Asset Prices, Monetary Policy and Determinacy

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Abstract

We study whether central banks should respond to asset prices in their policy rules. Using Bernanke, Gertler and Gilchrist's (1999) framework–a standard dynamic stochastic general equilibrium New Keynesian model with a financial accelerator–we explore how equilibrium determinacy is impacted when the policymaker reacts to asset prices. Our results indicate that, depending on the magnitude of the financial accelerator effect, by reacting to asset price movements a central bank can potentially reduce macroeconomic volatility by making determinacy of the rational expectations equilibrium more likely relative to a standard policy rule where the central bank does not react to asset prices. Contrary to other findings in the related literature, these results suggest that by incorporating asset prices in the monetary policy rule, central banks will not induce indeterminacy and could even induce macroeconomic stability.

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1 Introduction

1.1 Asset prices and the macroeconomy

Large fluctuations in asset prices have been observed in most economies.[1] Such episodes are typically characterised by a rapid increase in asset prices followed by a sharp decline in these prices. These fluctuations can be associated with substantial inflation and output volatility. For example, sharp declines in asset prices often coincide with severe contractions in real economic activity. In a recent study of financial crises Cecchetti, Kohler and Upper (2009) find that in one quarter of these financial crises the cumulative output loss relative to the pre-crisis GDP was more than 25 per cent. Moreover, they find that one third of all the contractions associated with a financial crisis lasted for three or more years. Given these outcomes, can the central bank alleviate some of the adverse effects of asset prices on the macroeconomy?

It is generally agreed that monetary policy *alone* is insufficient to overcome all the undesirable effects of asset price fluctuations on key macroeconomic variables.[2] But even within the context of monetary policy, there is no consensus in the literature regarding the role central banks should play when asset prices fluctuate. Focussing on Taylor-type nominal interest rate feedback rules, proponents of a policy rule that incorporates a measure of asset prices, such as Cecchetti et al. (2000), have argued that using such a policy rule will improve macroeconomic performance and induce stability by reducing distortions in investment and consumption. However, Bernanke and Gertler (1999) argue that monetary policy should only respond asset price changes when these variations signal changes in expected inflation. According to the authors, reacting to asset prices over and

[1] Some recent examples include Australia (1980s and 2000s), Finland (1980s), Japan (late 1980s and early 1990s), Norway (1980s), the United Kingdom (1980s and 2000s) and the United States (1980s and 2000s). Emerging economies have also experienced substantial fluctuations in asset prices, such as Mexico and several East Asian economies in the 1990s.

[2] Most often central banks can only affect asset prices by adjusting the nominal interest rate. However, fluctuations in asset prices can be reduced by using alternative policy instruments, for example fiscal policy, the legal system and regulation of the financial sector.

above this will result in few, if any, additional gains.

In this paper we study how the determinacy of the rational expectations equilibrium is impacted by a Taylor-type rule where the central bank adjusts the nominal interest rate to asset price movements. The rationale for responding to asset prices is to improve macroeconomic stability. However, if adopting such a monetary policy rule induces equilibrium indeterminacy, then such a policy rule could potentially induce *more* volatility by making the economy susceptible to sunspot shocks.

1.2 Main ideas and findings

We use a modified version of the Bernanke, Gertler and Gilchrist (1999) model–a standard dynamic stochastic general equilibrium New Keynesian model with a financial accelerator–to study the interaction between monetary policy and asset prices. A key feature of this framework is that it incorporates credit market frictions, giving rise to a financial accelerator effect. The credit market frictions arise because of costly state verification which creates asymmetric information between borrowers/entrepreneurs and lenders/households. As a result, the cost of raising funds externally is greater than the cost of internal funds. In this framework, when asset prices rise, the value of entrepreneurs' net worth increases thereby reducing the external finance premium. Such a reduction in the cost of external finance encourages investment and causes asset prices to increase in the subsequent period. The process then repeats itself. This is the financial accelerator effect in Bernanke, Gertler and Gilchrist (1999).[3]

We first show how the determinacy of the New Keynesian equilibrium is impacted when credit market frictions are embedded in a standard New Keynesian model, the financial accelerator effect is turned off and the nominal interest rate feedback rule does not respond to asset price movements. Using this as a benchmark, our main objective is to study how equilibrium

[3] Note that the indirect effects of a favourable fundamental shock via an increase in asset prices are in addition to the direct effects of the shock on output and employment. Therefore, the financial accelerator effect in fact enhances the effects of fundamental shocks.

determinacy changes when the policymaker does in fact directly respond to asset prices. We then study what happens to the determinacy region when the financial accelerator effects are turned on.

Our results indicate that adding credit market frictions in an otherwise standard New Keynesian model where the policymaker does not respond to asset prices does not alter the determinacy conditions significantly relative to a model without credit market frictions. Given this benchmark, our main finding is that central banks can *decrease* the likelihood of indeterminacy of the rational expectations equilibrium by reacting to asset prices.[4] This is in stark contrast to the findings of Bullard and Schaling (2002) and Carlstrom and Fuerst (2007).

Moreover, our results show that when the central bank does not respond to output gap deviations and its response to inflation deviations is passive, it can still induce determinacy by reacting to asset prices.[5] Intuitively, this is because by responding to asset prices, the central bank is indirectly responding to inflation deviations. Consider an increase in asset prices due to a fundamental shock. Due to an increase in the return to capital, investment increases which in turn stimulates aggregate demand. This increase in aggregate demand drives up the price level, fuelling inflation. Therefore, in a way, responding directly to asset prices is similar to responding more aggressively to inflation.

However, when the financial accelerator effect it turned on, the amplification effects of a change in asset prices are so large that a response to asset prices of any magnitude does not affect determinacy.

1.3 Recent related literature

The question of whether central banks should directly respond to asset price movements has been studied extensively. However, two papers–

[4]In this paper we consider a contemporaneous Taylor-type nominal interest rate rule. Focussing on this policy rule allows us to stay consistent with the related literature, for example Bullard and Schaling (2002) and Carlstrom and Fuerst (2007).

[5]A passive rule is where the coefficient on inflation deviation is less than 1.

