JOB MATCHING, COMPETITION AND MANAGERIAL INCENTIVES

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Abstract

When a manager's principal task is to organize production more efficiently, the intensity of the product market competition is crucial in determining the nature of firm-manager matching as well as the structure of managerial incentive. The firm-manager market is modeled as a two-sided matching game. If greater competition leads to increasing (decreasing) returns to cost-reduction, then a firm that faces more intense competition employs a manager with higher (lower) wealth, offers higher (lower) bonus and compensation, and has lower (higher) managerial slack. We further analyze the effects of entry on equilibrium matching and executive compensation. (JEL: C78, D82, J33, O31)

I Introduction

The study of the relationship between the product market and the market for managers is at the heart of the labor economics literature. The issue has gained even more importance as the worldwide liberalization wave in the late twentieth century has witnessed the emergence of firms with immense growth opportunities (such as investment banking, biotechnology and information technology), and an unprecedented demand for high-quality managers both in the developed and emerging economies. The popular belief is that the competitive pressure in the product market reduces agency costs, fosters innovation, provides high-powered incentives, and lures high-quality managers who organize production more efficiently. Nevertheless, there is a plethora of theoretical models and empirical evidence that have both dismissed or supported such a view.[1]

The purpose of this paper is to propose a framework that sheds light on two key issues: how market mechanisms sort heterogeneous managers into heterogeneous firms, and how managerial incentives and executive compensations are structured across firms. The heterogeneity among firms stems from the differences in the competitive environments to which they belong. Some

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[1]See the works of Oliver D. Hart (1983), David Scharfstein (1988), Klaus M. Schmidt (1997), Darius Palia (2000), and Vicente Cuñat and Maria Guadalupe (2005) for the analyses of the relationship between competition and managerial incentive.

markets are more competitive than others because the product substitutability is higher, the regulated price cap is tighter, or the entry barrier is lower in these markets. Firms compete in the labor market to hire managers, and offer incentive contracts for undertaking R&D activities that make their production processes more efficient. A manager's actions such as effort, investment decision, etc. influence the probability that the firms end up being more efficient. Such actions cannot be contracted upon since they cannot be verified by the firms. Managers are heterogenous due to the differences in their wealth endowment. In a situation that is characterized by such moral hazard problems, differences in wealth imply differences in liability. A wealthier manager puts more effort, and hence is more efficient since higher effort implies a higher probability of achieving a more efficient technology. Two-sided heterogeneity of the market induces matching between firms and managers.[2] Such a matching is endogenous since the contract offers of all the firms influence it. On the other hand, the contract for a firm-manager pair also depends on the matching. Thus one would expect that, in equilibrium, contracts and matching are determined simultaneously.

The need for a more efficient technology is the principal motive for offering high-powered incentives. If the firms benefit by reducing their costs, then it is desirable for all firms to hire better managers by offering them higher compensation. How the marginal benefit of cost-reduction relates to the intensity of product market competition depends on the form of competition one has in mind. For example, if the competition is in strategic substitutes such as price competition, a price cut by a particular firm via cost-reduction is retaliated by a price-reduction by its rival. In a market with almost similar products such a price war washes away the benefits of being "low-cost". In such firms the returns to managerial wealth/efficiency will be lower, and only the less wealthy managers will be lured into them. In other words, managers with higher wealth will be matched with firms that face less intense competition following a negatively assortative matching pattern. One may observe a reversal of matching pattern if the nature of product market competition is altered. We give sufficient conditions for a monotone matching, and analyze situations in which non-monotonicity may emerge.

The existing literature has employed the tools of the traditional agency theory to relate the product market competition to the structure of managerial incentive and executive compensation. A key feature of these studies is that a firm-manager relationship is treated as an isolated entity, and the outside option of the manager is exogenously given. Thus, such models are essentially a partial equilibrium approach. In a general equilibrium model that considers a firm-manager market, as in ours, the outside option of a manager is endogenous, which is the payoff from switching from one job to the other. We show that if more competition leads to an increasing return to cost-reduction, then due to positively assortative matching, the firms that belong to a more intense competitive environment end up offering steeper incentive, higher compensation, and managerial slack is lower in these firms. The relationships of the degree of product market competition with managerial incentive, executive compensation and effort are reversed if more competition implies a decreasing return to cost-reduction, or these relationships can even be non-monotone.

We aim at consolidating two different strands of the literature that study the relationship between the product and the labor markets. The first is how the product market competition is related to managerial efficiency via firm-manager matching. Works of Judith Chevalier and Glenn Ellison (1999), and Darius Palia (2000) show that, in the United States, the firms that belong to a more competitive environment or the firms with higher private holding were able to attract better edu-

[2]See Alvin E. Roth and Marilda A. Oliveira Sotomayor (1990) for discussions on two-sided matching markets.

cated managers. Our approach is closely related to that of Timothy Besley and Maitreesh Ghatak (2005), who show that the matching of "motivated" agents with proper organizational goals works as a substitute to high-powered incentives in mission-oriented firms. These authors do not consider the effects of product market competition on the shaping of incentives; rather, they consider how the possibility of matching between principals and agents reinforces the competition for specific types of agents in an organization. In contrast with the work of Besley and Ghatak (2005), our contracting environment resembles the analyses of Benjamin E. Hermalin (1992) and Klaus M. Schmidt (1997). In several other contexts one can observe the monotonicity of matching between firms and managers/workers such as the "superior" managers working for the "superior" firms (Sherwin Rosen, 1982), the more talented managers or the managers with better schooling managing the bigger firms (Xavier Gabaix and Augustin Landier, 2008, and John E. Garen, 1985), the small firms that enjoy efficiency advantages in process innovation employing the high-quality engineers (Todd R. Zenger and Sergio G. Lazzarini, 2004), and the higher-ranked hospitals being able to lure the higher-ranked residents (Jeremy Bulow and Jonathan Levin, 2006).

The second strand of the labor literature we explore is the relationship between competition and incentives. Oliver D. Hart (1983) asserts that competition provides greater incentives to the managers, and helps reduce the managerial slack if the firms' environments are correlated. David Scharfstein (1988) shows that more intense competition may aggravate the managerial incentive problem if a manager's marginal utility of income is strictly positive, a reversal of Hart's (1983) result. Hermalin (1992) and Schmidt (1997) find that the effects of an increased intensity of competition on managerial incentive are, in general, ambiguous. Michael Raith (2003) shows that the positive relationship between competition and incentives, as predicted by many empirical studies, is quite robust if one considers risk-averse managers. Our results are driven by the complementarity/substitutability between cost and competition in generating firm's profit, which is the same as Hermalin's "change-in-the-relative-value-of-actions" effect and Schmidt's "value-of-a-cost-reduction" effect.

