Understanding the Economic Dynamics Behind Growth-Inequality Relationships

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Abstract

We calibrate a Dynamic General Equilibrium (DGE) model of growth-inequality relationships, with missing credit markets, knowledge spillover and self-employed islanders, to New Zealand data. The model explains how two distinct policy shocks involving redistribution and immigration imply, subsequently, two completely opposite outcomes. Islanders' inability to borrow aggravates a negative macroeconomic effect of heterogeneity on growth. Redistribution mitigates that effect but creates microeconomic disincentives on saving and work-effort. Consequently, immigration shocks that perturb variance of efficiency induce a negative growth-inequality relationship, while redistribution shocks, in New Zealand's case, produce larger fluctuations in incentives than in macro benefits, implying a positive growth-inequality relationship.

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1 Introduction

"(A) careful reassessment of the relationship between these two variables (growth rate and income inequality) needs further theoretical and empirical work evaluating the channels through which inequality, growth, and any other variables are related." — Forbes (2000)

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The debate over the relationship between income inequality and economic growth is far from settled. Prior to the 1990s, it was generally believed that a greater income inequality is sometimes a necessary price to pay for raising output because of the Kaldor (1956) hypothesis that greater income inequality promotes saving and hence growth. Consequently, the Kaldorian channel leads to a positive relationship between income inequality and growth. Similar correlations arise in other models such as Bourguignon (1981) and Li and Zou (1998). Forbes (2000) concludes from her extensive panel data studies of a large number of countries, that income inequality and growth rates are most likely to be positively related. Frank (2009) presents similar evidence based on the data across states in the United States. However, a large body of literature in the 1990s popularized the idea of an alternative channel of interaction between income inequality and the growth rate of per capita income, producing a negative relationship between the two variables. Solow (1992) outlined a new hypothesis that more "equity" could actually promote more growth. Persson and Tabellini (1994) report that past inequality is negatively related to the current growth rate of per capita income by using Ordinary Least Square (OLS) regressions over a cross-section of nations and offer a political explanation for that result. Aghion, Caroli and Garcia-Penalosa (1999) argue that in the presence of credit barriers, inequality may limit the ability of low-income households to make investments in human or physical capital, thus reducing the subsequent economic growth. More recently, Glomm and Kaganovich (2008) reported that interactions between public education and public pensions can yield a non-monotonic growth-inequality relationship. The above literature brings the policy debate on the growth-inequality relationship to a stalemate by arming both sides with empirical support. That status-quo makes policy-making a challenging task. Our objective in this paper is to alleviate that problem by demonstrating, using New Zealand as a case study, how to extract the relevant lessons from history for future policy making.

To achieve that objective, we make a few special assumptions to modify a standard textbook Dynamic General Equilibrium (DGE) model. In particular, we follow the tradition of Galor and Zeira (1993), Benabou (1996, 2000, 2002), and Zhang (2005) to assume a missing credit market, which sustains Pareto inefficient interpersonal differences in productivity, coupled with knowledge spillover, which bring in endogenous growth by overcoming the standard textbook assumption of diminishing returns in the process of human capital accumulation and in the technology of production. A special feature of our model economy is a redistributive policy framework with progressive income taxation and public subsidy in education. This feature seems both appropriate for studying issues concerning inequality and growth and quite common in the developed countries, including New Zealand. For analytical tractability, we focus on home production and add a note to motivate why we expect our key results to survive even if consider a perfectly competitive market for organising income and production. Further motivation for home production environment comes from our desire to capture a distinctive history of the 1990s New Zealand economy, which records an unprecedented increase in self-employment and, es-

pecially, among women.[1] With the above caveat of special assumptions, we now describe the essential results, which our paper delivers in two distinctive dimensions.

First, it provides a modified DGE model from the standard textbook that rationalizes multiple facets of the growth-inequality relationship reported in the earlier empirical literature as alternative scenarios of the equilibrium outcome in response to alternative perturbations of the model's parameters. Second, it reports new findings with significant policy implications from a set of quantitative experiments based on analyzing the simulated outcomes of the above model, calibrated to the 1990s New Zealand economy. In particular, we discover that changes in the immigration policy that altered the degree of heterogeneity in the distribution of human capital contributed to a negative growth-inequality relationship subsequently, while shifts in the redistributive policies from a sufficiently large degree of progressive redistribution contributed to a positive growth-inequality relationship but for a quite subtle reason that is specific to the history of the New Zealand economy. To bring out the underlying intuitions that drive those multifaceted simulation outcomes, we return to our model's assumptions and the dynamics that follow from it and focus on two distinct scenarios.

In one scenario, the degree of heterogeneity in the human capital distribution increases (or decreases) exogenously due to an exogenous change in immigration policy, for example, which corresponds to the New Zealand experience. The above shock increases (or decreases) heterogeneity of expertise among the owner-operated production units in the economy. The combined assumptions of no credit market and a convex home production technology, which is subject to diminishing returns on combined inputs of physical and human capital, makes such an increase in inequality produce a growth-retarding effect. Benabou (2002) and Zhang (2005) find similar results in a different context. Consequently, this scenario results in a negative growth-inequality relationship consistent with the empirical observations of Persson and Tabellini (1994). This finding put a new twist in the policy debate suggesting that a negative growth-inequality relationship may not necessarily call for income redistribution. Rather it may ask us to explore suitable mechanisms to reduce heterogeneity in the population characteristics. A suitably controlled immigration policy could fulfill that task.

In the other scenario, we change the rate of progressivity in the income tax structure and allow automatic and proportionate adjustment in public subsidy to education, which is in a way similar to the New Zealand's policy framework following the Fiscal Responsibility Act of 1993. We find that if the above shock accompanies a higher (or

[1]Between 1981 and 1990 the size of the self-employed in the labour force increased from 13.7% to 19.3%. The 56% of the self-employed were women operating from home and a significant proportion of part-time workers are self-employed (New Zealand Yearbook, 1993, pp 278). Moreover, Bandyopadhyay (2001) presents evidence to show that the above trend continued till late 1990s and, more importantly, that the earnings of the self-employed and the workers move together in New Zealand. That finding supports the motivation behind using a simpler model with islanders rather than workers in a competitive market, at least for understanding the dynamics behind the fluctuations in income data that give rise to a multifaceted growth-inequality relationship, without losing the generality of intuition that fundamentally governs it.

lower) progressivity then it pushes down (or up) the long-run steady state income inequality relative to its initial state. We explain this result using the analytical properties of the model. The model's steady-state inequality is unique and its transitional dynamics satisfy a unique monotone convergence property. Consequently, income inequality monotonically decreases (or increases) to its new steady state following the fiscal policy shock involving an increase (or a decrease) of progressivity. The growth rate, however, gets pushed from two opposite directions. A higher (or lower) progressivity provides an immediate boost upwards (or downwards) by lowering (or by raising) the interpersonal productivity differentials and thereby raising (or lowering) the macroeconomic productivity. At the same time, a higher (or lower) progressivity discourages (encourages) work effort, saving and investment in education, and thereby retards (or promotes) economic growth. The macroeconomic effect of raising progressivity on productivity dominates the associated microeconomic disincentives if and only if the progressivity does not exceed a critical threshold. The quantitative estimate of this threshold progressivity for an economy depends on, among others, the fiscal policy regime that utilizes government subsidy on education and other measures to offset typical disincentives of progressive redistribution. Consequently, the net effect of increasing (lowering) progressivity can only be determined quantitatively because the relative strengths of competing economic forces, mentioned above, govern the ultimate outcome. That's where the importance of our quantitative experiment lies.

The threshold progressivity for New Zealand turns out to fall well below the range of progressivity rates that we observe in New Zealand during the period of our experiment. Consequently, fiscal policy shocks during that period give rise to a positive growth-inequality relationship. However, this positive growth-inequality relationship does not mean that an increasing inequality must be a necessary price to pay for faster growth. By examining analytical properties of the model we conclude that with suitable adjustments in the redistributive policy package such as exempting tax on saving or subsidy for work similar to recently introduced "working for families benefit" that rewards hours of work the New Zealand Government can push this threshold up and above sufficiently to turn the positive growth-inequality relationship into a negative one. Afterwards, increased progressivity would lead the economy to a path of faster growth with lower income inequality. Consequently, contrary to popular interpretation of data, a positive growth-inequality relationship does not mean that an increasing inequality must be a necessary price to pay for faster growth.

Following the introduction, Section 2 describes the New Zealand data. Section 3 presents the model. Section 4 provides analytical results for the model's growth-inequality relationship. Section 5 presents the quantitative analysis of the model's make-up. Section 6 concludes. The Appendix, including proofs of lemmas and propositions that are not included in the body of the paper, follows before the list of references.

2 The New Zealand Data and Related Literature

Following a series of economic reforms in the late 1980s and early 1990s (see Evans et al., 1996, for a quick summary), New Zealand's income inequality and GDP growth rate have changed dramatically over the last two decades compared to its long history of a relatively stable path of development in earlier decades. At the same time, we note that New Zealand immigration policies in the 1990s led to discrete changes in the net inflow of skilled and unskilled immigrants (see, e.g., Winkelmann and Winkelmann, 1998). One would expect such changes in the mix of immigrants to residents to alter the underlying degree of heterogeneity in the distribution of human capital as well. The government has also changed the average marginal income tax rate significantly on more than one occasion in the 1990s and 2000s and that, according to Pearce and Dorling (2006), has altered the distribution of human capital as well as income inequality in New Zealand significantly. In our model economy, both of the above changes would likely produce significant changes in income inequality and growth rate. Consequently, we expect that the New Zealand experience would shed light on the origin and relative strengths of various channels of growth-inequality relationships if we filter the relevant data using a benchmark model for the New Zealand economy.

With that objective we first examine data from the New Zealand economy on: (1) income inequality, (2) growth rate of real GDP per capita, (3) the proportion of skilled and unskilled labor in the labor force, and (4) the average marginal tax rate (MTR). We chose the period between 1992 and 2007 for our study, primarily because that was the time when most changes took place. Secondly, we calibrate the model's parameter values to match relevant statistics from the New Zealand economy with the model's outcome. Thirdly, we discuss effects on the relationship between inequality and growth through impulse response analysis of shocks to the model's unique balanced growth state, (i) from changes in the degree of heterogeneity in the human capital distribution due to discrete changes in net immigration flows that capture New Zealand experience, and (ii) from changes in the progressivity of a redistributive policy package around values comparable to the data for the 1990s New Zealand economy.

2.1 Income Inequality

First, we present data on income inequality. Figure 1 below plots annual data on the measure of income inequality defined in Benabou (2002).[2] It shows how New Zealand income inequality has changed during the last two decades. The solid and dotted lines in the following Figure represent the actual and HP[3] filter-smoothed data respectively.

[2]Income inequality, following Benabou (2002), is defined as the logarithm of the ratio of mean to median income. Source: This annual data is constructed from New Zealand Income Survey and Census Data for 1992, 1996.

[3]In Figures 1 and 2, we include the smoothed line to identify changes in the trend of inequality and growth rate using the HP filter with the smoothing parameter value set to 10.

