Why Do Similar Countries Choose Different Policies ? Endogenous Comparative Advantage and Welfare Gains*

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Abstract

Similar countries often choose very different policies and specialize in very distinct industries. This paper proposes a mechanism to explain policy diversity among similar countries from an open economy perspective. I study optimal policies in a two country model when policies affect determinants of trade patterns. I show that welfare gains from trade can provide sufficient incentive for asymmetric equilibrium policies, even if the two countries have identical economic fundamentals. Any asymmetric equilibrium exhibits greater production specialization than the common autarky optimum; this is the source of welfare gains. For this same reason, a more asymmetric Nash equilibrium Pareto dominates a less asymmetric one. I characterize necessary and sufficient conditions for existence of such asymmetric equilibrium. All equilibria are asymmetric if income is sufficiently convex in policy, and production technologies and consumer preferences are not strongly biased in favor of one of the factors of production. As an application, I consider a model where skill distribution is the determinant of trade patterns and the policy in question is education policy. When heterogeneous agents choose their skill levels optimally, optimal skill function is convex in government policy. In this application, the general sufficient condition requires that the education cost of agents is relatively inelastic with respect to skill. (JEL Classification: F11, E62.)

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1 Introduction

National policies of countries display important and persistent differences. Countries that are significantly different in terms of various socioeconomic institutions choose different policies, but such observed diversity can be explained in terms of differences in economic fundamentals. However, significant policy diversity is observed even among countries that are similar in terms of factor endowments, technology and general levels of economic progress. For example, the economic development strategy of China is characterized by a high rate of physical capital accumulation and an emphasis on basic education whereas India has instead emphasized tertiary education relatively more.[1] Also, such similar countries often tend to specialize in different industries and have experienced massive growth in international trade.[2] If we consider policies that affect the determinants of trade patterns, choice of diverse policies allow countries to specialize in different industries and gain from international trade.

In fact, many countries emphasize the international comparative advantage they derive from domestic policies in an increasingly integrated world. The education policy of India serves as a case in point. International competitiveness of the knowledge-intensive service sector is an important factor in formulating education policy in India.[3] In the formulation of the national education policy, policy-makers in India take into account details of the domestic policies of other large economies.[4,5] Thus, as opposed to each country choosing in isolation what policy is best for itself, countries interact in the open economy in their optimal design of national policies that affect international comparative advantage. In this paper I consider countries that are completely identical in terms of economic fundamentals to isolate the role of policy diversity in encouraging international trade, and study an optimal policy problem in the open economy in a general equilibrium model of trade. I explore whether welfare gains from trade can explain ex-post policy heterogeneity among ex-ante identical countries.

In section 2, I consider a two-good, two-factor, two-country model in which both good and factor

[1] IMF World Economic Outlook (2006), Bosworth and Collins (2008), Panagariya (2006), and He-Kuijis (2007) highlight important policy differences between India and China. Arora and Gambardella (2005, Chapter 3) discuss differences in economic policies between Ireland and Greece or Spain. Important differences in economic policies between USA and Europe have received considerable attention in the literature (Alesina, Glaeser and Sacerdote (2001), Krueger and Kumar (2004)).

[2] Bosworth and Collins (2008, Table 6, page 20) report that between 1995 and 2004 annual growth in exports (in goods and services) is 18.1% in China as opposed to 12.6% in India. Over this time, share of goods in exports has increased from 87% to 90% for China, and has reduced from 82% to 67% in India. Similar specialization is observed in other large countries in the world. Baumol and Gomory (2000) document that for the three largest economies– Germany, Japan and the United States– the correlations (and rank correlations) in production shares by industry are either negative or close to zero, and the cross-industry pattern of specialization is remarkably stable.

[3] The Indian task force report on Human Resource Development in Information Technology (2000) makes 47 specific recommendations with a view "to create a sustainable competitive advantage" in the knowledge-led businesses.

[4] For example, the Indian task force report (2000) recommends training in the language-cultural skills of potential export destinations like Japan, Germany, France and Korea in the regular engineering programs, citing China's current emphasis on English education.

[5] Ireland is the largest exporter of knowledge-intensive software goods and services in the world economy (OECD Information Technology Outlook, 2000). A very similar picture of the education policy of Ireland emerges from the Human Capital Priority program of Ireland's National Development Plan (2007).

markets are perfectly competitive. The planner in each country chooses a single policy that affects productivity differently for different goods.[6] For example, the policy in question affects relative technological progress across sectors or factor composition of a country. A difference in government policy is the only source of comparative advantage. The framework, at this level of generality, can handle any policy problem as long as the policy in question affects comparative advantage in the open economy.[7]

Given identical homothetic demand, policy affects comparative advantage from the supply side of the economy via its influence on domestic production possibilities. By varying the policy, I can define a production possibility set of an economy as an upper envelope of various production possibility sets, each corresponding to a different policy choice. Constant returns to scale technologies imply that for any given policy, the production possibility set is a convex set. However, the upper envelope of different convex sets is not necessarily a convex set. This nonconvexity in the envelope production possibility set is crucial for the existence of asymmetric policies in an equilibrium.

In the non - cooperative optimal - policy problem, the social planner in each country chooses national policy to maximize aggregate welfare. If these countries choose different policies, they gain from trade. This is the gain from asymmetry. Both countries have identical optimal policies in autarky and at least one of them has to deviate from the autarky optimal policy in order to trade. Hence, gains from trade can only arise at the cost of choosing a policy that would be suboptimal in the absence of trade opportunities. This is the cost of asymmetry. If the envelope production possibility set is sufficiently nonconvex, and consumer preferences are not very biased in favor of one of the goods, an asymmetric pure - strategy Nash Equilibrium (PSNE) exists.

In an asymmetric equilibrium, ex-ante identical countries are ex-post different with endogenous comparative advantage in different industries. An increase in policy diversity improves the scope for production specialization and exchange. Hence, both countries attain higher aggregate welfare in an equilibrium with greater asymmetry. In particular, any asymmetric PSNE is associated with higher aggregate welfare compared to the common autarky optimum. As a result, all cooperative equilibria are asymmetric whenever an asymmetric PSNE exists. I also explore the cooperative optimal - policy problem to understand when the Pareto optimum of a symmetric world is asymmetric. I show that all equilibria in the open economy are asymmetric if aggregate production is sufficiently convex in government policy, under suitable restrictions on consumer preferences.[8]

I illustrate the general abstract policy problem in a specific example. Educational investment by the government improves skills. The government policy decision is to allocate resources between ba-

[6] Clarida and Findlay (1991) consider a similar type of policy.

[7] A distinguished branch of international trade theory literature predicts comparative advantage on the basis of differences in factor endowment, technology, and domestic institutions (Levchenko (2007), Costinot (2009), Cunat and Melitz (2007)). Based on this literature many different supply-side policies, such as domestic institutional reforms, labor market policies, R & D policies, education policy, and capital accumulation policy satisfy my requirement.

[8] Note that when the production set is convex, the frontier of the production set plotted in output space is concave, and vice versa. Thus, convexity in aggregate production with respect to policy is intuitively similar to nonconvexity of the envelope production possibility set.

sic and higher education.[9] I consider three specifications of competitive economies to demonstrate different restrictions needed to satisfy the condition for existence of an asymmetric equilibrium in the general framework. The setup in section 3.1 is a classic Heckscher-Ohlin economy consisting of two goods with different skill intensities and two types of workers with different inherent skill levels. In this economy with standard production functions, an asymmetric equilibrium exists only if the government policy affects skills in a convex fashion. If government policy affects skills in a sufficiently convex fashion, all equilibria in the open economy are asymmetric under suitable restrictions on consumer preferences. In the presence of redistributive concerns, an asymmetric equilibrium exists under similar conditions, provided both political preferences and consumer preferences are not biased towards the same sector. In section 3.2, I consider a Heckscher-Ohlin model with endogenous skill choice by agents with heterogeneous abilities. From agents' optimality conditions, the optimal skill function is convex in government policy, even if government policy affects education cost in a linear fashion. If cost of education is relatively inelastic with respect to skill, an asymmetric equilibrium exists under suitable restrictions on technologies and preferences. In section 3.3, I consider another setup in which different workers' skill levels complement or substitute each other in the production of different goods, as studied by Grossman and Maggi (2000). In the presence of submodularity in production in this setup, even if policy affects skills in a linear fashion, aggregate production may respond in a convex manner to policy. An asymmetric equilibrium exists if the degree of submodularity in production is relatively high, provided consumers prefer the supermodular good relatively strongly.

In section 4, I allow for some initial differences among countries in my setup of section 3.1 and show that, in this case, these countries optimally choose to magnify initial differences by investing relatively more in their respective areas of comparative advantage. But in the absence of sufficient convexity in the skill function, equilibrium difference in the policies depends crucially on the magnitude of initial differences. Thus, convexity in aggregate production with respect to policy is crucial to explain substantial policy differences among similar countries via welfare gains from trade.

The analysis in this paper challenges the general assumption that globalization can be expected to exert pressure for convergence in the cross-national economic policies towards a common optimum. This is an assumption shared by scholars in comparative political science and economics (Cerny, 1997; Garrett, 1998; Kitschelt et al., 1999; Berger and Dore, 1996). However, focusing on one particular aspect of globalization, trade integration, diversity in domestic policy can arise and amplify in an open economy equilibrium, if comparative advantage is at least partly endogenous to national policy. The difference between the usual assumption in the literature and the classical trade theory point of view outlined in this paper is that the institutions and policies (of the type studied here) are not absolutely good or bad but rather are relatively good or bad for different

[9] Ansell (2003, 2004, 2008) empirically documents the rising public education expenditure with increase in economic openness and how economic openness pushes the balance between tertiary and primary education towards states' particular comparative advantage from a panel dataset covering more than hundred countries and a time span of 1960-2000.

types of production activities.

Related literature: Traditionally, the question of why similar countries choose different policies is studied in a closed economy setting in which the existence of multiple equilibria and coordination failure explains such observed diversity.[10] However, the mere existence of different types of equilibrium in a closed economy does not imply that two symmetric countries choose asymmetric policies in an equilibrium. In this sense, multiple equilibria and coordination failure are consistent with the existence of diversity but do not explain it. In contrast in an open economy setup the identical countries may choose diverse policies precisely because such policies allow them to gain from trade. Directly related to this explanation of policy diversity among similar countries is the "symmetry-breaking" approach pioneered by Matsuyama (2002) in which existence of an asymmetric equilibrium in a symmetric strategic setting is offered as an explanation for observed diversity among identical agents.[11] The literature on asymmetric equilibrium in symmetric games (Matsuyama (2002), Amir, Garcia and Knauff (2006)) proposes properties of the payoff function sufficient for symmetry-breaking. However, in my general optimal policy problem when the payoff function is an aggregate indirect utility function which incorporates agents' optimization, the existing sufficient conditions for symmetry-breaking are not satisfied. In the context of this literature my contribution is to characterize new necessary and sufficient conditions for the general optimal policy problem.

Another distinct branch of related literature studies the effects of globalization on domestic policy and institutions.[12] However, from the literature on comparative advantage, domestic factor endowments, technology and institutions also affect volumes of trade and influence trade patterns. If domestic policies affect these determinants of trade, a benevolent social planner should take that effect into consideration in designing optimal policy.[13] I, for the first time in the literature, incorporate a comparative - advantage motive for domestic policy in an open economy optimal policy problem.

In sum, this paper makes two distinct contributions. First, it provides a new answer to the well-researched question, why do similar countries choose very different policies, from an open-economy perspective in a simple, tractable and general framework. Second, I exploit properties of a standard general equilibrium model of international trade to characterize the conditions for and welfare-implications of asymmetric equilibrium in a symmetric optimal policy problem.

[10] See Alesina and Angeletos (2005), Benabou and Tirole (2006) for example.

[11] See Matsuyama (2004), Maggi (1996), Amir (1996) for important applications of the symmetry-breaking approach.

[12] See Brügemann (2003), Levchenko and Do (2007a) for references in economics literature and Watson (2003), Hall and Soskice (2001), Mosher and Franzese (2001, 2002)) in comparative political economy literature.

[13] Gomory and Baumol (2000) emphasize acquired comparative advantage and mention several examples of diverse economic policies of countries aimed at promoting comparative advantage in different industries. See Redding (1999) for an example of the role of government policy in acquiring comparative advantage.

2 Optimal Policy in a General Framework

I consider a two-good, two-factor, perfectly competitive world comprising two large economies. Each government chooses a domestic policy γ that is the only source of comparative advantage in the open economy. Without loss of generality, let an increase in γ confer a comparative advantage in good 1.

Let u(.) denote the direct utility function, p(.) the relative price of good 1 and Y(.) the aggregate income in the country. Government policy γ affects welfare directly through its effects on economic fundamentals and indirectly through the equilibrium price. Aggregate indirect utility is denoted by $V(p,\gamma)$. Also, $F_i(p, \gamma)$ stands for aggregate production of good i and $C_i(p, \gamma)$ denotes the aggregate demand of good i. From the equality of aggregate value of production and aggregate income,

$$Y(p,\ \gamma) = pF_1(p,\ \gamma) + F_2(p,\ \gamma). \tag{1}$$

I assume homothetic preferences which ensure that aggregate demand of each good is linearly homogenous in income,

$$C_i(p,\ \gamma) = f_i(p)Y(p,\ \gamma). \tag{2}$$

From the definition of aggregate welfare and homotheticity of u(.),

$$V(p,\gamma) = f(p)Y(.),$$

where $f(p) \equiv u(f_1(p),\ f_2(p))$. I denote autarky variables by a superscript A and trade variables by a superscript T. All foreign country variables are designated by an asterisk.

In the competitive equilibrium of the closed economy,

$$C_i(p^A,\ \gamma) = F_i(p^A,\ \gamma)\ ,\ i = 1, 2, \tag{3}$$

and hence the equilibrium price, p^A, depends only on own policy γ. In the open economy both countries' policies determine the equilibrium terms-of-trade $p^T(.)$ from goods market clearing of the world,

$$C_i(p^T,\ \gamma) + C_i^*(p^T,\ \gamma^*) = F_i(p^T,\ \gamma) + F_i^*(p^T,\ \gamma^*) \tag{4}$$

and the foreign policy γ^* affects domestic welfare through the terms-of-trade externality. Hence,

$$U^A(\gamma) \equiv V(p^A(\gamma), \gamma),\ \text{and}\ U^T(\gamma, \gamma^*) \equiv V(p^T(\gamma,\ \gamma^*), \gamma).$$

I assume that $U^A(\gamma)$ and $U^T(\gamma,\ \gamma^*)$ are differentiable upto second order. The world welfare is denoted by $W(.)$,

$$W(\gamma, \gamma^*) \equiv U^T(\gamma, \gamma^*) + U^T(\gamma^*, \gamma).$$

In closed economy government chooses γ to maximize aggregate welfare, $U^A(\gamma)$. In the non - cooperative optimal - policy problem in the open economy, each government maximizes $U^T(\gamma, \gamma^*)$ taking the other country's policy as given. In the cooperative optimal - policy problem in the open economy, the social planner maximizes the world welfare, $W(\gamma, \gamma^*)$.

Suppose that policy γ lies in a bounded policy space $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$. When there is more than one autarky optimum policy, at least one asymmetric equilibrium exists in both the non - cooperative and cooperative optimal - policy problem of the open economy. Hence, I restrict attention to the case where the autarky problem has a unique optimum, $\widetilde{\gamma}$.

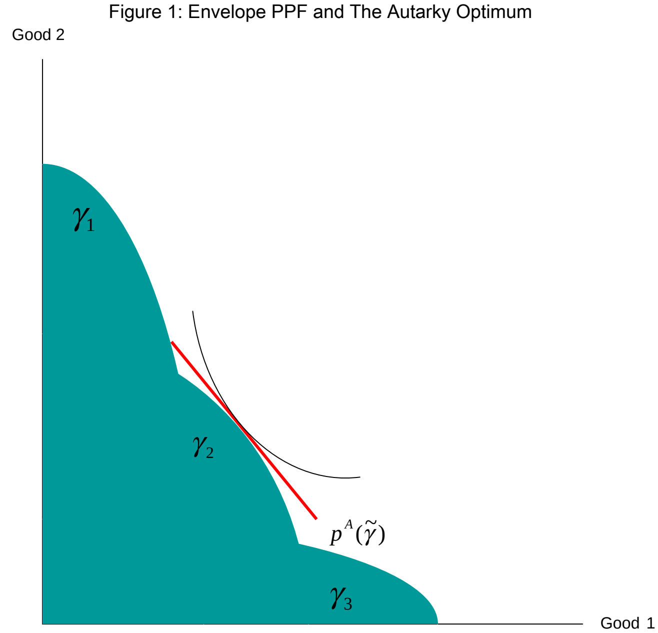
I interpret γ as a policy that affects sector-specific technologies or the relative factor endowments. For any given γ, one can define the production possibility frontier as the frontier of the output pairs that can be produced using the economy's available technologies and factor inputs. Let us denote this set as PPF(γ). A change in γ affects the production possibility frontier. Let us define the upper envelope of PPF(γ) as the PPF,

$$PPF = \max_{\gamma} PPF(\gamma). \tag{5}$$

When γ is a choice variable of the government, PPF describes the frontier of the true production possibilities of a country.

In Figure 1 for illustration purposes I consider three possible policy options $\gamma_1 < \gamma_2 < \gamma_3$. The shaded region denotes the envelope PPF. For the set of consumer preferences described in Figure 1, the autarky optimal policy $\widetilde{\gamma}$ is given by γ_2.

Figure 1: Envelope PPF and The Autarky Optimum



In the open-economy the aggregate welfare function satisfies the gains from trade property. By this property, a country with a given policy, can gain in welfare terms by trading with a partner with a different policy,

$$U^T(\gamma,\ \gamma^*) \geqslant U^T(\gamma,\ \gamma)\ \forall\gamma^*,\ \forall\gamma, \tag{6}$$

with equality at $\gamma = \gamma^*$. From the gains from trade property, for any two arbitrary policies γ_i and γ_j , the relative welfare of choosing γ_i compared to γ_j , conditional on the trading partner choosing γ_j , improves in the open economy over that in autarky.[14] In this sense, trade favors asymmetry in policy. But when do these symmetric countries choose asymmetric policies in the open economy?

To answer this question, it is important to understand an important property of asymmetric equilibrium. Suppose that $(\gamma',\ \gamma'^{*})$ is an asymmetric pure strategy Nash equilibrium. By (6),

$$U^T(\widetilde{\gamma},\ \widetilde{\gamma}) \leq U^T(\widetilde{\gamma},\ \gamma'^{*}).$$

But since γ' is the best response to γ'^{*},

$$U^T(\widetilde{\gamma},\ \gamma'^{*}) \leq U^T(\gamma',\ \gamma'^{*}).$$

Thus, at any asymmetric PSNE both countries gain in aggregate welfare compared to the common autarky optimum. Note that this result is independent of the countries experiencing any price change at the open economy asymmetric equilibrium compared to the autarky optimum.[15] Here the welfare gain in the asymmetric equilibrium is due to an increase in production specialization, which allows for an expansion of the consumption possibility frontier through trade.

In this framework with identical homothetic demands in both countries, comparative-advantage in any industry arises from the supply side. Thus, the role of policy γ in affecting production possibilities of a country is crucial for the existence of an asymmetric equilibrium with endogenous comparative advantage in different industries. In a perfectly competitive world with constant returns to scale technologies, the production set for any given γ, PPF(γ), is a convex set. However, constant returns to scale technologies do not imply that the production set described by PPF is a convex set. If the production set described by PPF is a convex set, no two different points on the envelope PPF can satisfy producers' optimality condition at the same free trade price, and hence an asymmetric PSNE does not exist.[16] Below I illustrate graphically the importance of nonconvexity of PPF for the existence of an asymmetric equilibrium.

