Transition accounting for India in a multi-sector dynamic general equilibrium model

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Abstract

Using a quantitative methodology designed specifically for emerging economies, we measure the components of India's economic growth over the period 1960-2005. Our approach accounts for time-varying parameters, transitional dynamics and non-linear trends. We find that increased productivity in the service sector, facilitated by a structural shift toward services, is the principal driver of India's economic growth. Our measures also suggest that the allocation of inputs across sectors has not improved over this period, and in the case of labor appears to have significantly worsened. We further find that fluctuations in output around its trend are due primarily to fluctuations in sector-specific total factor productivity, with fluctuations in labor market distortions and labor taxes also playing important roles. In the period 1960-1980, productivity fluctuations in the agricultural sector are the dominant source of cycles. Since then, productivity fluctuations in the manufacturing and service sectors have been more important.

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1 Introduction

The post-colonial history of the Indian economy is a study in contrasts. In the first three decades following its independence in 1947, India's real per capita output grew at the anemic rate of 1.6 percent per year. This dormant period was followed by a series of reforms, beginning in the early 1980s. Since then, the Indian economy has grown rapidly, with per capita output increasing 6 to 7 percent per year, and services replacing agriculture as the dominant sector.

In this paper we construct and implement a quantitative accounting procedure to measure the components of India's economic growth. Following the business cycle accounting (BCA) procedure developed by Chari, Kehoe and McGrattan (2007; also see Mulligan, 2005), we interpret the data through the lens of a dynamic general equilibrium model. This allows us to calculate a series of "wedges", which capture frictions or policies that alter the economy's equilibrium dynamics. Chari et al.'s (2007) equivalence results show that many types of frictions can be expressed in terms of these wedges. Consistent with Lahiri and Yi (2006, 2008), we use a multi-sector model to calculate three types of wedges: sector-specific efficiency wedges, which are modelled as productivity shifters; allocation wedges, which are modelled as sector-specific taxes on capital or labor; and fiscal policy wedges, which are modelled as aggregate labor income taxes, aggregate capital income taxes, or government spending shocks.

We depart from the existing literature, however, in that our accounting methodology is tailored for emerging economies. Our approach allows for non-linear trends; it recognizes that India's economy is transitioning to, rather than residing on, a balanced growth path; and it allows for time-varying parameters that capture sectoral shifts.[1] These features lead us to label our approach "transition accounting".[2]

The Indian economy provides a compelling setting to apply our methodology. Prior to the 1980s, the Indian economy was characterized by the notorious "Hindu rate of growth". Import-substituting industrial policies sustained state monopolies in "core sectors". To prevent the concentration of wealth, small-scale industries were protected by a combination of quota, licensing and permits. To encourage self-reliance, foreign capital and foreign technology were shunned. The reallocation of labor across sectors was restricted.

[1] Although sectoral shifts are a feature of developed economies as well, they are considered central to the process of development.

[2] In the approach most similar to ours, Lahiri and Yi (2006) find non-linear transition paths for a deterministic, constant-parameter economy. Verma (2008) solves a constant-parameter model that generates sectoral shifts through unbalanced growth; her analysis, however, considers only the effects of productivity growth.

All these policies led to low productivity, lost competitiveness and inefficient resource use. (Williamson and Zagha, 2002.)

In late 1970s, the second oil shock caused India's terms of trade to worsen. The conditions attached to the resulting IMF loans led to the "pro-business"reforms that began the second phase of economic growth in the 1980s. Controls over capacity utilization and capital imports were relaxed, price controls in key industrial products like cement and aluminium were dismantled, and investment in infrastructure doubled the growth rate of the public sector. The central government's fiscal deficit rose, however, reaching 8.5 percent of GDP in 1986-87 and depleting foreign reserves. These macroeconomic imbalances made the economy vulnerable to shocks, and finally the high oil prices caused by the first Gulf War in 1991, along with domestic political instabilities, pushed the country to the verge of defaulting on its external loans. In lieu of help from IMF, in 1991 India started "pro-market" reforms, consisting of currency devaluation, fiscal contraction and public sector divestment, financial sector and tax system reform, and the liberalization of domestic investment and foreign capital inflows. Ever since, per capita real GDP has grown 6 to 7 percent per year. The resulting sectoral changes have been just as compelling. Beginning in the 1980s, India transformed itself from a predominantly agrarian economy to one that was service-sector-based. (Williamson and Zagha, 2002; Rodrik and Subramanian, 2004.)

While it is unanimous that the right kind of policies accelerated growth in the Indian economy, the transmission channels through which these policies affected the various sectors have yet to be captured in a unified framework. Rodrik and Subramanian (2004) and Virmani (2004) corroborate the findings of Bai and Perron (1998, 2003) that a structural break occurred in the economy around 1980 from a variety of sources. They, along with Panagariya (2006), conclude that the policy shifts in the early 1980s induced large productivity responses because India was far from its production frontier.

A number of studies have argued that the manufacturing sector experienced a surge in productivity, which in turn led to a rise in aggregate productivity (Ahluwalia, 1995; and Unel, 2003). On the other hand, the IT sector is considered to be the most prominent channel of India's growth since the 1990s (Singh, 2004). Conducting traditional growth accounting exercises, Bosworth, Collins and Virmani (2007) and Bosworth and Collins (2008) also conclude that post-1980 growth is largely due to an increase in service sector productivity. Chakraborty (2006) uses BCA to measure the productivity, investment, and labor supply shocks behind India's growth since 1982. She finds that TFP shocks were the "primary conduit" through which India's policy reforms stimulated its economy.

Using a multi-sector accounting framework, Lahiri and Yi (2006, 2008) identify the wedges that explain why the state of Maharashtra grew so much more quickly than West Bengal. Verma (2008) uses a multi-sector model to study the rapid growth of India's service sector.

To extend these results, we use our approach to estimate the Indian economy's long-term, non-linear trend, and its fluctuations around this trend. We find that over the period 1960-2005, India's trend growth is due largely to higher service sector productivity; if the service sector had not become more productive, output in 2005 would be about half its actual level. This increase in service productivity, however, was facilitated by a structural shift that increased the importance of services to aggregate output. Without this shift, output in 2005 would have been 23 percent lower. Our findings are consistent with the argument that India's policy reforms benefited the economy primarily by allowing it to operate more efficiently. Nonetheless "direct" changes in fiscal policy also contributed; if tax rates and government spending remained at their 1960 levels, output would have been 26 percent lower. In contrast, the improved intersectoral allocation of capital had very little effect, and the intersectoral allocation of labor actually worsened over the sample period—India's service sector appears to be facing a shortage of labor. Although this apparent shortage may reflect conceptual measurement problems, if real it will pose a barrier to India's continued economic development.

Turning to business cycles, we find that fluctuations in output around its trend are due primarily to fluctuations in sector-specific total factor productivity, with fluctuations in labor market distortions and labor taxes also playing important roles. In the period 1960-1980, prior to the reforms, productivity fluctuations in the agriculture sector were the dominant source of cycles. Since then, productivity fluctuations in the manufacturing and service sectors have been more important, with fluctuations in service-sector productivity being the largest single source of volatility.

The rest of the paper proceeds as follows. Section 2 develops our multi-sector model. Section 3 describes how we measure and parameterize the model's wedges and time-varying parameters. Section 4 discusses the solutions to the model's trend and business cycle components. Section 5 discusses the data we use for our analyses. Sections 6 and 7 present the analyses of the India's trend growth and business cycle fluctuations, respectively. Section 8 concludes.

2 The Model

2.1 Firms

Consider a closed economy with four sectors of production—a final goods sector and three intermediate goods sectors: agriculture (a), manufacturing (m), and services (s). The production technologies for intermediate goods are given by the following Cobb-Douglas production functions:

$$
\begin{aligned}
Y_a &= A_a K_a^{\alpha} L_a^{1-\alpha}, && \text{(1a)}\\
Y_m &= A_m K_m^{\mu} L_m^{1-\mu}, && \text{(1b)}\\
Y_s &= A_s K_s^{\sigma} L_s^{1-\sigma}, && \text{(1c)}
\end{aligned}
$$

where: Y_j denotes total output of good $j = a, m, s$; K_j ($j = a, m, s$) denotes capital inputs in sector j; L_j denotes labor inputs; and A_j are exogenous and stochastic productivity shifters. All of these variables can vary over time; in the interest of notational simplicity, we will suppress time subscripts whenever possible.

The three intermediate goods are combined in a Cobb-Douglas aggregator to produce a single non-traded final good, Y:

$$
\begin{aligned}
Y &= A_f Y_a^{\psi} Y_m^{\eta} Y_s^{\theta}, && \text{(2)}\\
\theta &= 1 - \psi - \eta.
\end{aligned}
$$

To account for structural change, the share parameters ψ, η and θ can vary over time.

Each sector is populated by perfectly competitive firms, which maximize profits:

$$
\begin{aligned}
\Pi_a &= p_a Y_a - w_a L_a - r K_a,\\
\Pi_m &= p_m Y_m - w_m L_m - r K_m + r\frac{\kappa_m}{1+\kappa_m} K_m,\\
\Pi_s &= p_s Y_s - w_s L_s - r K_s + r\frac{\kappa_s}{1+\kappa_s} K_s,\\
\Pi &= pY - p_a Y_a - p_m Y_m - p_s Y_s,
\end{aligned}
$$

where w_j, r and p_j are real wages, rental rates and prices, respectively, in the j^{th} sector; final goods are the numeraire, so that $p = 1$. Following Lahiri and Yi (2008), we introduce the parameters κ_m and κ_s to capture market frictions, adjustment costs, and other factors that might alter the allocation of capital: although we formally model κ_m and κ_s as subsidies/taxes, we follow Chari et al. (2007) and Mulligan (2005) and interpret them as "wedges" that can embody a wide range of frictions. We normalize κ_a to zero, so that we are looking only at sectoral misallocations. In this light, positive values of κ_m and κ_s imply that the frictions divert capital *to* the manufacturing and service sectors. To capture sector-specific frictions in labor markets, we allow wages to vary across sectors, and introduce sector-specific taxes in the consumer's budget constraint.[3]

The first order conditions for the final goods sector are

$$p\psi Y = p_a Y_a, \tag{3a}$$
$$p\eta Y = p_m Y_m, \tag{3b}$$
$$p\theta Y = p_s Y_s. \tag{3c}$$

Combining these conditions with the first order conditions for intermediate goods producers (see Appendix 9.1) yields:

$$\psi(1-\alpha)\frac{Y}{L_a} = w_a, \tag{4a}$$
$$\eta(1-\mu)\frac{Y}{L_m} = w_m, \tag{4b}$$
$$\theta(1-\sigma)\frac{Y}{L_s} = w_s, \tag{4c}$$

for labor inputs, and

$$\alpha\psi\frac{Y}{K_a} = r, \tag{5a}$$
$$\eta\mu\frac{Y}{K_m} = r\frac{1}{1+\kappa_m}, \tag{5b}$$
$$\theta\sigma\frac{Y}{K_s} = r\frac{1}{1+\kappa_s}, \tag{5c}$$

for capital.

2.2 Households

The representative family receives utility from consumption and leisure. The flow

[3] We model capital and labor frictions in different ways only to facilitate our derivations; in the end the frictions' algebraic (as well as economic) effects are completely symmetric.

utility function for a family of size N is:

$$u(C_t, L_t, N_t) = N_t \left[\ln(C_t/N_t) - \chi \frac{(L_t/N_t)^{1+\gamma}}{1+\gamma} \right],$$

where C is consumption, L is the labor supply, χ is the weight on leisure in the utility function and γ is the inverse of the intertemporal elasticity of substitution for labor. In contrast to the production technologies, we assume that preferences are constant over time.

The family faces the following budget constraint

$$\begin{aligned} C_t + (1+\tau_{kt})K_{t+1} &= (1-\tau_{at})w_{at}L_{at} + (1-\tau_{mt})w_{mt}L_{mt} + (1-\tau_{st})w_{st}L_{st} \\ &\quad + [1 + r_t - \delta_t] K_t + \Pi_t + \Pi_{at} + \Pi_{mt} + \Pi_{st} + Tr_t, \end{aligned}$$

where K is the total capital stock and δ is the depreciation rate. As above, w_i is the wage in sector i, r is the interest rate and Π_t, Π_{at}, Π_{mt} and Π_{st} are dividends from firms. As with the capital frictions, the tax rates τ_{kt}, τ_{at}, τ_{mt}, τ_{st} can be interpreted literally, or as wedges that embody all market frictions.[4] Tr_t denotes government transfers. As with capital, it is useful to express the sector-specific labor tax rates as the product of an aggregate rate and two sector-specific effects. This allows us to rewrite the budget constraint as

$$\begin{aligned} C_t + (1+\tau_{kt})K_{t+1} &= (1-\tau_{lt}) \left[w_{at}L_{at} + (1+\ell_{mt})w_{mt}L_{mt} + (1+\ell_{st})w_{st}L_{st} \right] \\ &\quad + [1 + r_t - \delta_t] K_t + \Pi_t + \Pi_{at} + \Pi_{mt} + \Pi_{st} + Tr_t, \\ 1 + \ell_{mt} &\equiv \frac{1-\tau_{mt}}{1-\tau_{lt}}; \quad 1 + \ell_{st} \equiv \frac{1-\tau_{st}}{1-\tau_{lt}}; \quad \ell_{at} \equiv 0 \Leftrightarrow \tau_{lt} = \tau_{at}. \end{aligned}$$

Positive values of ℓ_m and ℓ_s imply that labor in these sectors is "taxed" less heavily, so that frictions promote the reallocation of labor *to* the manufacturing and service sectors. It also bears noting, however, that if labor is measured as raw employment, ℓ_m and ℓ_s will reflect differences in skill and utilization across sectors.

In addition to the budget constraint, the family faces the time endowment constraint

$$L_t = L_{at} + L_{mt} + L_{st},$$

[4] To facilitate our analysis of the model's trends, our treatment of capital taxes—taxes are levied on capital itself—differs from the canonical BCA approach (Chari et al., 2007), where taxes are levied on investment.

and the standard boundary conditions.

The first order conditions for the family's problem are

$$\frac{N_t}{C_t} = \beta E_t \left(\frac{N_{t+1}}{C_{t+1}} [1 + r_{t+1} - \delta_{t+1}] \right) \frac{1}{1 + \tau_{kt}}, \tag{6}$$

$$w_{at} = w_{mt}(1 + \ell_{mt}) = w_{st}(1 + \ell_{st}) = \frac{\chi}{1 - \tau_{lt}} \left(\frac{C_t}{N_t} \right) \left(\frac{L_t}{N_t} \right)^{\gamma}. \tag{7}$$

Equation (6) is the inter-temporal Euler equation determining savings, with $E_t(\cdot)$ denoting expectations based on time-t information. Equation (7) describes the optimal labor-leisure allocation.

2.3 The Government

Finally, there is a government, collecting taxes and purchasing goods and services. The government also makes lump-sum transfers, which are set to balance its budget:

$$\tau_{at} w_{at} L_{at} + \tau_{mt} w_{mt} L_{mt} + \tau_{st} w_{st} L_{st} + \tau_{kt} K_{t+1} = G_t + Tr_t + r_t \frac{\kappa_m}{1 + \kappa_m} K_m + r_t \frac{\kappa_s}{1 + \kappa_s} K_s,$$

where G_t denotes government purchases. Government purchases have no effect on production or on household utility.[5]

2.4 Aggregation

Let κ denote the average capital market distortion:

$$\begin{aligned} \kappa &= \frac{1}{\zeta} \left[\eta \mu \kappa_m + \theta \sigma \kappa_s \right], \\ \zeta &\equiv \alpha \psi + \mu \eta + \sigma \theta. \end{aligned}$$

(Recall that we normalize κ_a to zero.) Similarly, let ℓ denote the average labor market distortion:

$$\ell = \frac{1}{1 - \zeta} \left[\eta (1 - \mu) \ell_m + \theta (1 - \sigma) \ell_s \right].$$

Using these definitions, we show in Appendix 9.2 that in equilibrium equations (2)

[5] Any effects that government spending might have on utility or production will be captured by correlations between government spending and the other wedges. Chari et al. (2007) provide several useful examples.

and (7) can be written as

$$
\begin{aligned}
Y &= A^{1-\zeta} K^{\zeta} L^{1-\zeta}, && (8)\\
(1-\zeta)(1+\ell)(1-\tau_l)\frac{Y}{C} &= \chi\left(\frac{L}{N}\right)^{1+\gamma}. && (9)
\end{aligned}
$$

where

$$
\begin{aligned}
A &\equiv \left[A_f A_a^{\psi} A_m^{\eta} A_s^{\theta} \Omega \Upsilon \Delta\right]^{1/(1-\zeta)}, && (10)\\
\Omega &\equiv \frac{(1+\kappa_m)^{\eta\mu}(1+\kappa_s)^{\theta\sigma}}{(1+\kappa)^{\zeta}},\\
\Upsilon &\equiv \frac{(1+\ell_m)^{\eta(1-\mu)}(1+\ell_s)^{\theta(1-\sigma)}}{(1+\ell)^{1-\zeta}},\\
\Delta &\equiv \left(\psi\left(\frac{\alpha}{\zeta}\right)^{\alpha}\left(\frac{1-\alpha}{1-\zeta}\right)^{1-\alpha}\right)^{\psi}\left(\eta\left(\frac{\mu}{\zeta}\right)^{\mu}\left(\frac{1-\mu}{1-\zeta}\right)^{1-\mu}\right)^{\eta}\left(\theta\left(\frac{\sigma}{\zeta}\right)^{\sigma}\left(\frac{1-\sigma}{1-\zeta}\right)^{1-\sigma}\right)^{\theta}.
\end{aligned}
$$

As noted above, $K = K_a + K_m + K_s$ and $L = L_a + L_m + L_s$ denote aggregate capital and labor, respectively. A denotes aggregate productivity, expressed here in labor-enhancing form. Ω measures the efficiency lost due to *sectoral* misallocations of capital, while Υ measures the efficiency lost due to sectoral misallocations of labor. Note that when there are no sectoral misallocations, $\kappa = \kappa_m = \kappa_s = 0$, $\ell = \ell_m = \ell_s = 0$, and $\Omega = \Upsilon = 1$.