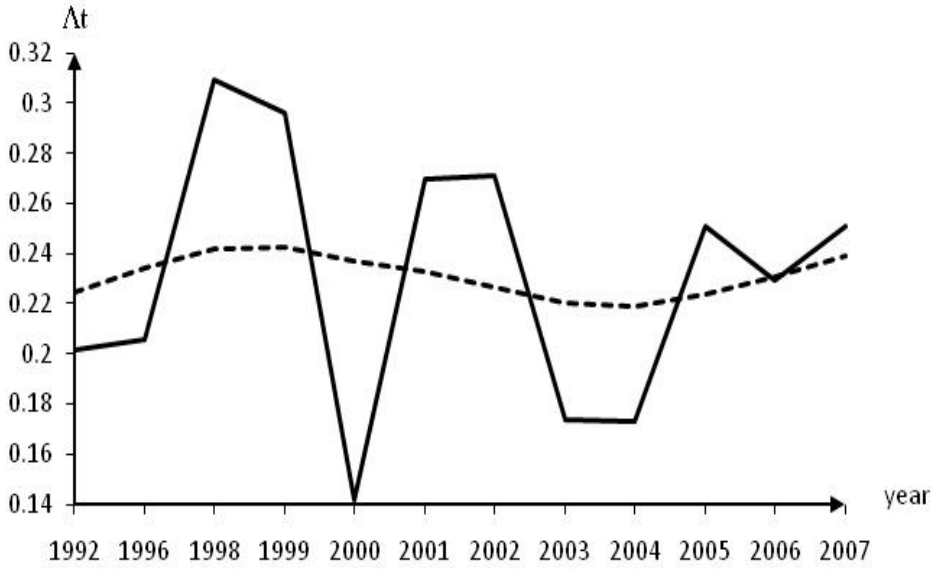


Figure 1—Annual data on the income inequality in New Zealand from 1992 to 2007.

In Figure 1 above, we use the dotted trend line, smoothed by HP filter, to identify three different phases. We note that income inequality increases by about 50%, from a low value of 0.20 in 1992 to a high value of 0.30 in 1999 (phase 1), and then decreases by about 40% until 2004 (phase 2) to a low value of 0.18, and has been increasing again since 2004, reaching a mark of about 0.25 in 2007 by about 45% (phase 3).

2.2 The Growth Rate

We also note significant and discrete jumps in growth rates of per capita income from Figure 2 below. The solid and dotted lines represent actual and HP filter-smoothed data respectively.

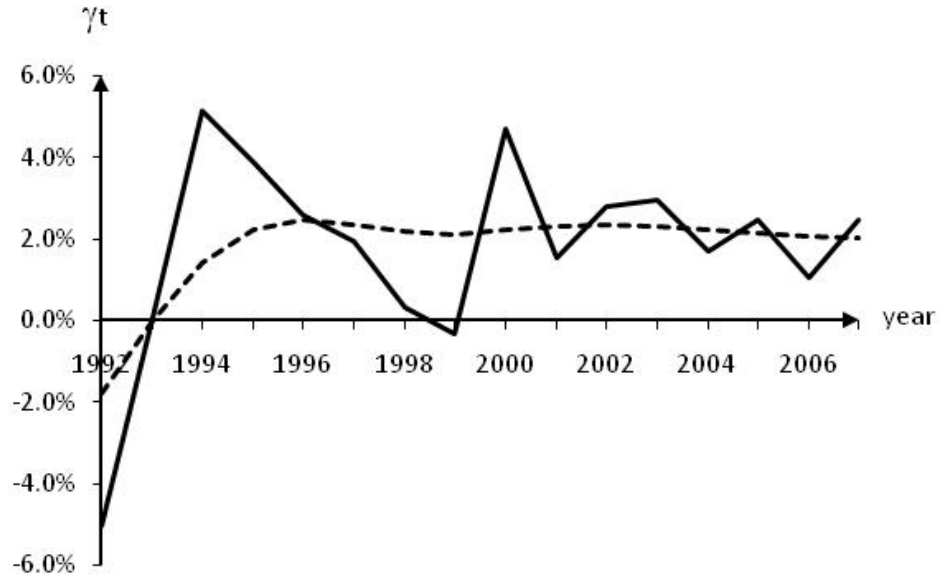


Figure 2—The average annual growth rate of New Zealand GDP from 1992 to 2006.[4]

We also note from Figure 2 that the growth rate increased in the first half of phase 1, from 1992 to 1994, from -5% to 5.2%, and then decreased in the second half of phase 1, from 1994 to 1999, from 5.2% to -0.3%. But, overall, from the dotted line, we can see the growth rate increases significantly. In phase 2, the growth rate increases from 1999 to 2003, followed by a small fluctuation.

2.3 Heterogeneity in Human Capital Distribution

The degree of heterogeneity in the human capital distribution requires a proxy. In this paper, following Borjas' (2003) finding that both schooling and work experience determine the immigrant's human capital, we choose the variance of skill composition of net immigrants to New Zealand as the proxy for that economic parameter, assuming that such heterogeneity can be largely identified with the profile of the distribution of occupational skill.

Based on the information on the occupations of the migrants available from the arrival or departure cards, Statistics New Zealand provide data on migrants after associating them into various occupational categories. Some of the occupations are basic and do not require specialized or high levels of education, while others do. Based on such information on the nature of different occupations provided by Statistics New Zealand, we divided all occupations into two discrete categories: skilled and unskilled. In the following two

[4]Source: These annual data are from Statistics New Zealand, Table 6.1, Series: SNCA.S6RB01NZ.

graphs, we use annual data to show how the distribution of residents and net immigrants into skilled and unskilled categories changed between 1992 and 2007.

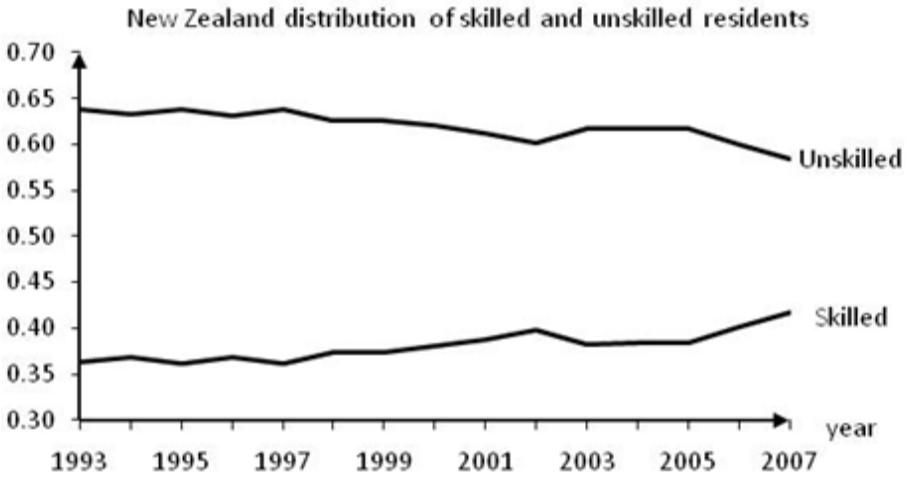
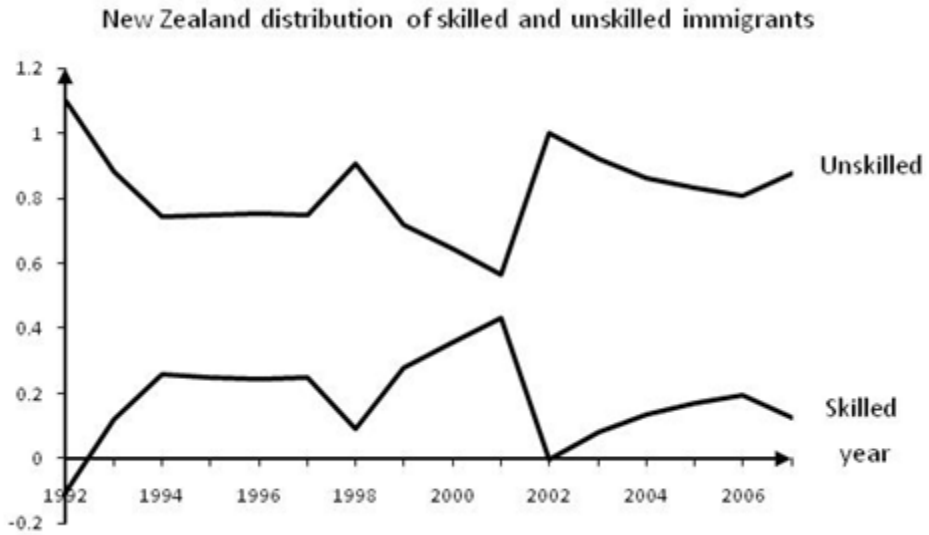


Figure 3a—Distribution of skilled and unskilled residents of New Zealand in each year ranging from 1992 to 2007[5].



[5]Source: These annual data are from Household Labour Force Survey, New Zealand, Table 4.05.

Figure 3b—Distribution of skilled and unskilled immigrants to New Zealand in each year ranging from 1992 to 2007[6].

First, by comparing Figures 3a and 3b, we conclude that the changes in the distribution of skilled and unskilled immigrants correspond to similar changes in the distribution of skill among New Zealand residents. That is because New Zealand is a small country with relatively large immigration flows. The composition of immigrants has had a significant influence on the distribution of income in New Zealand as well. Hence, the change in the distribution of skill among the immigrants could have had a significant impact on the underlying degree of heterogeneity in the human capital distribution in New Zealand.

Second, in phase 1 from 1992 to 1999, we can see that the percentage gap between skilled and unskilled labor decreases. The size of the decline was about 5% among all resident labor and more than 100% among the immigrants. After 2001, the gap increased significantly and discretely.

2.4 Progressivity

Data on the progressivity of various redistributive policy packages pursued in New Zealand is measured by the income weighted average marginal income tax rate for each year, the data for which is readily available from the OECD database.[7]

In Figure 4 below we plot the income weighted average marginal income tax rates for each year to show how progressivity in the redistributive policy package makes discrete jumps more than once during the period of our study: 1992-2007.

[6]Note that immigrants here refer to the net immigrants. It equals the number of arrivals minus the number of departures. Source: This annual data is from Statistics New Zealand, Series: EMIA.S13EZ1–EMIA.S13EZ9 for arrivals, EMIA.S2ZEZ1–EMIA.S2ZEZ9 for departures.

[7]We construct the data using Table 1.4 of the following link to the OECD tax database: http://www.oecd.org/document/60/0,3343,en_2649_34533_1942460_1_1_1_1,00.html

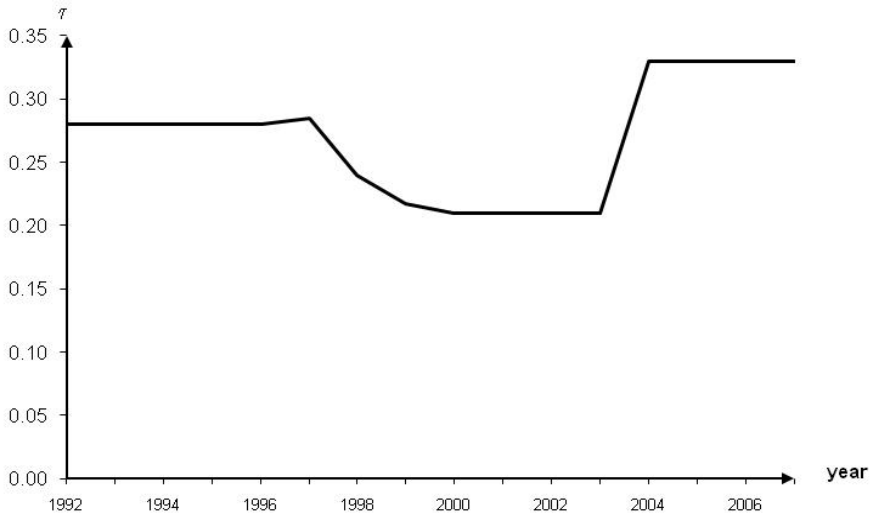


Figure 4—New Zealand average marginal tax rate from 1992 to 2007[8].