As before, suppose that there are three possible policy options $\gamma_1 < \gamma_2 < \gamma_3$, and $\widetilde{\gamma} = \gamma_2$. I ask, when can $(\gamma_1,\ \gamma_3)$ be a Pareto-improving asymmetric NE in the open economy? For illustration purposes, I completely shut down the traditional channel of gains from trade due to price movement from autarky to free trade. Hence, the free trade price at the asymmetric equilibrium is the same as the autarky price at $\widetilde{\gamma}$. In Figure 2, a Pareto-improving asymmetric NE exists since the production

[14] From (6), we know that

$$\frac{U^T(\gamma_i, \gamma_j)}{U^T(\gamma_j\ , \gamma_j\ )} > \frac{U^A(\gamma_i)}{U^A(\gamma_j\ )}.$$

[15] The traditional gains from trade derive from the fact that countries face different prices in the open economy compared to in autarky.

[16] Note that this result is true irrespective of the nature of political preferences.

possibilities described by the PPF is a sufficiently nonconvex set. In the open economy, a country that chooses $\gamma_3 > \gamma_2$, does not suffer a major adverse terms-of-trade movement, since in the open economy equilibrium price is less responsive to any country's policy movements.[17] Thus by specializing in two distinct industries each country realizes an expansion in its consumption possibility frontier. This opens the door for welfare gains in an asymmetric equilibrium in the open economy. In contrast, in Figure 3 when the production possibility set does not show enough nonconvexity, no such asymmetric PSNE exists.

Figure 2: Envelope PPF and An Asymmetric Equilibrium

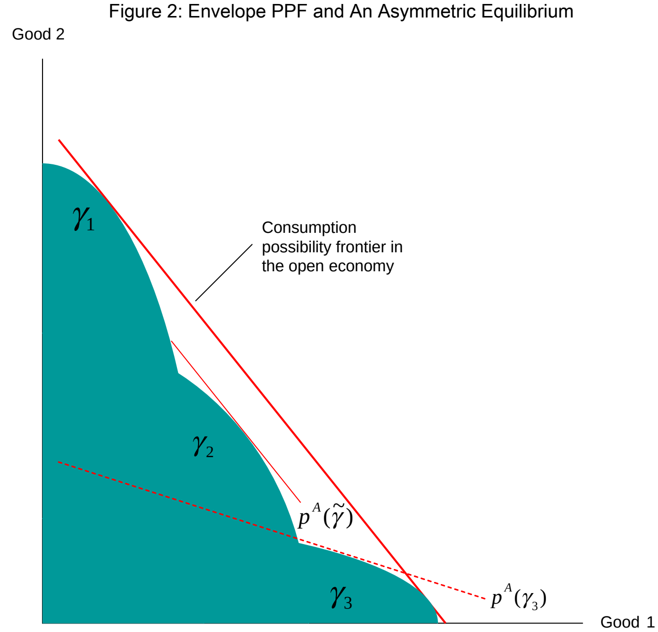
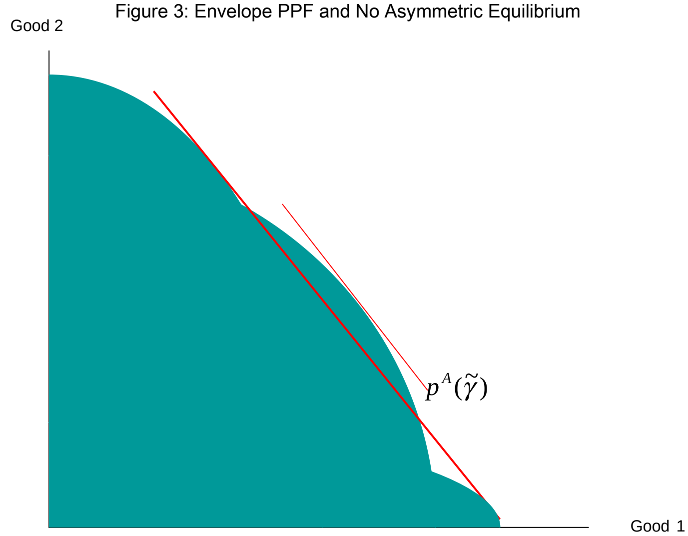


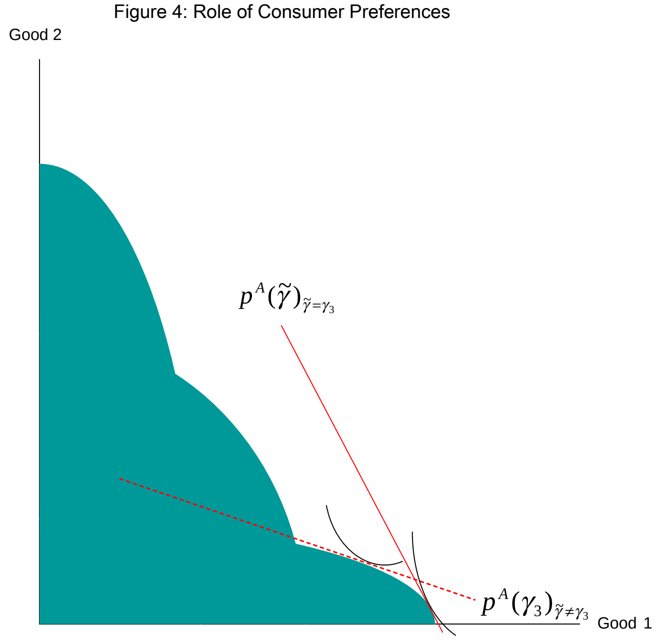
Figure 3: Envelope PPF and No Asymmetric Equilibrium



[17] This is due to two reasons. First, $\frac{\partial p^T}{\partial \gamma} = .5\frac{dp^A}{d\gamma}$ at a point of symmetry. Thus change in own policy affects equilibrium price relatively less in the open economy. Moreover, in the asymmetric equilibrium the foreign country's choice of policy actually improves the TOT for the home country.

However, a sufficiently nonconvex production possibility set alone is not sufficient for the existence of an asymmetric PSNE. In Figure 4, I consider a situation where the PPF is same as in Figure 2, and consumer preferences are significantly biased towards good 1. This makes γ_3 the relevant autarky optimal policy. This presence of significant bias in consumption implies that even by choosing a higher γ, the relative price of good 1 is not very low. The dotted line corresponds to $p^A(\gamma_3)_{\widetilde{\gamma}\neq\gamma_3}$ under consumer preferences in Figure 2 and the solid line corresponds to $p^A(\gamma_3)_{\widetilde{\gamma}=\gamma_3}$ under consumer preferences biased towards good 1. Note that for such biased consumer preferences there is no asymmetric equilibrium in the open economy, and γ_3 becomes the dominant strategy.

Figure 4: Role of Consumer Preferences



The figures, though great tools for illustration, leave a number of questions unanswered. Why is (γ_1, γ_1) not a symmetric equilibrium ? If (γ_1, γ_2) and (γ_1, γ_3) are both equilibria, what can we say about different properties of these equilibria? Do the existence conditions with three policies generalize? To answer these question, I consider a more general, bounded policy space.

First, I study the cooperative optimal - policy problem. It is straightforward to show that $(\widetilde{\gamma}, \widetilde{\gamma})$ is the only candidate for a symmetric Pareto optimum. If $\widetilde{\gamma}$ lies in the interior of the policy space, all the Pareto optima are asymmetric if $W(\gamma, \gamma^*)$ is quasiconvex in (γ, γ^*) at $(\widetilde{\gamma}, \widetilde{\gamma})$. I summarize these properties of the cooperative solution in the following proposition.

Proposition 1 (Cooperative Welfare Maximization) *A symmetric strategy profile in which both countries choose the autarky optimum, $(\widetilde{\gamma}, \widetilde{\gamma})$, is the only candidate for a symmetric Pareto optimum. If $\widetilde{\gamma}$ lies in the interior of the policy space and $W(\gamma, \gamma^*)$ is quasiconvex in (γ, γ^*) at $(\widetilde{\gamma}, \widetilde{\gamma})$, all the Pareto optima are asymmetric.*

Proof. The result that $(\widetilde{\gamma}, \widetilde{\gamma})$ is the only candidate for a symmetric Pareto optimum follows directly from the symmetry of the setup which ensures $W(\gamma, \gamma) = 2U^A(\gamma)$, and optimality of $\widetilde{\gamma}$.

If $\widetilde{\gamma}$ is an interior autarky optimum, first order condition (FOC) of maximization must be satisfied at $\widetilde{\gamma}$. From the definition of $U^A(.)$,

$$\frac{dU^A(.)}{d\gamma} = \frac{\partial V}{\partial p}\frac{\partial p^A}{\partial \gamma} + \frac{\partial V}{\partial \gamma}.$$

From Roy's identity,

$$\frac{\partial V}{\partial p} = f(p^A)(F_1(.) - C_1(.)),$$

which equals zero from (3). Therefore,

$$\frac{dU^A(.)}{d\gamma} = \frac{\partial V}{\partial \gamma} = 0 \text{ at } \widetilde{\gamma} \text{ from the FOC.}$$

Similarly from the definition of $W(.)$ and using (4),

$$\frac{\partial W(.)}{\partial \gamma} = \frac{\partial V}{\partial \gamma}.$$

Therefore, the FOC of optimization of the world welfare is satisfied at $(\widetilde{\gamma},\, \widetilde{\gamma})$. If $W(\gamma, \gamma^*)$ is quasiconvex in $(\gamma,\, \gamma^*)$ at $(\widetilde{\gamma},\, \widetilde{\gamma})$, every Pareto optimum is asymmetric since $(\widetilde{\gamma},\, \widetilde{\gamma})$ is the unique candidate for symmetric Pareto optimum. ■

Next, I consider the non - cooperative optimal policy problem. Each government simultaneously chooses its policy to maximize aggregate welfare. The optimal policy of a country depends on the policy of its trading partner. I restrict attention to the pure strategy Nash equilibrium (PSNE), and assume that an equilibrium exists in the policy game.[18]

Welfare properties of different asymmetric PSNEs follow from a simple generalization of the gains from trade property. Not only does a country gain in welfare by trading with a partner who has a different policy, but a country gains more from trade, given her own policy, the more different is the policy in the trading partner.[19] I summarize this property of the welfare function in the following lemma.

Lemma 2 (Greater the Difference, Greater the Gains) *For any given own policy, the welfare of a country increases with an increase in the foreign policy, if the foreign policy is greater than the own policy. Thus,*

$$\frac{\partial U^T(\gamma, \gamma^*)}{\partial \gamma^*} > 0 \; for \; \gamma^* > \gamma.$$

Similarly,

$$\frac{\partial U^T(\gamma, \gamma^*)}{\partial \gamma^*} < \; 0 \; for \; \gamma^* < \gamma.$$

[18] In an application in the next section, the optimal policy of a country decreases in the policy of the other country. Such strategic substitutability ensures the existence of at least one PSNE in the optimal policy problem by Topkis (1978).

[19] Ethier (2008) highlights a similar result "the greater the differences , the greater the gains" in comparative advantage-driven trade.

Proof. Since γ^* affects welfare of the home country through terms-of-trade,

$$\frac{\partial U^T(\gamma, \gamma^*)}{\partial \gamma^*} = \frac{\partial V}{\partial p} \frac{\partial p^T}{\partial \gamma^*}.$$

From Roy's identity,

$$\frac{\partial U^T(\gamma, \gamma^*)}{\partial \gamma^*} = f(p^T)(F_1(.) - C_1(.)) \frac{\partial p^T}{\partial \gamma^*}.$$

Since an increase in γ confer comparative advantage in good 1,

$$\frac{\partial p^T(\gamma,\ \gamma^*)}{\partial \gamma^*} < 0.$$

Suppose $\gamma^* > \gamma$, which implies that the home country is an importer of good 1,

$$F_1 < C_1, \text{ and } \frac{\partial U(\gamma,\ \gamma^*)}{\delta \gamma^*} > 0.$$

Similarly, $\gamma^* < \gamma$ implies $\frac{\partial U(\gamma,\ \gamma^*)}{\delta \gamma^*} < 0$. A similar proof works if an increase in γ confer comparative advantage in good 2. ■

This property of the welfare function simply says that given a country's own policy, the greater is the difference with the trading partner, the greater are the welfare gains. This generic property of comparative-advantage driven trade and the definition of NE allow us to rank various asymmetric PSNEs in terms of the associated welfare. Consider two asymmetric PSNEs (γ_1, γ_1^*) and (γ_2, γ_2^*) such that the home country is an exporter of good 1 in both of these equilibria ($\gamma_i > \gamma_i^*$, $i = 1, 2$). Also, the countries are more different in the first PSNE than in the second one, $\gamma_1^* < \gamma_2^* < \gamma_2 < \gamma_1$. Both countries attain a higher welfare in (γ_1, γ_1^*) compared to (γ_2, γ_2^*). Hence, an equilibrium with greater asymmetry generates greater welfare for both countries. I describe this property in Proposition 3. Proposition 3 provides a welfare-ranking of multiple PSNEs on any given side of the diagonal of the strategy space.

Proposition 3 (Welfare Ranking of Asymmetric PSNEs) *Consider two pairs of asymmetric PSNEs – $(\gamma_1,\gamma_1^*), (\gamma_1^*, \gamma_1)$ and $(\gamma_2, \gamma_2^*), (\gamma_2^*, \gamma_2)$ such that $\gamma_1^* < \gamma_2^* < \gamma_2 < \gamma_1$. Both countries have a higher welfare at (γ_1, γ_1^*) compared to (γ_2, γ_2^*) and at (γ_1^*, γ_1) compared to (γ_2^*, γ_2).*

Proof. Given $\gamma_1^* < \gamma_2^* < \gamma_2$ and $\frac{\partial U^T(\gamma,\gamma^*)}{\partial \gamma^*} < 0$ for $\gamma > \gamma^*$, (by Lemma 2),

$$U^T(\gamma_2, \gamma_2^*) < U^T(\gamma_2, \gamma_1^*).$$

But,

$$U^T(\gamma_2, \gamma_1^*) < U^T(\gamma_1, \gamma_1^*)$$

since γ_1 is the best response to γ_1^*.

Similarly, given $\gamma_2^* < \gamma_2 < \gamma_1$ and $\frac{\partial U^T(\gamma^*,\ \gamma)}{\partial \gamma} > 0$ for $\gamma^* < \gamma$,

$$U^T(\gamma_2^*, \gamma_2) < U^T(\gamma_2^*, \gamma_1).$$

But γ_1^* is the best response to γ_1, which implies

$$U^T(\gamma_2^*, \gamma_1) < U^T(\gamma_1^*, \gamma_1).$$

■

As a corollary of Proposition 3, whenever an asymmetric PSNE exists, the Pareto optimum is asymmetric. Alternatively, if $(\widetilde{\gamma}, \widetilde{\gamma})$ is the unique Pareto optimum, a Pareto-improving asymmetric equilibrium cannot exist. If world welfare $W(., .)$ is strictly quasiconcave, the symmetric strategy profile at $(\widetilde{\gamma}, \widetilde{\gamma})$ is the unique Pareto optimum. Hence, if the world welfare $W(., .)$ is strictly quasiconcave, an asymmetric equilibrium does not exist.

Next, I investigate properties of the welfare function that are sufficient for the existence of an asymmetric PSNE. The policy game does not satisfy the sufficient conditions for the existence of an asymmetric PSNE in a symmetric game in the literature. In Matsuyama (2002) an asymmetric PSNE exists if the symmetric PSNE is Cournot unstable. In this two-good model with the terms-of-trade externality as the only source of strategic interaction, the symmetric equilibrium, if it exists, is stable. Stability of the symmetric equilibrium follows from the substitutability of the two goods in consumption. Amir, Garcia and Knauff (2006) rule out any symmetric equilibrium since in their game the payoff function does not satisfy the necessary condition for an interior optimum at any interior point of symmetry. In this case by the gains from trade property the welfare function, $U^T(\gamma, \gamma^*)$, has slope 0 at $(\widetilde{\gamma}, \widetilde{\gamma})$.[20] Hence the sufficient condition outlined in Amir, Garcia and Knauff (2006) is not satisfied in my framework.

However, in this game the unique autarky optimum policy $\widetilde{\gamma}$ is also the unique candidate for a symmetric equilibrium. I prove this claim in the next Proposition. Any unilateral deviation from $(\widetilde{\gamma}, \widetilde{\gamma})$ comes with a gain- from- trade component and a loss- in- autarky- utility component. If the home country can profitably deviate to a $\gamma' \neq \widetilde{\gamma}$ when the partner is choosing $\widetilde{\gamma}$, only asymmetric PSNEs exist in this game. Existence of such a profitable deviation implies

$$U^T(\gamma', \ \widetilde{\gamma}) - U^A(\gamma') > U^A(\widetilde{\gamma}) - U^A(\gamma').$$

Thus, the symmetric PSNE at $(\widetilde{\gamma}, \widetilde{\gamma})$ is ruled out if the gains from trade at the strategy profile $(\gamma', \widetilde{\gamma})$ exceed the loss in autarky utility from choosing γ'.

In general, the problem of ruling out any symmetric PSNE is technically equivalent to finding a global maximum of $U^T(\gamma, \widetilde{\gamma})$ at a $\gamma' \neq \widetilde{\gamma}$, even though $\widetilde{\gamma}$ is a local maximum. The welfare function in the open economy, $U^T(\gamma, \widetilde{\gamma})$, is quasi-convex in own policy over some part of the action space for this separation of the global maximum (γ') from the autarky optimum. In fact if $U^T(\gamma, \gamma^*)$ is strictly quasiconvex in own policy and $\widetilde{\gamma}$ is an interior optimum in autarky, there is a unique pair

[20] The necessary first order condition of maximization of $U^T(\gamma, \widetilde{\gamma})$ is satisfied at $\widetilde{\gamma}$ since $U^T(\gamma, \widetilde{\gamma})$ is an upper envelop of $U^A(\gamma)$ with equality $\gamma = \widetilde{\gamma}$, where $\widetilde{\gamma}$ is the critical point of $U^A(\gamma)$.

of asymmetric NE given by the extremes $(\underline{\gamma},\ \overline{\gamma})$.and $(\overline{\gamma},\ \underline{\gamma})$.[21, 22]

Both the gain from trade, given that the foreign country is choosing the autarky optimal policy,

$$U^T(\gamma,\widetilde{\gamma}) - U^A(\gamma), \tag{7}$$

and the loss in autarky utility by choosing an autarky suboptimal policy,

$$U^A(\widetilde{\gamma}) - U^A(\gamma) \tag{8}$$

are positive for all γ, given (6) and the optimality of $\widetilde{\gamma}$. Also, both functions attain a global interior minimum value of zero at $\gamma = \widetilde{\gamma}$. Hence, both of these curves are quasiconvex in the neighborhood of $\gamma = \widetilde{\gamma}$. If every policy $\gamma^{'} \neq \widetilde{\gamma}$ constitutes a profitable unilateral deviation from $(\widetilde{\gamma},\widetilde{\gamma})$, (7) lies above (8) for all $\gamma^{'} \neq \widetilde{\gamma}$ and only asymmetric PSNEs exist. This situation is illustrated in Figure 5a. If there exists a policy $\gamma^{''} \neq \widetilde{\gamma}$, $\gamma^{''} \in (\underline{\gamma},\overline{\gamma})$ such that the net gain to unilaterally deviating from $(\widetilde{\gamma},\widetilde{\gamma})$ is zero,

$$U^T(\gamma^{''},\widetilde{\gamma}) - U^A(\gamma^{''}) = U^A(\widetilde{\gamma}) - U^A(\gamma^{''}),$$

only asymmetric PSNEs exist if (7) is steeper than (8) at $\gamma^{''}$,

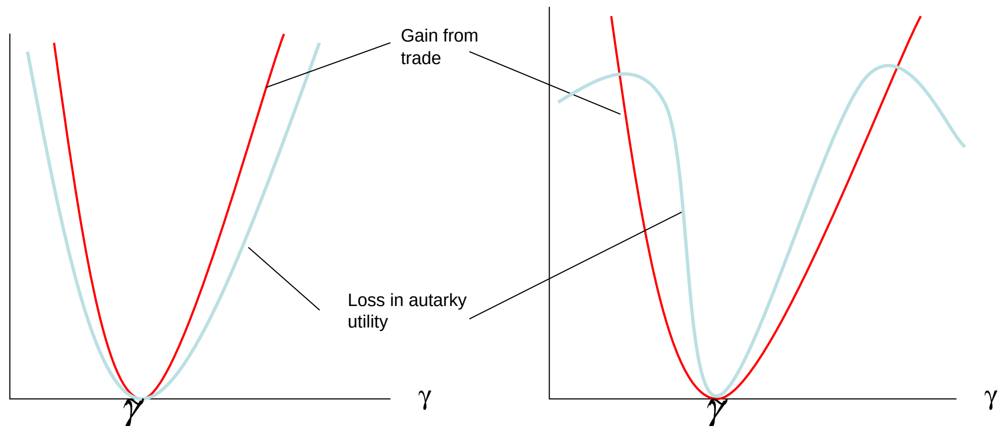
$$\left|\frac{\partial(U^T(\gamma^{''},\widetilde{\gamma}) - U^A(\gamma^{''}))}{\partial\gamma}\right| > \left|\frac{\partial(U^A(\widetilde{\gamma}) - U^A(\gamma^{''}))}{\partial\gamma}\right|. \tag{9}$$

This situation is illustrated in Figure 5b. I summarize the results for the non - cooperative optimal - policy problem in the next proposition.