Bullard and Schaling (2002) and Carlstrom and Fuerst (2007)–are closely related to our analysis. Both these papers ask a question similar to what is posed in this paper–how does equilibrium determinacy change when central banks react to asset price movements? These two studies conclude that central banks can inadvertently induce equilibrium indeterminacy by responding to asset prices, a conclusion that is strikingly at odds with our findings.

Bullard and Schaling (2002) use Rotemberg and Woodford's (1999) framework and abstract from financial market frictions. They study the impact on equilibrium determinacy by exploiting the asset arbitrage condition of standard New Keynesian models. In their framework, equity prices and the short-term nominal interest rate (the nominal interest rate on a one-period bond) are inversely related. By incorporating a response to deviations in equity prices in the policy rule, and exploiting the inverse relationship between equity prices and the nominal interest rate, the policy rule in their framework can be re-written such that it resembles a standard Taylor-type rule, one without a direct response to equity prices. However, in such a rule, the coefficients on inflation and output gap deviations are inversely related to the response coefficient on equity price deviations. Therefore, by responding to equity prices, the monetary authorities in effect reduce their response to the output gap and inflation deviations. It is this mechanism that leads them to conclude that by responding very strongly to equity price deviations, a central bank could induce indeterminacy.

A related paper, Carlstrom and Fuerst (2007) employs a sticky price, discrete time money-in-the-utility-function model. In this model, an increase in asset prices is indicative of a potential increase in current and future profits. A rise in inflation lowers profits and hence asset prices. In such a framework, the authors argue that by responding to asset prices, the central bank weakens its response to inflation. As a result, there can be instances when, by responding strongly to asset prices, the central bank violates the *Taylor Principle*[6] and the equilibrium is no longer determinate.

[6]See Woodford (2001, 2003) for more details.

Therefore, both papers, Carlstrom and Fuerst (2007) and Bullard and Schaling (2002), are of the view that central banks should not respond to asset prices–a conclusion which is distinct from our findings. However, using a different approach Cecchetti et al. (2000) also suggest that if a central bank responds to asset price movements, it could potentially achieve "superior performance." Bernanke and Gertler (1999) argue that monetary policy should only respond to changes in asset prices when this signals a change in expected inflation.[7] Both of these studies assess the performance of the two types of monetary policy rules–one without and the other with a reaction to asset prices–by simulating data from their calibrated models. The approach adopted in this paper is different. We study whether central banks should respond to asset prices from the perspective of equilibrium determinacy.

1.4 Organisation

The remainder of this paper is structured as follows. In the next section we discuss the model. The following section, Section 3, presents the equilibrium determinacy conditions.

[7] Cecchetti et al. (2000) and Bernanke and Gertler (1999) reach different conclusions even when they use a similar model–a modified version of Bernanke, Gertler and Gilchrist (1999) which allows for exogenous bubbles in asset prices. There are two main reasons reasons for this difference.

First, Bernanke and Gertler (1999) omit the output gap from the monetary policy rule for their simulations, which Cecchetti et al. (2000) argue merely demonstrates that reacting to stock prices instead of the output gap results in an inferior economic performance. Cecchetti et al. (2000) find that after adding the output gap, the perverse impact of an asset price bubble when a central bank is inflation-accommodating is completely eliminated.

Second, as Bernanke and Gertler (2001) point out in a subsequent paper, Cecchetti et al. (2000) do not take into account the probabilistic nature of the bubble but rather consider the single scenario of a bubble lasting five periods. In contrast, Bernanke and Gertler (2001) optimise the policy rule by considering an entire probability distribution of shocks, including shocks other than those caused by a bubble. Bernanke and Gertler (2001) argue that the method in Cecchetti et al. (2000) will only yield an optimal policy if a central bank knows that stock prices are being driven by non-fundamentals and knows when it will burst.

2 Model

2.1 A New Keynesian model with credit market frictions and a financial accelerator

In this paper we use Bernanke, Gertler and Gilchrist's (1999) framework to understand the implications for equilibrium determinacy when central banks react to asset prices. The model itself is not the focus of this paper however we present it here for completeness. It is a standard New Keynesian model with one distinguishing feature: it incorporates credit market frictions giving rise to a financial accelerator effect.

The model consists of four types of agents: households, entrepreneurs, retailers and government. Households consume, work and hold monetary and non-monetary assets. Firms are owned by entrepreneurs: producers of wholesale goods who own the stock of physical capital and face capital market frictions. Entrepreneurs acquire physical capital each period by using either internal funds or by borrowing externally from households and use it in the subsequent period. Retailers buy the wholesale goods, differentiate them costlessly, and sell them in a monopolistically competitive retail market which is characterised by Calvo-type staggered nominal price setting. There is a government sector that conducts fiscal and monetary policy.

2.1.1 Households

The representative household is infinitely lived. Households utility is defined over consumption C_t[8], real balances $\frac{M_t}{P_t}$ and leisure $1 - L_t$ such that

$$E_t \sum_{k=0}^{\infty} \beta^k \left[\log(C_{t+h}) + \zeta \log\left(\frac{M_{t+h}}{P_{t+h}}\right) + \xi \frac{(1 - L_{t+h})^{1-\frac{1}{\eta}}}{1 - \frac{1}{\eta}} \right] \tag{1}$$

[8]Here C_t is the CES aggregate of differentiated retail goods such that $C_t \equiv (\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di)^{\frac{\varepsilon}{\varepsilon-1}}$ where $\varepsilon > 1$ is the constant elasticity of substitution between the different goods.

where $\zeta > 0$, $\eta > 0$ and $\beta \in (0, 1)$. The period budget constraint is given by

$$C_t + D_t + \frac{M_t}{P_t} = W_t L_t - T_t + \Pi_t + R_{t-1} D_{t-1} + \frac{M_{t-1}}{P_t} \tag{2}$$

where D_t are real deposits at a financial intermediary, R_t is the real interest rate earned on these deposits, W_t is the real wage rate and T_t are real lump sum taxes. Dividends received from retailers are given by Π_t.