The difference in the degrees of product market competition is the root of the heterogeneity among the firms. We stick to this interpretation of heterogeneity since our main objective is to study how competitive pressure in the product market influences managerial incentives and executive compensation. Our analysis easily extends to other forms of heterogeneity at the firm level. Firms may be different because some are owned by private equity investors and others are purely public. In this context, one can reinterpret the parameter that describes the degree of product market competition as the fraction of private holding in a firm. A second source of heterogeneity is the difference in efficiency at the firm level. Some firms are inherently more efficient than others because they have better working conditions such as bigger offices, air conditioning, etc. Each of these factors enhances productivity within a firm and affects a firm's profitability. Firm size may be another source. Think of the production of a single output that is carried out by many production units of a firm, and there are increasing returns to size. Then smaller size (fewer units) would imply lower profit. Thus, our model can be used to analyze assortative matching between firm size and managerial quality, and how managerial incentive and executive compensation relate to firm size as in Gabaix and Landier (2008), Garen (1985), and Zenger and Lazzarini (2004). The set of firms we consider can also be interpreted as the set of various tasks in a single firm, and so, one can suppose that the profit is more sensitive to managerial efficiency in some tasks than in others. Then our model yields conclusions similar to those drawn by the O-ring theory of production (Michael Kremer, 1993, and Gilles Saint-Paul, 2001), which suggests that workers of similar skills tend to be matched together, and the equilibrium wage structure typically depends on the equilibrium skill segregation.

In a market with homogeneous managers, competition may be positively, negatively, or non-monotonically related to incentives depending on the assumptions of the specific models.[3] In our model, the relationship between competition and incentive not only depends on the moral hazard problem associated with the managerial effort, but also on the nature of sorting. In other words, firms in a more competitive sector may offer steeper incentive and better compensation not only because they are able to solve moral hazard problem more efficiently, but because they may end up hiring better managers. This has been the main message conveyed in the empirical work by Daniel A. Ackerberg and Maristella Botticini (2002), who analyze historical data on tenancy contracts, and show that the regression results are subject to selection bias if one does not take the endogenous nature of landlord-tenant matching into account. For empirical purposes, however, our results should be carefully interpreted. Observability may be a problematic issue. In most of the situations, the intensity of product market competition is easily measured. One can actually observe how many firms are there in a market, or how similar the products are. But measuring managerial wealth may not be easy. Experience, education, etc. can be used as proxies since such attributes are expected to have high correlation with managerial wealth.

A sufficient condition for monotone matching in a very general economic environment has been proposed by Patrick Legros and Andrew F. Newman (2007). In the current paper, the condition that greater competition leads to increasing/decreasing returns to cost-reduction implies the "generalized difference condition" of Legros and Newman (2007). Vincent P. Crawford and Elsie Marie Knoer (1981) analyze the matching between heterogeneous firms and workers under symmetric information. Konstantinos Serfes (2008) asserts that when principal-agent matching exists, the relationship between risk and incentive is non-monotone. The general equilibrium impact of principal-agent matching on optimal incentive contracts has also been analyzed by Archishman Chakraborty and Alessandro Citanna (2005), Kaniska Dam and David Pérez-Castrillo (2006), and Ulf von Lilienfeld-Toal and Dilip Mookherjee (2007).

II The Model

A Firms and Managers

Consider a market for managers which consists of a set $\mathscr{M} = \{m_1, \ldots, m_N\}$ of N risk-neutral firms and a set $\mathscr{W} = \{w_1, \ldots, w_N\}$ of N risk-neutral managers, with $N \geq 2$. Let $m, m' \in \mathscr{M}$, etc. denote the characteristics of the firms, which are determined by the conditions prevailing in the product markets. We assume that $m_1 \geq \ldots \geq m_N$. Each of the N firms represents a different industry. A higher value of m implies a higher intensity of industry-wide competition (corresponding to greater degree of product substitutability, more competitors, lower entry cost, etc.). The parameters $w, w' \in \mathscr{W}$, etc. represent managers' wealth. We assume that $w_1 \geq \ldots \geq w_N$. Several factors such as schooling and experience may attribute to differences in managerial wealth.

The production technology of each firm is characterized by a constant average cost $c \in \{c_L, c_H\}$

[3] Hart (1983), Scharfstein (1988), Hermalin (1992), Schmidt (1997), all draw different conclusions in this regard.

with $c_H > c_L > 0$. Initially all firms are endowed with the inefficient technology, i.e., $c = c_H$. Firms hire one manager apiece, whose principal task is to exert R&D effort to bring down the average cost to the lower level.[4] Each manager can reduce the average cost with probability $e \in [0, 1]$, which is his effort. A manager incurs a cost of effort that is given by $\psi(e)$, with $\psi'(e) > 0$, $\psi''(e) > 0$ and $\psi'''(e) > 0$. A market for manager is denoted by $\xi = (\mathscr{M}, \mathscr{W}, c_L, c_H, \psi)$, all elements being publicly known. We assume that cost-reduction by a firm does not alter the competition dynamics of the product market.

A firm is matched with a manager to form a partnership. Prior to the choice of effort, the firm and the manager in a match write binding contracts that specify state-contingent transfers to the manager. Manager's effort choice is followed by the realization of the cost parameter. The effort is not verifiable, and hence is not contractible. A firm's gross profit, $\pi(c, m)$ for $c \in \{c_L, c_H\}$, depends on the cost realization and on the degree of product market competition. If the manager is successful in reducing the cost (which happens with probability e), then firm m's marginal benefit is $\pi(m) \equiv \pi(c_L, m) - \pi(c_H, m)$. We assume the following.

ASSUMPTION 1 *For a given degree of competition, a firm's marginal benefit of cost reduction is positive, i.e.,* $\pi(m) > 0$.

ASSUMPTION 2 *For a given realization of cost, more competition leads to lower gross profit, i.e.,* $m > m'$ *implies* $\pi(c, m) < \pi(c, m')$*, for* $c \in \{c_L, c_H\}$.

Assumption 1 says that all firms gain from cost-reduction. Also for any realization of cost, a firm has strictly positive profit, and hence there is no concern for bankruptcy. Assumption 2 implies that as the degree of competition increases firm's gross profit decreases for any value of c. Following this assumption, one may give a second interpretation to m which describes heterogeneity across firms within a given industry. The parameter m may represent the fraction of private equity ownership in a firm, a firm's initial market size, or m may simply be an efficiency parameter that affects a firm's profitability. However, throughout the paper, we stick to the first interpretation, in which m is an industry-specific characteristic, and in Section V put forward an example of a situation where it is firm-specific.

B Contracts

A contract $t(m, w) = (R(m, w), b(m, w))$ between a firm m and a manager w specifies the state-contingent transfers to the manager: $R(m, w)$ is a fixed salary, and $b(m, w)$ is a bonus if the manager is successful in reducing the cost. The expected payoffs of firm m and manager w, when they sign a contract $t(m, w)$, are respectively given by

$$\begin{aligned}
\Pi(t(m, w)) &\equiv e(m, w)\pi(c_L, m) + [1 - e(m, w)]\pi(c_H, m) - R(m, w) - e(m, w)b(m, w), \\
V(t(m, w)) &\equiv R(m, w) + e(m, w)b(m, w) - \psi(e(m, w)).
\end{aligned}$$

We first describe the set of feasible contracts for the firm-manager pair (m, w). Since the effort is not contractible, a manager will choose the effort level that maximizes his expected payoff. This is

[4] A manager carries out several tasks to organize production more efficiently, which include product and process innovation, finding out profitable investment opportunities, laying off unproductive workers, etc. For tractability, we restrict attention only to the task of process innovation.

the *incentive compatibility* constraint. Since the expected payoff is strictly concave in effort, one can replace it by the first order condition of the maximization problem as follows.

$$(IC_w) \qquad \psi'(e(m,w)) = b(m,w).$$

A manager would accept a contract if it satisfies the following *participation* constraint.