[21] Because the game is symmetric, any asymmetric PSNE appears in pairs. When there are only asymmetric PSNEs in this game, the total number of PSNEs is even. In a game with continuum action space, usually there are an odd number of PSNEs by Wilson's Oddness Theorem (1971). This result is based on the degree theory and requires continuity of the best response form. Ruling out symmetric equilibrium in this game involves a robust jump of the best replies across the diagonal of the strategy space. Hence, my results are consistent with the Wilson's Oddness Theorem (1971).

[22] In this setup a symmetric mixed strategy NE always exists, by the Folk Theorem (Dasgupta and Maskin 1986). However, in a game characterized by strategic substitutability, MSNE is usually unstable. Also, in my case countries attain greater welfare in any asymmetric PSNE compared to a symmetric MSNE. Moreover, it is standard in the policy literature to focus on the PSNE as the relevant solution concept.

Figure 5a Figure 5b



Proposition 4 (Non - cooperative Welfare Maximization) *A symmetric strategy profile in which both countries choose the autarky optimum, $(\widetilde{\gamma},\ \widetilde{\gamma})$, is the only candidate for a symmetric PSNE. Suppose that $\widetilde{\gamma}$ is an interior optimum. If $W(\gamma, \gamma^{*})$ is strictly quasiconcave, an asymmetric PSNE does not exist. If $U^{T}(\gamma,\ \gamma^{*})$ is strictly quasiconvex in own strategy, there is a unique pair of asymmetric NE given by the extremes $(\underline{\gamma},\ \overline{\gamma})$.and $(\overline{\gamma},\ \underline{\gamma})$.*

Proof. Suppose that $(\widehat{\gamma},\ \widehat{\gamma})$, $\widehat{\gamma} \neq \widetilde{\gamma}$ is a PSNE. The home country attains a payoff of $U^{T}(\widetilde{\gamma},\ \widehat{\gamma})$ by deviating to $\widetilde{\gamma}$, given that the foreign country is choosing $\widehat{\gamma}$. The resulting change in payoff $(U^{T}(\widetilde{\gamma},\widehat{\gamma}) - U^{T}(\widehat{\gamma},\widehat{\gamma}))$ consists of a gain from trade component $((U^{T}(\widetilde{\gamma},\widehat{\gamma}) - U^{A}(\widetilde{\gamma}))$ and an increase in autarky utility $(U^{A}(\widetilde{\gamma}) - U^{A}(\widehat{\gamma}))$. Hence, the autarky optimum is a profitable unilateral deviation from the strategy profile $(\widehat{\gamma},\ \widehat{\gamma})$. Thus $(\widehat{\gamma},\ \widehat{\gamma})$, $\widehat{\gamma} \neq \widetilde{\gamma}$ cannot be a PSNE.

From the proof of Proposition 1, the strategy profile, $(\widetilde{\gamma},\ \widetilde{\gamma})$, satisfies the necessary condition for an interior maximum of $W(\gamma, \gamma^{*})$. If $W(\gamma, \gamma^{*})$ is strictly quasiconcave in $(\gamma,\ \gamma^{*})$, there is a unique interior Pareto optimum at $(\widetilde{\gamma},\ \widetilde{\gamma})$. From the Proposition 3, an asymmetric PSNE, if it exists, is a Pareto improvement over the autarky optimum. Hence, if $W(\gamma, \gamma^{*})$ is strictly quasiconcave, a Pareto improvement over $(\widetilde{\gamma},\ \widetilde{\gamma})$ is not possible. Hence, if $W(\gamma, \gamma^{*})$ is strictly quasiconcave, an asymmetric PSNE does not exist.

From the definition of strict quasiconvexity,

$$U^{T}(\lambda\gamma^{1} + (1-\lambda)\gamma^{2}) < \max\{U^{T}(\gamma^{1}),\ U^{T}(\gamma^{2})\}, \forall\gamma^{1},\ \gamma^{2} \in [\underline{\gamma},\ \overline{\gamma}]^{2}.$$

Let $\gamma^{1} = (\underline{\gamma},\ \gamma)$ and $\gamma^{2} = (\overline{\gamma},\ \gamma)$. Any $\gamma' \in (\underline{\gamma},\ \overline{\gamma})$ yields a lower pay off than $\gamma' = \underline{\gamma}$ or $\gamma' = \overline{\gamma}$, for any given foreign policy $\gamma \in [\underline{\gamma},\ \overline{\gamma}]$. Hence, $\forall\gamma \in [\underline{\gamma},\ \overline{\gamma}]$, the best response is either $\underline{\gamma}$ or $\overline{\gamma}$. Neither $\underline{\gamma}$ nor $\overline{\gamma}$ is an autarky optimum. Therefore, $\underline{\gamma}$ $(\overline{\gamma})$ is not the best response to $\underline{\gamma}$ $(\overline{\gamma})$. Thus, the best response to $\underline{\gamma}$ is $\overline{\gamma}$ and vice versa. The only two PSNEs are $(\overline{\gamma},\ \underline{\gamma})$ and $(\underline{\gamma},\ \overline{\gamma})$. ■

From proposition 4 it is clear that quasiconvexity in the welfare function with respect to own policy plays an important role for the existence of an asymmetric equilibrium. To derive analytically the sufficient and necessary conditions for the existence of asymmetric NE in terms of economic fundamentals, I adopt a cardinal interpretation of the policy, γ. Suppose that $\varpi = \frac{\varpi_1}{\varpi_2}$ is the original determinant of trade, and an increase in ϖ confers comparative advantage in good 1. Here ϖ may represent relative technology parameters of the two sectors as in a Ricardian model. Alternatively, ϖ may stand for relative endowment of two factors as in a Heckscher-Ohlin model, where sector 1 is more intensive in factor 1. To give a cardinal interpretation to γ, I consider γ as the fraction of available resources invested in ϖ_1, and $(1-\gamma)$ is the fraction invested in $\varpi_{2.}$ I show in the appendix that

$$\frac{\partial^2 Y(p^T,\gamma)}{\partial\gamma^2}|_{(\widetilde{\gamma},\widetilde{\gamma})} = (p^T\frac{\partial^2 F_1(p^T,\gamma)}{\partial\gamma^2} + \frac{\partial^2 F_2(p^T,.)}{\partial\gamma^2})|_{(\widetilde{\gamma},\widetilde{\gamma})}$$

plays a crucial role in satisfying the sufficient and necessary conditions for existence of an asymmetric NE as described in Proposition 4. In particular, given an interior $\widetilde{\gamma}$, if $\frac{\partial^2 Y(p^T,\gamma)}{\partial\gamma^2}|_{(\widetilde{\gamma},\widetilde{\gamma})}$ is sufficiently high, we can have a situation where $U^T(\gamma,\, \gamma^*)$ is quasiconvex at $(\widetilde{\gamma},\, \widetilde{\gamma})$, even though $U^A(\gamma)$ is quasiconcave at $\widetilde{\gamma}$, under suitable restrictions on technologies and preferences. This imply that all equilibria in the open economy are asymmetric. The proof makes use of the fact that the equilibrium price p^T is less responsive to changes in own policy than the equilibrium autarky price p^A. In the applications, restrictions on technologies and preferences imply that the production technologies of the goods are not very similar and consumer preferences are not very biased towards one of the goods.

Convexity in production also plays a crucial role to satisfy the necessary condition for existence of an asymmetric equilibrium. I show in the appendix that if $\frac{\partial^2 Y(p^T,\gamma)}{\partial\gamma^2} = 0$ there is no asymmetric equilibrium in the open economy. Since convexity in production with respect to policy plays such an important role for ensuring equilibrium asymmetry, I explore this property of the economy in detail in the applications.

Before considering a specific application, I discuss briefly a natural extension of the current pure welfare maximizing optimal policy problem to a situation where governments also care for redistributive equity. In such a situation different agents may receive different weights in the objective function of the policymakers.[23] Alternatively, policymakers may face political constraints in policy choice of the type $c(p,\gamma) \geqslant 0$, even if they maximize aggregate welfare.[24] The competitive economy and therefore the market clearing conditions remain unchanged. In general I denote the objective function of the government in presence of political concerns by a subscript P. The world welfare, $W_P(\gamma,\gamma^*)$ is similarly defined as the sum of $U^T_P(\gamma,\gamma^*)$ and $U^T_P(\gamma^*,\gamma)$. I continue to denote the optimal policy in absence of trade opportunities as $\widetilde{\gamma}$.

It is straightforward to show that $(\widetilde{\gamma},\, \widetilde{\gamma})$ is the only candidate for a symmetric Pareto optimum. If $\widetilde{\gamma}$ lies in the interior of the policy space, all the Pareto optima are asymmetric if $W_P(\gamma,\gamma^*)$ is quasiconvex in $(\gamma,\, \gamma^*)$ at $(\widetilde{\gamma},\, \widetilde{\gamma})$. Even in presence of redistributive concerns, convexity in $Y_P(p,\gamma)$ remains important for existence of an asymmetric equilibrium. I show in the appendix that under

[23] For example, different agents receiving different political weights may arise due to varying lobbying efforts of different special interest groups or due to some agents belonging to the majority group in a median voter model.

[24] One example of this type of constraint is if the policymaker has to ensure a minimum welfare for some special interest groups due to domestic political constraints.

reasonable conditions on the competitive economy if $\frac{\partial^2 Y_P(p^T,\gamma)}{\partial\gamma^2} = 0$ there is no asymmetric PSNE in the open economy under weighted welfare maximization.[25] This is consistent with my graphical illustration. Note that if the production possibility set described by the envelope PPF is a convex set, an asymmetric equilibrium does not exist under free trade, independent of the nature of government preferences.

However, if changes in equilibrium price affect indirect utility ($\frac{\partial V_P(p^T(\widetilde{\gamma},\widetilde{\gamma}),\widetilde{\gamma})}{\partial p^T} \neq 0$), then $(\widetilde{\gamma},\, \widetilde{\gamma})$ is not a symmetric PSNE. Under pure welfare maximization changes in equilibrium price does not affect aggregate indirect utility at a point of symmetry. However, in general this is not true in the current scenario. Again, this inefficiency of symmetric PSNE arises because in the open economy a change in own policy affects the equilibrium price less than in the absence of trade opportunities. But in the presence of political frictions $U_P^T(\gamma,\gamma^*)$ does not satisfy the gains from trade property (6) in general. As a result, there can be a symmetric PSNE $(\gamma',\, \gamma')$ while $\gamma' \neq \widetilde{\gamma}$. At any such symmetric PSNE, these countries cannot trade and the resulting welfare $U_P^A(\gamma')$ is strictly less than the welfare under optimal policy in autarky, $U_P^A(\widetilde{\gamma})$.[26] Also, in the presence of political concerns, it is no longer true that both countries attain higher social welfare at an asymmetric PSNE compared to any symmetric PSNE.[27]

3 An Application to Education Policy

I consider a specific application of the abstract general policy problem to education policy. Specifically, I suppose that governments allocate a fixed education budget to higher and primary education and that these investments complement original endowments of high and low skilled labor. I consider three specifications to illustrate how the key sufficient condition for existence of an asymmetric equilibrium is satisfied in different frameworks. In the simplest Heckscher-Ohlin model I consider a more general social welfare function to illustrate implications of redistributive concerns for this type of optimal-policy problem.

3.1 Education Policy in the Heckscher - Ohlin Model

My first application is to a canonical Heckscher-Ohlin model with two goods and two factors. The two factors are high- and low-skilled labor. Good 2 is the numeraire and p denotes the relative price of good 1. I assume that factor price equalization holds over the entire strategy space at the

[25] Specifically, under weighted welfare maximization government objective function reduces to

$$V_P(p^T(\gamma,\gamma^*),\gamma) = f(p^T(\gamma,\gamma^*))Y_P(p^T(\gamma,\gamma^*),\gamma),$$

where $Y_P(p^T(\gamma,\gamma^*),\gamma) = c_1 p^T F_1(p^T,\gamma) + c_2 F_2(p^T,.)$. Here, c_1, c_2 stand for a function of weights attached to different factors. When $c_1 > c_2$, and $\frac{\partial^2 F_1(p^T,\gamma)}{\partial p \partial\gamma} > 0$, the condition $\frac{\partial^2 Y(p^T,\gamma)}{\partial\gamma^2} = 0$ implies existence of a unique symmetric equilibrium.

[26] Note that this type of inefficient symmetric policy is unlikely to arise if governments have other instruments to address the political concerns more directly.

[27] For example, if $U_P^T(\gamma,\gamma^*)$ differs from $U^T(\gamma,\gamma^*)$ due to a political concern for increases in inequality, it is possible that one of the countries prefer a symmetric PSNE over an asymmetric PSNE because of the increase in inequality under trade.

equilibrium price for the specified set of parameters and functional forms.[28] The production and utility functions have analytically simple Cobb-Douglas forms. Here u denotes the direct utility function, $u(C_1,\, C_2) = C_1^{\mu} C_2^{1-\mu}$, where μ is the expenditure share of good 1. The Cobb-Douglas production function of good i is represented by $F_i(H_i,\, L_i) = H_i^{\eta_i} L_i^{1-\eta_i}$, where η_i denotes the share of high-skilled labor per unit cost of good i. Good 1 is relatively more intensive in high-skilled labor, $\eta_1 > \eta_{2.}$

The initial endowments of high and low skilled labor are H and L, respectively. The government has a total education budget $T = 1$, and chooses a fraction $\gamma \in [0,\, 1]$ to spend on higher education. The remainder goes to primary education. The government expenditure on higher education, γ, complements the initial endowment of high-skilled labor, H, and forms an effective endowment of high-skilled labor, H^e,

$$H^e = H g_H(\gamma).$$

An increase in the government spending increases the effective endowment, $g_H'(\gamma) > 0$, and the government can not reduce the effective endowment below the initial endowment by not spending on higher education, $g_H(\gamma) \geqslant 1$ for all γ. Similarly,

$$L^e = L g_L((1-\gamma)),$$

where $g_L(1-\gamma) \geqslant 1$ for all γ, and $g_L^{'}(1-\gamma) \geqslant 0$.

The consumers optimally choose consumption of the two goods to maximize utility, subject to the usual budget constraint, taking as given the equilibrium price, wages and government policy. The producers of a good choose how much high- and low-skilled labor to employ to maximize profit, taking as given the equilibrium price, wages and government policy. In the competitive equilibrium both producers and consumers optimize and all labor and goods markets clear.

The aggregate indirect utility is a function of the equilibrium price and the aggregate income,

$$V(p,\gamma) = \; c p^{-\mu} Y(p,\gamma).$$

Aggregate income, $Y(p,\gamma)$, is given by $w_H(p)H^e(\gamma) + w_L(p)L^e(\gamma)$, where

$$w_L(p) = \; c_h p^{\eta_2/(\eta_2-\eta_1)},\; w_H(p) = c_l p^{-(1-\eta_2)/(\eta_2-\eta_1)}.$$

In the competitive equilibrium of the closed economy only domestic policy determines the equilibrium price, where from (3),

$$p^A(\gamma) = c_p(H^e(\gamma)/L^e(\gamma))^{\eta_2-\eta_1},$$

and in the open economy both domestic and foreign policy determine the equilibrium terms-of-trade, where from (4),

[28] Factor price equalization is not necessary for our results but makes the analytical and numerical problem simpler.

$$p^T(\gamma,\ \gamma^*) = c_p((H^e(\gamma) + H^{*^e}(\gamma^*))/(L^e(\gamma) + L^{*^e}(\gamma^*)))^{\eta_2-\eta_1}.$$

Here c, c_h, c_l, and c_p denote constants that depend on the economic fundamentals.

I consider a simple social welfare function in which each government faces a trade-off between aggregate efficiency and distributive equity,

$$U^T_P(\gamma, \gamma^*) = U^T(\gamma, \gamma^*) - \lambda(U^T_H(\gamma, \gamma^*) - U^T_L(\gamma, \gamma^*)).$$

Here $U^T_H(\gamma, \gamma^*)$, and $U^T_L(\gamma, \gamma^*)$ stand for aggregate welfare of the high- and low-skilled labor, respectively. This is equivalent to a social welfare function in which different types of agents receive different political weights in the government objective function,

$$U^T_P(\gamma, \gamma^*) = (1-\lambda)U^T_H(\gamma, \gamma^*) + (1+\lambda)U^T_L(\gamma, \gamma^*),\ 1 \geqslant \lambda \geqslant -1.$$

The government's concern for redistribution is captured by the parameter λ, where $\lambda > 0$ implies that the low skilled agents receive relatively more weight in the social welfare function and vice versa.[29] The case $\lambda = 0$ represents pure welfare maximization. The social planner takes market clearing, and incentive compatibility conditions of the producers and consumers as given.

For the rest of the section, I focus on understanding conditions on economic fundamentals that ensure the existence of an asymmetric equilibrium in the open economy. In this economy, with standard production functions, the only way an asymmetric equilibrium may arise is if government policy affects the skill endowments in a sufficiently convex manner. Note that

$$\frac{\partial^2 Y(.,\gamma)}{\partial \gamma^2} = w_H(p)g''_H(\gamma) + w_L(p)g''_L(1-\gamma). \tag{10}$$

If $g_H(.)$ and $g_L(.)$ are both concave, the joint welfare $W(.,.)$ is globally concave in $(\gamma,\ \gamma^*)$. Hence, by the necessary condition discussed in Proposition 4, if $g_H(.)$ and $g_L(.)$ are both concave an asymmetric PSNE does not exist. If $g''_t(.)$ is sufficiently high, $Y(.,\gamma)$ is sufficiently convex in policy. However, as I discussed in section 2, sufficiently convex $Y(.,\gamma)$ alone is not sufficient for existence of an asymmetric equilibrium.

What restrictions do we need on the rest of the parameters for the existence of an asymmetric equilibrium? Let us define the relative welfare under a policy γ' compared to γ'', given that the foreign country is choosing γ^*, as

$$r^T(\gamma',\ \gamma'',\ \gamma^*) = \frac{U^T_P(\gamma', \gamma^*)}{U^T_P(\gamma'', \gamma^*)}. \tag{11}$$

[29] This simple generalization from pure welfare maximization can be justified by political economy considerations. In a standard voting model, $\lambda > 0$ may arise from a positively skewed skill distribution in a setting in which every agent has equal voting power. On the contrary $\lambda < 0$ can be justified by political economy considerations if the high skilled agents are the political elites with relatively higher voting power, even though the skill distribution is positively skewed. Levchenko (2008) and Benabou (2000) consider such unequal distribution of voting power.

Using a similar notation,

$$r^A(\gamma',\ \gamma'') = \frac{U_P^A(\gamma')}{U_P^A(\gamma'')}, \tag{12}$$

stands for the relative welfare under a policy γ' compared to γ'' in autarky. For any $\gamma' > \gamma''$, (12) increases in μ and η_i and does not depend on H, L or λ. Hence, the autarky optimal policy ($\widetilde{\gamma}$) does not depend on original factor endowments or redistributive concern and it monotonically increases in the expenditure share of the skill intensive good and in the skill intensities of production. For any $\gamma' > \gamma''$, (11) increases in μ and η_1, which ensures that the non - cooperative best reply correspondence in the open economy, BR(γ^*), shifts up in μ and η_1. Also, for any $\gamma' > \gamma''$, (11) decreases in λ, which ensures that BR(γ^*), shifts down in λ. I summarize this result in the next lemma. Since all the proofs of this section are algebraic in nature, I relegate the proofs to the appendix.