The first order conditions for the household's problem are given below

$$\frac{1}{C_t} = \beta E_t \left(\frac{1}{C_{t+1}} R_t \right) \tag{3}$$

$$W_t \frac{1}{C_t} = \eta \frac{1}{(1 - L_t)} \tag{4}$$

$$\frac{M_t}{P_t} = \zeta C_t \left(\frac{R_t^n - 1}{R_t^n} \right)^{-1} \tag{5}$$

where $R_t^n = R_t \frac{P_{t+1}}{P_t}$ is the gross nominal interest rate.

2.1.2 Entrepreneurs

Entrepreneurs are risk neutral, face credit market frictions, and are finitely lived.[9] Each entrepreneur faces a survival probability τ and new entrepreneurs $(1 - \tau)$ enter to replace exiting entrepreneurs such that there is a unit measure of entrepreneurs at any date. Exiting entrepreneurs make a small transfer to the new entrepreneurs and then consume what remains.[10] Each period, entrepreneurs purchase capital and use it in the subsequent period. Capital K_t, purchased in period $(t - 1)$ and hired labour L_t are used to produce wholesale goods Y_t. The production technology is given by

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha} \tag{6}$$

[9]The finite lives assumption precludes cases in which entrepreneurs acquire sufficient wealth and are self-sufficient thereby making credit markets redundant.

[10]The transfer to new entrepreneurs is a technical assumption that ensures that each new entrepreneur starts off with some initial capital in order to produce wholesale goods. While Bernanke and Gertler (1999) make this assumption, Bernanke, Gertler and Gilchrist (1999) approach this indirectly by assuming that entrepreneurs supplement their income by working in the general labour market. As noted in these papers, these assumptions are not critical to the analysis. Therefore, following Hirose (2008), we ignore them in our analysis.

where $\alpha \in (0,1)$ and Z_t is an exogenous technology parameter such that $\log Z_t = \rho_z \log Z_{t-1} + \varepsilon_t^z$ with $\varepsilon_t^z \sim i.i.d.N(0, \sigma_z^2)$ and $0 \leq \rho_z \leq 1$.

At any date t, each risk neutral entrepreneur maximises expected discounted profits by optimally choosing capital and labour

$$E_t \sum_{h=0}^{\infty} \Lambda_{t,h}(Y_{t+h} - R_{t+h}^k K_{t+h} - W_{t+h} L_{t+h}) \tag{7}$$

subject to equation (6). Note that the stochastic discount factor $\Lambda_{t,h}$ is $(\tau^h \beta^h C_t)/C_{t+h}$. The first order conditions are given below

$$R_t^k = \alpha \frac{Y_t}{K_t} MC_t \tag{8}$$

$$W_t = (1-\alpha)\frac{Y_t}{L_t} MC_t \tag{9}$$

where the real marginal cost of production MC_t is the Lagrangian multiplier on date t constraint.

In this framework, entrepreneurs are also capital producers. The aggregate capital stock evolves according to the following equation

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right) K_t + (1-\delta) K_t \tag{10}$$

where $\delta \in (0,1)$ is the depreciation rate of capital, I_t is aggregate investment expenditure and $\Phi'(\cdot) > 0$, $\Phi''(\cdot) < 0$ and $\Phi(0) = 0$. Here aggregate investment expenditure I_t gives $\Phi(I_t/K_t)K_t$ new capital goods and the term $\Phi(I_t/K_t)K_t$ captures the increasing marginal adjustment costs in the production of capital.

In equilibrium, the price of a unit of capital Q_{t+1} is given by[11]

$$E_t Q_{t+1} = E_t \left[\Phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\right]^{-1} \tag{11}$$

[11] This relationship is derived by solving the following profit maximization problem in each period: $E_t[Q_{t+1}\Phi\left(\frac{I_{t+1}}{K_{t+1}}\right) K_{t+1} - I_{t+1}]$. This specification assumes that investment expenditures are chosen one period ahead; capturing the idea of investment delays. A consequence of this assumption is that even though shocks to the economy impact asset prices immediately, investment and output experience a delayed effect. See Bernanke, Gertler and Gilchrist (1999) for more details.

and the gross return from holding a unit of capital from $t-1$ to t is given by

$$R_t^q = \frac{R_t^k + Q_t(1-\delta)}{Q_{t-1}} \tag{12}$$

where R_t^k is the marginal product of capital. Substituting equations (6), (8) in (12) gives the demand curve for new capital.

Bernanke, Gertler and Gilchrist (1999) assume asymmetric information between borrowers and lenders and credit market frictions arise because of costly state verification. Therefore, the cost of raising funds externally is greater than the cost of internal funds. In their framework, capital expenditure, $Q_t K_{t+1}$, of any entrepreneur is proportional to his/her net worth N_{t+1} and external borrowing is given by $(Q_t K_{t+1} - N_{t+1})$. The supply curve for investment finance is given by

$$E_t R_{t+1}^q = s\left(\frac{N_{t+1}}{Q_t K_{t+1}}\right) R_t \tag{13}$$

where $s(\cdot)$ is the ratio of the costs of external finance, $E_t R_{t+1}^q$, and internal finance, R_t, $s'(\cdot) < 0$ and $s(1) = 1$.

In this framework, the dynamic behavior of capital demand and the return to capital depends on how net worth evolves. Net worth depends on gross earnings of equity holdings less repayments of borrowings.

