Effect of piracy on innovation in the presence of network externality

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Abstract:

The joint impact of piracy and network externality increases the R&D investment of a single firm if the network effect dominates the piracy effect. With R&D competition, if the firms "significantly" differ with respect to the efficiency in R&D investment and if the piracy effect dominates the network effect then the less efficient firm's investment increases and that of the more efficient firm's decreases. The reverse holds if the network effect dominates the piracy effect. If the firms are "less" asymmetric then their R&D investment either increases or decreases depending on the relative strengths of the piracy and network effects.

JEL Classification: D21, D43, L13, L21, L26, O3.

Keywords: Innovation, market uncertainty, network effect, piracy effect, technological uncertainty.

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I would like to thank Lata Gangadharan, Birendra Rai, Ranjan Ray, and Vai-Lam Mui for useful comments and suggestions. The usual disclaimer applies.

1. Introduction

Piracy has generally been perceived as having a damaging influence on software and media industries due to the high magnitude of loss in retail sale since such products can be copied at a low cost (Marshall, 1999; Straub and Nance, 1990) and also due to the possible detrimental effects on the incentive to innovate.[1] That piracy retards the incentive to innovate is supported in a recent study by Banerjee et.al. (2010), in the context of a single innovating firm facing technological uncertainty. Similar support has been rendered by Jaisingh (2009), Qiu (2006), and Novos et.al., (1984) under different contexts but with a single innovating firm.

An important feature of the above mentioned industries is the presence of network externality that plays the role of a medium of advertisement for legitimate products. Thus Shy et.al. (1999), and Takeyama (1994) show that firms can benefit from lax copyright enforcement due to the presence of network externality and no protection against piracy is an equilibrium. Conner et.al. (1991), and Nascimento et.al. (1988) discusses the role of network externalities on the marketing of software. However, this literature does not consider innovation and treats it as a sunk cost.

This paper bridges the gap between the literature on piracy and innovation, and that on piracy and network externality by analysing the joint impact of piracy and network externality on innovation. An accurate assessment of the impact of piracy on innovation need to account for the presence of network externality and may be relevant in the design of appropriate anti-piracy policies which however, is not the focus of this paper.

The model consists of a two stage game where firm/s choose a level of R&D investment to develop a new product in stage one and if successful, competes in price with a

[1] In the Piracy Study (2005), Business Software Alliance (BSA) mentions that “local software industries crippled from competition with high-quality pirated software”. The International Federation of the Phonographic Industry (IFPI) in its 2005 Commercial Piracy Reports argues that, “The illegal music trade is destroying creativity and innovation, ...”

pirating firm who illegally copies the innovating firm's product, in stage two. An increase in piracy reduces the stage two realised profit of the innovating firm which is referred to as the *piracy effect*. An increase in the network externality increases the innovating firms' stage two realised profit and this is referred to as the *network effect*.

Without R&D competition a single innovating firm faces technological uncertainty which implies that the R&D investment resulting in a new product is stochastic and depends on the level of investment. In this case increases in piracy and network externality increase the R&D investment of the innovating firm if the network effect is stronger than the piracy effect. This is because the domination of the network effect over the piracy effect results in an increase in the stage two realised profit of the innovating firm which provides the incentive to increase the R&D investment in stage one. The reverse is true if piracy effect dominates the network effect. Hence, an increase in piracy does not necessarily retard a firm's incentive to innovate. It depends on the relative strengths of the piracy and network effects.

The introduction of R&D competition in stage one generates market uncertainty on top of technological uncertainty.[2] Market uncertainty arises when multiple firms are involved in R&D competition and thus a firm's success in developing a new product does not necessarily imply its success in obtaining a patent. In this case I show that, if the innovating firms are "highly" asymmetric with respect to their efficiency in R&D investment and if the piracy effect dominates the network effect then the less efficient firm's R&D investment increases and that of the more efficient firm's decreases. In this case, the overall probability of a successful innovation increases. If the network effect dominates the piracy effect then the more efficient firm increases its R&D investment and the less efficient firm decreases it.[3]

[2] The terms technological uncertainty and market uncertainty have been introduced by Shy (2000).

[3] Banerjee et.al. (2010) show that in the presence of R&D competition, piracy may enhance the R&D incentive of the less efficient firm only.

If the two firms are "less" asymmetric with respect to the efficiency in R&D investment then both firms either increase or decrease their R&D investment when there is an increase in piracy and network externality. The outcome depends on the relative strengths of the piracy effect and the network effect. The above stated results show that an increase in piracy may not necessarily have a negative impact on the incentive to innovate and the intuition is provided in the main text.

While this paper focuses on piracy and innovation through the demand side via network externality, El Harbi et.al. (2008) and Easley et al. (2003) considers the supply side and show that piracy can have a beneficial impact on innovation due to a positive feedback effect that provides direction to innovating firms for further R&D.[4] There can also be tacit reciprocity (Kolm 2006) in knowledge exchange between the innovating and pirating firms in which case the innovating firm accepts piracy (Barnett, 2005; Raustiala et.al., 2006; Barnett et al., 2010).

The issue of piracy has not been explored in the literature on innovation and patent races. This literature shows that patents and innovations can have a two-way relationship. For more on this see Bessen et.al. (2003), Hunt (2004, 2006), Kultti et. al. (2006), Maurer et.al. (2003) Shapiro (2006).

This paper is organized as follows. Section 2 contains the analysis with a single innovating firm. In Section 3 I analyse the case with R&D competition between two innovating firms. Section 4 contains the concluding remarks.

[4] When hackers used Valve Software's Half Life game engine to develop a game called Counter Strike, Valve, a gaming company, took the illegal game software and marketed it themselves, selling over 1.5 million copies (Barnes, 2005). Apple Computer, in a strategic reaction to P2P file sharing technologies, launched the iTunes online music library that was easy to navigate and explore, with free music previews, and allowed flexible download and copying for personal use. See Choi and Perez (2007) for anecdotal evidences on legal firms adopting technologies used by illegal P2P file sharers. In design based industries, a good being pirated is a signal of the high quality of the legal product, and products which 'are not faked are considered too weak to generate consumer demand and are consequently not produced' (Whitehall, 2006). Ritson (2007) says that pirated goods are indicative of heralding a brand's renaissance and a brand dies if no copies appear in the market.

