Does Government Spending on Education Promote Growth and Schooling Returns?

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Work in Progress, Comments Welcome

Abstract

Since human capital is a major driver of growth, the conventional wisdom suggests that the government should direct more resources to education. However, surprisingly the cross country data show little positive correlation between growth and public spending on education. In fact, the pattern is rather puzzling. The public spending on education tends to lower growth and schooling returns. In this paper, we revisit this issue and try to understand these puzzling facts in terms of an endogenous growth model. We model return to schooling by adopting an asset pricing approach and show the explicit linkage between government intervention and growth via this schooling return.

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1 Introduction

The effect of public expenditure on growth is an unresolved issue. Greater government spending on education creates an unproductive bureaucracy that may not have any bearing on growth and productivity (Pritchet, 2001). Sylwester (2000) demonstrates that the contemporaneous education expenditure has a negative effect on growth. Temple (2001) revisits Pritchet's empirical evidence and shows with alternative statistical procedures that the link between education expenditure and growth is "clouded with uncertainty."

In this paper, we revisit this issue and focus specifically on the relationship between public expenditure on education, returns to schooling and growth. The cross country development facts summarized in Table 1 show some curious patterns. Countries with a higher share of public education spending in GDP have lower growth rates and lower returns to schooling while growth and returns to schooling are positively correlated.

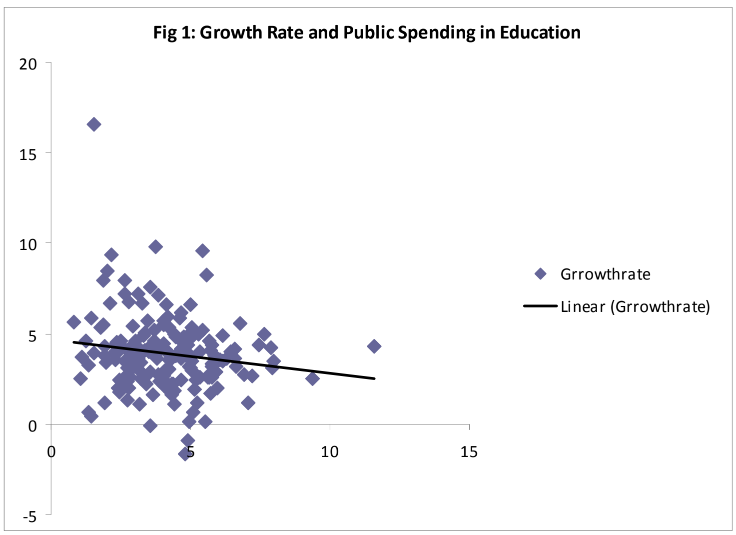
Table 1: Correlation coefficients among education spending, growth and returns to schooling

	Education Spending	Growth	Returns to Schooling
Education Spending	1	-.29	-.39
Growth		1	.39
Returns to Schooling			1

The data encompass a sample of 44 countries for which the returns to schooling data are available.[1] The negative correlation between public spending on education and growth survives for a bigger sample of countries although the correlation is sensitive to the choice of the sample of countries as pointed out by Bosworth and Collins (2003). Figure 1 plots the relationship for a larger sample of 180 countries. Despite the tremendous cross country growth dispersion, the correlation coefficient is -.15 and significant at the 6% level. The negative correlation is weak for countries in the upper echelon

[1] The data encompass a sample of 44 countries for which the returns to schooling data are available. Rate of return data used in this analysis were constructed from the rate of return education database of the World Bank available at their website. The growth and public spending on education series were compiled from the World Development Indicators.

of education spending. In fact, for countries in the top quartile range of education spending, the correlation turns positive at .04 and for the bottom quartile range, the correlation is -.11. We looked into possible nonlinearity in the relationship between growth and education spending by adding a quadratic term in the growth-education regression. The quadratic term is positive and significant at the 20% level. This means a U shaped relationship between growth and education which suggests that growth-education spending relationship reverses sign from negative to positive for countries with higher education spending.