2.1.3 Retailers

A continuum of retailers buy wholesale goods from entrepreneurs in a competitive market at price $P_t MC_t$, the nominal marginal cost. They differentiate these goods at no cost and sell them to households in a monopolistically competitive retail market. Prices are set on a staggered basis following Calvo (1983). In any period, a retailer can change price with probability $1-\theta$. Therefore, retailer i chooses price $P_t^*(i)$ to maximise expected discounted profits

$$E_t \sum_{h=0}^{\infty} \theta^h \Lambda_{t,h} \left[\left(\frac{P_t^*(i) - P_{t+h} MC_{t+h}}{P_{t+h}}\right) Y_{t+h}(i)\right] \tag{14}$$

subject to the retailers' demand curve

$$Y_{t+h}(i) = \left[\frac{P_t^*(i)}{P_{t+h}}\right]^{-\epsilon} Y_{t+h} \tag{15}$$

where ϵ is the price elasticity of the retail good i and the discount factor of the retailer is $\Lambda_{t,h} = (\beta^h C_t)/C_{t+h}$.[12] The first order condition for retailer i is given by

$$P_t^*(i) = \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{E_t \sum_{h=0}^{\infty} \theta^h \Lambda_{t,h} \frac{Y_{t+h}(i) P_{t+h} MC_{t+h}}{P_{t+h}}}{E_t \sum_{h=0}^{\infty} \theta^h \Lambda_{t,h} \frac{Y_{t+h}(i)}{P_{t+h}}}. \tag{16}$$

The aggregate price in period t is given by

$$P_t = [\theta P_{t-1}^{1-\epsilon} + (1-\theta) P_t^{*1-\epsilon}]^{\frac{1}{1-\epsilon}} \tag{17}$$

2.1.4 Government

The government sector in this model conducts fiscal and monetary policy. Real government expenditure G_t is financed by creating real money $\frac{M_t - M_{t-1}}{P_t}$ and collecting lump sum taxes T_t such that

$$G_t = \frac{M_t - M_{t-1}}{P_t} + T_t \tag{18}$$

and the process of G_t is given by

$$\log(G_t) = \rho_g \log(G_{t-1}) + \varepsilon_t^g \tag{19}$$

where $0 \leq \rho_g < 1$ and $\varepsilon_t^g \sim i.i.d.N(0, \sigma_g^2)$.

The monetary policy feedback rule is such that the central bank responds to inflation deviations, the output gap, and deviations of asset prices from their steady state value by changing the nominal interest rate R_t^n

$$\log\left(\frac{R_t^n}{R^n}\right) = \phi_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \phi_y \log\left(\frac{Y_t}{Y}\right) + \phi_q \log\left(\frac{Q_t}{Q}\right) \tag{20}$$

[12] Similar to a standard New Keynesian model, aggregate demand in this model is the composite of individual retail demands $Y_t(i)$ and is given by $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$. The corresponding price index is $P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$.

where the response coefficients of the central bank are $\phi_\pi \geq 0, \phi_y \geq 0$ and $\phi_q \geq 0$. Note that capital letters without a time subscript denote the steady state value of that variable. The monetary policy rule has been modified to allow central banks to respond to asset price movements.

2.2 Linearised equations

Following Bernanke, Gertler and Gilchrist (1999), we present the log-linearised model in four blocks of equations.

Aggregate demand

$$y_t = \frac{C}{Y}c_t + \frac{C^e}{Y}c_t^e + \frac{I}{Y}i_t + \frac{G}{Y}g_t \tag{21}$$

$$c_t = -r_t + E_t c_{t+1} \tag{22}$$

$$c_t^e = q_t + k_{t+1} \tag{23}$$

$$r_t^q = (1-\vartheta)(mc_t + y_t - k_t) + \vartheta q_t - q_{t-1} \tag{24}$$

$$E_t r_{t+1}^q = r_t - \psi(n_t - q_t - k_{t+1}) \tag{25}$$

$$E_t q_{t+1} = \varphi(E_t i_{t+1} - k_{t+1}) \tag{26}$$

Aggregate supply

$$y_t = z_t + \alpha k_t + (1-\alpha) l_t \tag{27}$$

$$y_t - l_t + mc_t - c_t = \eta^{-1} l_t \tag{28}$$

$$\pi_t = \kappa mc_t + \beta E_t \pi_{t+1} \tag{29}$$

Evolution of state variables

$$k_{t+1} = \delta i_t + (1-\delta) k_t \tag{30}$$

$$n_t = R^q[\frac{K}{N}(r_t^q - E_{t-1} r_t^q) + \frac{(1-\tau R^k)}{\tau} y_t + n_{t-1}] \tag{31}$$

Monetary policy rule and shock processes

$$r_t^n = \phi_\pi \pi_t + \phi_y y_t + \phi_q q_t \tag{32}$$

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g \tag{33}$$

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z \tag{34}$$

In the aggregate demand block, equation (21) is the economy-wide resource constraint. Equation (22) is the intertemporal Euler equation. From equation (23) entrepreneurial consumption varies proportionately with net worth. The next three equations in this block capturing investment demand arise because of the additional features–credit market constraints and the financial accelerator–of Bernanke, Gertler and Gilchrist (1999). In this model, the gross asset (capital) return, equation (24), depends not only on the marginal product of capital $[(1-\vartheta)(mc_t + y_t - k_t)]$ but also on changes in asset prices $[\vartheta q_t - q_{t-1}]$. Equation (25) is critical and captures the financial accelerator effect. If credits markets were perfect, in equilibrium at the optimal level of investment, the expected return on the asset would equal the risk-free rate. Due to credit market frictions, borrowing externally is costly and the cost of external funds depends on entrepreneurs' net worth relative to the gross value of capital $[n_t - (q_t + k_{t+1})]$. If net worth increases relative to the total value of capital, the external finance premium falls. Here ψ is the elasticity of the external finance premium to leverage. The last equation in this block, equation (26), gives the relationship between asset prices and investment.[13]

The second block, the aggregate supply block, is relatively standard. Equation (27) is the log-linearised wholesale production function[14], the labour market equilibrium condition is given by equation (28) and the New Keynesian Phillips curve is given by equation (29).[15]

In the model, the two state variables are capital and net worth. The evolution of capital is standard and is given by equation (30). Net worth, equation (31), depends primarily on the net return on assets (the first term) and the lagged value of net worth.

[13]Equations (22), (24), (25) and (26) are obtained by log-linearising equations (3), (12), (13) and (11) respectively. In these equations $\vartheta = (1-\delta)/(\alpha MC\frac{Y}{K} + 1 - \delta)$ and $\varphi = [\Phi(I/K)^{-1}]'/[\Phi(I/K)^{-1}]''$.

[14]Equation (27) is obtained by log-linearizing equation (6).

