Inequality may retard growth but sometimes progressive redistribution makes it worse

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Abstract

We provide an empirically plausible endogenous growth model to prove analytically that sometimes a progressive redistribution from rich to poor lowers the growth rate of consumption per capita in all subsequent periods. The model accommodates the growth retarding effect of income inequality by combining the assumptions of no credit market and a production technology with diminishing returns to the combined inputs of physical and human capital. Also, to make the model's assumptions consistent with the evidence reported by leading labor economists, we assume that the parental human capital sufficiently improves the effectiveness of expenditure on a child's education, in order to induce increasing returns to scale in the education technology. A reduction in the progressivity of redistribution, under such education technology, enhances the average human capital of all future cohorts of parents, which in turn boosts the growth rate of average human capital. The immediate resulting gain in the growth rate of consumption per capita sufficiently outweighs the subsequent growth loss due to the decline in TFP brought about by the associated increase in income inequality. Consequently, in our model, a policy of progressive redistribution is dynamically inefficient.

Keywords: heterogeneous ability, education technology, endogenous growth, progressive income tax rate.

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1 Introduction

Galor and Zeira (1993), and a large body of subsequent literature, provide a dynamic microfoundation for the systematic examination of a wide variety of interactions between income distribution and growth within the neoclassical framework. These developments have motivated policy discussions (see, e.g., Glomm and Ravikumar, 1992; Benabou, 1996; and Fernandez and Rogerson, 1998) on the design of the optimal scheme of income redistribution that will allow the economy to achieve its maximal growth path.

The body of literature mentioned above mostly confines itself to a specialized environment characterized by the absence of a credit market and an output technology with diminishing returns to capital which could be defined broadly to include both human and physical capital. In such an environment, a policy of income redistribution could potentially be justified not only on a normative ground but also on a positive ground and, in particular, on the ground of promoting economic growth. The question that immediately follows is: what level of income redistribution would bring about the maximum economic growth?

In his pioneering work, Benabou (2002) provides an answer to this question. He reports that in his model, which was calibrated to the US economy, and in a wide range of other economies around that benchmark, a significant degree of progressive redistribution would be necessary to maximize the long-run per capita output. We ask whether this result could be extended to an environment that accommodates endogenous growth. In this endeavour we begin by carefully designing an endogenous growth model that preserves the harmful effects of inequality on growth, and hence would provide a reason to expect redistribution to promote economic growth. Afterward, we ask what degree of progressivity would help the economy to achieve its maximal growth path.

Clearly, some models of endogenous growth, such as that of Romer (1986), which requires constant returns to capital in the production technology or a model that allows a credit market for trading capital goods would rule out one of the two key assumptions of Benabou (2002), and hence would eliminate some of the gains from redistribution by assumption. We therefore focus on models that maintain the two key assumptions: an output technology with diminishing returns to capital and a model without a credit market.

Also, some other models, such as those popularized by Lucas (1988) and Tamura (1991), produce endogenous growth by relying on knowledge spillover or human capital externality. However, we argue that such models allow a costless spillover of knowledge to overcome the barriers of credit constraints. In particular, following the ideas explored by Galor and Tsiddon (1997), we note two mutually counterbalancing effects on income inequality from two different types of externality. First, there is the income equalizing effect of "global technological externality" which arises from knowledge spillover. Second, there is the income widening effect of "local home environment externality" which arises from the complementarity between parental human capital and expenditure on the child's education. The complementarity between the two inputs above is implicit in our education technology. Thus, allowing knowledge spillover to act as an income equalizing force in the model is likely to offset the income widening effect of the second externality, and hence could mitigate the potentially harmful effects of an overall increase in income inequality on economic growth.

Moreover, a model of endogenous growth with knowledge spillover makes it necessary, for technical reasons alone, to assume diminishing returns to scale in the above education technology. However, this assumption could be problematic when modelling economic growth, especially in developing countries, where, as Trostel (2004) reports, there is strong empirical evidence of increasing returns to scale in the technology for human capital accumulation. It is also well known, from an influential study by Rosenzweig and Wolpin (1994), that returns to an investment in education significantly increase with parental human capital, which results in increasing returns to human capital through using maternal schooling to augment the production of children's human capital. Trostel (2004) reports that this type of complementarity induced increasing returns in the production of human capital is especially relevant for low income countries.

We provide a model of endogenous growth that addresses the above challenges by combining an education technology with increasing returns to scale with a production technology with diminishing returns to capital. As a consequence, our model environment fulfils two essential tasks. Like that of Benabou (2002), it does not automatically rule out any growth promoting role that might be played by a policy of progressive redistribution. Also, it accommodates endogenous growth without violating the restrictions imposed on the model's assumptions by the empirical evidence reported above. Moreover, we allow investment subsidies to offset any distortionary effects of redistribution on capital accumulation, and consider an abstraction where even the labor supply does not fall after redistribution. In this specialized set-up, designed to allow the maximum possible beneficial effect of progressive redistribution on economic growth, we ask what degree of progressivity in redistribution helps the economy to reach its maximal growth path of consumption per capita. We find that the answer is unambiguously zero, or no progressivity at all.

The intuition behind the above result is as follows: the combination of the assumptions of no credit market and diminishing returns production technology results in an economy which is stuck with rigid interpersonal differences in marginal products. Redistribution from more wealthy agents with a lower marginal product to less wealthy agents with a higher marginal product typically helps to improve allocative efficiency, and hence improves the total factor productivity (TFP) in such an economy. With no market for trading capital, however, wealthier people also represent more educated parents while empirical rationale requires us to consider an education technology with increasing returns to scale. Consequently, the above redistribution amounts to a transfer of resources from more educated parents to less educated parents and that decreases, in all subsequent periods, the average stock of parental human capital. This decrease lowers the effectiveness of educational expenditure on children for raising the human capital of the future generations of producers in the economy. Thus, more progressivity in redistribution reduces the growth rate of average human capital. It turns out that if the economy grows endogenously, then the above reduction in the growth rate of human capital must always outweigh any improvement in the total factor productivity brought about by a more progressive redistribution of resources to imply overall a lower growth rate of output per capita. Consequently, the degree of progressivity that allows the economy to reach its maximal attainable growth path must be zero.

To facilitate future work involving redistribution, endogenous growth and income inequality, we provide explicit analytical expressions for the growth rate of per capita output under different regimes of redistribution. In each case the balanced growth rates turn out to be functions of the degree of income inequality, as well as of the degree of progressivity in the underlying scheme of redistribution. The growth rate and income inequality together exhibit a special block recursive transitional dynamics, which is a relatively unexplored area in the growth-inequality literature to date. The economy generates a unique path of evolution for income inequality and the rate of growth, and they converge monotonically to their respective steady states. We take the complete transitional dynamics into account to prove that the growth-maximizing degree of progression must always be zero.

In Section 2 we present the model environment, individual optimization and the model's equilibrium outcome. In Section 3 we focus on the issues related to income inequality and economic growth and in Section 4 we turn to the merits of redistributive policies and the key proposition of our paper. In Section 5, we discuss the consistency of our model with the data. In Section 6 we add a few concluding remarks and summarize our contributions. An Appendix containing the proofs of our lemmas and propositions follows, and the paper closes with the list of references cited.

2 The Model

The environment used here is similar to that of Benabou (2002), but we have broadened it by including capital goods in the production technology and by allowing bequests for transferring wealth. The motivation for including bequests comes from Kotlikoff and Summers (1981), who, using historical U.S. data, find that most of aggregate capital accumulation is due to intergenerational transfers. Also, Laitner (1979a-c) argues that bequests are an important source of capital, and finds that bequest behaviors have important effects on the national distribution of wealth. Altonji and Doraszelski (2005) discuss the important effects of bequests in explaining the income gap between the white and black populations.

2.1 *Preference, Technology and Endowments*

There is a continuum of infinitely lived decision makers or agents indexed by $i \in [0,1]$ with the preference at period t given by:

$$\ln U_t^i = E_t \left[\sum_{n=0}^{\infty} \rho^n \left(\ln c_{t+n}^i - \left(l_{t+n}^i \right)^{\eta} \right) \right], \eta > 1, \tag{1}$$

where $c_t^i > 0$ and $l_t^i \geq 0$ denote the consumption and labor supply of agent i in period t, respectively, and $\rho \in (0,1)$ denotes the discount factor.

We allow both physical and human capital to affect the output as complementary inputs, in the same way as Barro, Mankiw and Sala-i-Martin (1995), such that the output of the self-employed agent i as a function of her physical and human capital k_t^i, h_t^i and labor l_t^i satisfies

$$y_t^i = \left(k_t^i \right)^{\lambda} \left(h_t^i \right)^{\mu} \left(l_t^i \right)^{\varepsilon}, \text{ where } \varepsilon = 1 - \lambda - \mu. \tag{2}$$

The government redistributes investments in education among pupils, either indirectly, through a scheme of progressive income taxation and transfer, or directly, through a scheme of a progressive subsidy of private expenditure e_t^i on education. Under both schemes, the government subsidises private expenditure on education at the average rate $d \in [0,1]$ and private expenditure on bequests at the average rate $\upsilon \in [0,1]$,[1], and finances these subsidies with consumption tax at a rate $\theta \in [0,1]$, such that

$$\theta \int_0^1 c_t^i di = d \int_0^1 e_t^i di + \nu \int_0^1 b_t^i di. \tag{3}$$

Under the income tax scheme, the disposable income $\hat{y}_t^i$ of a typical agent i at a given date t satisfies

$$\ln \hat{y}_t^i = (1 - \tau) \ln y_t^i + \tau \ln \tilde{y}_t, \tag{4}$$

where $0 \leq \tau < 1$ measures the average degree of progression in the scheme for income redistribution and $\tilde{y}_t$ represents the break-even level of income such that

$$\int_0^1 \hat{y}_t^i di = \int_0^1 y_t^i di \equiv y_t, \tag{5}$$

where, y_t denotes the per-capita output or income at a given period t. The post-subsidy expenditure on education is $\hat{e}_t^i = (1 + d) e_t^i$.

[1]This could be regarded as a kind of subsidy on the purchase of capital goods or as an exemption for estate taxation. Cremer and Pestieau, (2001) discuss positive or negative taxations on bequests as a tool for influencing intergenerational equity within each family and intragenerational equity across families of different wealth. We allow a bequest subsidy in our model also as a hypothetical tool for offsetting the distortionary effect of redistribution on physical capital accumulation, in order to find the upper limit for the optimal degree of progressivity in the underlying scheme of redistribution.

Under the education finance scheme, income is not taxed but the education subsidy varies with income, such that

$$\hat{y}_t^i = y_t^i, \qquad \hat{e}_t^i = (1+d)\left(\tilde{y}_t / y_t^i\right)^{\tau} e_t^i, \tag{6}$$

where $0 \leq \tau < 1$ measures the rate at which the education subsidy decreases as the agent's income increases relative to the 'break-even' level $\tilde{y}_t$ such that $\int_0^1 \hat{e}_t^i di = (1+d)\int_0^1 e_t^i di.$

Without access to a credit market, the disposable income $\hat{y}_t^i$ of the agent i at each date must equal the total expenditure on consumption c_t^i, consumption tax θc_t^i, private education expenditure e_t^i, and bequest b_t^i; in other words,

$$\hat{y}_t^i = (1+\theta)\, c_t^i + e_t^i + b_t^i. \tag{7}$$

The human capital h_{t+1}^i of the agent i at the date $t+1$, as a function of an idiosyncratic shock ξ_{t+1}^i, which is $i.i.d.$ with $\ln \xi_t^i \sim N\left(-\sigma^2/2, \sigma^2\right)$, and the post-subsidy expenditure $\hat{e}_t^i$ on her education in the previous period, is given by,

$$h_{t+1}^i = \kappa \xi_{t+1}^i \left(h_t^i\right)^{\alpha} \left(\hat{e}_t^i\right)^{\beta}, \kappa > 0, \alpha \in (0,1), \beta \in (0,1). \tag{8}$$

Note that α can be interpreted as the elasticity of children's human capital with respect to parents' human capital, and β as the elasticity of children's human capital with respect to expenditure on their educations. Rosenzweig and Wolpin (1994) report that the sum of these two elasticities could be greater than one.