The objective of this paper is to understand these stylized facts. Our paper is novel because we model returns to schooling by integrating two factors, (i) public policy in education and (ii) genetically determined talents of pupils which one may call cognitive skills. We take an asset pricing approach to formulate the returns to schooling. Human capital is viewed as an asset which yields flow returns which are earnings from the goods sector. We show that (i) and (ii) have very different effects on returns to education and growth. Increase in the size of the government in the education sector has a nonmonotonic U- shaped effect on growth and schooling returns. This happens because two opposing effects are at work when the government spends more on education by diverting resources from the goods sector. Since the goods sector is more capital intensive, this transfer of resource acts as an implicit tax on capital. Since along a balanced growth path, return to physical capital equals the return to human capital, agents have to reallocate more time to goods production and less to education to preserve this long run arbitrage condition. An increase in government spending on education thus crowds out private schooling efforts along a balanced growth path. This crowding out effect has an adverse consequence for schooling returns and growth. At early stage of government intervention in education, this distortionary effect dominates. Once a threshold level of government intervention is reached, the positive growth effect picks up. If the economy is subject to greater adjustment cost of physical investment, it takes more intervention by the government to have this positive growth effect.

A different picture emerges when one varies cognitive skills. Greater talents of pupils means higher returns to education, and growth. Countries with a bigger cohort of talented pupils will thus put more efforts at schooling to augment human capital. This will create a larger tax base for the government to spend on education. The steady state share of government spending on education will thus be higher. When one looks at the cross country data for growth and share of government spending in education, these two opposing effects may be at work. Countries with a greater degree of government intervention in the education sector will show a negative correlation between public education spending and growth while countries with more talented pupils will show the opposite pattern.

Our paper is related to a growing literature that shows the connection between growth and education spending (Basu, 2009, Krueger and Lindhal, 2001, Glomm and Ravikumar, 1997, Zhang, 1997). The closest paper is Blankeau et al. (2007) who also establish a nonmonotonic relationship between growth and public spending on education using an endogenous growth model. There are a several major differences between our approach and Blanekeau et al. First, Blankeau et al. do not explicitly model returns to schooling by integrating time allocation between work and education. Second, they use an overlapping generations model while we use an infinite horizon setup making the model amenable to calibration. Third, unlike

Blankeau et al. in our model there is a physical adjustment cost for changing the capital stock that is crucial for understanding how long it takes for the government spending to have a positive effect on growth and schooling returns.

In the following section, we lay out the theoretical model. Section 3 performs some quantitative analysis on the balanced growth equations of the model. Section 4 concludes.

2 The Model

2.1 Schooling Technology

The model is an adaptation of the Lucas-Uzawa (Lucas, 1988) model. There are two sectors, goods and education. A fixed time (normalized at unity) is allocated between schooling and goods production. Time l_{Ht} allocated to schooling at date t creates effective labour or human capital (h_{t+1}) in the following period. The productivity of schooling effort which is the same as the quality of schooling depends on individual cognitive skills (A_H) that is exogenous and the public spending on education (g_t) :

The human capital thus evolves following the technology:

$$h_{t+1} = (1 - \delta_h)h_t + A_H g_t^{\eta}(l_{Ht}h_t)^{1-\eta} \tag{1}$$

where$0 < \eta < 1$. g_t is the government provided input for human capital acquisition. This input that resembles Barro (1990) takes the form of public school infrastructure facilities, expenditure on teachers, school lunch programme and other aids to promote learning. Without this government support there is diminishing returns to human capital. There is a fixed rate of depreciation, δ_h of human capital.

The parameter η represents the degree of government intervention in the education sector. This government intervention parameter is modelled as a schooling technology. This intervention depends on the institutional features of a country which we do not model in this paper. Absent the government role in the education (η equals zero), the schooling technology reverts to the Lucas (1988) form.

2.2 Goods Production

Final goods (y_t) are produced with the help of human and physical capital via the Cobb-Douglas production technology:

$$y_t = A_G {k_t}^{\alpha} (l_G h_t)^{1-\alpha} \tag{2}$$

where l_G(that equals $1 - l_{Ht}$) is the remaining time allocated to the production of goods and A_G is the total factor productivity (TFP) in the goods sector.[2]

2.3 Investment Technology

The investment goods technology as follows:

$$\frac{k_{t+1}}{k_t} = \left[1 - \delta_k + \frac{i_t}{k_t}\right]^{\theta} \tag{3}$$

where δ_k is a fixed rate of depreciation of physical capital. . The parameter $\theta \in (0,1)$ represents the extent of adjustment cost. For $\theta = 1$, the investment technology reduces to a standard linear depreciation rule. This adjustment cost is a parametric version of the Lucas and Prescott (1971) adjustment cost function. Similar parametric form with 100% depreciation of capital has been used in the literature (Basu, 1987, Hercowitz and Sampson, 1991).

2.4 Government

The home country's government finances the education spending (g_t) by levying lump-sum taxes on the households. The education spending is then set at an efficient level that maximizes societal utility.