[15]Substituting equation (16) in (17) and linearising the latter around the steady state gives the standard New Keynesian Phillips Curve where $\kappa = \frac{(1-\theta\beta)(1-\theta)}{\theta}$ and $\pi_t = p_t - p_{t-1}$.

As in a standard New Keynesian model, the nominal interest rate is pinned down by an ad hoc monetary policy rule, equation (32). In order to allow comparability with other related papers, we use a contemporaneous rule. In addition, since the objective of the paper is to understand how equilibrium determinacy conditions change when central banks respond to deviations of asset prices from their steady state value, the nominal interest rate responds to asset price movements as well. Equations (33) and (34), characterise the exogenous disturbances to government consumption and technology.

2.3 The dynamic system

The reduced form representation of the dynamic system is given below

$$B_{11}P_t = B_{12}E_tP_{t+1} + B_{13}X_t \tag{35}$$

$$X_t = RP_{t-1} + SX_{t-1} + U_t \tag{36}$$

where $P_t = [c_t, i_t, \pi_t, r_t^q, q_t]$ is a vector of free variables, $X_t = [k_t, n_{t-1}, g_t, z_t, q_{t-1}, E_{t-1}r_t^q]$ is a vector of predetermined variables, $U_t = [0, 0, \varepsilon_t^g, \varepsilon_t^z, 0, 0]$ is a vector of fundamental disturbances and R and S are conformable matrices.

Let $B_1 = (B_{11})^{-1}B_{12}$ and $C = (B_{11})^{-1}B_{13}$, then the dynamic system can be rewritten as

$$P_t = B_1E_tP_{t+1} + CX_t \tag{37}$$

$$X_t = RP_{t-1} + SX_{t-1} + U_t \tag{38}$$

We follow Evans and Honkapohja (2001) and Honkapohja and Mitra (2004) and write the dynamic system as a vector autoregressive process.

$$P_t = B_1P_{t+1} + CX_t - B_1\eta_{t+1} \tag{39}$$

$$X_{t+1} = RP_t + SX_t + U_{t+1} \tag{40}$$

where $\eta_{t+1} = P_{t+1} - E_tP_{t+1}$. Simplifying further, we get

$$D_1 \begin{bmatrix} P_t \\ X_t \end{bmatrix} = D_2 \begin{bmatrix} P_{t+1} \\ X_{t+1} \end{bmatrix} + E \begin{bmatrix} U_{t+1} \\ \eta_{t+1} \end{bmatrix} \tag{41}$$

where

$$D_1 = \begin{bmatrix} I & -C \\ R & S \end{bmatrix},$$

$$D_2 = \begin{bmatrix} B_1 & 0 \\ 0 & I \end{bmatrix}$$

and

$$E = \begin{bmatrix} 0 & -B_1 \\ -I & 0 \end{bmatrix}$$

Equilibrium determinacy, following Blanchard and Kahn (1980), requires that the number of eigenvalues of $D_1^{-1}D_2$ inside the unit circle equals the number of free variables in the model. See Appendix A for more details.

3 Determinacy of equilibrium

In this section we numerically compute the determinacy region for our benchmark case–a New Keynesian model with credit market frictions where the central bank does not respond to asset prices.[16] We then study how the determinacy region changes relative to the benchmark case when the central bank responds to asset prices. Finally, we study the effect introducing the financial accelerator has on the determinacy regions.

3.1 Calibrated values

In calibrating the model, we choose parameters values that are consistent with Bernanke, Gertler and Gilchrist (1999) and Bernanke and Gertler (1999). The technology and preference parameters are standard where $\beta = 0.99$ is the quarterly discount rate, the depreciation rate per quarter, δ, is set to 0.025, the capital share in total income $\alpha = 0.33$, labour supply elasticity $\eta = 3.0$, the probability that a retail firm cannot change its price within a period, $\theta = 0.75$ such that $\kappa = 0.086$ and ϵ, the price elasticity of the retail good, equals 6. The autoregressive coefficients on technology and government process, ρ_z and ρ_g, are set to 1.0 and 0.95 respectively. The additional

[16] In this model, D_1 is singular. Therefore, we use the Schur Decomposition to obtain the eigenvalues.

parameters relate to the entrepreneurial sector and the financial accelerator effect in the model. The elasticity of the price of capital with respect to the investment capital ratio, φ, equals 0.25, and $\vartheta = 0.32$.[17] We set the entrepreneur's survival probability τ to 0.95.

In both Bernanke, Gertler and Gilchrist (1999) and Bernanke and Gertler (1999) the elasticity of the external finance premium to leverage is $\psi = 0.05$. We also adopt this value when we study the effects of the financial accelerator mechanism on equilibrium determinacy. However, for our baseline case with credit market frictions but no financial accelerator effect, we set $\psi = 0$.

The steady state risk spread is assumed to be given by $R^q = 0.005 + R$ implying the risk spread is equal to approximately two percentage points. The steady state share of government expenditure in total income, $\frac{G}{Y}$ is 0.2 and $\frac{N}{K}$ the steady state ratio of net worth to capital is set to 0.5.

3.2 A New Keynesian model with credit market frictions

In the benchmark case, we explore how introducing credit market frictions in a standard New Keynesian model changes the determinacy region when the central bank does not react to asset prices. We first need to compute the determinacy region consistent with a standard New Keynesian model without credit market frictions. For this, we use Bullard and Mitra's (2002) model where the central bank responds to deviations of current inflation and output. In addition, to allow comparability across different models considered here, the parameter values are calibrated using values described in the earlier section.[18]

[17] $\vartheta = \frac{(1-\delta)}{\alpha MC \frac{Y}{K} + 1 - \delta}$ where $MC = \frac{\epsilon - 1}{\epsilon}$. Using $\epsilon = 6$ (to yield a fairly standard value for steady state marginal cost of approximately 0.83) we can obtain $\vartheta = 0.32$.

[18] For example, in Bullard and Mitra (2002) $\sigma = 0.157$ and $\kappa = 0.024$. These have been changed to $\sigma = 1$ and $\kappa = 0.086$ to stay consistent with Bernanke, Gertler and Gilchrist (1999).