Capital goods are complementary to human capital and become obsolete at the end of each generation. A tool loses value when its user dies. Parents buy new tools for their children at a subsidized rate set by the government and leave them as a bequest. To capture this feature we assume that they depreciate completely in the production process. Thus, in generation $t+1$, agent i's physical capital k_{t+1}^i consists only of her parent's bequest b_t^i and a bequest subsidy from the government at the rate of v per unit of the bequest, such that

$$k_{t+1}^i = (1+v)\, b_t^i. \tag{9}$$

Initial endowments of human and physical capital h_0^i and k_0^i are jointly lognormally distributed, and the adult receives one unit of labor endowment in each period.

2.2 *Individual Optimization*

At each date t, let m_{ht} and m_{kt} denote the means and Δ_{ht}^2 and Δ_{kt}^2 the variances of $\ln h_t^i$ and $\ln k_t^i$, respectively, and let cov_t denote the covariance between $\ln h_t^i$ and $\ln k_t^i$. Suppose that $M_t \equiv \left(m_{ht}, m_{kt}, \Delta_{ht}^2, \Delta_{kt}^2, cov_t\right)$. Then, for the agent's dynamic optimization problem, the progressively redistributive policy regime is $P \equiv (\tau, d, v, \theta)$, the state variables are (h_t^i, k_t^i, M_t), the control variables are $(c_t^i,\ l_t^i,\ e_t^i,\ b_t^i)$, and the Bellman equation is

$$\ln U\left(h_t^i, k_t^i, M_t; P\right) = \max_{c_t^i, l_t^i, e_t^i, b_t^i} \left\{ \begin{array}{l} (1-\rho)\left[\ln c_t^i - (l_t^i)^{\eta}\right] \\ +\rho E_t\left[\ln U(h_{t+1}^i, k_{t+1}^i, M_{t+1}; P)\right] \end{array} \right\}, \tag{10}$$

subject to (2), (4) or (6), (7), (8) and (9).

The first order conditions associated with the Bellman equation described by (10) yield complete solutions to the agent's problem. We first discuss the labor supply, followed by investments in education and bequests.

Lemma 1: *The optimal labor supply remains invariant to time and personal characteristics, and decreases with the average marginal income tax rate, τ, such that*:

$$l^Y(\tau) = \left(\frac{(1-\rho\alpha)\left((1-\lambda-\mu)/\eta\right)(1-\tau)}{(1-\rho\alpha)(1-\rho\lambda(1-\tau)) - \rho\beta\mu(1-\tau)} \right)^{1/\eta}, \tag{11}$$

$$l^E(\tau) = \left(\frac{(1-\rho\alpha)(1-\lambda-\mu)/\eta}{(1-\rho\alpha)(1-\rho\lambda) - \rho\beta\mu(1-\tau)} \right)^{1/\eta}, \tag{12}$$

where a superscript Y or E denotes the case under the income tax scheme or the education finance scheme, respectively.

Proof: See Appendix.

In the special case where agents do not get disutility from work, i.e., $\eta = 0$, the labor supply will be independent of the redistributive policy, such that

$$l^Y(\tau) = l^E(\tau) = l > 0. \tag{13}$$

Next, we consider the investment propensities of the two forms of capital. We denote by s_{jt}^i, $j = 1, 2$, respectively, the fraction of her disposable income that agent i invests in her children's educations and her bequests, such that $s_{1t}^i \equiv e_t^i/\hat{y}_t^i$, $s_{2t}^i \equiv b_t^i/\hat{y}_t^i$.

Lemma 2: *The education investment rate s_{1t}^i and the bequest rate s_{2t}^i are time invariant, and decrease with the average marginal income tax rate τ*:

$$s_1^Y(\tau) = \frac{\rho\beta\mu(1-\tau)}{1-\rho\alpha} \equiv (1-\tau)\bar{s}_1, \tag{14}$$

$$s_2^Y(\tau) = \rho\lambda(1-\tau) \equiv (1-\tau)\bar{s}_2, \tag{15}$$

$$s_1^E(\tau) = \frac{\rho\beta\mu}{1-\rho\alpha+\rho\beta\mu\tau} \equiv \frac{\bar{s}_1}{1+\tau\bar{s}_1}, \tag{16}$$

$$s_2^E(\tau) = \frac{\rho\lambda(1-\rho\alpha)}{1-\rho\alpha+\rho\beta\mu\tau} \equiv \frac{\bar{s}_2(1-\rho\alpha)}{1-\rho\alpha+\rho\beta\mu\tau}, \tag{17}$$

where $\bar{s}_1 = \frac{\rho\beta\mu}{1-\rho\alpha}$ and $\bar{s}_2 = \rho\lambda$ denote the laissez-faire saving rates.

Proof: See Appendix.

Lemmas 1 and 2 explicitly spell out the negative effect of redistribution on the incentives to supply labor and capital inputs under both the income tax scheme and the education finance scheme. Typically, governments attempt to offset some of the distortionary effects of income taxes using a package of redistributive policies, as discussed by Benabou (2002). In particular, we assume that the government chooses the subsidy rates d and v such that for each policy regime $j = Y, E$:

$$(1+d)\, s_1^j = \bar{s}_1, \tag{18}$$

$$(1+v)\, s_2^j = \bar{s}_2. \tag{19}$$

By the government budget constraint (3) and Lemma 2, it follows that

$$\frac{\theta\left(1 - s_1^j - s_2^j\right)}{1+\theta} = d s_1^j + v s_2^j, \tag{20}$$

and from (18), (19) and (20), the subsidy rates d and v and the consumption tax rate θ satisfy

$$d = \frac{\tau}{1-\tau},\ v = \frac{\tau}{1-\tau} \text{ and } \theta = \frac{\bar{s}_1 + \bar{s}_2}{1 - \bar{s}_1 - \bar{s}_2}\tau. \tag{21}$$

We can switch on the intertemporal distortions simply by setting either d or v (or both) equal to zero, and adjusting θ according to (20).

By (21), the redistributive policy package can be summarized by the parameter τ alone. We refer to τ as the degree of progressivity of redistribution, or simply the degree of redistribution. Note that, contrary to common practice, we allow the above two subsidies in sufficient amount to completely eliminate all distortionary effects of progressive redistribution on the two investment rates. We consider the above scenario to determine the upper limit for the degree of progressivity that would maximize economic growth. In the same vein, we switch off the distortionary effect of redistribution on the labor supply by first focussing on the case where agents get no disutility from work and supply labor inelastically, as indicated by (13). Afterwards, we argue that if we reduce the subsidies to allow the distortions implied by (14)-(17), then the upper limit will still hold.

2.3 *The Equilibrium Dynamics*

The optimization problem (10) yields (11)-(17) and other decision rules as follows:

$$\begin{aligned} \ln c_t^{i,Y} &= \ln(1-\bar{s}_1-\bar{s}_2) - \ln(1+\theta) + (1-\tau)\ln y_t^i + \tau \ln \tilde{y}_t, \\ \ln c_t^{i,E} &= \ln(1-\bar{s}_1-\bar{s}_2) - \ln(1+\theta) + \ln y_t^i, \end{aligned} \tag{22}$$

$$\begin{aligned} \ln e_t^{i,Y} &= \ln \bar{s}_1 + (1-\tau)\ln y_t^i + \tau \ln \tilde{y}_t, \\ \ln e_t^{i,E} &= \ln \bar{s}_1 + \ln y_t^i, \end{aligned} \tag{23}$$

$$\begin{aligned}\ln b_t^{i,\,Y} &= \ln \bar{s}_2 + (1-\tau)\ln y_t^i + \tau \ln \tilde{y}_t, \\ \ln b_t^{i,\,E} &= \ln \bar{s}_2 + \ln y_t^i.\end{aligned} \tag{24}$$

Together with the government's budget constraints (3) and (4), the above decision rules imply that there is a unique sequence of aggregate state variables $\{M_t\}$ that coincides with what agent i takes as given in (10), such that at each date $t = 0, 1, 2, ..,$ the following aggregate consistency condition holds:

$$\int_0^1 y_t^i di = \int_0^1 c_t^i di + \int_0^1 e_t^i di + d\int_0^1 e_t^i di + \int_0^1 b_t^i di + v\int_0^1 b_t^i di. \tag{25}$$

3 Inequality and Growth

In this section we characterize the equilibrium interactions between measures of income inequality and various economic factors that contribute to the growth of per capita income.

3.1 *Intergenerational persistence*

Under the income tax scheme, by (9) or (8) combined with (13)-(15) yields the dynamics of physical and human capital for the dynasty i as follows:

$$\begin{aligned}\ln k_{t+1}^{i,\,Y} = {} & \ln \bar{s}_2 + (1-\lambda-\mu)(1-\tau)\ln l + \lambda(1-\tau)\ln k_t^{i,\,Y} \\ & + \mu(1-\tau)\ln h_t^{i,\,Y} + \tau \ln \tilde{y}_t,\end{aligned} \tag{26}$$

$$\begin{aligned}\ln h_{t+1}^{i,\,Y} = {} & \ln \kappa + \beta \ln \bar{s}_1 + \beta(1-\lambda-\mu)(1-\tau)\ln l + \ln \xi_{t+1}^i \\ & + \beta\lambda(1-\tau)\ln k_t^{i,\,Y} + (\alpha + \beta\mu(1-\tau))\ln h_t^{i,\,Y} + \beta\tau \ln \tilde{y}_t.\end{aligned} \tag{27}$$

Under the education finance scheme, by (9), (13) and (17), the decision rule for physical capital accumulation satisfies

$$\ln k_{t+1}^{i,\,E} = \ln \bar{s}_2 + \lambda \ln k_t^{i,\,E} + \mu \ln h_t^{i,\,E} + (1-\lambda-\mu)\ln l^E, \tag{28}$$

and the decision rule for human capital accumulation is the same as (27).

Substituting (26) or (28), and (27) into the logarithm of (2) yields the equilibrium path of income for agent i.

Under the income tax scheme, they satisfy

$$\begin{aligned}\ln y_{t+1}^{i,Y} = {} & \psi + \mu \ln \kappa + (1-\alpha)(1-\lambda-\mu)\ln l + \mu \ln \xi_{t+1}^i \\ & + (\lambda + \beta\mu)\tau \ln \tilde{y}_t - \alpha\lambda\tau \ln \tilde{y}_{t-1} \\ & + (\alpha + (\lambda+\beta\mu)(1-\tau))\ln y_t^i - \alpha\lambda(1-\tau)\ln y_{t-1}^i,\end{aligned} \tag{29}$$

and under the education finance scheme

$$\ln y_{t+1}^{i,E} = \psi + \mu \ln \kappa + (1-\alpha)(1-\lambda-\mu)\ln l + \mu \ln \xi_{t+1}^{i} + \mu\beta\tau \ln \tilde{y}_t + (\alpha+\lambda+\mu\beta(1-\tau))\ln y_t^i - \alpha\lambda \ln y_{t-1}^i, \tag{30}$$

where $\psi \equiv \mu\beta \ln \bar{s}_1 + \lambda(1-\alpha)\ln \bar{s}_2$.

