Recursive Contracts, Rat Races, and Rent Dissipation

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Abstract

We study contract design within an overlapping generations partnership firm. The old generation owns the firm, and designs the contract under which the young generation is hired. Young employees become partners with ownership rights, and the right to write to the contract with the next generation, when the old generation dies. This leads to a recursive problem in contract design. We show that this can lead to low wages and severe rent dissipation through excessive effort. Experimental results confirm the predictions of the model.

Preliminary and incomplete. Please do not cite or circulate.

1 Introduction

We study the firm as an overlapping generations structure, in which young workers enter as employees, or agents, and then move on at a later stage in their career to become principals who oversee the work of a new generation of young agents. We are interested in the structure of the contracts that are likely to emerge in an organization where each generation can choose the contract that will be offered to the next generation. The canonical example of such a firm is the law partnership [9], but any firm in which there is an insider bias to promotion will have some of the characteristics of our model.

Since the expected reward to an agent derives in part from the benefit of being a future principal, the contracts are interlinked, leading to an exercise in recursive mechanism design. In order to isolate the incentive structure that is implicit in such firms we adopt the very simplest environment, with complete information, homogeneous agents, and no moral hazard or adverse selection, so that we are left with only the recursive nature of the contracting environment. Our model is thus distinct from the the Akerlof Miyazaki rat race [2], which is driven by adverse selection, and the Lazear Rosen tournament [12], which is driven by moral hazard. Our model is related to Bardsley Sherstyuk [1], which studies OLG recursive contracting in a more complex environment with adverse selection, heterogeneous agents and a richer contracting space. In contrast to [1] we assume complete information and homogeneous agents, and we study firms with a finite life: they fail with a constant hazard rate α. This allows us to isolate a very simple rent dissipation mechanism that we are able to study in the laboratory.

2 The Model

There are n individuals born each period, and each individual lives for two periods. Utility is separable between periods and there is no discounting. All individuals are risk neutral.

There is a single firm, composed of an owner and a worker. At the beginning of each period the owner of the firm, the principal, seeks to hire a worker by posting a take it or leave it offer specifying a non-negative wage w and and an output level b, which the agent can produce at effort cost $e(b)$. Individuals who are not hired receive a reservation utility normalised to zero in both periods. The principal exerts no effort and receives the surplus $W = b - w$ generated by the firm after paying the wage and selling the output (the price of output is normalised to 1).

At the end of the period the firm continues with probability $1 - \alpha$, and goes out of existence with probability α. If it continues then the agent inherits the firm, with the right to choose the contract in the following period. There is complete information, with no moral hazard or adverse selection, and the contract can be enforced costlessly. There is no lending or borrowing (the agent is born with no assets other than their endowment of labour). Agents have rational expectations, and anticipate the future equilibrium when choosing their actions. We assume that the effort function $e(b)$ is smooth, increasing, convex, that $e(0) = 0$ and $0 \leq e'(0) < 1$, and that there exists an output level b such that $b > e(b)$; otherwise the optimal production level will always be 0.

The principal will choose $w \geq 0$ and $b \geq 0$ to maximise her rent

$$W = b - w$$

subject to the individual rationality constraint and a non-negativity constraint on the wage

$$\begin{aligned} w + (1-\alpha) W_+ &\geq e(b) \\ w &\geq 0, \end{aligned}$$

where we write W_+ for the next period's rent. After eliminating w, we see that the principal chooses $b \geq 0$ subject to the constraints

$$\begin{aligned} W &\leq b \\ W &\leq (1-\alpha) W_+ + b - e(b) \\ &= (1-\alpha) W_+ + \phi(b) \end{aligned}$$

where we have written $\phi(b) = b - e(b)$ for the surplus generated by the firm. The first of these constraints is the wage non-negativity constraint[1], while the second is the participation constraint.

Thus the rent obeys the Bellman equation

$$W = \max_{b \geq 0} \min \left[b, (1-\alpha) W_{+} + \phi(b) \right]. \tag{1}$$

This equation reflects the basic dilemma of the agent, who may be prepared to accept a harsh employment contract today if they expect that they can impose a similar contract on the next generation tomorrow. This recursion determines the contract dynamics. As is usual with dynamic programming problems, there are two environments where the recursion can be solved easily. The first is the finite horizon case. If the firm will cease to exist, if it has not already done so, at some definite date that is independant of and unrelated to the fixed hazard rate α, then we can solve for the optimal contract by backward induction. The other is a stationary environment, where there is a unique stationary contract described explicitly, if $\alpha > 0$, by

$$W = \max_{b \geq 0} \min \left[b, \frac{\phi(b)}{\alpha} \right]. \tag{2}$$

This is the environment that we will study. For a proof that Equation 1 reduces to Equation 2 in a stationary environment see the Appendix.