Lemma 5 *For any $\gamma' > \gamma''$, (12) increases in μ and η_i and does not depend on H, L, and λ. The autarky optimal policy $\widetilde{\gamma}$ is increasing in μ, η_i and does not depend on H, L, and λ. For any $\gamma' > \gamma''$, (11) increases in μ and η_1, and decreases in λ. In the open economy $BR(\gamma^*)$ shifts up in μ and η_1, and shifts down in λ.*

Proof. See appendix. ■

In this model the policy game is submodular and hence, by Topkis (1978), a PSNE exists. Let us consider the case of pure welfare maximization. If an increase in the government education expenditure affects the effective endowment of skill in a strongly convex fashion, if skill intensities of production are sufficiently different, and if consumers do not prefer either of the two goods too strongly, only asymmetric PSNEs exist. The next proposition summarizes the necessary and sufficient conditions for the existence of an asymmetric PSNE in the current setting.

Proposition 6 (Fundamentals for Existence of Asymmetric Equilibrium) *Suppose that $\lambda = 0$ and that governments maximize aggregate welfare. If both $g_H''(.) < 0$ and $g_L''(.) < 0$, $(\widetilde{\gamma}, \widetilde{\gamma})$ is the unique symmetric PSNE and is the unique symmetric Pareto optimum. If $g_t''(.)$ is sufficiently high, skill intensities of production are sufficiently different and consumers do not prefer either of the two goods too strongly, only asymmetric PSNEs and hence, only asymmetric Pareto optima exist.*

Proof. See appendix. ■

If $g_t''(.)$ is sufficiently high such that $U^T(.,.)$ is quasiconvex, there are only asymmetric PSNEs if $\widetilde{\gamma}$ is an interior optimum. By Lemma 6, $\widetilde{\gamma}$ is an interior optimum if consumers do not prefer either of the two goods too strongly. Since (12) is also increasing in η_i, an increase in $(\eta_1 - \eta_2)$ ensures that the intermediate range of μ for which an asymmetric PSNE exists lies in the interior of the bounded parameter space $[0, 1]$. If $g_t''(.)$ is sufficiently high such that $U^A(.)$ is quasiconvex, $\gamma = 0$ or $\gamma = 1$ are the only possible autarky equilibrium policies. Here, η_1 and μ represent preference for higher γ in the competitive economy.[30] In this case also only asymmetric equilibria exist under similar restrictions on technologies and preferences.

Now I turn to the case in which governments care about redistributive equity, represented by $\lambda \neq 0$. Simple algebra shows that if $\eta_2 < .5$,

$$\frac{\partial V_P(p^T(\widetilde{\gamma}, \widetilde{\gamma}), \widetilde{\gamma})}{\partial p^T} < 0 \text{ if } \lambda > 0.$$

Since $\frac{\partial p^T}{\partial \gamma} < 0$, each country has an incentive to deviate to an inefficient overinvestment in basic education ($\gamma < \widetilde{\gamma}$), given that the other country is choosing $\gamma = \widetilde{\gamma}$, if $\lambda > 0$ and $\eta_2 < .5$. The opposite is true for $\lambda < 0$.

I can use the comparative static properties of (12) and (11) and convexity properties of $U_P^T(.,.)$ to establish conditions for the existence of an asymmetric PSNE. If both $g_H''(.) \leq 0$ and $g_L''(.) \leq 0$, the absolute slope of the best response is less than unity everywhere, which implies that there is a unique PSNE. In the symmetric setting, any asymmetric PSNE exist in pairs. Thus, if both $g_H''(.) \leq 0$ and $g_L''(.) \leq 0$, there is a unique symmetric PSNE. In addition, if both $g_H''(.) \leq 0$ and $g_L''(.) \leq 0$ and $\eta_2 < .5$, the unique symmetric PSNE $(\widetilde{\gamma}', \widetilde{\gamma}')$ is inefficient $(\widetilde{\gamma}' \neq \widetilde{\gamma})$.

If $g_t''(.) > 0$ and sufficiently high, $U_P^T(.,.)$ is quasiconvex. For a quasiconvex $U_P^T(.,.)$ the $BR(\gamma^*)$ is a subset of $\{0, 1\}$ from the definition of quasiconvexity. Hence, for $g_t''(.)$ sufficiently high, the only possible symmetric PSNEs are at the extremes. If $(\eta_1 - \eta_2)$ is sufficiently high, consumers do not prefer either of the two goods too strongly and political preferences in favor of either of the two types of agents is not very strong, one can rule out a symmetric PSNE at the extremes. Hence, given sufficiently convex $g_t(.)$ functions and suitable restrictions on technologies and consumer and political preferences, only asymmetric PSNEs exist. I summarize these results in the next proposition.

Proposition 7 (Inefficient Symmetry and Asymmetry in PSNE) *Suppose that $\lambda \neq 0$ and that governments face a trade-off between aggregate welfare and distributive equity. If $\eta_2 < .5$ and if $\lambda > 0$, each country has an incentive to deviate to an inefficient overinvestment in basic education $(\gamma < \widetilde{\gamma})$, given that the other country is choosing $\gamma = \widetilde{\gamma}$. If both $g_t''(.) \leq 0$ and $\eta_2 < .5$, $(\widetilde{\gamma}', \widetilde{\gamma}')$ is the unique symmetric PSNE and $(\widetilde{\gamma}, \widetilde{\gamma})$ is the unique symmetric Pareto optimum, $\widetilde{\gamma}' \neq \widetilde{\gamma}$. If $g_t''(.)$ is sufficiently high, skill intensities of production are sufficiently different, consumers do not prefer either of the two goods too strongly, and political preferences in favor of either of the two types of agents is not very strong, only asymmetric PSNEs exist.*

Proof. See appendix. ■

The comparative static properties of (11) help us to understand the comparative static properties of an asymmetric PSNE. For any $\gamma' > \gamma''$ (11) is increasing in both μ and η_1. Suppose that μ' is the minimum value of μ such that $\gamma' > \widetilde{\gamma}$ is a profitable unilateral deviation from $(\widetilde{\gamma}, \widetilde{\gamma})$. An increase

[30] Preference for higher γ in the competitive economy is represented by parameter θ in the appendix. Both (12) and (11) increase in θ, for $\gamma' > \gamma''$.

in η_1 increases $\mathrm{r}^T(\gamma', \widetilde{\gamma}, \widetilde{\gamma})$ and ensures that γ' is a profitable unilateral deviation from $(\widetilde{\gamma}, \widetilde{\gamma})$ for values of $\mu < \mu'$. In general, if $\underline{\mu}$ and $\overline{\mu}$ are respectively the minimum and maximum values of μ for which only asymmetric PSNEs exist, then both $\underline{\mu}$ and $\overline{\mu}$ decline in η_1. Note that η_1 is the high-skill intensity in the production of good 1, and μ is the expenditure-share of the more skill-intensive good 1. Thus, given a sufficiently high $g_t''(.)$, only asymmetric PSNEs exist if consumer preferences and production technologies are not biased towards the same factor of production. Similarly, both $\underline{\mu}$ and $\overline{\mu}$ increase in λ. Here, λ is the political weight attached to the low-skilled agents. Thus, given a sufficiently high $g_t''(.)$, only asymmetric PSNEs exist if consumer preferences and political preferences are not biased towards the same factor of production.

In the asymmetric PSNE, both countries attain greater aggregate welfare compared to the autarky optimum, from Proposition 3. However, the opening of trade has implications for inequality. In the Heckscher-Ohlin economy, wages are functions of only price. Both countries have the same price in the identical autarky optimum and face the free trade price in the open economy equilibrium. Comparing the autarky optimum with the open economy asymmetric equilibrium, wage inequality in both countries change in the same direction depending on the price-movement. The skill-exporting country is more likely to experience an increase in welfare inequality since this country invests relatively more in higher education in the open economy equilibrium. I summarize the comparative static results for inequality in the next proposition.

Proposition 8 *Consider the values of the preference parameter μ for which an asymmetric PSNE exists. Comparing an asymmetric PSNE (γ, γ^*) and $(\widetilde{\gamma}, \widetilde{\gamma})$, both countries may experience higher wage inequality $(w_H(p) - w_L(p))$ at the asymmetric PSNE for relatively high values of μ. If $\gamma > \gamma^*$, the home country experiences a relatively larger increase in welfare inequality $(U_H(\gamma, \gamma^*) - U_L(\gamma, \gamma^*))$ at the asymmetric PSNE.*

Proof. See appendix. ■

Thus, in presence of redistributive concerns a gain in social welfare is not guaranteed in the asymmetric PSNE compared to the autarky optimum. In the numerical section in the appendix 6.4 I illustrate how the gain in social welfare in the asymmetric PSNE varies with changes in economic fundamentals. Also, I demonstrate numerically the parameter space for which an asymmetric equilibrium exists and how key convexity parameters affect the optimal policy outcome.

3.2 Education Policy in the Heckscher - Ohlin Model: Endogenous Skill

What changes when agents choose their skill optimally in the previous model, given government policy? I consider a simple model of endogenous skill choice by agents with heterogeneous abilities. In each country there are two types of labor. High skill types are born with ability h and low skill types are born with ability l, $h > l$. There are n_L low type workers and n_H high type workers. A positively skewed skill distribution implies $n_L > n_H$. The total initial endowment of high-skilled labor is $H = hn_H$. Producers of the two goods employ high- and low-skilled labor with different intensities as before. The assumptions concerning functional forms of production and direct utility remain unchanged. As before, good 2 is the numeraire and p is the relative price of good 1.

High skill types choose a skill h^e, and c_1, and c_2 to maximize

$$\begin{aligned} u(c_1, c_2) - \beta_H(\frac{h^e}{h})^\epsilon,\ \beta_h &> 0, \epsilon > 1, \\ s.t.\ pc_1 + c_2 &\leq w_H h^e. \end{aligned}$$

Here, w_t, $t = H, L$ refer to the wage of high- and low-skilled workers, and c_1 and c_2 refer to individual consumption. Also, $\beta_H(\frac{h^e}{h})^\epsilon$ is the welfare cost of education. Both total and marginal cost of education rise in the ability h and fall in the skill h^e. Note that a fall in β_t, $t = H, L$ reduces the cost of education. I define β_t as the quality of educational institutions. Here, ϵ is the elasticity of cost of education with respect to skill. Note that given an ability h, the relative cost of acquiring higher skill $h'' > h'$, $\frac{\beta_H(\frac{h''}{h})^\epsilon}{\beta_H(\frac{h'}{h})^\epsilon}$, increases in ϵ. I define ϵ as the progressivity of the education system, following the interpretation in Benabou (2002).

Agents take the equilibrium wage, price and educational institutional parameters as given. The condition $\epsilon > 1$ ensures that the second order condition of optimality of the agents' skill choice is satisfied. Producers maximize profit given the equilibrium wage, price and effective endowments of skill, $H^e = h^e n_H$ and $L^e = l^e n_l$. In equilibrium all the optimization conditions hold and both labor and goods markets clear. The optimal skill choice function is given by

$$\begin{aligned} H^e &= c_{h'}(\frac{H^\epsilon p^{\frac{1-a}{\eta_1-\eta_2}}}{n_H\beta_H})^{\frac{1}{\epsilon-1}},\ c_{h'} = f(\eta_1, \eta_2, \mu, \epsilon),\ a = \mu\eta_1 + (1-\mu)\eta_2, \qquad (13) \\ \text{and } L^e &= c_{l'}(\frac{L^\epsilon p^{\frac{-a}{\eta_1-\eta_2}}}{n_l\beta_L})^{\frac{1}{\epsilon-1}},\ c_{l'} = f(\eta_1, \eta_2, \mu, \epsilon). \end{aligned}$$

The government has a total education budget $T = 1$, and chooses a fraction $\gamma \in [0, 1]$ to spend on higher education. The remainder goes to primary education. The government expenditure on higher education, γ, improves the higher educational institutions,

$$\beta_H = g(\gamma),\ g' < 0.$$

Similarly, $\beta_L = g(1-\gamma)$.[31] The condition $\epsilon > 1$ ensures that H^e is a convex function of γ, given p, provided $g' < 0$ and $g'' \leq 0$. Thus when agents choose their skill levels optimally, optimal skill function in the economy is convex in government policy, even though government policy affects education costs in a simple linear fashion.

For simplicity, I assume that initially higher education and basic education institutions have same quality ($\beta_H = \beta_L = b > 1$), and government policy affects the institutions in a simple linear

[31] The assumptions of education system essentially mean that education is publicly funded and government decides which type of institutions to emphasize relatively more.

fashion,

$$\begin{aligned}\beta_H &= b - c\gamma,\ b > c > 0,\\ \beta_L &= b - c(1-\gamma).\end{aligned}$$

Note that the government always has the option of improving both types of institutions equally ($\gamma = .5$), but may choose to attach different priorities to different institutions. The government takes the optimal response of the agents and market clearing conditions as given and maximizes the aggregate indirect utility,

$$u(C_1, C_2) - n_H\beta_H(\frac{h^e}{h})^{\epsilon} - n_L\beta_L(\frac{l^e}{l})^{\epsilon}.$$

The following proposition summarizes the condition for the existence of an asymmetric PSNE. Note that in this model,

$$\frac{\partial^2 Y(.,\gamma)}{\partial\gamma^2} = w_H(p)s_H(p)\frac{\partial\beta_H^{\frac{1}{1-\epsilon}}}{\partial\gamma^2} + w_L(p)s_L(p)\frac{\partial\beta_L^{\frac{1}{1-\epsilon}}}{\partial\gamma^2}, \tag{14}$$

where $s_t(p)$, $t = H, L$, are given from (13). Given $\epsilon > 1$, a fall in ϵ increases the convexity of (14) provided $b \leq 1$. Hence, if ϵ is sufficiently low, $(\eta_1 - \eta_2)$ is sufficiently high and consumers do not prefer either of the goods too strongly, an asymmetric PSNE exists. The proof uses a similar logic as in Proposition 6.

Proposition 9 *If ϵ is sufficiently low and $b \leq 1$, $(\eta_1 - \eta_2)$ is sufficiently high and consumers do not prefer either of the goods very strongly, an asymmetric PSNE exists in the open economy optimal policy problem.*

Proof. See appendix. ■

What value of the crucial parameter, ϵ, is sufficiently low for our purposes? Note that ϵ has similarity to the standard exponent of disutility of effort in the macroeconomics literature. Even though the interpretation is not exact, I use a fairly standard value of ϵ for illustrative purposes. If I express ϵ as $1 + \frac{1}{\alpha}$, α is known as disutility of constant labor supply elasticity. Consistent with empirical estimates surveyed in Tuomala (1990, Chapter 3) and used in Saez (2001, 2002), and Itskhoki (2009), a standard value of α in the literature is .5. This gives a value of ϵ at 3. I fix $\mu = .5$ to rule out any bias in consumer preferences. I fix η_1 at .8 and η_2 at .2 to allow for sufficient differences in the skill intensities, and fix $\frac{H}{L}$ at 1.27.[32] For these set of parameters the two symmetric countries choose maximal asymmetric policies at the PSNE and the Pareto optimum.

[32] See appendix 6.4 for a discussion on how the production and relative endowment parameters are chosen.

3.3 Education Policy in the Grossman - Maggi Model

Next, I consider an economy that has a similar specification of consumer preferences and government policy as my application in section 3.1, but shares the production structure of Grossman and Maggi (2000). The submodular production technology implies that even when government policy affects skills in a linear fashion, aggregate production and income can be sufficiently convex in policy. Since the only crucial difference with the Heckscher-Ohlin model is on the production side , I present the assumptions on the production technologies briefly here, and present the rest of the model in the appendix.

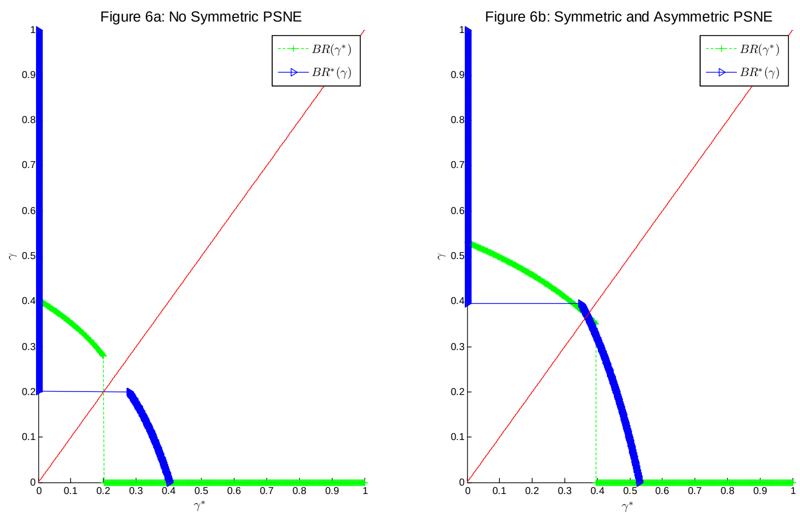
There are two sectors denoted by $i = 1, 2$ and two tasks denoted by $j = A, B$. The tasks are indivisible. Each must be performed by exactly one worker. For good i, $F^i(z_A, z_B)$ be the output from the process when the first task is performed by a worker with skill z_A and the second by a worker with skill z_B. Production of each good is homogenous of degree one in the skill levels of the two workers. Good 1 is supermodular in worker skill and good 2 is submodular in worker skill. Task A is the manager's task. The tasks are symmetric in sector 1. I allow for task A to be more skill-sensitive in sector 2. Good 2 is the numeraire good and p is the relative price of good 1. I consider the simplest functional forms for the production functions that satisfy above assumptions of the production technology,

$$
\begin{aligned}
F^1(z_A, z_B) &= (z_A^{\theta_1} + z_B^{\theta_1})^{1/\theta_1},\ \theta_1 < 1, \\
F^2(z_A, z_B) &= (A * z_A^{\theta_2} + z_B^{\theta_2})^{1/\theta_2},\ \theta_2 > 1,\ A \geqslant 1.
\end{aligned}
$$

In this model, $F + F^*$ essentially plays the role of $H^e + H^{e*}$ compared to the application in section 3.1, where $F = (A * H^{e\theta_2} + L^{e\theta_2})^{\frac{1}{\theta_2}} \equiv f(\gamma)^{\frac{1}{\theta_2}}$. Note that,

$$
\begin{aligned}
\frac{\partial^2 F}{\partial \gamma^2} &= \frac{1}{\theta_2}(\frac{1}{\theta_2} - 1) f^{\frac{1}{\theta_2}-2} (\frac{\partial f}{\partial \gamma})^2 \\
&\quad + \frac{1}{\theta_2} f^{\frac{1}{\theta_2}-1} \{A\theta_2(\theta_2 - 1) H^{e^{\theta_2 - 1}} (\frac{\partial H^e}{\partial \gamma})^2 + A\theta_2 H^{e^{\theta_2-1}} \frac{\partial^2 H^e}{\partial \gamma^2} \\
&\quad + \theta_2(\theta_2 - 1) L^{e^{\theta_2-1}} (\frac{\partial L^e}{\partial \gamma})^2 + \theta_2 L^{e^{\theta_2-1}} \frac{\partial^2 L^e}{\partial \gamma^2}\}.
\end{aligned}
\tag{15}
$$

Evidently, $\frac{\partial^2 F}{\partial \gamma^2}$ is a positive function of $\frac{\partial^2 H^e}{\partial \gamma^2} = H g_H''$ and $\frac{\partial^2 L^e}{\partial \gamma^2} = L g_L''$. Thus, g_H'' and g_L'' play a similar role as in Proposition 6. If g_H'' and $g_L'' > 0$ and sufficiently high, the welfare function is quasiconvex. From (23) given g_H'' and $g_L'' > 0$ and sufficiently high, an asymmetric PSNE exists for a high value of the degree of submodularity, θ_2, provided the consumers prefer the supermodular good sufficiently strongly. However, in this case convexity of the $g_t(.)$ functions is not necessary for existence of an asymmetric equilibrium. From (15) note that $g_t''(.) \leq 0$ does not imply that $\frac{\partial^2 F}{\partial \gamma^2} \leq 0$ in presence of submodularity ($\theta_2 > 1$). Here I consider an example of existence of an asymmetric PSNE even though both of the $g_t(.)$ functions are linear. In the next figure I plot the best responses of the two countries. In Figure 6a I fix $\theta_2 = 2$, and in Figure 6b $\theta_2 = 2.5$.[33] In the first case there is no symmetric PSNE, and in the second case there are both symmetric and asymmetric PSNEs.