Note that, by (26)-(30), the intergenerational persistence of human capital, $p_h(\tau) \equiv \alpha + \beta\mu(1-\tau)$, and physical capital, which is $p_k(\tau) \equiv \lambda(1-\tau)$, under the income tax scheme or $p_k(\tau) \equiv \lambda$ under the education finance scheme, together imply the intergenerational persistence of income, $p_y^Y(\tau) \equiv \alpha + (\lambda+\beta\mu)(1-\tau)$ or $p_y^E(\tau) \equiv \alpha+\lambda+\beta\mu(1-\tau)$, between parents and children. It has either one or two structural components that cannot be lowered by redistribution: α under the income tax scheme or both α and λ under the education finance scheme. The other component of intergenerational persistence decreases with the degree of redistribution τ, and a policy of redistribution enhances intergenerational social mobility through this channel.[2] Next, we characterize the dynamic path of the aggregate state variables.

3.2 *Endogenous Growth-Inequality Dynamics*

In line with Benabou (2002), we define an index of income inequality Λ_t for each date t as the logarithm of the ratio of the mean to median income. Also, for notational convenience we drop the superscript j wherever it is necessary to describe general results that do not vary across redistributive regimes, except for specific formulas which we can clearly determine from the context.

Lemma 3: *The evolution of earnings of adults is governed by a lognormal distribution such that* $\ln y_t^i \sim N(\lambda m_{kt} + \mu m_{ht} + (1-\lambda-\mu)\ln l, 2\Lambda_t)$. *At each date* $t = 0, 1, 2, ...$, *the inequality index* Λ_t *equals half of the variance of the logarithmic earnings of agents, such that* $\Lambda_t = \left(\lambda^2\Delta_{kt}^2 + \mu^2\Delta_{ht}^2 + 2\lambda\mu cov_t\right)/2$.

Proof. See Appendix.

Lemma 4: *The break-even level of income* $\tilde{y}_t$ *at which an agent's net tax obligation is zero satisfies:*

$$\ln \tilde{y}_t = \ln y_t + (1-\tau)\Lambda_t. \tag{31}$$

Proof: See Appendix.

[2] Note also that the income of the parents does not sufficiently determine the children's income, which is consistent with the results of Becker and Tomes (1979).

Given the initial distributions of human and physical capital, under the income tax scheme, by (26) and (27), physical and human capital remain jointly lognormally distributed over time, such that under the income tax scheme, at each date t, M_t satisfies

$$m_{kt+1} = \ln \bar{s}_2 + (1-\lambda-\mu)\ln l + \lambda m_{kt} + \mu m_{ht} + \tau(2-\tau)\left(\lambda^2\Delta_{kt}^2 + \mu^2\Delta_{ht}^2 + 2\lambda\mu cov_t\right)/2, \tag{32}$$

$$\Delta_{kt+1}^2 = (1-\tau)^2\left(\lambda^2\Delta_{kt}^2 + \mu^2\Delta_{ht}^2 + 2\lambda\mu cov_t\right), \tag{33}$$

$$m_{ht+1} = \ln\kappa - \sigma^2/2 + \beta\ln\bar{s}_1 + \beta(1-\lambda-\mu)\ln l + \beta\lambda m_{kt} + (\alpha+\beta\mu)\, m_{ht} + \beta\tau(2-\tau)\left(\lambda^2\Delta_{kt}^2 + \mu^2\Delta_{ht}^2 + 2\lambda\mu cov_t\right)/2, \tag{34}$$

$$\Delta_{ht+1}^2 = \sigma^2 + \beta^2\lambda^2(1-\tau)^2\Delta_{kt}^2 + (\alpha+\beta\mu(1-\tau))^2\Delta_{ht}^2 + 2\beta\lambda(1-\tau)(\alpha+\beta\mu(1-\tau))\, cov_t, \tag{35}$$

$$cov_{t+1} = \beta\lambda^2(1-\tau)^2\Delta_{kt}^2 + \mu(1-\tau)(\alpha+\beta\mu(1-\tau))\Delta_{ht}^2 + \lambda(1-\tau)(\alpha+2\beta\mu(1-\tau))\, cov_t, \tag{36}$$

and under the education finance scheme, (34)-(35) hold, and by (27) and (28), M_t satisfies

$$m_{kt+1} = \ln\bar{s}_2 + (1-\lambda-\mu)\ln l + \lambda m_{kt} + \mu m_{ht}, \tag{37}$$

$$\Delta_{kt+1}^2 = \lambda^2\Delta_{kt}^2 + \mu^2\Delta_{ht}^2 + 2\lambda\mu cov_t, \tag{38}$$

$$cov_{t+1} = \beta\lambda^2(1-\tau)\Delta_{kt}^2 + \mu(\alpha+\beta\mu(1-\tau))\Delta_{ht}^2 + \lambda(\alpha+2\beta\mu(1-\tau))\, cov_t, \tag{39}$$

Equations (32) and (34) are obtained by substituting the expression (31) for the break-even income into (26) and (27).

Given M_0, using (32)-36), or (34)-(35) and (37)-(39), we can compute a unique sequence $\{M_t\}_{t=1,2,..\infty}$. In particular, the following lemma characterizes the sequence of income inequality and its convergence to a stationary state.

Lemma 5: *Irrespective of the initial conditions, income inequality converges monotonically to its unique ergodic limit, which is determined by the degree of progressivity τ in the underlying policy regime of redistribution.*

Proof: Writing the system of linear equations (33), (35) and (36) in a matrix form, for the redistributive scheme with income tax, we get

$$\Delta_{t+1} = A_0 + A_1 * \Delta_t, \tag{40}$$

where

$$\Delta_{t+1} \equiv \begin{bmatrix} \Delta^2_{kt+1} \\ \Delta^2_{ht+1} \\ \mathrm{cov}_{t+1} \end{bmatrix}, A_0 \equiv \begin{bmatrix} 0 \\ \sigma^2 \\ 0 \end{bmatrix},$$

$$A_1 \equiv \begin{bmatrix} (1-\tau)^2 \lambda^2 & (1-\tau)^2 \mu^2 & 2\lambda\mu (1-\tau)^2 \\ (1-\tau)^2 \beta^2 \lambda^2 & (\alpha + \beta\mu (1-\tau))^2 & 2\beta\lambda (1-\tau) (\alpha + \beta\mu (1-\tau)) \\ (1-\tau)^2 \beta\lambda^2 & \mu (1-\tau) (\alpha + \beta\mu (1-\tau)) & \lambda (1-\tau) (\alpha + 2\beta\mu (1-\tau)) \end{bmatrix},$$

or,

$$A_1 \equiv \begin{bmatrix} x^2 & y^2 & 2xy \\ \beta^2 x^2 & z^2 & 2\beta xz \\ \beta x^2 & yz & x(z+\beta y) \end{bmatrix},$$

where $x \equiv (1-\tau)\lambda$, $y \equiv (1-\tau)\mu$, $z \equiv \alpha + \beta\mu(1-\tau)$.

Then, rearranging (40) yields

$$(I - A_1 L)\,\Delta_{t+1} = A_0, \tag{41}$$

where *I* is an identity matrix and *L* is a lag operator.

All eigenvalues of A_1 in (40) are positive real and less than unity. Consequently, $\{\Delta_t\}$ monotonically converges to Δ, where

$$\Delta = [I - A_1]^{-1} A_0. \tag{42}$$

Now, by Lemma 3, income inequality $\Lambda_t = (\Delta_t)^T w$, where $(\Delta_t)^T$ denotes the transpose of Δ_t and $w \equiv 0.5\left(\lambda^2,\ \mu^2,\ 2\lambda\mu\right)^T$. It follows, therefore, that from any arbitrarily specified initial state Δ_0, as Δ_t monotonically converges to Δ, the income inequality $\Lambda_t = (\Delta_t)^T w$ also monotonically converges to $\Lambda = (\Delta)^T w$.

A similar conclusion follows in a straightforward way when the redistribution scheme changes from the income tax to the education finance scheme. We skip that repetitive exercise as it does not add any additional insights relevant to our main result. □

Next, we discuss an important channel through which the increase in income inequality hinders economic growth.

3.2.1 *How Inequality Lowers TFP*

Following Solow (1957), the total factor productivity (*TFP*) of the economy is equal to the ratio of the average output to the weighted average of inputs. We use a Cobb-Douglas production technology similar to those assigned to each individual to compute the *TFP* for the economy as follows

$$TFP \equiv \frac{\int_0^1 y^i di}{\left(\int_0^1 k^i di\right)^{\lambda} \left(\int_0^1 h^i di\right)^{\mu} \left(\int_0^1 l^i di\right)^{1-\lambda-\mu}}, \tag{43}$$

such that, by Lemma 1, the period t growth rate γ_t of per capita income $y_t = \int_0^1 y_t^i di$, as a function of the growth rate of *TFP,* g_{TFPt}, and the growth rates of the average capital stock $k_t = \int_0^1 k_t^i di$ and the average human capital stock $h_t = \int_0^1 h_t^i di$, g_{kt} and g_{ht}, respectively, is given by,

$$\gamma_t = g_{TFPt} + \lambda g_{kt} + \mu g_{ht}, \tag{44}$$

where $\gamma_t \equiv \ln y_{t+1} - \ln y_t$, $g_{TFPt} \equiv \ln TFP_{t+1} - \ln TFP_t$, $g_{kt} \equiv \ln k_{t+1} - \ln k_t$, and $g_{ht} \equiv \ln h_{t+1} - \ln h_t$.

Clearly, by (2) and (43), the negative effect of income inequality on the level of TFP arises from the combination of the diminishing returns to capital in the production technology (i.e., $0 < \lambda + \mu < 1$) and the absence of a credit market that precipitates allocative inefficiency due to the persistence of unexploited arbitrage opportunities from interpersonal differences in the marginal products of capital.

Lemma 6: *The period t TFP for this economy satisfies*

$$LnTFP_t = -0.5(\Delta_t)^T(\lambda(1-\lambda), \mu(1-\mu), -2\lambda\mu)^T, \tag{45}$$

and it converges to a constant $TFP(\Delta)$ when Δ_t converges to Δ. Also, if $\lambda = 0$ then a higher value of Δ corresponds to a lower value of $TFP(\Delta)$.

Proof: See Appendix.

Simulations of (45) for a large number of cases when $\lambda > 0$ show that an increase in income inequality would correspond to a lower level of TFP. By Lemma 5, as income inequality monotonically converges to a steady state so does the TFP. Consequently, an increase in income inequality by itself would be likely to decrease TFP in all future periods.

3.2.2 ***Endogenous Growth***

Intuitively, to accommodate perpetual growth such that both the capital-output ratio and the marginal product of physical capital remain constant, from Lemma 6 and (44), $\gamma_t = \left(\frac{\mu}{1-\lambda}\right) g_{ht}$. However, by (8), the above definition of g_{ht} implies, by Lemma 2, that $g_{ht} = \left(\frac{\beta}{1-\alpha}\right)\gamma_t$. The consistency of the above two conditions implies that $\frac{\beta}{1-\alpha} = \frac{1-\lambda}{\mu}$. The following Proposition formally establishes the conditions for persistent endogenous growth.

PROPOSITION 1: *The model exhibits endogenous growth if and only if* $\frac{\beta}{1-\alpha} = \frac{1-\lambda}{\mu}$.