2. The model with technological uncertainty

Let us consider the market for a digital/information good, like software, that has positive network externality and also faces piracy. I first consider the case where there is only one firm investing in R&D in order to increase its profit above a reservation level, $\bar{\pi}$. For simplicity we assume $\bar{\pi} = 0$. The innovating and the pirating firms play a sequential game. In stage 1 the innovating firm chooses a level of R&D investment R and in the second stage engages in price competition with the pirating firm who makes unauthorized copies of the innovating firms product and sells it in the market.

The probability that the innovating firm is successful in developing the product is $k\alpha(R)$ such that $0 \le k\alpha(R) \le 1$ with the properties $\alpha'(R) > 0$ and $\alpha''(R) < 0$.[5] Thus, technological uncertainty in the model is captured by $k\alpha(R)$. k can be viewed as the R&D efficiency parameter. I further assume that $-\dfrac{\alpha''(R)}{\alpha'(R)}$ is decreasing in *R* meaning that the curvature of $\alpha(R)$ is decreasing in *R*.

As in Banerjee (2003), the difference between the innovating firm's product (hereafter, referred to as the original product) and the pirated one is with respect to the reliability of the pirated product which is captured by the parameter *q*. The pirating firm operates in an illegal manner using a makeshift arrangement. So if the pirated software turns out to be defective it cannot be replaced. That is, the pirated product comes with no warranty. The original product receives full warranty. The parameter *q* is the probability that the pirated software is operational which is common knowledge and we assume $q \in (0,1)$.[6] If $q = 0$ it means there is no piracy, so *q* can be a proxy for the degree of piracy. Henceforth, an increase in *q* which reflects an increase in the reliability of the pirated product will be referred to as an *increase in piracy*.

[5] These properties ensure that the second order condition for profit maximization holds.
[6] We set this bound to ensure that the profits are not indeterminate.

There is a continuum of consumers indexed by $\theta, \theta \in [0,1]$. θ is assumed to follow a uniform distribution and represents the consumer's valuation of the product. Each consumer is assumed to buy only one unit of a product, if at all, and can either buy the original product, or the pirated product or nothing. Also there is the presence of network externality which means that a consumer's utility is increasing in the number of other consumers using either the original or the pirated product. A consumer buying the original product enjoys θ, the network externality generated by consumers who buys the original product, and the network from those who buys the pirated product only if the latter is operational which occurs with probability *q*. A consumer buying the pirated product enjoys θ and the network externality only if the pirated product is operational.

Following Banerjee (2003), the utility of a type-θ consumer is as follows.

$$U(\theta) = \begin{cases} \theta - p_m + \beta D_m + q\beta D_c\,, & if\ the\,consumer\,buys\,the\,original\,product, \\ q(\theta + \beta D_m + \beta D_c) - p_c\,, & if\ the\,consumer\,buys\,the\ pirated\,copy, \\ 0, & if\ the\,consumer\,buys\,nothing. \end{cases} \qquad (1)$$

p_m and p_c are the prices of the original and the pirated products. The coefficient β measures the extent of network externality and it lies in the interval $\beta \in (0,1)$. Henceforth, we will refer to an increase in β within the interval $\beta \in (0,1)$ as an *increase in the network externality*. The demand functions for the original and the pirated products are as follows.[7]

[7] The marginal consumer indifferent between purchasing the original product and the pirated product satisfies, $\theta_m = \dfrac{p_m - p_c - \beta D_m(1-q)}{(1-q)}$. The marginal consumer indifferent between purchasing the pirated product and not buying anything satisfies, $\theta_c = \dfrac{p_c}{q} - \beta(D_m + D_c)$. Using the expressions for θ_{cm} and θ_{c0} we get the demand functions. Note that in the absence of network externality the demand functions are $D_m = 1 - \theta_m = 1 - \dfrac{p_m - p_c}{(1-q)}$, and $D_c = \theta_m - \theta_c = \dfrac{qp_m - p_c}{q(1-q)}$.

$$D_m = 1-\theta_m = \frac{1}{1-\beta} - \frac{p_m - p_c}{(1-q)(1-\beta)},$$
$$D_c = \theta_m - \theta_c = \frac{qp_m - p_c}{q(1-q)(1-\beta)}. \tag{2}$$

The expected profit of the innovating and the pirating firms are,

$$E\pi = k\alpha(R)\left(\frac{1}{1-\beta} - \frac{p_m - p_c}{(1-q)(1-\beta)}\right)p_m - R,$$
$$E\pi_c = k\alpha(R)\frac{qp_m p_c - p_c{}^2}{q(1-q)(1-\beta)}. \tag{3}$$

Let $r = \left(\frac{1}{1-\beta} - \frac{p_m - p_c}{(1-q)(1-\beta)}\right)p_m$ denote the realised second stage profit of the innovating firm if its innovation is successful in stage 1. The pirating firm can compete with the innovating firm only if the latter is successful in innovation. The reaction functions of the two firms in stage 2 of the game are $p_m = \frac{1-q+p_c}{2}$ and $p_c = \frac{qp_m}{2}$. Solving the reaction functions yield the equilibrium prices and the innovating firm's realised second stage profit, r^{**}, as;

$$p_m{}^{**} = \frac{2(1-q)}{4-q},\ p_c{}^{**} = \frac{q(1-q)}{4-q}, \text{ and } r^{**}(q,\beta) = \frac{4(1-q)}{(1-\beta)(4-q)^2}. \tag{4}$$

The rate of change of $r^{**}(q,\beta)$ due to an increase in q, that is $\frac{dr^{**}}{dq}$, is referred to as the *piracy effect* and the rate of change of $r^{**}(q,\beta)$ due to an increase in β, that is $\frac{dr^{**}}{d\beta}$, is referred to as the *network effect*. Lemma 1 summarizes the results for the comparative static analysis of $r^{**}(q,\beta)$ with respect to q and β, and the proof is given in the Appendix.

Lemma 1. *(i) The equilibrium realised stage 2 profit of the innovating firm is decreasing in q and increasing in* β. *(ii) The network effect exceeds the piracy effect if* $q < \bar{q}$ *where* $\bar{q} = \frac{6-\beta-\sqrt{28-20\beta+\beta^2}}{2}$. *The reverse is true if* $q > \bar{q}$.