We note that in phase 1 and, in particular, between 1996 and 1999, progressivity decreases from 0.285 to 0.218, by about 24%. In phase 2 from 2002 to 2004, it increases by more than 50%.

2.5 Growth-Inequality Relationships

To sum up, in phase 1 (1992 to 1999) we see that income inequality increases by about 50%. This increase corresponds mainly to the decrease in progressivity. In general, the growth rate increases from a negative average to a positive average in this period. Thus we observe a positive relationship between inequality and the growth rate in phase 1.

In phase 2 (1999 to 2004), income inequality decreases by about 40% while the growth rate increases. This outcome corresponds to a discrete increase in progressivity as well as an increase in the heterogeneity of the human capital distribution. The relationship between inequality and growth appears to be negative in this phase.

In phase 3 (2004 to 2007), income inequality increases by about 45%, while the growth rate decreases slightly. The relationship between inequality and growth is weakly negative.

[8]Source: These annual data are from OECD, Table 1.4.

3 The Model

This section provides a theoretical foundation for the key empirical findings and, in particular, derives an explicit dynamic relationship between income inequality and the growth rate of per capita income as an equilibrium outcome of a standard dynamic general equilibrium model popularly used in the literature (e.g., Benabou, 2002, and Zhang, 2005).

3.1 Endowment, Technology and Preference

We consider a model with a continuum of infinitely lived islanders indexed by $i \in [0,1]$ with initial endowments of expertise or human capital h_0^i and intermediate capital inputs k_0^i for production such that the distribution of endowments is jointly lognormally distributed. Each islander i is also endowed with one unit of labor in each period $t = 0, 1, 2,....$which she divides between leisure and work-effort e_t^i from home as a self-employed unit. She uses intermediate capital inputs such as fertilizer, seeds or pencils and papers that perish completely during the production process and operates a technology from home such as working from home or farming in her backyard. We assume that the output y_t^i of a self-employed islander i, at each date t, as a function of her work-effort e_t^i, her intermediate capital input k_t^i and her expertise or human capital h_t^i is given by:

$$y_t^i = \left(k_t^i\right)^{\lambda} \left(h_t^i\right)^{\mu} \left(e_t^i\right)^{\varepsilon}, \text{ where, } \varepsilon = 1 - \lambda - \mu. \tag{1}$$

She invests in her schooling to update her expertise, part of which loses relevance over time. She also benefits from the accumulation of human capital by others, which we call knowledge spillover. We assume that as a function of her date t expenditure on schooling s_{1t}^i, which includes government subsidy and the knowledge spillover externality κ_t, her human capital stock evolves over time as follows:

$$h_{t+1}^i = \phi \kappa_t \xi_{t+1}^i \left(h_t^i\right)^{\alpha} \left(s_{1t}^i\right)^{\theta}, \alpha, \theta \in (0,1), \tag{2}$$

where ξ_t^i is an $i.i.d.$shock to her efficiency in using or accessing her own human capital for benefiting from knowledge spillover with $\ln \xi_t^i \sim N\left(-\sigma^2/2, \sigma^2\right)$, where σ^2 is a constant. The knowledge spillover externality[9] κ_t increases with the average stock of human capital in the economy such that

$$\kappa_t \equiv \left(\int_0^1 \left(h_t^i\right)^{\mu} di\right)^{\delta/\mu}, \delta \geq 0. \tag{3}$$

Being self-employed with no durable assets to mortgage against, like most small-scale, owner-operated firms in New Zealand, she faces severe borrowing constraints, and we

[9]Benabou (1996), Tamura (1991) and Davies, Zhang and Zeng (2005) uses similar externality in human capital accumulation process.

assume that she uses her own savings from the previous period to get intermediate capital inputs for home production.[10]

$$k_{t+1}^i = s_{2t}^i. \tag{4}$$

Following the progressive tax system in New Zealand and in most other countries, the government in our model economy has a scheme of progressive income taxation and transfer such that the disposable income of a typical agent at a date t satisfies

$$\hat{y}_t^i \equiv \left(y_t^i\right)^{1-\tau} \left(\tilde{y}_t\right)^{\tau}, \tag{5}$$

where τ measures the average marginal income tax rate and $\tilde{y}_t$ represents the break-even level of income such that the balanced-budget constraint satisfies

$$\int_0^1 \hat{y}_t^i di = \int_0^1 y_t^i di \equiv y_{t,} \tag{6}$$

where y_t denotes the per-capita output or income at a given period t. The government also provides sufficient subsidy to schooling to offset adverse effect of income tax on schooling and finance it with consumption taxes such that the total amount of schooling subsidy and consumption tax are monotone functions of τ which serves as the policy parameter in our model.

Preferences are given by:

$$v_0^i = E_0 \left[\sum_{t=0}^{\infty} \beta^t u_t^i\right], \text{ where, } u_t^i = \ln z_t^i - (e_t^i)^{\eta}, \eta > 1, \tag{7}$$

$z_t^i > 0$ and $e_t^i \geq 0$ denote, respectively, consumption and work effort of the agent i in period t and $\beta \in (0,1)$ denotes the discount factor. The date t budget constraint of the agent i satisfies

$$\ln z_t^i = \ln(\hat{y}_t^i - s_{1t}^i - s_{2t}^i) - z_0, \tag{8}$$

where z_0 denotes a constant consumption tax rate.

3.2 *Individual Optimization*

Let m_{ht}, m_{kt} denote the means and Δ_{ht}^2, Δ_{kt}^2 denote the variances of $\ln h_t^i$ and $\ln k_t^i$, respectively, cov_t denote the covariance between $\ln h_t^i$ and $\ln k_t^i$ and let M_t denote the vector $(m_{ht}, m_{kt}, \Delta_{ht}^2, \Delta_{kt}^2, cov_t)$. By (7)-(8), each agent faces a concave optimization

[10] We assume that our economy is a capital scarce economy and, like what we observe in a small open economy such as New Zealand, capital may come from foreign countries. We only require that capital goods are perishable and that islanders cannot borrow. Consequently, they must use their own saving to buy capital goods, wherever they come from, home or abroad.

exercise such that each date t, given the state variables, $(h_t^i, k_t^i, M_t; \tau)$, and the control variables, $(e_t^i,\ s_{1t}^i,\ s_{2t}^i)$, the Bellman equation associated with the above exercise satisfies**:**

$$v\left(h_t^i, k_t^i, M_t; \tau\right) = \max_{e_t^i, s_{1t}^i, s_{2t}^i} \left\{ \begin{array}{l} (1-\beta)\left[\ln(\hat{y}_t^i - s_{1t}^i - s_{2t}^i) - z_0 - (e_t^i)^\eta\right] \\ +\beta E_t\left[v(h_{t+1}^i, k_{t+1}^i, M_{t+1}; \tau)\right] \end{array} \right\}, \tag{9}$$

subject to (1)-(5).

Lemma 1: *The first order conditions of the above Bellman equation imply:*

$$e \equiv \left(\frac{\left((1-\lambda-\mu)/\eta\right)(1-\beta\alpha)(1-\tau)}{(1-\beta\alpha)(1-\beta\lambda(1-\tau)) - \beta\theta\mu(1-\tau)} \right)^{1/\eta}, \tag{10}$$

$$s_{1t}^i = \frac{\theta\beta\mu}{1-\beta\alpha}\hat{y}_t^i, \tag{11}$$

$$s_{2t}^i = \beta\lambda(1-\tau)\,\hat{y}_t^i. \tag{12}$$

Proof: See Appendix.

We now describe the model's equilibrium and the associated dynamics before going to the section on quantitative analysis.

3.3 Implications of the Optimal Decision Rules

The above optimization exercise implies that at each date t, by (4), (1), (5), (10) and (12), and by (2), (1), (5), (10) and (11), respectively, the model's two agent-specific state variables k_t^i and h_t^i satisfy:

$$\begin{aligned} \ln k_{t+1}^i = {} & \ln\beta\lambda(1-\tau) + (1-\lambda-\mu)(1-\tau)\ln e + \lambda(1-\tau)\ln k_t^i \\ & + \mu(1-\tau)\ln h_t^i + \tau\ln\tilde{y}_t, \end{aligned} \tag{13}$$

$$\begin{aligned} \ln h_{t+1}^i = {} & \ln\phi + \ln\kappa_t + \theta\ln\frac{\theta\beta\mu}{1-\beta\alpha} + \theta(1-\lambda-\mu)(1-\tau)\ln e + \ln\xi_{t+1}^i \\ & + \theta\lambda(1-\tau)\ln k_t^i + (\alpha + \theta\mu(1-\tau))\ln h_t^i + \theta\tau\ln\tilde{y}_t. \end{aligned} \tag{14}$$

We define, for each date t, an index of income inequality Λ_t as the logarithm of the ratio of mean to median income, following Benabou (2002). The following lemma describes the implication of the individual decision rules together with the government budget constraint on the distribution of income, income inequality and the break-even level of income.

Lemma 2: *a. The evolution of earnings of adults must be governed by a lognormal distribution such that* $\ln y_t^i \sim N\left(\lambda m_{kt} + \mu m_{ht} + (1-\lambda-\mu)\ln e, 2\Lambda_t\right)$*, where,*

$$\Lambda_t = \left(\lambda^2\Delta_{kt}^2 + \mu^2\Delta_{ht}^2 + 2\lambda\mu cov_t\right)/2. \tag{15}$$

b. The break-even level of income $\tilde{y}_t$ *at which an agent's net tax obligation is zero satisfies:*

$$\begin{aligned} \ln\tilde{y}_t &= \ln y_t + (1-\tau)\Lambda_t \\ &= \lambda m_{kt} + \mu m_{ht} + (1-\lambda-\mu)\ln e + (2-\tau)\Lambda_t. \end{aligned} \tag{16}$$

Proof: See Appendix.