[2] We assume that lesure time is fixed.

The home country receives instantaneous utility $U(c_t)$ from consumption (c_t) at date t and has an infinite horizon with a subjective utility discount factor β. It chooses the sequences $\{c_t\}, \{i_t\}, \{l_{Ht}\}, \{g_t\}$ that maximizes

$$Max \sum_{t=0}^{\infty} \beta^t U(c_t)$$

subject to the resource constraint:

$$c_t + i_t + g_t = y_t \tag{4}$$

and (1) through (3).

3 Balanced Growth Properties

Hereafter we specialize to a logarithmic utility function, $U(c_t) = \ln c_t$, to analyze the balanced growth properties of the model. The appendix details the solution of the balanced growth path. There are three key balanced growth equations.

Based on the first order condition for the physical capital stock we get:

$$G = \left[\frac{\beta\theta \left(A_G(\alpha y/k) + 1 - \delta_k \right)}{1 - \beta \left(1 - \theta \right)} \right]^{\theta}, \tag{5}$$

From the first order condition for the human capital stock, one gets:

$$G = \beta(1-\delta_h) + \beta A_H \eta^{\eta} (1-\eta)^{1-\eta} (1-\alpha)^{\eta} . (1-l_H)^{-\eta} (y/h)^{\eta} \tag{6}$$

where y/h and l_H are constants along the balanced growth path.

Finally, using the human capital technology (1), we get a third balanced growth equation:

$$G = 1 - \delta_h + A_H \left[\frac{(1-\alpha)\eta}{(1-\eta)} . \frac{l_H}{l_G} \right]^{\eta} l_H^{1-\eta} A_G l_G^{(1-\alpha)\eta} (k/h)^{\alpha\eta} \tag{7}$$

These three equations solve for three unknowns, namely k/h, l_H and G. The appendix provides full expression for the steady state levels of these two variables.

The share of public spending in GDP is given by:

$$\frac{g_t}{y_t} = \frac{1-\alpha}{1-\eta} . \frac{\eta l_H}{l_G} \tag{8}$$

A higher intervention of government in he education sector (meaning larger η) raises this ratio. On the other, greater time to school (l_H) also raises this share because the positive growth effect of education increases the tax base for the economy.

3.1 Return to Schooling

In this model, the human capital is an asset which yields a flow return. Think of human capital as a Lucas tree whose valuation is q_t^h which is akin to Tobin's q of physical capital. This valuation is driven by the return and opportunity cost of going to school.

It is easy to verify that this value of human capital is the same as the ratio of the shadow price of consumption to that of investment in schooling. In other words,

$$q_t^h = \frac{\mu_t}{\lambda_t} \tag{9}$$

where μ_t and λ_t are the lagrange multipliers associated with the schooling

technology (1) and the flow resource constraint (4). using the Euler equation for human capital (see (A.7), one gets the following valuation equation for the human capital:

$$q_t^h = m_{t+1}\{q_{t+1}^h[1-\delta_h+A_H g_{t+1}^{\eta}(1-\eta)(1-l_{Gt+1})^{1-\eta}h_{t+1}{}^{-\eta}]+\{A_G(1-\alpha)k_{t+1}^{\alpha}h_{t+1}^{-\alpha}l_{Gt+1}^{1-\alpha}\} \tag{10}$$

where m_{t+1} is the intertemporal marginal rate of substitution in consumption given by λ_{t+1}/λ_t.

Next verify from (A.5) in the appendix that

$$q_t^h = \frac{MPH_t^G}{MPH_t^E} \tag{11}$$

where MPH_t^G and MPH_t^E are the marginal products of effective labour in the goods and education sectors respectively.

Rewrite (10) as

$$q_t^h = m_{t+1}\left[q_{t+1}^h(1-\delta_h + l_{Ht+1}MPH_{t+1}^E) + l_{Gt+1}MPH_{t+1}^G\right] \tag{12}$$

The valuation equation for human capital looks similar to a Lucas (1978) tree valuation equation. The value of this tree at date t is the discounted next period marginal product of human capital in the goods sector, $l_{Gt+1}MPH_{t+1}^G$ and the imputed next period value of unused portion of the tree $(1-\delta_h)q_{t+1}^h$ plus the replenishment of it, $l_{Ht+1}MPH_{t+1}^E$ due to new education.