$$(PC_w) \qquad R(m,w) + e(m,w)b(m,w) - \psi(e(m,w)) \geq u_w,$$

where $u_w > 0$ is the manager's payoff if he does not accept the contract offered by the firm. Also, a firm would not accept a contract if it generates negative expected payoffs, i.e., the contract $t(m,w)$ must satisfy the following *individual rationality* constraint of firm m

$$(IR_m) \qquad e(m,w)\pi(c_L,m) + [1 - e(m,w)]\pi(c_H,m) - R(m,w) - e(m,w)b(m,w) \geq 0,$$

Finally, *limited liability* requires that, for any realization of cost, a manager's net income must be positive, which gives rise to the following constraints.

$$(LLC_w) \qquad R(m,w) + b(m,w) \geq -w, \qquad R(m,w) \geq -w.$$

Let $\Omega(m,w)$ be the set of $(e(m,w), R(m,w), b(m,w))$ that satisfy (IC_w), (PC_w), (IR_m) and (LLC_w). This is the set of feasible contracts for a pair (m,w).

C Allocations

Firms and managers are matched according to a rule μ. The rule specifies a manager w (a firm m) can only be matched with a firm m (a manager w). In this case we write $\mu(w) = m$ ($\mu(m) = w$). If a manager w or a firm m is unmatched, we write $\mu(w) = w$ or $\mu(m) = m$. We assume that if a firm or a manager is unmatched, then this individual signs a null contract, denoted by t^{null}, such that $\Pi\left(t^{null}\right) = V\left(t^{null}\right) = 0$ for all m and w unmatched. Given a matching rule μ, a list of compatible contracts $\mathscr{C}$ is a set of feasible contracts, one for each pair. An allocation for the market ξ is a pair $(\mu, \mathscr{C})$.

III Optimal Contracts

As a benchmark, we analyze the optimal incentive scheme for a given firm-manager pair taking the outside option of the manager, u_w as given. Our final objective is to see how the formation of other pairs influences the contracts within a partnership. Thus, the optimal contract $t(m,w,u_w) = (e(m,w,u_w), R(m,w,u_w), b(m,w,u_w))$ for a pair (m,w) is the solution to the following maximization problem.

$$(\mathscr{P}) \qquad \phi(m,w,u_w) = \max_{t(m,w)\in\Omega(m,w)} \Pi(t(m,w)).$$

The function $\phi(m,w,u_w)$ is the Pareto frontier for the pair (m,w). In other words, it represents the maximum payoff firm m can obtain if it is to guarantee a minimum of u_w to manager w. When the limited liability for $c = c_H$ binds, it is typically costly for the firm to provide incentive to the risk-neutral manager. In this case the moral hazard problem bites, and the first-best is not achieved.[5] The optimal second-best contracts $(e(m, w, u_w), R(m, w, u_w), b(m, w, u_w))$ solve the program ($\mathscr{P}$) with the limited liability at the high-cost state binding, i.e., $R(m, w, u_w) = -w$. In the following proposition we analyze the comparative statics of the optimal incentive contract with respect to managerial wealth and the degree of product market competition.

PROPOSITION 1 *Higher managerial wealth implies a higher optimal effort and a higher bonus for the manager. If more competition leads to increasing (decreasing) returns to cost-reduction, i.e., if $\pi(m) > (<) \pi(m')$ for $m > m'$, then more intense competition implies a higher (lower) optimal effort and a higher (lower) bonus.*

Higher wealth implies that the limited liability is less likely to bind, and hence the moral hazard problem is less stringent. Consequently, optimal effort is higher following an increase in manager's wealth. A higher-wealth manager reduces the average production cost with higher probability, and hence is more efficient. The bonus offered to the manager is designed to compensate for the incremental cost of exerting an additional unit of effort. Higher wealth implies higher effort, and hence greater marginal cost of effort. Thus, the manager is compensated with a higher bonus. More intense competition may have a positive or a negative impact on effort and bonus. To see this, suppose that $\pi(m)$ is increasing in m. As the firm benefits more (at the margin) by reducing its cost, it provides greater incentive to the manager. Consequently, manager's bonus and effort are higher following an increase in the intensity of competition.[6] The effect is exactly the opposite if $\pi(m)$ is a decreasing function of m.

IV Market Equilibrium

In this section we focus on two key issues. The first is the nature of firm-manager matching. We pose the question: if a particular firm belongs to a more intense competitive environment, then under what conditions it would employ a wealthier/more efficient manager? Such a question is important since hiring the "best" manager may be the most attractive option for a firm, but for some it may be very costly to do so. The second important aspect of the market equilibrium is how managerial incentive and executive compensation are associated with the degree of competition.

[5]The first-best is a situation in which the firm can contract on the manager's effort. If the limited liability in no state binds, then risk-neutrality leads to the optimality of the first-best contracts. In this case the Pareto frontier is linear with a slope equal to -1, i.e., the equation of the frontier can be expressed as $\phi(m, w, u_w) = h(m, w) - u_w$, where $h : \mathscr{M} \times \mathscr{W} \longrightarrow \mathbf{R}_+$. Since we aim at studying the relationship between competition and incentives, we skip the analysis of the first-best. In the second-best situation, the incentive constraint together with the limited liability at $c = c_H$ imply that the limited liability at $c = c_L$ is always satisfied, and hence this constraint can be ignored.

[6]The property that $\pi(m)$ is an increasing (decreasing) function of m is equivalent to that $\pi(c, m)$ has decreasing (increasing) differences in (c, m), i.e., $\pi(c_H, m) - \pi(c_L, m) > (<) \pi(c_H, m') - \pi(c_L, m')$ for $c_H > c_L$ and $m > m'$. This is a standard complementarity/substitutability property of the profit function. See Donald M. Topkis (1998) for various general complementarity conditions. Suppose that the decreasing differences condition holds. This implies that cost and competition are substitutes in generating firm's profit. In other words, the marginal benefit of cost-reduction is higher if competition is more intense. We do not explicitly model the (product) market game that follows the realizations of firms' cost parameters. In Section V we construct examples of market games in which such differences conditions naturally arise.

This analysis is different from the results stated in Proposition 1 in the following sense. In Section III we have taken a manager's outside option as exogenously given since the formation of other partnerships did not impose externality on the optimal contracts for a particular pair. In a firm-manager market, the payoff/compensation a manager can earn from an alternative employment is endogenous and influences the equilibrium contracts in the market. Consequently, the entire set of incentive compatible contracts obtained in the previous section may not be optimal in a market equilibrium. We use stability as the solution concept.

DEFINITION 1 *An allocation* $(\mu, \mathscr{C})$ *is in the market equilibrium or is stable if there do not exist any firm-manager pair* (m, w) *and a feasible contract* $t'(m, w) \in \Omega(m, w)$ *such that* $\Pi(t'(m, w)) > \Pi(t(m, \mu(m)))$ *and* $V(t'(m, w)) > V(t(\mu(w), w))$, *for* $t(m, \mu(m))$ *and* $t(\mu(w), w)$ *in* $\mathscr{C}$.