Proof: First, considering the redistributive scheme with income tax, we write the system of linear equations (32) and (34) in a matrix form, and get

$$m_{t+1} = Bm_t + D\Lambda_t + E, \tag{46}$$

where

$$m_{t+1} \equiv \begin{bmatrix} m_{kt+1} \\ m_{ht+1} \end{bmatrix},$$

$$B \equiv \begin{bmatrix} \lambda & \mu \\ \beta\lambda & \alpha + \beta\mu \end{bmatrix}, D \equiv \begin{bmatrix} \tau(2-\tau) \\ \beta\tau(2-\tau) \end{bmatrix},$$

$$E \equiv \begin{bmatrix} \ln \bar{s}_2 + (1-\lambda-\mu)\ln l \\ \ln \kappa - \sigma^2/2 + \beta \ln \bar{s}_1 + \beta(1-\lambda-\mu)\ln l \end{bmatrix}.$$

Rearranging (46), we get

$$(I - BL)\, m_{t+1} = E + D\Lambda_t, \tag{47}$$

$\Leftrightarrow$

$$m_{t+1} = [I-B]^{-1} E + [I - BL]^{-1} D\Lambda_t. \tag{48}$$

It follows from the above equation that if $|I - B| \neq 0$, then m_t converges to a stationary state. Thus, the endogenous growth of m_t requires $|I - B| = 0$, i.e.,

$$\left| \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & \mu \\ \beta\lambda & \alpha + \beta\mu \end{bmatrix} \right| = 0.$$

$\Rightarrow$

$$\left| \begin{bmatrix} 1-\lambda & -\mu \\ -\beta\lambda & 1-(\alpha+\beta\mu) \end{bmatrix} \right| = 0.$$

$\Rightarrow (1-\alpha)(1-\lambda) - \beta\mu = 0$; or, equivalently, $\frac{\mu}{1-\lambda} = \frac{1-\alpha}{\beta}$.

Note that the above condition is also sufficient for perpetual endogenous growth.

Rearranging (46), we get

$$m_{t+1} - m_t = E + [B - I]\, m_t + D\Lambda_t. \tag{49}$$

Recall that, m_t denotes the vector consisting of the means of $\ln k^i_{t+1}$ and $\ln h^i_{t+1}$; we denote $m^k_{t+1} - m^k_t$ by $g_{m^k}(t)$ and $m^h_{t+1} - m^h_t$ by $g_{m^h}(t)$, respectively, and decompose the singular matrix $[B - I]$ to rewrite (49) as follows:

$$\begin{bmatrix} g_{m^k}(t) \\ g_{m^h}(t) \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} \lambda - 1 \\ \beta\lambda \end{bmatrix} \begin{bmatrix} m_{kt} - \frac{\mu}{(1-\lambda)} m_{ht} \end{bmatrix} + \begin{bmatrix} \tau(2-\tau) \\ \beta\tau(2-\tau) \end{bmatrix} \Lambda_t, \tag{50}$$

where $e_1 \equiv \ln \bar{s}_2 + (1-\lambda-\mu)\ln l$ and $e_2 \equiv \ln \kappa - \sigma^2/2 + \beta \ln \bar{s}_1 + \beta(1-\lambda-\mu)\ln l$.

The period t growth rates g_{kt} and g_{ht} of the average stocks of physical and human capital, respectively, are given by

$$g_{kt} = g_{m^k}(t) + \left(\Delta_{kt+1}^2 - \Delta_{kt}^2\right)/2, \tag{51}$$

and

$$g_{ht} = g_{m^h}(t) + \left(\Delta_{ht+1}^2 - \Delta_{ht}^2\right)/2. \tag{52}$$

According to Lemmas 5 and 6, both the income inequality and the TFP converge to their respective steady states. It follows, therefore, that g_{kt} and g_{ht} approach $g_{m^k}(t)$ and $g_{m^h}(t)$, respectively, and, by (44), (45) and (50)-(52), we get an endogenous growth of per capita income that lasts forever.[3]

In the case of the education finance scheme, we repeat the above exercise after replacing (32) with (37). $\square$

3.2.3 *How Inequality Retards Growth*

Next, we continue our discussion of the way in which the rise in income inequality hinders economic growth. The adverse effect of income inequality on TFP discussed earlier also explains why the growth rate of output declines when income inequality rises. We turn to this issue after presenting Lemma 7.

Lemma 7: *If $\alpha + \beta\mu/(1-\lambda) = 1$, then, under both schemes of redistribution, the per capita income growth rate γ_t evolves as follows:*

$$\begin{aligned} \gamma_t &= \Phi + (\lambda + \mu\beta)(1 - \lambda - \mu)\ln l \\ &\quad + (1 - \Psi_1 L)\,\Lambda_{t+1} + \alpha\lambda\left(\mu m_{ht} - (1-\lambda)m_{kt}\right), \end{aligned} \tag{53}$$

where $\Phi \equiv \lambda \ln \bar{s}_2 + \mu\left(\ln\kappa - \sigma^2/2 + \beta\ln\bar{s}_1\right)$, under the income tax scheme $0 < \Psi_1 \equiv \left(1 - \left(1 - \alpha\left(1-\lambda\right)\right)\tau\left(2-\tau\right)\right) < 1$ or under the education finance scheme $0 < \Psi_1 \equiv 1 - \beta\mu\tau\left(2-\tau\right) < 1$, and L denotes the lag operator.

Proof: See Appendix.

Clearly, as the income inequality converges to a steady state following Lemmas 3 and 5, by (53), the growth rate of per capita income also approaches a balanced growth state.

Equation (53), together with (33) or (38), (35) and (36) or (39) describe the model's unique transitional dynamics for income inequality and the rate of growth.

[3]Clearly, on a balanced growth path, $m_{kt} - \frac{\mu}{1-\lambda}m_{ht}$ must be constant, or, equivalently, $g_{m^k}(t) = \frac{\mu}{1-\lambda}g_{m^h}(t)$, and hence, by (51), (52) and (44), $\gamma_t = g_{kt} = \frac{\mu}{1-\lambda}g_{ht}$.

Special Case: Without Physical Capital ($\lambda = 0$) For the special case without bequests and physical capital (i.e., $\lambda = 0$), when the model resembles the environment of Benabou (2002), we get a clear and insightful result regarding the growth-inequality transitional dynamics. In particular, if $\lambda = 0$, by (53), it follows that

$$\begin{aligned} \gamma_t &= \mu\left(\ln\kappa - \sigma^2/2 + \beta\ln\bar{s}_1 + \beta(1-\mu)\ln l\right) \\ &\quad + (1 - \Psi_2 L)\,\Lambda_{t+1}, \end{aligned} \tag{54}$$

where $0 < \Psi_2 \equiv (1 - (1-\alpha)\,\tau\,(2-\tau)) < 1$. Also, from Lemma 3 and equation (35), it follows that

$$\Lambda_{t+1} = \frac{\mu^2\sigma^2}{2} + (1 - \beta\mu\tau)^2\,\Lambda_t. \tag{55}$$

Next, substituting equation (55) into (54) yields

$$\gamma_t = \mu\left(\ln\kappa - (1-\mu)\,\sigma^2/2 + \beta\ln\bar{s}_1 + \beta(1-\mu)\ln l\right) - \alpha\beta\mu\tau^2\Lambda_t. \tag{56}$$

Clearly, we note from equation (56),[4] that income inequality retards economic growth along the transitional growth path. To uncover the underlying economic intuition, let us separately examine how the rising inequality affects the growth rates of input and productivity. By (45) and (52), if $\lambda = 0$, then the growth rates of TFP and human capital are given by

$$g_{TFPt} = -\frac{(1-\mu)}{\mu}\left(\Lambda_{t+1} - \Lambda_t\right), \tag{57}$$

and

$$g_{ht} = g_h - Q(\tau)\Lambda_t, \tag{58}$$

where $g_h = \ln\kappa - (1-\mu)\,\sigma^2/2 + \beta\ln\bar{s}_1 + \beta(1-\mu)\ln l$ denotes the stationary growth rate of human capital without any redistributive taxes and $Q(\tau) \equiv \alpha\beta\tau^2 > 0$ denotes the magnitude of the adverse effect of inequality on the growth rate of average human capital stock. In the presence of a redistributive tax regime, as inequality increases, possibly after a one-time reduction in progressivity, the economy's TFP declines, and its average stock of human capital grows at a lower rate. Consequently, the economy's output growth decelerates as well. In other words, the model preserves the typical harmful effect of income inequality on economic growth that arises from a combination of a production technology with diminishing returns to capital and the absence of a credit market for trading capital.

Note also that the coefficient of Λ_t in (55) is strictly less than unity, and hence, following a reduction in τ, income inequality monotonically increases to a new steady state. Also, as income inequality increases over time, by (56), γ_t declines monotonically.

[4] Equation (56) also provides a dynamic restriction on the growth-inequality relationship which is comparable to what Persson and Tabellini (1994) estimated, but provides an alternative interpretation of the data, since, unlike Persson and Tabellini (1994), the effect of income inequality on the growth rate does not work only through changes in the income tax rate, but also works through the model's transitional dynamics.

General Case: With Physical Capital $(\lambda > 0)$ We extend the above discussion to the general case with illustrative simulations as well as with the following Lemma:

Lemma 8: *Along the transition path, a rise in income inequality in period t, by itself, decreases the growth rate γ_t of per capita income between periods t and $t+1$. Also, if income inequality increases because of a reduction in τ, then once the growth rate begins to decline it must continue to decline for all subsequent periods. In other words, if for some period t, $\gamma_{t+1} < \gamma_t$,* then for all $n = 0, 1, 2...$, $\gamma_{t+n+1} < \gamma_{t+n}$.

Proof. By comparing Ψ_2 with Ψ_1 from equations (53) and (54), we get $\Psi_2 - \Psi_1 = \alpha\lambda\tau(2-\tau) > 0$, or, $\Psi_2 > \Psi_1$ under the income tax scheme, and $\Psi_1 = \Psi_2$ under the education finance scheme. Consequently, if $\lambda > 0$, the partial negative effect of the period t income inequality on the growth rate of per capita income between periods t and $t+1$ continues to exist; but its magnitude may either get smaller or remain the same, as the special case with $\lambda = 0$, depending upon the nature of the underlying scheme of redistribution. Note that under the income tax scheme, the negative effect of inequality on growth decreases as the output share λ of physical capital increases. Thus, unlike the case when $\lambda = 0$, it may take a few additional periods before the partial negative effects of rising inequality on growth outweigh the one-time boost in the growth rate that comes from the reduction in τ. However, once $\gamma_{t+1} < \gamma_t$ for some period t, $\gamma_{t+n+1} < \gamma_{t+n}$ for all $n = 0, 1, 2...$, according to the monotonicity property described by Lemma 5 and the endogenous relationship between income inequality and growth rate described by Lemma 7. □

A numerical illustration To illustrate the above lemma using an empirically reasonable numerical example, we choose a set of plausible values for the model's parameters from Barro, Mankiw and Sala-i-Martin (1995) and Benabou (2002), except for the education technology parameters, which we calibrate to yield an endogenous growth rate of 2% per annum, to match the long-run growth experience in the US. In the above benchmark model for the US economy, we simulate the changes in growth rate sequence in response to a permanent reduction of τ from 30% to 10% using equation (53), and present the result in Figure 1 below. We note that, following a reduction in progressivity τ, the growth rate of per capita income jumps up (by about two and a half times in this simulation) immediately. However, as the income inequality increases sufficiently following the policy change, the growth rate eventually declines in all subsequent periods before converging to a new balanced growth state.