The return to schooling (R_{t+1}^h) is thus given by:

$$R_{t+1}^h = \frac{\left[q_{t+1}^h(1-\delta_h + l_{Ht+1}MPH_{t+1}^E) + l_{Gt+1}MPH_{t+1}^G\right]}{q_t^h}$$

Along the balanced growth path, q_t^h is constant. Using (11) one obtains the following expression for the steady state return to human capital:

$$R^h = 1 - \delta_h + MPH^E \tag{13}$$

Using (13) one can rewrite the balanced growth equation as follows:

$$1 + g = \beta R^h \tag{14}$$

3.2 Cognitive Skills, Returns to Schooling and Growth

The model has sharp implications for the relationship between returns to schooling, growth and cognitive skills. Long run growth is proportional to return to schooling as evident from (14). The return to schooling is entirely driven by the marginal product of effective labour in the education sector as seen from (13). This marginal product is a function of the combination of the cognitive skills of pupils A_H , the degree of government intervention (η) in the education sector as well as the private schooling efforts, l_H.

3.3 The Physical Capital Adjustment Cost Wedge

Generally, with $\theta \in (0,1)$, the physical capital adjustment cost drives a wedge between the returns to physical capital and return to schooling. This wedge depends on the human capital investment. To see this note the following useful relationship:

$$R^h = (1+\rho)\left[\frac{\theta}{\rho+\theta}\right]^{\theta}[1+MPK-\delta_k]^{\theta}. \tag{15}$$

where MPK is the marginal product of capital in the goods sector. In the benchmark case of no adjustment cost ($\theta = 1$), it follows from equation (6) that the traditional Euler equation holds, meaning

$$1+g = \beta(1-\delta_k+MPK), \tag{16}$$

In the present setting, the adjustment cost wedge or the user cost of capital depends non-trivially on the long run growth rate. To see this, use (15) and (6) to obtain the following expression for the user cost of capital:

$$\frac{1-\delta_h+MPH^E}{1-\delta_k+MPK} = \left\{(1+\rho).\frac{\theta}{\rho+\theta}\right\}.\frac{1}{G^{(1-\theta)/\theta}} \tag{17}$$

For any growing economy, the right hand side of (17) is always a positive fraction. This means that $1-\delta_h+MPH^E < 1-\delta_k+MPK$. This inequality result can be interpreted as implying that the physical capital adjustment cost creates a user cost wedge that causes a lower physical capital to effective labor ratio in equilibrium than when $\theta = 1$. And this is consistent with our notion that accumulating physical capital is more costly in the presence of adjustment cost, as in Lucas (1967). What is novel in the present setting is the interaction between this user cost wedge and the investment in human capital via the long run growth rate, g.

4 Quantitative Analysis

In this section, we carry out some quantitative analysis of the model's balanced growth properties to understand the stylized facts reported in Table 1. The model has 10 parameters, namely $\alpha, \delta_h, \delta_k, \beta, \eta, A_H, \theta, A_G$. Using the US economy as the benchmark, α is fixed at .36, β at .99, δ_k =.1 and $\delta_h = .05$. The government intervention parameter η is chosen at .05 which is approximately the cross country average share of education spending in GDP for our sample of countries. For the baseline economy we assume that there is no adjustment cost which means $\theta = 1$.The remaining productivity parameters, A_G and A_H are varied in a range to target about a 4 to 6% growth rate of world GDP and k:h ratio around unity. [3] Doing so we fix $\mathrm{A}_G = 2.2$, $\mathrm{A}_H = .15$.

Figure 2 plots the effect of government intervention (η) in education on growth and schooling returns. Greater intervention lowers growth and returns until a threshold level of intervention is reached. After this the positive growth effect picks up. Decline in growth and schooling returns in response to government intervention is sharper if a 20% adjustment cost is added ($\theta = .8$). In the presence of adjustment cost, it takes more intervention by the government before the positive growth effect picks up.

Fig 4 plots the effect of government intervention on schooling efforts $l_{Ht.}$ Greater degree of government interference in the education sector unambiguously crowds out private schooling efforts.[4] A fundamental arbitrage condition is at work to engender this crowding out effect. To see this, ignore adjustment cost. Along the balanced growth path, the return to human capital equals the returns to physical capital. Based on (A.9) it is easy to see that ceteris paribus, an increase in η diverts resources from the goods to the education sector. This is an implicit tax on capital because the goods sector is capital intensive. If the social planner cannot change time alloca-

[3]Since there is no availble estimate of physical to human capital ratio, we take unity as a reasonable benchmark.

[4]The effect is quantitatively very similar in positive adjustmet cost scenario which we do not report here.