Note that the intuition for this result is quite general. To see this, let us consider the general framework in section 2. In the general framework where ϖ is the original determinant of trade and γ is the relevant policy,

$$\frac{\partial^2 Y(p^T,\gamma)}{\partial \gamma^2} = (p^T \frac{\partial^2 F_1(p^T,\gamma)}{\partial \gamma^2} + \frac{\partial^2 F_2(p^T,.)}{\partial \gamma^2})$$

plays the crucial role for existence of asymmetric equilibrium. But note that

$$\frac{\partial^2 F_i(p^T,\gamma)}{\partial \gamma^2} = \frac{\partial^2 F_i(p^T,\gamma)}{\partial \varpi^2}(\frac{\partial \varpi}{\partial \gamma})^2 + \frac{\partial F_i(p^T,\gamma)}{\partial \varpi}(\frac{\partial^2 \varpi}{\partial \gamma^2}).$$

In the presence of submodularity in production essentially $\frac{\partial^2 F_i(p^T,\gamma)}{\partial \varpi^2} > 0$, and hence $\frac{\partial^2 \varpi}{\partial \gamma^2} > 0$ is not required for the convexity in production. In this specification effective skill endowments, H^e and L^e, play the role of ϖ.

4 Asymmetric Countries

But how important is the convexity assumption for practical purposes when countries are originally asymmetric? In this case the relevant question is whether countries optimally choose to magnify

[33] Suppose that $g_t(z) = 1 + \beta_1 z$. I fix the rest of the parameters at the following values.

Table 1

Parameters	θ_1	H/L	n_L/n_H	β_1	A
Values	.8	1.27	2	1.2	2

pre-existing national differences by investing relatively more in their respective areas of comparative advantage.

I consider my simplest specification in section 3.1. A natural way to introduce initial differences is to consider countries that have different initial factor endowments. In the current framework differences in the initial factor distributions are the sources of comparative advantage, and government policies can affect these original determinants of trade.[34] Suppose that the home country is initially more abundant in high skill labor ($H > H^*$ and $L = L^*$). Both countries have the same autarky optimum $\widetilde{\gamma}$, since $\widetilde{\gamma}$ does not depend on the initial factor endowments. If $g_H^{'}(.)$ is sufficiently large, home country invests relatively more in higher education in a PSNE. Thus if the government policy is sufficiently effective in increasing skill, countries optimally amplify initial sources of comparative advantage in the open economy equilibrium. In such a PSNE both countries attain larger aggregate welfare compared to their respective autarky optima.

However, for the relatively skill abundant country, an increase in the higher education investment leads to a terms-of-trade deterioration. When these terms-of-trade considerations are very important, an inefficient symmetric non cooperative outcome may exist.[35] In such a symmetric PSNE, the world welfare improves if each country invests more in their areas of comparative advantage. This possible inefficiency of the PSNE raises the same concern of international policy cooperation as in the familiar case of tariff/tax policies. The difference is that in this case the countries should focus in their relative areas of comparative advantage to attain a Pareto improvement. When comparative advantage is at least partly endogenous to national policy, an international policy coordination requires the countries to agree to disagree. I summarize these results in the next proposition.

Proposition 10 *If $g_H^{'}(.) > 0$, $g_L^{'}(.) > 0$ and $g_H^{'}(.)$ is sufficiently large, in the asymmetric PSNE home country invests more in higher education, $\gamma_{NE} > \gamma_{NE}^*$. In such a PSNE both countries attain larger aggregate welfare compared to the autarky optimum. If a symmetric PSNE exists, a Pareto improvement requires further investment in higher education in the home country and further investment in basic education in the foreign country.*

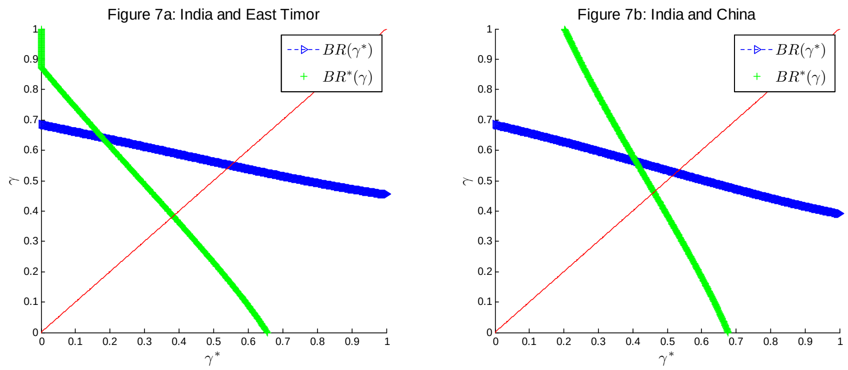
Proof. See appendix. ■

Thus, if countries are initially different, convexity of $g_t(.)$ is no longer necessary for amplification of initial comparative advantage in the open economy optimal policy outcome. However, in absence of sufficient convexity in $g_t(.)$ one can explain significant policy differences among countries via welfare gains from trade only if the countries are originally significantly different.

[34] Another interesting way to consider initial asymmetry is to consider countries that are identical except $\lambda \neq \lambda^*$. I can show that countries with different political preferences have identical autarky optimal policy $\widetilde{\gamma} = \widetilde{\gamma^*}$. However, in the open economy $g_t'(.) > 0$ and $\lambda < \lambda^*$ ensures that in the NE $\gamma_{NE} > \gamma_{NE}^*$. This follows from the properties of (12) and (11). Recall that (12) does not depend on λ, and (11) falls in λ. In this case allowing countries to differ in political preferences generates endogenous comparative advantage in the open economy, even though these countries choose the same autarky optimal policy.

[35] Note the tariff policies are the first-best instruments for terms-of-trade manipulation. Hence, if governments also choose tariff instruments optimally, this kind of inefficient symmetric choice in γ is unlikely to arise.

To illustrate this point, I fix the rest of the parameters, and consider two pairs of countries.[36] First, I allow for only limited convexity in the skill function ($\beta_2 = .5$). In our numerical illustration in section 3.2, the symmetric countries have $\frac{H}{L} = 1.27$. Here my first pair of countries are substantially different, $\frac{H}{L} = 3$, and $\frac{H^*}{L^*} = 1.27$. I denote this country pair as India and East Timor. The best responses for India and East Timor are plotted in Figure 7a. The second pair of countries are only marginally different, $\frac{H}{L} = 1.8$, and $\frac{H^*}{L^*} = 1.27$. I denote this country pair as India and China. The best responses for India and China are plotted in Figure 7b. Comparing figure 7a and 7b it is clear that difference in optimal policies is a monotonic function of initial difference between the countries, in absence of sufficient convexity in how policy affects skills.



The intuition of this result follows from the logic of the proof of proposition 10. To prove proposition 10, I first consider two countries that are completely identical. From proposition 6, given $g_t''(.) = 0$ there is a unique symmetric PSNE. Now I allow the home country to have relatively higher endowment of high skilled labor, $H > H^*$. Under the conditions of Proposition 10, best response of the home country shifts up, and best response of the foreign country shifts down following such a comparative static exercise. These shifts in the best responses give rise to an asymmetric PSNE in which the home country invests relatively more in higher education. But in absence of any convexity in $g_t(.)$, magnitude of the initial difference determines the extent of the shifts in best responses, and hence degree of asymmetry in equilibrium.

In Figure 8a and Figure 8b I consider the same two pairs of countries, but allow for sufficient convexity in the skill function ($\beta_2 = 4$). In this case the extremal PSNE in both cases involve substantial differences in equilibrium policies. Thus in presence of sufficient convexity, economic openness can amplify policy diversity, irrespective of the magnitude of initial differences.

[36] I assume a quadratic $g_t(z) = 1 + \beta_1 z + \beta_2 z^2$. The parameters are fixed at the following values.

Table 2

Parameters	η_1	η_2	μ	β_1	λ
Values	.8	.2	.5	1.2	0

Figure 8a: India and East Timor

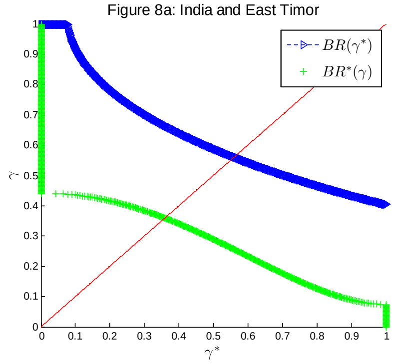
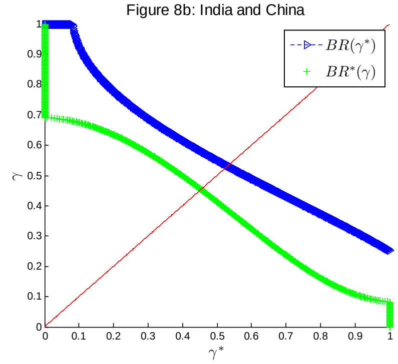


Figure 8b: India and China



Hence, in absence of convexity in $g_t(.)$ I can explain significant differences in cross-country policies via welfare-gains from trade only if the countries are significantly different to begin with. But when the countries are significantly different, one can easily explain associated asymmetric policy choices by studying two separate closed economies. Thus, welfare gains from trade can be an explanation of substantial policy differences among similar countries only in presence of sufficient convexity in how policy affects aggregate production.

5 Discussion

In this paper I explore whether similar countries may choose different policies because these policies allow them to specialize in different industries and gain from international trade. I find that even identical countries may optimally choose different policies in an open economy, and both of these symmetric countries gain in aggregate welfare in any asymmetric equilibrium compared to the autarky optimum. In an application in the competitive economy an asymmetric equilibrium arises if education policy affects the determinants of trade, namely effective endowments of skill, in a strong convex fashion. I construct a model where agents optimally choose their skill levels given government policy, and show that optimal skill is a convex function of government policy. I also study countries that are similar but not identical and find that these countries may optimally choose to invest more in their respective areas of comparative advantage to magnify initial differences.

The mechanism outlined in this paper has quantitative implications for growth in world trade. The magnitude of growth in world trade has been puzzling given modest reductions in measured transport costs and tariffs. In my work in progress Chatterjee (2009) I develop a quantitative multi-country framework with a continuum of goods in which countries optimally choose a policy that can affect their comparative advantage and explore how much of the growth in trade arises from optimal adjustment in domestic policies, given exogenous changes in transport costs and tariffs.

This paper outlines a general mechanism that applies to many different policies which can potentially affect comparative advantage in the open economy. For any particular application, it is important to model the domestic economic environment more carefully. For example, education policy is an important policy in encouraging trade, but there are several reasons why education policy is important for the domestic economy itself. Since future human capital is typically not accepted as collateral, availability of credit for financing educational expenditure is limited. This aspect of the education policy has received attention in both trade and macro policy literature (Benabou (2002), Ranjan (2000), Chesnokova and Krishna (2008)). Also, skills learnt in the earlier stages of academic development are complementary in acquiring advanced skills. In future work I intend to incorporate these aspects of education in a more complete application to study the interplay between the domestic and the international motives of optimal policy, and to study the implications for inequality.

Moreover, understanding the political economy implications of the type of policy problem emphasized in this paper is an important next step. When we view domestic policy as a source of comparative advantage, domestic political institutions also become a source of comparative advantage. Implications of different political economy mechanisms for comparative advantage in the open economy, and feedback effects of political changes through international transmission remain to be explored.

In general, how countries should (normative) and do (positive) form domestic policies to encourage international trade given the constraints imposed by the domestic and the world economy, and the implications of such policy for cross-national diversity in macroeconomic policies, inequality, and growth in trade are some exciting questions for future research.

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6 Appendix

6.1 Derivations for Section 2

From homothetic demand aggregate demand for good i C_i is linearly homogenous in income,

$$C_i = f_i(p) * Y.$$

Here Y is aggregate income,

$$Y = pF_1 + F_2,$$

where F_i stands for aggregate production of good i. The aggregate indirect utility is given by,

$$V(.,.) = U(f_1(p) * y,\ f_2(p) * y) = Yf(p),$$

where $f(p) = U(f_1(p),\ f_2(p))$. From Roy's identity,

$$\frac{f'(p)Y}{f(p)} = -C_1 = -f_1(p)Y. \tag{16}$$

The marginal effect of p on indirect utility is given by,

$$\frac{\partial V}{\partial p} = f'(p)Y + f(p)F_1 + f(p)(p\frac{dF_1}{dp} + \frac{dF_2}{dp}).$$

The condition $p = MRT = \frac{-dF_2}{dF_1}$ implies

$$\frac{\partial V}{\partial p} = f(p)(F_1 + \frac{f'(p)}{f(p)})Y).$$

From (16),

$$f(p)(F_1 + \frac{f'(p)}{f(p)}Y) = f(p) * (F_1 - f_1(p)Y) = f(p) * (F_1 - C_1).$$

Hence from (3), $\frac{\partial V}{\partial p} = 0$ in autarky. Assume that the policy instrument γ provides absolute advantage in good 1 and absolute disadvantage in good 2. Since total value of consumption equals income,

$$f_1(p) * p + f_2(p) = 1. \tag{17}$$

To prove $\frac{\partial^2 V(.,.)}{\partial\gamma\partial p} \gtrless 0$, note that,

$$\frac{\partial^2 V(.,.)}{\partial\gamma\partial p} = f'(p)\frac{\partial F_2}{\partial\gamma} + f(p)\frac{\partial F_1}{\partial\gamma}f_2(p), \text{ using (16) and (17).}$$

Indirect utility is decreasing in p given Y,

$$f'(p) < 0$$

Since γ gives absolute advantage in good 1 and absolute disadvantage in good 2,

$$\delta F_2/\delta\gamma < 0,\ \delta F_1/\delta\gamma > 0.$$

Hence, $\frac{\partial^2 V(.,.)}{\partial\gamma\partial p} \geqslant 0$.

The sufficient condition in Proposition 4 requires a quasiconvex $U^T(\gamma,\,\gamma^*)$, even though $\widetilde{\gamma}$ is an interior autarky optimum. Let us consider the difference between the curvatures of $U^T(\gamma,\,\gamma^*)$ at $(\widetilde{\gamma},\,\widetilde{\gamma})$ and $U^A(\gamma)$ at $\widetilde{\gamma}$. Note that the curvature of $U^A(\gamma)$ is given by,

$$\frac{d^2U^A(\gamma)}{d\gamma^2} = \frac{dp^A}{d\gamma}(2\frac{\partial^2 V(p^A,.)}{\partial\gamma\partial p} + \frac{\partial^2 V(p^A,.)}{\partial p^2}\frac{dp^A}{d\gamma}) + \frac{\partial^2 V(p^A,.)}{\partial\gamma^2}. \tag{18}$$

We know that $\frac{\partial^2 V(p^A,.)}{\partial\gamma\partial p} > 0$. Now let us consider $\frac{\partial^2 V(p^A,.)}{\partial p^2}$. Note that,

$$\frac{\partial^2 V(p^A,.)}{\partial p^2} = f(p^A)(\underset{+}{\frac{\partial F_1}{\partial p}} - f_1'(p)Y) + \underset{-}{f'(p^A)}F_1.$$

Thus the sign of $\frac{\partial^2 V(p^A,.)}{\partial p^2}$, in general, is ambiguous and $\frac{\partial^2 V(p^A,.)}{\partial\gamma^2}$ is given by,

$$f(p^A)\frac{\partial^2 Y(p^A,\gamma)}{\partial\gamma^2} = f(p^A)[p^A\frac{\partial^2 F_1(p^A,\gamma)}{\partial\gamma^2} + \frac{\partial^2 F_2(p^A,.)}{\partial\gamma^2}].$$

The first principal minor of the Hessian of $U^T(\gamma,\,\gamma^*)$ has a similar expression as (18) given by,

$$\frac{\partial p^T}{\partial\gamma}(2\frac{\partial^2 V(p^T,\gamma)}{\partial\gamma\partial p} + \frac{\partial^2 V(p^T,\gamma)}{\partial p^2}\frac{\partial p^T}{\partial\gamma}) + \frac{\partial^2 V(p^T,\gamma)}{\partial\gamma^2} + \frac{\partial V(p^T,\gamma)}{\partial p}\frac{\partial^2 p^T}{\partial\gamma^2}. \tag{19}$$

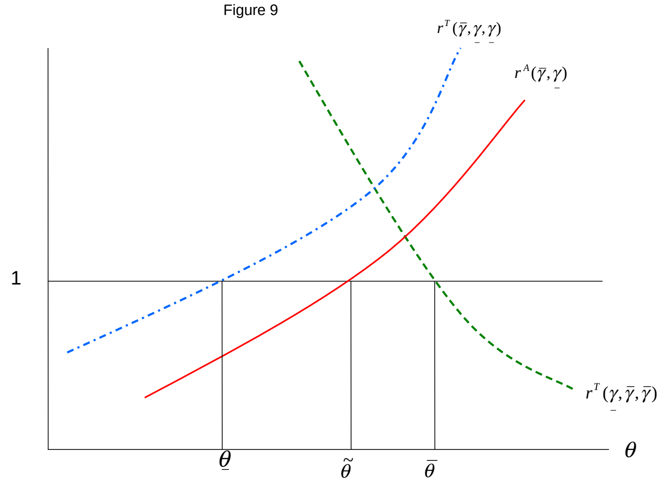
At a point of symmetry, $\frac{\partial p^T}{\partial\gamma} = .5\frac{dp^A}{d\gamma}$, $p^T(\gamma,\gamma) = p^A(\gamma)$, and $\frac{\partial V(p^T,\gamma)}{\partial p} = 0$. If $|\frac{\partial^2 V(p^T,\gamma)}{\partial p^2}|_{(\widetilde{\gamma},\widetilde{\gamma})}$ is sufficiently low, $\frac{\partial^2 Y(p^T,\gamma)}{\partial\gamma^2} > 0$ and sufficiently high, it is possible that (18) is negative, while (19) is positive. The second principal minor of the Hessian of $U^T(\gamma,\,\gamma^*)$ at a point of symmetry is given by,

$$\frac{\partial^2 V(p^T,\gamma)}{\partial\gamma^2}\left(2\frac{\partial p^T}{\partial\gamma}(2\frac{\partial^2 V(p^T,\gamma)}{\partial\gamma\partial p} + \frac{\partial^2 V(p^T,\gamma)}{\partial p^2}\frac{\partial p^T}{\partial\gamma}) + \frac{\partial^2 V(p^T,\gamma)}{\partial\gamma^2}\right)$$

Hence, sufficiently high values of $\frac{\partial^2 Y(p^T,\gamma)}{\partial\gamma^2}$ also ensures that the second principal minor of the Hessian of $U^T(\gamma,\,\gamma^*)$ is positive. Thus sufficient convexity in production with respect to the policy in question ensures that there are only asymmetric equilibria in the open economy, even though $\widetilde{\gamma}$ is an interior autarky optimum.

Now let us consider a situation in which convexity in production is high enough to rule out any interior autarky optimum. Suppose that both (11) and (12) increase in a parameter θ. This ensures both the best response in the open economy and the autarky optimum increase θ. I define θ as the parameter representing relative preference for γ in the competitive economy. Define $\widetilde{\theta}$ such that the autarky optimum is $\underline{\gamma}$ for $\theta \leq \widetilde{\theta}$, and the autarky optimum is $\overline{\gamma}$ for $\theta \geqslant \widetilde{\theta}$. Also, define $\underline{\theta}$ such that $r^T(\overline{\gamma}, \underline{\gamma}, \underline{\gamma}) = 1$ at the $\underline{\theta}$. By the gains from trade property (6), $r^T(\gamma', \underline{\gamma}, \underline{\gamma})$ lies above $r^A(\gamma', \underline{\gamma})$ for any θ. Hence, $\underline{\theta}$ is strictly less than $\widetilde{\theta}$. Thus, for $\theta \in [\underline{\theta}, \widetilde{\theta}]$ there is only asymmetric PSNE. Similarly I can define $\overline{\theta}$ such that $r^T(\underline{\gamma}, \overline{\gamma}, \overline{\gamma}) = 1$ at the $\overline{\theta}$, and for $\theta \in [\widetilde{\theta}, \overline{\theta}]$ there is only asymmetric PSNE. Thus when convexity in production is high enough to rule out any interior autarky optimum, there is only asymmetric PSNEs if relative preference for γ in the competitive economy is not very extreme.