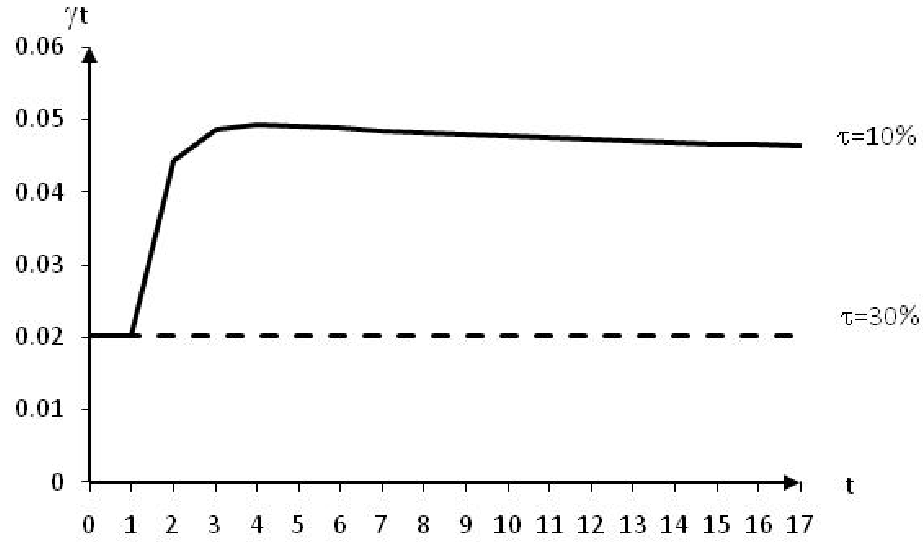

Figure 1: Under the income tax scheme, $\lambda = 0.3$, $\mu = 0.5$, $\alpha = 0.71$, $\beta = 0.4$, $\sigma^2 = 1$, $\rho = 0.4$ and $\kappa = 4.905$.

Also note that the post-tax reduction limiting growth rate settles down at about two percentage points higher than the growth rate prior to the tax reduction. This latter feature turns out to be a robust property of this model for any arbitrary parameter values, as we prove in the following section.

3.2.4 *Long-run Inequality and the Balanced Growth State*

We define the balanced growth state of this model to be the one where the endogenous growth rate is time invariant. By (53) the endogenous growth rate approaches such a balanced growth state over time. Also, from Lemma 5, we can see that, for all $0 < \tau < 1$, there exists a constant $\Lambda^j < \infty$, $j = Y$ or E, depending upon whether the scheme of redistribution is with income tax or education finance, respectively, such that, if the initial degree of income inequality $\Lambda_0 = \Lambda^j$, then at all periods t, $\Lambda_t = \Lambda^j$, where

$$\Lambda^j = \frac{\mu^2 \left(1 + \lambda\alpha \left(1 - \tau\right)\right)}{\tau \left(1 - \alpha\right) \left(1 - \lambda\alpha \left(1 - \tau\right)\right) \left(2 + 2\alpha\lambda \left(1 - \tau\right) - \tau \left(1 - \alpha\right)\right)} \frac{\sigma^2}{2}, j = Y, \qquad (59)$$

$$\Lambda^j = \frac{\left(1 + \lambda\alpha\right) \mu^2}{\left(1 - \lambda\alpha\right) \left(\left(1 + \lambda\alpha\right)^2 - \left(\left(1 + \alpha\lambda\right) \left(1 - \tau\right) + \left(\alpha + \lambda\right) \tau\right)^2\right)} \frac{\sigma^2}{2}, j = E. \qquad (60)$$

PROPOSITION 2: *If the initial income inequality Λ_0 equals its steady state value Λ^j given by (59)-(60) and the endogenous growth condition holds, such that* $\frac{\beta}{1-\alpha} = \frac{1-\lambda}{\mu}$, *then the economy follows a balanced growth path such that the growth rate of per capita income $\gamma_t = \gamma^j$, $j = Y, E$, where,*

$$\gamma^j = \frac{1}{1-\alpha\lambda}\left(\psi + \mu \ln \kappa + (1-\alpha)(1-\lambda-\mu)\ln l + \left((\lambda+\mu)^2 - \mu\right)\sigma^2/2 - \Omega^j \Lambda^j\right), \tag{61}$$

and the negative trade-off Ω^j between income inequality and the growth rate as a function of the degree τ of progressivity are given by

$$\Omega^j \equiv \frac{\Theta_1}{\mu^2\left(1+\lambda\alpha(1-\tau)\right)} > 0,\ j = Y, \tag{62}$$

$$\Omega^j \equiv \frac{\Theta_2}{\mu^2(1+\lambda\alpha)} > 0,\ j = E, \tag{63}$$

where

$$\begin{aligned}\Theta_1 \equiv\ & \mu^2\left(1+\lambda\alpha(1-\tau)\right)\left(\alpha + (1-\alpha)(1-\tau)^2\right) - \mu^2\left(1+\lambda\alpha(1-\tau)\right) \\ & + (\lambda+\mu)^2\left(1-\lambda\alpha(1-\tau)\right)\left(\left(1+\lambda\alpha(1-\tau)\right)^2 - \left(\alpha + (\alpha\lambda-\alpha+1)(1-\tau)\right)^2\right),\end{aligned}$$

$$\begin{aligned}\Theta_2 \equiv\ & \mu^2(1+\lambda\alpha)\left(\alpha + (1-\alpha)\lambda + (1-\alpha)(1-\lambda)(1-\tau)^2\right) - \mu^2(1+\lambda\alpha) \\ & + (\lambda+\mu)^2(1-\lambda\alpha)\left((1+\lambda\alpha)^2 - \left(\alpha+\lambda+(1-\alpha)(1-\lambda)(1-\tau)\right)^2\right).\end{aligned}$$

Proof: See Appendix.

From the above proposition we conclude that, for a given redistributive regime, across different steady states a higher income inequality corresponds to a lower balanced growth rate. Thus we establish a negative relationship between income inequality and the growth rate of per capita income both along the transitional path and across different balanced growth states that share a common redistributive regime.

A question then arises naturally: can we redress this problem of growth deceleration with rising income inequality by progressively redistributing income from the rich to the poor? The following section takes up this issue.

4 Progressive Redistribution and Growth

We consider the merit of redistributing resources from richer to poorer people in a progressive manner, such that, in the underlying redistributive scheme, the degree of progressivity $\tau \geq 0$. We ask what value of τ would be consistent with a dynamically efficient consumption path for risk-neutral agents. Clearly, as τ decreases, by (59) and (60), income inequality increases and approaches infinity.[5] Naturally, a related question arises if the growth rate could nevertheless remain finite. The following lemma addresses this concern.

Lemma 9: *As τ goes to zero, the balanced growth rate, γ^j, $j = Y$ or E, approaches a finite limit γ such that*

$$\gamma = \frac{1}{1-\alpha\lambda}\left(\psi + \mu \ln \kappa + (1-\alpha)(1-\lambda-\mu)\ln l - \frac{(1-\alpha\lambda-\mu)\,\mu}{1-\alpha\lambda}\frac{\sigma^2}{2}\right) < \infty. \tag{64}$$

Proof: Substituting (59) or (60) into (61) and setting $\tau = 0$, we can get (64). □

Special Case: $\lambda = 0$

A lower value of τ increases income inequality, which in turn retards economic growth, as the previous section discusses. In particular, if $\lambda = 0$ then by (56)-(58), rising inequality, due to a lower progressivity in redistribution, by itself clearly lowers TFP and retards the growth rate of average human capital, and hence that of the per capita output. Nevertheless, by (56), a lower degree τ of progressivity implies a higher growth rate of per capita income for any given level of income inequality. By (58), this positive effect on the growth rate is a result of the improvement in the average human capital of the parents, which influences the quality of the education received by the children, who are the producers in the next generation.[6] A lower progressivity means returning resources from those with a lower human capital to parents with higher human capital. Because the education technology exhibits increasing returns to scale, such a transfer of resources results in an overall improvement in the average quality of the cohort of parents, and hence in the effectiveness of the education expenditure in all subsequent periods. Thus, a less progressive redistribution would raise the growth rate of the average human capital, which in turn would boost the growth rate of the per capita income. The above gain shows up as an immediate increase in the growth rate of the average human capital, which partly offsets the growth-retarding effect of the subsequent increase in income inequality discussed earlier, following (58) and which is summarized in Lemma 8.

However, without further analysis, the overall effect on growth of lowering the progressivity of redistribution would appear to be ambiguous. In the next section, we prove

[5] A risk-averse agent may not like this scenario, and it is well-known that the welfare-maximizing τ in that context would be a function of the controversial and subjective risk-aversion parameter. We leave this normative issue aside and focus on the positive question posed above.

[6] Note that $\alpha + \beta > 1$ implies that an increase in the curvature of the education technology with higher values of α and β would enhance the beneficial effect on the growth of average human capital obtained by lowering the progressivity parameter τ.

unambiguously that a reduction in the degree of redistribution τ both increases the long run balanced growth rate and pushes the whole transition path upward to that state, such that the growth rates remain higher in every period under the new regime with reduced progressivity.

Lemma 10: *(Special Case, $\lambda = 0$)* $\frac{\partial \gamma}{\partial \tau} < 0$.

Proof. If $\lambda = 0$ then, by (56),

$$\gamma = \mu \left(\ln \kappa - (1-\mu)\,\sigma^2/2 + \beta \ln \bar{s}_1\right) + (1-\alpha)\,(1-\mu)\ln l - \alpha\beta\mu\tau^2\Lambda, \tag{65}$$

where, by (59) or (60), the steady state income inequality Λ satisfies,

$$\Lambda = \frac{\mu^2}{1-(1-\beta\mu\tau)^2}\frac{\sigma^2}{2}. \tag{66}$$

Note that, by (66), $\tau^2\Lambda = \frac{\tau\mu\sigma^2/2}{\beta(2-\tau\beta\mu)}$, which is a smooth function of τ for all $0 \le \tau < 1$, and equals 0 when $\tau = 0$. This means that as τ decreases to zero, γ approaches a finite limit. Consequently, by equation (65), if $\tau = 0$ then

$$\gamma = \mu \left(\ln \kappa - (1-\mu)\,\sigma^2/2 + \beta \ln \bar{s}_1\right) + (1-\alpha)\,(1-\mu)\ln l. \tag{67}$$

Differentiating (65) with respect to τ yields

$$\frac{\partial \gamma}{\partial \tau} = -\frac{(1-\beta\mu)\mu^2\sigma^2}{(2-\beta\mu\tau)^2} < 0. \ \square \tag{68}$$

General Case: $\lambda > 0$

Lemma 11: *(General Case, $\lambda > 0$)* $\frac{\partial \gamma}{\partial \tau} < 0$.

Proof. See Appendix.

4.1 *A Note on the Growth Rate of Consumption Per Capita*

The above discussions establish that the balanced growth rate always increases in response to a lower degree of progressivity. Note also that this increase comes as a 'free-lunch', since we allow a subsidy sufficient to keep the ratio of saving to disposable income constant, at the laissez-faire rate. Consequently, by (5)-(7) and Lemma 2, it follows that the growth rate of consumption per capita must be equal to the growth rate of output per capita.