Similarly, the equilibrium Euler equation becomes:

$$
\frac{N_t}{C_t} = \beta E_t\left(\frac{N_{t+1}}{C_{t+1}}\left[1+\zeta_{t+1}(1+\kappa_{t+1})\frac{Y_{t+1}}{K_{t+1}} - \delta_{t+1}\right]\right)\frac{1}{1+\tau_{kt}}. \tag{11}
$$

Finally, we have the capital accumulation equation:

$$
K_{t+1} = (1-\delta_t)K_t + Y_t - C_t - G_t. \tag{12}
$$

2.5 A Stationary Transformation

The next step is to express the model in intensive quantities suitable for numerical analysis. We assume, consistent with the data, that productivity follows a stationary process around the trend A^*. Let lower case variables denote upper case variables divided by population and this productivity trend, with $c_t \equiv C_t/(A_t^* N_t)$, and so on. Labor hours are normalized by population, so that $l_t = L_t/N_t$. With these definitions, we can rewrite the Euler and capital accumulation equations as

$$\frac{1}{c_t} = \beta \frac{N_{t+1}}{N_t} E_t \left(\frac{1}{c_{t+1}} (G^*_{t+1})^{-1} \left[1 + \zeta_{t+1}(1+\kappa_{t+1}) \frac{y_{t+1}}{k_{t+1}} - \delta_{t+1} \right] \right) \frac{1}{1+\tau_{kt}}, \tag{13}$$

$$G^*_{t+1} k_{t+1} = (1-\delta_t)k_t + y_t - k_t - g_t, \tag{14}$$

$$y_t = \left(k_t^{\zeta_t} a_t^{1-\zeta_t} \right)^{\phi_t} c_t^{1-\phi_t} \Gamma_t^{\phi_t - 1}. \tag{15}$$

where

$$G^*_{t+1} \equiv \frac{N_{t+1} A^*_{t+1}}{N_t A^*_t},$$

$$\Gamma_t \equiv \frac{1}{\chi}(1-\zeta_t)(1+\ell_t)(1-\tau_{lt}),$$

$$a_t \equiv \frac{A_t}{A^*_t}, \quad \phi_t \equiv \frac{1+\gamma}{\gamma+\zeta_t} = 1 + \frac{1-\zeta_t}{\gamma+\zeta_t}.$$

(See Appendix 9.3.) It bears emphasizing that the trend term G^*, although deterministic, can vary over time.

3 Finding the Time-varying Parameters and the Wedges

The key to the accounting procedures developed by Chari et al. (2007), Mulligan (2005) and others is that the economy's distortions, or "wedges" can be calculated by rearranging the equations of the model and applying them to the data. In our variant of the accounting methodology, we proceed in two steps. First we find the time-varying parameters. Then we use the parameter values to calculate the wedges.

3.1 Time-Varying Parameters

To derive the sectoral shares, rewrite equations (3a)-(3c) as

$$\psi_t = \frac{p_{at} Y_{at}}{p_t Y_t}, \tag{16a}$$

$$\eta_t = \frac{p_{mt} Y_{mt}}{p_t Y_t}, \tag{16b}$$

$$\theta_t = 1 - \eta_t - \psi_t = \frac{p_{st} Y_{st}}{p_t Y_t},. \tag{16c}$$

In making this derivation, we are assuming that there are no distortions in the choice of intermediate goods, so that changes in sectoral shares are due only to changes in the aggregate production function. Lahiri and Yi (2006) take the opposite position, assuming that η, θ, and ψ are constant, and that any variation in sectoral shares reflects distortions faced by final goods producers. Although the two approaches are observationally equivalent in terms of equations (16a) - (16c), our approach is more consistent with the prevailing view that India has experienced major sectoral shifts.

Similarly, we can estimate a series of depreciation rates from equation (12):

$$\delta_t = \frac{1}{K_t}\left[I_t + K_t - K_{t+1}\right].$$

3.2 Fixed Parameters

Calculating the wedges also requires several other parameters, which we assume are fixed throughout our time period.

Parameter	**Description**	**Value**
α	agriculture capital share	0.375
μ	manufacturing capital share	0.4
σ	services capital share	0.4
β	time discount factor	0.96
γ	$1/IES_{\text{labor}}$	1

Table 1. Calibrated Parameters

The capital shares are taken from Bosworth et al. (2007).[6] The value for β, the rate of time preference, is standard. The value for γ implies an intertemporal supply elasticity for labor of 1. Although this value is lower than the value taken in many macro studies, it is consistent with more recent micro-level studies: Hall (2008) concludes that recent estimates imply an elasticity of 0.9.

[6] Bosworth et al.'s (2007) factor shares for agriculture include a component for land (0.25), which we divided evenly between capital and labor.

3.3 Wedges

The sectoral productivity levels can be computed from equations (1a)-(1b), as

$$\begin{aligned} A_{at} &= \frac{Y_{at}}{K_t^{\alpha} L_{at}^{1-\alpha}}, \\ A_{mt} &= \frac{Y_{mt}}{K_t^{\mu} L_{mt}^{1-\mu}}, \\ A_{st} &= \frac{Y_{st}}{K_t^{\sigma} L_{st}^{1-\sigma}}, \end{aligned}$$

while the aggregate productivity shifter A_f can be computed from equation (2), as

$$A_{ft} = \frac{Y_t}{Y_a^{\psi_t} Y_{mt}^{\eta_t} Y_{st}^{\theta_t}}.$$

In addition, when $p_{at} = p_{mt} = p_{st} = p_t$, it follows from equations (16a)-(16c) that

$$Y_t = A_{ft} \left(\psi_t Y_t\right)^{\psi_t} \left(\eta_t Y_t\right)^{\eta_t} \left(\theta_t Y_t\right)^{\theta_t} \Rightarrow A_{ft} = \left[\psi_t^{\psi_t} \eta_t^{\eta_t} \theta_t^{\theta_t}\right]^{-1}. \tag{18}$$

Because India's price data show no pronounced trends in relative prices, we assume that p_{mt}, p_{st} and p_t are constant, and employ this simplifying approximation, both here and in calculating the sectoral shares.

Equations (5a)-(5c) show that the capital distortions κ_m and κ_s solve:

$$\begin{aligned} 1 + \kappa_{mt} &= \frac{\psi_t \alpha}{\eta_t \mu} \frac{K_{mt}}{K_{at}} = \frac{MPK_{at}}{MPK_{mt}}, \\ 1 + \kappa_{st} &= \frac{\psi_t \alpha}{\theta_t \sigma} \frac{K_{st}}{K_{at}} = \frac{MPK_{at}}{MPK_{st}}. \end{aligned}$$

Finding the aggregate distortion κ_t is straightforward. The labor distortions are found in a similar manner:[7]

$$\begin{aligned} 1 + \ell_{mt} &= \frac{\psi_t (1-\alpha)}{\eta_t (1-\mu)} \frac{L_{mt}}{L_{at}} = \frac{MPL_{at}}{MPL_{mt}}, \\ 1 + \ell_{st} &= \frac{\psi_t (1-\alpha)}{\theta_t (1-\sigma)} \frac{L_{st}}{L_{at}} = \frac{MPL_{at}}{MPL_{st}}. \end{aligned}$$

It bears repeating that if labor is measured as raw employment, ℓ_{mt} and ℓ_{st} could reflect differences in skill—which affect the marginal product of raw employment—as well as

[7] Lahiri and Yi (2006, 2008) calculate the same labor distortions under the names $\omega^{l,am}$ and $\omega^{l,as}$. They also calculate capital distortions, under the names $\omega^{k,am}$ and $\omega^{k,as}$.

misallocation. Let q_{at}, q_{mt} and q_{st} denote skill measures. We can then decompose ℓ_{mt} and ℓ_{st} into

$$\begin{aligned}
1+\ell_{mt} &= (1+\ell_{1mt})(1+\ell_{2mt}), \\
1+\ell_{2mt} &= \frac{q_{at}}{q_{mt}}; \quad 1+\ell_{1mt} = \frac{1+\ell_{mt}}{1+\ell_{2mt}}, \\
1+\ell_{st} &= (1+\ell_{1st})(1+\ell_{2st}), \\
1+\ell_{2st} &= \frac{q_{at}}{q_{st}}; \quad 1+\ell_{1st} = \frac{1+\ell_{st}}{1+\ell_{2st}}.
\end{aligned}$$

If q_{at}, q_{mt} and q_{st} are accurate, ℓ_{1mt} and ℓ_{1st} provide "true" measures of misallocation.

The parameter χ and the series $\{\tau_{lt}\}$ solve the following version of equation (9):

$$\chi l_t^{1+\gamma} = (1-\zeta_t)(1+\ell_t)(1-\tau_{lt})\frac{Y_t}{C_t}.$$

Because this approach provides T equations to identify $T+1$ parameters, additional information must be imposed. We utilize Poirson (2006, Table 2), who calculates India's effective tax rate on labor income, τ_{lt}, to be about 16% over the period 1993-2000. Taking averages over the same period, we find that $\chi = 0.90825$, and with χ in hand, we can back out $\{\tau_{lt}\}$.

Government spending can be inferred as

$$G_t = Y_t + (1-\delta_t)\,K_t - C_t - K_{t+1}.$$

Because we model the Indian economy as closed, these "government spending" shocks will reflect any changes in net exports.

The capital tax τ_{kt} can be calculated by rearranging the Euler equation. Because of the expectation on the right-hand side of the Euler equation, finding τ_{kt} is tricky (Chakraborty, 2006; Chari et al., 2006, 2007; Bäurle and Burren, 2007.) We use the approximation adopted by Kobayashi and Inaba (2006), and replace expectations with realized values. We therefore estimate the approximate tax $\widetilde{\tau}_{k,t+1}$:

$$\begin{aligned}
\widetilde{\tau}_{k,t+1} &= \beta\frac{N_{t+1}}{N_t}\left(\frac{C_t}{C_{t+1}}\left[1+\zeta_{t+1}(1+\kappa_{t+1})\frac{Y_{t+1}}{K_{t+1}} - \delta_{t+1}\right]\right) - 1, \\
\tau_{kt} &= E_t\left(\widetilde{\tau}_{k,t+1}\right).
\end{aligned}$$

Because $\{\widetilde{\tau}_{k,t+1}\}$ differs from $\{\tau_{kt}\}$ only by a sequence of uncorrelated forecast errors, it should provide a reasonable basis for estimating the trend $\{\tau^*_{kt}\}$. The measurement of the deviation $\tau_{kt} - \tau_{kt}^*$ is more involved; we discuss this point in section 7.1 below.

4 Solving the Model

Once we have calculated the wedges, we can solve the model numerically to assess their importance. This is the "accounting" part of the BCA methodology.

The full model has 10 stochastic state variables, as well as capital, and four time-varying parameters. Moreover, the model cannot be expressed as the solution to a social planner's problem. We therefore adopt the common practice of separating the model into "trend" and "cycle" components, and solving the model in steps. The first step is to solve the perfect foresight "trend" model. In particular, we estimate the trends $\{G_t^*, \Gamma_t^*, \kappa_t^*, \tau_{kt}^*, g_t^*, \zeta_t^*, \delta_t^*\}$, set $a_t^* = 1$, and solve a deterministic version of the model using these series. The second step is to use the trend series $\{k_t^*, c_t^*\}$ generated in the first step, along with the exogenous trends, as the base points for a time-varying linearization.

4.1 Finding the Trend

We find the trend series by using equations (13) - (15) to produce the sequence $\{k_t^*, c_t^*\}_{t=1960}^{2094}$. Because $\{G_t^*, \Gamma_t^*, \kappa_t^*, \tau_{kt}^*, g_t^*, \zeta_t^*, \delta_t^*\}$ are treated as known, there are no expectations involved, and the recursion is simple.[8] This leaves the problem of finding the initial pair (k_{60}^*, c_{60}^*). To find c_{60}, we assume that for $t \geq 2035$, $\{G_t^*, \Gamma_t^*, \kappa_t^*, \tau_{kt}^*, g_t^*, \zeta_t^*, \delta_t^*\}$ are constant. (For the period 2006-2035, we extrapolate from the data trends shown below.) Because our model has the usual stability properties, it follows that by 2094 the deterministic economy will have converged to a steady state (in intensive quantities), and that for any initial capital stock k_{60} there is a unique initial consumption level c_{60} that takes the economy to this steady state. The stable value $c_{60}^* = c^*(k_{60})$ is straightforward to find. To find k_{60}^*, we find the value of k_{60} that minimizes the total squared log deviations between the trend series $\{k_t^*, c_t^*, l_t^*\}$ and their data counterparts.

4.2 Finding Deviations from Trend

We model the effects of "non-trend" movements in $\{G_t, \Gamma_t, \kappa_t, \tau_{kt}, g_t, \zeta_t, \delta_t\}$ on the model's endogenous variables by linearizing the model around the trend values described

[8] The only wrinkle is that with endogenous labor supply, the choice of c_{t+1} in equation (13) also affects y_{t+1}, so that even when c_t, k_t, and k_{t+1} are known, there is no closed form solution for c_{t+1}. However, it follows from equation (15) that y_{t+1} is decreasing in c_{t+1}, so that given k_{t+1}, the right-hand-side of equation (13) is monotonically decreasing in c_{t+1}, and a numerical search is straightforward.

immediately above. This is an extension of the approach used by King et al. (1988, 2002) and many others.[9] The only substantive difference is that the matrices that describe our solution vary over time; the standard approach, which linearizes around a steady state, yields time-invariant matrices. Given that the parameters of the Indian economy appear to have changed significantly over the past 45 years, we view this time variation as a valuable feature.[10] Appendix 9.4 provides the log-linearized equations, and Appendix 9.5 describes our solution method.

5 Data

The data for the Indian economy that we use are annual observations from 1960 through 2005 (or 2006). The data include: real output by expenditure; real output and capital by sector; employment and worker quality by sector; and population.

Almost all these data were compiled and constructed by Bosworth et al. (2007), who provide a detailed description. With the exception of employment, their data are largely standard. Bosworth et al. derive sectoral employment by combining population data from the census and total workforce data from quinquennial household surveys. The same surveys also allow Bosworth et al. to calculate the average years of schooling for workers in each sector. Assuming that each year of schooling increases earnings by 7 percent, Bosworth et al. convert the schooling data into indices of worker quality.

Two data series come from other sources. Our measure of consumption is the one in the national accounts, rescaled to be consistent with Bosworth et al.'s measures of total output and investment. We define population, N, to be the number of people between ages 15 and 70. Our historical population measures are found by combining data from the United Nations (Population Division, 2008) and national accounts data. To solve our model, we also need population projections, for which we use the United Nations' (Population Division, 2008) "medium variant" forecast.

[9] Our approach most closely follows that of Klein (2000) and Blanchard and Kahn (1980). We also incorporate elements of the approaches used by Broze, Gourieroux and Szafarz (1985, 1995), Farmer (1993), Farmer and Guo (1994), and Sims (2002).

[10] Allowing time variation introduces a small amount of imprecision into our solution; Appendix 9.5 provides a detailed discussion. Ignoring time variation, however, would arguably introduce more inaccuracy. For example, the parameter θ, the share of services, rises 27 percentage points over the sample period. In contrast, the magnitude of the solution errors appears to be less than 1 percent of the consumption deviations.

6 Trend Results

6.1 Trends in wedges

To find the trends for the time-varying parameters and wedges, we utilize a flexible curve-fitting approach. With relatively little theoretical guidance, the specifications were chosen to fit the data well and make reasonable projections.[11]

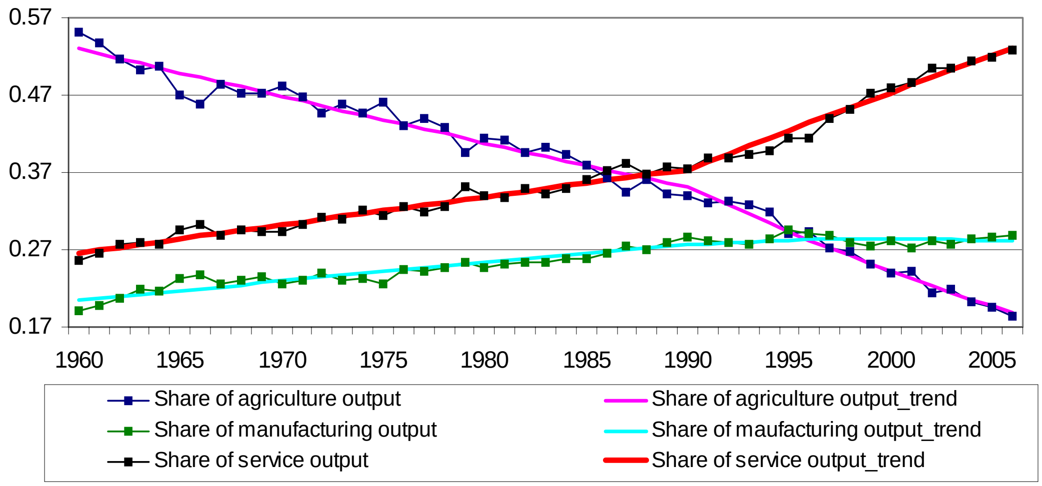

Figure 1. Sectoral Shares: Data and Trends

Figure 1 shows the estimated sectoral shares (ψ_t, η_t and θ_t) and their trends, which are estimated as logistic functions of time, with a trend break in 1990. The share of agricultural output declines through the whole sample period, falling from 55 to 18 percent. Offsetting this is a rapid increase in the service sector share, from 26 to 53 percent. Manufacturing, the smallest of the three sectors at the beginning of 1960, also surpassed agriculture, in the mid-nineties. The structural shifts appear to have accelerated since 1990, suggesting that the "pro-market" reforms beginning at that time have facilitated the economy's transformation.

[11] In general, we assume that the trends follow our estimated trend equations until 2035, and then stay stable. The two exceptions are fiscal policies and depreciation, for which we did not feel comfortable making extended projections. We simply assume these variables stay at their 2005 (or 2006) trend values for the foreseeable future. Appendix 9.6 shows the projected trends. Our results do not appear sensitive to these assumptions.