Thus, if $(\mu, \mathscr{C})$ is a stable allocation, then there is no firm-manager pair that can propose among themselves a feasible contract, different from the one assigned to them at $(\mu, \mathscr{C})$, to make both of them better-off. In other words, if one such feasible contract existed for a pair (m, w), then with this contract the pair would have "blocked" the allocation. A market equilibrium consists of allocations that are immune to such pairwise blocking. Stability implies that all contracts in a market equilibrium must be Pareto optimal, i.e., they must solve program ($\mathscr{P}$). Suppose, in a stable allocation, that $\mu(w) = m$. If the manager receives u_w, then the payoff to his partner must be $\phi(m, w, u_w)$.

A Matching

In a market equilibrium, matching and contracts are determined simultaneously and endogenously. Further, in a stable allocation, not only are the contracts optimal, but the matching itself is optimal in the sense that no other matching may generate a higher aggregate surplus. It is worth noting that, given the provision of signing null contracts, in a stable allocation there is always full employment.

DEFINITION 2 *Consider any two firms* m *and* m' *with* $m > m'$. *A matching is said to be positively (negatively) assortative if* $\mu(m) > (<) \mu(m')$.

A positively assortative matching means that a higher value of w is matched with a higher value of m. This should no way be confused with the fact that a "better" manager is hired by a "better" firm. The following proposition analyzes under what conditions the equilibrium exhibits assortative matching.

PROPOSITION 2 *If more intense competition in the product market leads to increasing (decreasing) returns to cost-reduction, i.e., if for any two firms* m *and* m' *with* $m > m'$ *one has* $\pi(m) > (<) \pi(m')$, *then a firm that faces more competition employs a manager with higher (lower) wealth following a positively (negatively) assortative matching pattern.*

If more competition implies a higher marginal benefit of cost-reduction, then a firm with greater m would have a higher marginal gain from employing a wealthier manager. Unlike the first-best situation, one main consequence of moral hazard is that the surplus cannot be transfered on a one-to-one basis between a firm and a manager, i.e., the Pareto frontier is not linear. In our setup, the condition that $\pi(m)$ is an increasing function of m induces a positively assortative matching in equilibrium via two channels: a wealthier manager produces higher surplus along with a firm that faces more competition, and for such a firm it is easier to provide incentives to such a manager (through the transfer of utility). It is worth noting that, any matching pattern is optimal in the first-best situation, and this does not depend on whether $\pi(m)$ is monotone in m. Also notice that we do not discard the possibility of a non-monotone matching as $\pi(m)$ may not be globally increasing in m.[7]

The above type of equilibrium matching can also be obtained as part of a Walrasian allocation in a vertically differentiated market. For a manager w, u_w can be thought of as his "price".[8] Had the prices been equal, all firms would have liked to employ the wealthiest manager since $\phi_2(m, w, u_w) > 0$, i.e., there is vertical differentiation in the market for managers.[9] Hence, the firm that is willing to pay the most (at the margin) for the best manager would get him. How much a firm is willing to sacrifice (in terms of utility) to hire a better manager? Consider a firm m and two managers with $w > w'$. The maximum amount this firm is willing to sacrifice to get the better manager is $\phi(m, w, u_w) - \phi(m, w', u_{w'})$. This is the firm's "willingness to pay" for w. If this amount is increasing in m, then the firm that belongs to the most competitive industry has the highest willingness to pay for the best manager, and thus the matching is positively assortative. In our model, $\pi(m)$ being increasing in m is a sufficient condition for increasing willingness to pay.[10]

B Incentive

We have noted earlier that the contract externality in the firm-manager market makes the outside option endogenous, and competition for the managers determines its equilibrium value. The equilibrium outside option in turn determines the equilibrium payoff or the executive compensation. Notice that all the firms have the same preferences over managers. Each firm ranks the wealthiest manager as first, the manager with second-highest wealth as second, etc. Hence, it is natural to expect that, in equilibrium, a particular manager cannot be worse-off than his less wealthy counterpart. This gives rise to the question that which firm offers higher bonus and compensation, and has lower managerial slack. Obviously, the answers to these questions depend on the nature of the firm-manager matching in equilibrium.

PROPOSITION 3 *If more intense competition in the product market leads to increasing (decreasing) returns to cost-reduction, then a firm that faces more competition offers higher (lower) bonus and compensation, and has lower (higher) managerial slack.*

[7] See Section V for an example of this situation.

[8] See von Lilienfeld-Toal and Mookherjee (2007) for the equivalence between the Walrasian allocations and the set of stable allocations in a model with homogeneous principals and heterogeneous agents.

[9] For the function $\phi(m, w, u_w)$, $\phi_i(m, w, u_w)$ is the partial derivative with respect to the i-th argument.

[10] The "increasing willingness to pay" is same as the "generalized increasing differences" property of the Pareto frontier in Legros and Newman (2007), which asserts that competition and managerial efficiency are complements in producing as well as transferring the surplus. Complementarity in the production of surplus implies $\phi_{12}(m, w, u_w) > 0$, and that in transferring the surplus is equivalent to $\phi_{13}(m, w, u_w) > 0$. The function $\pi(m)$ being increasing in m guarantees both of the above conditions.

To prove the above proposition, first we show that a wealthier manager is always offered a higher compensation. In a stable allocation, if a manager with higher wealth is offered a lower compensation than his less wealthy counterpart, then the firm that is matched with the less wealthy manager can always offer a slightly better contract to the wealthier manager and form a blocking pair. Such a blocking is feasible since a wealthier manager is always preferred to a less wealthy one. Having established the ranking of the managerial compensations, the proposition immediately follows from Proposition 2. Suppose first that $\pi(m)$ is increasing in m. Then the equilibrium matching is positively assortative, i.e., a wealthier individual manages a firm that faces steeper competition in the product market. Thus, executive compensation is higher in such a firm. In case of a negatively assortative equilibrium matching, managerial compensation is inversely related to competition. The equilibrium may exhibit a non-monotone relationship between payoff and competition if $\pi(m)$ is not globally monotone with respect to m. As regard to the relationship between incentive and competition, notice that both matching and incentive affect the contracts in equilibrium. Suppose that $\pi(m)$ is increasing in m. Then the equilibrium matching is positively assortative, and one is interested in comparing the contracts in these two pairs (m, w) and (m', w'), with $m > m'$ and $w > w'$. There are two effects at work.[11] The first is the *matching effect*. When the matching is positively assortative, bonus and effort in the partnership (m, w) are higher since higher values of both m and w imply higher effort and bonus (by Proposition 1). In addition to this, there is an *incentive effect* that works through the endogenous outside option. A higher value of w implies a higher outside option, and hence a higher equilibrium compensation. This entails higher effort and bonus. As both effects go in the same direction in determining optimal contract, managerial slack is lower and bonus is higher in the partnership (m, w). One might also be interested in the payoffs of the firms in a market equilibrium. The following corollary analyzes that.

COROLLARY 1 *In a market equilibrium, a firm that faces more intense competition consumes a lower payoff.*

More competition lowers a firm's profit for every realization of cost, and thus each manager prefers to work for a firm that faces less competition since there is more to share. Thus in equilibrium, a firm with lower m gets higher payoff. Had the firms been identical, they would have obtained equal payoffs. The same would also be true had all managers been homogeneous.

