Agglomeration and Growth with Endogenous Expenditure Shares

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Abstract

We develop a New Economic Geography and Growth model which, by using a CES utility function in the second-stage optimization problem, allows for expenditure shares in industrial goods to be endogenously determined. The implications of our generalization are quite relevant. In particular, we obtain the following novel results: 1) two additional non-symmetric interior steady states emerge for some intermediate values of trade costs. These steady-states are stable if the industrial and the traditional goods are either good or *very* poor substitutes, while they are unstable for intermediate (yet lower than one) values of the intersectoral elasticity of substitution. In the latter case, the model displays three interior steady states - the symmetric and the core-periphery allocations - which are stable at the same time; 2) catastrophic agglomeration may always take place, whatever the degree of market integration, provided that the traditional and the industrial goods are sufficiently good substitutes; 3) the regional rate of growth is affected by the interregional allocation of economic activities even in the absence of localized spillovers, so that geography always matters for growth and 4) the regional rate of growth is affected by the degree of market openness: in particular, depending on whether the traditional and the industrial goods are good or poor substitutes, economic integration may be respectively growth-enhancing or growth-detrimental.

Key words: agglomeration, growth, endogenous expenditure shares, elasticity of substitution

JEL Classifications: O41, F43, R12.

1 Introduction

The recent Nobel Prize assigned to Paul Krugman "for his analysis of trade patterns and location of economic activity"witnesses the important role that the scientific community gives to the insights of the New Economic Geography (NEG) literature. This field of economic analysis has always been particularly appealing to policy makers, given the direct link between its results and regional policy rules. For the same reason it is useful to deepen the analysis of its most important outputs by testing the theoretical robustness of some of its more relevant statements. This paper tries to offer a contribution in this direction by focusing on a particular sub-field of NEG literature, the New Economic Geography and Growth (NEGG, henceforth), which basically adds endogenous growth to a version of Krugman's core-periphery model (Krugman 1991).

In this paper, we develop a NEGG model whose main deviation from the standard approach is the adoption of a Constant Elasticity Function (henceforth CES) instead of a Cobb-Douglas utility function in the second-stage optimization problem, thereby allowing the elasticity of substitution between manufacture and traditional good (*intersectoral* elasticity henceforth) to diverge from the unit value. The main effect of these departures is that the share of expenditure on manufactures is no longer exogenously fixed (as in the Cobb-Douglas approach) but it is endogenously determined via agents' optimization. By endogenizing the expenditure shares in manufacturing goods, we are able to test the robustness of several well-established results in the NEGG literature and we show that the validity of such results, and of the associated policy implications, crucially depends on the particular Cobb-Douglas functional form used by this class of models.

Our deviations from the standard NEGG literature act at two different levels: a) the dynamic pattern of equilibrium allocation of economic activities and b) the equilibrium growth prospect.

As for the first level, the main result of our analysis is the emergence of a completely new multiple equilibria pattern. In particular, our analysis shows that, for some intermediate values of the trade costs, two new non-symmetric interior steady states emerge. These steady states turn out to be stable when the intersectoral elasticity of substitution is either larger than 1 (and then the two kinds of commodities are good substitutes) or very low (i.e. very poor substitution), while they are unstable otherwise, i.e., when the traditional and the industrial goods are not-too-poor substitutes. In the latter case, a very interesting equilibrium pattern arises: the two emerging non-symmetric equilibria remain unstable until they collapse, for a higher value of the transport costs, to the core-periphery equilibria. The result is a a multiple equilibria pattern with three equilibria (the symmetric and the two core-periphery allocations) stable at the same time. In other words, if the economy starts from a non-symmetric equilibrium and trade costs are neither too low or too high, a very small shock can give rise to a catastrophic agglomeration or to a catastrophic dispersion. Citare Baldwin book