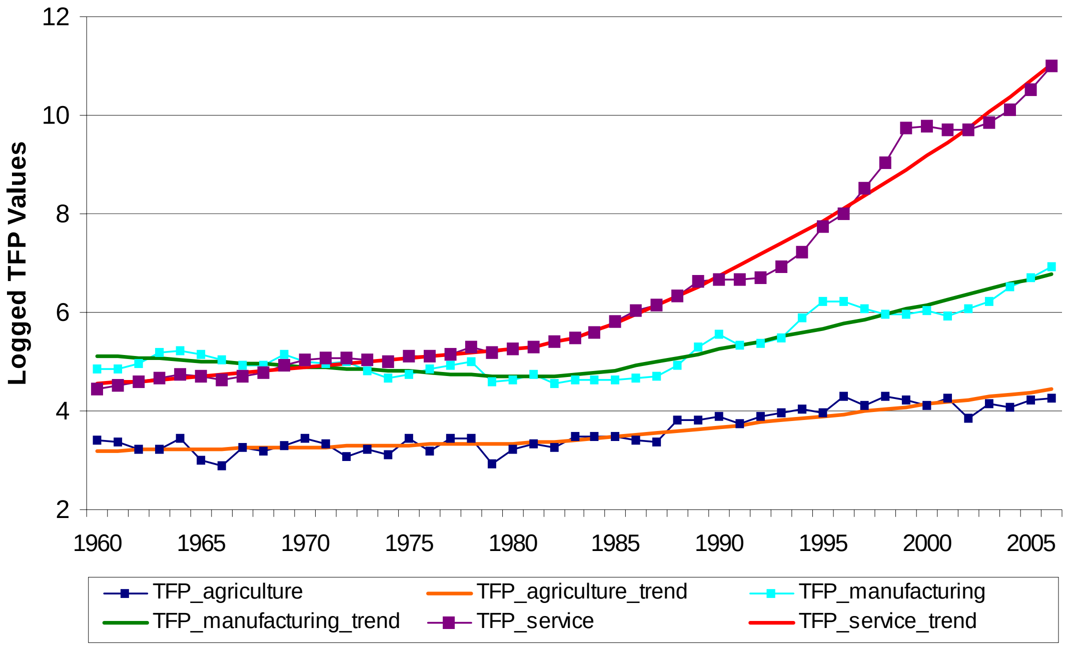

Figure 2. Sector-Specific Total Factor Productivity: Data and Trends

The three sectoral productivity wedges (A_{at},A_{mt} and A_{st}) are estimated as logarithmic functions of a time trend, with a trend break around 1982;[12] most observers conclude that the Indian economy experienced a structural shift around that time (Bai and Perron,1998, 2003; Williamson and Zagha, 2002; Virmani, 2004; Rodrik and Subramanian, 2004). Figure 2 shows the productivity wedges and their trends. The trends all slope more steeply after 1980, with the most pronounced increase in the service sector.[13] This change in trend is consistent with the argument that reforms begun in the early 1980s were at least in part responsible for the higher growth rates of the past 25 years.

Figure 3 shows the capital market distortions, (κ_{mt} and κ_{st}), the labor market distortions (ℓ_{mt} and ℓ_{st}), and their respective trends. Both capital distortions are positive, implying that India's capital-related policies favor the service and especially the manufacturing sectors over agriculture. While the capital distortion in the manufacturing sector, κ_{mt}, has grown over the sample period, the service sector distortion, κ_{st}, has shrunk. Both labor market distortions are negative, implying that not enough labor is supplied to

[12] We are assuming that the productivity trends shifted slowly over the course of 6 years, rather than in a 1-period break. In all other respects, however, we treat the trend break as known in advance; our ability to divide the data into trend and deviations relies on this assumption. In contrast, Aguiar and Gopinath (2007) argue that trend breaks are the primary source of business cycles in emerging economies.

[13] Bosworth et al. (2007, p. 39) find the amount of growth in India's service sector productivity to be "quite puzzling", and perhaps exaggerated by measurement error.

the manufacturing and service sectors, and the shortage appears to be getting worse over time. Lahiri and Yi (2006, 2008) find similar trends in their labor distortion measures.

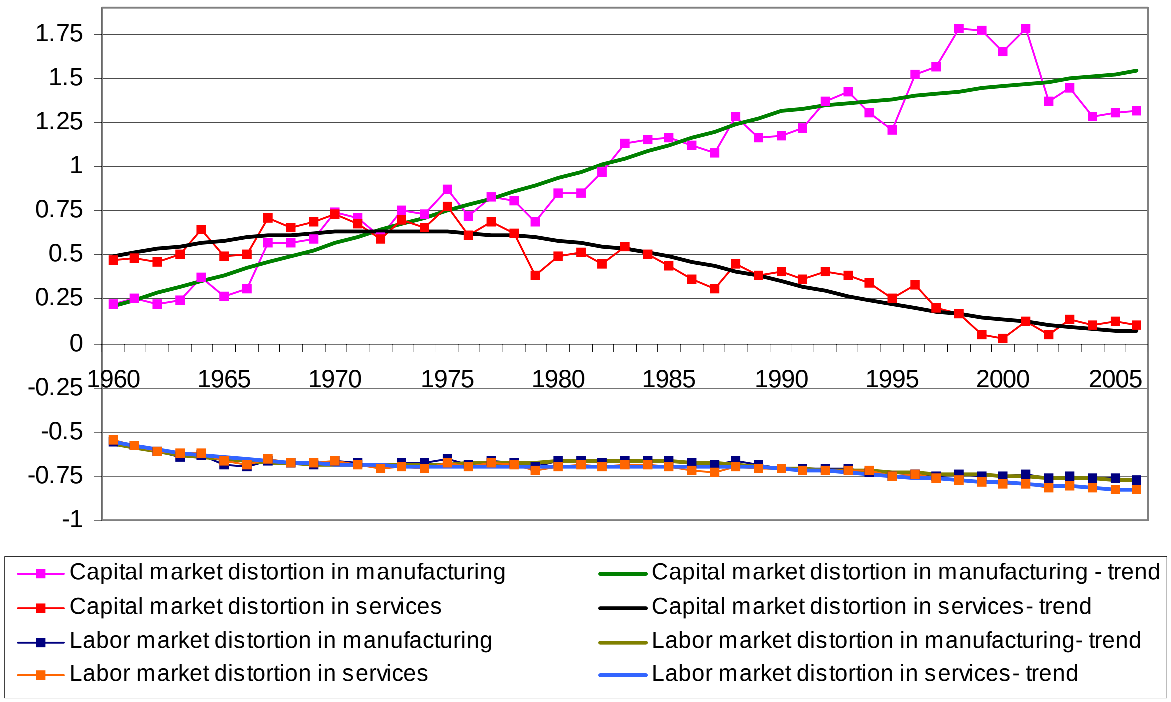

Figure 3. Capital and Labor Market Distortions: Data and Trends

Because ℓ_{mt} and ℓ_{st} reflect ratios of the marginal product of raw labor hours, it is possible that these negative values arise because non-agricultural labor is more skilled. To account for this possibility, we use the education measure constructed by Bosworth, et al. (2007) to construct the quality-adjusted distortions described in Section 3, ℓ_{1mt} and ℓ_{1st}. These measures are somewhat less extreme than the measures shown in Figure 2, but they are nonetheless negative, large and growing larger. As Lahiri and Yi (2006, p. 17) point out, large negative values of ℓ_m and ℓ_s "reflect a well known characteristic of developing countries, the concentration of the workforce in agriculture, a sector with low productivity." They are also consistent with the belief that India's markets face many barriers, both regulatory (Besley and Burgess, 2004) and social (Verma, 2008, section 3), to labor mobility. The *growth* in these distortions is perhaps more surprising, but is consistent with Bosworth et al. (2007, p. 4), who "find evidence of shortages among the group of highly-educated workers (university graduates) who have done so well in recent years."

Figures 4 and 5 show the fiscal policy wedges and the estimated depreciation rates. The capital tax rate (τ_{kt}) has shrunk over the sample period, while intensive government spending (g_{2t}) has risen. Neither the labor tax rate (τ_{lt}) nor the depreciation rate (δ_t) show any pronounced trends.

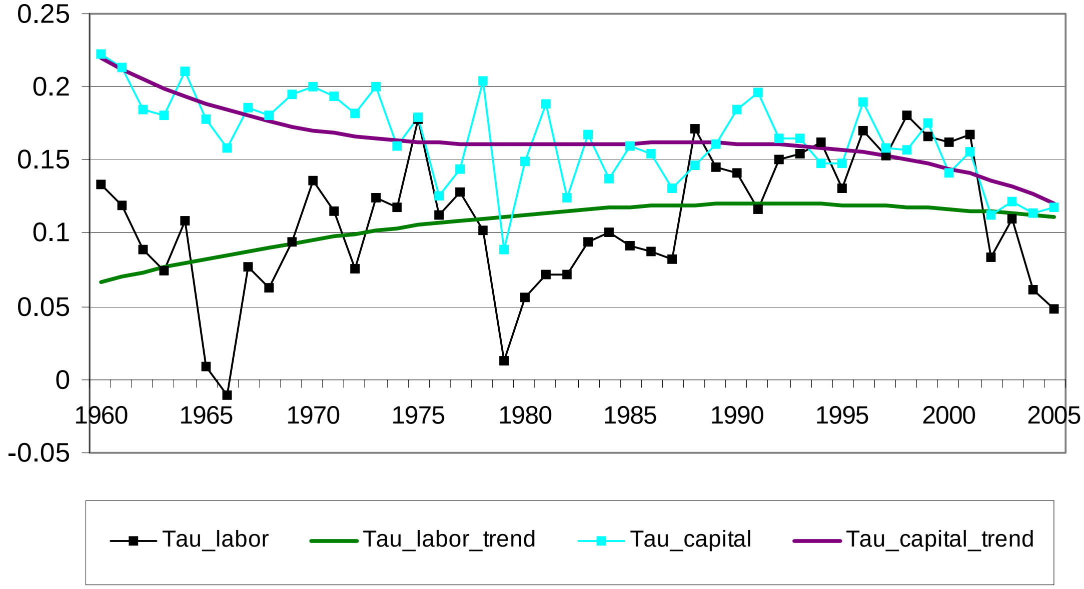

Figure 4. Capital and Labor Taxes: Data and Trends

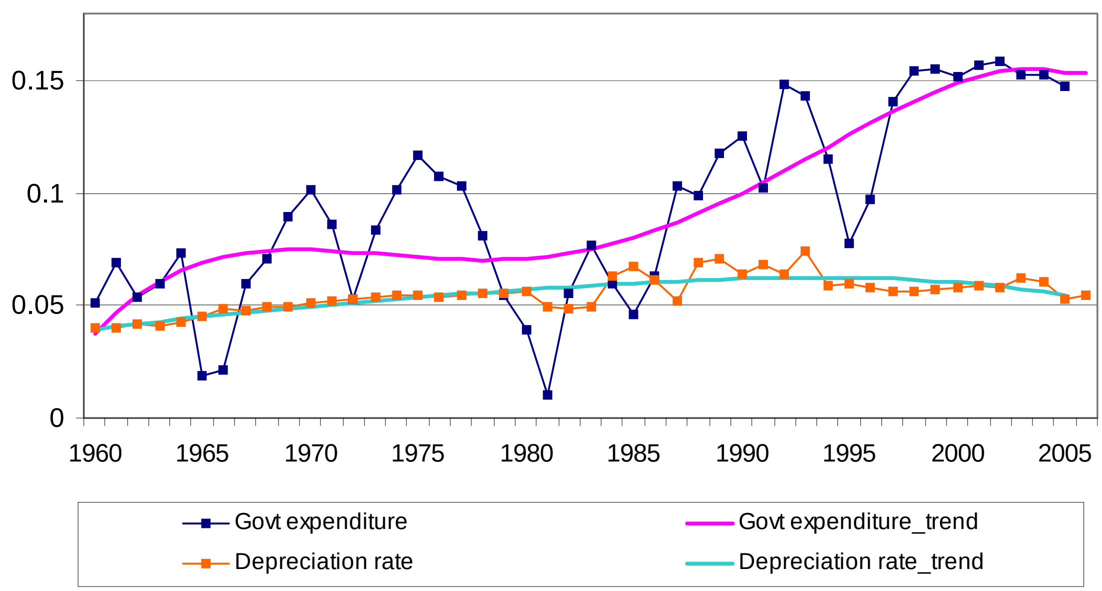

Figure 5. Government Spending and Depreciation Rate: Data and Trends

6.2 Benchmark model and counterfactuals

The benchmark model uses the wedge trends estimated in the preceding section to generate time paths for the endogenous variables; transitional dynamics aside, any trends in the endogenous variables are attributable to trends in the wedges. Figure 6 compares the model-generated trends in logged per-capita capital, consumption and output to their data counterparts. Figure 7 makes the same comparison for employment rate. In general the fits are good.

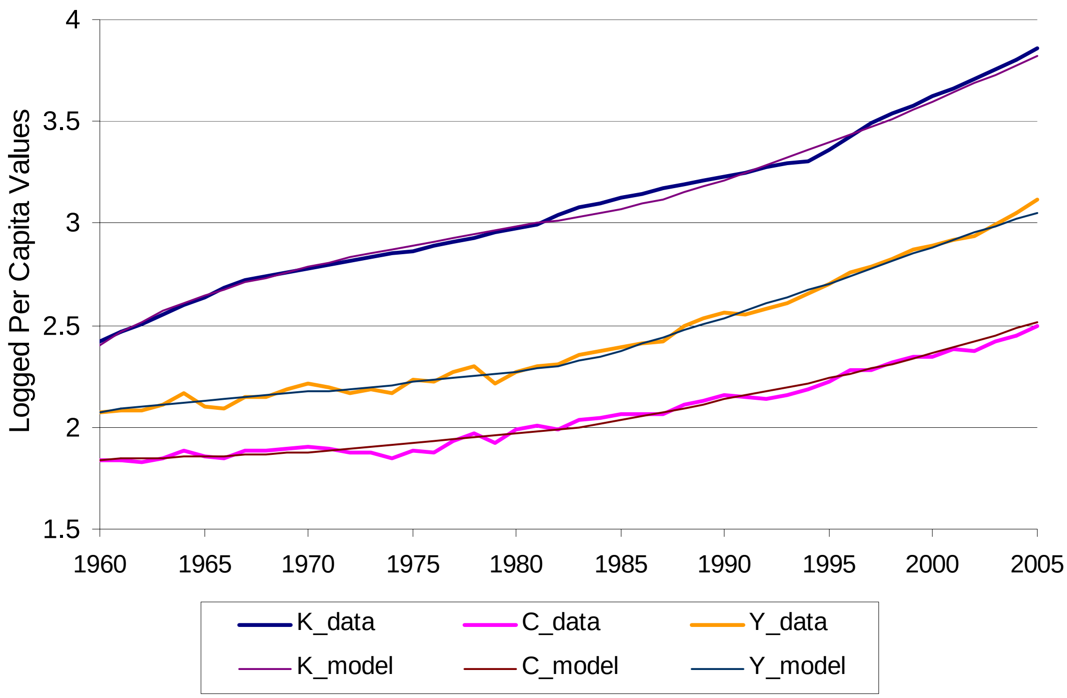

Figure 6. Data and Model-Generated Trends in Capital, Consumption and Output

To determine which wedges had the biggest impacts on the Indian economy's trend path, we do several counterfactual exercises. For instance, we allow no growth in the trend wedges, both individually and collectively, and measure how output and employment change from their benchmark trends. Table 2 summarizes fifteen such counterfactual experiments. Figures 8 and 9 show per capita output and employment rates for the data, the baseline model, and all the counterfactual experiments.

The increase in total factor productivity (TFP) in the service sector is arguably the single most important trend. When TFP in this sector is locked at its 1960 value for rest of the period, output in 2005 falls from 21.35 (thousand rupees) in the benchmark case to

10.88. The annual growth rate declines from 2.31% to 0.70%. Given that our flow utility function is separable and logarithmic in consumption, persistent changes in TFP have relatively little effect on employment: the employment rate in 2005 declines from 0.61 to 0.58. Collectively, when there is no growth in any of the TFP trends, output in 2005 falls even further, to 8.79, and output growth declines to 0.23% per year.

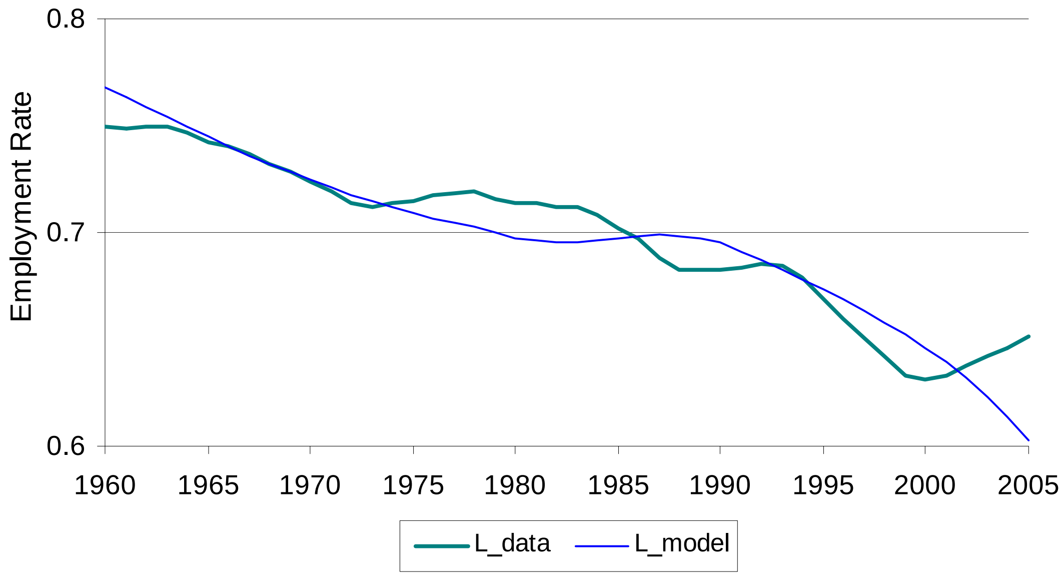

Figure 7. Data and Model-Generated Trend in the Employment Rate

The aggregate effect of improved service sector productivity would have been much smaller, however, had the Indian economy not been willing to accept more services, either as final goods, or as tradable exports. If the sectoral shares in the aggregate production function had not changed, output in 2005 would have been 16.45, 20% less than its actual value. Our results suggest that without sectoral shifts, India's annual output growth rate would have been about 0.3 percentage points lower over the period 1960-1980, and about 0.8 percentage points lower over the period 1980-2005. Adapting the standard growth accounting approach, Bosworth et al. (2007) estimate that "reallocation effects" increase annual output growth by 0.4 and 1.0 percentage points over the periods 1960-1980 and 1980-2004, respectively.