Figure 1 illustrates how the common growth rate of output and consumption per capita jumps up immediately after a reduction in the progressivity parameter τ. Clearly, if $\lambda = 0$

then, by (56), as τ decreases, the adverse effect of inequality decreases and the growth rate increases. From Lemma 8 and equation (53), similar conclusions follow for the case with $\lambda > 0$. The following lemma establishes that claim explicitly, and elaborates the channel through which this growth boost occurs.

Lemma 12: *For a given level of inequality, a once and for all reduction in τ increases the common growth rate of output and consumption per capita.*

Proof. See Appendix.

Lemma 13: *Suppose that the economy is on a balanced growth path such that for all t, the common time invariant growth rate $\gamma_t^j(\tau_1) = \gamma^j(\tau_1), j = Y, E$. If the government reduces the degree of progressivity of redistribution by setting a new policy $\tau_2 < \tau_1$ at period $t = T$ then for all $n = 0, 1, 2, ..., \gamma_{T+n}^j(\tau_2) > \gamma^j(\tau_1)$.*

Proof. By Lemma 12, the growth rate jumps up immediately to a higher level in response to a reduction in τ. By Lemmas 3, 5, 8 and 11, the growth rate must converge to a limit that lies strictly above the value prior to the reduction of τ. By Lemma 8, if the growth rate declines, it must do so monotonically. Consequently, it must stay above the pre-tax reduction balanced growth path. □

4.2 *Dynamic Efficiency*

We define an equilibrium as *dynamically inefficient* if a social planner can redistribute resources to increase the equilibrium growth rate of per capita consumption in some period t. An equilibrium is *dynamically efficient* if and only if it is not *dynamically inefficient*.

PROPOSITION 3: *The economy's endogenous growth rate follows a dynamically efficient path if and only if the degree of progressivity in either the income tax regime or the education finance regime is set to zero.*

Proof: For any arbitrarily small degree of progressivity, a reduction in τ to lower progressivity would, according to Lemma 13, lift the entire sequence of growth rates above its previous state. In other words, the optimal degree of progression must be set to zero for the economy to reach its maximal possible growth path, or, equivalently, its dynamically efficient equilibrium. □

If we consider the case where agents get disutility from work, by (11)-(12), the above change in policy would encourage each agent to increase the labor supply, which would boost the growth rate further. If we consider risk-neutral agents then the above policy would not only be dynamically efficient but would also maximize welfare.

5 Consistency of the Model with the Data

Gordon and Li (2009) report that the countries with per capita incomes less than $745 collect only about 36% of their revenue from income taxes, as opposed to the 54% that countries with per capita incomes above $9,206 collect. Also, according to the same study, the tax to GDP ratio in the less developed countries is only about 50% of that of their developed counterparts. These findings may simply reflect that the less developed countries do not have an efficient institutional framework for collecting income taxes. However, they may also signal that the low income countries have a lower degree of progressivity than their developed counterparts. Sicat and Virmani (1988) report that less developed countries tend to have a relatively low average marginal income tax rate. This paper provides one reason why such strategy may be helpful for fostering economic growth in less developed countries. Using a panel data across a large number of countries, Jha (1999) reports that an increase in the average marginal tax rate decreases growth at a low level of GDP, while it increases growth at a high level of GDP. This is consistent with our model's prediction together with the finding of Trostel (2004) that the evidence of increasing returns to scale in education technology is only strong among the less developed countries. Park (1998) also reports a negative effect of redistributive tax on economic growth.

On the impact of redistribution on growth, the work of Easterly and Rebelo (1993) and Perotti (1996) are worth noting. Using several measures of redistribution (marginal tax rates, average tax rates, social spending), Easterly and Rebelo (1993) find that redistribution is likely to have a positive impact on growth. Perotti (1996) also reports that a higher marginal tax rate has a positive impact on growth. These reports are in contrast with the evidence gathered from less developed countries, indicating that the endogenous growth model that we provide here is possibly more suitable for studying economic problems in the developing countries rather than the developed countries.

6 Concluding Remarks

We assume a production technology with diminishing returns to capital and no credit markets in order to demonstrate that an exogenous decrease in income inequality always lifts the growth rate of per capita income up in all subsequent periods. However, we prove that if the same model generates growth endogenously, using an "increasing return to scale" (IRS) education technology, a progressive redistribution to reduce income inequality would unambiguously push the economy down to a dynamically inefficient growth path. We provide empirical evidence from recent studies in labor economics that the education technology does exhibit such behaviour, especially in less developed countries.

Note, however, that the use of an IRS education technology is neither necessary nor sufficient for establishing no progressivity as the dynamically efficient policy of redistribution. A sufficient condition for the above outcome to hold is that the economy exhibits endogenous growth such that returns to capital or, equivalently, its marginal product approaches a constant. This condition requires in turn that an increment of the average stock of physical capital must be accompanied by a proportionate increase in the average stock of human capital. A progressive redistribution raises the average stock of both types of capital if and only if both production and education technology exhibit diminishing returns but endogenous growth cannot occur under such condition. Consequently, a policy of progressive redistribution ends up slowing down the engine of endogenous growth. The above result is our main contribution. We elaborate the specific details of our model's result below.

Following a reduction in the degree of progressivity from any arbitrary value, the common growth rate of consumption and output per capita always jumps up, primarily because the reduced progressivity, coupled with increasing returns to scale education technology, implies an increase in the growth rate of the average human capital. Subsequently, income inequality increases, raising interpersonal differences in productivity, which in turn lowers TFP, and hence lowers economic growth. Nevertheless, the policy-induced boost in the growth rate of average human capital enhances the common growth rate of output and consumption per capita sufficiently to outweigh any slow-down in subsequent periods due to the accompanying increase in income inequality. Consequently, the whole consumption path moves to a higher level in all periods. The above result implies that the policy of eliminating progressivity from a redistributive policy altogether would be dynamically efficient.

If we relax the model's assumptions and allow for a fully functional credit market, a competitive market for trading capital goods, disutility from work and redistributive policies without offsetting subsidies of education and bequests, then the above result will only be strengthened. Finally, if we focus our attention on risk-neutral agents only, then a progressive redistribution also lowers economic welfare in our model.

Besides establishing this key result relating to the merit of progressive redistribution, our contributions include various features that may benefit future research in more than one dimension. We offer a new model of endogenous growth and income distribution which is consistent with the evidence reported by prominent labor economists. The model should be especially useful in less developed countries for examining macroeconomic outcomes related to growth and income inequality. Our extensive discussion of the transitional dynamics in a model where growth and income inequality arise and evolve endogenously makes a special contribution to a relatively unexplored area of the literature. In particular, we provide an explicitly derived analytical expression to characterize growth-inequality dynamics which can aid future empirical research in characterizing theoretical underpinning behind the so-called growth-inequality relationship.

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Appendix

PROOFS OF LEMMAS 1 AND 2:

Under the income tax scheme: By (7), (8) and (9), we rewrite (10) as follows:

$$\ln U\left(h_t^i, k_t^i, M_t; T\right) = \max_{s_{1t}^i, s_{2t}^i, l_t^i} \left\{ \begin{array}{l} (1-\rho)\left[\ln\left(1 - s_{1t}^i - s_{2t}^i\right) - \ln\left(1+\theta\right) + \ln \hat{y}_t^i - \left(l_t^i\right)^\eta\right] \\ +\rho E_t\left[\ln U\left(h_{t+1}^i, k_{t+1}^i, M_{t+1}; T\right)\right] \end{array} \right\}. \tag{A.1}$$

Agent solves (A.1) subject to (2) and (4) and

$$h_{t+1}^i = \kappa\left(\left(1+d\right) s_{1t}^i\right)^\beta \xi_{t+1}^i \left(k_t^i\right)^{\beta\lambda(1-\tau)} \left(h_t^i\right)^{\alpha+\beta\mu(1-\tau)} \left(l_t^i\right)^{\beta(1-\lambda-\mu)(1-\tau)} \left(\tilde{y}_t\right)^{\beta\tau}, \text{ and} \tag{A.2}$$

$$k_{t+1}^i = \left(1+v\right) s_{2t}^i \left(k_t^i\right)^{\lambda(1-\tau)} \left(h_t^i\right)^{\mu(1-\tau)} \left(l_t^i\right)^{(1-\lambda-\mu)(1-\tau)} \left(\tilde{y}_t\right)^\tau. \tag{A.3}$$

We guess the value function as: $\ln U\left(h_t^i, k_t^i, M_t; T\right) = Z_1 \ln h_t^i + Z_2 \ln k_t^i + B_t$. Then by substituting this value function into (A.1), we get

$$\begin{aligned} Z_1 \ln h_t^i + Z_2 \ln k_t^i + B_t = \; & (1-\rho)\left(\begin{array}{l} \ln\left(1 - s_{1t}^i - s_{2t}^i\right)/\left(1+\theta\right) \\ +(1-\lambda-\mu)\left(1-\tau\right)\ln l_t^i + \tau \ln \tilde{y}_t - \left(l_t^i\right)^\eta \end{array} \right) \\ & + \left(1 - \rho + \rho\beta Z_1 + \rho Z_2\right)\lambda\left(1-\tau\right)\ln k_t^i \\ & + \left(\left(1 - \rho + \rho\beta Z_1 + \rho Z_2\right)\mu\left(1-\tau\right) + \rho\alpha Z_1\right)\ln h_t^i \\ & + \rho\left(\begin{array}{l} Z_1\left(\begin{array}{l} \ln\kappa + \beta\ln(1+d)s_{1t}^i - \sigma^2/2 \\ +\beta\left(1-\lambda-\mu\right)\left(1-\tau\right)\ln l_t^i + \beta\tau\ln\tilde{y}_t \end{array} \right) \\ +Z_2\left(\ln\left(1+v\right)s_{2t}^i + \left(1-\lambda-\mu\right)\left(1-\tau\right)\ln l_t^i + \tau\ln\tilde{y}_t\right) + B_{t+1} \end{array} \right). \end{aligned} \tag{A.4}$$

Taking partial differentials with respect to $\ln k_t^i$ and $\ln h_t^i$ yields

$$Z_1 = \left(1 - \rho + \rho\beta Z_1 + \rho Z_2\right)\mu\left(1-\tau\right) + \rho\alpha Z_1, \tag{A.5}$$

$$Z_2 = \left(1 - \rho + \rho\beta Z_1 + \rho Z_2\right)\lambda\left(1-\tau\right). \tag{A.6}$$

Rearranging equations (A.5) and (A.6) yields

$$Z_1 = \frac{(1-\rho)\mu\left(1-\tau\right)}{\left(1-\rho\alpha\right)\left(1-\rho\lambda\left(1-\tau\right)\right) - \rho\beta\mu\left(1-\tau\right)}, \tag{A.7}$$

$$Z_2 = \frac{(1-\rho\alpha)(1-\rho)\lambda(1-\tau)}{(1-\rho\alpha)(1-\rho\lambda(1-\tau)) - \rho\beta\mu(1-\tau)}. \tag{A.8}$$

The values of human and physical capital, as expressed by their utility elasticities, are given by Z_1 and Z_2, respectively. Note that the tax rate τ can alter these values individually but does not alter the relative value of human to physical capital, $\frac{\mu}{\lambda(1-\rho\alpha)}$, which increases with the output elasticity of human capital μ, the neighborhood effect α and patience ρ, but remains unaffected by the quality of education β. We can then verify the guess and confirm the existence of (A.4).