While the multiplicity of equilibria is due to the non-linear form of the optimal-investment relation, the dynamic properties can be viewed as the result of a new force, which we dub as the expenditure share effect. This force, which is a direct consequence of the dependence of the expenditure shares on the location of economic activities, is neutralized in the standard NEGG model by the unitary intersectoral elasticity of substitution. Our model "activates"this force which turns to be an agglomeration or a dispersion force depending on whether the traditional and the differentiated commodities are respectively good or poor substitutes. In the first case we show that, unlike the standard model, catastrophic agglomeration may always take place whatever the degree of market integration, provided that such force is strong enough. This result, which is also a novelty in the NEGG literature, has important implications in suggesting that policy makers should be aware of the fact that policies affecting the degree of market integration can affect the equilibrium location of economic activities only for a restricted set of values for the parameters describing the economy. More generally, the emergence of the expenditure share effect suggests that the intersectoral elasticity of substitution has a crucial and unexpected role in shaping the agglomeration or the dispersion process of economic activities.

As for the equilibrium growth prospect, results are even more striking. We show that, due to the endogenous expenditure shares: 1) the regional rate of growth is affected by the interregional allocation of economic activities even in the absence of localized spillovers, so that geography *always* matters for growth and 2) the regional rate of growth is affected by the degree of market openness: in particular, according to whether the intersectoral elasticity of substitution is larger or smaller than unity, economic integration may be respectively growth-enhancing or growth-detrimental. These results are novel with respect to the standard NEGG literature according to which geography matters for growth only when knowledge spillovers are localized and, moreover, trade costs never affect the growth rate in a direct way. They can also be all the more appreciated by viewing them as the dynamic counterparts of endogenous expenditure shares in static models. In the static model of Murata (2008), trade costs have *level effects* since the mass of varieties depends on trade costs via endogenous expenditure share generated by a Stone-Geary non-homothetic utility function. By adding the time-dimension, our model allows to uncover the emergence of an additional *growth* effect of trade costs as they also affect the rate of growth via the endogenous expenditure share. This second set of results is characterized by even more important policy implications: first, our results suggest that interregional allocation of economic activity can always be considered as an instrument able to affect the rate of growth of the economy. In particular, when the average interregional expenditure share on industrial goods are higher in the symmetric equilibrium than in the core-periphery one, then each policy aiming at equalizing the relative size of the industrial sector in the two regions will be good for growth, and vice-versa. Second, each policy affecting economic integration will also affect the rate of growth and the direction of such influence is crucially linked to the value of the intersectoral elasticity of substitution.

As already anticipated, the literature we refer to is basically the New Economic Geography and Growth (NEGG) literature, having in Baldwin and Martin (2004) and Baldwin *et al.* (2004) the most important theoretical syntheses. These two surveys collect and present in an unified framework the works by Baldwin, Martin and Ottaviano (2001) - where capital is immobile and spillovers are localized - and Martin and Ottaviano (1999) where spillovers are global and capital is mobile. Other related papers are Baldwin (1999) which introduces forward looking expectations in the Footloose Capital model developed by Martin and Rogers (1995); Baldwin and Forslid (1999) which introduces endogenous growth by means of a q-theory approach; Baldwin and Forslid (2000) where spillovers are localized, capital is immobile and migration is allowed. Some more recent developments in the NEGG literature can be grouped in two main strands. One takes into consideration factor price differences in order to discuss the possibility of a non-monotonic relation between agglomeration and integration (Bellone and Maupertuis (2003) and Andres (2007)). The other one assumes firms heterogeneity in productivity (first introduced by Eaton and Kortum (2002) and Melitz (2003)) in order to analyse the relationship between growth and the spatial selection effect leading the most productive firms to move to larger markets (see Baldwin and Okubo (2006) and Baldwin and Robert-Nicoud (2008)). These recent developments are related to our paper in that they introduce some relevant departures from the standard model.

All the aforementioned papers, however, work with exogenous expenditure shares. A first attempt to introduce endogenous expenditure shares in a NEGG model has been carried out by Cerina and Pigliaru (2007), who focused on the effects on the balanced growth path of introducing such assumption. The present paper can be seen as an extension of the latter, considering that we deepen the analysis of the implications of endogenous expenditure shares by fully assessing the dynamics of the model, the mechanisms of agglomeration and the equilibria growth rate.