Next, we describe what the optimal decision rules imply on the sequence of vector $\{M_t\}$ of the economy wide state variables, as defined earlier in the description of the Bellman equation. Given the initial condition, M_0, implied by the initial jointly lognormal distribution of physical and human capital, by (13), (14), and the optimal decision rules, (10)–(14), imply that, at each date t, the sequence of vector M_t must satisfy:

$$m_{kt+1} = \ln\beta\lambda(1-\tau) + (1-\lambda-\mu)\ln e + \lambda m_{kt} + \mu m_{ht} + \tau(2-\tau)\Lambda_t, \tag{17}$$

$$\Delta_{kt+1}^2 = 2(1-\tau)^2\Lambda_t, \tag{18}$$

$$\begin{aligned} m_{ht+1} = \ln\phi - \sigma^2/2 + \theta\ln\frac{\theta\beta\mu}{1-\beta\alpha} + \theta(1-\lambda-\mu)\ln e \\ + \theta\lambda m_{kt} + (\alpha+\theta\mu+\delta)m_{ht} + \delta\mu\Delta_{ht}^2/2 + \theta\tau(2-\tau)\Lambda_t, \end{aligned} \tag{19}$$

$$\begin{aligned} \Delta_{ht+1}^2 = \sigma^2 + \theta^2\lambda^2(1-\tau)^2\Delta_{kt}^2 + (\alpha+\theta\mu(1-\tau))^2\Delta_{ht}^2 \\ + 2\theta\lambda(1-\tau)(\alpha+\theta\mu(1-\tau))cov_t, \end{aligned} \tag{20}$$

$$\begin{aligned} cov_{t+1} = \theta\lambda^2(1-\tau)^2\Delta_{kt}^2 + \mu(1-\tau)(\alpha+\theta\mu(1-\tau))\Delta_{ht}^2 \\ + \lambda(1-\tau)(\alpha+2\theta\mu(1-\tau))cov_t, \end{aligned} \tag{21}$$

3.4 Definition of Equilibrium

We define the model's equilibrium as the set of decision rules described by (10)–(14) for the islanders and the sequence of vector $\{M_t\}$ of state variables described by (17) - (21) for the economy such that, for each islander, the decision rules satisfy the optimization

problem given by the Bellman equation (9) and the sequence of vector $\{M_t\}$ that the agent i takes as given in (9) coincides with the solutions to the dynamical system defined by (17) - (21). In other words, the equilibrium sequence of vector of aggregate state variables $\{M_t\}$ solves a fixed point problem such that, at each date $t = 0, 1, 2, ..,$ individual decision rules as functions of $\{M_t\}$ meet the following aggregate consistency condition:

$$\int_0^1 y_t^i di = \int_0^1 z_t^i di + \int_0^1 s_{1t}^i di + \int_0^1 s_{2t}^i di. \tag{22}$$

3.4.1 A note on Competitive Equilibrium

Our definition of equilibrium can be mapped to that of a competitive equilibrium for a quick comparison. The two alternative cases for comparison are: the case I with self-employed islanders who work from home with a backyard technology (BT) and the case II with wage earners who supply their human capital weighted labor and rent their capital acquired by last period's saving to a fictitious firm which maximizes profit in a competitive environment (CE). To map BT into CE such that we preserve the mean value of the per capita output to be the same under both cases and we make the following transformation: by omitting t subscript for simplicity and by denoting, $l^i = (e^i)^{\varepsilon}$ as one's effective labor in production and $\tilde{h}^i \equiv \left(\frac{h^i}{l^i}\right)^{\frac{\mu}{1-\lambda}}$ as the efficiency unit of labor, we rewrite (1) with a subscript BT to denote the context of an islander's technology as

$$y_{BT}^i = \left(k^i\right)^{\lambda} \left(\tilde{h}^i l^i\right)^{1-\lambda}, \tag{23}$$

and assume that the representative firm operates in the competitive environment (CE) as if it uses the following technology:

$$y = A(M)k^{\lambda}H^{1-\lambda}, \text{ where, } k = \int_0^1 k^i di,\ H = \int_0^1 \tilde{h}^i l^i di \text{ and, } y = \int_0^1 y_{BT}^i di. \tag{24}$$

and, $A(M)$ denotes the endogenous total factor productivity (TFP) for the representative firm in the economy as a function of economy's state variable M.[11]

Given a rental price of r per unit of capital goods, and a wage rate w per efficiency unit of labor, the representative firm solves the following exercise:

$$\max_{k,H} \left(y - rk - wH\right), \tag{25}$$

and the first order conditions of the above exercise satisfy $r = \lambda y/k$ and $w = (1-\lambda)\, y/H$. Consequently, in period t the earning of the agent i in the CE environment would be given

[11] It turns out that only the variances and the covariance but not the means of the state vector M determine the value of the TFP $A(M)$ of the competitive firm in our model economy.

by,

$$y^i_{CE} = rk^i + w_t\tilde{h}^i l^i \equiv \chi^i y, \text{ where, } \chi^i \equiv \lambda\left(\frac{k^i}{k}\right) + (1-\lambda)\left(\frac{\tilde{h}^i l^i}{H}\right). \tag{26}$$

To summarize, the islander i's earning from the backyard technology is given by $y^i = y^i_{BT}$, which is described by (23) and, her earning from a competitive environment is given by $y^i = y^i_{CE}$, which is described by (26) such that by construction per capita income does not vary across BT and CE. Thus we complete our mapping of BT into CE.

We keep the remaining part of the model economy identical for the two cases, BT and CE and, in particular, assume that the joint distribution of the initial endowments of human and physical capital among the islanders is lognormal. We motivate this assumption, for empirical reasons. The above assumption reproduces the commonly observed right-skewed income distribution as the model's equilibrium outcome. However, the sum of the two lognormal distributions is not lognormal and for that reason alone the income process given by (26) makes it impossible to get explicit analytical expressions of the CE outcome that we need for interpreting our findings from various quantitative exercises in a transparent way and that violates our key objective for this paper. At the same time, our key results only concern the growth rate of per capita output and its relationship with income inequality and even though not exact, the sum of the two lognormal distributions does closely mimic a lognormal distribution.[12] Consequently, to meet our objective of providing a consistent and transparent interpretation to the simulation results of our quantitative analysis of the 1990s New Zealand history concerning the so-called growth-inequality relationship, we focus on deriving explicitly analytical properties of equilibrium in a BT set-up rather than settling only for numerical simulations of a CE economy. Our strategy successfully delivers new insights regarding economic dynamics behind multifaceted growth-inequality relationships without any loss of generality but which we typically fail to discern in a purely simulation based results.[13]

[12]The distribution of the sum of lognormal random variables has no exact expression. It is widely accepted that the sum of two log-normal distributions is approximated by another lognormal distribution, such as Naus (1969, 1973), Safak (1993), Cardieri and Rappaport (2000), Yue (2000) and Mehta, Molisch, Wu and Zhang (2006). It confirms our conclusion that the dynamic property of the correlation between inequality and growth under BT and CE environment does not vary significantly so that we can use BT, which can give us explicit analytical solutions, to characterize inequality-growth correlation. The policy implications arisen could also shed light on CE environment.

[13]Also, note that as long as we maintain our missing credit market assumption we can allow a free flow of capital across national border for a small open economy like New Zealand to extend the scope of our analysis, but without compromising our key results. In particular, the borrowing constraints imply that capital must be acquired with past saving, such that $k^i_t = s^i_{t-1}$ and $k_t = s_{t-1} \equiv \int s^i_{t-1}di$. Clearly, in the BT set-up the key results remain unchanged. In the CE set-up the world interest rate r^*_t pins down the equilibrium capital stock per capita k^*_t while the islander's saving remains unaffected by r^*_t as our log utility assumption ensures exactly offsetting income and substitution effects of r^*_t. Consequently, the date t net capital inflow $nci^*_t = k^*_t - s_{t-1}$ ensures the CE firm maintains its profit-maximizing capital stock and allows islanders to earn a competitive return by investing abroad if $k^*_t < s_{t-1}$. As the capital income of

4 Analytical Results

In this section we present a few key analytical results. They help us to explore how our model's assumptions influence our conclusions. These analytical results also help us later to identify specifically various economic mechanisms that drive our findings based on numerical simulations.

4.1 Model's Growth-Inequality Relationship

Recall from Section 3.3 that, by Lemma 2, the income inequality index Λ equals the variance of pre-tax income. The following Lemma states the key analytical result for this paper involving a generalized dynamic relationship between that above measure of income inequality together with variance of human and physical capital on one hand and the growth rate of per capita income on the other hand.

Lemma 3: If and only if $(1-\alpha-\delta)(1-\lambda)-\mu\theta=0$[14] *then the equilibrium outcome gives rise to perpetual endogenous growth accompanied by an explicit and endogenous relationship between income inequality and the growth rate of per capita income,* $\gamma_t \equiv \ln y_{t+1} - \ln y_t$*, such that*

(27)

$$\gamma_t = \frac{1}{1-(\alpha+\delta)\lambda L}\left(\begin{array}{l} \Phi+(1-\alpha-\delta)\lambda\ln(1-\tau)+(1-\alpha-\delta)(1-\lambda-\mu)\ln e \\ +\Lambda_{t+1}-\left(\delta+\alpha+(\lambda+\mu\theta)(1-\tau)^2\right)\Lambda_t \\ +\alpha\lambda\Delta_{kt}^2/2+\delta\left(\lambda\Delta_{kt}^2/2+\mu^2\Delta_{ht}^2/2\right) \end{array}\right),$$

where $\Phi \equiv \mu\left(\ln\phi-\sigma^2/2+\theta\ln\frac{\theta\beta\mu}{1-\beta\alpha}\right)+(1-\alpha-\delta)\lambda\ln\beta\lambda$*,* e *is given by (10) and* L *denotes lag operator.*

Proof: See Appendix.

Equations (15)-(21), together with (27), describe the model's unique dynamic relationship between income inequality and the rate of growth. In the long run, the above transitional dynamics approaches a unique ergodic limiting distribution where both the income inequality and the per capita income growth rate remain stationary. We summarize this feature in the following Proposition.

---

the islanders remains $r_t^* s_{t-1}^i$, by (26), the key results remain unchanged with one caveat. We assume that the variable nci_t^*, the difference between the GNP, which our BT set-up accounts for, and the GDP, which the CE set-up measures, is either stationary or the fluctuations in its growth rate do not depend significantly on the income distribution of the home country.

[14] Substituting this condition into (11), we can reach the same conclusion as Zhang (2003), in which he found that the private saving rate for education decreases with the degree of externality.

PROPOSITION 1: *From any arbitrary initial state, the equilibrium sequence of* Δ^2_{kt+1}, Δ^2_{ht+1}, cov_{t+1}, Λ_t *and* γ_t, *described by (18), (20), (21), (15) and (27) converges monotonically to its unique ergodic steady state.*

Proof: See Appendix.

Next, we elaborate on the equation (27) to establish a few analytical results on dynamics of the growth-inequality relationship that arise as equilibrium outcomes of our model economy. In particular, we show how our missing credit market assumption influences a negative growth-inequality relationship and how a scheme of progressive redistribution of income which utilizes public subsidy to education to offset accompanying economic distortion contributes to a positive growth-inequality relationship. Also, we explore the importance of other assumptions such as external spillover of knowledge and non-tradable capital goods by examining if and how they may influence the logical foundation of our analysis regarding these multifaceted growth inequality relationships.

4.1.1 A Negative Growth-Inequality Relationship

We choose the parameter σ^2, which directly influences the variance of efficiency of human capital and thereby variance of expertise, to proxy for the degree of heterogeneity in the human capital distribution. As discussed earlier a discrete change in immigration policy (an immigration shock) could conceivably change the value of σ^2, as it may fundamentally alter the characteristics of the population.

Importance of the missing credit market The following lemma brings out the implication for our assumption of the missing credit market together with diminishing returns technologies for production of output and human capital. The absence of a credit market creates rigid and inefficient interpersonal differences in marginal productivity of human and physical capital. In the presence of diminishing returns technology for accumulating human capital and production of output as the variance of productivity increases the average output decreases. Consequently, in such an environment any economic factor that increases heterogeneity among the production units would lower per capita output and hence its growth rate in the present context. At the same time a greater heterogeneity would imply larger income inequality. Thus the outcome for changes in income inequality and growth rate are exactly opposite. In presence of a credit market, on the other hand, a larger inequality or a larger productivity difference would foster among the agents mutually beneficial exchanges and higher growth. Thus inequality and growth will be positively related in the presence of a credit market. The following lemma illustrates how the missing credit market plays a key role in giving rise to a negative growth-inequality relationship.