The effects of holding the quality-adjusted labor market distortions, ℓ_{1mt} and ℓ_{1st}, at their 1960 values are also significant. If the distortions are held fixed, output in 2005 rises from 21.35 to 29.37, while the employment rate rises from 0.61 to 0.72. If the labor market distortion in the service sector alone could be restrained at its 1960 value, then the annual growth rate would have increased to 2.68%, and employment in 2005 would have increased to 0.68. Conversely, if the sectoral shares had remained at their 1960 values, the labor shortage in the manufacturing and service sectors would have been much less important to the overall economy. With the service share held fixed, employment in 2005 rises from 0.61 in the benchmark model to 0.77.

	Per Capita Output				**Employment Rate**		
	(1000s of 1993-94 Rs)			**CAGR**†			
	1960	**1980**	**2005**	**1960-05**	**1960**	**1980**	**2005**
Data	7.93	9.65	22.64	2.31%	0.75	0.71	0.65
Benchmark Model	7.90	9.73	21.35	2.18%	0.76	0.70	0.61
No TFP growth							
All sectors	7.91	9.13	8.79	0.23%	0.77	0.70	0.58
Service sector	7.91	9.08	10.88	0.70%	0.77	0.70	0.58
Manufacturing sector	7.90	10.08	19.10	1.94%	0.76	0.70	0.60
Agricultural sector	7.90	9.46	19.26	1.96%	0.77	0.70	0.61
Labor distortions at 1960 values‡							
All sectors	7.88	10.78	29.37	2.90%	0.76	0.72	0.72
Service sector	7.89	10.36	26.60	2.68%	0.76	0.71	0.68
Manufacturing sector	7.89	10.12	23.20	2.37%	0.77	0.70	0.64
Capital distortions at 1960 values							
All sectors	7.84	8.76	20.59	2.12%	0.76	0.68	0.60
Service sector	7.88	9.66	24.40	2.49%	0.76	0.70	0.63
Manufacturing sector	7.86	8.84	18.19	1.84%	0.76	0.68	0.59
Fiscal policies at 1960 values							
All policies	7.77	8.42	15.73	1.54%	0.75	0.68	0.54
Capital tax	7.82	8.45	17.05	1.71%	0.75	0.68	0.58
Labor tax	7.90	9.95	21.91	2.24%	0.76	0.72	0.62
Government expenditures	7.85	9.49	19.44	1.99%	0.76	0.68	0.56
Sectoral shares at 1960 values	7.88	9.12	16.45	1.61%	0.76	0.73	0.77

†Annual growth rates

‡Labor distortions adjusted for education level

Table 2. Per Capita Output: Data and Model-Generated Trends

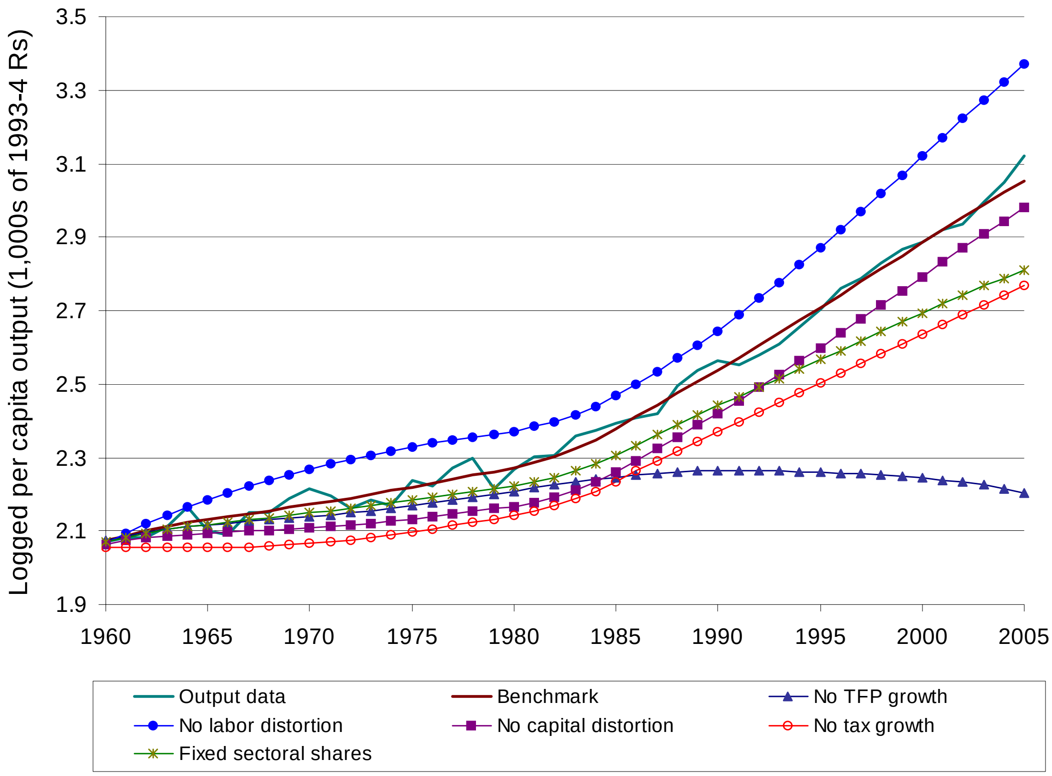

Figure 8. Data and Model-Generated Trends in Per Capita Output

Although we have adjusted our labor distortion measures for education levels, using the index constructed by Bosworth et al. (2007), it is possible that we have not adequately controlled for skill or effort. Our findings are consistent, however, with Bosworth and Collins' (2008, p. 63) claim that "India faces serious deficiencies in the education of the bulk of its youth population," and the widespread belief that India's labor markets operate inefficiently (Verma, 2008). The long-run effects of the labor distortions are potentially quite large: if the distortions are held at their 1960 values, rather than our projections, output in 2035 will be nearly two and a half times as large. Such a long-range prediction is of course speculative, but even over the sample period the worsening of the labor distortions has reduced output by over 27%.

The effects of the capital market distortions are more modest. Recall that positive values of κ_{mt} and κ_{st} imply that capital in the manufacturing and service sectors is being subsidized. As a result, setting κ_{mt} at its 1960 value, which is the lowest observed, reduces capital accumulation and output growth. Conversely, setting κ_{st} at its 1960 value, which is among the highest observed, stimulates growth. When both distortions are set to their 1960 values, their effects offset and output and employment both stay near their benchmark values.

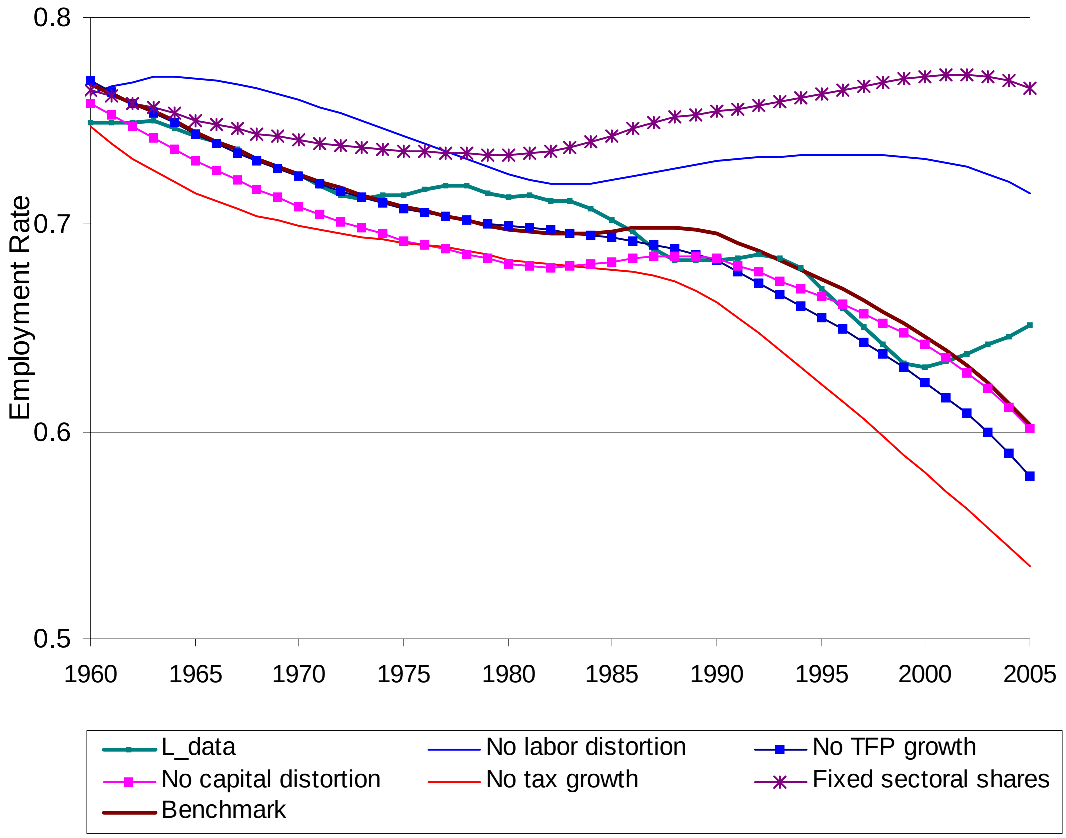

Figure 9. Data and Model-Generated Trends in the Employment Rate

The aggregate effect of direct fiscal policies—labor taxes, capital taxes and government expenditures—is mixed. Capital taxes have fallen over the sample period, so that keeping them at their 1960 value lowers output in 2005. Government expenditures have grown over the sample period. Because the wealth effect of government expenditures is to increase labor supply, if g_{2t} is set to its 1960 value, the year-2005 employment rate falls from 0.61 to 0.56, and output falls as well. Labor taxes, on the other hand, have risen over the sample period, so that setting these taxes at their 1960 values would raise both employment and output. Collectively, when the three fiscal wedges are held at their respective 1960 values, output in 2005 falls from 21.35 to 15.73, a 26% decrease, while the employment rate falls from 0.61 to 0.54.

Put together, the "direct "effect of India's fiscal policy changes is higher output. Taken as a whole, however, the counterfactual trend experiments indicate that growth in service sector productivity is most important source of output growth in the Indian economy over the last four decades. Bosworth et al. (2007) and Bosworth and Collins (2008), using standard growth accounting, reach a similar conclusion. Like Chakraborty (2006), we conclude that the main contribution of India's policy reforms has almost surely been through indirect channels, as changes in the regulatory environment manifested themselves as changes in TFP or sectoral composition.

7 Business Cycle Results

7.1 VAR estimation of wedge deviations

We model the trend deviations of the wedges and the time-varying parameters as a first-order vector autoregression:

$$\begin{aligned} \mathbf{w}_{t+1} &= \mathbf{P}\mathbf{w}_t + \boldsymbol{\epsilon}_{t+1}, \qquad (22) \\ \boldsymbol{\epsilon}_{t+1} &= \mathbf{Q}\boldsymbol{\xi}_{t+1}, \\ \mathbf{w}_t &= \left[\begin{array}{ccccccccccccc} \widehat{a}_{at} & \widehat{a}_{mt} & \widehat{a}_{st} & \widehat{\kappa}_{mt} & \widehat{\kappa}_{st} & \widehat{\ell}_{mt} & \widehat{\ell}_{st} & \widehat{\widetilde{\tau}}_{kt} & \widehat{\tau}_{lt} & \widehat{g}_t & \widehat{\delta}_t & \widehat{\psi}_t & \widehat{\theta}_t \end{array} \right]'. \end{aligned}$$

where the elements of $\boldsymbol{\xi}_t$ are unit-variance and uncorrelated, and $\mathbf{Q}$ is the lower triangular Cholesky decomposition of the covariance matrix of $\boldsymbol{\epsilon}_{t+1}$. The coefficient matrix $\mathbf{P}$ is restricted to be diagonal, but $\mathbf{Q}$ is unrestricted, so that the wedges are not independent. We found that this parsimonious specification did a good job of capturing the correlations observed in the data. Table 3 presents the coefficient matrix $\mathbf{P}$. In general, the wedge deviations are weakly correlated across time; service sector TFP has the largest autocorrelation, at 0.78. Table 4 presents the Cholesky decomposition $\mathbf{Q}$.

Recall from Section 3.3 that the measurement of the capital tax τ_{kt} is complicated by the expectation in the Euler equation. We work instead with the realized tax $\widetilde{\tau}_{k,t+1}$, where $\tau_{kt} = E_t\left(\widetilde{\tau}_{k,t+1}\right)$, which allows us to calculate the trend series τ^*_{kt}. By assuming that $\widetilde{\tau}_{k,t+1}$ follows an exogenous univariate AR(1) process around this trend, we can estimate $\widehat{\tau}_{kt}$. In particular, if $\widetilde{\tau}_{k,t+1} - \tau^*_{kt}$ follows a univariate AR(1) process, then

$$\widehat{\tau}_{kt} = \tau_{kt} - \tau^*_{kt} = E_t\left(\widetilde{\tau}_{k,t+1} - \tau^*_{kt}\right) = \rho_{\tau k}\left(\widetilde{\tau}_{kt} - \tau^*_{k,t-1}\right), \qquad (23)$$

where $\rho_{\tau k}$ is the autoregressive coefficient. Given that $\rho_{\tau k} = -0.03$ (Table 3), the end result is that capital taxes play no meaningful role in our cycle analyses.

	$\widehat{a}_a$	$\widehat{a}_m$	$\widehat{a}_s$	$\widehat{\kappa}_m$	$\widehat{\kappa}_s$	$\widehat{\ell}_m$	$\widehat{\ell}_s$	$\widehat{\tau}_k$	$\widehat{\tau}_l$	$\widehat{g}$	$\widehat{\delta}$	$\widehat{\psi}$	$\widehat{\theta}$
$\widehat{a}_a$	0.23												
$\widehat{a}_m$		0.65											
$\widehat{a}_s$			0.78										
$\widehat{\kappa}_m$				0.55									
$\widehat{\kappa}_s$					0.36								
$\widehat{\ell}_m$						0.01							
$\widehat{\ell}_s$							0.11						
$\widehat{\tau}_k$								−0.03					
$\widehat{\tau}_l$									0.54				
$\widehat{g}$										0.49			
$\widehat{\delta}$											0.36		
$\widehat{\psi}$												0.27	
$\widehat{\theta}$													0.39

Table 3. Vector AR(1) Coefficients

$\widehat{a}_a$	4.8												
$\widehat{a}_m$	0.9	2.4											
$\widehat{a}_s$	0.7	0.3	1.6										
$\widehat{\kappa}_m$	7.9	−6.3	1.5	5.5									
$\widehat{\kappa}_s$	5.6	0.4	−2.6	1.7	3.6								
$\widehat{\ell}_m$	0.9	−0.3	−0.2	−0.2	0.5	0.6							
$\widehat{\ell}_s$	1.1	0.1	−0.2	0.2	0.4	0.1	0.7						
$\widehat{\tau}_k$	−0.8	−0.5	0.1	0.0	−0.1	−0.1	0.0	2.1					
$\widehat{\tau}_l$	3.1	−0.7	−0.1	0.6	0.5	0.4	−0.3	0.3	1.1				
$\widehat{g}$	12.7	11.1	1.5	−2.5	13.3	2.2	−4.9	8.7	6.6	31.5			
$\widehat{\delta}$	0.0	0.0	−0.1	0.0	0.1	0.1	−0.1	0.0	−0.1	−0.1	0.4		
$\widehat{\psi}$	0.8	−0.2	−0.1	0.1	0.3	0.2	0.5	0.0	0.2	0.0	0.0	0.3	
$\widehat{\theta}$	−0.6	−0.1	0.2	−0.2	−0.2	0.1	−0.2	0.0	−0.1	0.0	0.0	−0.1	0.1

Table 4. Cholesky Decomposition for AR(1) Innovations (in percent)

7.2 Benchmark model and counterfactuals

Combining the wedge deviations estimated in the preceding section with the log-linearized model described in section 4.2, we simulate the fluctuations of the model's endogenous variables, and compare them to the data.

Figure 10 compares the model-generated output series with its data counterpart.