Fig 2: Growth, Schooling Returns and Government Intervention: No Adj Cost Scenario

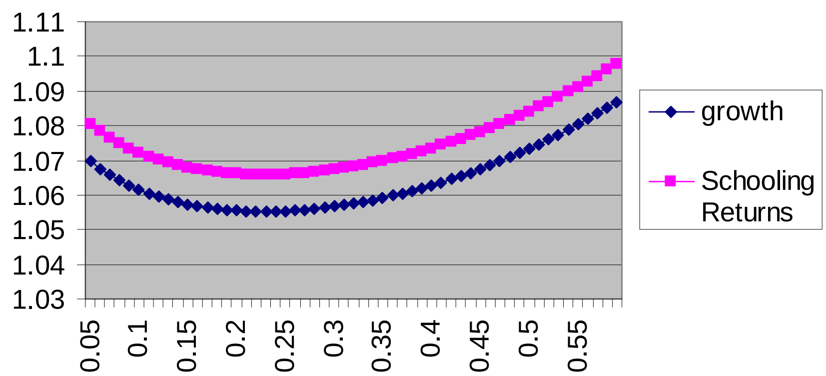
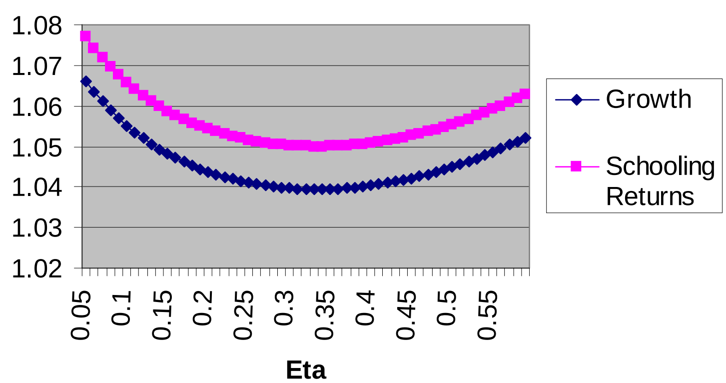
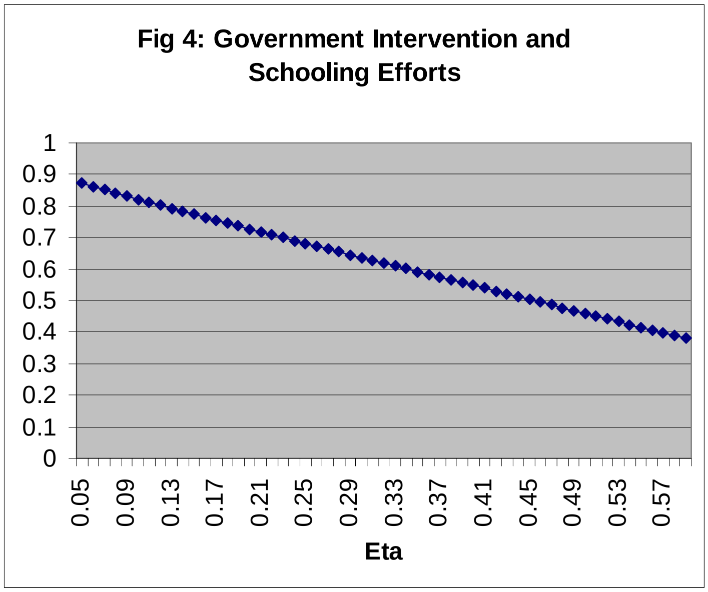


Fig 3: Government Intervention, Growth and Schooling Returns: Positive Adjustment Cost Scenario