V Examples of Market Game

In the previous sections we have not explicitly modeled the market game that follows the realization of cost parameters of the firms, and we have shown that the nature of the equilibrium matching depends on the behavior of $\pi(m)$ with respect to the changes in the degree of product market competition. In this section we consider examples of various market games where the optimal matching pattern and the implications for managerial incentives are different as a result of a change in the intensity of competition. The last example is of a Cournot duopoly where m is a firm-specific attribute.

[11] Besley and Ghatak (2005) also characterize the equilibrium contracts in terms of these two effects.

A Competition in Differentiated Goods

There are N separate markets for differentiated goods, each of which consists of two firms, i and j. The degree of competition in a particular market is determined by the degree of product substitutability, $m \in (0, 1)$. Let the demand functions in market m be given by

$$p_i = 1 - q_i - mq_j \text{ for } i, j = 1, 2, \text{ and } i \neq j.$$

First, we consider the situation in which both firms do R&D activities in order to reduce costs, and may compete either in quantities or in prices. The symmetric Cournot and Bertrand profits of each firm are given by

$$\pi(c, m) = \begin{cases} \left(\frac{1-c}{2+m}\right)^2, & \text{for quantity competition,} \\ \frac{(1-m)(1-c)^2}{(1+m)(2-m)^2}, & \text{for price competition.} \end{cases}$$

It is easy to show that $\pi(m) = \pi(c_L, m) - \pi(c_H, m)$ is decreasing in m under both quantity and price competitions. Hence, the equilibrium matching is negatively assortative, firms in a market with higher degree of product substitutability offer lower bonus and compensation, and have higher managerial slack. Cost-reduction has two effects on a firm's profit. The direct effect is that lower cost increases the profit of a particular firm. There is also a strategic effect in which cost-reduction indirectly affects a firm's profit. In a Cournot market, cost-reduction by a firm reduces the marginal profit of its rival through an increase in output since the quantities are strategic substitutes. On the other hand, in a Bertrand market, due to the strategic complementarity between prices, cost-reduction by a firm results in a decrease in its price, and hence a decrease in the marginal profit of its rival. Since both the firms are involved in cost reducing R&D, only the strategic effect is at work. Thus in both sorts of competition, the reduction in the rival's marginal profit due to cost-reduction is higher if the products are more homogeneous. Hence, more competition leads to decreasing returns to cost-reduction.

Next, we modify the above example to introduce asymmetry in the R&D activities in each Cournot market, and show that the monotonicity of equilibrium matching may not hold since $\pi(m)$ may not be globally monotone in m.[12] In each industry one of the two firms (say, firm i) is engaged in the R&D activity to bring down its average cost from c_H to c_L, whereas the average cost of the other firm is fixed at $c^j = 0$. The N innovators, one from each industry, compete in the managerial labor market. For the existence of an interior Cournot equilibrium we assume that $m < 2(1 - c_H)$. We only consider the Cournot profits of the innovators across N markets. In market m, the profit of the innovator is given by

$$\pi(c, m) = \left[\frac{2(1-c) - m}{4 - m^2}\right]^2.$$

For $c_L + c_H \geq 1/4$, $\pi(m)$ is globally decreasing in m, and the matching is negatively assortative. For $c_L + c_H < 1/4$, there is a cutoff level of the degree of substitutability, $\bar{m} \equiv \bar{m}(c_L, c_H)$ below which $\pi(m)$ is decreasing in m, and above $\bar{m}$ it is increasing. In this case, monotonicity of matching breaks down. For very high degrees of product differentiation, firms gain more from cost-reduction since they enjoy quasi-monopoly rents. As firms become more similar in terms of the products they sell, they have higher marginal benefits of cost-reduction, and hence, over this range of values of m, the optimality is achieved by assigning wealthier managers to firms from more competitive

[12] This example is adapted from Helmut Bester and Emmanuel Petrakis (1993).

industries. The equilibrium matching, bonus, compensation and managerial effort thus have U-shaped relationships with the degree of product substitutability.

B Price Cap Regulation

There are N distinct and non-competing product markets, and each of the N firms is a monopolist in one of these markets.[13] A market m faces a demand function $D(p)$ with $D'(.) < 0$, where p is the market price. Further, each market m is subject to a price cap regulation: $p \leq 1/m$. We assume that the values of m are such that the price cap is set below the monopoly price at any realization of cost. The price in market m thus will be set at the price cap. A tighter price cap (corresponding to a higher value of m) implies a market price closer to marginal cost, and hence more competition. The profit of firm/industry m at the price cap is given by

$$\pi(c,m) = \left(\frac{1}{m} - c\right) D\left(\frac{1}{m}\right), \quad \text{for} \quad c \in \{c_L, c_H\}.$$

Given $D' < 0$, $\pi(m)$ is globally increasing in m, i.e., the firm that is subject to a tighter price cap has a higher incentive for cost-reduction. A lower average cost does not change the market price since it is pegged at the price cap. But the marginal benefit of cost-reduction is higher in the market with the tighter price cap since the marginal gain to reduce the cost applies to a higher quantity, and hence is higher. Thus in a market equilibrium a higher degree of competition implies higher compensation, higher bonus, lower managerial slack, and a positively assortative matching.

C Varying Degree of Efficiency

In this example the parameter m is a firm-specific characteristic that influences the profitability of a firm by altering production efficiency. There are two firms 1 and 2 which compete in both the product and the labor markets. The competition in the product market is in quantity where the inverse market demand is $P(q_1+q_2) = 1-(q_1+q_2)$. The cost function of firm $i = 1,2$ is given by $C_i(q_i) = (m_i c)q_i$, where $c \in \{c_H, c_L\}$. Both firms undertake the R&D activities to reduce their average cost to the lower level. We assume that $m_1 > m_2$. Thus, firm 2 enjoys efficiency advantage over firm 1. The Cournot profits are given by

$$\pi(c, m_i) = \frac{1}{9}\left[1-(2m_i - m_j)c\right]^2, \text{ for } i, j = 1,2 \text{ and } i \neq j.$$

The above functions are decreasing in c and it is easy check that $\pi(c,m_1) < \pi(c,m_2)$ for $c \in \{c_H, c_L\}$, i.e., Assumption 2 is satisfied. For the existence of an interior Cournot equilibrium, we assume that $2-(m_i+m_j)(c_L+c_H) > 0$. The marginal benefit of cost-reduction for firm $i = 1,2$ is given by

$$\pi(m_i) \equiv \pi(c_L, m_i) - \pi(c_H, m_i) = \frac{1}{9}(c_H - c_L)(2m_i - m_j)[2-(m_i+m_j)(c_L+c_H)].$$

[13] This example is adapted from Luis M. B. Cabral and Michael H. Riordan (1989), Hermalin (1992), and Schmidt (1997).

Notice that $\pi(m_1) > \pi(m_2)$, i.e., $\pi(m)$ is increasing in m. Thus the more efficient firm (firm 2) has lower incentives to reduce its cost since, prior to the R&D investments, this firm has already been enjoying an efficiency advantage. Hence, in equilibrium firm 1 employs a wealthier manager, offers higher bonus and higher executive compensation, and has lower managerial slack.