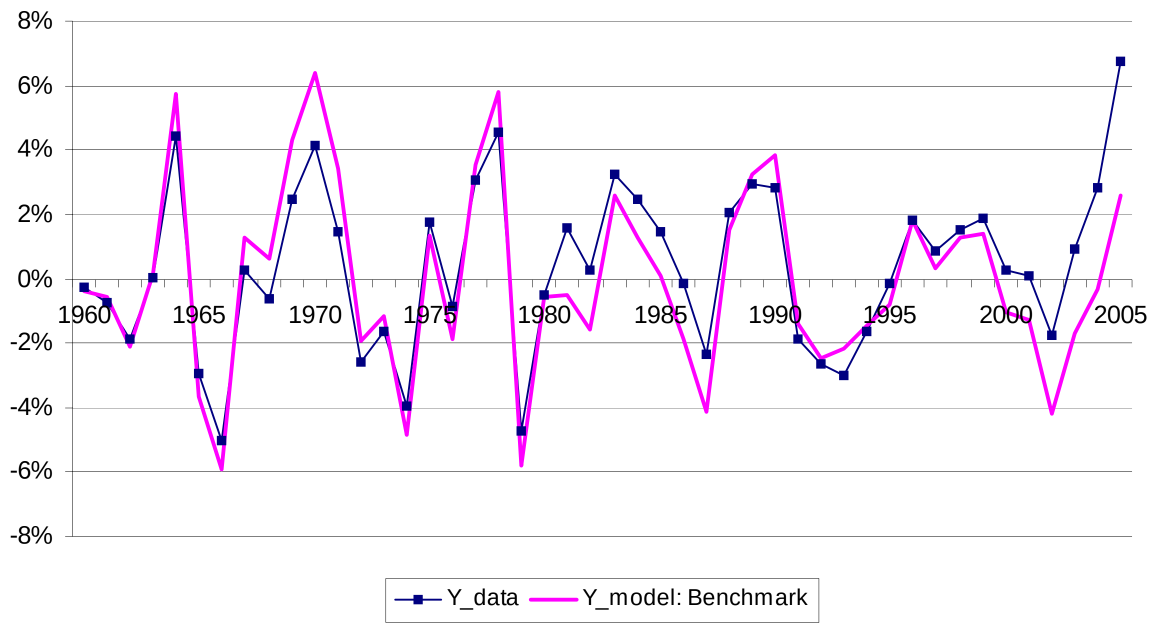

Figure 10. Output Fluctuations: Data and Model with All Wedges

The first row of Table 5 shows the standard deviation of the output fluctuations observed in the data. To allow for the possibility that India's reforms have changed the nature of its business cycles, we consider results both for the entire sample, 1960-2005, and for the subperiods 1960-1980 and 1981-2005. The first row of Table 5 shows that the standard deviation drops about 31%, from 2.93% to 2.01%, between the two subsamples. In contrast to Chari et al. (2007), who capture capital distortions in a "investment wedge" that makes the model fit the data, our model is not guaranteed to reproduce the data. Nonetheless, the fit is good, especially during the first half of the sample period. The second ("All wedges") row of Table 5 shows that during the 1960-1980 subsample, the fit between the observed output series and its model-generated counterpart is almost perfect: the correlation between the two series is about 98%. Although the correlation during the second subsample drops to 80%, the overall fit of the model for the entire sample period is reasonably good at about 87%. The model is also able to generate as much volatility as observed in the data; in fact, the model generates too much volatility.

	Standard deviation (percent)			Correlation with data (percent)		
	1960-1980	**1981-2005**	**1960-2005**	**1960-1980**	**1981-2005**	**1960-2005**
Data	2.93	2.01	2.52			
Model: All Wedges	3.63	2.17	2.86	97.6	79.7	87.1
Model: TFP Wedges Only						
All sectors	3.67	3.21	3.37	94.5	48.1	71.1
Service sector	0.61	1.93	1.50	43.8	28.2	22.4
Manufacturing sector	0.82	1.42	1.20	44.9	28.9	26.3
Agricultural sector	3.37	1.80	2.59	84.1	32.4	67.2
Model: Labor Distortion Wedges Only						
All sectors	1.42	1.15	1.26	83.0	12.8	50.2
Service sector	0.82	0.96	0.89	89.7	12.3	45.7
Manufacturing sector	0.66	0.50	0.57	67.7	5.7	38.6
Model: Capital Distortion Wedges Only						
All sectors	0.14	0.49	0.37	−50.7	7.0	−6.2
Service sector	0.12	0.30	0.24	−45.4	10.6	−4.3
Manufacturing sector	0.08	0.35	0.27	−19.2	0.8	−6.6
Model: Fiscal Policy Wedges Only						
All policies	1.62	1.92	1.78	−51.4	−2.2	−25.2
Capital tax	0.06	0.02	0.05	20.7	29.3	9.7
Labor tax	1.94	1.93	1.92	−60.1	−4.0	−32.5
Government expenditures	0.71	0.72	0.71	46.1	3.8	22.5
Model: Depreciation Wedges Only	0.07	0.29	0.22	−12.5	3.4	−2.4
Model: Sectoral Share Wedges Only						
All sectors	0.52	0.55	0.54	−31.0	6.7	−15.1
Service sector	0.51	0.30	0.40	79.3	20.6	53.1
Agricultural sector	0.94	0.76	0.83	−59.9	−3.3	−35.9

Table 5. Output Deviations from Trend: Data and Model

Next we conduct a number of counterfactual experiments to determine which wedges can best account for India's output fluctuations. Following standard BCA practice, we set various wedge series to zero, individually or jointly, and re-solve the model. Table 5 summarizes eighteen such counterfactual exercises; row headings show which wedges have *not* been shut down.[14]

The three sector-specific TFP shocks are the most significant wedges. Figure 11 compares the data to the output series generated by the model with all three TFP wedges in action.

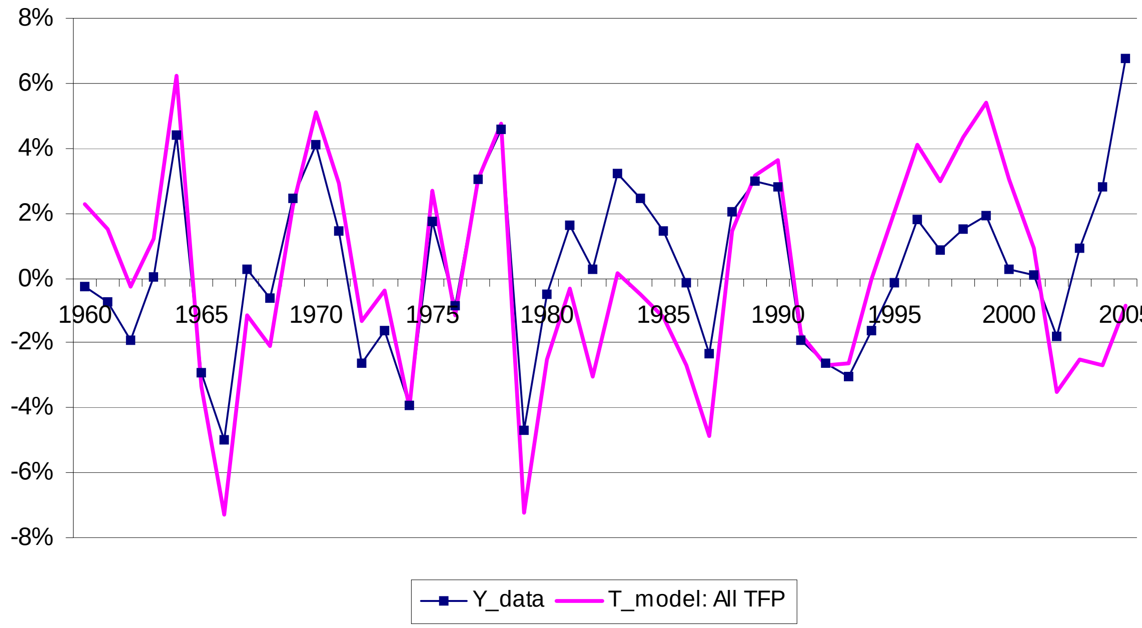

Figure 11. Output Fluctuations: Data and Model with All TFP Wedges

Table 5 shows that the TFP-only output series correlate with the data at 95% and 48% in the first and second sub-periods, respectively. Table 5 also shows that the TFP-only model generates by far the most volatility. Figures 12 and 13 illustrate the effects of the sector-specific TFP shocks. During the first subsample, TFP shocks in the agricultural sector generated 93% as much volatility as all the TFP wedges combined. Moreover, the correlation between agricultural-TFP-only output and observed output was 84%. The prominent role of agricultural TFP shocks may in part reflect the Green Revolution, consisting mainly of the spread of high-yield rice and wheat varieties, which started in 1967/68. In the second subsample, on the other hand, agricultural TFP shocks generate

[14] It bears noting that because the wedge series are not orthogonal, these experiments do not produce a true variance decomposition.

much less volatility; service sector TFP shocks appear to be more important. Much of the shift is simply due to the shift from agriculture to services in the overall economy. With our time-varying linearization, changes in sectoral shares translate immediately into changed effects.

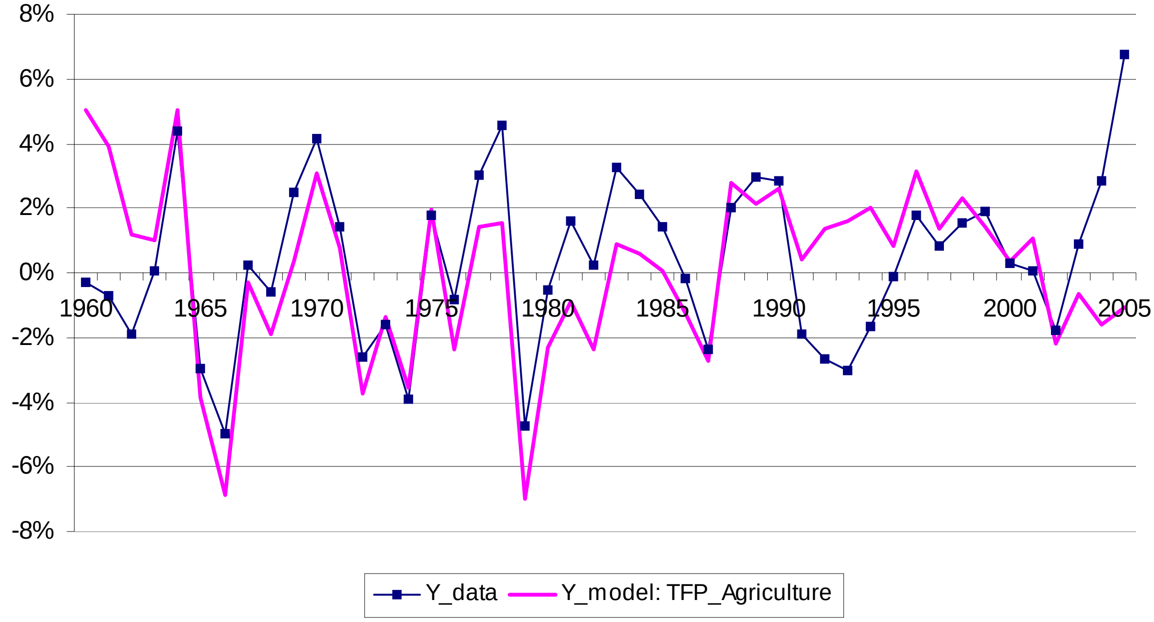

Figure 12. Output Fluctuations: Data and Model with Agricultural TFP Wedge

Labor market distortions appear significant during both periods of our analysis (Figure 14). When combined, labor market distortions can account for 48% of the output volatility observed in the first subsample and 57% of the volatility observed in the second. Like their associated trends, many fluctuations in the labor market distortions probably reflect changes in skill or effort that affect the marginal product of labor. A striking result is that that while the labor market distortions have a strong positive correlation with output in the first subsample (83%), they have virtually no correlation in the second. It is possible that this difference reflects changes in India's labor market institutions.

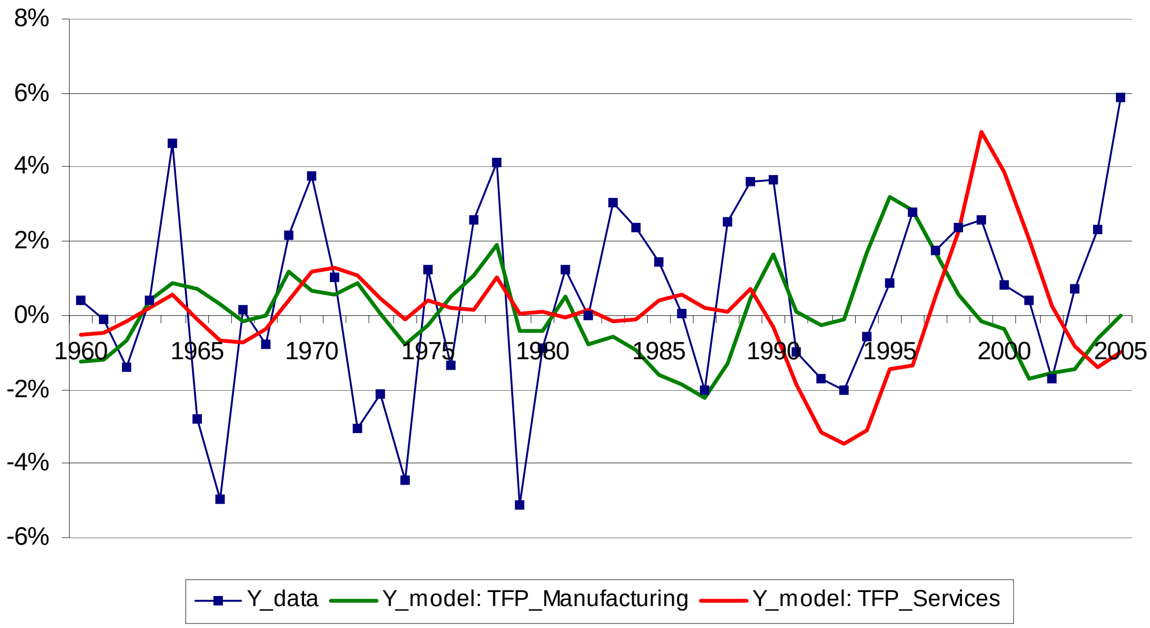

Figure 13. Output Fluctuations: Data and Model with Manufacturing and Service TFP Wedges

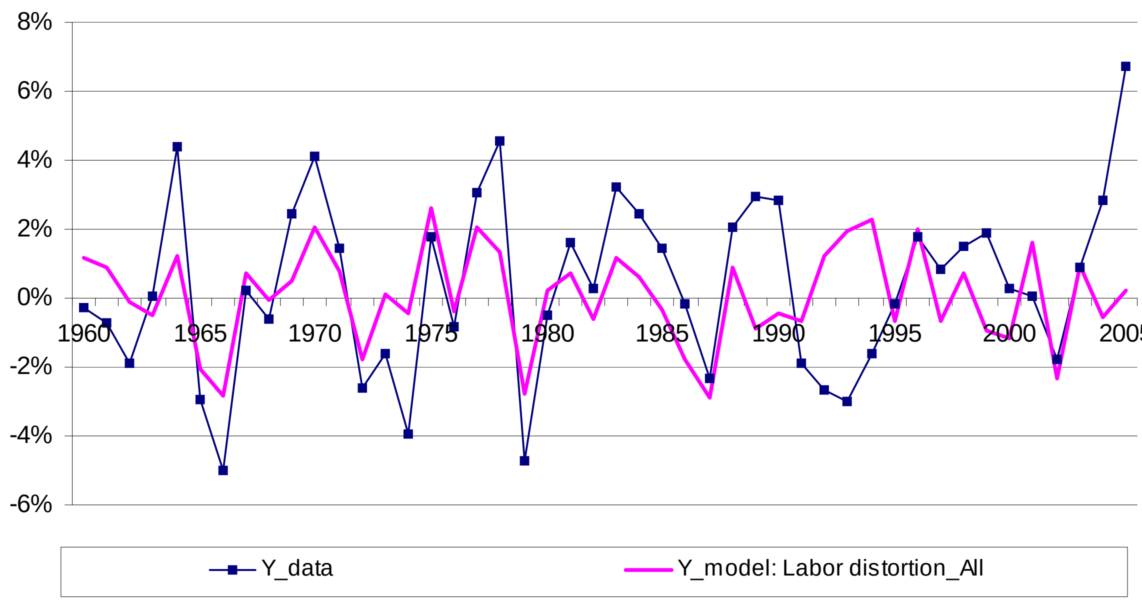

Figure 14. Output Fluctuations: Data and Model with All Labor Market Distortions

Capital market distortions (Figure 15) and our measure of aggregate capital taxes play a very small role in accounting for India's business cycles. This does not necessarily mean that capital market frictions are unimportant in explaining India's business cycles. As Chakraborty (2006) points out, because we do not set the investment wedge to make the model fit the data, any residual movements in the data not captured by our model can be interpreted as reflecting investment wedges of the type envisioned by Chari et al. (2007). These residual movements—the gaps between the data and complete-model-generated output shown in Figure 10 and Table 5—imply that the effects of such investment wedges, although small, are not completely negligible.

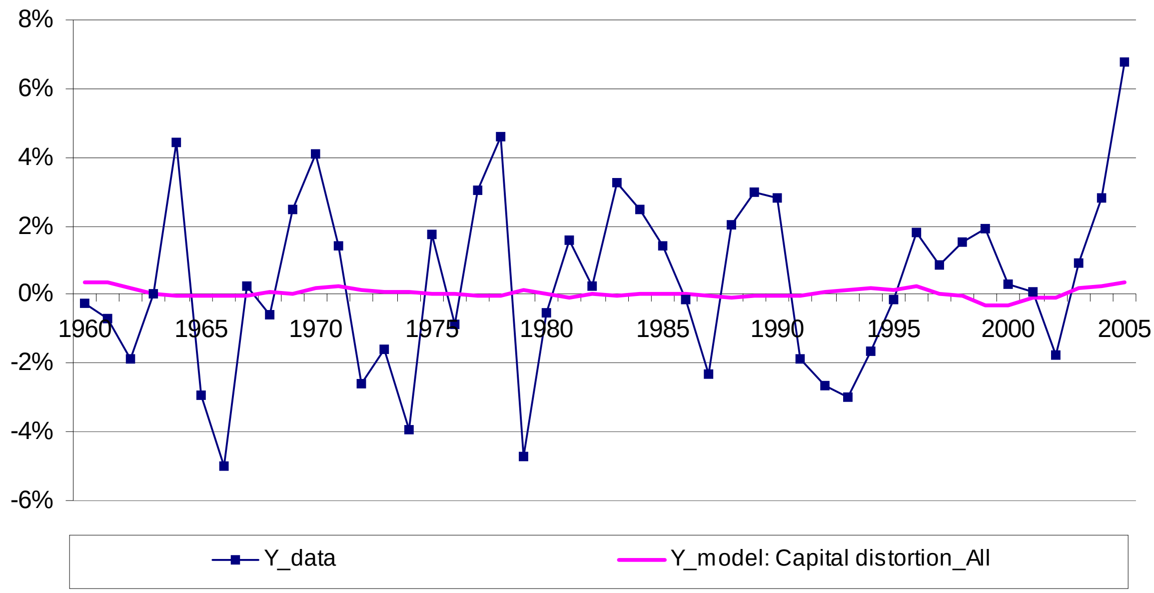

Figure 15. Output Fluctuations: Data and Model with All Capital Market Distortions

Labor taxes play a major role. In both subsamples, the model with fiscal policy shocks is the second most volatile, and in both subsamples labor taxes are the dominant fiscal policy shock. As Figure 16 and Table 5 show, labor taxes often move in the opposite direction of output, especially during the first subsample, where the correlation between labor taxes-only output and the data is -60.1%. Given that an increase in labor taxes depresses labor supply, this is not surprising. A second, subtler reason for a negative correlation is that the labor tax wedge embodies aggregate labor market frictions. (See the extensive discussion in Mulligan, 2005.) India has stringent labor laws (Besley and Burgess, 2004), as well as social barriers to job mobility (Verma, 2008). One implication of such labor market frictions is that when a shock to TFP or another wedge occurs, the response of the labor market is smaller than predicted by a frictionless model. To fit the data, a shock that moves output will have to be accompanied by a "labor tax" shock of the opposite direction that dampens the labor response; the end result is that the taxes are negatively correlated with output (Jones, 2002).