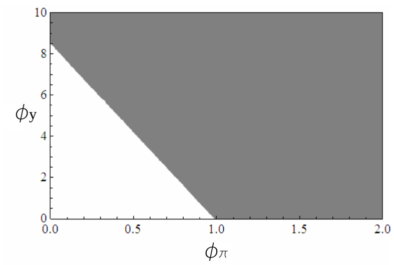


Figure 1: Bullard and Mitra (2002)

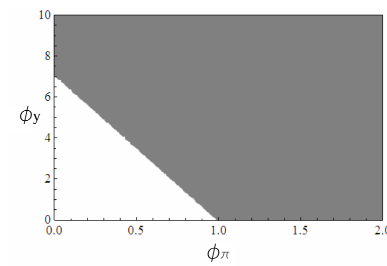


Figure 2: Credit market frictions

The shaded region in Figure 1 gives the coefficients on inflation deviations and the output gap that are consistent with equilibrium determinacy. Here, unlike our benchmark case, there is no capital or credit market frictions. Comparing the results of our benchmark case in Figure 2 with Figure 1, we see that determinacy region does not change much. Note in both cases, the Taylor Principle holds. As noted earlier in the literature, if $\phi_y = 0$, then the central bank's response to inflation deviations is passive when $\phi_\pi < 1$. However, when $\phi_\pi > 1$ the central bank's monetary policy response is characterised as active.[19] When $\phi_y > 0$ and $\phi_\pi > 0$, the policymaker can exploit the trade-off between responding to inflation deviations and the output gap. For example, if $\phi_\pi < 1$, a central bank can still induce determinacy if it responds strongly to output gap deviations. The trade-off is not one-for-one due to the relationship between inflation deviations and the output gap implied by the New Keynesian Phillips Curve.[20]

What is responsible for the slight difference in the determinacy regions?

[19] Intutively the passive monetary policy rule corresponds to the the following scenario. Suppose there is an increase in expected inflation. Since the response of the central bank towards inflation is less than one-for-one, the real ineterst rate falls. The lower real interest rate in turn stimulates aggregate demand and fuels inflation. Therefore, inflationary expectations are self-fulfilling. Such a policy rule is therefore undesirable.

[20] According to the New Keynesian Phillips Curve, each percentage point of permanently higher inflation *implies* a permanently higher output gap of more than one per cent. Therefore, if one considers the interest rate rule purely in terms of a change in inflation, the coefficient on the output gap will need to be larger to have the same effect on the nominal interest rate as responding to inflation.

We find that incorporating capital is responsible for the decrease in the slope. If the role of capital in production is reduced[21], the slope becomes closer to the result in Bullard and Mitra (2002). If capital's contribution to output is reduced via the elasticity parameter α, the slope increases.

3.2.1 Asset prices in the monetary policy rule

This section examines whether a central bank can affect equilibrium determinacy by reacting to asset prices. Figure 3 is the determinacy region consistent with a monetary policy rule where the central bank reacts to deviations in inflation and asset prices from their steady state values ($\phi_y = 0$).

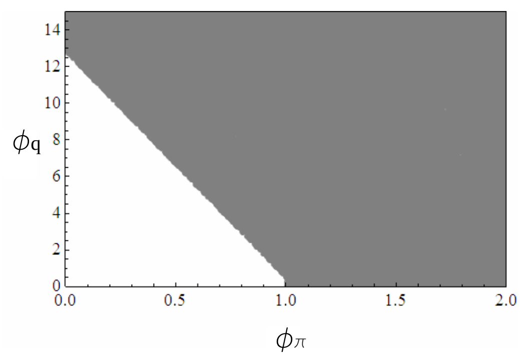


Figure 3: Reaction to asset prices and $\phi_y = 0$

Figure 3 shows that a central bank can respond to deviations in asset prices without inducing additional volatility. In other words, our findings suggest that following a monetary policy rule which incorporates asset price deviations can do no harm. Moreover, the trade-off at the boundary of the determinacy region can be exploited to the extent that values of $\phi_\pi < 1$ can still lead to determinacy when there is a response to asset prices.

Similar to the trade-off between responding to the output gap and inflation deviations, the trade-off between responding to asset price and in-

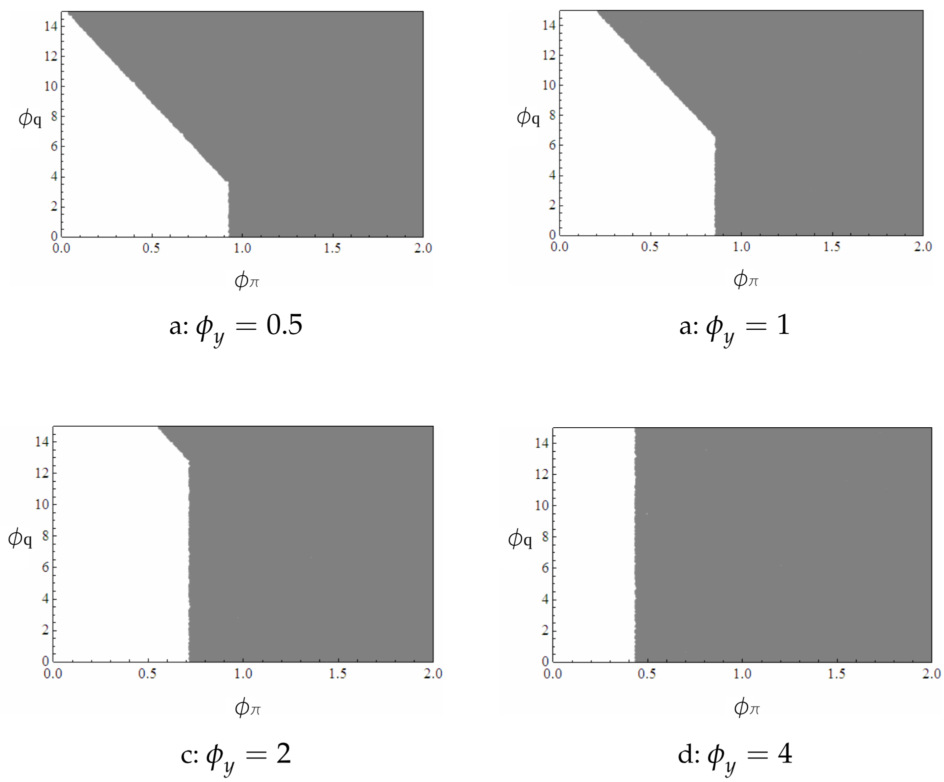
[21]It has been observed in the literature that incorporating capital has a small effect on determinacy for contemporaneous monetary policy rules (Carlstrom and Fuerst 2005; Duffy and Xiao 2008).