The first-order conditions of (A.1) with respect to the saving rates and labor supply are

$$\frac{1-\rho}{1-s_{1t}^i - s_{2t}^i} = \rho\left(\frac{\partial \ln U_{t+1}^i}{\partial \ln h_{t+1}^i}\frac{\partial \ln h_{t+1}^i}{\partial s_{1t}^i} + \frac{\partial \ln U_{t+1}^i}{\partial \ln k_{t+1}^i}\frac{\partial \ln k_{t+1}^i}{\partial s_{1t}^i}\right), \tag{A.9}$$

$$\frac{1-\rho}{1-s_{1t}^i - s_{2t}^i} = \rho\left(\frac{\partial \ln U_{t+1}^i}{\partial \ln h_{t+1}^i}\frac{\partial \ln h_{t+1}^i}{\partial s_{2t}^i} + \frac{\partial \ln U_{t+1}^i}{\partial \ln k_{t+1}^i}\frac{\partial \ln k_{t+1}^i}{\partial s_{2t}^i}\right), \tag{A.10}$$

$$\begin{aligned}(1-\rho)\,\eta\left(l_t^i\right)^{\eta-1} &= (1-\rho)(1-\lambda-\mu)(1-\tau)/l_{1t}^i \\ &\quad + \rho\left(\frac{\partial \ln U_{t+1}^i}{\partial \ln h_{t+1}^i}\frac{\partial \ln h_{t+1}^i}{\partial l_t^i} + \frac{\partial \ln U_{t+1}^i}{\partial \ln k_{t+1}^i}\frac{\partial \ln k_{t+1}^i}{\partial l_t^i}\right),\end{aligned} \tag{A.11}$$

where $\partial \ln k_{t+1}^i/\partial s_{1t}^i = 0$, $\partial \ln k_{t+1}^i/\partial s_{2t}^i = 1/s_{2t}^i$, $\partial \ln h_{t+1}^i/\partial s_{1t}^i = \beta/s_{1t}^i$, $\partial \ln h_{t+1}^i/\partial s_{2t}^i = 0$, $\partial \ln k_{t+1}^i/\partial l_{1t}^i = (1-\lambda-\mu)(1-\tau)/l_{1t}^i$ and $\partial \ln h_{t+1}^i/\partial l_{1t}^i = \beta(1-\lambda-\mu)(1-\tau)/l_{1t}^i$.

The above optimization problem (A.4) is strictly concave. Consequently, (A.9)-(A.11) are sufficient for the optimization exercise and Lemmas 1 and 2 follow immediately after we substitute (A.7) and (A.8) into (A.9)-(A.11).

Under the education finance scheme: By (7), (8) and (9), the Bellman equation is as follows

$$\ln U\left(h_t^i, k_t^i, M_t; T\right) = \max_{s_{1t}^i, s_{2t}^i, l_t^i}\left\{\begin{array}{l} (1-\rho)\left[\ln\left(\left(1-s_{1t}^i - s_{2t}^i\right)/(1+\theta)\right)\right. \\ \left. +\lambda \ln k_t^i + \mu \ln h_t^i + (1-\lambda-\mu)\ln l_t^i - \left(l_t^i\right)^\eta\right] \\ +\rho E_t\left[\ln\left(U\left(h_{t+1}^i, k_{t+1}^i, M_{t+1}; T\right)\right)\right] \end{array}\right\}, \tag{A.12}$$

where h_{t+1}^i and k_{t+1}^i are given by (27) and (28), respectively. Suppose that $\ln U\left(h_t^i, k_t^i, M_t; T\right) =$

$Z_1 \ln h_t^i + Z_2 \ln k_t^i + B_t$. Substituting into (A.12) yields

$$\begin{aligned} Z_1 \ln h_t^i + Z_2 \ln k_t^i + B_t = \; & \rho \left(Z_1 \left(\ln \kappa + \beta \ln(1+d) - \sigma^2/2 \right) + B_{t+1} \right) - (1-\rho) \ln (1+\theta) \\ & + (1-\rho) \ln \left(1 - s_{1t}^i - s_{2t}^i\right) + \rho \left(\beta Z_1 \ln (1+d)\, s_{1t}^i + Z_2 \ln (1+v)\, s_{2t}^i \right) \\ & + \rho\beta Z_1 \tau \ln \tilde{y}_t + (1-\rho+\rho\beta Z_1 (1-\tau) + \rho Z_2)(1-\lambda-\mu) \ln l_t^i \\ & + (1-\rho+\rho\beta Z_1 (1-\tau) + \rho Z_2) \lambda \ln k_t^i - (1-\rho) \left(l_t^i\right)^\eta \\ & + \left((1-\rho+\rho\beta Z_1 (1-\tau) + \rho Z_2) \mu + \rho\alpha Z_1 \right) \ln h_t^i. \end{aligned} \tag{A.13}$$

Taking the partial differentials of (A.13) with respect to $\ln k_t^i$ and $\ln h_t^i$ yields

$$Z_1 = \frac{(1-\rho)\mu}{(1-\rho\lambda)(1-\rho\alpha) - \rho\beta\mu(1-\tau)}, \tag{A.14}$$

$$Z_2 = \frac{(1-\rho\alpha)(1-\rho)\lambda}{(1-\rho\lambda)(1-\rho\alpha) - \rho\beta\mu(1-\tau)}. \tag{A.15}$$

The differences between equations (A.7), (A.8) and (A.14), (A.15) are the absence of $1-\tau$ multiplying the numerator and $\rho\lambda$ in the denominator. The first-order conditions for the saving rates are unchanged and are still given by equations (A.9) and (A.10). The first-order condition for the labor supply becomes

$$(1-\rho)\eta \left(l_t^i\right)^{\eta-1} = (1-\rho)(1-\lambda-\mu)/l_t^i + \rho \left(\frac{\partial \ln U_{t+1}^i}{\partial \ln h_{t+1}^i} \frac{\partial \ln h_{t+1}^i}{\partial l_t^i} + \frac{\partial \ln U_{t+1}^i}{\partial \ln k_{t+1}^i} \frac{\partial \ln k_{t+1}^i}{\partial l_t^i} \right). \tag{A.16}$$

The strict concavity of equation (A.13) implies that equations (A.9), (A.10) and (A.16) are sufficient for optimality, and thus Lemmas 1 and 2 are established, after substituting (A.14) and (A.15) into (A.9), (A.10) and (A.16). □

PROOF OF LEMMA 3: By assumption, at the initial date $t = 0$, the physical and human capitals are lognormally distributed. By (26) and (27), it follows that k_t^i and h_t^i remain lognormally distributed over time, and hence, by (2), y_t^i is lognormal and is given by,

$$\ln y_t^i = \lambda \ln k_t^i + \mu \ln h_t^i + (1-\lambda-\mu) \ln l_t^i. \tag{A.17}$$

By (11), it follows that the mean of the lognormal distribution of y_t^i is given by,

$$\int_0^1 \ln y_t^i di = \lambda m_{kt} + \mu m_{ht} + (1-\lambda-\mu) \ln l. \tag{A.18}$$

The variance of $\ln y_t^i$ is the sum of the variances of $\ln k_t^i$ and $\ln h_t^i$ plus the covariance of these two variables

$$\text{var}\left[\ln y_t^i\right] = \lambda^2 \Delta_{kt}^2 + \mu^2 \Delta_{ht}^2 + 2\lambda\mu cov_t. \tag{A.19}$$

The income per capita y_t, following Crow and Shimizu's (1988) description of properties of the moment generating function on a lognormal distribution, is

$$y_t = \int_0^1 y_t^i di = \exp\left(\int_0^1 \ln y_t^i di + \frac{1}{2}\operatorname{var}\left[\ln y_t^i\right]\right). \tag{A.20}$$

The median income is

$$y_{t,median} = \exp\left(\int_0^1 \ln y_t^i di\right). \tag{A.21}$$

In line with Benabou (2002), therefore, the inequality index is

$$\Lambda_t \equiv \log\left(\frac{y_t}{y_{t,median}}\right) = \frac{1}{2}\operatorname{var}\left[\ln y_t^i\right] = \left(\lambda^2\Delta_{kt}^2 + \mu^2\Delta_{ht}^2 + 2\lambda\mu cov_t\right)/2. \tag{A.22}$$

This proves Lemma 3. □

PROOF OF LEMMA 4: To derive the expression for the break-even point defined in (5), we note that the mean of y_t^i in logarithm, according to equation (A.20), satisfies

$$\ln y_t = \lambda m_{kt} + \mu m_{ht} + (1-\lambda-\mu)\ln l + \Lambda_t, \tag{A.23}$$

and the mean of $\left(y_t^i\right)^{1-\tau}$ in logarithm is

$$\ln \int_0^1 \left(y_t^i\right)^{1-\tau} di = (1-\tau)\left(\lambda m_{kt} + \mu m_{ht} + (1-\lambda-\mu)\ln l\right) + (1-\tau)^2\Lambda_t. \tag{A.24}$$

Taking the difference between the income before and after tax yields

$$\ln y_t - \ln \int_0^1 \left(y_t^i\right)^{1-\tau} di = \lambda\tau m_{kt} + \mu\tau m_{ht} + (1-\lambda-\mu)\tau\ln l + \tau(2-\tau)\Lambda_t. \tag{A.25}$$

It means that

$$\ln \tilde{y}_t = \lambda m_{kt} + \mu m_{ht} + (1-\lambda-\mu)\ln l + (2-\tau)\Lambda_t. \tag{A.26}$$

From the proof of Lemma 3, we know that the physical and human capital are distributed lognormally. Then, from the property of the moment generating function for lognormal distribution, we get

$$\ln k = m_k + \Delta_k^2/2 \text{ and } \ln h = m_h + \Delta_h^2/2. \tag{A.27}$$

Then substituting (A.27) into (A.26) yields (31). This proves Lemma 4. □

PROOF OF LEMMA 6: Substituting (A.27) into (A.23) yields

$$\begin{aligned} y &= k^\lambda h^\mu l^{1-\lambda-\mu}\exp\left(\left((\lambda-1)\lambda\Delta_k^2 + (\mu-1)\mu\Delta_h^2 + 2\lambda\mu cov\right)/2\right) \\ &\equiv TFP * k^\lambda h^\mu l^{1-\lambda-\mu}. \end{aligned} \tag{A.28}$$

Then we get (45). If $\lambda = 0$, by (A.22), we get

$$\ln TFP = -\frac{1-\mu}{\mu}\Lambda. \tag{A.29}$$

The above equation shows that a higher Δ or Λ leads to a lower TFP. Thus the proof of Lemma 6 is completed. □

PROOF OF LEMMA 7: Under the income tax scheme, by (A.23), we get the growth rate of per capita income as follows

$$\ln y_{t+1} - \ln y_t = \lambda\left(m_{kt+1} - m_{kt}\right) + \mu\left(m_{ht+1} - m_{ht}\right) + \Lambda_{t+1} - \Lambda_t. \tag{A.30}$$

Then, substituting (32)-(36) into (A.30) and by Lemma 3 and endogenous condition, we get (53). Similarly, under the education finance scheme, by substituting (34), (35), and (37)-(39) into (A.30) and by Lemma 3 and the endogenous condition, we get (53). This proves Lemma 7. □

PROOF OF PROPOSITION 2: Integrating both sides of (29) across all agents i we get,

$$\begin{aligned}\int_0^1 \ln y_{t+1}^i di &= \psi + \mu\ln\kappa + (1-\alpha)(1-\lambda-\mu)\ln l - \mu\sigma^2/2 + (\lambda+\beta\mu)\tau\ln\tilde{y}_t - \alpha\lambda\tau\ln\tilde{y}_{t-1} \\ &\quad + (\alpha + (\lambda+\beta\mu)(1-\tau))\int_0^1 \ln y_t^i di - \alpha\lambda(1-\tau)\int_0^1 \ln y_{t-1}^i di.\end{aligned} \tag{A.31}$$