VI Entry of New Firms

In this section we analyze the impact of the entry of new firms on the market equilibrium. Competition is determined via two different channels. The degree of product substitutability, the barriers to entry, the extent of regulation, etc. determine the level of the product market competition. A second source of competition is the number of firms that compete in the managerial labor market, which is crucial in determining the payoffs of the managers by changing their outside options. More firms competing for a fixed set of managers implies the outside option of each manager is broader. Thus one can expect that the entry of new firms into the labor market should increase the compensation received by each manager. In the previous section, we have been working with the same number of firms and managers. A simple modification of that would help analyze the consequences of the entry on the market equilibrium.

PROPOSITION 4 *The entry of new firms into the managerial labor market leads to increases in executive compensation and bonus for all managers, and lowers the managerial slack.*

Suppose that a new product market opens up in which the intensity of competition is $\hat{m}$, and this firm intends to compete with the N existing firms in the market for managers. Given the restriction of one-to-one matching, only as many N pairs can be formed in the new equilibrium. It is easy to show that, if there are more firms than managers in the labor market, only the firms belonging to the N least competitive markets get matched in any equilibrium. Hence, the new equilibrium depends on the value of $\hat{m}$. If this new firm faces more competition in the product market than any of the existing firms, then it will stay unmatched, and hence the equilibrium will remain the same as before. If the entrant faces less product market competition than at least one of the existing firms, then the new market equilibrium will be different from the old one. Suppose that $\hat{m} < m' \equiv \sup\{\mathscr{M}\}$, where $\mathscr{M}$ is the set of incumbent firms. Then in the new equilibrium, entry will drive m' out of the market for managers. Otherwise, $\hat{m}$ can write a blocking contract with the manager who was matched with m'. The possibility of blocking implies that, in the new equilibrium, this manager has to be offered a higher compensation by the entrant firm, which is higher than that he was receiving in the old equilibrium. For the remaining of the managers, executive compensation will either improve or remain the same. Further, as a consequence of the rise in the payoff, bonus will be higher and managerial slack will be lower with each manager. Within the set of matched firms and managers, the matching may be assortative or non-monotone depending on the behavior of $\pi(m)$ with respect to m.

Economic integration increases the number of potential competitors in the labor market, the effect of which on the equilibrium is analyzed in the above proposition. A direct consequence of the presence of more firms in the economy is that the average market power of the firms competing for a fixed set of managers increases, since only the firms in the N least competitive industries survive the competition. Proposition 4 suggests that increase in the market power has favorable consequences for executive compensation, bonus and effort of the managers, which conforms to the findings in the labor literature that workers' compensation is higher if the market power increases since a part of the monopoly profit may accrue to the workers or a monopoly pass-through to the consumers is easier in a more concentrated industry.[14] In the recent years, the emerging market economies have witnessed the opening up of new business opportunities and huge rises in the managerial compensation. The overall managerial slack has also been reduced due to the new corporate culture. On the other hand, the high-quality managers remained scarce, which justifies the consideration of a fixed set of managers in the analysis of this section.

VII Conclusions

This paper analyzes the nature of the equilibrium matching between the firms that are heterogeneous with respect to the competitive environments they belong to and the managers that differ in initial wealth, and how such matching influences the ways in which executive compensation, bonus and managerial slack are related to the intensity of the product market competition. Whether the firms in a more competitive industry employ wealthier managers and offer higher incentives depend crucially on the nature of competition one has in mind.

Casual empiricism suggests that there are relatively more competitive R&D activities in the young and high-growth industries such as software and biotechnology than the matured sectors like power generation. One reason may lie in the fact that the marginal benefits of process innovation are more or less constant across firms in a matured industry, whereas a higher degree of competition leads to an increasing return to cost-reduction in young industries. Thus our results help explain the heterogeneity in the compensation for the top executives in the high-tech sectors as opposed to the very little variation in the pays of the managers of the utility firms.

The introduction of free trade changes the intensity of competition in some sectors, whereas the level of competition in some others may remain the same as that in autarky. In many developing countries, the manufacturing and the services industries have faced relatively higher competition in the world market following the removal of the trade barriers. On the other hand, in many specific industries such as food products, the globalization did not alter the level of competition much because these firms either cater to a local clientele or enjoy the same comparative advantage before and after free trade. If one assumes that more competition leads to an increasing return to cost-reduction, then our analysis may yield useful insights in explaining the recent rise in the executive compensation in the newly integrated sectors like manufacturing and services.

Ours is a stylized model that analyzes how incentives are related to the intensity of the product market competition in the presence of moral hazard. As proposed by Marcel Boyer and Jean-Jacques Laffont (2003), there are various other channels through which competition may affect incentives. The view taken by these authors is that greater product-market competition ameliorates informational asymmetry within a firm, and works as a substitute for high-powered incentives when principal-agent relationships are subject to an adverse selection problem. A complete general equilibrium analysis of competition and incentives in the presence of both moral hazard and adverse selection will be an interesting future research agenda.

[14] See James E. Long and Albert N. Link (1983) for similar arguments.

Appendix

Analysis of the second-best The second-best optimal contracts are obtained by solving the maximization problem ($\mathscr{P}$) with the limited liability binding for $c = c_H$, i.e., $R(m, w) = -w$. If the incentive constraint is satisfied together with limited liability for $c = c_H$, then the limited liability for $c = c_L$ is also satisfied, and hence can be ignored throughout. Incorporating (IC_w) and (LLC_w), the problem ($\mathscr{P}$) reduces to:

$$(\mathscr{P}') \qquad \max_{e(m,w)} e(m, w)\pi(c_L, m) + [1 - e(m, w)]\pi(c_H, m) + w - e(m, w)\psi'(e(m, w)),$$

subject to

$$(PC_w) \qquad e(m, w)\psi'(e(m, w)) - \psi(e(m, w)) - w \geq u_w\,.$$

We identify two disjoint sets of parameters over which the solution is optimal. First consider the case when (PC_w) does not bind at the optimum. The optimal effort is obtained by setting the derivative of the objective function equal to zero, which is given by

$$(1) \qquad \psi'(e(m, w, u_w)) + e(m, w, u_w)\psi''(e(m, w, u_w)) = \pi(m).$$

Now consider the case when (PC_w) binds at the optimum. The maximum value of effort is obtained from the binding participation constraint, which is given by

$$(2) \qquad e(m, w, u_w)\psi'(e(m, w, u_w)) - \psi(e(m, w, u_w)) = w + u_w\,.$$

From (IC_w) we get the optimal bonus as $b(m, w, u_w) = \psi'(e(m, w, u_w))$. The optimal payoff of the firm is given by $\phi(m, , w, u_w) = e(m, w, u_w)[\pi(m) - \phi'(e(m, w, u_w))] + \pi(c_H, m) + w$, and that of the manager is $\sigma(m, w, u_w) = e(m, w, u_w)\psi'(e(m, w, u_w)) - \psi(e(m, w, u_w)) - w$. Let

$$\phi(m, , w, u_w) = \begin{cases} \bar{\phi}(m, w) & \text{when } (PC_w) \text{ is not binding,} \\ \hat{\phi}(m, w, u_w), & \text{when } (PC_w) \text{ is binding.} \end{cases}$$

And

$$\sigma(m, , w, u_w) = \begin{cases} \bar{\sigma}(m, w) & \text{when } (PC_w) \text{ is not binding,} \\ u_w, & \text{when } (PC_w) \text{ is binding.} \end{cases}$$

Since the payoffs of both the firm and the manager do not depend on u_w when (PC_w) does not bind at the optimum, we omit u_w from the arguments of both $\phi(.)$ and $\sigma(.)$. ||