Figure 9



In figure 9 I plot the change in relative welfare as a function of θ. Here $r^A(\overline{\gamma}, \underline{\gamma})$ is represented by the solid line; $r^T(\underline{\gamma}, \overline{\gamma}, \overline{\gamma})$ is represented by the dashed line; and $r^T(\overline{\gamma}, \underline{\gamma}, \underline{\gamma})$ is represented by the dotted line. The slopes of the line are given by the definition of θ. The ordering of the lines are given by the gains from trade property. Clearly, $\underline{\theta} < \widetilde{\theta} < \overline{\theta}$. Hence, for intermediate values of θ only asymmetric equilibria exist. In the application the preference and production parameters satisfy the definition of θ. Of course, this analysis also uses the convexity in production with respect to policy, since only in presence of enough convexity in production I can rule out an interior autarky optimum.

Convexity in production also plays a crucial role to satisfy the necessary condition for existence of an asymmetric equilibrium. To see this, suppose that production of goods is linear in policy, $\frac{\partial^2 F_1(p^T,\gamma)}{\partial \gamma^2} = \frac{\partial^2 F_2(p^T,\gamma)}{\partial \gamma^2} = \frac{\partial^2 Y(p^T,\gamma)}{\partial \gamma^2} = 0$. This ensures that the second principal minor of the Hessian of $U^T(\gamma, \gamma^*)$ is zero, and the first principal minor is negative. Hence, the Hessian of $U^T(\gamma, \gamma^*)$ is negative semidefinite, and $U^T(\gamma, \gamma^*)$ is strictly quasiconcave. Hence, if $\frac{\partial^2 Y(p^T,\gamma)}{\partial \gamma^2} = 0$ there is no asymmetric equilibrium in the open economy. An alternative way to prove this is the following.

Note that when $\frac{\partial^2 F_1(p^T,\gamma)}{\partial \gamma^2} = \frac{\partial^2 F_2(p^T,\gamma)}{\partial \gamma^2} = 0$, from (4) $\frac{\partial p}{\partial \gamma} = \frac{\partial p}{\partial \gamma^*}$ and $\frac{\partial^2 p}{\partial \gamma^2} = \frac{\partial^2 p}{\partial \gamma^* \partial \gamma}$, for all (γ, γ^*). Given $\frac{\partial^2 V(p^T,\gamma)}{\partial \gamma \partial p} > 0$, we can show that $|\frac{d^2 U}{d\gamma d\gamma^*}| < |\frac{d^2 U}{d\gamma^2}|$ for all (γ, γ^*). This implies that the best response has absolute slope less than unity everywhere which ensures existence of a unique symmetric equilibrium. Note that, $\frac{d^2U}{d\gamma d\gamma^*}$ is given by,

$$\frac{\partial p^T}{\partial \gamma^*}(\frac{\partial^2 V(p^T,\gamma)}{\partial \gamma \partial p} + \frac{\partial^2 V(p^T,\gamma)}{\partial p^2}\frac{\partial p^T}{\partial \gamma^*}) + \frac{\partial V(p^T,\gamma)}{\partial p}\frac{\partial^2 p^T}{\partial \gamma \partial \gamma^*}$$

and $\frac{d^2U}{d\gamma^2}$ is given by (19).

Next, I consider the case of presence of political concerns in the government objective.

Proposition 11 *Symmetric strategy in which both countries choose the autarky optimum,* ($\widetilde{\gamma}_P$, $\widetilde{\gamma}_P$)*, is the only candidate for a symmetric Pareto optimum. If* $\widetilde{\gamma}_P$ *lies in the interior of the policy space, all the Pareto optima are asymmetric if* $W_P(\gamma,\gamma^*)$ *is quasiconvex in* (γ, γ^*) *at* ($\widetilde{\gamma}_P$, $\widetilde{\gamma}_P$)*. If equilibrium price changes affect indirect utility* $V_P(p^T(\gamma,\gamma^*),\gamma)$*,at* ($\widetilde{\gamma}_P$, $\widetilde{\gamma}_P$), ($\frac{\partial V_P(p^T(\widetilde{\gamma}_P,\widetilde{\gamma}_P),\widetilde{\gamma}_P)}{\partial p^T} \neq 0$), ($\widetilde{\gamma}_P$, $\widetilde{\gamma}_P$) *cannot be a symmetric PSNE.*

Proof. The result that ($\widetilde{\gamma}_P$, $\widetilde{\gamma}_P$) is the only candidate for a symmetric Pareto optimum follows directly from the symmetry of the setup which ensures $W_P(\gamma,\gamma) = 2U_P^A(\gamma)$, and optimality of $\widetilde{\gamma}_P$.

At the interior autarky optimum $\widetilde{\gamma}_P$, the first order condition of maximization in autarky is satisfied,

$$\frac{dU_P^A(\gamma)}{d\gamma} = \frac{\partial V_P(p^A(\gamma),\gamma)}{\partial p}\frac{dp^A(\gamma)}{d\gamma} + \frac{\partial V_P(p^A(\gamma),\gamma)}{\partial \gamma} = 0.$$

Since a change in γ affects the welfare of the foreign country only through terms-of-trade, the first order derivative of $W_P(\gamma,\gamma^*)$ is,

$$\frac{\partial W_P(\gamma,\gamma^*)}{\partial \gamma} = \frac{\partial V_P(p^T(\gamma,\gamma^*),\gamma)}{\partial \gamma} + \frac{\partial V_P(p^T(\gamma,\gamma^*),\gamma)}{\partial p}\frac{\partial p^T(\gamma,\gamma^*)}{\partial \gamma} + \frac{\partial V_P(p^T(\gamma,\gamma^*),\gamma^*)}{\partial p}\frac{\partial p^T(\gamma,\gamma^*)}{\partial \gamma}.$$

At a point of symmetry,

$$\begin{aligned}\frac{\partial W_P(\gamma,\gamma^*)}{\partial \gamma} &= \frac{\partial V_P(p^T(\gamma,\gamma),\gamma)}{\partial \gamma} + \frac{\partial V_P(p^T(\gamma,\gamma),\gamma)}{\partial p}\frac{\partial p^T(\gamma,\gamma)}{\partial \gamma} + \frac{\partial V_P(p^T(\gamma,\gamma),\gamma)}{\partial p}\frac{\partial p^T(\gamma,\gamma)}{\partial \gamma} \\ &= 2\frac{\partial V_P(p^A(\gamma),\gamma)}{\partial p}\frac{\partial p^T(\gamma,\gamma)}{\partial \gamma} + \frac{\partial V_P(p^A(\gamma),\gamma)}{\partial \gamma},\ \text{since } p^T(\gamma,\gamma) = p^A(\gamma).\end{aligned}$$

At a point of symmetry change in own policy affects equilibrium terms-of-trade less than the autarky price,

$$\frac{\partial p^T(\gamma,\gamma)}{\partial \gamma} = .5\frac{dp^A(\gamma)}{d\gamma},$$

which ensures

$$\frac{\partial W_P(\gamma,\gamma)}{\partial \gamma} = \frac{dU_P^A(\gamma)}{d\gamma}.$$

Therefore, the FOC of optimization of the world welfare is satisfied at ($\widetilde{\gamma}_P$, $\widetilde{\gamma}_P$). Following the logic of Proposition 1, all the Pareto optima are asymmetric if $W_P(\gamma,\gamma^*)$ is quasiconvex in (γ, γ^*) at ($\widetilde{\gamma}_P$, $\widetilde{\gamma}_P$).

The first order condition of a PSNE is given by,

$$\frac{\partial U_P^T(\gamma,\gamma^*)}{\partial \gamma} = \frac{\partial V_P(p^T(\gamma,\gamma^*),\gamma)}{\partial \gamma} + \frac{\partial V_P(p^T(\gamma,\gamma^*),\gamma)}{\partial p}\frac{\partial p^T(\gamma,\gamma^*)}{\partial \gamma} = 0.$$

If $\frac{\partial V_P(p^T(\widetilde{\gamma}_P,\widetilde{\gamma}_P),\widetilde{\gamma}_P)}{\partial p^T} \neq 0$, necessary condition of a PSNE cannot be satisfied at $(\widetilde{\gamma}_P, \widetilde{\gamma}_P)$. ■

Now let us consider $\frac{\partial^2 F_1(p^T,\gamma)}{\partial \gamma^2} = \frac{\partial^2 F_2(p^T,\gamma)}{\partial \gamma^2} = 0$. The goods market clearing (4) remains unchanged. Therefore, $\frac{\partial p}{\partial \gamma}=\frac{\partial p}{\partial \gamma^*}$ and $\frac{\partial^2 p}{\partial \gamma^2}=\frac{\partial^2 p}{\partial \gamma^* \partial \gamma}$, for all (γ,γ^*). To prove existence of a unique symmetric equilibrium, we need to prove that $\frac{\partial^2 V_P(p^T,\gamma)}{\partial \gamma \partial p} > 0$. Consider the case of weighted welfare maximization which implies that,

$$V_P(p^T,\gamma) = f(p^T)Y_P(p^T,\gamma),$$

where $Y_P(p^T,\gamma) = c_1 p F_1 + c_2 F_2$. Here, $c_1, c_2 > 0$ are constants representing different weights. Now $\frac{\partial^2 V_P(p^T,\gamma)}{\partial \gamma \partial p}$ is given by,

$$f'(p)c_2\frac{\partial F_2}{\partial \gamma} + f(p)c_1\frac{\partial F_1}{\partial \gamma}f_2(p) + f(p)(c_1-c_2)\frac{\partial^2 F_1}{\partial \gamma \partial p}.$$

This expression is positive if $c_1 > c_2$, and $\frac{\partial^2 F_1}{\partial \gamma \partial p} > 0$. Here, $\frac{\partial^2 F_1}{\partial \gamma \partial p} > 0$ implies that price of good 1 is complementary to policy γ in improving production of good 1.

6.2 Proofs for Section 3

In the competitive economy, p^T and U^T are given by,

$$p^T = (\frac{H^e + H^{e*}}{L^e + L^{e*}})^{\eta_2-\eta_1}c_p(\mu,\eta_1,\eta_2), \tag{20}$$

$$U_P^T = gq^a(\frac{H^e + H^{e*}}{L^e + L^{e*}})^a(L^e(1+\lambda) + (1-\lambda)\frac{H^e}{qf(\mu,\eta_1,\eta_2)}(\frac{H^e + H^{e*}}{L^e + L^{e*}})^{-1}), \tag{21}$$

where $a = \eta_2 + \mu(\eta_1 - \eta_2)$, $qf = ((1-\mu)f_1 + \mu f_2)/(\mu f_2(\eta_1/(1-\eta_1)) + (1-\mu)f_1(\eta_2/(1-\eta_2))$, $f = (((1-\eta_1)/(1-\eta_2))(\eta_2/(1-\eta_2))^{-\eta_2}(\eta_1/(1-\eta_1))^{\eta_1})^{1/(\eta_2-\eta_1)}$, $f_2 = (\eta_2/(1-\eta_2))^{\eta_2}f^{\eta_2-1}/(\eta_1/(1-\eta_1) - \eta_2/(1-\eta_2))$, $f_1 = (\eta_1/(1-\eta_1))^{\eta_1}f^{\eta_1-1}/(\eta_1/(1-\eta_1)-\eta_2/(1-\eta_2))$, and $g = (1-\eta_1)(\eta_1/(1-\eta_1))^{\eta_1}f^{\eta_1}$ are constants depending on the production and demand parameters.

Proof of Lemma 5. I prove the lemma for $\lambda = 0$. The same result goes through for $\lambda \neq 0$.Consider a pair of policies (γ_1, γ_2) such that $\gamma_1 > \gamma_2$. Relative welfare of higher education, (12), is given by,

$$r^A(\gamma_1,\gamma_2) = \frac{(\frac{g_H(\gamma_1)}{g_L(1-\gamma_1)})^a g_L(1-\gamma_1)}{(\frac{g_H(\gamma_2)}{g_L(1-\gamma_2)})^a g_L(1-\gamma_2)}.$$

Evidently given $g_t' > 0$, (12) increases in the demand share of the skill intensive good (μ) and in the skill intensities of production,

$$\frac{dr^A(\gamma_1,\gamma_2)}{d\kappa} > 0 \text{ for } \kappa = \mu,\ \eta_1,\ \eta_2.$$

Given the rest of the parameters, consider an increase in μ' to μ''. Let the original autarky optimal policy be $\widetilde{\gamma}'$ and the new autarky optimal policy be $\widetilde{\gamma}''$. Since $\widetilde{\gamma}'$ is the original autarky optimal,

$$\frac{U^A(\widetilde{\gamma}')}{U^A(\gamma)}|_{(\mu=\mu')} > 1\ \nabla\gamma \neq \widetilde{\gamma}'.$$

With increase in μ, $r^A(\widetilde{\gamma}',\gamma)$ increases $\nabla\ \gamma < \widetilde{\gamma}'$. Thus,

$$U^A(\widetilde{\gamma}')/U^A(\gamma)|_{(\mu=\mu'')} \geqslant U^A(\widetilde{\gamma}')/U^A(\gamma)|_{(\mu=\mu')} > 1\ \nabla\gamma < \widetilde{\gamma}'.$$

Hence, $\widetilde{\gamma}'' \geqslant \widetilde{\gamma}'$. This implies that the autarky optimal policy $(\widetilde{\gamma})$ is increasing in μ, η_i. Also, (12) does not depend on H, L. Hence, $\widetilde{\gamma}$ does not depend on H, L.

The relative welfare of higher education in the open economy, (11), is given by,

$$\begin{aligned} r^T(\gamma_1,\gamma_2,\gamma^*) \quad = \quad & \underbrace{\frac{(\frac{g_H(\gamma_1)+g_H(\gamma^*)}{g_L(1-\gamma_1)+g_L(1-\gamma^*)})^a}{(\frac{g_H(\gamma_2)+g_H(\gamma^*)}{g_L(1-\gamma_2)+g_L(1-\gamma^*)})^a}}_{\frac{U_L^T(\gamma_1,\gamma^*)}{U_L^T(\gamma_2,\gamma^*)}} \times \\ & \underbrace{\frac{(g_L(1-\gamma_1)(1+\lambda)+(1-\lambda)\frac{g_H(\gamma_1)}{qf(\mu,\eta_1,\eta_2)}(\frac{g_H(\gamma_1)+g_H(\gamma^*)}{g_L(1-\gamma_1)+g_L(1-\gamma^*)})^{-1})}{(g_L(1-\gamma_2)(1+\lambda)+(1-\lambda)\frac{g_H(\gamma_2)}{qf(\mu,\eta_1,\eta_2)}(\frac{g_H(\gamma_2)+g_H(\gamma^*)}{g_L(1-\gamma_2)+g_L(1-\gamma^*)})^{-1})}}_{\frac{S_L^T(\gamma_1,\gamma^*)}{S_L^T(\gamma_2,\gamma^*)}}. \end{aligned}$$

where U_L^T is the aggregate indirect utility of the low-skilled workers and S_L^T stands for $\frac{Y}{w_L}$. From $\frac{\partial a}{\partial \mu} > 0$ and $\frac{\partial qf}{\partial \mu} < 0$, I can prove that both $\frac{U_L^T(\gamma_1,\gamma^*)}{U_L^T(\gamma_2,\gamma^*)}$ and $\frac{S_L^T(\gamma_1,\gamma^*)}{S_L^T(\gamma_2,\gamma^*)}$ increases in μ, if $\gamma_1 > \gamma_2$ and $g_t' > 0$. A similar proof works for increase in η_1. Hence using a similar logic as before, $BR(\gamma^*)$ shifts up in μ and η_1. ■

Proof of proposition 6. **Step 1 (** If both $g_H''(.) < 0$ and $g_L''(.) < 0$, $(\widetilde{\gamma},\widetilde{\gamma})$ is the unique symmetric PSNE and is the unique symmetric Pareto optimum.)

The Hessian of $W(\gamma,\, \gamma^*)$ is given by,

$$\begin{aligned} sign(\frac{d^2W}{d\gamma^2}) &= sign\{aHg''_H(.)(\frac{g_H(\gamma)+g_H(\gamma^*)}{g_L(1-\gamma)+g_L(1-\gamma^*)})^{a-1} \\ &+a(a-1)H^2(g'_H(\gamma))^2(\frac{(g_H(\gamma)+g_H(\gamma^*))^{a-2}}{(g_L(1-\gamma)+g_L(1-\gamma^*))^{a-1}}) \\ &-2a(a-1)HLg'_H(\gamma)g'_L(\gamma)(\frac{(g_H(\gamma)+g_H(\gamma^*))^{a-1}}{(g_L(1-\gamma)+g_L(1-\gamma^*))^{a}}) \\ &+(1-a)Lg''_L(\gamma)(\frac{g_H(\gamma)+g_H(\gamma^*)}{g_L(1-\gamma)+g_L(1-\gamma^*)})^{a} \\ &-a(1-a)(Lg'_L(\gamma))^2(\frac{(g_H(\gamma)+g_H(\gamma^*))^{a}}{(g_L(1-\gamma)+g_L(1-\gamma^*))^{-a-1}})\}, \end{aligned} \tag{22}$$

and

$$\begin{aligned} sign(\frac{d^2W}{d\gamma d\gamma^*}) &= sign\{a(a-1)H^2g'_H(\gamma)g'_H(\gamma^*)(\frac{(g_H(\gamma)+g_H(\gamma^*))^{a-2}}{(g_L(1-\gamma)+g_L(1-\gamma^*))^{a-1}}) \\ &-a(a-1)HLg'_H(\gamma)g'_L(\gamma^*)(\frac{(g_H(\gamma)+g_H(\gamma^*))^{a-1}}{(g_L(1-\gamma)+g_L(1-\gamma^*))^{a}}) \\ &-a(a-1)HLg'_H(\gamma^*)g'_L(\gamma)(\frac{(g_H(\gamma)+g_H(\gamma^*))^{a-1}}{(g_L(1-\gamma)+g_L(1-\gamma^*))^{a}}) \\ &-a(1-a)L^2g'_L(\gamma)g'_L(\gamma^*)(\frac{(g_H(\gamma)+g_H(\gamma^*))^{a}}{(g_L(1-\gamma)+g_L(1-\gamma^*))^{-a-1}})\} \end{aligned}$$

By assumption, $e < 1$, and $g'_L(\gamma) < 0$, $g'_H(\gamma) > 0$. If both $g''_H(\gamma) \leqslant 0$, and $g''_L(1-\gamma) \leqslant 0$,

$$\frac{d^2W}{d\gamma^2} < 0, \;\; \frac{d^2W}{d\gamma^{*2}} < 0, \;\; \text{and} \; (\frac{d^2W}{d\gamma^2}\frac{d^2W}{d\gamma^{*2}} - (\frac{d^2W}{d\gamma d\gamma^*})^2) \geqslant 0.$$

This ensures that the Hessian of W is negative semi-definite. Hence, W is quasiconcave. By Proposition 4, there is a unique Pareto optimum and a unique PSNE at $(\widetilde{\gamma}, \widetilde{\gamma})$.

Step 2: (If $g''_t(.)$ is sufficiently high, skill intensities of production are sufficiently different and consumers do not prefer either of the two goods too strongly, only asymmetric PSNEs and hence, only asymmetric Pareto optima exist.)

Note that

$$\frac{\partial^2 Y(.,\gamma)}{\partial \gamma^2} = w_H(p)g''_H(\gamma) + w_L(p)g''_L(1-\gamma).$$

From the derivation in appendix 6.1 $U^T(\gamma,\gamma^*)$ is quasiconvex in γ, if $\frac{\partial^2 Y(.,\gamma)}{\partial \gamma^2}$ is sufficiently high. Hence, if $g''_t(.)$ is sufficiently high, $U^T(\gamma,\gamma^*)$ is quasiconvex in γ. Now there are two possible cases.