Lemma 1 specifies the conditions for which the piracy effect dominates the network effect and vice versa.[8] These conditions will be used for deriving the impact of increase in piracy and network externality on R&D investment. Substituting the expressions from (4) in equation (3) and maximising the innovating firm's expected profit with respect to R yields the equilibrium R&D investment. Let R^P and $E\pi^P$ denote the equilibrium R&D investment and expected profit. The results are summarized in Proposition1.

Proposition 1. *(i) The equilibrium R&D investment of the innovating firm satisfies* $\alpha'(R^P) = \dfrac{1}{kr^{**}}$ *and its expected profit is* $E\pi^P = k\alpha(R^P)r^{**} - R^P$*.*[9] *(ii) Increase in piracy and network externality result in an increase (a decrease) in the equilibrium R&D investment and the expected profit of the innovating firm if the network effect (piracy effect) dominates the piracy effect (network effect). The equilibrium R&D investment and the expected profit of the innovating firm remain unchanged if the two effects are the same.*

Proposition 1 shows that an increase in piracy may not necessarily result in a decrease in the R&D investment even when there is a single innovating firm. The intuition behind this result is as follows. Increases in piracy and network externality have opposing effects on the second stage realised profit of the innovating firm. If the network effect dominates the piracy effect, it means there is a net increase in the second stage profit. Thus there is the incentive for the innovating firm to increase its R&D investment in stage 1. The reverse is true when the piracy effect dominates the network effect. The finding summarized in Proposition 1 part

[8] $\dfrac{dr^{**}}{dq}$ and $\dfrac{dr^{**}}{d\beta}$ can be compared because *q* and β are unit free numbers in the same domain $(0,1)$. See Currier (2000).

[9] $k \geq \dfrac{R^P}{\alpha(R^P)r^{**}}$ ensures non-negative profit.

(ii) contradicts the claim that piracy retards the incentive to innovate as shown by Banerjee et.al. (2010), Jaisingh (2009), and Qiu (2006) in the context of a single innovating firm.

3. R&D competition

Let us now introduce R&D competition between two innovating firms *i* and *j*. In stage 1, the two innovating firms compete in R&D investment and the winner of the patent engages in price competition with the pirating firm in stage 2. The stage 2 results are the same as given in (4).

In stage 1 of the game an innovating firm can win the patent if it is successful in innovation and the rival firm is unsuccessful. If both firms are successful then each firm receives the patent with equal probability. So the probability of a firm *i*, successful in receiving the patent is,

$$\mu_i(R_i, R_j) = k_i\alpha(R_i)(1-k_j\alpha(R_j)) + \frac{k_i k_j \alpha(R_i)\alpha(R_j)}{2}. \tag{5}$$

Hence, the expected profit of firm *i* is,

$$E\pi_i = \left(k_i\alpha(R_i)(1-k_j\alpha(R_j)) + \frac{k_i k_j \alpha(R_i)\alpha(R_j)}{2} \right) r^{**} - R_i. \tag{6}$$

The first order conditions yield the reaction function of firm *i* as given in equation (7).[10]

$$\frac{dE\pi_i}{dR_i} = k_i\alpha'(R_i)\left((1+\frac{k_j\alpha(R_j)}{2} \right) r^{**} - 1 = 0 \Rightarrow k_i\alpha'(R_i)(2-k_j\alpha(R_j)) = \frac{2}{r^{**}} \tag{7}$$

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[10] The second order conditions require $\frac{\partial^2 E\pi_i}{\partial R_i^{\,2}} < 0$ and $|D| \equiv \begin{vmatrix} \frac{\partial^2 E\pi_i}{\partial R_i^{\,2}} & -\frac{\partial^2 E\pi_i}{\partial R_i \delta R_j} \\ -\frac{\partial^2 E\pi_i}{\partial R_i \delta R_j} & \frac{\partial^2 E\pi_j}{\partial R_j^{\,2}} \end{vmatrix} > 0.$ We have $\frac{\partial^2 E\pi_i}{\partial R_i^{\,2}} = k_i\alpha''(R_i)\left(1-\frac{k_j\alpha(R_j)}{2} \right) r^{**} < 0$ because, by assumption, $\alpha''(R_i) < 0$.

The effect of increases in piracy and network externality on the reaction functions of the innovating firms is summarized in Lemma 2 and the proof is given in the Appendix.

Lemma 2. *Increase in piracy and network externality shifts the reaction functions of the two firms: (i) downward if the piracy effect dominates the network effect (* $q > \bar{q}$ *); (ii) upward if the network effect dominates the piracy effect (* $q < \bar{q}$ *).*

Lemma 2 implies that if both piracy and network externality increases, and if the piracy effect dominates the network effect then either the R&D investments of both firms decrease or the R&D investment of one firm increases and that of the other decreases. If the network effect dominates the piracy effect then either the R&D investments of both firms increase or the R&D investment of one firm increases and that of the other decreases. These two cases are diagrammatically represented in Figures 1 and 2. A represents the initial equilibrium, and B, C and D represents the possible equilibrium after incorporating the piracy and network effects.