Lemma 4: If, in a specific period, the variance of expertise among the production units increases, then, in that same period, the growth rate decreases while income inequality increases.

Proof: Substituting (18), (20) and (21) into (15) yields

$$\begin{aligned}\Lambda_{t+1} &= \mu^2\sigma^2/2 + \left((\lambda+\mu\theta)(1-\tau)+\alpha\right)^2\Lambda_t \\ &\quad -\alpha\left(\alpha/2 + (\lambda+\mu\theta)(1-\tau)\right)\lambda^2\Delta_{kt}^2 \\ &\quad -\alpha\left(\alpha + (\lambda+\mu\theta)(1-\tau)\right)\lambda\mu cov_t.\end{aligned} \tag{28}$$

Then substituting (28) into (27) and differentiating w.r.t σ^2, we get

$$\frac{\partial\gamma_t}{\partial\sigma^2} = -\mu(1-\mu)/2 < 0. \tag{29}$$

The above equation shows that if σ^2 increases, γ_t will decrease. In contrast, by (28), we see that if σ^2 increases, Λ_{t+1} increases. It then implies that the correlation between inequality and growth is negative when there is an immigration shock. □

Importance of the knowledge spillover externality We make a concluding observation for this section to shed light on how our modeling assumptions influence our results. Our assumption of a positive knowledge spillover parameterized by $\delta > 0$ turns out to be critical for generating a negative growth-inequality relationship. If $\delta = 0$ then our endogenous growth condition, as stated in Lemma 3, would require that either the production technology or the human capital accumulation technology must exhibit increasing returns; but that would offset the possibility of a negative growth-inequality relationship which typically requires a unique combination of assumptions of a missing credit market and technologies with diminishing returns.

Thus our model accommodates scenarios in which an increase in income inequality retards future economic growth. These special scenarios of growth-retarding inequality coincide with the empirical findings of Persson and Tabellini (1994). However, they provide alternative and wider interpretations of the data since, unlike Persson and Tabellini (1994), the effect of past income inequality on the future growth rate does not just work through changes in the income tax rate parameter τ. The equation (27) demonstrates explicitly that the effect of past income inequality also works through changes in various other economic factors, including the variance σ^2 of efficiency in the human capital usage, the knowledge spillover externality δ and, other parameters that influence the extent of diminishing returns in the production technologies for output and human capital. As per furthering future empirical research, the model's explicit transitional dynamics as described by (27) provide an alternative restriction on the growth-inequality data compared to what Persson and Tabellini (1994) used for their regressions.

4.1.2 A Positive Growth-Inequality Relationship

The economic dynamics summarized in (27) sometimes generate also a positive growth-inequality relationship. Typically, they do so in response to a fiscal policy shock that changes the progressivity parameter τ which accompanies some offsetting subsidy to private expenditure on education. An increase in τ decreases steady state income inequality and thereby pushes it below its current state. Consequently, by Proposition 1, income inequality monotonically decreases to its new steady state. Next, we make it clear that our income inequality index does indeed decrease with an increased degree τ of progressivity of redistribution.

Lemma 5: A higher value of progressivity parameter τ corresponds to a monotonically decreasing sequence of income inequality which converges to a new steady state characterized by a lower index Λ of income inequality.

Proof: By (18), (20) and (21), we establish that in response to a higher value of τ in a specific period, the values of Δ_{kt}^2, Δ_{ht}^2 and cov_t decrease in all subsequent periods and, by Lemma 2, income inequality follows the same path. Also, by Proposition 1, it decreases monotonically to its new steady state. □

However, an increase in τ creates two opposing effects on the growth rate. On one hand, by lowering income inequality, it fosters growth but, on the other hand, by discouraging work-effort and saving, it slows down growth. The overall effect is ambiguous. The following lemma characterizes a condition when a change in τ implies a positive growth-inequality relationship in subsequent periods.

Importance of government subsidy to investment in education In most developed countries, including New Zealand, public subsidy on education goes hand in hand with a progressive redistribution of income as an integral component for redistribution. The public policy of education subsidy is designed in the model economy to offset the negative incentive effect of income tax on investment in education. A greater progressivity accompanied by such offsetting subsidy works to redistribute income in the model economy by increasing investment in education for those whose disposable income lies below the break-even income and by decreasing investment for those whose disposable income is above the break-even income. The following lemma summarizes this point.

Lemma 6: $\frac{\partial s_{1t}^i}{\partial \tau} \leq 0$ *if and only if* $y_t^i \geq \tilde{y}_t$.

Proof. By (11) the gross investment in education that includes public subsidy is a constant fraction of one's disposable income and by the design of this subsidy that fraction is independent of the progressivity parameter τ.

Differentiate $\ln s_{1t}^i$ w.r.t τ yields

$$\begin{aligned}\frac{\partial \ln s_{1t}^i}{\partial \tau} &= \frac{\partial \ln \hat{y}_t^i}{\partial \tau} \\ &= \frac{\partial \left((1-\tau)\ln y_t^i + \tau \ln \tilde{y}_t\right)}{\partial \tau} \\ &= \ln \tilde{y}_t - \ln y_t^i\end{aligned}$$

The above equation shows that if and only if $y_t^i \geq \tilde{y}_t$, then $\frac{\partial \ln s_{1t}^i}{\partial \tau} \leq 0$. □

In other words, the redistributive package consisting of a progressive income tax and a proportional education subsidy together redistributes allocation of resources to education from the rich with income above the threshold $\tilde{y}_t$ to the poor with income below $\tilde{y}_t$. The following two lemmas show that in the absence of a subsidy on saving or on work, the negative effect of redistribution on these two variables is more pervasive in the population than on the investment in education.

Lemma 7: A greater progressivity reduces work effort and the desired saving for a larger fraction of the population.

Proof. By (10) the negative effect of τ on work effort follows in a straightforward way. By (12), it follows that unlike investment in education, both rate of saving and gross saving are functions of the progressivity parameter τ.

Differentiate $\ln s_{2t}^i$ w.r.t τ

$$\begin{aligned}\frac{\partial \ln s_{2t}^i}{\partial \tau} &= -\frac{1}{1-\tau} + \frac{\partial \ln \hat{y}_t^i}{\partial \tau} \\ &= -\frac{1}{1-\tau} + \frac{\partial \left((1-\tau)\ln y_t^i + \tau \ln \tilde{y}_t\right)}{\partial \tau} \\ &= -\frac{1}{1-\tau} + \ln \tilde{y}_t - \ln y_t^i \\ \frac{\partial \ln s_{2t}^i}{\partial \tau} &\leq 0 \text{ if and only if } \ln y_t^i \geq (\ln \tilde{y}_t - \frac{1}{1-\tau}) = \tilde{y}_t^-\end{aligned}$$

The above equation shows that the negative effect of income taxation not only lowers the saving of the rich, for whom the income exceeds $\tilde{y}_t$, but it also lowers saving of the lower middle class, whose income fall in the range between $\tilde{y}_t$ and $\tilde{y}_t$, and the size of this class increases with progressivity. Thus a higher progressivity increases the size of the population who respond with a decision to reduce saving. □

Lemma 8: There exists a threshold limit for progressivity τ^* such that if $\tau > \tau^*$ then $\frac{\partial \gamma_t}{\partial \tau} < 0$ and a change in τ in period t would imply a positive growth-inequality relationship subsequently.[15]

[15] $\tau^* = -\frac{1}{2\alpha\lambda}\left(\lambda + \theta\mu - 1 - 2\alpha\lambda + \sqrt{\lambda\left(\lambda + 2\left(2\alpha^2 + \theta\mu - 1\right)\right) + \left(1-\theta\mu\right)^2}\right)$

Proof: To analyze how an increase of progressivity parameter τ in period t affects the growth rate in that period, we differentiate γ_t, given by (27), w.r.t τ, and get

$$
\begin{aligned}
(30)\frac{\partial \gamma_t}{\partial \tau} &= -\frac{(1-\alpha-\delta)\,\lambda}{1-\tau} + (1-\alpha-\delta)\,(1-\lambda-\mu)\,\frac{\partial \ln e}{\partial \tau} \\
&\quad + \frac{\partial \Lambda_{t+1}}{\partial \tau} + 2\,(\lambda+\mu\theta)\,(1-\tau)\,\Lambda_t \\
&= -\frac{(1-\alpha-\delta)\,\lambda}{1-\tau} + (1-\alpha-\delta)\,(1-\lambda-\mu)\,\frac{\partial \ln e}{\partial \tau} \\
&\quad + \frac{\mu^2\,(\lambda+\theta\mu)\,((1-\tau)\,(1+\alpha\lambda\,(1-\tau)) - (\alpha+(\lambda+\theta\mu)\,(1-\tau)))\,\sigma^2}{(1-\lambda\alpha\,(1-\tau))\left((1+\lambda\alpha\,(1-\tau))^2 - ((\lambda+\theta\mu)\,(1-\tau)+\alpha)^2\right)}
\end{aligned}
$$

We note from the above equation that the first two terms are negative. They measure microeconomic disincentives of redistribution on saving and work-effort, respectively. The third term measures macroeconomic benefit from redistribution led productivity enhancement due to relaxing the credit constraints of the islanders with low stocks of physical and human capital but in an economy with diminishing returns in both production technology and human capital accumulation. This term could be positive with low progressivity and, in particular, if $(1-\tau)\,(1+\alpha\lambda\,(1-\tau)) - (\alpha+(\lambda+\theta\mu)\,(1-\tau)) > 0$. Clearly, if $\tau > \tau^*$ then the third term is also negative and hence $\frac{\partial \gamma_t}{\partial \tau} < 0$. We conclude that too large progressivity would be counterproductive, since the mechanism of relaxing the credit constraints ceases to be effective once everyone in the economy can afford to reach the unconstrained optimum. Any additional transfer to the poor would be simply consumed by them and that would be harmful to growth. Consequently, in an economy with too large progressivity, growth rates and progressivity would be inversely related and, by Lemma 5, inequality is always negatively related to progressivity. It follows, therefore, that if $\tau > \tau^*$ then we get a positive growth-inequality relationship. □

It is important to note, however, that the above lemma only provides a sufficient and not a necessary condition. Indeed, we can get a positive growth-inequality relationship even if $\tau < \tau^*$, provided the microeconomic disincentives on work-effort and saving outweigh the macroeconomic growth benefit from a reduction of interpersonal differences in productivity with income redistribution.

Importance of capital goods as complementary inputs to human capital The above conclusion that in an economy with a sufficiently high progressivity prior to a change in τ would generate subsequently a positive growth-inequality relationship does have one important caveat as outlined in the following lemma.

Lemma 9: Without physical capital (i.e., $\lambda = 0$) in the model, an increase in income inequality always retards future economic growth regardless the value of τ.