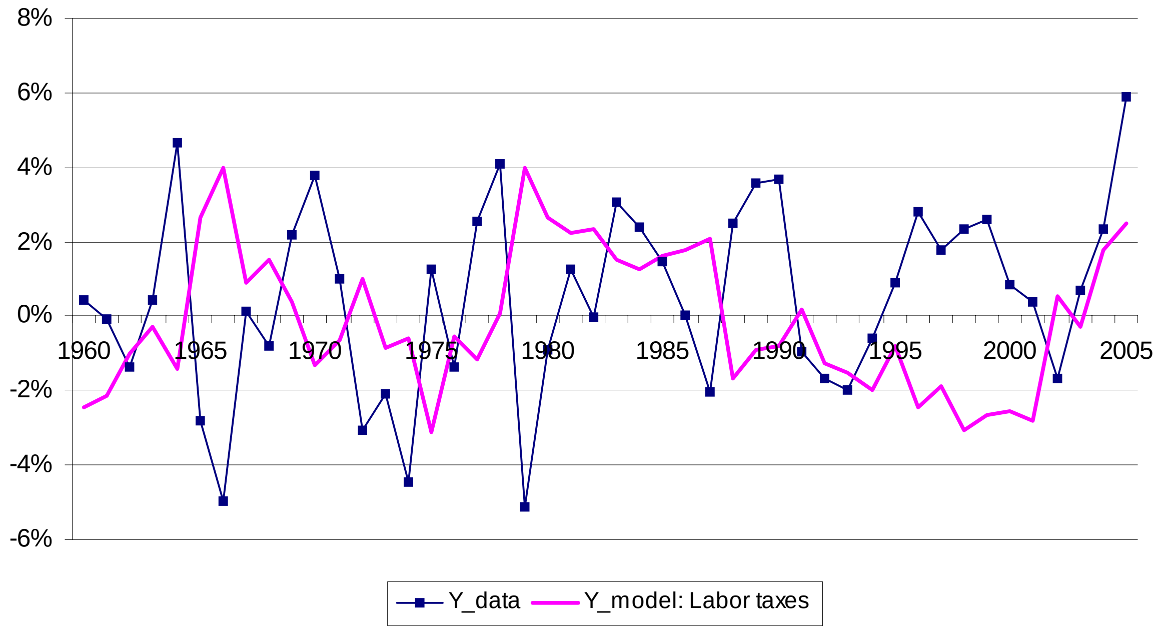

Figure 16: Output Fluctuations: Data and Model with Aggregate Labor Tax Wedge

Fluctuations in government expenditures, depreciation rates, and sectoral shares all have very modest effects. One interesting feature is that fluctuations in the service sector share are positively correlated with output, while fluctuations in the agricultural share are negatively correlated. This reinforces our finding from the trend model that India's transition from agriculture to services was an important contributor to its recent growth.

In short, our BCA exercises suggest that just as changes in sector-specific TFP are the principal driver of India's trend growth, they are the principal driver of its business cycles. In the period 1960-1980, TFP shocks in the agricultural sector were the dominant source of fluctuations. As the economy transitioned away from agriculture, output volatility fell and TFP shocks to manufacturing and services became more important. Labor market distortions and aggregate labor taxes are both significant as well, suggesting that labor market frictions are an important component of India's business cycle dynamics.

8 Conclusion

This paper develops a quantitative methodology specifically designed for analyzing the economic dynamics of developing economies. Our approach accounts for time-varying parameters, transitional dynamics and non-linear trends. We apply this methodology to the Indian economy over the period 1960-2005 to study both its long-run trends and its fluctuations around these trends.

Our findings indicate that increased total factor productivity in the service sector, facilitated by a structural shift toward services, has been the principal driver of India's growth. We also find that the apparent misallocation of labor has hindered output growth for several decades, and is growing worse. Although it is possible that these distortions reflect unmeasured differences in skill and/or effort, they suggest large inefficiencies. If the distortions continue their current trend, future growth will be significantly constrained.

Our analysis also suggests that short-run fluctuations in the Indian economy have been caused mainly by fluctuations in sector-specific productivity. During the period 1960-1980, fluctuations in agricultural productivity dominated India's business cycles. Over time, however, India has shifted from an agricultural economy to a service economy. Since 1980, total output volatility has been lower, and manufacturing and service sector productivity shocks have been the leading source of output fluctuations. Labor market distortions, both between sectors and collectively, have also had a significant effect on output fluctuations.

Despite its reliance on a formal model, our approach is an accounting methodology that does not provide structural interpretations. Our results instead identify areas where structural analysis should be most productive. Three topics appear especially promising: (i) Is the rapid growth of service sector productivity due to the removal of technology barriers, a la Parente and Prescott (2000), or improved input allocation across existing technologies? (ii) What are the mechanisms that allow rapid productivity growth in the service sector to translate into a higher share of output: substitution away from goods or trade?[15] and (iii) Do the large sectoral differences in labor productivity reflect frictions or differences in skill? Investigations into any of these questions should be quite useful.

[15] Verma (2008) considers this issue, and concludes (p. 30) "an export-led growth hypothesis of service sector growth is difficult to support."

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9 Appendix: Background Calculations

9.1 Input Prices

Assuming interiority, the first-order conditions for intermediate goods producers are

$$(1-\mu)\frac{p_m Y_m}{L_m} = w_m, \tag{24a}$$

$$(1-\sigma)\frac{p_s Y_s}{L_s} = w_s, \tag{24b}$$

$$(1-\alpha)\frac{p_a Y_a}{L_a} = w_a, \tag{24c}$$

$$\alpha\frac{p_a Y_a}{K_a} = \mu\frac{p_m Y_m}{K_m}(1+\kappa_m) = \sigma\frac{p_s Y_s}{K_s}(1+\kappa_s). \tag{24d}$$

Under perfect competition and constant returns, the average cost of the final good will equal its price, and it follows from equations (3a) to (3c) that

$$\begin{aligned} p &= \frac{1}{Y}\left[p_a Y_a + p_m Y_m + p_s Y_s\right] \\ &= \frac{1}{Y}\left[p\psi Y + p\eta Y + p\theta Y\right] = 1. \end{aligned} \tag{25}$$

Combining these results with equations (3a) to (3c) produces equations (4a)-(5c).

9.2 Input Aggregation

Combining equations (1a)-(2), and inserting equations (5a)-(5c) produces

$$\begin{aligned} K_a &= \frac{\alpha\psi}{\zeta(1+\kappa)} K, \qquad (26) \\ K_m &= \frac{\eta\mu(1+\kappa_m)}{\zeta(1+\kappa)} K, \\ K_s &= \frac{\theta\sigma(1+\kappa_s)}{\zeta(1+\kappa)} K, \\ \kappa &\equiv \frac{1}{\zeta}\left[\eta\mu\kappa_m + \theta\sigma\kappa_s\right], \\ \zeta &\equiv \alpha\psi + \eta\mu + \theta\sigma. \end{aligned}$$

and we can rewrite equation (2) as:

$$Y = \widehat{A}\Omega K^{\zeta} L_a^{\psi(1-\alpha)} L_m^{\eta(1-\mu)} L_s^{\theta(1-\sigma)}, \tag{27}$$

where

$$\begin{aligned} \widehat{A} &\equiv A_f A_a^{\psi} A_m^{\eta} A_s^{\theta} \zeta^{-\zeta} \left(\alpha\psi\right)^{\alpha\psi} \left(\eta\mu\right)^{\eta\mu} \left(\theta\sigma\right)^{\theta\sigma}, \\ \Omega &\equiv \frac{\left(1+\kappa_m\right)^{\eta\mu}\left(1+\kappa_s\right)^{\theta\sigma}}{\left(1+\kappa\right)^{\zeta}}. \end{aligned}$$

Combining equations (4a) to (4c) with equation (7) produces

$$\psi(1-\alpha)\frac{Y}{L_a} = \eta(1-\mu)\frac{Y}{L_m}(1+\ell_m) = \theta(1-\sigma)\frac{Y}{L_s}(1+\ell_s).$$

Letting ℓ denote the average labor market "distortion",

$$\ell = \frac{1}{1-\zeta}\left[\eta(1-\mu)\ell_m + \theta(1-\sigma)\ell_s\right],$$

we can rewrite the preceding equations as

$$\begin{aligned} L_a &= \frac{\psi(1-\alpha)}{(1-\zeta)(1+\ell)}L, \\ L_m &= \frac{\eta\,(1-\mu)\,(1+\ell_m)}{(1-\zeta)(1+\ell)}L, \\ L_s &= \frac{\theta(1-\sigma)(1+\ell_s)}{(1-\zeta)(1+\ell)}L. \end{aligned} \tag{28}$$

Inserting these results into equation (27) and rearranging to find equation (8):

$$Y = A^{1-\zeta}K^{\zeta}L^{1-\zeta},$$

with the term A defined as in the main text.

Combining equations (4a), (7) and (28), we can rewrite the labor-leisure allocation condition as

$$\begin{aligned} \chi\frac{CL^{\gamma}}{N^{1+\gamma}} &= \psi(1-\alpha)\frac{Y}{L_a} \\ &= (1-\zeta)(1+\ell)(1-\tau_l)\frac{Y}{L}. \end{aligned}$$

This is equation (9) in the main text.

Combining equations (3a) and (26) produces

$$\zeta\,(1+\kappa)\,\frac{Y}{K} = r. \tag{29}$$

Combining equations (6) and (29) produces equation (11) in the main text.

9.3 Normalized Production

Let lower case variables denote upper case variables divided by population and trend productivity, with $c_t = C_t/\left(A_t^* N_t\right)$, and so on. The one exception is labor hours, where $l_t = L_t/N_t$. Inserting these definitions into equations (8) and (9), we get equation (15) in the main text:

$$
\begin{aligned}
y_t &= k_t^{\zeta_t}(a_t l_t)^{1-\zeta_t}, \\
&= \frac{\chi c_t l_t^{1+\gamma}}{(1-\zeta_t)(1+\ell_t)(1-\tau_{lt})}, \\
&= \left(k_t^{\zeta_t} a_t^{1-\zeta_t}\right)^{\phi_t} c_t^{1-\phi_t} \left(\frac{1}{\chi}(1-\zeta_t)(1+\ell_t)(1-\tau_{lt})\right)^{\phi_t - 1},
\end{aligned}
$$

with a_t and ϕ_t defined as in the main text.

9.4 Linearization

Let hats ("$\widehat{\ }$") denote deviations around the transition path. The tax rates, the allocation wedges (the κ's and the ℓ's), the production parameters (ζ and ϕ), and the depreciation rate (δ) are expressed as level deviations; in these cases $\widehat{x}_t = x_t - x_t^*$. All other variables are expressed as log deviations; in these cases, $\widehat{x}_t = \ln(x_t/x_t^*)$.

Consider the Euler equation

$$
\frac{1}{c_t} = \beta \frac{N_{t+1}}{N_t} E_t \left(\frac{1}{c_{t+1}} (G_{t+1}^*)^{-1} \left[1 + \zeta_{t+1}(1+\kappa_{t+1}) \frac{y_{t+1}}{k_{t+1}} - \delta_{t+1} \right] \right) \frac{1}{1+\tau_{kt}},
$$

We can rewrite this expression as

$$
\begin{aligned}
\frac{1}{c_t^*} \exp(-\widehat{c}_t) = \frac{1}{1+\tau_{kt}^* + \widehat{\tau}_{kt}} \times \beta E_t \Bigg(& \frac{1}{c_{t+1}^*} \exp(-\widehat{c}_{t+1}) \left(G_{A,t+1}^*\right)^{-1} \times \\
& \left[1 + (\zeta_{t+1}^* + \widehat{\zeta}_{t+1})(1+\kappa_{t+1}^* + \widehat{\kappa}_{t+1}) \frac{y_{t+1}^*}{k_{t+1}^*} \exp\left(\widehat{y}_{t+1} - \widehat{k}_{t+1}\right) - \left(\delta_{t+1}^* + \widehat{\delta}_{t+1}\right) \right] \Bigg),
\end{aligned}
$$

where $G_{A,t+1}^* = A_{t+1}^*/A_t^*$. Logging both sides, and assuming the deviations are small, one gets

$$
\begin{aligned}
-\widehat{c}_t \approx{} & \ln(\lambda_{2,t+1}) - \ln(1+\tau_{kt}^* + \widehat{\tau}_{kt}) - E_t\left\{\widehat{c}_{t+1}\right\} \\
& + E_t \left\{ \ln\left(1 + (\zeta_{t+1}^* + \widehat{\zeta}_{t+1})(1+\kappa_{t+1}^* + \widehat{\kappa}_{t+1})\lambda_{1,t+1} \exp\left(\widehat{y}_{t+1} - \widehat{k}_{t+1}\right) - \left(\delta_{t+1}^* + \widehat{\delta}_{t+1}\right)\right) \right\}, \\
\lambda_{1t} \equiv{} & \frac{y_t^*}{k_t^*}; \quad \lambda_{2t} \equiv \beta \frac{c_{t-1}^*}{c_t^*} \left(G_{A,t}^*\right)^{-1}.
\end{aligned}
$$

Implicitly differentiating around trend values ("stars", with "hats" set equal to zero), and noting that $1/\left[1+\zeta_{t+1}(1+\kappa^*_{t+1})\lambda_{1,t+1}-\delta_{t+1}\right]=\lambda_{2,t+1}/(1+\tau^*_{kt})$, we get

$$\begin{aligned}\widehat{c}_t &\approx E_t\left(\widehat{c}_{t+1}-\lambda_{3,t+1}\left(\frac{1}{\zeta^*_{t+1}}\widehat{\zeta}_{t+1}+\widehat{y}_{t+1}-\widehat{k}_{t+1}+\frac{1}{1+\kappa^*_{t+1}}\widehat{\kappa}_{t+1}\right)-\frac{\lambda_{2,t+1}}{1+\tau^*_{kt}}\widehat{\delta}_{t+1}\right)+\frac{1}{1+\tau^*_{kt}}\widehat{\tau}_{kt},\\ \lambda_{3t} &\equiv \frac{\zeta^*_{t+1}\lambda_{1,t+1}\lambda_{2,t+1}(1+\kappa^*_{t+1})}{1+\tau^*_{kt}}.\end{aligned}$$

Following equation (23), we replace $\widehat{\tau}_{kt}$ with $\rho_k\widehat{\widehat{\tau}}_{k,t-1}$.

Next, consider the capital accumulation equation

$$G^*_{t+1}k_{t+1}=(1-\delta_t)\,k_t+y_t-c_t-g_t,$$

which can be rewritten as

$$G^*_{t+1}k^*_{t+1}\exp\left(\widehat{k}_{t+1}\right)=\left(1-\left(\delta^*_t+\widehat{\delta}_t\right)\right)k^*_t\exp\left(\widehat{k}_t\right)+y^*_t\exp\left(\widehat{y}_t\right)=-c^*_t\exp\left(\widehat{c}_t\right)-g^*_t\exp\left(\widehat{g}_t\right).$$

Implicit differentiation yields

$$G^*_{t+1}k^*_{t+1}\widehat{k}_{t+1}=k^*_t\left(1-\delta^*_t\right)\widehat{k}_t-k^*_t\widehat{\delta}_t+y^*_t\widehat{y}_t-c^*_t\widehat{c}_t-g^*_t\widehat{g}_t,$$

or

$$\begin{aligned}\lambda_{4,t+1}\widehat{k}_{t+1} &= \left(1-\delta^*_t\right)\widehat{k}_t+\lambda_{1t}\widehat{y}_t-\lambda_{5t}\widehat{c}_t-\lambda_{6t}\widehat{g}_t-\widehat{\delta}_t,\\ \lambda_{4t} &\equiv \frac{k^*_t}{k^*_{t+1}}G^*_t;\quad \lambda_{5t}\equiv\frac{c^*_t}{k^*_t};\quad \lambda_{6t}\equiv\frac{g^*_t}{k^*_t}.\end{aligned}$$

To fill out these two difference equations, we substitute for output, using

$$y_t=\left(\frac{1}{\chi}\left(1-\zeta_t\right)\left(1-\tau_{lt}\right)\left(1+\ell_t\right)\right)^{\phi_t-1}\left(k_t^{\zeta_t}a_t^{1-\zeta_t}\right)^{\phi_t}c_t^{1-\phi_t}. \tag{30}$$

Linearizing this equation requires us to consider the effects of the exponent deviations $\widehat{\phi}_t$ and $\widehat{\zeta}_t$. To see how this works, consider another expression for output:

$$y_t=k_t^{\zeta_t}(a_tl_t)^{1-\zeta_t}.$$

This equality can be rewritten as

$$\frac{y_t}{y_t^*} = \frac{k_t^{\zeta_t^*+\widehat{\zeta}_t}(a_t l_t)^{1-(\zeta_t^*+\widehat{\zeta}_t)}}{(k_t^*)^{\zeta_t^*}(a_t^* l_t^*)^{1-\zeta_t^*}} = \left(\frac{k_t}{k_t^*}\right)^{\zeta_t^*}\left(\frac{a_t l_t}{a_t^* l_t^*}\right)^{1-\zeta_t^*} k_t^{\widehat{\zeta}_t}(a_t l_t)^{-\widehat{\zeta}_t},$$

and taking logs yields

$$\begin{aligned}\widehat{y}_t &= \zeta_t^*\widehat{k}_t + (1-\zeta_t^*)(\widehat{a}_t + \widehat{l}_t) + \widehat{\zeta}_t \ln(k_t) - \widehat{\zeta}_t(\ln(l_t) + \ln(a_t)) \\ &\approx \zeta_t^*\widehat{k}_t + (1-\zeta_t^*)(\widehat{a}_t + \widehat{l}_t) + [\ln(k_t^*) - \ln(l_t^*)]\,\widehat{\zeta}_t,\end{aligned}$$

as $\ln(a_t^*) = 0$.