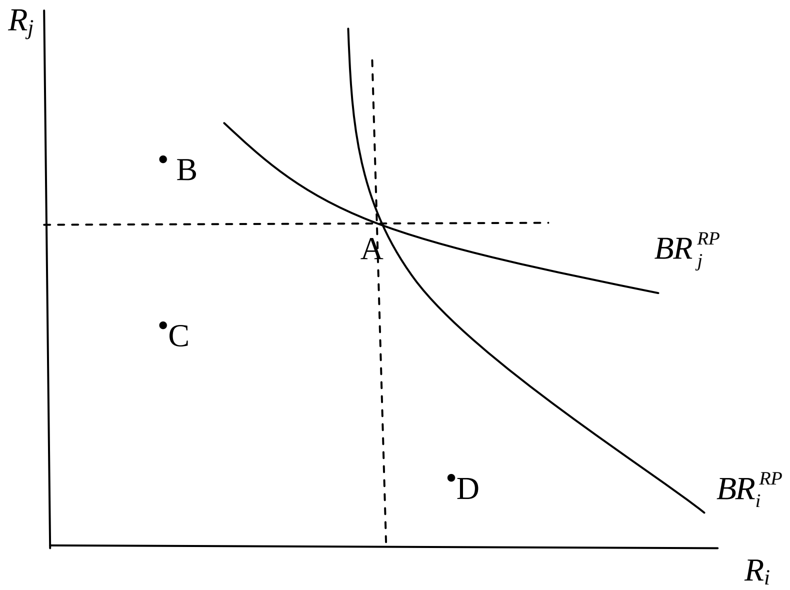

Figure 1: The piracy effect dominates the network effect

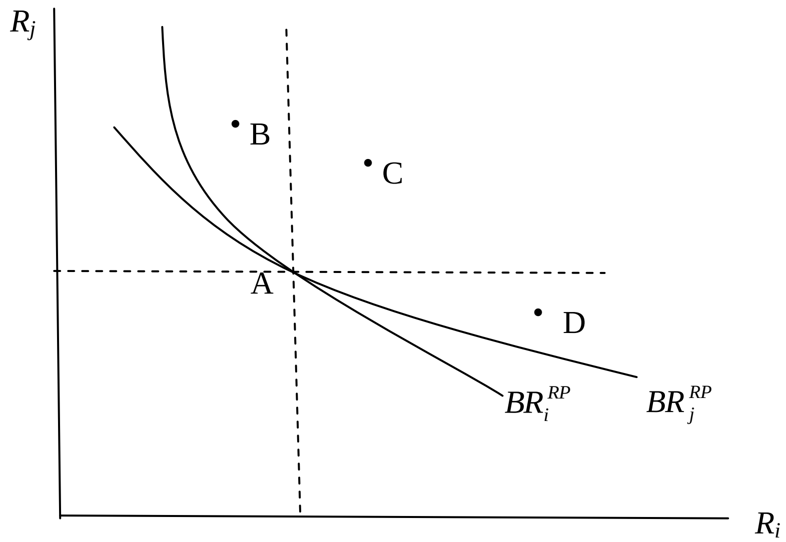

Figure 2: The network effect dominates the piracy effect

Let (R_i^{RP}, R_j^{RP}) be the initial the equilibrium R&D investments of the two firms in stage 1 which is obtained by solving the reaction functions in equation (7) and is represented as the point A in Figures 1 and 2. If the firms are symmetric with respect to their R&D efficiency then an increase in piracy and network externality increases or decreases the equilibrium R&D investments of both firms depending on whether the network effect or the piracy effect dominate.

Let us consider the case where the firms are asymmetric with respect to the R&D efficiency, that is, $k_i > k_j$. To determine which firm's R&D investment increases when piracy and network externality increases requires the comparative static analysis of the firms' equilibrium R&D investment with respect to the efficiency parameters. The result is summarized in Lemma 3 and the proof is provided in the Appendix.

Lemma 3. *For a given level of piracy and network externality, an increase in a firm's efficiency parameter increases its equilibrium R&D investment. An increase in the rival firm's efficiency parameter decreases the firm's equilibrium R&D investment.*

Intuitively, an increase in a firm's efficiency parameter shifts its reaction function to the right. That is, the marginal gain from an increase in investment outweighs the loss from doing so. Consequently, the firm's equilibrium R&D investment increases. The opposite is true when the rival firm's efficiency parameter increases.

I now turn to the analysis of whether the more or the less efficient firm's equilibrium R&D investment increases or decreases when the network effect dominates the piracy effect and vice versa. Such an analysis requires the derivation of the expressions for $\frac{dR_i^{RP}}{dq}$, $\frac{dR_i^{RP}}{d\beta}$,and $\frac{dR_i^{RP}}{d\beta} - \left(-\frac{dR_i^{RP}}{dq} \right)$ which are provided in the Appendix, and are given in the following equations.

$$\frac{dR_i^{RP}}{dq} = A\frac{dr^{**}}{dq}, \tag{8}$$

$$\frac{dR_i^{RP}}{d\beta} = A\frac{dr^{**}}{d\beta}, \tag{9}$$

where $A = \frac{2k_j}{r^{**2}} \frac{1}{|D|} \alpha'(R_j^{RP})(2 - k_i\alpha(R_i^{RP}) \left(-\frac{\alpha''(R_j^{RP})}{\alpha'(R_j^{RP})} - \frac{k_i\alpha'(R_i^{RP})}{(2 - k_i\alpha(R_i^{RP}))} \right)$.

From equations (8) and (9) we observe that it is not possible to have the equilibrium R&D investment of a firm increasing (decreasing) due to increases in piracy and network externality. That is, $\frac{dR_i^{RP}}{d\beta}$ and $\frac{dR_i^{RP}}{dq}$ have opposite signs. This is because $\frac{dr^{**}}{dq} < 0$ and $\frac{dr^{**}}{d\beta} > 0$.

The difference between the effects of an increase in piracy and an increase in the network externality on a firm's equilibrium R&D investment is given in equation (10).

$$\frac{dR_i^{RP}}{d\beta}-\left(-\frac{dR_i^{RP}}{dq}\right)=A\left(\frac{dr^{**}}{d\beta}-\left(-\frac{dr^{**}}{dq}\right)\right) \tag{10}$$

The sign of the expression in equation (10) depends on the signs of A and $\left(\frac{dr^{**}}{d\beta}-\left(-\frac{dr^{**}}{dq}\right)\right)$.

In A we know that $|D|>0$ from the second order condition (shown in footnote 12), and

$2k_j(\alpha'(R_j^{RP})(2-k_i\alpha(R_i^{RP}))>0$. So the sign of A depends on the sign of

$\left(-\frac{\alpha''(R_j^{RP})}{\alpha'(R_j^{RP})}-\frac{k_i\alpha'(R_i^{RP})}{(2-k_i\alpha(R_i^{RP}))}\right)$. Thus the sign of $\frac{dR_i^{RP}}{d\beta}-\left(-\frac{dR_i^{RP}}{dq}\right)$ depends on the sign

of $\left(-\frac{\alpha''(R_j^{RP})}{\alpha'(R_j^{RP})}-\frac{k_i\alpha'(R_i^{RP})}{(2-k_i\alpha(R_i^{RP}))}\right)$ and $\left(\frac{dr^{**}}{d\beta}-\left(-\frac{dr^{**}}{dq}\right)\right)$.