Proof: If $\lambda = 0$, by (28), we can get

$$\Lambda_{t+1} = \mu^2\sigma^2/2 + (\alpha + \mu\theta(1-\tau))^2 \Lambda_t. \tag{31}$$

Substituting (31) into (27) and rearranging yields

$$\begin{aligned} \gamma_t &= \Phi + (1-\alpha-\delta)(1-\mu)\ln e + \Lambda_{t+1} - \left(\alpha + \mu\theta(1-\tau)^2\right)\Lambda_t \\ &= \Phi + (1-\alpha-\delta)(1-\mu)\ln e + \mu^2\sigma^2/2 \\ &\quad - \left(\left(\alpha + \mu\theta(1-\tau)^2\right) - (\alpha + \mu\theta(1-\tau))^2\right)\Lambda_t, \end{aligned} \tag{32}$$

where $\Phi \equiv \mu\left(\ln\phi - \sigma^2/2 + \theta\ln\frac{\theta\beta\mu}{1-\beta\alpha}\right)$. As $\alpha + \mu\theta(1-\tau)^2 - (\alpha + \mu\theta(1-\tau))^2 > 0$, the above equation shows that the growth-inequality relationship is always negative. □

Lemma 9 justifies our inclusion of physical capital in the model in a way that significantly distinguishes our work from Benabou (1996, 2002) and others who find only a negative growth-inequality relationship in their model. In other words, by Lemma 9, we suggest that the absence of a positive growth-inequality relationship in those models follows from that the fact that they violate a necessary condition by not allowing the presence of complementary capital goods for augmenting marginal productivity of human capital.

In the following section, we do impulse response analysis of the model to discuss how two specific policy shocks, which receive enormous attention in New Zealand's policy debates (see, Evans et al., 1996), make their impact on income inequality and growth outcome and on the subsequent growth-inequality relationships in New Zealand's history of the late 1990s and early 2000s. The two specific policy shocks that stand out in the media debate arise from sudden, discrete and controversial changes in public policies related to immigration and income redistribution. In the following section we carefully design a benchmark model for the New Zealand economy. Within that calibrated model economy, we conduct controlled experiments based on numerical simulations to derive new insights that could potentially advance the quality of the economic policy debate regarding the so-called growth-inequality relationship.

5 Quantitative Analysis

In this section, we first calibrate the parameters of our model so as to reproduce key statistics from the New Zealand economy as our model's equilibrium outcome. In that benchmark model economy of New Zealand, we design a set of controlled experiments to carry out impulse response analysis of the model's equilibrium path corresponding to one specific policy shock at a time. Each shock corresponds to the specific history of events in New Zealand during the period ranging from 1992 to 2007. One advantage of our experiment is that we know everything of our model economy to be able to clearly isolate the effects of one shock from another. Also, following the description of each experiment, we

present interpretations of the results with an internally consistent storyline based on the analytical properties of the equilibrium of the model economy of New Zealand. Clearly, the model's interpretation of the impulse responses reflects a storyline which can have reasonable alternatives from other models that focus on other important shocks with different storylines. However, developing such an alternative that would utilize multiple shocks including those we consider in our DGE model of endogenous growth and income inequality remains an ongoing challenge for the seriously interested researchers.

5.1 Calibration of parameters

The key parameters needed are: the output elasticities of unskilled labor ε and of physical and human capital λ and μ, respectively; the autocorrelation of human capital parameter α; the effectiveness of the education system θ; the variance of human capital efficiency, σ^2; the human capital index, ϕ; the intertemporal elasticity of labor supply ϵ which identifies the preference parameter η; and each agent's discount factor β.

Production Parameters

Statistics New Zealand provides data on incomes for the self-employed and wage and salary earners. The sum of these values is regarded as the total labor income, while government transfers, benefits, business and rent losses are excluded. The ratio of the labor income to GDP was, on average, 0.45 during the period of our study. In our discussion on the equilibrium of our model, we show a 1-1 mapping between the economy of the self-employed with a backyard technology to that of a corresponding technology with the identical capital and labor shares but in a competitive environment. Consequently, the above methodology for estimating the backyard technology of our model is justified.

Next, we follow the methodology of Mankiw, Romer and Weil (1992) to specify shares of human capital μ and raw labor ε. Mankiw, Romer and Weil (1992) regard laborers who earn the minimum wage as having no human capital. The share of human capital is calculated by multiplying one minus the ratio of the minimum wage to the average wage by the labor share, i.e., $\mu = (1 - ratio) * labor\ share$. The ratio of the minimum wage to the average wage, according to the data from *Statistics New Zealand in 1996*, was 0.42. Therefore, we get $\lambda = 0.55$, $\mu = 0.26$ and $\varepsilon = 0.19$.

Inequality

Following Benabou (2002), we measure income inequality by using the logarithm of the ratio of the mean to median income. In New Zealand, according to the data from Statistics New Zealand's Household Economic Surveys, the logarithm of the ratio of the mean to median income decreased from 0.31 to 0.23 between 1998 and 2006. Using (15), we set $\sigma^2 = 4$, so that the feasible range of income inequality is [0.14, 0.33].

Balanced Growth Rate

According to Statistics New Zealand[16], the average GDP growth rate, based on 1995/96 prices per capita, between 1994 and 2004 was 2.4%, while, according to the New Zealand Treasury, the average marginal income tax rate in the corresponding period was about 26.3%. Then, by (27), to match the 2.4% long run growth rate with $\tau = 26.3\%$, we set human capital index $\phi = 23.7$ and $\delta = 0.58$.

Earning Return to Education

Our model provides Chiswick and Mincer (1972) type earning as a function of schooling as follows:

$$\ln y_{t+1}^i = \lambda \ln \left(s_{2t}^i\right) + \mu \ln \phi \kappa_t \xi_{t+1}^i \left(h_t^i\right)^\alpha + \mu\theta \ln \left(s_{1t}^i\right) + \varepsilon \ln e_t^i. \tag{33}$$

The earning return to education is $\mu\theta$. From Maani (1996), we know the rate of return to education was around 10%. Therefore, given $\mu = 0.26$, we get $\theta = 0.38$.

Labor Supply

Kalb and Scutella (2003), by using four separately estimated sets of discrete choice labor supply models, found that the average wage elasticities in New Zealand were 0.24, 0.40, 0.63 and 0.82, for married men, and women, and single men, and women, respectively. From the Statistics New Zealand 2001 Household Economic Survey, we have found that married men, married women, single men and single women account for more than 90% of the total labor in the market. Therefore, after multiplying the wage elasticities with their corresponding percentage of the labor population and taking the sum, the average labor supply elasticity of New Zealand is around 0.45. Equivalently, this means that $\eta = 3.2$.

Discount Factor

To have a time period t comparable to the average duration of the three distinct phases mentioned earlier we choose a time discount factor $\beta = 0.82$ that would be consistent with a 4% real interest rate.

Table 1 gives the benchmark parameter values[17].

[16]Data source: Statistics New Zealand, Table 6.1, Series SNCA.S6RB01NZ.

[17]Note that to satisfy the condition for endogenous growth we chose $\alpha = 1 - \delta - \frac{\mu\theta}{1-\lambda}$ =0.20.

TABLE 1
BENCHMARK PARAMETERS

λ	0.55
μ	0.26
ε	0.19
θ	0.38
ϕ	23.7
δ	0.58
σ^2	4
η	3.20
β	0.82

5.2 Numerical Experiments and Results

In this subsection, we report on the impulse response analysis of the income inequality and growth to two specific shocks. One is an immigration shock and the other one is a fiscal policy shock.

5.2.1 Immigration shock

First, through impulse response analysis, we discuss the effects of an immigration shock on the dynamic path of income inequality and the growth rate. Clearly, a change in immigration policy would most likely alter the distribution of characteristics of a country's population significantly. In particular, it could change the mean and the variance of the exogenous component of a country's human capital distribution. In our model economy a change in the mean would have no effect on income inequality. However, a change in the mean preserving spread of the distribution of efficiency of human capital usage would have non-trivial impact on both income inequality and the future growth rates in our model economy. This property of the model find strong empirical support from the recent work of Borjas (2003) and Card (2009) as both of them confirm significant changes in income inequality and wage growth following large swings in immigration. The parameter σ^2, which measures the variance of efficiency in human capital usage, captures such effect and, therefore, we identify an immigration shock with a change in σ^2 and we focus on the impulse response analysis of a change in σ^2 to understand how changes in immigration policies may impact on the growth-inequality dynamics. We now describe such an experiment.

Suppose that before period $t = 3$, the economy was on a balanced growth path and $\tau = 26.3\%$ such that $\gamma = 2.4\%$. Suppose that, in period $t = 3$, a change in the government's immigration policies led to a one-off decrease in the variance of human capital efficiency σ^2. We regard the drop in σ^2 as an immigration shock that allows immigration flows to vary in a way that reduces the variance of skill in the labor force.

Using the benchmark values from Table 1, (18), (20), (21) and (15) for simulating inequality and equation (27) for growth, Figure 5 shows the impulse response of the growth rate and income inequality to the immigration shock when the redistributive income tax rate is set equal to 26.3%.

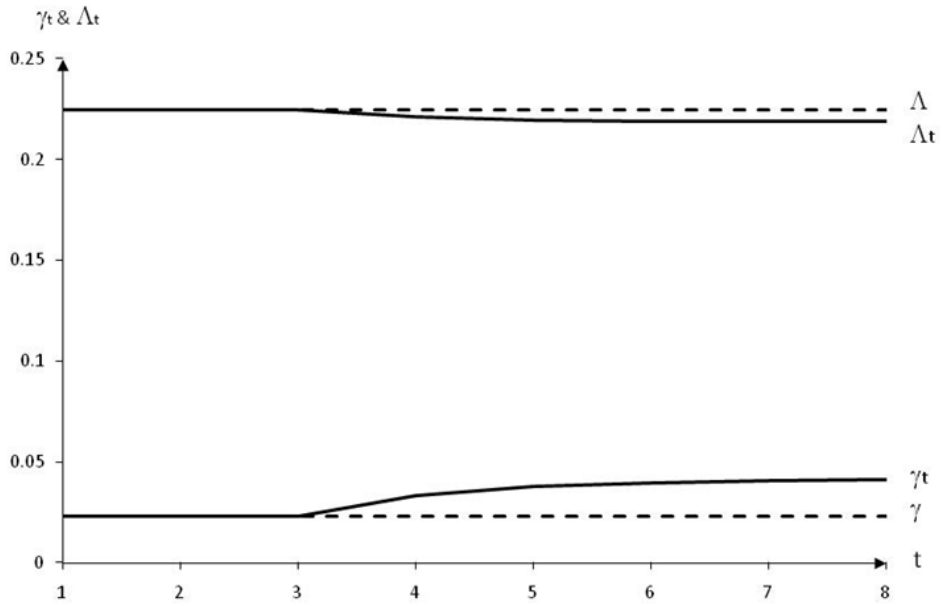


Figure 5—Impulse response of the growth rate and income inequality to a drop in σ^2, reflecting reduced heterogeneity in the skill distribution of the work force due to policy induced changes in immigration flows.

Figure 5 shows that in period three, the growth rate increases from its old balanced growth state by following a decelerating transition path while income inequality decreases implying a negative growth-inequality relationship subsequent to the immigration shock. In the long run, the growth rate converges to its newly enhanced balanced growth state due to a lower variance human capital.

The above simulations result is consistent with the data presented in Figure 3a and Figure 3b. In particular, the data show that in Phase 1, the gap of frequency between skilled and unskilled labor becomes smaller while the growth rate, shown in Figure 2, increases in that period. Figure 5 shows that income inequality converges to a lower steady state than pre-shock state. This is also consistent with Figure 1 once we can separate the partial effect of the tax cut which occurred during that period but is assumed to be constant in this simulation. Note, however, during a period of stable tax rate, between 1999 and 2003 (phase 2), income inequality generally decreases and the growth rate increases, although slightly, which is consistent with what the model's simulation result suggests. Note the correlation between inequality and growth following the immigration shock is unambiguously negative.