Before discussing the nature of the optimal contract, it may be worth discussing the extent to which our assumptions may be relaxed. If we maintain the hypothesis of risk neutrality, then the size of the firm is not restricted to one worker. The principal may employ m workers, promoting one of them at random[2]. If there are m workers then the return to the principal is scaled up by m (we assume constant returns to scale). Since all workers are identical[3], they each face the same promotion probability. Under risk neutrality the expected reward from promotion is scaled by the promotion probability $\frac{1}{m}$. These two effects cancel out, leaving the effect on the agents' incentives unchanged. We can also accommodate multi-principal firms, such as professional partnerships. If there are k partners then the incentives are the same as if there were k single principal firms. Since the principals exert no effort, but simply collect rents, they have no difficulty in agreeing on the objectives of the firm. We remark that the assumption that the firm faces a fixed hazard rate and may go out of business has an alternative interpretation. The model would look the same if the firm lives forever, but there is a probability α that the manager is appointed from outside the firm rather than being appointed internally. The assumption of no discounting is inessential. Agents could be allowed to discount the future; the effect is equivalent to increasing the hazard rate.

[1] Without such a constraint an equilibrium does not exist.

[2] Since agents are homogeneous and indistinguishable, any promotion rule must be random.

[3] In a related model Bardsley and Sherstyuk have studied an OLG model with heterogeneous agents in which the principal can use promotion probability, as well as the wage and the effort level, as a contract variable.

If agents are risk averse then none of these equivalences will hold. In particular, a hazard rate is not necessarily equivalent to discounting. In this case the most straight forward interpretation of our model is one where agents discount the future, but the firm does not face a failure risk. In that case there is no uncertainty, and the model holds precisely. Bardsley WP?? has shown that the structure of the equilibrium and the conclusions of the model are robust to the introduction of some risk aversion. Risk aversion introduces an additional participation cost, which must be accommodated in the participation constraint. Otherwise the effects are of second order.

2.1 The equilibrium contract

The equilibrium contract is shown in Figure **??**. On the horizontal axis is the agent's output b. On the vertical axis is the rent W paid to the principal. A contract is a pair (b, W), which implicitly determines the effort $e(b)$ and the wage $w = e(b) - W$ paid to the agent. The agent's cost of effort curve $e(b)$ is shown as a dashed line, and the revenue b is shown as a diagonal line. The current period surplus generated by the firm, given output level b, is $\phi(b) = b - e(b)$. The present value of the firm, the sum of future surpluses discounted at the hazard rate α, is $\frac{\phi(b)}{\alpha}$. This is shown on the graph for a range of hazard rates from $\alpha = 1$ (the one period firm, which we will loosely refer to as the myopic firm), to $\alpha = 0$ (the infinitely long lived firm). As the firm becomes more long lived, the present value increases. This can be seen on the graph as the value curve bows upwards.

The principal solves the optimisation described by Equation 2, maximising her rent W subject to two constraints. The individual rationality constraint $W \leq \frac{\phi(b)}{\alpha}$ can be interpreted to mean that the principal can extract no more than the present value of the future surpluses generated by the firm. This constraint requires that the contract point must be on or below the value curve $\frac{\phi(b)}{\alpha}$. The wage constraint requires that the contract point must be on or below the diagonal line.

We begin by considering the myopic or one period firm, with hazard rate $\alpha = 1$. In this case the wage constraint does not bind (this can be checked by noting that $e(0) = 0$, $e'(0) < 1$, and $e(b)$ is convex) and the principal chooses the efficient output level $\tilde{b}$ that maximises the value of the firm. The wage is set equal to the effort cost, and the principal extracts all the rents. We notice that the wage $\tilde{w} = e\left(\tilde{b}\right)$ is strictly positive, and that the outcome is efficient.