The result for the effects of an increase in piracy and an increase in the network externality on a firm's R&D investment is summarized in Proposition2. The proof is given in the Appendix.

Proposition 2. *Consider an increase in piracy and an increase in network externality. (i) If the two firms are "highly" asymmetric with respect to their R&D efficiencies and if the piracy effect dominates the network effect then the less efficient firm's equilibrium R&D investment increases and that of the more efficient firm decreases. The reverse is true if the network effect dominates the piracy effect. (ii) If the of the two firms are "less" asymmetric then the equilibrium R&D investment of both firms either increases or decreases depending on the relative strengths of the piracy and the network effects.*

The intuition behind the result summarized in Proposition 2 is as follows. When the piracy effect dominates the network effect it reduces the stage 2 realised profit of an innovating firm. The market uncertainty, which is the probability of winning the patent race if both firms are successful, is the same for both firms. The only difference between the two firms is with respect to the R&D efficiency parameters which is a measure of the probability of success when there is technological uncertainty. Thus a firm's stage 1 expected profit will be dictated by the relative strengths of the probability of success and the cost of R&D. An increase in the R&D investment decreases its expected profit because the cost goes up. However, an increase in the R&D investment increases the stage 1 expected profit because the probability of success increases.

In the symmetric case, ($k_i = k_j$) the equilibrium R&D investments of both firms decrease. Starting from the symmetric situation we allow k_i to take smaller values and k_j to take higher values which capture the asymmetric situation. From Lemma 3 we know that lower values of k_i and higher values of k_j implies lower R_i^{RP} and higher R_j^{RP}. By assumption $\alpha(R)$ is concave and that its curvature is decreasing in *R*. So for firm *i* a marginal increase in its R&D investment (R_i^{RP}) results in a higher increase in its probability of success ($k_i\alpha(R_i^{RP})$) compared to firm *j*. So lower the value of k_i, higher is the increase in the probability of success of firm *i* for a marginal increase in its R&D investment. Similarly, higher the value of k_j, lower is the increase in the probability of success of firm *j* for a marginal increase in its R&D investment.

This means that for firm *i* if its efficiency parameter is significantly low then the increase in its profit from a higher R&D investment outweigh the cost of doing so. The opposite is true for firm *j*. So if the piracy effect dominates the network effect then the less efficient firm's equilibrium investment increases and that of the more efficient firm decreases. The reverse argument holds for the case where the network effect dominates the piracy effect.

Proposition 2 implies that increase in piracy result in an increase in the R&D investment of one of the firms if they are sufficiently asymmetric. The dominance of the piracy effect or the network effect dictates whether the R&D investment of the less or the more efficient firm will increase. Both firms reduce their R&D investment if the piracy effect dominates the network effect and if the firms are less asymmetric. Thus the standard claim that piracy retards innovation incentives may not necessarily hold in the presence of network externality

We now discuss the impact of increases in piracy and network externality on the overall probability of a successful innovation. If the two firms are less asymmetric with respect to the efficiency parameters and if the piracy effect dominates the network effect then from Proposition 2 we know that both firms will reduce their R&D investment. Thus the overall probability of a successful innovation decreases. If the network effect dominates the piracy effect then both firms will increase their R&D investment levels and therefore the overall probability of a successful innovation increases.

Proposition 3 summarizes the overall probability of a successful innovation when the firms are sufficiently asymmetric. We discuss the proof in the main text.

Proposition 3. *Consider an increase in piracy and an increase in the network externality. If the firms are sufficiently asymmetric with respect to the R&D efficiency parameters and the piracy effect dominates the network effect, then there is an overall increase in the probability of a successful innovation. If the network effect dominates the piracy effect then the overall probability of successful innovation decreases.*

If the firms are sufficiently asymmetric and the piracy effect dominates the network effect, then from Proposition 2 we know that the less efficient firm will increase its R&D investment and the more efficient one will reduce it when there is an increase in piracy and network externality. The increase in the probability of success of the less efficient firm exceeds the decrease in the probability of success of the more efficient firm because $\alpha(R)$ is increasing and concave in R and that its curvature is decreasing in R. This means that the domination of piracy effect over network effect may result in an increase in the overall probability of success. The reverse is true when the network effect dominates the piracy effect.

4. Conclusions

This paper analysed the impact of an increase in piracy and network externality on the incentive to innovate. The piracy effect is the decrease in the innovating firm's realised profit due to an increase in piracy. The network effect is the increase in the realised profit due to an increase in the network externality. In the case of a single innovating firm facing technological uncertainty, I showed that the incentive to innovate increases when the network effect is stronger than the piracy effect.

However, with R&D competition, which generates market uncertainty on top of technological uncertainty, I showed that an increase in piracy and network effect results in an increase in the R&D investment of one firm if the competing firms significantly differ with respect to the efficiency in R&D investment. Specifically, if the piracy effect (network effect) is stronger than the network effect (piracy effect) then the less (more) efficient firm's R&D investment increases and that of the more (less) efficient firm's investment decreases.

I also showed that in the above case if the piracy effect dominates the network effect then the overall probability of successful innovation of a new product increases. The reverse is true when the network effect dominates the piracy effect. This paper thus showed that increases in piracy do not necessarily have a negative impact on innovation

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Appendix

Proof of Lemma 1. (i) $\frac{\delta r^{**}(q,\beta)}{\delta q} = \frac{-4(2+q)}{(1-\beta)(4-q)^3} < 0$ and

$\frac{\delta r^{**}(q,\beta)}{\delta \beta} = \frac{4(1-q)}{(1-\beta)^2(4-q)^2} > 0$. (ii) The difference between the absolute values of these two expressions are $\frac{\delta r^{**}(q,\beta)}{\delta \beta} - \frac{-\delta r^{**}(q,\beta)}{\delta q} = \frac{4(q^2-q(6-\beta)+2(1+\beta))}{(1-\beta)^2(4-q)^3}$. The sign depends on that of the numerator because the denominator is always positive. Now the numerator is decreasing in *q*. Solving for $q^2 - q(6-\beta) + 2(1+\beta) = 0$ yields $\bar{q} = \frac{6-\beta-\sqrt{28-20\beta+\beta^2}}{2}$.

