sigma}(\frac{\omega(1-\gamma)}{\gamma})^{-\gamma}(1-\gamma)[\frac{(1-\gamma)\beta\delta\{1-\frac{(1-\psi)\omega}{\psi y}\}}{\gamma(1-\psi)\{\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)}{\psi}\}}+\frac{(1-\psi-\beta\delta)}{(1-\psi)}]-\frac{\rho}{\sigma}$$

$$-m(\frac{(1-\psi)\omega}{\psi})^{(1-\psi)}{E_0}^{\delta}y^{-\beta\delta}[\frac{\{\frac{(1-\gamma)\omega}{\gamma y}-1\}}{\frac{\omega}{y}\{\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)}{\psi}\}}]; \tag{22}$$

$$\frac{\dot{y}}{y} = A(\frac{\omega(1-\gamma)}{\gamma})^{-\gamma}\frac{(1-\gamma)}{\gamma}\frac{\{1-\frac{(1-\psi)\omega}{\psi y}\}}{\{\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)}{\psi}\}}-\frac{x}{y}-m(\frac{(1-\psi)\omega}{\psi})^{(1-\psi)}{E_0}^{\delta}y^{-\beta\delta}[\frac{\{\frac{(1-\gamma)\omega}{\gamma y}-1\}}{\frac{\omega}{y}\{\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)}{\psi}\}}]; \tag{23}$$

and

$$\frac{\dot{\omega}}{\omega} = \frac{\psi}{(\gamma-\psi)}m(\frac{(1-\psi)\omega}{\psi})^{(1-\psi)}{E_0}^{\delta}y^{-\beta\delta}+$$

$$A(\frac{\omega(1-\gamma)}{\gamma})^{-\gamma}\frac{(1-\gamma)}{(1-\psi)}[\frac{\beta\delta}{\gamma}\frac{\{1-\frac{(1-\psi)\omega}{\psi y}\}}{\{\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)}{\psi}\}}+\frac{(\beta\delta-1+\psi)}{(\gamma-\psi)}]-\frac{\beta\delta x}{(\gamma-\psi)y} \tag{24}$$

3.1 Optimal policy

Gomez (2003) and Garcia-Castrillo and Sanso (2000) have designed optimal fiscal policies in the Lucas-type model. In both the models the dynamic system of equations obtained in decentralized economy is compared to that obtained in centrally planned economy to find out the optimal policies. Comparing $\frac{\dot{x}}{x}$ of competitive economy and centrally planned economy using equations (15) and (22)we have

$$\frac{A\omega^{-\gamma}\gamma^{\gamma}(1-\gamma)^{1-\gamma}}{m\sigma(\frac{(1-\psi)}{\psi})^{1-\psi}\omega^{-\psi}{E_0}^{\delta}y^{1-\beta\delta}\{\frac{(1-\gamma)\omega}{\gamma y}-1\}}$$

$$= \frac{[\frac{1}{\{\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)}{\psi}\}}-\frac{\{\frac{(1-\tau_l-s)}{(1-\tau_k)}\}^{(1-\psi)}}{\{\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)(1-\tau_l-s)}{\psi(1-\tau_k)}\}}]}{[\frac{(1-\gamma)\beta\delta}{\gamma(1-\psi)}\frac{\{1-\frac{(1-\psi)\omega}{\psi y}\}}{\{\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)}{\psi}\}}+\frac{(1-\psi-\beta\delta)}{(1-\psi)}-(1-\tau_K)]} \tag{25}$$

Comparing $\frac{\dot{y}}{y}$ from equations (16) and (23)we have

$$\frac{A\omega^{-\gamma}\gamma^{\gamma-1}(1-\gamma)^{1-\gamma}}{m(\frac{(1-\psi)}{\psi})^{1-\psi}\omega^{-\psi}{E_0}^{\delta}y^{1-\beta\delta}\{\frac{(1-\gamma)\omega}{\gamma y}-1\}}$$

$$=\frac{[\frac{1}{\{\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)}{\psi}\}}-\frac{\{\frac{(1-\tau_l-s)}{(1-\tau_k)}\}^{(1-\psi)}}{\{\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)(1-\tau_l-s)}{\psi(1-\tau_k)}\}}]}{\frac{\{1-\frac{(1-\psi)\omega}{\psi y}\}}{\{\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)}{\psi}\}}-\frac{\{1-\frac{(1-\psi)(1-\tau_l-s)\omega}{\psi(1-\tau_k)y}\}}{\{\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)(1-\tau_l-s)}{\psi(1-\tau_k)}\}}} \tag{26}$$