After reproducing New Zealand history with our simulations, we explain those simulations as a part of our interpretation of what happened in New Zealand history. In particular, we interpret the above simulation result by using Lemma 4 as well as by (32). Lemma 4 explains the increase in the growth rate and the gradual decline in income inequality following a one-off decrease in the variance of human capital. In the subsequent periods of transitional dynamics the continued increase in the growth rate comes from the declining income inequality in the preceding period via (32) while the deceleration of the growth rate mimics the declining rates of reduction in income inequality as it approaches its new steady state. This transition dynamics capture a growth promoting role of declining inequality that arises from the combination of assumptions of the missing credit market and diminishing returns technologies.

Figure 5 together with our model's explanation for it brings out a key policy conjecture as well. We hypothesize that an immigration policy which successfully reduces heterogeneity in the country's labor force, leads the economy over time to a path of faster growth with lower income inequality. Such a policy requires filtering of immigrants according to special skills criteria to fill in the gaps in the skill spectrum of the New Zealand residents in a way that would lower the variance of skills.

The above policy insight coincides with the findings of Friedberg and Hunt (1995) in which, by studying the skill composition of immigration flows to the United States from 1950s to 1980s, they conclude that economic benefits increase when immigrants brings skills that are not very different from the local residents. Borjas (2003) makes a similar conclusion by examining both schooling and work experience of the immigrants, identifying their significant skill-gap with the residents and, noting subsequently, that a large influx of immigrants in recent decades has substantially worsened economic opportunities in the labor market.

5.2.2 Fiscal policy shock

Next, we discuss the effects of a fiscal policy shock on the dynamic path of income inequality and the growth rate through impulse response analysis. The objective of this experiment is to capture various changes in the whole redistributive policy package during economic reforms in New Zealand. Evans et al., 1996 and others document a decline of the average marginal tax rate (AMTR) under the National Government in early 1990s' New Zealand economy and a subsequent reversal of that policy under the labor Government in the late 1990s. Those changes accompanied changes in the budgetary provisions for various subsidies and, in particular, education subsidies to schools and universities to keep the expenditure in line with revenue in the spirit of the Fiscal Responsibility Act (FRA) of 1993.[18] In our model economy we identify these changes by reducing education subsidies endogenously corresponding to a lower AMTR and vice versa. Also, as Benabou (2002), argues a modern government such as the one in New Zealand designs such

[18]The growth of budget surplus following the FRA of 1993 is typically attributed to the so-called Cullen Fund, without which the budget would have been more or less balanced.

endogenous subsidy to education in the context of a wider redistributive policy package also to mitigate the distortionary effects of redistribution and we capture that spirit in our model economy as well. In particular, an increase in AMTR discourages work-efforts, saving and investment in education while a corresponding increase in public subsidy to education would offset, at least partially, the negative effect of a tax-hike on the investment in education. However, governments do not typically provide subsidy to work or saving and hence we do not put them in our model economy either. Consequently, disincentives to saving and work from increased AMTR continue to dampen economic growth in our model.

We now discuss how the effect of a fiscal policy shock differs from that of an immigration policy shock. While the immigration shock can be characterized as purely exogenous because we can identify it with a single parameter σ^2, the fiscal shock is partly endogenous. Because it allows government subsidy to education as a built in automatic stabilizer for offsetting distortionary effects of redistribution and, in particular, the effects of changing the progressivity parameter τ. In other words, a discrete change in progressivity parameter τ and the accompanying automatic changes in the public subsidy to education is interpreted as a fiscal policy shock in the following simulation based experiment.

Figure 6 below shows that in period 3, the government increases the AMTR from 26.3% to 33% and both income inequality and growth rate decrease and monotonically converge to their new stationary states.

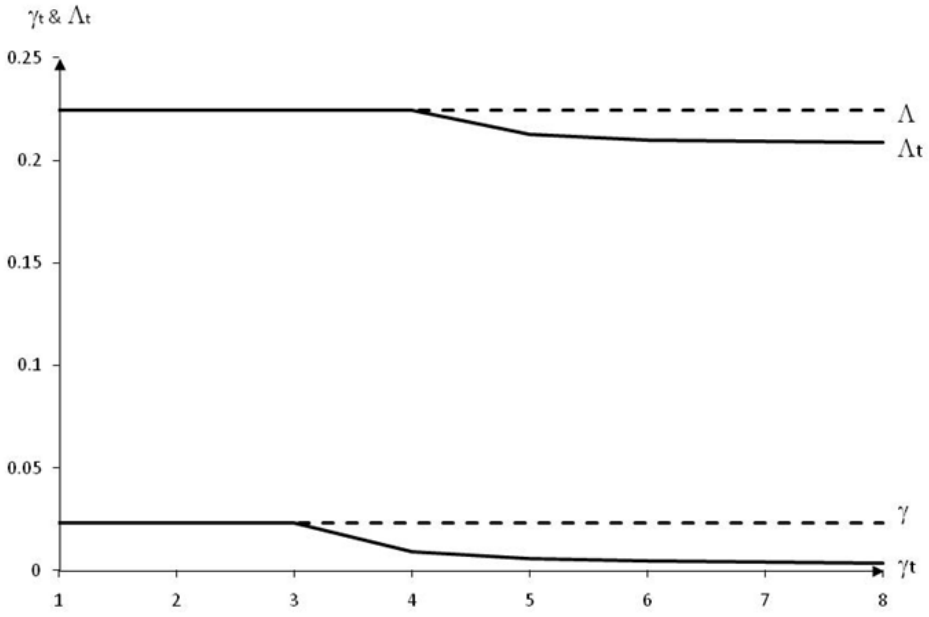


Figure 6 —Impulse response of the growth rate and income inequality to an increase in progressivity parameter τ of a redistributive policy package that includes a government subsidy to offset the distortionary effects of redistribution on the private investment in education.

We interpret this result based on our model as follows: First of all, by Lemma 5, a higher progressivity implies a declining sequence of income inequality in all subsequent periods and vice versa. Income redistribution in our model provides a general growth boost by relaxing the credit constraints of those with higher expected marginal productivity of capital (broadly defined to include both human and physical capital). However, as explained by Lemmas 8-9, any redistribution from high income people to low income people discourages work-effort, saving and investment in education. If the government does not sufficiently offset those disincentives of income redistribution, as has been the case for the New Zealand Government, then the overall effects of changing the degree of progressivity of redistribution would be ambiguous. By Lemma 8, we know that if τ is sufficiently high then an increase (or decrease) in τ would lower (or raise) both the steady state growth rate and the steady state income inequality. Facing such theoretical ambiguity we need to rely on our quantitative experiment to derive a conclusion.

For the calibrated model for the New Zealand economy we find the critical threshold for the progressivity parameter τ lies well below 10%. Given the high degree of progressivity in the 1990s relative to this threshold, this implies a positive correlation between income inequality and the growth rate. In addition, we conclude that the observed positive relationship between income inequality and the growth rate indicates that a reduction of AMTR in New Zealand would increase growth rate by Lemma 8 while increasing income inequality by Lemma 5 at the same time. In contrast, we find that if the government can subsidize saving or waive taxes on saving then the threshold progressivity that maximizes long run growth rate increases to 32%. Consequently, under that policy framework, a reduction of AMTR below 32% will reduce the long-run growth rate as well as increasing income inequality, giving rise to a negative growth-inequality relationship. Also, that would be clearly a sub-optimal policy. This result parallels Benabou (1996) and Glomm and Kaganovich (2008) in which they find that increasing tax rate could enhance or slow down growth depend whether the initial level of tax is low or high, even though increasing the tax rate always reduce income inequality.

Note also, from Figure 6 that, following the increase in progressivity parameter τ, the growth rate drops immediately while inequality decreases after one period. This is because by (27), we can see that increasing τ leads to immediate distortion in labor and saving as a tax rate hike raises leisure but reduces saving. It then leads to an immediate decrease in growth. In contrast, inequality is a state variable. By Lemma 5, it does not change in the concurrent period but in all subsequent periods. Also after the one-off fiscal policy shock, the decrease in the growth rate becomes relatively small because the decrease in income inequality in the preceding periods, by (32), provides an offsetting boost to the otherwise declining growth rate. Consequently, the correlation between income inequality and growth appears to be positive.

This experiment with a fiscal shock brings out new insights on the New Zealand economy together with a policy conjecture as well. The observed positive growth-inequality relationship implies, according to our model, that the progressivity in New Zealand is too high relative to its threshold. Consequently, a reduction of AMTR would lead the

economy to a new balanced growth path with a higher growth rate but only at the price of increasing income inequality during the transitional period.

Also, unlike what we discussed earlier under the immigration shock, the findings reported above appear to be in direct conflict with the regression result of Persson and Tabellini (1994), and Aghion, Caroli and Garcia-Penalosa (1999). Instead, they resemble, in a way, Kaldor (1957)'s hypothesis of a positive growth-inequality trade-off.

5.3 Multifaceted Growth-Inequality Relationship

From Figures 5 and 6, we discern that two distinct policy shocks can have two distinct implications for the growth-inequality relationship during the immediate time period following the policy shock. This theoretical underpinning behind growth-inequality empirics provides a caution against conventional regression analysis. It suggests that prior to conducting an empirical analysis, one may need to pay more attention to the quality of the data in order to determine whether any policy shocks occurred around the period of their study. This was also of concern to Banerjee and Duflo (2003), in which they emphasized that the quality of data may affect the estimation of the relationship between inequality and growth.

Regarding the policy debates our two experiments clearly demonstrate that the apparent conflict between two political parties, one promoting growth and the other equity cannot be satisfactorily resolved unless we discern the underlying economic factors that initiate an impulse to which our optimal response, in turn, gives rise to a unique dynamics of income inequality and growth. The optimal policy, as our experiments reveal, must take into account the specific channel that is quantitatively relevant for developing a specific growth-inequality relationship.

It turns out that without our "missing credit market" assumption the results from the above two experiments would not hold. In the scenario for fiscal policy shocks, if we allow a credit market for borrowing and lending then the threshold progressivity could not be greater than zero as the individuals would trade away their productivity differentials and hence the macroeconomic benefit from redistribution would not be there. Consequently, changes in progressivity would always create a positive growth-inequality relationship. In the scenario for immigration shocks, in response to increased heterogeneity, economic agents would try to utilize their tools of borrowing and lending to achieve a state of zero productivity differential, a Pareto optimal state. In the process, there would be a growth spurt as well as widening of income inequality to reflect greater heterogeneity of the population characteristics. The result again would be a positive growth-inequality relationship. We conclude that id credit markets are not a problem then increasing income inequality would be a necessary price to pay for a faster rate of growth.

On the other hand, if we relax other assumptions of our model but strictly maintain the "missing credit market" assumption then the qualitative nature of our results as discussed in the above two scenarios of impulse response analysis of our model would essentially remain unchanged. For example, if we allow competitive firms to higher workers and

rent capital to organize production in the economy but the firm's total factor productivity (TFP) varies inversely with heterogeneity of the workforce then the results discussed above would remain unchanged. The assumption of knowledge spillover allows us to bring in endogenous growth without violating the diminishing returns property of the output and human capital technologies. Without it we would not have endogenous growth but would still recover all the results by replacing the growth rate variable with per capita output. However, a change in the extent of knowledge spillover does affect the size of fluctuations of the model's equilibrium outcome in response to the two policy shocks we discussed above. Analyzing the quantitative differences and their implications for policies would be an interesting avenue for future research and falls outside the scope of our objectives for this paper.