To apply this approach to equation (30), we log both sides and implicitly differentiate:

$$\begin{aligned}\widehat{y}_t &= \phi_t^*\zeta_t^*\widehat{k}_t + \phi_t^*(1-\zeta_t^*)\widehat{a}_t + (1-\phi_t^*)\left(\widehat{c}_t + \frac{1}{1-\zeta_t^*}\widehat{\zeta}_t + \frac{1}{1-\tau_{lt}^*}\widehat{\tau}_{lt} - \frac{1}{1+\ell_t^*}\widehat{\ell}_t\right) \\ &\quad + \phi_t^*\ln\left(\frac{k_t^*}{a_t^*}\right)\widehat{\zeta}_t + \left[\zeta_t^*\ln(k_t^*) + (1-\zeta_t^*)\ln(a_t^*) + \ln\left(\frac{1}{\chi c_t^*}(1-\zeta_t^*)(1-\tau_{lt}^*)(1+\ell_t^*)\right)\right]\widehat{\phi}_t \\ &= \phi_t^*\zeta_t^*\widehat{k}_t + \phi_t^*(1-\zeta_t^*)\widehat{a}_t + (1-\phi_t^*)\left(\widehat{c}_t + \frac{1}{1-\tau_{lt}^*}\widehat{\tau}_{lt} - \frac{1}{1+\ell_t^*}\widehat{\ell}_t\right) \\ &\quad + \left[\frac{1-\phi_t^*}{1-\zeta_t^*} + \phi_t^*\ln(k_t^*)\right]\widehat{\zeta}_t + \left[\zeta_t^*\ln(k_t^*) + \ln\left(\frac{1}{\chi c_t^*}(1-\zeta_t^*)(1-\tau_{lt}^*)(1+\ell_t^*)\right)\right]\widehat{\phi}_t.\end{aligned}$$

Next, we substitute for the components of $\widehat{a}_t$, $\widehat{\kappa}_t$, $\widehat{\ell}_t$, $\widehat{\zeta}_t$ and $\widehat{\phi}_t$. Consider first the expression for total factor productivity, $\widehat{a}_t$. It follows from equation (10) that

$$\frac{A_t^{1-(\zeta_t^*+\widehat{\zeta}_t)}}{(A_t^*)^{1-\zeta_t^*}} \equiv \frac{A_{ft}A_{at}^{\psi_t^*+\widehat{\psi}_t}A_{mt}^{\eta_t^*+\widehat{\eta}_t}A_{st}^{\theta_t^*+\widehat{\theta}_t}\Omega_t\Upsilon_t\Delta_t}{A_{ft}^*(A_{at}^*)^{\psi_t^*}(A_{mt}^*)^{\eta_t^*}(A_{st}^*)^{\theta_t^*}\Omega_t^*\Upsilon_t^*\Delta_t^*}.$$

Taking logs yields

$$\begin{aligned}(1-\zeta_t^*)\,\widehat{a}_t - \widehat{\zeta}_t\ln(A_t) &= \psi_t^*\widehat{a}_{at} + \eta_t^*\widehat{a}_{mt} + \theta_t^*\widehat{a}_{st} + \widehat{\psi}_t\ln(A_{at}) + \widehat{\eta}_t\ln(A_{mt}) + \widehat{\theta}_t\ln(A_{st}) \\ &\quad + \widehat{a}_{ft} + \widehat{\Omega}_t + \widehat{\Upsilon}_t + \widehat{\Delta}_t,\end{aligned}$$

or

$$\begin{aligned}\widehat{a}_t \approx \frac{1}{1-\zeta_t^*}\Big(&\psi_t^*\widehat{a}_{at} + \eta_t^*\widehat{a}_{mt} + \theta_t^*\widehat{a}_{st} + \ln(A_{at}^*)\,\widehat{\psi}_t + \ln(A_{mt}^*)\,\widehat{\eta}_t + \ln(A_{st}^*)\,\widehat{\theta}_t + \ln(A_t^*)\,\widehat{\zeta}_t \\ &+ \widehat{a}_{ft} + \widehat{\Omega}_t + \widehat{\Upsilon}_t + \widehat{\Delta}_t\Big).\end{aligned}$$

Continuing, it follows from the definition of Ω_t that:

$$\frac{\Omega_t}{\Omega_t^*} \equiv \frac{(1+\kappa_{mt}^*+\widehat{\kappa}_{mt})^{(\eta_t^*+\widehat{\eta}_t)\mu}\left(1+\kappa_{st}^*+\widehat{\kappa}_{st}\right)^{\left(\theta_t^*+\widehat{\theta}_t\right)\sigma}}{(1+\kappa_t^*+\widehat{\kappa}_t)^{\zeta_t^*+\widehat{\zeta}_t}} \cdot \frac{(1+\kappa_t^*)^{\zeta_t^*}}{(1+\kappa_{mt}^*)^{\eta_t^*\mu}\left(1+\kappa_{st}^*\right)^{\theta_t^*\sigma}}.$$

Taking logs, and then implicitly differentiating, yields

$$\begin{aligned}
\widehat{\Omega}_t &= \eta_t^*\mu\ln\left(\frac{1+\kappa_{mt}^*+\widehat{\kappa}_{mt}}{1+\kappa_{mt}^*}\right)+\theta_t^*\sigma\ln\left(\frac{1+\kappa_{st}^*+\widehat{\kappa}_{st}}{1+\kappa_{st}^*}\right)-\zeta_t^*\ln\left(\frac{1+\kappa_t^*+\widehat{\kappa}_t}{1+\kappa_t^*}\right)\\
&\quad+\widehat{\eta}_t\mu\ln\left(1+\kappa_{mt}^*+\widehat{\kappa}_{mt}\right)+\widehat{\theta}_t\sigma\ln\left(1+\kappa_{st}^*+\widehat{\kappa}_{st}\right)-\widehat{\zeta}_t\ln\left(1+\kappa_t^*+\widehat{\kappa}_t\right)\\
&\approx \frac{\eta_t^*\mu}{1+\kappa_{mt}^*}\widehat{\kappa}_{mt}+\frac{\theta_t^*\sigma}{1+\kappa_{st}^*}\widehat{\kappa}_{st}-\frac{\zeta_t^*}{1+\kappa_t^*}\widehat{\kappa}_t\\
&\quad+\mu\ln\left(1+\kappa_{mt}^*\right)\widehat{\eta}_t+\sigma\ln\left(1+\kappa_{st}^*\right)\widehat{\theta}_t-\ln\left(1+\kappa_t^*\right)\widehat{\zeta}_t.
\end{aligned}$$

Similarly,

$$\begin{aligned}
\widehat{\Upsilon}_t &\approx \frac{\eta_t^*(1-\mu)}{1+\ell_{mt}^*}\widehat{\ell}_{mt}+\frac{\theta_t^*(1-\sigma)}{1+\ell_{st}^*}\widehat{\ell}_{st}-\frac{1-\zeta_t^*}{1+\ell_t^*}\widehat{\ell}_t\\
&\quad+(1-\mu)\ln\left(1+\ell_{mt}^*\right)\widehat{\eta}_t+(1-\sigma)\ln\left(1+\ell_{st}^*\right)\widehat{\theta}_t+\ln\left(1+\ell_t^*\right)\widehat{\zeta}_t.
\end{aligned}$$

Finally, it follows from equations (10) and (18) that

$$\Delta_t A_{ft} \equiv \left(\alpha^\alpha(1-\alpha)^{1-\alpha}\right)^{\psi_t}\left(\mu^\mu(1-\mu)^{1-\mu}\right)^{\eta_t}\left(\sigma^\sigma(1-\sigma)^{1-\sigma}\right)^{\theta_t}\zeta_t^{-\zeta_t}(1-\zeta_t)^{\zeta_t-1},$$

so that

$$\begin{aligned}
\widehat{a}_{ft}+\widehat{\Delta}_t &\approx \ln\left(\alpha^\alpha(1-\alpha)^{1-\alpha}\right)\widehat{\psi}_t+\ln\left(\mu^\mu(1-\mu)^{1-\mu}\right)\widehat{\eta}_t+\ln\left(\sigma^\sigma(1-\sigma)^{1-\sigma}\right)\widehat{\eta}_t\\
&\quad-\zeta_t^*\frac{1}{\zeta_t^*}\widehat{\zeta}_t-\widehat{\zeta}_t\ln\left(\zeta_t^*\right)+\left(\zeta_t^*-1\right)\frac{1}{1-\zeta_t^*}\left(-\widehat{\zeta}_t\right)+\widehat{\zeta}_t\ln\left(1-\zeta_t^*\right)\\
&= \ln\left(\alpha^\alpha(1-\alpha)^{1-\alpha}\right)\widehat{\psi}_t+\ln\left(\mu^\mu(1-\mu)^{1-\mu}\right)\widehat{\eta}_t+\ln\left(\sigma^\sigma(1-\sigma)^{1-\sigma}\right)\widehat{\eta}_t\\
&\quad+\ln\left(\frac{1-\zeta_t^*}{\zeta_t^*}\right)\widehat{\zeta}_t.
\end{aligned}$$

Next, we consider the sectoral distortions, κ_t and ℓ_t. Recall the capital distortion:

$$\kappa_t = \frac{1}{\zeta_t}\left[\eta_t\mu\kappa_{mt}+\theta_t\sigma\kappa_{st}\right].$$

Implicit differentiation yields

$$
\begin{aligned}
\widehat{\kappa}_t &\approx \frac{1}{\zeta_t^*}\left[\mu\kappa_{mt}^*\widehat{\eta}_t + \eta_t^*\mu\widehat{\kappa}_{mt} + \sigma\kappa_{st}^*\widehat{\theta}_t + \theta_t^*\sigma\widehat{\kappa}_{st}\right] - \frac{1}{(\zeta_t^*)^2}\left[\eta_t^*\mu\kappa_{mt}^* + \theta_t^*\sigma\kappa_{st}^*\right]\widehat{\zeta}_t \\
&= \frac{1}{\zeta_t^*}\left[\mu\kappa_{mt}^*\widehat{\eta}_t + \eta_t^*\mu\widehat{\kappa}_{mt} + \sigma\kappa_{st}^*\widehat{\theta}_t + \theta_t^*\sigma\widehat{\kappa}_{st} - \kappa_t^*\widehat{\zeta}_t\right].
\end{aligned}
$$

Similarly

$$
\widehat{\ell}_t \approx \frac{1}{1-\zeta_t^*}\left[(1-\mu)\ell_{mt}^*\widehat{\eta}_t + \eta_t^*(1-\mu)\widehat{\ell}_{mt} + (1-\sigma)\ell_{st}^*\widehat{\theta}_t + \theta_t^*(1-\sigma)\widehat{\ell}_{st} + \ell_t^*\widehat{\zeta}_t\right].
$$

Finally, we express the composite parameters ζ_t and ϕ_t as functions of the share parameters ψ_t, η_t and θ_t. Recall that

$$
\begin{aligned}
\zeta_t &\equiv \alpha\psi_t + \mu\eta_t + \sigma\theta_t, \\
\phi_t &\equiv \frac{1+\gamma}{\gamma+\zeta_t}, \\
1 &= \psi_t + \eta_t + \theta_t,
\end{aligned}
$$

yielding

$$
\begin{aligned}
\widehat{\zeta}_t &\equiv \alpha\widehat{\psi}_t + \mu\widehat{\eta}_t + \sigma\widehat{\theta}_t, \\
\widehat{\phi}_t &\equiv -\frac{1+\gamma}{(\gamma+\zeta_t^*)^2}\widehat{\zeta}_t, \\
\widehat{\eta}_t &= -\widehat{\psi}_t - \widehat{\theta}_t.
\end{aligned}
$$

9.5 Solving Time-Varying Linear Expectational Difference Equations

Our approach most closely follows that of Klein (2000) (and to a lesser extent Sims, 2002), although we use the eigenvalue-eigenvector decomposition introduced by Blanchard and Kahn (1980) (as well as their notation), rather than the generalized Schur decomposition. We also incorporate elements of the MDS approaches used by Broze, Gourieroux and Szafarz (1985, 1995), Farmer (1993) and Farmer and Guo (1994), as well as insights from King, Plosser and Rebelo (2002).

9.5.1 The basic solution

Consider the following system:

$$E_t\left(\mathbf{v}_{t+1}\right)=\mathbf{A}_t\mathbf{v}_t, \quad t=0,1,2..., \tag{31}$$

where $\mathbf{v}_t$ is an $(n\times 1)$ vector, $\mathbf{A}_t$ is an $(n\times n)$ time-dependent matrix, and $E_t\left(.\right)$ is the usual conditional expectations operator. The vector $\mathbf{v}_t$ can be decomposed into the $(n_1\times 1)$ control vector $\mathbf{x}_t$ and the $(n_2\times 1)$ state vector $\mathbf{p}_t$. In particular, $\mathbf{p}_t$ is restricted by

$$\begin{aligned}&\mathbf{d}_{t+1}\equiv\mathbf{p}_{t+1}-E_t(\mathbf{p}_{t+1}) \text{ given}, \quad t=0,1,2...,\\&\mathbf{p}_0 \text{ given},\end{aligned} \tag{32}$$

where $\{\mathbf{d}_{t+1}\}_{t=0}^{\infty}$ is a covariance stationary Martingale difference sequence. In contrast, $\mathbf{x}_t$ needs only to obey some standard boundedness conditions; $\mathbf{g}_{t+1}\equiv\mathbf{x}_{t+1}-E_t\left(\mathbf{x}_{t+1}\right)$ is otherwise unrestricted. Using these definitions, we can rewrite equation (31) as

$$\begin{pmatrix}\mathbf{x}_{t+1}\\ \mathbf{p}_{t+1}\end{pmatrix}=\mathbf{A}_t\begin{pmatrix}\mathbf{x}_{t}\\ \mathbf{p}_{t}\end{pmatrix}+\begin{pmatrix}\mathbf{g}_{t+1}\\ \mathbf{d}_{t+1}\end{pmatrix}, \quad t=0,1,2..., \tag{33}$$

subject to the restrictions in equation (32). In our case, the control vector $\mathbf{x}_t$ has one variable, the consumption deviation $(\widehat{c}_t)$, so that $n_1=1$. The remaining variables, the capital and wedge deviations, are elements of $\mathbf{p}_t$.

The next step is to diagonalize $\mathbf{A}_t$:

$$\mathbf{A}_t=\mathbf{B}_t\mathbf{J}_t\mathbf{C}_t,$$

where: the matrix $\mathbf{B}_t$ contains the eigenvectors of $\mathbf{A}_t$; the matrix $\mathbf{J}_t$ is a diagonal matrix holding the associated eigenvalues; and $\mathbf{C}_t=\mathbf{B}_t^{-1}$. (In our case, a simple diagonalization always works.) Assume that the eigenvalues are sorted by size in descending order, and let m_1 denote the number of eigenvalues of magnitude greater than 1. In the standard saddle-path case, $m_1=n_1$. We will hold this assumption throughout our analysis; readers interested in other configurations can consult the references listed above. With saddle-path stability, we can partition $\mathbf{B}_t$, $\mathbf{J}_t$, and $\mathbf{C}_t$, as:

$$\begin{aligned}
\mathbf{B}_t &= \begin{bmatrix} \underset{(n\times n_1)}{\mathbf{B}_{1t}} & \underset{(n\times n_2)}{\mathbf{B}_{2t}} \end{bmatrix} \equiv \begin{bmatrix} \underset{(n_1\times n_1)}{\mathbf{B}_{11t}} & \underset{(n_1\times n_2)}{\mathbf{B}_{12t}} \\ \underset{(n_2\times n_1)}{\mathbf{B}_{21t}} & \underset{(n_2\times n_2)}{\mathbf{B}_{22t}} \end{bmatrix}, \\
\mathbf{J}_t &= \begin{bmatrix} \underset{(n_1\times n_1)}{\mathbf{J}_{1t}} & \underset{(n_1\times n_2)}{\mathbf{0}} \\ \underset{(n_2\times n_1)}{\mathbf{0}} & \underset{(n_2\times n_2)}{\mathbf{J}_{2t}} \end{bmatrix}, \\
\mathbf{C}_t &= \begin{bmatrix} \underset{(n_1\times n)}{\mathbf{C}_{1t}} \\ \underset{(n_2\times n)}{\mathbf{C}_{2t}} \end{bmatrix}.
\end{aligned}$$

Premultiplying equation (33) by $\mathbf{C}_t$ yields the transformed system

$$\begin{aligned}
\widetilde{\mathbf{w}}_{t+1} \equiv \begin{pmatrix} \widetilde{\mathbf{y}}_{t+1} \\ \widetilde{\mathbf{q}}_{t+1} \end{pmatrix} &= \begin{bmatrix} \mathbf{J}_{1t} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_{2t} \end{bmatrix} \begin{pmatrix} \mathbf{y}_t \\ \mathbf{q}_t \end{pmatrix} + \begin{pmatrix} \widetilde{\mathbf{h}}_{t+1} \\ \widetilde{\mathbf{e}}_{t+1} \end{pmatrix}, \quad t = 0, 1, 2..., \qquad (34) \\
\mathbf{y}_t &= \mathbf{C}_{1t}\mathbf{v}_t; \quad \mathbf{q}_t = \mathbf{C}_{2t}\mathbf{v}_t, \\
\widetilde{\mathbf{y}}_{t+1} &= \mathbf{C}_{1t}\mathbf{v}_{t+1}; \quad \widetilde{\mathbf{q}}_t = \mathbf{C}_{2t}\mathbf{v}_{t+1}, \\
\widetilde{\mathbf{h}}_{t+1} &= \mathbf{C}_{1t}\begin{pmatrix} \mathbf{g}_{t+1} \\ \mathbf{d}_{t+1} \end{pmatrix}; \quad \widetilde{\mathbf{e}}_{t+1} = \mathbf{C}_{2t}\begin{pmatrix} \mathbf{g}_{t+1} \\ \mathbf{d}_{t+1} \end{pmatrix}.
\end{aligned}$$

Because the timing of the transformation is important, we use tildes to denote transformed variables with time "mismatches".