So $\frac{\delta r^{**}(q,\beta)}{\delta \beta} - \frac{-\delta r^{**}(q,\beta)}{\delta q} > 0$ if $q < \bar{q}$ and the reverse is true if $q > \bar{q}$. *Q.E.D.*

Proof of Proposition 1. $\frac{\partial E\pi}{\partial R} = k\alpha'(R)r^{**} - 1 = 0 \Rightarrow \alpha'(R^P) = \frac{1}{kr^{**}}$.

$\alpha''(R^P)\frac{dR^P}{dq} = \frac{-1}{r^{**2}}\frac{dr^{**}}{dq} \Rightarrow \frac{dR^P}{dq} = \frac{-1}{r^{**2}\alpha''(R^P)}\frac{dr^{**}}{dq} < 0$ and

$\alpha''(R^P)\frac{dR^P}{d\beta} = \frac{-1}{r^{**2}}\frac{dr^{**}}{d\beta} \Rightarrow \frac{dR^P}{d\beta} = \frac{-1}{r^{**2}\alpha''(R^P)}\frac{dr^{**}}{dq} > 0$. This follows from Lemma 1 and the property that $\alpha''(R) < 0$. Now $\frac{dR^P}{d\beta} - \frac{-dR^P}{dq} = \frac{-1}{r^{**2}\alpha''(R^P)}\left(\frac{dr^{**}}{d\beta} - \frac{-dr^{**}}{dq}\right)$. The first expression is positive because $\alpha''(R) < 0$ hence the sign depends on that of the expression in parenthesis. The sign of this expression follows from Lemma 1. So

$\frac{dE\pi^P}{dq} = k\alpha'(R^P)\frac{dR^P}{dq} + k\alpha(R^P)\frac{dr^{**}}{dq} < 0$ since $\frac{dR^P}{dq} < 0$ and $\frac{dr^{**}}{dq} < 0$.

$\frac{dE\pi^P}{d\beta} = k\alpha'(R^P)\frac{dR^P}{d\beta} + k\alpha(R^P)\frac{dr^{**}}{d\beta} > 0$ since $\frac{dR^P}{d\beta} > 0$ and $\frac{dr^{**}}{d\beta} > 0$. So

$\frac{dE\pi^P}{d\beta} - (-\frac{dE\pi^P}{dq}) = k\alpha'(R^P)\left(\left(\frac{dR^P}{d\beta} - \frac{-dR^P}{dq}\right) + \left(\frac{dr^{**}}{d\beta} - \frac{-dr^{**}}{dq}\right)\right) =$

$k\alpha'(R^P)\left(1 - \frac{-1}{r^{**2}\alpha''(R^P)}\right)\left(\frac{dr^{**}}{d\beta} - \frac{-dr^{**}}{dq}\right)$**.** Since $k\alpha'(R^P) > 0$ and $\left(1 - \frac{-1}{r^{**2}\alpha''(R^P)}\right) > 0$

hence the sign depends on the sign of $\left(\frac{dr^{**}}{d\beta} - \frac{-dr^{**}}{dq}\right)$ which is stated in Lemma 1. *Q.E.D.*

Proof of Lemma 2. To find the effects of changes in q and β on the reaction function of firm *i* we differentiate its reaction with respect to q and β assuming that R_j remains unchanged. Differentiation with respect to q yields,

$$k_i\alpha''(R_i)\left((1+\frac{k_j\alpha(R_j)}{2}\right)r^{**}\frac{dR_i}{dq}+k_i\alpha'(R_i)\left((1+\frac{k_j\alpha(R_j)}{2}\right)\frac{dr^{**}}{dq}=0$$

$$\Rightarrow\frac{dR_i}{dq}=\frac{-k_i\alpha'(R_i)\left((1+\frac{k_j\alpha(R_j)}{2}\right)\frac{dr^{**}}{dq}}{k_i\alpha''(R_i)\left((1+\frac{k_j\alpha(R_j)}{2}\right)r^{**}}<0,$$

because the numerator is positive since $\frac{dr^{**}}{dq}<0$ from Lemma 1 and by assumption $\alpha''(R)<0$. Similarly, $\frac{dR_i}{d\beta}=\frac{-k_i\alpha'(R_i)\left((1+\frac{k_j\alpha(R_j)}{2}\right)\frac{dr^{**}}{d\beta}}{k_i\alpha''(R_i)\left((1+\frac{k_j\alpha(R_j)}{2}\right)r^{**}}>0,$ because from Lemma 1 we know that $\frac{dr^{**}}{d\beta}>0$ and by assumption $\alpha''(R)<0$. This means that an increase in piracy shifts down firm *i*'s reaction function and the opposite is true for an in increase in the network effect. The overall effect of an increase in piracy and network externality is obtained by taking the difference of the absolute value of the effects which is,

$\frac{dR_i}{d\beta}-\left(-\frac{dR_i}{dq}\right)=\frac{-k_i\alpha'(R_i)\left((1+\frac{k_j\alpha(R_j)}{2}\right)}{k_i\alpha''(R_i)\left((1+\frac{k_j\alpha(R_j)}{2}\right)r^{**}}\left(\frac{dr^{**}}{d\beta}-\left(-\frac{dr^{**}}{dq}\right)\right)$. The sign of this depends on the sign of $\left(\frac{dr^{**}}{d\beta}-\left(-\frac{dr^{**}}{dq}\right)\right)$ which we get from Lemma 1. That is $\left(\frac{dr^{**}}{d\beta}-\left(-\frac{dr^{**}}{dq}\right)\right)>0$ if $q<\bar{q}$ and $\left(\frac{dr^{**}}{d\beta}-\left(-\frac{dr^{**}}{dq}\right)\right)<0$ if $q>\bar{q}$. *Q.E.D.*

Proof of Lemma 3. Total differentiation of $k_i\alpha'(R_i^{RP})(2-k_j\alpha(R_j^{RP}))=\frac{2}{r_i^{**}}$ and

$k_j\alpha'(R_j^{RP})(2-k_i\alpha(R_i^{RP}))=\frac{2}{r_j^{**}}$ with respect to k_i and solving for $\frac{dR_i^{RP}}{dk_i}$ and $\frac{dR_j^{RP}}{dk_i}$ using