From equations (25) and (26) we have

$$\frac{\hat{\omega}}{\hat{y}}=\frac{\gamma\{\frac{(1-\psi-\beta\delta)}{(1-\psi)}-(1-\tau_K)\}-\frac{\sigma-\frac{\beta\delta(1-\gamma)}{(1-\psi)}}{\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)}{\psi}}+\frac{\sigma}{\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)(1-\tau_l-s)}{\psi(1-\tau_k)}}}{\frac{(1-\psi)}{\psi}[\frac{\frac{\sigma(1-\tau_l-s)}{(1-\tau_k)}}{\{\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)(1-\tau_l-s)}{\psi(1-\tau_k)}\}}-\frac{\{\sigma-\frac{\beta\delta(1-\gamma)}{(1-\psi)}\}}{\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)}{\psi}\}}]} \tag{27}$$

Comparing $\frac{\dot{\omega}}{\omega}$ from equations (17) and (24)we have

$$\frac{A\omega^{-\gamma}\gamma^{\gamma}(1-\gamma)^{1-\gamma}}{m(\frac{(1-\psi)}{\psi})^{1-\psi}\omega^{1-\psi}{E_0}^{\delta}y^{-\beta\delta}}=\frac{A_2}{A_1} \tag{28}$$

where

$$A_1=\frac{\beta\delta}{\gamma}\frac{(1-\frac{(1-\tau_l-s)(1-\psi)\omega}{(1-\tau_k)\psi y})}{(\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)(1-\tau_l-s)}{\psi(1-\tau_k)})}-(1-\tau_K)-\frac{\beta\delta}{\gamma}\frac{(1-\frac{(1-\psi)\omega}{\psi y})}{(\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)}{\psi})}-\frac{(\beta\delta-1+\psi)}{(\gamma-\psi)}$$

and

$$A_2=\psi-\{\frac{(1-\tau_l-s)}{(1-\tau_K)}\}^{1-\psi}\frac{\psi(1-\tau_l)}{(1-\tau_l-s)}+\beta\delta\frac{\{\frac{(1-\gamma)\omega}{\gamma y}-1\}}{\{\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)(1-\tau_l-s)}{\psi(1-\tau_k)}\}}\{\frac{(1-\tau_l-s)}{(1-\tau_k}\}^{1-\psi}\frac{y}{\omega}$$

Using equations (25) and (28) we have

$$\sigma[\frac{1}{\{\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)}{\psi}\}}-\frac{\{\frac{(1-\tau_l-s)}{(1-\tau_k)}\}^{1-\psi}}{\{\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)(1-\tau_l-s)}{\psi(1-\tau_k)}\}}]A_1\{\frac{(1-\gamma)}{\gamma}-\frac{y}{\omega}\}$$

$$=A_2[\frac{(1-\gamma)\beta\delta\{1-\frac{(1-\psi)\omega}{\psi y}\}}{\gamma(1-\psi)(\frac{(1-\gamma)}{\gamma}-\frac{(1-\psi)}{\psi})}+\frac{(1-\psi-\beta\delta)}{(1-\psi)}-(1-\tau_k)]$$

If we substitute $\frac{\omega}{y}$ by $\frac{\hat{\omega}}{\hat{y}}$ given by the equation (27) we have a relationship among τ_k, τ_l, s that specify the optimal policy.

Proposition 3 *There exists a set of optimal policy parameters that can lead decentralized economy solution to command economy solution.*

4 Conclusion

We have developed an endogenous growth model where the environmental quality varies negatively with the stock of physical capital and varies positively with the size of human capital. The rate of human capital accumulation is positively affected by the external effect emanating from environment. The interesting results obtained in this model are as follows. Firstly, there exists a unique growth rate. Secondly, the effect of tax rate imposed on on capital income on the steady state equilibrium rate of growth is uncertain when tax revenue is spent as lumpsum payment in both modified Rebelo (1990) and modified Lucas (1988) model. In Lucas (1988), this rate of growth is independent of the tax rate imposed on labour income or subsidy given on education. However, the growth rate in modified Rebelo (1991) model varies negatively with the tax rate imposed on labour income and may vary in either direction with change in subsidy given on education. Thirdly, we have found that there exists a set of optimal policy parameters that can lead decentralized economy solution to command economy solution.

In Rebelo (1991), Mohtadi (1995) etc. the rate of growth varies inversely with the tax rate imposed on capital. Garcia Castrillo Sanso (2000), Gomez (2003) etc. find the optimal physical capital tax rate to be zero and the optimal labour tax rate to be positive in the Lucas (1988) model when tax revenue is spent as educational subsidy. However, none of these models considers the negative effect of environmental degradation on the human capital accumulation. However, in this model, we have considered the negative effect of environmental degradation on the human capital accumulation and have shown that the steady state equilibrium growth rate may receive a positive effect from taxation on capital income. Hence positive optimal tax rate on capital may be obtained. Existing literature does not point out such a possibility.

The present work is subject to various limitations. The dynamic effects of taxes are yet to study. We should also consider the welfare effects of taxation. We have not studied the effects of taxation when tax revenue is not spent as lumpsum transfer but spent for providing educational subsidy or spent for financing abatement expenditure. We intend to consider these aspects in our future research.

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