We now consider what happens if α falls, and the firm becomes more long-lived. We let α decrease a little, but not so much that the wage constraint binds. Once again, the principal chooses the efficient output level that maximises the value of the firm but there is now scope to reduce the wage. Since the firm will continue into the future with positive probability, the agent expects to earn some rent as a future principal. The output level b has not changed, so the agent's effort cost has not changed. Thus the wage can be reduced without violating the participation constraint. As α falls further, the value of being a

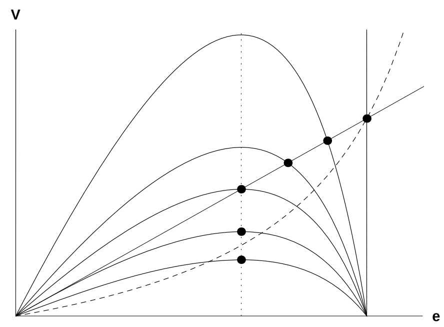

Figure 1:

principal in the firm increases, and the principal continues to reduce the wage to extract the surplus. This continues until the hazard rate reaches a critical value $\tilde{\alpha} = 1 - \frac{e(\tilde{b})}{\tilde{b}}$ where the wage constraint just binds. The principal offers, and the agent accepts, a wage of zero.

If the hazard rate falls below this critical value, then cutting the wage is no longer an option. The principal is now willing to increase effort beyond the efficient level even though this reduces the value of the firm, as no other instrument is available with which to extract rents. In the limit, as the firm becomes very long lived, effort rises to the "rat race" point where all rents are dissipated.

If we imagine the firm being founded at some point in time and continuing until it fails, then all principals, including the first, earn $W = \frac{\phi(b)}{b}$ when they act as principal. However all except the first have this surplus extracted in the preceding period, and ex ante all individuals except for the entrepreneurial founder earn zero. If equilibrium effort is inefficiently high then a policy that reduces effort, for example a binding output ceiling, can increase the total surplus and improve the welfare of all individuals except the firm founder. Such a ceiling reduces the rent paid to principals, but increases the total surplus available to share between agents and principals, so all agents are made strictly better off. A wage floor shifts down the wage constraint so that it binds more tightly. In equilibrium it increases effort and reduces surplus. All rent is still extracted from the agents. Figures **??** to **??** show the predicted wage, effort and value of the firm as a function of the hazard rate.

3 Experimental design

- Overlapping generations labor market with 21 participants, split into three roles.
- 3 Employers hire workers each period through a labor market.

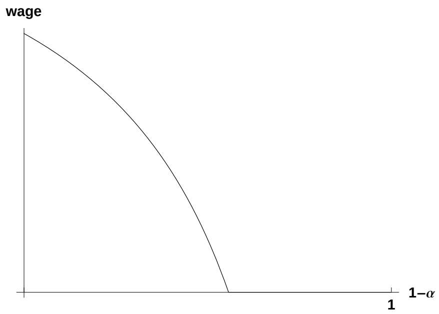

Figure 2:

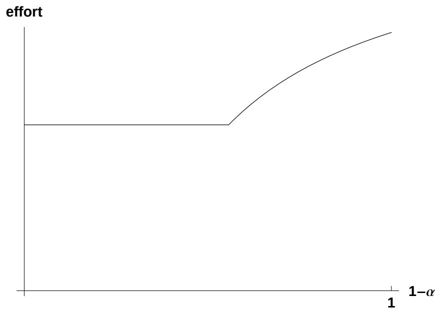

Figure 3:

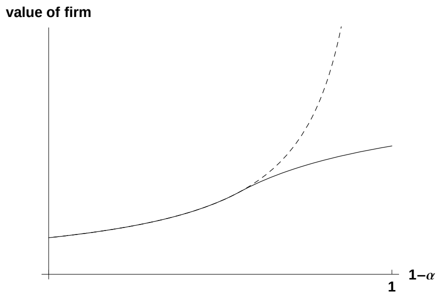

Figure 4:

- They offer contracts to workers in a continuous-time posted-offer market.
  - Contracts specify a wage and an effort amount.
- 6 Workers decide whether to accept any one of the contracts offered by the employers.
  - Workers observe the contracts and can accept them at any point in time.
  - Each employer needs only one worker. (3 out of 6 workers are employed.)
- 12 Observers act as the next generation.
- Each period ends when all firms have hired a worker (or after 3 minutes).
- Continuation in each period is probabilistic:
  - At the end of each period, a (virtual) dice is rolled. If the outcome is below a threshold, the firm continues.
  - All hired workers become employers in the following period.
  - Six observers are randomly selected to be workers.
  - Employers and unemployed workers become observers.
- If the die is at or above the threshold, the existing firms stop operating and new firms come into existence.
  - 21 participants are randomly given new roles (employer, worker, observer).
- Each session went for at least 75 minutes. After that, the experiment ended when the existing set of firms ended.
- Wages could be any integer amount from 0 to 150.
- Effort was in increments of 20.
- Principal's payoff: $1.5e - w$
- Agent's payoff: $w - c(e)$

3.1 Treatments and Predictions

There are two treatments. In the first, it is predicted that wages re driven to zero, but that effort is efficient. In the second, wages are driven to zero and effort is predicted to be double the efficient level, leading to severe rent dissipation.