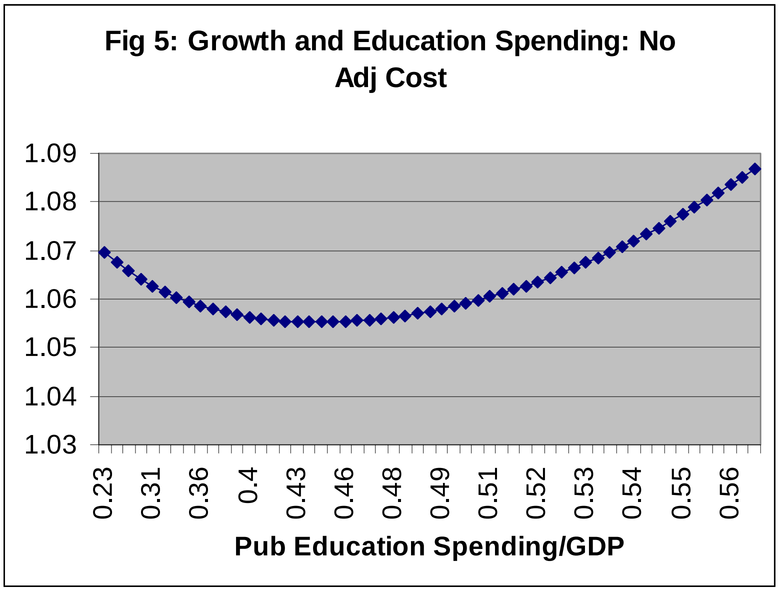


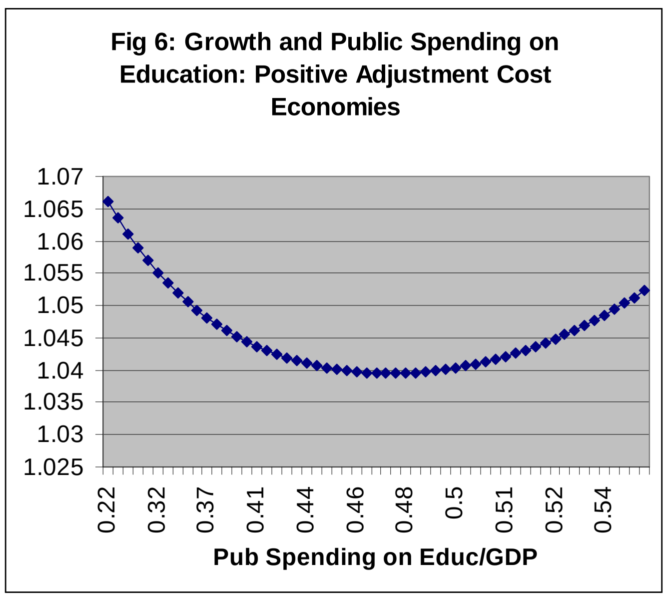


tion l_H, this results in a lower k/h. Once the planner has control over the time allocation, time will be reallocated from education to goods sector to preserve this long run arbitrage condition.

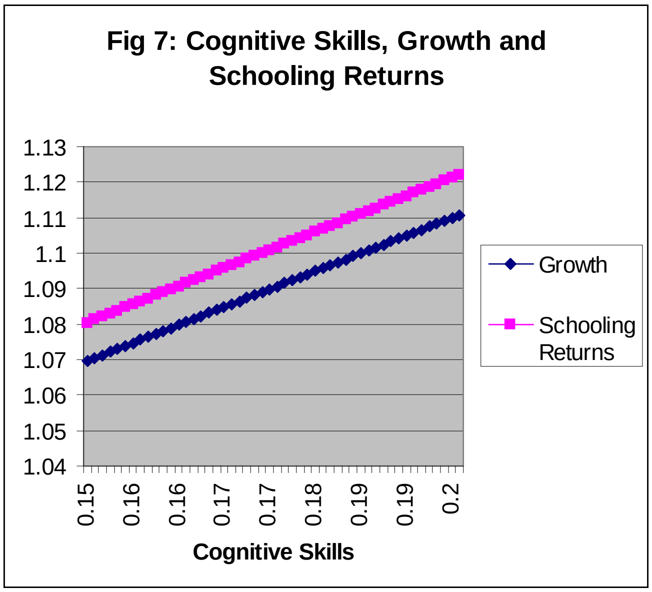
Two opposing effects are at work when the government intervenes more in the education sector. It distorts private sector incentive to put time in schooling. This has adverse effects on growth and schooling returns. On the other hand, complementarity between human capital and government provided services boosts growth and schooling returns. The former negative effect dominates when the size of the government in the education sector is small.

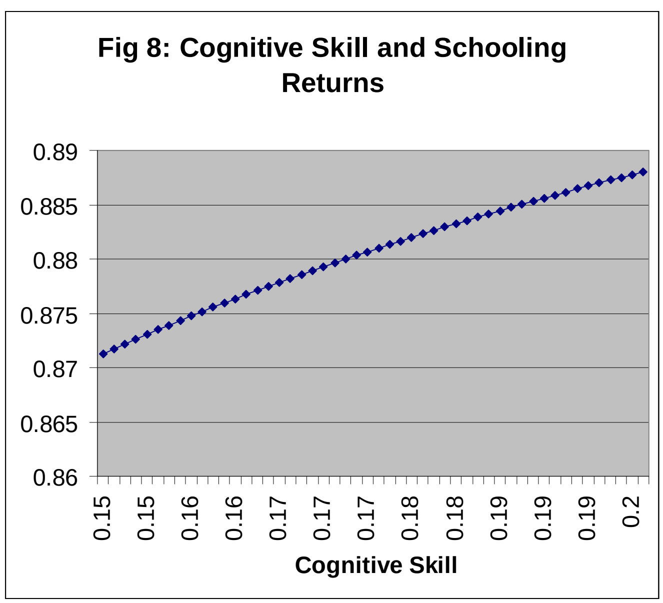
Figure 5 plots the relation between growth and the share of public education spending in GDP. The relationship between education and GDP is non-monotonic and resembles Figure 2. Note that public spending on GDP is endogenous in the model and is determined by direct intervention of the government in education and the private sector response to this intervention (see equation (8)).