Case 1: If $g''_l(.)$ is such that both U^A and U^T is quasiconvex, by the derivation in appendix 6.1, only asymmetric equilibria exist if preference for higher γ in the competitive economy represented by parameter θ is not very high. In this economy, μ plays the role of parameter θ. From the derivation in appendix 6.1, $\underline{\mu} < \overline{\mu}$. To have a non empty interval of $[\underline{\mu}\ , \overline{\mu}] \sqsubseteq [0,1]$, we need to check rest of the parameters of the economy. The sufficient condition for $\underline{\mu} \geqslant 0$ is

$$(1 + g_L(0))^{\eta_2} - ((1 + g_L(0))/(1 + (g_L(0))\eta_2)) < 0.$$

For a given $g_L(0) > 0$, this condition depends only on η_2 and is more likely to be satisfied for a fall in η_2. A fall in η_2 makes the low-skill intensive good even more intensive in lower skill and hence in primary education. This increase in low-skill intensity of production for good 2 makes it more likely that investing only in primary education is the symmetric equilibrium for non-negative values of μ. In a similar vein, $\overline{\mu} \leq 1$ essentially means that for some values of the preference parameter investing only in higher education is the symmetric equilibrium. For a given $g_H(0) > 0$, this sufficient condition depends only on η_1 and is more likely to be satisfied for an increase in η_1. Thus higher is the difference in the skill intensities of production of the two goods, more likely is the existence of a non empty subset of the parameter space for the preference parameter $[\underline{\mu}, \overline{\mu}] \subseteq [0, 1]$ for which an asymmetric PSNE exists.

Case 2: If $g_t''(.)$ is sufficiently high such that U^T is quasiconvex at $(\widetilde{\gamma}, \widetilde{\gamma})$, by Proposition 4 there is only asymmetric PSNEs at the extremes provided there is a interior $\widetilde{\gamma}$. Since (12) increases in μ, there is an interior $\widetilde{\gamma}$ for values of $\mu \in [\mu_0, \mu_1]$, where μ_0, μ_1 are respectively the maximum and the minimum values of μ for which $\widetilde{\gamma}$ is not interior. Since (12) increases in η_1 and η_2, I can show that μ_0 rises in η_2 and μ_1 falls in η_1. Hence, a rise in $(\eta_1 - \eta_2)$ ensure that $[\mu_0, \mu_1] \subseteq [0, 1]$.

Step 3: (A PSNE exists)

From (21), $U^T(\gamma, \gamma^*)$ is continuous in γ. To prove submodularity of the game it is necessary and sufficient to prove that $\frac{d^2U^T(\gamma,\gamma^*)}{d\gamma d\gamma^*} < 0$. For $g_t(.)$ linear, one can directly sign this derivative. Let us consider the case of nonlinear $g_t(.)$. From previous derivations $\frac{d^2V}{dpd\gamma} > 0$. From the definition of $V(.,.)$,

$$\frac{d^2V}{dpd\gamma} = \frac{\partial^2 V}{\partial p^2}\frac{\partial p}{\partial \gamma} + \frac{\partial^2 V}{\partial p \partial \gamma}.$$

From the definition of $U^T(\gamma, \gamma^*)$,

$$\frac{d^2U^T(\gamma, \gamma^*)}{d\gamma d\gamma^*} = \underset{-}{\frac{\partial p}{\partial \gamma^*}}\underset{+}{\frac{d^2V}{dpd\gamma}} + \frac{\partial V}{\partial p}\frac{d^2p^T(\gamma, \gamma^*)}{d\gamma d\gamma^*},$$

and $\frac{\partial V}{\partial p} < (>) 0 \; for \; \gamma > (<)\gamma^*$. So the sufficient condition for $\frac{d^2U^T(\gamma,\gamma^*)}{d\gamma d\gamma^*} < 0$ is that $\frac{d^2p^T(\gamma,\gamma^*)}{d\gamma d\gamma^*} > (<)0$ for $\gamma > (<)\gamma^*$. This condition is satisfied if,

$$g_H''(\gamma) > \frac{g_H'(\gamma)|g_L'(\gamma)|}{g_L(\gamma)}.$$

To be completed. ■

Proof of Proposition 8. In this economy wage inequality is a function of p. If in the asymmetric PSNE $p^T(\gamma, \gamma^*) > p^A(\widetilde{\gamma})$, wage inequality is higher in the asymmetric PSNE. Equilibrium price is given by (20), and a rise in (γ, γ^*) reduces p. From (12) μ enters the autarky optimal policy only as a, and a rise in μ raises a which increases $\widetilde{\gamma}$. From (11), μ enters the best responses as a and qf. A rise in μ raises a which increases the best responses. Also, a rise in μ reduces qf, which then increases the best responses. Thus, for higher values of μ, $\frac{g_H(\gamma)+g_H(\gamma^*)}{g_L(1-\gamma)+g_L(1-\gamma^*)}$ in the asymmetric PSNE is likely to be higher than $\frac{g_H(\widetilde{\gamma})}{g_L(1-\widetilde{\gamma})}$. Hence, both countries may experience higher wage inequality in the asymmetric PSNE compared to the autarky optimum.

The difference in welfare is given by,

$$U_H - U_L = f(p)(w_H H^e - w_L L^e).$$

Both countries experience the same p in a free-trade equilibrium, but the country that invests more in higher education increases H^e more relative to L^e. ■

Proof of Proposition 7. **Step 1:** (If $\eta_2 < .5$ and if $\lambda > 0$, each country has an incentive to deviate to an inefficient overinvestment in basic education ($\gamma < \widetilde{\gamma}$), given that the other country is choosing $\gamma = \widetilde{\gamma}$. The opposite result holds for $\lambda < 0$. Thus if $\eta_2 < .5$ and $\lambda \neq 0$, both countries attain smaller welfare in any symmetric PSNE compared to the autarky optimum.)

The politically constrained objective function is,

$$U_P = U + \lambda p^{-\mu}(w_L * L^e - w_H * H^e).$$

Note that $\frac{\partial V}{\partial p}|_{\widetilde{\gamma},\widetilde{\gamma}} = 0$. Now consider $\frac{\partial p^{-\mu}(w_L * L^e - w_H * H^e)}{\partial p} =$

$$\underset{-}{\frac{\partial p^{-\mu}}{\partial p}}(w_L * L^e - w_H * H^e) + p^{-\mu}\frac{\partial(w_L * L^e - w_H * H^e)}{\partial p}$$

Now $(w_L * L^e - w_H * H^e)|_{(\widetilde{\gamma},\widetilde{\gamma})} > 0$ ensures that,

$$\frac{\partial p^{-\mu}(w_L * L^e - w_H * H^e)}{\partial p}|_{(\widetilde{\gamma},\widetilde{\gamma})} < 0.$$

One can show that,

$$\frac{w_L * L^e}{w_H * H^e}|_{(\widetilde{\gamma},\widetilde{\gamma})} = q(\mu, \eta_1, \eta_2) > 1 \text{ if } \eta_2 < .5.$$

Hence,

$$\frac{\partial V_P}{\partial p}|_{(\widetilde{\gamma},\widetilde{\gamma})} < (>)0 \ if \ \lambda \ > (<)0.$$

From the definition of $U_P(\gamma,\ \gamma^*)$,

$$\frac{dU_P(\gamma,\ \gamma^*)}{d\gamma}|_{(\widetilde{\gamma},\ \widetilde{\gamma})} = \frac{\partial V_P(\widetilde{\gamma},\ p(\widetilde{\gamma},\ \widetilde{\gamma})}{\partial \gamma} + \frac{\partial V_P(\widetilde{\gamma},\ p(\widetilde{\gamma},\ \widetilde{\gamma})}{\partial p}\frac{\partial p(\gamma,\ \gamma^*)}{\partial \gamma}|_{(\widetilde{\gamma},\ \widetilde{\gamma})}$$

$$= -\frac{\partial V_P(\widetilde{\gamma},\ p(\widetilde{\gamma},\ \widetilde{\gamma})}{\partial p}\frac{\partial p(\gamma,\ \gamma^*)}{\partial \gamma}|_{(\widetilde{\gamma},\ \widetilde{\gamma})} \quad \text{, by FOC of Autarky Optimum.}$$

Hence, $\frac{dU_P(\gamma,\, \gamma^*)}{d\gamma}|_{(\widetilde{\gamma},\, \widetilde{\gamma})} < (>) 0$ if $\lambda > (<) 0$.

Step 2: (If both $g''_H(.) \leq 0$ and $g''_L(.) \leq 0$, $(\widetilde{\gamma}', \widetilde{\gamma}')$ is the unique symmetric PSNE and $(\widetilde{\gamma}, \widetilde{\gamma})$ is the unique symmetric Pareto optimum, $\widetilde{\gamma}' \neq \widetilde{\gamma}$.)

Expression for the sign of the Hessian of $W(.,.)$ remains unchanged from (22). From Proposition 11, $(\widetilde{\gamma}, \widetilde{\gamma})$ satisfies the necessary FOC for a Pareto optimum. From Proposition 6, if both $g''_H(.) \leq 0$ and $g''_L(.) \leq 0$ $W(.,.)$ is globally concave.

Now let us consider the non cooperative optimization problem. First, I consider linear $g(.)$ functions, $g''_H(.) = 0$, $g''_L(.) = 0$, and $g'_H(\gamma) = \beta_h$ and $g'_L(1-\gamma) = \beta_l$. The FOC of a NE is given by,

$$\begin{pmatrix} a(\frac{\beta_h H}{H^e+H^{*e}} + \frac{\beta_l L}{L^e+L^{*e}})(\frac{1-\lambda}{qf}\frac{H^e(L^e+L^{*^e})}{H^e+H^{*^e}} + (1+\lambda)L^e) \\ +\frac{1-\lambda}{qf}(\frac{H\beta_h(L^e+L^{*e})}{H^e+H^{*^e}} + H'\frac{\partial \frac{L^e+L^{*^e}}{H^e+H^{*e}}}{\partial \gamma}) - (1+\lambda)\beta_l L \end{pmatrix} = 0.$$

One can show that,

$$|\frac{d\gamma}{d\gamma^*}| = \frac{|\frac{\partial^2 U^T}{\partial\gamma\partial\gamma^*}|}{|\frac{\partial^2 U^T}{\partial\gamma\partial\gamma^*}| + |C|},$$

where

$$sign(C) = sign\left(\frac{e-1}{(H^e + H^{*e})^2} - (1+\lambda)\beta_l L\right) < 0.$$

Hence, $|\frac{d\gamma}{d\gamma^*}| < 1$, which implies that there is a unique symmetric PSNE.

Step 3: (If $g''_t(.)$ is sufficiently high, skill intensities of production are sufficiently different and consumers do not prefer either of the two goods too strongly, only asymmetric PSNEs exist.)

The proof of this result is very similar to the proof of Proposition 6. If $g''_t(.)$ is sufficiently high, $U^T_P(.,.)$ is quasiconvex. The only difference is that in this case even if $U^T_P(.,.)$ is quasiconvex and $\widetilde{\gamma}$ is an interior optimum, one can not rule out symmetric PSNEs. I need to explicitly make sure that $(1,1)$ and $(0,0)$ are not symmetric PSNEs in this case. But $(1,1)$ and $(0,0)$ are not symmetric PSNEs if skill intensities of production are sufficiently different and consumers do not prefer either of the two goods too strongly. One can easily prove that under such parameter conditions 0 is a profitable unilateral deviation from $(1,1)$ and vice versa, by using properties of (11). ■

Derivation for the endogenous skill case

Let us first derive (13). A more able agent with initial ability h chooses h^e by maximizing

$$p^{-\mu} w_H(p) h^e - \beta_h (\frac{h^e}{h})^\epsilon,$$

where $w_H(p) = c_h p^{\frac{1-\eta_2}{\eta_1-\eta_2}}$. From the FOC, one can derive

$$h^e = (\frac{c_h p^{\frac{1-a}{\eta_1-\eta_2}}}{\epsilon\beta_h} h^\epsilon)^{\frac{1}{\epsilon-1}}.$$

Multiplying both sides by n_H I arrive at (13). The second order condition of optimality requires $\epsilon > 1$. Similarly one can solve the optimal skill-choice problem of the low-skilled agents. Note that,

$$\frac{\partial H^e}{\partial \gamma} = -(\frac{c_h p^{\frac{1-a}{\eta_1-\eta_2}}}{\epsilon n_H} h^{\epsilon})^{\frac{1}{\epsilon-1}} (\frac{1}{\epsilon-1}) \beta_h^{-\frac{1}{\epsilon-1}-1} \underbrace{\frac{\partial \beta_h}{\partial \gamma}}_{-} > 0$$

and $\frac{\partial^2 H^e}{\partial \gamma^2}$ is equal to

$$(\frac{c_h p^{\frac{1-a}{\eta_1-\eta_2}}}{\epsilon n_H} h^{\epsilon})^{\frac{1}{\epsilon-1}} \left((\frac{1}{\epsilon-1})(\frac{1}{\epsilon-1}+1) \beta_h^{-\frac{1}{\epsilon-1}-2} (\underbrace{\frac{\partial \beta_h}{\partial \gamma}}_{-})^2 - (\frac{c_h p^{\frac{1-e}{\eta_1-\eta_2}}}{\epsilon} h^{\epsilon})^{\frac{1}{\epsilon-1}} (\frac{1}{\epsilon-1}) \beta_h^{-\frac{1}{\epsilon-1}-1} \frac{\partial^2 \beta_h}{\partial \gamma^2} \right) > 0,$$

provided $\frac{\partial^2 \beta_h}{\partial \gamma^2} \leq 0$. A similar result hold for L^e. The equilibrium price in the open economy is given by,

$$p^T(\gamma, \gamma^*) = \{\frac{\beta_h^{\frac{1}{1-\epsilon}} + \beta_h^{*\frac{1}{1-\epsilon}}}{\beta_l^{\frac{1}{1-\epsilon}} + \beta_l^{*\frac{1}{1-\epsilon}}}\}^{\frac{(\eta_1-\eta_2)(1-\epsilon)}{\epsilon}} c_p(\eta_1, \eta_2, \mu, H, L, \epsilon, n_H, n_l),$$

where c_p denotes a constant that does not depend on policy.

Proof of Proposition 9. Note that a fall in ϵ increases the magnitude of $\frac{\partial^2 H^e}{\partial \gamma^2}$. I can show that for sufficiently small ϵ objective function of the government is convex in $\dot{\gamma}$. If $\{0, 1\}$ are the only values equilibrium policies can take, the condition on μ and $(\eta_1 - \eta_2)$ follows in a manner similar to the proof of proposition 6. ■

Derivation in the Grossman-Maggi economy

In each country there are two types of labor. High skill types are born with ability H and low skill types are born with ability L, $H > L$. There are n_L low type workers and n_H high type workers. A positively skewed skill distribution implies $n_L > n_H$. The government educational investment complements the ability of the individuals in forming the effective skill levels H^e and L^e. The government has a fixed resource T=1 and chooses which fraction $\gamma \in [0, 1]$ to invest in higher education. The per student expenditure in higher education is $\frac{\gamma}{n_H}$, and the resulting effective per individual skill levels H^e is,

$$H^e = Hg(\frac{\gamma}{n_H}).$$

Consumer preferences are captured in a Cobb - Douglas utility as before.

Agents choose which sector to work in, whom to match with and their consumption levels to maximize utility subject to the usual budget constraint. In the presence of a homothetic demand the indirect utility is a linear function of income. Hence, agents choose optimal matching and occupation by maximizing income. Given this production specification, Grossman and Maggi (2000) show that self-matching is optimal in sector 1 and cross matching is optimal in sector 2. I consider an occupation structure in which the high skilled workers work in sector 2 and they cross match with n_H low skilled workers. Rest of the low skilled workers self-match in sector 1.[37] The occupation structure is optimal if at the equilibrium prices, the low skilled workers are indifferent between working in sector 1 and sector 2, and the high skilled workers do not have an incentive to work in sector 1,

$$(A * H^{e\theta_2} + L^{e\theta_2})^{1/\theta_2} - p * 2^{(1/\theta_1 - 1)} * L^e > p * 2^{(1/\theta_1 - 1)} * H^e.$$

In this economy a country with a more dispersed skill distribution has comparative advantage in the submodular good.

I explore how the relative welfare under different policies changes with changes in the competitive economy. I consider the general social welfare function,

$$U_S = U_H(\frac{n_H}{n_L + n_H} - \lambda) + U_L(\frac{n_L}{n_L + n_H} + \lambda),\ 1 \geqslant \lambda \geqslant -1,$$

where U_H and U_L are the indirect utilities of an individual high and low skilled worker. An increase in the degree of submodularity, θ_2, or, in the expenditure share of the submodular good, $1 - \mu$, and a decrease in the political preference for low skilled labor, λ, improves the welfare trade-off of higher education in both autarky and the open economy,

$$\begin{aligned} \frac{\partial r^A(\gamma_1,\ \gamma_2)}{\partial \kappa} &> 0 \text{ for } \kappa = 1 - \mu,\ \theta_2, -\lambda,\ \gamma_1 > \gamma_2, \\ \frac{\partial r^T(\gamma_1,\ \gamma_2,\ \gamma^*)}{\partial \kappa} &> 0 \text{ for } \kappa = 1 - \mu,\ \theta_2, -\lambda,\ \gamma_1 > \gamma_2. \end{aligned} \tag{23}$$

A change in the degree of submodularity, θ_1, does not affect the relative welfare under different policies. Hence, both the autarky optimal policy and the best response in the open economy increase in θ_2, decrease in μ and λ, and does not depend on θ_1. Now the welfare function is given by,

$$\begin{aligned} U^T &= c(\mu,\ n_H,\ n_{L,}\ \theta_1)(\frac{F + F^*}{L^e + L^{e*}})^{1-\mu}(F(\frac{F + F^*}{L^e + L^{e*}})^{-1} + \frac{\mu}{1-\mu} L^e), \\ \text{and } p &= \frac{2^{(\frac{1}{\theta_1} - 1)} \mu n_H}{(1-\mu)(n_L - n_H)}(\frac{F + F^*}{L^e + L^{e*}}) \end{aligned}$$

where $F = (A * H^{e\theta_2} + L^{e\theta_2})^{\frac{1}{\theta_2}} \equiv f(\gamma)^{\frac{1}{\theta_2}}$. The proof of sufficient conditions of existence is similar as before.

[37] Kremer and Maskin (1996) suggest a similar occupation structure in a matching framework with discreet skill types.

6.3 Proof for Section 4

Proof of Proposition 10. **Step 1:** To prove (If $g_H^{'}(.) > 0$, $g_L^{'}(.) > 0$ and $g_H^{'}(.)$ is sufficiently large, in the asymmetric PSNE home country invests more in higher education, $\gamma_{NE} > \gamma^*_{NE}$.)