flation deviations is not one-for-one because of the relationship between inflation and asset price deviations implied by the New Keynesian Phillips Curve.[22] This substitutability between responding to inflation and asset price deviations is somewhat consistent with Bernanke and Gertler's (1999) view that a strong commitment to stabilising inflation renders a response to asset prices unnecessary.

The question of importance is then what is driving this substitutability in the model. Suppose there is an increase in fundamental asset prices. This increases the return to capital, causing investment demand to increase and stimulating aggregate demand. This increase in aggregate demand will increase the output gap, which in turn will drive up the price level and fuel inflation. Therefore, in a way, responding directly to asset prices is similar to responding more aggressively to inflation.

Figure 4 shows how the determinacy region changes as the weight on the output gap coefficient ϕ_y is increased.

[22] It is difficult to show this algebraically without an analytical condition for determinacy. Using the expression for marginal cost from the labour equilibrium equation, the New Keynesian Phillips Curve can be expressed as: $\pi_t = \kappa[\frac{\chi}{1-\alpha}(y_t - z_t - \alpha k_t) + c_t - y_t] + \beta E_t \pi_{t+1}$. Where $y_t = \frac{C}{Y}c_t + \frac{C^e}{Y}c_t^e + \frac{I}{Y}i_t + \frac{G}{Y}g_t$. Asset price deviations affect investment and capital which in turn affect inflation via the above expression for the New Keynesian Phillips Curve.



a: $\phi_y = 0.5$

a: $\phi_y = 1$

c: $\phi_y = 2$

d: $\phi_y = 4$

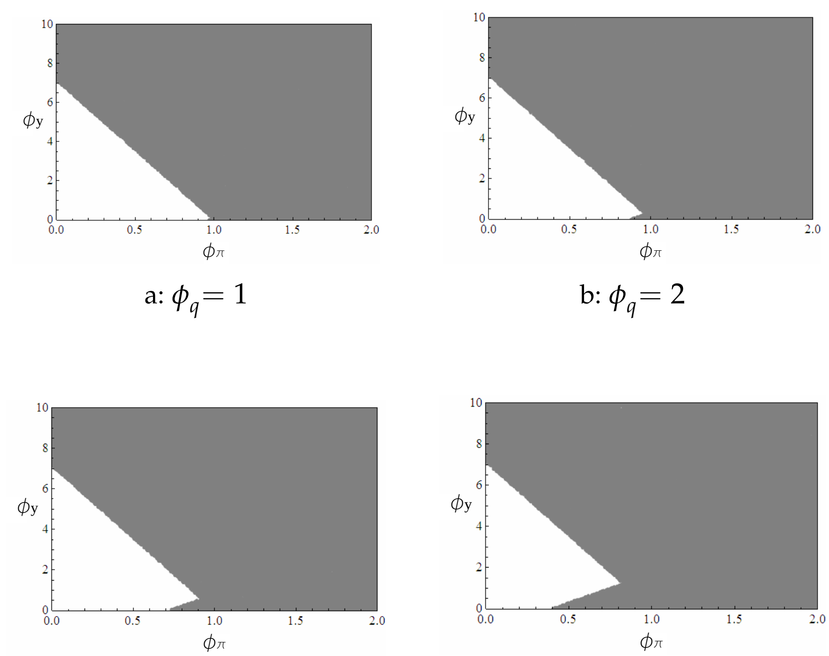
Figure 4: The determinacy region as the weight on the output gap coefficient is increased

The substitutability between ϕ_π and ϕ_q becomes increasing difficult to exploit (see bottom panels c and d in Figure 4). The intuition is that as the central bank increases the weight on the output gap, when $\phi_\pi < 1$ the *additional* gain from responding to asset price deviations declines because both responding to the output gap and asset price deviations are in effect different ways to respond to inflationary pressures in this model.

When the central bank is increasing the weight on the output gap coefficient, it cannot further exploit the substitutability between responding to inflation and asset price deviations by increasing the weight on ϕ_q.

3.2.2 Implications for the conduct of monetary policy

Given the substitutability between responding to deviations in asset prices and inflation, the underlying question of importance is whether this can be exploited. Figure 5 shows how the determinacy region changes as the weight on the asset price coefficient, ϕ_q is increased.



a: $\phi_q = 1$

b: $\phi_q = 2$

c: $\phi_q = 4$

d: $\phi_q = 8$

Figure 5: The determinacy region as the weight on the asset price coefficient ϕ_q is increased

Figure 5 shows that indeterminacy becomes less likely when the central bank reacts to asset price deviations. As the value of ϕ_q increases, the determinacy region becomes more determinate, and the Taylor principle when $\phi_y = 0$ no longer holds.

However, Figure 5 shows that the substitutability between responding to deviations in asset prices and inflation can only be exploited when the coefficient on the output gap is small. This is consistent with the reasoning that increasing the weight of the output gap coefficient diminishes the gain from responding to deviations in asset prices from a determinacy perspective, as seen in Figure 4.

The key implication of the preceding results for the conduct of monetary policy is that if a central bank elects to respond to asset prices it can do so without inducing additional volatility. This contrasts the results in Bullard and Schaling (2002) and Carlstrom and Fuerst (2007) where the opposite is found. The difference in the results can be attributed to the way asset prices have been incorporated in the models.