We believe that the results obtained in this paper are important because they shed light on some mechanism which are neglected by the literature and which might be empirically relevant. From this viewpoint, the main message of our paper is probably that of highlighting how a more relevant effort on the empirical assessment of the intersectoral elasticity of substitution is strongly needed. Moreover, from a purely theoretical perspective, a tractable endogenous expenditure share approach, being more general than an exogenous one, represents a theoretical progress in the NEG literature and it can be extended to several other NEG models in order to assess their robustness. Finally, from a policy perspective, our paper suggests that policy makers should not trust too much on implications drawn from standard NEGG models because of their limited robustness.

The rest of the paper is structured as follows: section 2 presents the analytical framework, section 3 deals with the equilibrium location of economic activities, section 4 develops the analysis of the growth rate and section 5 concludes.

2 The Analytical Framework

2.1 The Structure of the Economy

The model structure is closely related to Baldwin, Martin and Ottaviano (2001). The world is made of 2 regions, North and South, both endowed with 2 factors: labour L and capital K. 3 sectors are active in both regions: manufacturing M, traditional good T and a capital producing sector I. Regions are symmetric in terms of: preferences, technology, trade costs and labour endowment. Labour is assumed to be immobile across regions but mobile across sectors within the same region. The traditional good is freely traded between regions whilst manufacture is subject to iceberg trade costs following Samuelson (1954). For the sake of simplicity we will focus on the northern region[1].

Manufactures are produced under Dixit-Stiglitz monopolistic competition (Dixit and Stiglitz, 1975, 1977) and enjoy increasing returns to scale: firms face a fixed cost in terms of knowledge capital[2] and a variable cost a_M in terms of labour. Thereby the cost function is $\pi + wa_M x_i$, where π is the rental rate of capital, w is the wage rate and a_M are the unit of labour necessary to produce a unit of output x_i.

Each region's K is produced by its I-sector which produces one unit of K with a_I unit of labour. So the production and marginal cost function for the I-sector are, respectively:

$$\dot{K} = Q_K = \frac{L_I}{a_I} \tag{1}$$

$$F = wa_I \tag{2}$$

Note that this unit of capital in equilibrium is also the fixed cost F of the manufacturing sector. As one unit of capital is required to start a new variety, the number of varieties and of firms at the world level is simply equal to the capital stock at the world level: $K + K^* = K^w$. We denote n and n^* as the number of firms located in the north and south respectively. As one unit of capital is required per firm we also know that: $n + n^* = n^w = K^w$. As in Baldwin, Martin and Ottaviano (2001), we assume capital immobility, so that each firm operates, and spends its profits, in the region where the capital's owner lives. In this case, we also have that $n = K$ and $n^* = K^*$. Then, by defining $s_n = \frac{n}{n^w}$ and $s_K = \frac{K}{K^w}$, we also have $s_n = s_K$: the share of firms located in one region is equal to the share of capital owned by the same region[3].

To individual I-firms, the innovation cost a_I is a parameter. However, following Romer (1990), endogenous and sustained growth is provided by assuming that the marginal cost of producing new capital declines (i.e., a_I falls) as the sector's cumulative output rises. In the most general form, learning spillovers are assumed to be localised. The cost of innovation can be expressed as:

$$a_I = \frac{1}{AK^w} \tag{3}$$

where $A \equiv s_K + \lambda (1 - s_K)$, $0 < \lambda < 1$ measures the degree of globalization of learning spillovers and $s_K = n/n^w$ is share of firms allocated in the north. The south's cost function is isomorphic, that is,

[1] Unless differently stated, the southern expressions are isomorphic.

[2] It is assumed that producing a variety requires a unit of knowledge interpreted as a blueprint, an idea, a new technology, a patent, or a machinery.

[3] We highlight that our results on the equilibrium growth rate holds even in the case of capital mobility.

$F^* = w^*/K^w A^*$ where $A^* = \lambda s_K + 1 - s_K$. However, for the sake of simplicity, we focus on the case of global spillovers, i.e., $\lambda = 1$ and $A = A^* = 1$[4]. Moreover, in the model version we examine, capital depreciation is ignored[5].