Because the elements of $\mathbf{J}_{1t}$ are bigger than 1 in magnitude, the non-explosive solution to the first row of equation (34) is to set

$$\mathbf{y}_t = \widetilde{\mathbf{h}}_{t+1} = \widetilde{\mathbf{y}}_{t+1} = \mathbf{0}.$$

It immediately follows that

$$\begin{aligned}
\mathbf{v}_t &= \mathbf{B}_t\mathbf{w}_t = \mathbf{B}_{2t}\mathbf{w}_t, \\
\mathbf{v}_{t+1} &= \mathbf{B}_{2t}\widetilde{\mathbf{w}}_{t+1}, \\
\begin{pmatrix} \mathbf{f}_{t+1} \\ \mathbf{d}_{t+1} \end{pmatrix} &= \mathbf{B}_{2t}\widetilde{\mathbf{e}}_{t+1} = \begin{bmatrix} \mathbf{B}_{12t} \\ \mathbf{B}_{22t} \end{bmatrix} \widetilde{\mathbf{e}}_{t+1}.
\end{aligned}$$

But because $\mathbf{d}_{t+1}$ is given, it must be the case that

$$\begin{aligned}\mathbf{B}_{22t}\widetilde{\mathbf{e}}_{t+1} &= \mathbf{d}_{t+1},\\ \widetilde{\mathbf{e}}_{t+1} &= \mathbf{B}_{22t}^{-1}\mathbf{d}_{t+1},\end{aligned}$$

and

$$\begin{aligned}\begin{pmatrix}\mathbf{f}_{t+1}\\ \mathbf{d}_{t+1}\end{pmatrix} &= \mathbf{H}_t\mathbf{d}_{t+1}, \qquad (35)\\ \mathbf{H}_t &\equiv \begin{bmatrix}\mathbf{B}_{12t}\mathbf{B}_{22t}^{-1}\\ \mathbf{I}_{n_2}\end{bmatrix},\end{aligned}$$

where $\mathbf{I}_{n_2}$ is an identity matrix of size n_2.

9.5.2 The effects of time variation

Equation (35) implies that the innovation to the control variable $\mathbf{x}_t$ is a linear function of the innovations to the state variable $\mathbf{p}_t$. The same logic, however, applies to the variables $\mathbf{x}_t$ and $\mathbf{p}_t$ themselves. The fact that $\mathbf{p}_t$ is pre-determined at time t, along with the non-explosiveness restriction $\mathbf{y}_t = \mathbf{0},$ implies that

$$\begin{pmatrix}\mathbf{x}_t\\ \mathbf{p}_t\end{pmatrix} = \mathbf{H}_t\mathbf{p}_t. \qquad (36)$$

Comparing equations (35) and (36) reveals a timing inconsistency: time-t innovations are "stabilized" using time-$t-1$ coefficients, while the variables themselves are stabilized using time-t coefficients. To see how this plays out, consider the system:

$$\begin{pmatrix}\mathbf{x}_{t+1}\\ \mathbf{p}_{t+1}\end{pmatrix} = \mathbf{A}_t\begin{pmatrix}\mathbf{x}_t\\ \mathbf{p}_t\end{pmatrix} + \mathbf{H}_t\mathbf{d}_{t+1}. \qquad (37)$$

Suppose further that: $\mathbf{v}_0 = \mathbf{0}$; $\mathbf{d}_t = \mathbf{0}$, $\forall t \neq 1$; and $\mathbf{d}_1 \neq \mathbf{0}$. This yields:

$$\begin{aligned}\mathbf{v}_0 &= \mathbf{0},\\ \mathbf{v}_1 &= \mathbf{H}_0\mathbf{d}_1,\\ \mathbf{v}_2 &= \mathbf{A}_1\mathbf{v}_1 = \mathbf{A}_1\mathbf{H}_0\mathbf{d}_1,\\ \mathbf{v}_3 &= \mathbf{A}_2\mathbf{A}_1\mathbf{H}_0\mathbf{d}_1\end{aligned}$$

But it should also be the case that

$$\mathbf{v}_2 = \mathbf{A}_1\mathbf{H}_1\mathbf{p}_1 = \mathbf{A}_1\mathbf{H}_1\mathbf{d}_1.$$

Following Klein (2000), we can show that equation (36) generates a bounded solution. In particular,

$$\begin{aligned} \mathbf{A}_t\mathbf{H}_t &= \begin{bmatrix} \mathbf{B}_{1t} & \mathbf{B}_{2t} \end{bmatrix} \begin{bmatrix} \mathbf{J}_{1t} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_{2t} \end{bmatrix} \begin{bmatrix} \mathbf{C}_{1t} \\ \mathbf{C}_{2t} \end{bmatrix} \mathbf{H}_t \\ &= \left(\mathbf{B}_{1t}\mathbf{J}_{1t}\mathbf{C}_{1t} + \mathbf{B}_{2t}\mathbf{J}_{2t}\mathbf{C}_{2t}\right)\mathbf{H}_t. \end{aligned}$$

Moreover,

$$\mathbf{H}_t = \begin{bmatrix} \mathbf{B}_{12t}\mathbf{B}_{22t}^{-1} \\ \mathbf{I}_{n_2} \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{12t} \\ \mathbf{B}_{22t} \end{bmatrix} \mathbf{B}_{22t}^{-1} = \mathbf{B}_{2t}\mathbf{B}_{22t}^{-1}.$$

As a result,

$$\begin{aligned} \mathbf{A}_t\mathbf{H}_t &= \left(\mathbf{B}_{1t}\mathbf{J}_{1t}\mathbf{C}_{1t} + \mathbf{B}_{2t}\mathbf{J}_{2t}\mathbf{C}_{2t}\right)\mathbf{B}_{2t}\mathbf{B}_{22t}^{-1} \\ &= \left(\mathbf{B}_{1t}\mathbf{J}_{1t}\mathbf{0} + \mathbf{B}_{2t}\mathbf{J}_{2t}\mathbf{I}_{n_2}\right)\mathbf{B}_{22t}^{-1} \\ &= \mathbf{B}_{2t}\mathbf{J}_{2t}\mathbf{B}_{22t}^{-1}, \end{aligned}$$

because, as noted by Klein (2000, p. 1418), $\mathbf{C}_t\mathbf{B}_t = \mathbf{I}$.

Note that

$$\mathbf{A}_t\mathbf{H}_t = \mathbf{A}_t^*\mathbf{H}_t,$$

where the "stabilized" transition matrix $\mathbf{A}_t^*$ has been purged of its explosive eigenvalues:

$$\begin{aligned} \mathbf{A}_t^* &= \begin{bmatrix} \mathbf{B}_{1t} & \mathbf{B}_{2t} \end{bmatrix} \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_{2t} \end{bmatrix} \begin{bmatrix} \mathbf{C}_{1t} \\ \mathbf{C}_{2t} \end{bmatrix} \\ &= \mathbf{B}_{2t}\mathbf{J}_{2t}\mathbf{C}_{2t}. \end{aligned}$$

In short, applying equation (36) is equivalent to updating equation (33) with a non-explosive transition matrix. This result does *not* hold if we use $\mathbf{H}_{t-1}$ from equation (35), as $\mathbf{A}_t$ contains $\mathbf{C}_{1t}$, while $\mathbf{H}_{t-1}$ contains $\mathbf{B}_{2,t-1}$. On the other hand, using equation (35) bests captures the transition dynamics in effect at time t. Our solution is this:

1. Given $\mathbf{p}_t$, use equation (36) to find $\mathbf{x}_t$.

2. Given $(\mathbf{x}_t', \mathbf{p}_t')'$, use the bottom n_2 rows of equation (33) or (37) to find $\mathbf{p}_{t+1}$. Return

to step 1.

Using this approach means that $\mathbf{x}_{t+1}$ is not entirely consistent with the dynamics implied by equations (33) or (37). In our context, this means that consumption does not perfectly satisfy the linearized Euler equation. The error appears to be less than 1 percent of consumption, however, which is small relative to some observed parameter changes.

9.6 Estimated and Projected Trends

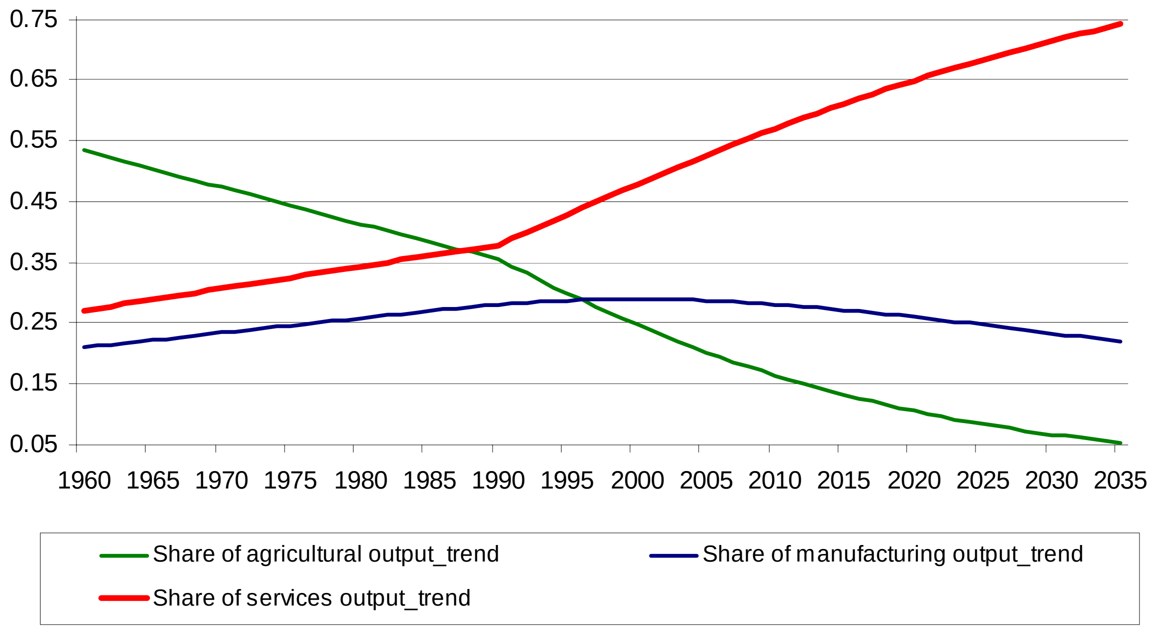

Figure 17. Estimated and Projected Trends: Sectoral Shares

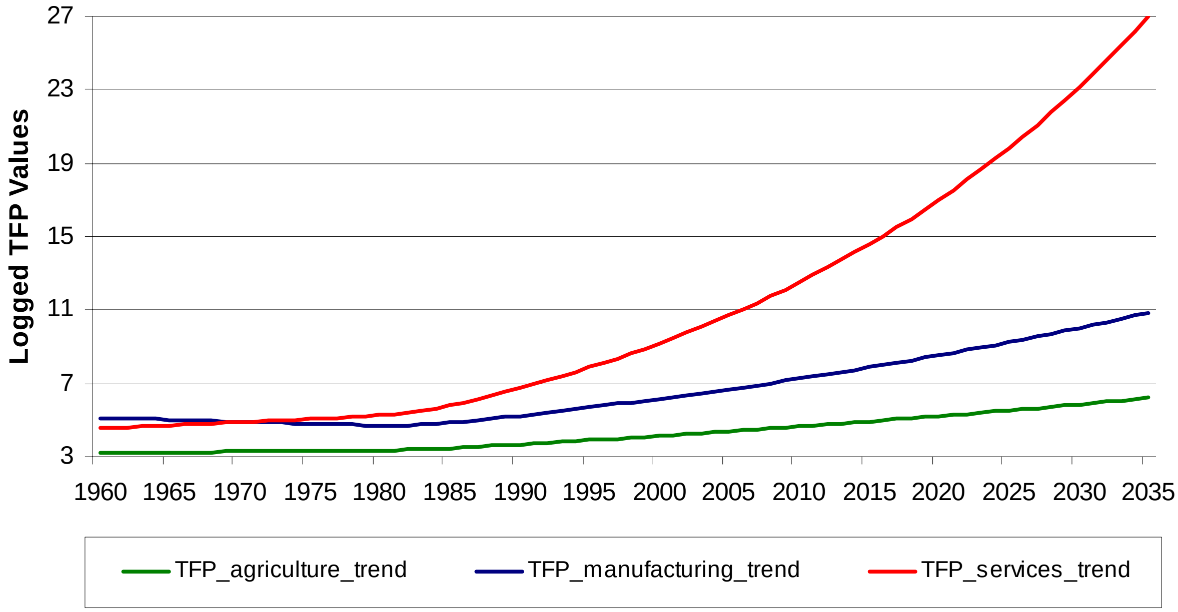

Figure 18. Estimated and Projected Trends: Sector-Specific Total Factor Productivity

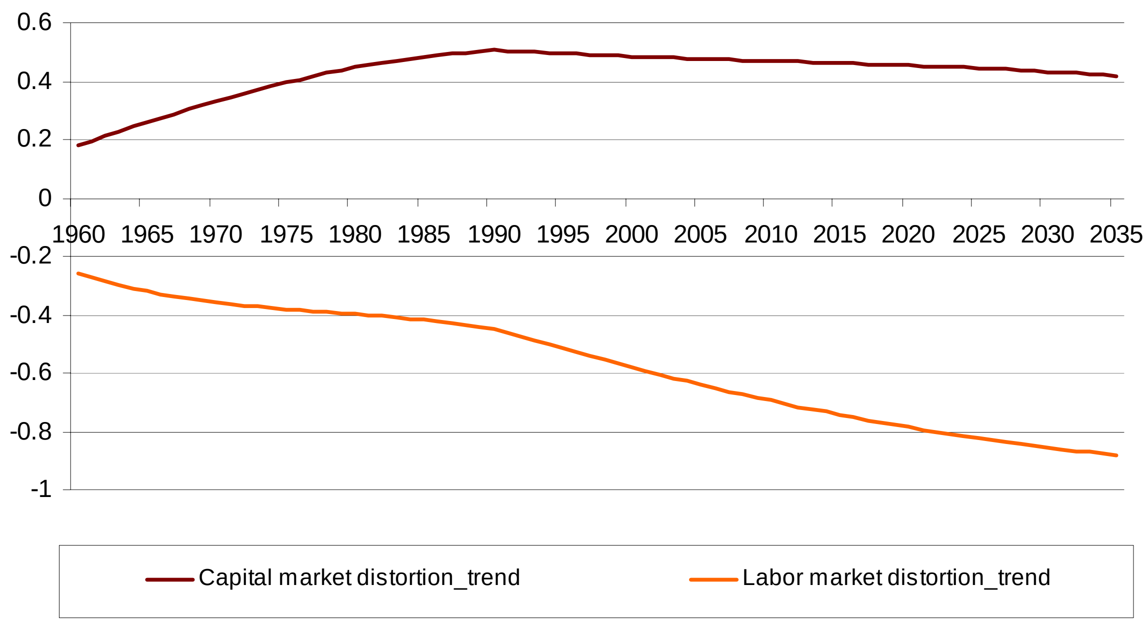

Figure 19. Estimated and Projected Trends: Capital and Labor Market Distortions

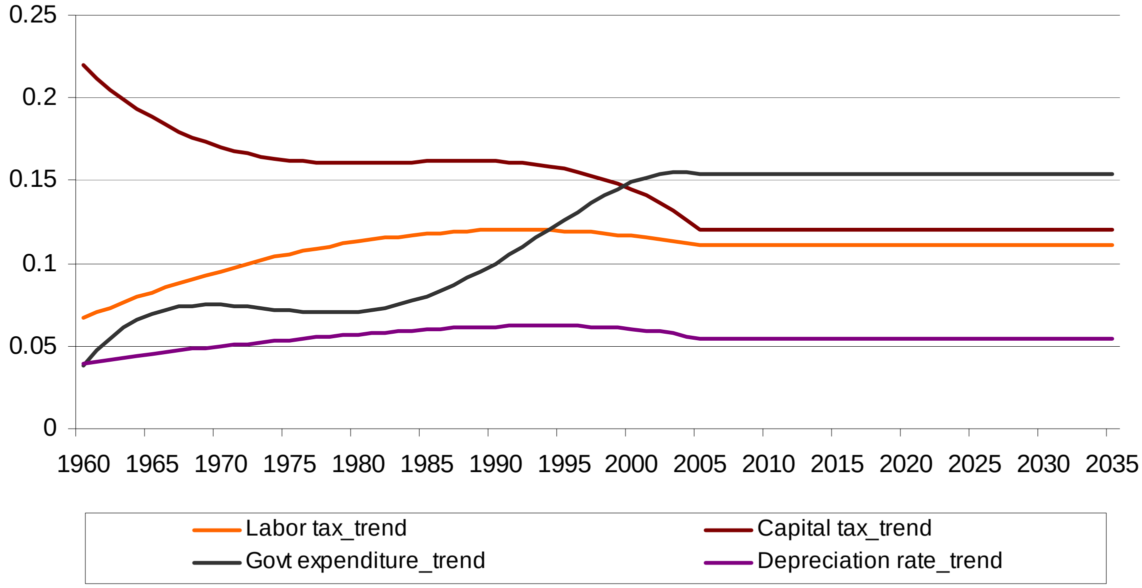

Figure 20. Estimated and Projected Trends: Fiscal Policies and Depreciation Rate