Proof of Proposition 1 From equation (1) we have the following

$$\begin{aligned}
\frac{\partial e(m, w, u_w)}{\partial w} &= \frac{\partial b(m, w, u_w)}{\partial w} = 0, \\
\frac{\partial e(m, w, u_w)}{\partial m} &= \frac{\pi'(m)}{2\psi'' + e\psi'''} \gtrless 0 \text{ as } \pi'(m) \gtrless 0, \\
\frac{\partial b(m, w, u_w)}{\partial m} &= \psi'' \left[\frac{\partial e(m, w, u_w)}{\partial m} \right] > 0 \text{ as } \pi'(m) \gtrless 0.
\end{aligned}$$

Now form equation (2) we have

$$\begin{aligned}
\frac{\partial e(m,w,u_w)}{\partial m} &= \frac{\partial b(m,w,u_w)}{\partial m} = 0, \\
\frac{\partial e(m,w,u_w)}{\partial w} &= \frac{1}{e\psi''} > 0, \\
\frac{\partial b(m,w,u_w)}{\partial w} &= \psi''\left[\frac{\partial e(m,w,u_w)}{\partial w}\right] = \frac{1}{e} > 0.
\end{aligned}$$

The above completes the proof of the proposition. ||

Proof of Proposition 2 Consider two firms with $m > m'$ and two managers with $w > w'$. We will show that if $\pi(m)$ is increasing (decreasing) in m, then the equilibrium matching is positively (negatively) assortative. We first prove that if $\pi(m) > \pi(m')$, then the matching is positively assortative. We omit the proof of negatively assortative matching since it is similar to the case of positively assortative matching. There are the three following cases.

CASE 1: Suppose first that the participation constraints for both the managers do not bind at the optimum and that the matching is negatively assortative, i.e., $\mu(w) = m'$ and $\mu(w') = m$. Then the payoffs of the principals are given by

$$\begin{aligned}
\Pi(t(m,w')) &= \bar{\phi}\left(m,w'\right), \\
\Pi(t(m',w)) &= \bar{\phi}\left(m',w\right).
\end{aligned}$$

Manager w gets $\bar{\sigma}(m',w)$. Since for both managers the participation constraints do not bind, we must have $e(m,w,u_w) = e(m,w',u_{w'})$. Notice that $\bar{\phi}(m,w) - \bar{\phi}(m,w') = w - w' > 0$ since $\bar{\sigma}(m,w) > u_w$. Also,

$$\frac{\partial \bar{\sigma}(m,w)}{\partial m} = \frac{e\psi''\pi'(m)}{2\psi'' + e\psi'''} > 0 \text{ since } \pi'(m) > 0.$$

Thus, $\bar{\sigma}(m,w) > \bar{\sigma}(m',w)$. Hence, there exists $\varepsilon \in \left(0, \frac{1}{2}[\bar{\phi}(m,w) - \bar{\phi}(m,w')]\right)$ such that

$$\begin{aligned}
\Pi(t(m,w)) &= \bar{\phi}(m,w) - \varepsilon > \bar{\phi}(m,w'), \\
V(t(m,w)) &\geq \bar{\sigma}(m',w) + \varepsilon > \bar{\sigma}(m',w).
\end{aligned}$$

Thus, firm m and manager w will block the initial allocation which contradicts stability. Hence, the stable matching must be positively assortative.

CASE 2: Now suppose that for one of the two managers the participation constraint binds at the optimum. This has to be the case with w, otherwise the optimal effort of w will be lower than that of w'. Suppose further that the equilibrium matching is not positively assortative. In the similar fashion as above, one can show that m and w can form a blocking pair which would contradict the stability of the initial allocation.

CASE 3: Finally suppose that the participation constraints for both the managers bind at the optima. The Pareto frontier for a given pair (m,w) is given by

$$\phi(m,w,u_w) = \hat{\phi}(m,w,u_w).$$

From the above, it is easy to check that

$$sign[\phi_{12}(m,w,u_w)] = sign[\phi_{13}(m,w,u_w)] = sign[\pi'(m)].$$

Given that $\pi'(m) > 0$, we have $\phi_{12}(m,w,u_w) > 0$ and $\phi_{13}(m,w,u_w) > 0$. Take $m > m'$, $w > w'$ and $u_w > u_{w'}$. Then the last two inequalities respectively imply

$$\phi(m,w,u_w) - \phi(m',w,u_w) > \phi(m,w',u_w) - \phi(m',w',u_w), \tag{3}$$
$$\phi(m,w',u_w) - \phi(m',w',u_w) > \phi(m,w',u_{w'}) - \phi(m',w',u_{w'}), \tag{4}$$

which together give

$$\phi(m,w,u_w) - \phi(m,w',u_{w'}) > \phi(m',w,u_w) - \phi(m',w',u_{w'}). \tag{5}$$

The above is the condition of increasing willingness to pay. Now suppose that, in a stable allocation, condition (5) holds and the matching is not positively assortative. Then there exist $m > m'$ and $w > w'$ such that $\mu(m) = w'$ and $\mu(m') = w$. Since the allocation is stable, neither (m,w) nor (m',w') can block the allocation. Hence, it must be the case that (i) $\phi(m,w',u_{w'}) \geq \phi(m,w,u_w)$ and (ii) $\phi(m',w,u_w) \geq \phi(m',w',u_{w'})$. These two inequalities together imply

$$\phi(m,w,u_w) - \phi(m,w',u_{w'}) \leq \phi(m',w,u_w) - \phi(m',w',u_{w'}), \tag{6}$$

which is a contradiction to the fact that condition (5) holds in a stable allocation. ||

Proof of Proposition 3 To prove this proposition, we first show that a wealthier manager consumes higher payoff/compensation in equilibrium. Consider any two managers w and w' with $w > w'$. Suppose further that $\mu(w) = m$ and $\mu(w') = m'$ in this outcome. First we prove that, for any firm m, at the second-best

$$\phi(m,w,u) > \phi(m,w',u'), \text{ if } w > w' \text{ and } u \leq u'.$$

To show the above, consider the optimal contracting problem ($\mathscr{P}$). It is easy to check that $\phi_2(m,w,u) > 0$ and $\phi_3(m,w,u) < 0$. Then these two together imply

$$\phi(m,w,u) > \phi(m,w',u), \text{ for } w > w', \tag{7}$$
$$\phi(m,w',u) > \phi(m,w',u'), \text{ for } u \leq u'. \tag{8}$$

Thus, from the above two inequalities we have

$$\phi(m,w,u) > \phi(m,w',u'), \text{ if } w > w' \text{ and } u \leq u'.$$

Now suppose that $w > w'$ and in a stable allocation $u_w \leq u_{w'}$. From the previous inequality we know that, for firm m',

$$\phi(m',w,u_w) > \phi(m',w',u_{w'}).$$

Define by $t + \varepsilon = (R + \varepsilon, R + b + \varepsilon)$ for $\varepsilon > 0$ and sufficiently small. In this contract a manager is paid the same additional amount for both cost realizations. It is easy to check that, compared to the contract t, incentive constraint remains unaltered under $t + \varepsilon$ as well. If the last inequality holds, then there exists a feasible contract for the pair (m',w), $t'(m',w) = t(m',w,u_w) + \varepsilon$ such that $\Pi(t'(m',w)) = \phi(m',w,u_w) - \varepsilon > \phi(m',w',u_{w'})$ and $V(t'(m',w)) \geq u_w + \varepsilon > u_w$. Thus, m' and w can form a blocking pair, which is a contradiction. Thus, we conclude that $u_w > u_{w'}$ for any two w and w' with $w > w'$.