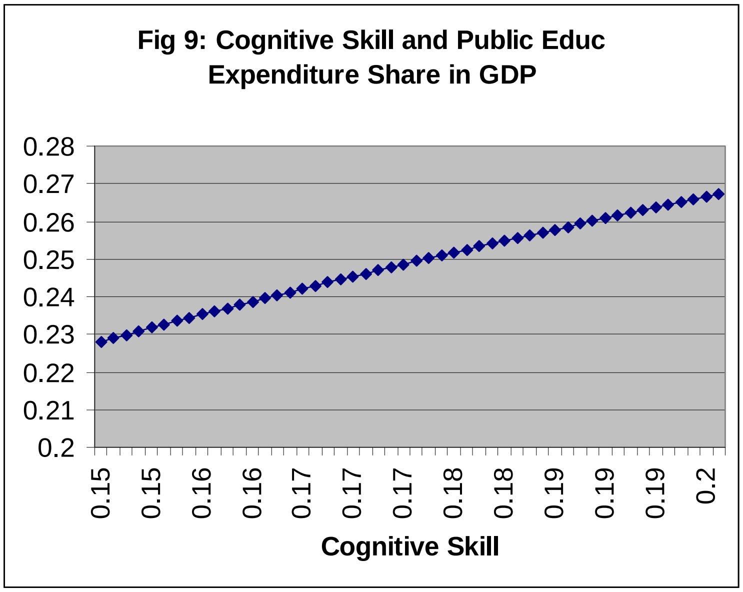




A very different picture emerges if one performs comparative statics by changing the pupil's talent parameter, A_H (Table 3). Countries with more talented pupils will show greater schooling participation. This happens because a higher A_H raises return to education, R^H. Agents reallocate more time to education. This lowers the ratio of physical to human capital thus raising the returns to the goods sector as well. The net outcome is a higher returns to schooling, higher growth, and greater education share in GDP. .







5 Conclusion

In this paper we revisit the issue whether public spending on education has favourable effects on growth and schooling returns. The model demonstrates that there is a clear nonmonotonic relationship bewteen public spending on education and growth. Positive effects of government spending appear for economies with a large size of the government in the education sector. It is also well known that richer countries spend more on education (Armellini and Basu, 2009). The immediate implication is that public spending on education has favorable effects on growth and schooling returns for rich countries with large share of public spending. Blankeau et al. (2007) also provide confirmation of this hypothesis. Our model demonstrates that despite this positive effect on growth greater government intervention in education lowers private schooling efforts. This disincentive effects of government education spending bears also on a different strand of literature which advocates for privatization of education in terms of auctioning off education vouchers. Future research can explore this implication further.

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A Appendix

Let $\lambda_t, \mu_t, \nu_t, \omega_t$ be the lagrange multipliers associated with the flow budget constraint (4), human capital technology, (1), the physical investment technology (3).

The lagrange is:

$$L = \sum_{t=0}^{\infty} \beta^t U(c_t) + \sum_{t=0}^{\infty} \lambda_t [A_{Gt} k_t^{\alpha} (l_{Gt} h_t)^{1-\alpha} - c_t - i_t - g_t]$$
$$+ \sum_{t=0}^{\infty} \mu_t [(1-\delta_h) h_t + A_{Ht} g_t^{\eta} (l_{Ht} h_t)^{1-\eta} - h_{t+1}]$$
$$+ \sum_{t=0}^{\infty} \nu_t \left\{ k_t \left[1 - \delta_k + \frac{i_t}{k_t} \right]^{\theta} - k_{t+1} \right\}$$