Consider the following comparative static exercise. Suppose that given H^* and L, the endowment of high-skilled labor in the home country, H, increases. If such a comparative static exercise shifts up the best response of the home country, $BR(\gamma^*)$, and shifts down the best response of the foreign country, $BR^*(\gamma)$, the home country invests more in higher education in NE ($\gamma > \gamma^*$). For the home country I study how this comparative static exercise affects (11) for any $\gamma_1 > \gamma_2$. In the asymmetric case,

$$r^T(\gamma_1,\gamma_2,\gamma^*) = \underbrace{\frac{(\frac{Hg_H(\gamma_1)+H^*g_H(\gamma^*)}{Lg_L(1-\gamma_1)+Lg_L(1-\gamma^*)})^a}{(\frac{Hg_H(\gamma_2)+H^*g_H(\gamma^*)}{Lg_L(1-\gamma_2)+Lg_L(1-\gamma^*)})^a}}_{\frac{U_L^T(\gamma_1,\gamma^*)}{U_L^T(\gamma_2,\gamma^*)}} \times \underbrace{\frac{(Lg_L(1-\gamma_1)+\frac{Hg_H(\gamma_1)}{qf(\mu,\eta_1,\eta_2)}(\frac{Hg_H(\gamma_1)+H^*g_H(\gamma^*)}{Lg_L(1-\gamma_1)+Lg_L(1-\gamma^*)})^{-1})}{(Lg_l(1-\gamma_2)+\frac{Hg_H(\gamma_2)}{qf(\mu,\eta_1,\eta_2)}(\frac{Hg_H(\gamma_2)+H^*g_H(\gamma^*)}{Lg_L(1-\gamma_2)+Lg_L(1-\gamma^*)})^{-1})}}_{\frac{S_L^T(\gamma_1,\gamma^*)}{S_L^T(\gamma_2,\gamma^*)}}. \tag{24}$$

It is straightforward to show that $\frac{U_L^T(\gamma_1,\gamma^*)}{U_L^T(\gamma_2,\gamma^*)}$ increases with H. Now let us consider $\frac{\partial \frac{S_L^T(\gamma_1,\gamma^*)}{S_L^T(\gamma_2,\gamma^*)}}{\partial H}$. I can show that,

$$\begin{aligned}
\frac{\partial \frac{S_L^T(\gamma_1,\gamma^*)}{S_L^T(\gamma_2,\gamma^*)}}{\partial H} &= \frac{a_1a_2(Hg_H(\gamma_1)+H^*g_H(\gamma^*))(\frac{Hg_H(\gamma_2)+H^*g_H(\gamma^*)}{Hg_H(\gamma_1)+H^*g_H(\gamma^*)}-1)+a_1a_2't(\gamma_1)-a_2a_1't(\gamma_2)}{(S_L^T(\gamma_2,\gamma^*))^2}, \\
where\ t(\gamma) &= \frac{g_H(\gamma)H^*g_H(\gamma^*)}{(Hg_H(\gamma)+H^*g_H(\gamma^*))^2}, a_1 = \frac{Lg_L(1-\gamma_1)+Lg_L(1-\gamma^*)}{qf}, \\
a_1' &= Lg_L(1-\gamma_1), \text{ and } a_2', a_2 \text{ similarly defined replacing } \gamma_1 \text{ by } \gamma_2.
\end{aligned}$$

Now $\frac{\partial \frac{S_L^T(\gamma_1,\gamma^*)}{S_L^T(\gamma_2,\gamma^*)}}{\partial H}$ is positive if $g_H'(\gamma)$ is sufficiently high. I can show that $a_1a_2' > a_2a_1'$. If $g_H'(\gamma)$ is sufficiently high, $t(\gamma_1) \gg t(\gamma_2)$, and sign of $(\frac{Hg_H(\gamma_2)+H^*g_H(\gamma^*)}{Hg_H(\gamma_1)+H^*g_H(\gamma^*)}-1)$ does not depend on the magnitude of $g_H'(\gamma)$. Thus, if $g_H'(\gamma)$ is sufficiently high, $BR(\gamma^*)$ shifts up in H. Now let us consider

(11) for any $\gamma_1^* > \gamma_2^*$,

$$r^{T*}(\gamma_1^*, \gamma_2^*, \gamma) = \underbrace{\frac{(\frac{H^* g_H(\gamma_1^*)+Hg_H(\gamma)}{Lg_L(1-\gamma_1^*)+Lg_L(1-\gamma)})^a}{(\frac{H^* g_H(\gamma_2^*)+Hg_H(\gamma)}{Lg_L(1-\gamma_2^*)+Lg_L(1-\gamma)})^a}}_{\frac{U_L^T(\gamma_1^*,\gamma)}{U_L^T(\gamma_2^*,\gamma)}} \times \underbrace{\frac{(Lg_L(1-\gamma_1^*) + \frac{H^* g_H(\gamma_1^*)}{qf(\mu,\eta_1,\eta_2)}(\frac{H^* g_H(\gamma_1^*)+Hg_H(\gamma)}{Lg_L(1-\gamma_1^*)+Lg_L(1-\gamma)})^{-1})}{(Lg_L(1-\gamma_2^*) + \frac{H^* g_H(\gamma_2^*)}{qf(\mu,\eta_1,\eta_2)}(\frac{H^* g_H(\gamma_2^*)+Hg_H(\gamma)}{Lg_L(1-\gamma_2^*)+Lg_L(1-\gamma)})^{-1})}}_{\frac{S_L^T(\gamma_1^*,\gamma)}{S_L^T(\gamma_2^*,\gamma)}}.$$

It is straightforward to show that $\frac{U_L^T(\gamma_1^*,\gamma)}{U_L^T(\gamma_2^*,\gamma)}$ decreases with H. Now let us consider $\frac{\partial \frac{S_L^T(\gamma_1^*,\gamma)}{S_L^T(\gamma_2^*,\gamma)}}{\partial H}$. I can show that,

$$\begin{aligned}
\frac{\partial \frac{S_L^T(\gamma_1^*,\gamma)}{S_L^T(\gamma_2^*,\gamma)}}{\partial H} &= \frac{b_1 b_2 (H^* g_H(\gamma_1^*) + Hg_H(\gamma))(\frac{H^* g_H(\gamma_2^*)+Hg_H(\gamma)}{H^* g_H(\gamma_1^*)+Hg_H(\gamma)} - 1) + b_1 b_2' t^*(\gamma_1^*) - b_2 b_1' t^*(\gamma_2^*)}{(S_L^T(\gamma_2, \gamma^*))^2}, \\
where\ t^*(\gamma^*) &= -\frac{g_H(\gamma) H^* g_H(\gamma_1^*)}{(Hg_H(\gamma) + H^* g_H(\gamma_1^*))^2}, b_1 = \frac{Lg_L(1-\gamma_1^*) + Lg_L(1-\gamma)}{qf}, \\
b_1' &= Lg_L(1-\gamma_1^*),\ \text{and } b_2',\ b_2 \text{ similarly defined replacing } \gamma_1 \text{ by } \gamma_2.
\end{aligned}$$

I can show that $g_H' > 0$ implies that $\frac{H^* g_H(\gamma_2^*)+Hg_H(\gamma)}{H^* g_H(\gamma_1^*)+Hg_H(\gamma)} < 1$. If $g_H'(\gamma)$ is sufficiently high, $t^*(\gamma_1^*) \ll t^*(\gamma_1^*)$ which ensures that $BR^*(\gamma)$ shifts down in H.

Step 2: To prove (In such a PSNE both countries attain larger aggregate welfare compared to the autarky optimum.)

The proof is very similar to the proof of Proposition 3. Hence I omit the proof.

Step 3: To prove (If a noncooperative symmetric PSNE exists, a Pareto improvement requires further investment in higher education in the home country.)

Let $(\overline{\gamma},\ \overline{\gamma})$ be a PSNE. If both $g_H(.)$ and $g_L(.)$ are both concave, the FOC characterizes the PSNE as well since $U^T(\gamma,\ \gamma^*)$ is concave in γ. Hence, at $(\overline{\gamma},\ \overline{\gamma})$,

$$\begin{aligned}
\frac{\partial U^T}{\partial \gamma} &= \frac{\partial V(p,\gamma)}{\partial p}\frac{\partial p}{\partial \gamma} + \frac{\partial V}{\partial \gamma} = 0 \\
\text{and } \frac{\partial U^{T*}}{\partial \gamma^*} &= \frac{\partial V^*(p,\gamma^*)}{\partial p}\frac{\partial p}{\partial \gamma^*} + \frac{\partial V^*(p,\gamma^*)}{\partial \gamma^*} = 0
\end{aligned}$$

From previous derivations,

$$sign(\frac{\partial V(p,\gamma)}{\partial p}) = sign(F_1 - C_1)$$

Given that the home country is relatively more abundant in high-skilled labor, at $(\overline{\gamma},\ \overline{\gamma})$,

$$\frac{\partial V(p,\gamma)}{\partial p} > 0,\ \frac{\partial V^*(p,\gamma^*)}{\partial p} < 0.$$

Since the policy confers comparative advantage in good 1, $\frac{\partial p}{\partial \gamma^*} < 0$, $\frac{\partial p}{\partial \gamma} < 0$. From the Pareto problem incorporating world goods market clearing (4),

$$\frac{\partial W}{\partial \gamma} = \frac{\partial V}{\partial \gamma},\ \text{and}\ \frac{\partial W}{\partial \gamma^*} = \frac{\partial V^*}{\partial \gamma^*}.$$

Hence, at $(\overline{\gamma},\ \overline{\gamma})$,

$$\frac{\partial W}{\partial \gamma} > 0,\ \frac{\partial W}{\partial \gamma^*} < 0.$$

■

6.4 Numerical Solution

Here, I illustrate comparative static and welfare properties of equilibria in the context of the Heckscher - Ohlin model. Similar properties are observed with the Grossman - Maggi (2000) production structure. Let us assume that both g_H and g_L are quadratic functions of the respective expenditure share and choose the same g(.) function for both skills,

$$g(z) = 1 + \beta_1 z + \beta_2 z^2.$$[38]

Properties of $g_t(.)$ require that $\beta_1 > 0$.

I first consider the case of pure welfare maximization, $\lambda = 0$, and fix $\mu = .5$ to rule out any bias in consumer preferences. Yeaple (2006) reports skill intensities of various industries. I fix η_1 at .8 which is the maximum value of the skill intensities and fix η_2 at .2 which is the minimum value of the skill intensities. In 1980 two of the major global players of today, India and China, were mostly a closed economy. Bosworth and Collins (2008) report that in 1980 on average, 44% of the population was without schooling in India and China, and relative share of manufacturing and services in domestic production was on average .5. I fix $\frac{H}{L}$ at $\frac{1-.44}{.44}$. I choose β_1 and β_2 to ensure that at the autarky optimal policy relative share of the two industries in GDP is roughly .5. This gives a value of $\beta_1 = 1.2$, $\beta_2 = 1$. The following table reports the values of the crucial parameters.

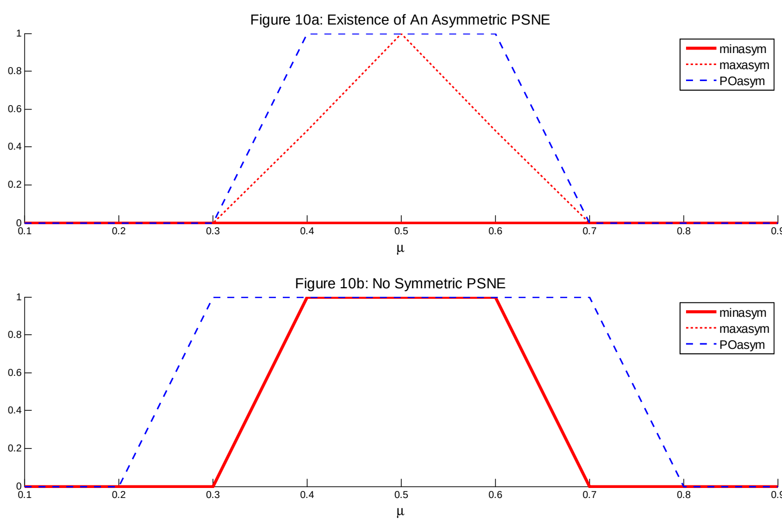
Table 3

Parameters	η_1	η_2	H/L	β_1	β_2
Values	.8	.2	1.27	1.2	1

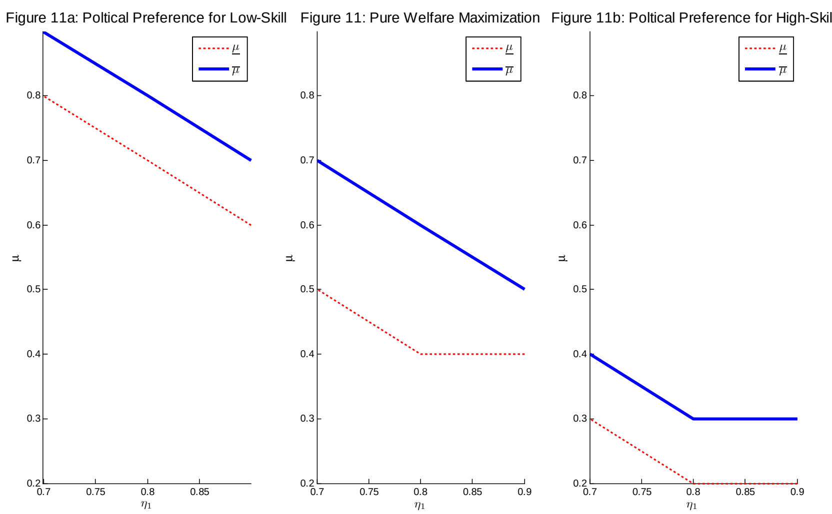
[38] Suppose an increase in the education subsidy relaxes the credit constraint in human capital acquisition. If low (high) ability individuals are more credit constrained, g_L (g_H) is relatively more responsive to education expenditure. By allowing for the same g function for both skill types, I demonstrate that my results are independent of this assumption.

In Figure 10 I plot 3 different measures of equilibrium asymmetry as a function of the demand parameter μ, keeping rest of the parameters as in Table 3. The solid line corresponds to minimum asymmetry in PSNE, denoted by minasym. When $(\widetilde{\gamma}, \widetilde{\gamma})$ is a symmetric PSNE, the minimum asymmetry in PSNE is 0. The dotted line corresponds to the maximum asymmetry in PSNE, denoted by maxasym. The game in this application is submodular. In a submodular game the maximum asymmetry PSNE is known as extremal equilibrium. This PSNE is always Cournot-stable (Echenique (2002)), has maximum welfare for both countries compared to any other PSNE (Proposition 3) and has nice comparative static properties (Milgrom and Roberts (1990)). If maxasym is nonzero even though minasym is zero, both an asymmetric PSNE and a symmetric PSNE at $(\widetilde{\gamma}, \widetilde{\gamma})$ exist. The dashed line corresponds to asymmetry in the Pareto optimum, referred to as POasym. By Proposition 3 whenever maxasym is nonzero, POasym is also nonzero. By definition whenever minasym is nonzero, maxasym is also nonzero.

By proposition 6, for $\beta_2 = 0$ there is no asymmetric equilibria and all the 3 different measures of equilibrium diversity is zero. In Figure 10a I consider $\beta_2 = .5$. This value of β_2 is not sufficiently high to rule out $(\widetilde{\gamma}, \widetilde{\gamma})$ as a symmetric PSNE. Hence, for all values of the demand parameter the minasym is zero. But for intermediate values of μ, both POasym and maxasym are nonzero implying existence of an asymmetric Pareto optimum and an asymmetric PSNE. In Figure 10b I consider $\beta_2 = 1$. Such value of β_2 is sufficient to rule out $(\widetilde{\gamma}, \widetilde{\gamma})$ as a symmetric PSNE for intermediate values of μ. In Figure 10b minasym and maxasym curves coincide. In both Figure 10a and Figure 10b, the Pareto optimum is asymmetric over a strictly larger parameter space, compared to the non cooperative solution.



Next, I focus on how comparative static properties of asymmetric equilibria change with change in technology parameters given a convexity parameter. To this end, I fix $\beta_2 = 1$, and vary μ and η_1. The rest of the parameters are fixed as in Table 3. As argued before, a rise in η_1 makes asymmetric equilibrium more likely for lower values of μ. The maximum and the minimum values of μ for which an asymmetric PSNE exists, referred to as $\overline{\mu}$ and $\underline{\mu}$, fall in η_1. In the next figure the solid line corresponds to $\overline{\mu}$, and the dotted line corresponds to $\underline{\mu}$. The area between $\overline{\mu}$ and $\underline{\mu}$ is the parameter space in which an asymmetric PSNE exists. This result is illustrated in Figure 11 for the case of pure welfare maximization.

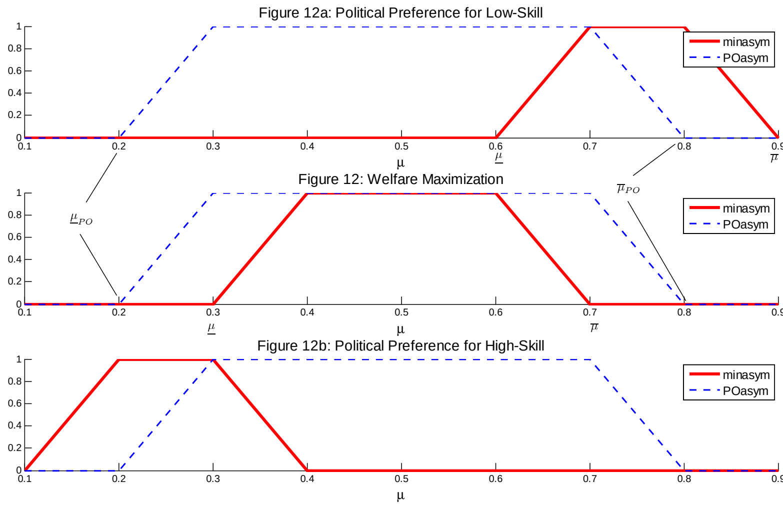


When governments care more for the low-skilled agents ($\lambda > 0$) an asymmetric PSNE exists only if demand preference for the skill-intensive good 1 is relatively stronger. In Figure 11a, I fix $\lambda = .5$ and illustrate that $\overline{\mu}(\eta_1)$ and $\underline{\mu}(\eta_1)$ shift up with increase in λ. The opposite case is illustrated in Figure 11b corresponding to $\lambda = -.5$.

In presence of $\lambda \neq 0$, it is no longer true that the Pareto optimum is asymmetric whenever an asymmetric PSNE exists. I illustrate this in the next figure. For the purpose of the next figure, I fix $\beta_2 = 1$, and the rest of the parameters as in Table 3. Let us define $\overline{\mu_{PO}}$ and $\underline{\mu_{PO}}$ as the maximum and minimum values of μ for which an asymmetric Pareto optimum exists. By proposition 3,

$$\underline{\mu_{PO}} \leq \underline{\mu} \leq \overline{\mu} \leq \overline{\mu_{PO}},$$

under pure welfare maximization. I describe this situation in Figure 12. The dashed line corresponds to asymmetry in the Pareto optimum, and the solid line represents the asymmetry in PSNE. When governments care more for the low-skilled agents ($\lambda > 0$), $\underline{\mu_{PO}} \leq \underline{\mu}$, and $\overline{\mu_{PO}} \leq \overline{\mu}$. Thus, for $\lambda > 0$, the Pareto optimum is asymmetric for relatively lower values of μ compared to the asymmetric PSNE. This is intuitive since by Proposition 8, for relatively larger values of μ both countries experience an increase in wage inequality in the asymmetric equilibrium, and for $\lambda > 0$ an increase in inequality reduces social welfare of both countries. I describe this situation in Figure 12a for $\lambda = .5$. When governments care more for the high-skilled agents ($\lambda < 0$), $\underline{\mu_{PO}} \geqslant \underline{\mu}$, and $\overline{\mu_{PO}} \geqslant \overline{\mu}$. Thus, for $\lambda < 0$, the Pareto optimum is asymmetric for relatively larger values of μ compared to the asymmetric PSNE. I describe this situation in Figure 12b for $\lambda = -.5$.



Next, I illustrate the comparative static properties of welfare gains in a PSNE compared to the autarky optimum in the presence of a redistributive concern. In Figure 13 I fix $\beta_2 = 1$, rest of the parameters as in Table 3 and vary $\lambda \in [-1, 1]$. I plot the gain in social welfare at the extremal PSNE compared to the autarky optimum for the skill-exporting ($\gamma \gtrless \gamma^*$) and importing country. The solid line represents the skill-exporting country's gain and the dotted line stands for the skill-importing country's gain. For the extreme values of λ, both countries suffer a welfare loss from the inefficient symmetric PSNE.[39] The skill-exporting country is more likely to gain for relatively low values of λ implying a higher political preference for high-skilled agents. Both countries experience similar wage movements but the skill-exporting country invests more in higher education, and hence experiences a larger increase in welfare inequality. Thus, when inequality concern is relatively low, the solid line lies above the dotted line and vice versa. For intermediate range of the weight on equity, both countries gain from trade in the extremal PSNE.

[39] By Proposition 7, any symmetric PSNE is inefficient compared to the autarky optimum if $\eta_2 < .5$.

Figure 13: Gain in Social Welfare in Presence of Redistributive Concern