5.4 Review of New Zealand Data

To sum up we review the New Zealand data presented earlier in light of the controlled experiments described above. In particular, to make some sense of the changes in income inequality and growth rate described in Figures 1-2, we apply the insights derived from our experiments as follows: By (18), (20), and (21) and then (15), we conclude that changes in income inequality could come either from an exogenous change in the variance σ^2 of human capital efficiency, or from a change in the average marginal income tax rate τ.

By (18), (20), (21) and then (15), we know that income inequality increases with σ^2 but decreases with τ. We can see from Figure 3a and Figure 3b that, in phase 1, σ^2 decreases implying a downward pressure on the trend of income inequality while we note from Figure 4 that the tax rate τ decreases in phase 1 and that exerts upward pressure on income inequality. In phase 2, the gap between skilled and unskilled residents becomes larger, implying a larger value of variance σ^2 while the tax rate increases sharply during that period. Figure 1 shows that income inequality increases in phase 1 but decreases in phase 2. Consequently, Figure 1 indicates that the tax effect dominates the immigration effect in determining income inequality trends. The effects on the growth rate from changes in the variance and tax parameters are not straightforward.

Thus our quantitative analysis of the calibrated model for the New Zealand economy helps us to make some sense of the New Zealand data and, in particular, to discern the economic dynamics that give rise to multifaceted growth-inequality relationships.

6 Conclusion

In this paper we set out to explore underlying economic dynamics that give rise to apparently contradictory growth-inequality relationships in the data. We proceed by building a unified theoretical framework based on modifications of a standard textbook version of a DGE model. Our modifications included a missing credit market and islands of produc-

tion units operated by self-employed individuals. We provide an explicit algorithm for replicating our model's analysis using a competitive market economy. However, transparency and analytical tractability considerations motivate us to avoid that alternative framework.

One contribution of this paper is that provides explicit analytical properties derived transparently from simple modification of a standard textbook version of a DGE model. Those properties motivate a new theoretical rationale for interpreting multifaceted growth-inequality relationships as alternative equilibrium outcomes. Our convenience with this new model for tracing in detail subtle but transparent links between various analytical properties equip us with necessary tools to design quantitative experiments to bring out additional information from the data. We calibrated the model's parameters suitably to create a benchmark model for the New Zealand economy and use it to interpret what happened in New Zealand history regarding the outcome of income inequality and growth rate following a series of major economic reforms that ended in the early 1990s. Additional insights from our quantitative analysis of the model economy provide important clues for solving existing policy puzzles which have a special relevance to the current policy debate in New Zealand.

For example, we learn from the simulations of the calibrated model's equilibrium that immigration and tax policy shocks may have significantly different implications for the growth-inequality relationships. In particular, we discover that changes in immigration policies that alter variance of skill distribution would subsequently lead to a negative growth-inequality relationship while changes in redistributive policies have an ambiguous effect on that relationship. If the minimum threshold for progressivity that maximizes the long-run growth rate is too low compared to existing progressivity, which has been the case for New Zealand all along, then a change in progressivity would lead to a positive growth-inequality relationship. If the threshold could be made too high relative to the current level, for example, by eliminating tax on saving, then a negative growth-inequality relationship would follow a change in progressivity. In particular, a higher progressivity in that case would bring faster economic growth with lower income inequality.

While trying to put all these findings together to get the big picture we realize that a negative growth-inequality relationship does not call for a reduction of inequality through increased redistribution, although that would be tempting. Instead, it asks us to find a mechanism to reduce heterogeneity in the population characteristics. A suitably controlled immigration policy could fulfill that task. Similarly, a positive growth-inequality relationship does not mean that increasing inequality must be a necessary price to pay for faster growth. It may simply signal that the country's threshold progressivity is quite low and that, with a suitable tax and subsidy scheme the government can raise the threshold above the current progressivity. Afterwards, increased progressivity would not only lower income inequality but also promote growth. Thus the seeds of a policy conjecture for "growth with equity" spring up ironically from a positive growth-inequality relationship while a negative growth-inequality relationship may lure politicians, unaware of these effects, to promote a policy of redistribution, unwittingly.

All our results stand or fall on the "missing credit market" assumption. In the presence of a complete credit market, increased heterogeneity could be traded away with borrowing and lending to foster growth and a redistributive policy would lose its utility in the absence of any interpersonal differences in productivity. Consequently, we should always expect in this case to find a positive growth-inequality relationship. If we relax any of the other assumptions the magnitude of the growth-inequality correlations would change but their sign would not. For example, if we allow a perfectly competitive market but where a firm's TFP inversely varies with the variance of the efficiency of labor then an increase in the variance of efficiency will increase income inequality but decrease growth, implying a negative growth-inequality relationship, similar to what we find in our model economy. Similarly, a greater progressivity in a competitive environment can create macroeconomic benefit by lowering income inequality and thereby raising TFP; but if the progressivity is sufficiently high then the microeconomic incentive costs of raising progressivity may outweigh that benefit, implying a positive growth-inequality relationship. Consequently, by allowing a competitive environment but by maintaining the "missing credit market assumption" we can reproduce our result with a clear economic intuition.

For future work it could be a worthwhile exercise to continue the spirit of our analysis by designing more quantitative experiments to interpret history of events associated with different channels through which a growth-inequality relationship emerges. Some of those channels are explicitly identified in this paper. Some remain unexplored but could be very easily pursued by exploiting a number of explicitly derived analytical expressions including the key equation of this paper that provides relatively broad characterization of the growth-inequality dynamics. For example, an unexplored avenue for tracing the growth-inequality relationship would be to examine the quantitative significance of shocks to the degree of knowledge spillover. It could be argued that the extent of this spillover effect varies significantly between countries with and without modern "communication highways". Also, in the process of development this spillover effect may increase or decrease depending on economic policies related to choice of institutions. By changing the externality parameter in our key equation we get ambiguous growth inequality relationships depending upon the context of the model economy to be identified by the other parameters. Consequently, an interesting question for future research would be to ask if different realizations of growth-inequality relationships, across countries or over time, could be significantly governed by the extent of knowledge spillover. We provide an explicit algorithm involving knowledge spillover in our paper for making conditional conjectures about that possibility. The task we leave for future research is to find a historically relevant context and then to design quantitative experiments based on that context to explore those conjectures further.

By presenting a theoretical framework, the findings in this paper also build an alternative foundation for future empirical research. Our findings suggest that paying attention to specific events that may have a significant impact on the data, and understanding the theoretical channel that spells out the impact within a general theoretical framework like ours, may be necessary to overcome serious concerns expressed in Banerjee and Duflo (2003)

about the current state of empirical research involving growth-inequality relationships.

Appendix

PROOF OF LEMMA 1: We guess the value function as: $v\left(h_t^i, k_t^i, M_t; T\right) = N_1 \ln h_t^i + N_2 \ln k_t^i + B_t$. Then, by substituting this value function, (2), (4) and (5) into (9), the first-order conditions with respect to the savings and labor supply imply Lemma 1. □

PROOF OF LEMMA 2: By assumption, at the initial date $t = 0$, physical and human capital are lognormally distributed. By (13) and (14), it follows that k_t^i and h_t^i remain lognormally distributed over time and hence by (1) y_t^i is lognormal. Then, by the property of lognormal distribution, the income per capita, y_t, is

$$y_t = \int_0^1 y_t^i di = \exp\left(\int_0^1 \ln y_t^i di + \frac{1}{2}\operatorname{var}\left[\ln y_t^i\right]\right). \tag{A.1}$$

The median income is $\exp\left(\int_0^1 \ln y_t^i di\right)$. Therefore, following Benabou (2002) we define for each date t an index of income inequality Λ_t as the logarithm of the ratio of mean to median income. Thus the inequality index (15) is proved. By (5), we can get (16). The proof of Lemma 2 is completed. □

PROOF OF LEMMA 3: Equations (17) and (19) show that the coefficients on m_{kt} and m_{ht} can be represented by a 2×2 matrix with an eigenvalue equal to one if $(1-\alpha-\delta)(1-\lambda) - \theta\mu = 0$. It implies that both m_{kt} and m_{ht} will go to infinity and grow at a constant growth rate in the long run.

Substituting (13) and (14) into (1) yields the equilibrium path of income for agent i. Then by the property of lognormal distribution described in (A.1), we get:

(A.2)

$$\begin{aligned}\int_0^1 \ln y_{t+1}^i di = {} & \mu\theta \ln\frac{\theta\beta\mu}{1-\beta\alpha} + \lambda(1-\alpha)\ln\beta\lambda(1-\tau) + \mu\left(\ln\phi - \sigma^2/2\right) \\ & + \mu\ln\kappa_t + (1-\alpha)(1-\lambda-\mu)\ln e + (\lambda+\theta\mu)\tau\ln\tilde{y}_t - \alpha\lambda\tau\ln\tilde{y}_{t-1} \\ & + \Gamma(\tau)\int_0^1 \ln y_t^i di - \alpha\lambda(1-\tau)\int_0^1 \ln y_{t-1}^i di.\end{aligned}$$

From (A.1), we know:

$$\int_0^1 \ln y_t^i di = \ln\int_0^1 y_t^i di - \frac{1}{2}\operatorname{var}\left[\ln y_t^i\right]. \tag{A.3}$$

Combining (A.3) with (A.2) yields:

(A.4)

$$\begin{aligned}\ln \int_0^1 y_{t+1}^i di - \frac{1}{2}\operatorname{var}\left[\ln y_{t+1}^i\right] = {} & \mu\theta \ln \frac{\theta\beta\mu}{1-\beta\alpha} + \lambda(1-\alpha)\ln\beta\lambda(1-\tau) + \mu\left(\ln\phi - \sigma^2/2\right) \\ & + \mu \ln \kappa_t + (1-\alpha)(1-\lambda-\mu)\ln e \\ & + (\lambda+\theta\mu)\tau \ln \tilde{y}_t - \alpha\lambda\tau \ln \tilde{y}_{t-1} \\ & + \Gamma(\tau)\left(\ln \int_0^1 y_t^i di - \frac{1}{2}\operatorname{var}\left[\ln y_t^i\right]\right) \\ & - \alpha\lambda(1-\tau)\left(\ln \int_0^1 y_{t-1}^i di - \frac{1}{2}\operatorname{var}\left[\ln y_{t-1}^i\right]\right).\end{aligned}$$

By the definition of κ_t we can get:

(A.5)
$$\kappa_t = \exp\left(\delta\left(\ln h_t + (\mu-1)\Delta_{ht}^2/2\right)\right).$$

Integrating (4), and by (6) and (12), we get:

(A.6)
$$\ln k_t = \ln \beta\lambda(1-\tau) + \ln y_{t-1}.$$

Substituting (16), (A.5) and (A.6) into (A.4), and rearranging, we can get (27). □

PROOF OF PROPOSITION 1: By equations (18), (20) and (21), we find that the coefficients on Δ_{kt}^2, Δ_{ht}^2 and cov_t can be represented by a 3×3 matrix. Moreover, the eigenvalues of the matrix are positive and less than one. It then implies that Δ_{kt}^2, Δ_{ht}^2 and cov_t will monotonically converge to a unique steady state.[19] Thus, the proof of Proposition 1 is completed. □

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[19]For a detailed discussion of this property, please see Reich (1949), Lorenz (1993) or Young (2003).

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