First order conditions are:

$$c_t : \beta^t U'(c_t) = \lambda_t \tag{A.1}$$

$$i_t : -\lambda_t + \nu_t \theta \left[1 - \delta_k + \frac{i_t}{k_t}\right]^{\theta - 1} = 0 \tag{A.2}$$

$$k_{t+1} : -\nu_t + \nu_{t+1} \left[\left(1 - \delta_k + \frac{i_{t+1}}{k_{t+1}}\right)^{\theta} - \theta \frac{i_{t+1}}{k_{t+1}}. \left(1 - \delta_k + \frac{i_{t+1}}{k_{t+1}}\right)^{\theta - 1}\right] + \lambda_{t+1} \alpha \frac{y_{t+1}}{k_{t+1}} = 0 \tag{A.3}$$

$$\begin{aligned} h_{t+1} \quad : \quad & \mu_t = \mu_{t+1}[1 - \delta_h + A_{Ht+1} g_{t+1}^{\eta} (1 - \eta) h_{t+1}{}^{-\eta} l_{Ht+1}^{1-\eta}] \\ & + \lambda_{t+1} \{ A_{Gt+1} (1 - \alpha) k_{t+1}^{\alpha} h_{t+1}^{-\alpha} l_{Gt+1}^{1-\alpha} \end{aligned} \tag{A.4}$$

$$l_{Gt} : \lambda_t (1 - \alpha) A_G l_{Gt}^{-\alpha} k_t^{\alpha} h_t^{1-\alpha} - \mu_t (1 - \eta) g_t^{\eta} A_H h_t^{1-\eta} l_{Ht}^{-\eta} = 0 \tag{A.5}$$

$$g_t : \lambda_t = \mu_t A_{Ht} \eta g_t^{\eta - 1} (h_t l_{Ht})^{1-\eta} \tag{A.6}$$

A.1 Derivation of the Balanced Growth Equations

Along the balanced growth path, we also exploit the fact that the raw labour allocation variables l_{Gt} and l_{Ht} are constant

Using (A.2), (A.3) and (3) one gets the balanced growth equation (5).

Rewrite (A.4) as:

$$\frac{\mu_t}{\lambda_t} = \frac{\mu_{t+1}}{\lambda_{t+1}}.\frac{\lambda_{t+1}}{\lambda_t}[1-\delta_h + A_H g_{t+1}^{\eta}(1-\eta)(1-l_{Gt+1})^{1-\eta} h_{t+1}{}^{-\eta}] \quad \text{(A.7)}$$
$$+\frac{\lambda_{t+1}}{\lambda_t}\{A_G(1-\alpha)k_{t+1}^{\alpha}h_{t+1}^{-\alpha}l_{Gt+1}^{1-\alpha}$$

Next use (A.6) to substitute out $\frac{\mu_t}{\lambda_t}$ and also use the balanced growth condition using (A.1) that $\frac{\lambda_{t+1}}{\lambda_t} = \beta/(1+g)$ which upon substitution above yields:

From (A.5), one can write

$$\frac{\mu_t}{\lambda_t} = \frac{(g_t/h_t)^{1-\eta}}{\eta A_H l_{Ht}^{1-\eta}}$$

Use (A.6) and (A.5) to get the following compact expression for g_t/h_t :

$$\frac{g_t}{h_t} = \frac{1-\alpha}{1-\eta}.\frac{\eta l_H}{l_G}.\frac{y_t}{h_t} \quad \text{(A.8)}$$

which immediately gives the following expression for the education share in GDP:

$$\frac{g_t}{y_t} = \frac{1-\alpha}{1-\eta}.\frac{\eta l_{Ht}}{l_{Gt}} \quad \text{(A.9)}$$

Note that along a steady state $\frac{(g_t/h_t)^{1-\eta}}{\eta A_H l_{Ht}^{1-\eta}}$ is a constant which means $\frac{\mu_t}{\lambda_t} = \frac{\mu_{t+1}}{\lambda_{t+1}}$. Use this fact in (A.7) and also use the fact that $\frac{\lambda_{t+1}}{\lambda_t} = \frac{\beta}{1+g}$ to get:

$$1 = \frac{\beta}{1+g}\left[1-\delta_h + A_H(1-\eta)l_H^{1-\eta}(g/h)^{\eta}\right] + \frac{\beta}{1+g}.\frac{\eta A_H l_H^{1-\eta}}{(g/h)^{1-\eta}}.\left\{(1-\alpha)A_G(k/h)^{\alpha}l_G^{1-\alpha}\right\} \quad \text{(A.10)}$$

Plugging (A.8) into (A.10) and simplifying one gets (6).

Using (1) one gets the third balanced growth equation (7).