Cramer's rule yields,

$$\frac{dR_i^{RP}}{dk_i}=\frac{k_j\alpha'(R_j^{RP})}{|D|}\{-\alpha''(R_j^{RP})(2-k_i\alpha(R_i^{RP}))(2-k_j\alpha(R_j^{RP}))+k_ik_jk_i\alpha(R_i^{RP})(\alpha'(R_j^{RP}))^2\}>0$$

and $\frac{dR_j^{RP}}{dk_i}=-\frac{k_ik_j\alpha'(R_j^{RP})(2-k_j\alpha(R_j^{RP}))}{|D|}\{-\alpha''(R_j^{RP})\alpha(R_j^{RP})-(\alpha'(R_i^{RP}))^2\}<0$. *Q.E.D.*

Derivation of $\frac{dR_i^{RP}}{dq}$, $\frac{dR_j^{RP}}{dq}$, $\frac{dR_i^{RP}}{d\beta}$, $\frac{dR_j^{RP}}{d\beta}$, $\frac{dR_i^{RP}}{d\beta}-\left(-\frac{dR_i^{RP}}{dq}\right)$ **and** $\frac{dR_i^{RP}}{d\beta}-\left(-\frac{dR_i^{RP}}{dq}\right)$.

1. To derive the expressions for $\frac{dR_i^{RP}}{dq}$ and $\frac{dR_j^{RP}}{dq}$ we substitute (R_i^{RP},R_j^{RP}) in the first order condition in equation (7) and perform total differentiation with respect to q. This yields,

$$k_i\alpha''(R_i^{RP})(2-k_j\alpha(R_j^{RP}))\frac{dR_i^{RP}}{dq}-k_ik_j\alpha'(R_i^{RP})\alpha'(R_j^{RP})\frac{dR_j^{RP}}{dq}=-\frac{2}{r^{**2}}\frac{dr^{**}}{dq}$$ and

$$k_j\alpha''(R_j^{RP})(2-k_i\alpha(R_i^{RP}))\frac{dR_j^{RP}}{dq}-k_ik_j\alpha'(R_i^{RP})\alpha'(R_j^{RP})\frac{dR_i^{RP}}{dq}=-\frac{2}{r^{**2}}\frac{dr^{**}}{dq}$$. Using

Cramer's rule to solve for $\frac{dR_i^{RP}}{dq}$ and $\frac{dR_j^{RP}}{dq}$ gives us the following two equations.

$$\frac{dR_i^{RP}}{dq}=-\frac{2k_j}{r^{**2}}\frac{dr^{**}}{dq}\frac{1}{|D|}\alpha'(R_j^{RP})(2-k_i\alpha(R_i^{RP})\left(\frac{\alpha''(R_j^{RP})}{\alpha'(R_j^{RP})}+\frac{k_i\alpha'(R_i^{RP})}{(2-k_i\alpha(R_i^{RP}))}\right)$$
$$\frac{dR_j^{RP}}{dq}=-\frac{2k_i}{r^{**2}}\frac{dr^{**}}{dq}\frac{1}{|D|}\alpha'(R_i^{RP})(2-k_j\alpha(R_j^{RP})\left(\frac{\alpha''(R_i^{RP})}{\alpha'(R_i^{RP})}+\frac{k_j\alpha'(R_j^{RP})}{(2-k_j\alpha(R_j^{RP}))}\right)$$

2. To derive the expressions for $\frac{dR_i^{RP}}{d\beta}$ and $\frac{dR_j^{RP}}{d\beta}$ we proceed analogously by performing the total differentiation with respect to β which yields,

$$k_i\alpha''(R_i^{RP})(2-k_j\alpha(R_j^{RP}))\frac{dR_i^{RP}}{d\beta}-k_ik_j\alpha'(R_i^{RP})\alpha'(R_j^{RP})\frac{dR_j^{RP}}{d\beta}=-\frac{2}{r^{**2}}\frac{dr^{**}}{d\beta}$$ and

$$k_j\alpha''(R_j^{RP})(2-k_i\alpha(R_i^{RP}))\frac{dR_j^{RP}}{d\beta}-k_ik_j\alpha'(R_i^{RP})\alpha'(R_j^{RP})\frac{dR_i^{RP}}{d\beta}=-\frac{2}{r^{**2}}\frac{dr^{**}}{d\beta}.$$ Using

Cramer's rule to solve for $\frac{dR_i^{RP}}{d\beta}$ and $\frac{dR_j^{RP}}{d\beta}$ gives us the following two equations.

$$\frac{dR_i^{RP}}{d\beta}=-\frac{2k_j}{r^{**2}}\frac{dr^{**}}{d\beta}\frac{1}{|D|}\alpha'(R_j^{RP})(2-k_i\alpha(R_i^{RP})\left(\frac{\alpha''(R_j^{RP})}{\alpha'(R_j^{RP})}+\frac{k_i\alpha'(R_i^{RP})}{(2-k_i\alpha(R_i^{RP}))}\right)$$

$$\frac{dR_j^{RP}}{d\beta}=-\frac{2k_i}{r^{**2}}\frac{dr^{**}}{d\beta}\frac{1}{|D|}\alpha'(R_i^{RP})(2-k_j\alpha(R_j^{RP})\left(\frac{\alpha''(R_i^{RP})}{\alpha'(R_i^{RP})}+\frac{k_j\alpha'(R_j^{RP})}{(2-k_j\alpha(R_j^{RP}))}\right).$$

$$\frac{dR_i^{RP}}{d\beta}-\left(-\frac{dR_i^{RP}}{dq}\right)=$$

$$\frac{2k_j(\alpha'(R_j^{RP})(2-k_i\alpha(R_i^{RP}))}{|D|r^{**2}}\left(-\frac{\alpha''(R_j^{RP})}{\alpha'(R_j^{RP})}-\frac{k_i\alpha'(R_i^{RP})}{(2-k_i\alpha(R_i^{RP}))}\right)\left(\frac{dr^{**}}{d\beta}-\left(-\frac{dr^{**}}{dq}\right)\right)$$

$$\frac{dR_j^{RP}}{d\beta}-\left(-\frac{dR_j^{RP}}{dq}\right)=$$

$$\frac{2k_i(\alpha'(R_i^{RP})(2-k_j\alpha(R_j^{RP}))}{|D|r^{**2}}\left(-\frac{\alpha''(R_i^{RP})}{\alpha'(R_i^{RP})}-\frac{k_j\alpha'(R_j^{RP})}{(2-k_j\alpha(R_j^{RP}))}\right)\left(\frac{dr^{**}}{d\beta}-\left(-\frac{dr^{**}}{dq}\right)\right)$$