From (A.20), we know

$$\int_0^1 \ln y_t^i di = \ln\int_0^1 y_t^i di - \frac{1}{2}\operatorname{var}\left[\ln y_t^i\right]. \tag{A.32}$$

Combining (A.32) with (A.31) yields

$$\begin{aligned}\ln\int_0^1 y_{t+1}^i di - \frac{1}{2}\operatorname{var}\left[\ln y_{t+1}^i\right] &= \psi + \mu\ln\kappa - \mu\sigma^2/2 + (1-\alpha)(1-\lambda-\mu)\ln l \\ &\quad + (\lambda+\beta\mu)\tau\ln\tilde{y}_t - \alpha\lambda\tau\ln\tilde{y}_{t-1} \\ &\quad + (\alpha+(\lambda+\beta\mu)(1-\tau))\left(\ln\int_0^1 y_t^i di - \frac{1}{2}\operatorname{var}\left[\ln y_t^i\right]\right) \\ &\quad - \alpha\lambda(1-\tau)\left(\ln\int_0^1 y_{t-1}^i di - \frac{1}{2}\operatorname{var}\left[\ln y_{t-1}^i\right]\right).\end{aligned} \tag{A.33}$$

Similarly, under the education finance scheme, integrating both sides of (30) across all agents i yields,

$$\int_0^1 \ln y_{t+1}^i di = \psi + \mu \ln \kappa - \mu\sigma^2/2 + (1-\alpha)(1-\lambda-\mu)\ln l + \beta\mu\tau \ln \tilde{y}_t + (\alpha+\lambda+\beta\mu(1-\tau))\int_0^1 \ln y_t^i di - \alpha\lambda \int_0^1 \ln y_{t-1}^i di. \tag{A.34}$$

Then, substituting (A.32) into (A.34) yields

$$\ln \int_0^1 y_{t+1}^i di - \frac{1}{2}\operatorname{var}\left[\ln y_{t+1}^i\right] = \psi + \mu \ln \kappa - \mu\sigma^2/2 + (1-\alpha)(1-\lambda-\mu)\ln l + (\alpha+\lambda+\beta\mu(1-\tau))\left(\ln \int_0^1 y_t^i di - \frac{1}{2}\operatorname{var}\left[\ln y_t^i\right]\right) - \alpha\lambda\left(\ln \int_0^1 y_{t-1}^i di - \frac{1}{2}\operatorname{var}\left[\ln y_{t-1}^i\right]\right) + \beta\mu\tau \ln \tilde{y}_t. \tag{A.35}$$

And by substituting (31) and the endogenous condition into (A.33) and (A.35), and by Lemma 5, we can prove (61). This proves Proposition 2. □

PROOF OF LEMMA 11: The balanced growth rate γ^E of per capita income under the education finance scheme is given by (61). Taking a differential w.r.t τ yields

$$\frac{\partial \gamma^E}{\partial \tau} = \frac{(1-\alpha)}{1-\alpha\lambda}\left((1-\lambda)(2-2\tau)\Lambda^E + \tau(2-\tau)(1-\lambda)\frac{\partial \Lambda^E}{\partial \tau}\right). \tag{A.36}$$

Taking the logarithm of (60) and taking a differential w.r.t τ yields

$$\frac{\partial \ln \Lambda^E(\tau)}{\partial \tau} = -\frac{2((1+\alpha\lambda)(1-\tau)+(\alpha+\lambda)\tau)(1-\alpha)(1-\lambda)}{(1+\lambda\alpha)^2 - ((1+\alpha\lambda)(1-\tau)+(\alpha+\lambda)\tau)^2}. \tag{A.37}$$

We know $\frac{\partial \Lambda}{\partial \tau} = \frac{\partial \ln \Lambda}{\partial \tau}\Lambda$. Then substituting (A.37) into (A.36) yields

$$\frac{\partial \gamma^E}{\partial \tau} = -\frac{1-\alpha}{1-\alpha\lambda}\left(\frac{2\tau(1-\alpha)(1-\lambda)(\alpha+\lambda)}{(1-\alpha)((1+\alpha\lambda)(2-\tau)+(\alpha+\lambda)\tau)}\right)\Lambda^E < 0. \tag{A.38}$$

The above equation shows that $\frac{\partial \gamma^E}{\partial \tau} < 0$ for all $\tau \geq 0$. And by Lemma 9, we can see that $\gamma^E(0)$ exists. This implies that γ^E decreases monotonically with $\tau \geq 0$. The balanced growth rate γ^E reaches a maximum at $\tau = 0$. This therefore implies that the optimal degree of redistribution is zero.

Similarly, under the income tax scheme, by taking a differential of (61) w.r.t τ, we get

$$\frac{\partial \gamma^Y}{\partial \tau} = \frac{(1-\alpha)}{1-\alpha\lambda}\left((2-2\tau)\Lambda^Y + \tau(2-\tau)\frac{\partial \Lambda^Y}{\partial \tau}\right). \tag{A.39}$$

Taking the logarithm of (59) and taking a differential w.r.t τ yields

$$\frac{\partial \ln \Lambda^Y(\tau)}{\partial \tau} = -\frac{2\alpha\lambda}{1-\lambda^2\alpha^2(1-\tau)^2} - \frac{2(1-\alpha)(\alpha\tau(1-\lambda)+(1+\alpha\lambda)(1-\tau))}{(1+\lambda\alpha(1-\tau))^2-((\lambda+\beta\mu)(1-\tau)+\alpha)^2}. \tag{A.40}$$

We know $\frac{\partial \Lambda}{\partial \tau} = \frac{\partial \ln \Lambda}{\partial \tau}\Lambda$. Then substituting (A.40) into (A.39) yields

$$\frac{\partial \gamma^Y}{\partial \tau} = -\frac{1-\alpha}{1-\alpha\lambda}\frac{2\alpha\tau^2(1-\alpha)(1+\alpha\lambda)\begin{pmatrix} 1-\lambda\alpha+\alpha\lambda^2 \\ +\lambda(3-\tau)(1-\tau)+\alpha\lambda\tau(2-\tau)(1-\lambda)\end{pmatrix}}{\left(1-\lambda^2\alpha^2(1-\tau)^2\right)\begin{pmatrix}(1+\lambda\alpha(1-\tau))^2 \\ -((\lambda+(1-\alpha)(1-\lambda))(1-\tau)+\alpha)^2\end{pmatrix}}\Lambda^Y < 0. \tag{A.41}$$

The above equation shows that $\frac{\partial \gamma^Y}{\partial \tau} < 0$ for all $\tau \geq 0$. And by Lemma 9, we can see that $\gamma^Y(0)$ exists. This implies that γ^Y decreases monotonically with $\tau \geq 0$. The balanced growth rate γ^Y reaches a maximum at $\tau = 0$. This therefore implies that the optimal degree of redistribution under the income tax scheme is zero. Hence, Lemma 11 is proved. □

PROOF OF LEMMA 12: Substituting (31) and the endogenous condition into (A.33) yield

$$\begin{aligned}(1-\alpha\lambda L)\gamma_t &= \psi+\mu\ln\kappa-\mu\sigma^2/2+(1-\alpha)(1-\lambda-\mu)\ln l \\ &+\Lambda_{t+1}^Y-\left(\alpha+(\lambda+\mu\beta)(1-\tau)^2\right)\Lambda_t^Y+\alpha\lambda\Delta_{kt}^2/2.\end{aligned} \tag{A.42}$$

By (33), (35), (36) and Lemma 3, we can get

$$\begin{aligned}\Lambda_{t+1} = {} & \mu^2\sigma^2/2+((\lambda+\mu\beta)(1-\tau)+\alpha)^2\Lambda_t \\ & -\alpha(\alpha/2+(\beta\mu+\lambda)(1-\tau))\lambda^2\Delta_{kt}^2 \\ & -\alpha(\alpha+(\beta\mu+\lambda)(1-\tau))\lambda\mu cov_t.\end{aligned} \tag{A.43}$$

Substituting (A.43) into (A.42) yields

$$\begin{aligned}(1-\alpha\lambda L)\gamma_t &= \psi+\mu\ln\kappa-\mu(1-\mu)\sigma^2/2+(1-\alpha)(1-\lambda-\mu)\ln l \\ &+\left(((\lambda+\mu\beta)(1-\tau)+\alpha)^2-\left(\alpha+(\lambda+\mu\beta)(1-\tau)^2\right)\right)\Lambda_t \\ &-\alpha(\alpha/2+(\beta\mu+\lambda)(1-\tau))\lambda^2\Delta_{kt}^2 \\ &-\alpha(\alpha+(\beta\mu+\lambda)(1-\tau))\lambda\mu cov_t+\alpha\lambda\Delta_{kt}^2/2.\end{aligned} \tag{A.44}$$

Taking a differential w.r.t τ yields

$$\begin{aligned}(1-\alpha\lambda L)\frac{\partial \gamma_t^Y}{\partial \tau} &= -2\alpha(\lambda(1-\tau)+\tau)(\lambda+\beta\mu)\Lambda_t \\ &+\alpha(\beta\mu+\lambda)\left(\lambda^2\Delta_{kt}^2+\lambda\mu cov_t\right).\end{aligned} \tag{A.45}$$

The variables Λ_t, Δ_{kt}^2 and cov_t are at steady state when the government announces a tax cut. Thus, we can substitute the steady state into the above equation. The steady state of Λ is given by (59) and steady states of Δ_k^2 and cov can be derived from (33), (35) and (36). They are shown below

$$\Delta_k^2 = \frac{\mu^2 (1-\tau)^2 (1+\lambda\alpha(1-\tau))}{(1-\lambda\alpha(1-\tau))\left((1+\lambda\alpha(1-\tau))^2 - ((\lambda+\beta\mu)(1-\tau)+\alpha)^2\right)}\sigma^2, \quad \text{(A.46)}$$

$$cov = \frac{\beta\lambda^2\mu^2(1-\tau)^4 + \mu(1-\tau)(\alpha+\beta\mu(1-\tau))\left(1-\lambda^2(1-\tau)^2\right)}{(1-\lambda\alpha(1-\tau))\left((1+\lambda\alpha(1-\tau))^2 - ((\lambda+\beta\mu)(1-\tau)+\alpha)^2\right)}\sigma^2. \quad \text{(A.47)}$$

Then substituting (59), (A.46), (A.47) and the endogenous condition into (A.45) yields

$$(1-\alpha\lambda L)\frac{\partial\gamma_t}{\partial\tau} = -\frac{\alpha\mu^2(\lambda+(1-\alpha)(1-\lambda))(1+\lambda(1-\tau))\tau}{(1-\lambda\alpha(1-\tau))\left((1+\lambda\alpha(1-\tau))^2 - ((\lambda+(1-\alpha)(1-\lambda))(1-\tau)+\alpha)^2\right)}\sigma^2 < 0. \quad \text{(A.48)}$$

This means that γ_t jumps following a tax cut.

Taking the integral of (22) yields

$$\ln c_t = \ln(1-\bar{s}_1-\bar{s}_2) - \ln(1+\theta) + \ln y_t, \quad \text{(A.49)}$$

which implies that the growth rate of consumption is equal to the growth rate of per capita income. We thus conclude that the growth rate of consumption jumps following a tax cut.

A similar conclusion follows in a straightforward way under the education finance scheme. We skip this repetitive exercise as it does not add any additional insights which are relevant to our main result. Thus Lemma 12 is proved. □

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