Now suppose that $\pi(m)$ is an increasing function of m. Then by Proposition 2, a higher w is matched with a higher m. This immediately implies that, between any two firms m and m' with $m > m'$, firm m ends up offering higher compensation, u_w. If $\pi(m)$ is decreasing in m, there is a decreasing relationship between executive compensation and competition.

Next, we compare the optimal contracts in two distinct pairs. Consider two firms $m > m'$ and two managers $w > w'$. First suppose that $\pi(m)$ is increasing in m. Then the matching is positively assortative. In this case, the optimal efforts for the two pairs are given by $e(m,w,u_w)$ and $e(m',w',u_{w'})$, and the bonuses are $b(m,w,u_w)$ and $b(m',w',u_{w'})$. From the optimization problem $(\mathscr{P})$ we have

$$e_1(m,w,u_w) \geq 0, \quad e_2(m,w,u_w) > 0, \quad e_3(m,w,u_w) \geq 0,$$
$$b_1(m,w,u_w) \geq 0, \quad b_2(m,w,u_w) > 0, \quad b_3(m,w,u_w) \geq 0$$

The above inequalities imply that

$$e(m',w',u_{w'}) \leq e(m,w',u_{w'}) \leq e(m,w',u_w) < e(m,w,u_w),$$
$$b(m',w',u_{w'}) \leq b(m,w',u_{w'}) \leq b(m,w',u_w) < b(m,w,u_w).$$

Hence, bonus is higher and managerial slack is lower for higher m. A similar argument leads to the fact that $\pi(m)$ is a decreasing function of m implies that bonus is higher and managerial slack is lower for lower m. Obviously, if $\pi(m)$ is non-monotone in m, then bonus and effort are non-monotone with respect to the degree of competition. ||

Proof of Corollary 1 We need to show that if $m > m'$, then $\phi(m',.,.) > \phi(m,.,.)$ irrespective of whether the equilibrium matching is positively or negatively assortative. Suppose first that the matching is positively assortative, i.e., for $m > m'$ and $w > w'$ we have $\mu(m) = w$ and $\mu(m') = w'$. Suppose, on the contrary, that

$$\phi(m,w,u_w) > \phi(m',w',u_{w'}). \tag{9}$$

We know that $\phi_1(m,w,u) \leq 0$, which implies

$$\phi(m,w,u_w) \leq \phi(m',w,u_w). \tag{10}$$

The above two inequalities together imply

$$\phi(m',w,u_w) > \phi(m',w',u_{w'}). \tag{11}$$

The above implies that there exists a blocking contract $t'(m',w) = t(m',w,u_w) + \varepsilon$ such that $\Pi(t'(m',w)) = \phi(m',w,u_w) - \varepsilon > \phi(m',w',u_{w'})$ and $V(t'(m',w)) \geq u_w + \varepsilon > u_w$. Thus, m' and w can form a blocking pair, which is a contradiction. Now, if the matching is negatively assortative, then one can construct a blocking contract $t'(m',w')$ in the similar fashion as above, which would contradict the stability of the initial allocation. ||

Proof of Proposition 4 Without loss of generality, we analyze the market equilibria for $N = 3$. Let the initial market be $\mathscr{M} = \{m', m'', m'''\}$, with $m' > m'' > m'''$ and $\mathscr{W} = \{w', w'', w'''\}$ with $w' > w'' > w'''$. Thus, $m' = \sup\{\mathscr{M}\}$. We use the following lemma to prove this proposition.

LEMMA **1** *If there are more firms than managers, i.e.,* $|\mathscr{M}|>|\mathscr{W}|$*, then only the firms in the three least competitive markets and all the managers are matched in a market equilibrium.*

Proof Suppose there are four firms with $m' > m'' > m''' > \hat{m}$ and three managers with $w' > w'' > w'''$, and in a stable allocation $(\mu, \mathscr{C})$, $\hat{m}$ is unmatched. So this firm consumes zero payoff by signing a null contract. Take any $m \in \{m', m'', m'''\}$. Since $\phi(m, w, u_w) < \phi(\hat{m}, w, u_w)$ for any w, firm $\hat{m}$ and manager $\mu(m)$ can sign a blocking contract $t'(\hat{m}, \mu(m)) = t\left(\hat{m}, \mu(m), u_{\mu(m)}\right) + \varepsilon$ such that $V(t'(\hat{m}, \mu(m))) \geq u_{\mu(m)} + \varepsilon > u_{\mu(m)}$ and $\Pi(t'(\hat{m}, \mu(m))) = \phi(\hat{m}, \mu(m), u_{\mu(m)}) - \varepsilon > 0$. This contradicts the fact that $(\mu, \mathscr{C})$ is stable. Following this logic, it is immediate to show that only firms m'', m''', $\hat{m}$ will be matched. □

The above lemma trivially extends to $|\mathscr{M}|>|\mathscr{W}| = N \geq 3$. Let there be an entrant firm $\hat{m}$. First suppose that $\pi(m)$ is increasing in m. Hence, the matching is positively assortative both in the initial and the final equilibria. Let the initial equilibrium allocation be $(\mu, \mathscr{C})$ with $\mu(m') = w'$, $\mu(m'') = w''$ and $\mu(m''') = w'''$, and u_w for $w \in \{w', w'', w'''\}$ be the equilibrium compensation of manager w. Let the new equilibrium allocation be $\left(\hat{\mu}, \hat{\mathscr{C}}\right)$ with the vector of equilibrium payoffs of the managers, $(\hat{u}_w)_{w\in\mathscr{W}}$. If $\hat{m} > m'$, then following the above lemma firm $\hat{m}$ is not matched, and $(\mu, \mathscr{C}) = \left(\hat{\mu}, \hat{\mathscr{C}}\right)$. Now suppose that $m' > \hat{m} > m''$. Then in the new equilibrium, $\hat{\mu}(\hat{m}) = w'$, $\hat{\mu}(m'') = w''$ and $\hat{\mu}(m''') = w'''$. This implies that $\hat{m}$ and w' could have formed a blocking pair has $\hat{m}$ been unmatched in the new equilibrium. Hence, w' must receive strictly higher payoff in the new equilibrium allocation, whereas w'' and w''' continues to receive the same. Thus compensations of the managers weakly improves in the final equilibrium. Next, suppose that $m'' > \hat{m} > m'''$. Thus in the new equilibrium allocation, we have $\hat{\mu}(m'') = w'$, $\hat{\mu}(\hat{m}) = w''$ and $\hat{\mu}(m''') = w'''$. This new allocation is stable since $\hat{m}$ can form blocking pair with either of w' and w'', and m'' can write a blocking contract with w'. Hence, in the new equilibrium both w' and w'' are strictly better-off, while w''' consumes the same payoff as in the initial equilibrium. It is easy to see that if $\hat{m} < m'''$, then all the three managers are strictly better-off. Similar logic goes through when the matching is negatively assortative in both the equilibria. ||

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