Proof of Proposition 2. We begin with the symmetric case where $k_i = k_j$. In this case $R_i^{RP} = R_j^{RP}$. Changes in piracy and network effect will have symmetric effects on these equilibrium values, thus, the signs of $\frac{dR_i^{RP}}{d\beta}-\left(-\frac{dR_i^{RP}}{dq}\right)$ and $\frac{dR_j^{RP}}{d\beta}-\left(-\frac{dR_j^{RP}}{dq}\right)$ are the same. So either they are positive or negative depending on whether $q<\bar{q}$ or $q>\bar{q}$ which determines which effect dominate as given in Lemma 2. Starting from the point of symmetry let us consider k_i taking smaller values and k_j taking higher values. Lower values of k_i and higher values of k_j implies lower R_i^{RP} and higher R_j^{RP}. Consequently, $-\frac{\alpha''(R_i^{RP})}{\alpha'(R_i^{RP})}$ becomes higher and $-\frac{\alpha''(R_j^{RP})}{\alpha'(R_j^{RP})}$ becomes lower because, by assumption, $-\frac{\alpha''(R)}{\alpha'(R)}$ is decreasing in *R*.

As $-\frac{\alpha''(R_j^{RP})}{\alpha'(R_j^{RP})}$ becomes lower, a lower k_i can sustain the inequality

$\frac{k_i\alpha'(R_i^{RP})}{(2-k_i\alpha(R_i^{RP}))} > -\frac{\alpha''(R_j^{RP})}{\alpha'(R_j^{RP})}$ because from the first order condition given in equation (7) we

know that k_i must satisfy $k_i < \frac{2}{\alpha(R_i^{RP})}$. So in this case $0 > -\frac{\alpha''(R_j^{RP})}{\alpha'(R_j^{RP})} - \frac{k_i\alpha'(R_i^{RP})}{(2-k_i\alpha(R_i^{RP}))}$

which is an important determinant in the sign of $\frac{dR_i^{RP}}{d\beta} - \left(-\frac{dR_i^{RP}}{dq}\right)$. As $-\frac{\alpha''(R_i^{RP})}{\alpha'(R_i^{RP})}$

becomes higher due to a small k_i, it needs a very high k_j to sustain the inequality

$\frac{k_j\alpha'(R_j^{RP})}{(2-k_j\alpha(R_j^{RP}))} > -\frac{\alpha''(R_i^{RP})}{\alpha'(R_i^{RP})}$. However, k_j is restricted by $k_j < \frac{2}{\alpha(R_j^{RP})}$ and a higher

R_j^{RP} means a lower $\frac{2}{\alpha(R_j^{RP})}$ which further restricts the possibility of a high k_j. Thus for

firm *j* $-\frac{\alpha''(R_i^{RP})}{\alpha'(R_i^{RP})} - \frac{k_j\alpha'(R_j^{RP})}{(2-k_j\alpha(R_j^{RP}))} > 0$ which is an important determinant of the sign of

$\frac{dR_j^{RP}}{d\beta} - \left(-\frac{dR_j^{RP}}{dq}\right)$. Suppose the increase in the network effect dominate the effect of an

increase in the piracy rate, that is, $\left(\frac{dr^{**}}{d\beta} - \left(-\frac{dr^{**}}{dq}\right)\right) > 0$ (that is, $q < \bar{q}$) which implies that

the reaction functions shift up. In this case for firm *i*,

$\frac{dR_i^{RP}}{d\beta} - \left(-\frac{dR_i^{RP}}{dq}\right) = \left(-\frac{\alpha''(R_i^{RP})}{\alpha'(R_i^{RP})} - \frac{k_j\alpha'(R_j^{RP})}{(2-k_j\alpha(R_j^{RP}))}\right)\left(\frac{dr^{**}}{d\beta} - \left(-\frac{dr^{**}}{dq}\right)\right) < 0$, and for firm *j*,

$\frac{dR_j^{RP}}{d\beta} - \left(-\frac{dR_j^{RP}}{dq}\right) = \left(-\frac{\alpha''(R_i^{RP})}{\alpha'(R_i^{RP})} - \frac{k_j\alpha'(R_j^{RP})}{(2-k_j\alpha(R_j^{RP}))}\right)\left(\frac{dr^{**}}{d\beta} - \left(-\frac{dr^{**}}{dq}\right)\right) > 0$. Thus if the network effect dominates the piracy effect then the more efficient firm's equilibrium R&D investment increases and that of the less efficient firm's decreases. When $q > \bar{q}$ which implies that $\left(\frac{dr^{**}}{d\beta} - \left(-\frac{dr^{**}}{dq}\right)\right) < 0$, that is the piracy effect dominates the network effect, then,

$$\frac{dR_i^{RP}}{d\beta} - \left(-\frac{dR_i^{RP}}{dq}\right) = \left(-\frac{\alpha''(R_i^{RP})}{\alpha'(R_i^{RP})} - \frac{k_j \alpha'(R_j^{RP})}{(2 - k_j \alpha(R_j^{RP}))}\right)\left(\frac{dr^{**}}{d\beta} - \left(-\frac{dr^{**}}{dq}\right)\right) > 0 \text{ for firm } i\text{, and}$$

$$\frac{dR_j^{RP}}{d\beta} - \left(-\frac{dR_j^{RP}}{dq}\right) = \left(-\frac{\alpha''(R_i^{RP})}{\alpha'(R_i^{RP})} - \frac{k_j \alpha'(R_j^{RP})}{(2 - k_j \alpha(R_j^{RP}))}\right)\left(\frac{dr^{**}}{d\beta} - \left(-\frac{dr^{**}}{dq}\right)\right) > 0 \text{ for firm } j.$$

That is, the less efficient firm's equilibrium R&D investment increases and that of the more efficient firm's decreases. *Q.E.D.*