In Carlstrom and Fuerst (2007), an increase in inflation is associated with a decline in firm profits and, as a result, share prices. Therefore, inflation and share prices move in opposite directions which means the overall response to inflation is weaker when a central bank responds to share prices. Similarly, in Bullard and Schaling (2002), the central bank's responses to deviations in inflation and equity prices move in opposite directions because any increase in the equity price coefficient is associated with a decrease in the coefficients on inflation and the output gap and thus increases the likelihood of indeterminacy.

The findings in this paper that a central bank will not necessarily induce volatility in the economy by adopting a policy rule which responds to deviations in asset prices also contrasts the view of Bernanke and Gertler (1999). The conclusion is more in line with that of Cecchetti *et al*. (2000). However, our results are a different vantage point from which to answer the question of whether central banks should respond to asset prices. From the perspective of equilibrium determinacy for a New Keynesian model with credit market frictions, not only do such monetary policy rules cause no harm, they can also make indeterminacy less likely.

3.3 A New Keynesian model with financial accelerator effects

In this section we consider what happens when the financial accelerator effect is turned on. In the Bernake, Gertler and Gilchrist (1999) model, when there is a financial accelerator effect, an increase in asset prices increases entrepreneurs' net worth which reduces the risk premium. This allows entrepreneurs to borrow more, increasing investment and causing asset prices to rise further. The process then repeats itself such that the financial accelerator amplifies the effect of the original shock to the economy.

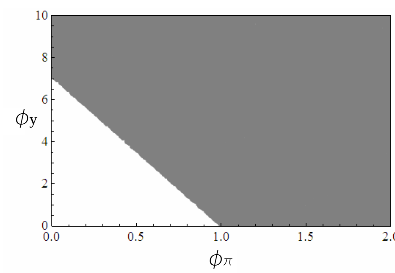


Figure 2: Credit market frictions and no reaction to asset prices

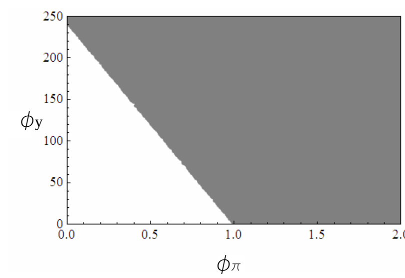


Figure 6: Financial accelerator and no reaction to asset prices

Figure 6 shows that introducing a financial accelerator significantly increases the likelihood of indeterminacy. While the substitutability between responding to inflation and the output gap still exists, it is infeasible to exploit.

Figure 7 shows the determinacy region when there is a financial accelerator effect and the central bank responds to asset prices.

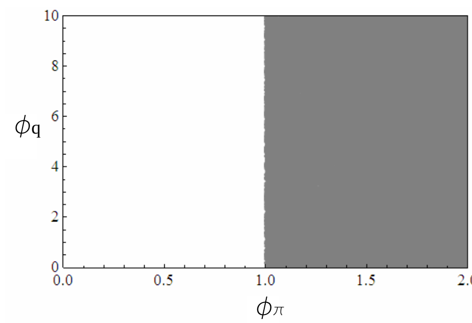


Figure 7: Financial accelerator and reaction to asset prices

This figure indicates that the amplification effects of an increase in asset prices are so large that a response to asset prices of any magnitude does not affect determinacy. Suppose asset prices increase and agents form an expectation that investment is going to increase, in response the central bank raises the interest rate. However, at the same time the increase in asset prices also increases net worth, reducing the premium for external finance causing investment to rise and asset prices to rise further. This gives rise to self-fulfilling expectations such that the likelihood of indeterminacy is much greater.

4 Conclusions

Our main finding is that a central bank will not induce additional volatility by responding to asset prices in a New Keynesian model with credit market frictions. In fact, responding to deviations in asset prices makes indeterminacy less likely. When there is a financial accelerator effect, a central bank cannot reduce the likelihood of indeterminacy by responding to asset prices. However, both results suggest that, by following a monetary policy rule which incorporates deviations in asset prices, a central bank will not induce *additional* volatility.

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A Method of decomposition

Following the Schur Decomposition, assume there exists square matrices G and H and some upper triangular matrices Λ and Ω such that:

$$GD_1 = \Lambda H' \tag{A.1}$$

$$GD_2 = \Omega H' \tag{A.2}$$

where $GG' = G'G = I$ and $HH' = H'H = I$ are unitary matrices.

$$D_1 \begin{bmatrix} P_t \\ X_t \end{bmatrix} = D_2 \begin{bmatrix} P_{t+1} \\ X_{t+1} \end{bmatrix} + E \begin{bmatrix} U_{t+1} \\ \eta_{t+1} \end{bmatrix} \tag{A.3}$$

To obtain the eigenvalues for the system the error vector can be omitted. Multiplying both sides by G yields:

$$GD_1 \begin{bmatrix} P_t \\ X_t \end{bmatrix} = GD_2 \begin{bmatrix} P_{t+1} \\ X_{t+1} \end{bmatrix} \tag{A.4}$$

Using A.1 and A.2:

$$\Lambda H' \begin{bmatrix} P_t \\ X_t \end{bmatrix} = \Omega H' \begin{bmatrix} P_{t+1} \\ X_{t+1} \end{bmatrix} \tag{A.5}$$

This can then be expressed as:

$$\Lambda \begin{bmatrix} V_t \\ W_t \end{bmatrix} = \Omega \begin{bmatrix} V_{t+1} \\ W_{t+1} \end{bmatrix} \tag{A.6}$$

where

$$\begin{bmatrix} V_t \\ W_t \end{bmatrix} = H' \begin{bmatrix} P_t \\ X_t \end{bmatrix} \tag{A.7}$$

The diagonal entries of $\Lambda^{-1}\Omega$ are the eigenvalues for the system such that:

$$\Lambda^{-1}\Omega = \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ 0 & \lambda_{22} \end{pmatrix}^{-1} \begin{pmatrix} \omega_{11} & \omega_{12} \\ 0 & \omega_{22} \end{pmatrix} \tag{A.8}$$

where $\Lambda^{-1}\Omega$ is an upper triangular matrix. Therefore, the eigenvalues are given by ω_{ii}/λ_{ii}.