- Treatment 1: $1 - \alpha = 5/6$ (6 sessions)
  - Wage = 0
  - Effort = 80
  - Principal's payoff = 120
  - Agent's payoff = -100
- Treatment 2: $1 - \alpha = 2/6$ (6 sessions)
  - Wage = 0
  - Effort = 40
  - Principal's payoff = 60

4 Summary of experimental results

Experiments have been concluded, but analysis is not yet completed. We report here a summary of the results.

- With a continuation probability of 5/6:
  - The rat-race equilibrium is observed in all sessions. Effort converges to 80 and wages fall to between 0 and 15. This would be consistent with risk or loss aversion.
  - Profits of the principals and the agents diverge.
  - The value of the firm stabilizes at 20. This is exactly the gain needed for an agent to accept the risk from the firm ending.
- With a continuation probability of 2/6:
  - Effort is efficient as predicted. Wages do not fall significantly below the zero profit level.
  - The value of each firm stabilizes at 40.
  - The market starts close to the equilibrium, so the convergence rates are much faster.
- Efficiency in both treatments is as predicted. As the probability of continuation increases, rents are dissipated.
- The rat-race equilibrium is extremely robust. It has occurred in almost all of the sessions.

5 Conclusion

- The experimental results are in line with the theoretical predictions.
- More so in the case of 5/6. The rat-race equilibrium is very robust.
  - May be because subjects cannot distinguish between 0 and 2/6.

6 Appendix

We must show that the equations

$$W = \max_{b\geq 0} \min \left[b, (1-\alpha) W + \phi(b)\right] \quad (3a)$$

$$W = \max_{b\geq 0} \min \left[b, \frac{\phi(b)}{\alpha}\right] \quad (3b)$$

are equivalent if $\alpha > 0$. Let $b_0 = \text{argmax}_{b\geq 0}\, \phi(b)$ and let b_1 solve $b = \frac{\phi(b)}{\alpha}$. Let $\left(\bar{b}, \bar{W}\right)$ solve problem 3b. We must show that $\left(\bar{b}, \bar{W}\right)$ also solves problem 3b.

We consider first the case $b_0 \leq b_1$. By convexity, only the constraint $W \leq b$ in 3b binds on $(0, b_0)$. Since this constraint is increasing, the optimal value $\bar{b}$ cannot lie in this interval. To the right of b_0 one constraint is increasing and one is decreasing, so both must bind. Thus $\bar{W} = \bar{b} = \frac{\phi(\bar{b})}{\alpha}$, and $\left(\bar{b}, \bar{W}\right) = \left(b_1, \frac{\phi(b_1)}{\alpha}\right)$. We note that $\left(\bar{b}, \bar{W}\right)$ also satisfies the constraints of optimisation 3a. We now consider a potential solution (b, W) to 3a. If $b < b_1$ then $W \leq b < b_1 = \bar{W}$, so (b, W) cannot be optimal. If $b \geq b_1$ then, for any W, the constraint b is increasing and the constraint $(1-\alpha) W + \phi(b)$ is decreasing in b so both constraints must bind. Thus $W = b = (1-\alpha) W + \phi(b)$ which implies that $b = \frac{\phi(b)}{\alpha}$. Thus $b = b_1$ and $(b, W) = \left(\bar{b}, \bar{W}\right)$.

We consider the case $b_1 < b_0$. Once again, we look first at 3b. To the left of b_0 both the constraints b and $\frac{\phi(b)}{\alpha}$ are increasing, so the optimal b cannot lie in $[0, b_1)$. But the constraint b crosses the constraint $\frac{\phi(b)}{\alpha}$ from below somewhere in this interval, so by convexity the constraint $W \leq b$ cannot bind to the right of b_1. Since only one constraint is binding, $\bar{b} = b_0 = \text{argmax}_{b\geq 0} \frac{\phi(b)}{\alpha}$ and $\bar{W} = \frac{\phi(b_0)}{\alpha}$. We now consider 3a. By a similar argument, the optimal b cannot be to the left of b_0, and if both constraints bind then $W = b = (1-\alpha) b + \phi(b)$ so $b = b_1$, which is impossible. Thus only one constraint binds and $b = b_0$, $W = \frac{\phi(b_0)}{b_0}$. Thus $(b, W) = \left(\bar{b}, \bar{W}\right)$.

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