Because the number of firms, varieties and capital units is equal, the growth rate of the number of varieties, on which we focus, is therefore:

$$g \equiv \frac{\dot{K}}{K}; g^* \equiv \frac{\dot{K}^*}{K^*}$$

Finally, traditional goods, which are assumed to be homogeneous, are produced by the T-sector under conditions of perfect competition and constant returns. By choice of units, one unit of T is made with one unit of L.

2.2 Preferences and consumers' behaviour

The preferences structure of the infinitely-lived representative agent is given by:

$$U_t = \int_{t=0}^{\infty} e^{-\rho t} \ln Q_t dt; \tag{4}$$

$$Q_t = \left[\delta\left(n^{w\frac{1}{1-\sigma}} C_M\right)^{\alpha} + (1-\delta) C_T{}^{\alpha}\right]^{\frac{1}{\alpha}}; \alpha < \frac{\sigma - 1}{\sigma} \tag{5}$$

$$C_M = \left[\int_{i=0}^{n+n^*} c_i^{1-1/\sigma} di\right]^{\frac{1}{1-1/\sigma}}; \sigma > 1. \tag{6}$$

Where α is the elasticity parameter related to the elasticity of substitution between manufacture and traditional goods and σ is the elasticity of substitution across varieties. We deviate from the standard NEGG framework in two respects.

First, we use a more general CES second-stage utility function instead of a Cobb-Douglas one, thereby allowing the elasticity of substitution between manufacture and traditional good (*intersectoral* elasticity henceforth) to diverge from the unit value. The intersectoral elasticity is equal to $\frac{1}{1-\alpha}$ which might be higher or lower than unity (albeit constant) depending on whether α is respectively negative or positive. In the intermediate case, when $\alpha = 0$, the intersectoral elasticity of substitution is equal to 1 and the second-stage utility function collapses to the Cobb-Douglas case. The main effect of this modification is that the share of expenditure on manufacture is no longer constant but it is affected by changes in the price indexes of manufacture. This consequence is the source of most of the result of this paper.

Second, as in Murata (2008) in the context of a NEG model and Blanchard and Kiyotaki (1987) in a macroeconomic context, we neutralize agents' love for variety by setting to zero its parameter. Notice that in the canonical NEGG framework the love for variety parameter takes the positive value $\frac{1}{\sigma-1}$, being tied to the elasticity of substitution across varieties σ (*intrasectoral* elasticity henceforth)[6]. An analytical consequence of

[4]Analysing the localised spillover case is possible, but it will not significantly enrich the results and it will obscure the object of our analysis.

[5]See Baldwin (1999) and Baldwin et al. (2004) for similar analysis with depreciation but with exogenous expenditure shares.

[6]Take an utility function $U(C_T.C_M)$ where $C_M = V_n(c_1, ..., c_n)$ is homogeneous of degree one, with n being the number of varieties. By adopting the natural normalization $V_1(q_1) = q_1$, we can define the following function:

$$\gamma(n) = \frac{V_n(c, ..., c)}{V_1(nc)} = \frac{V_n(1, ..., 1)}{n}$$

with $\gamma(n)$ representing the gain in utility derived from spreading a certain amount of expenditure across n varieties instead of concentrating it on a single one. The degree of love for variety v is just the elasticity of the $\gamma(n)$ function:

$$v(n) = \frac{n\gamma'(n)}{\gamma(n)}$$

In the standard NEGG framework $C_M = \left(\int_0^n c_i^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}}$ hence $\gamma(n) = \frac{1}{\sigma-1}$.

abstracting from the love of variety is the emergence of the term $n^{w^{\frac{1}{1-\sigma}}}$ in the second-stage utility function: this normalization neutralizes the dependence of the price index on the number of varieties allowing us to concentrate the analysis on the influence of firms' location and transport costs on the expenditure shares. We do this for several reasons: 1) by abstracting from the love of variety, we are able to focus on the effect that a non-unitary value of the intersectoral elasticity of substitution has on the equilibrium outcomes of the model; 2) as explained in detail in Cerina and Pigliaru (2007), by eliminating the love for variety when using a second-stage CES utility we are able to solve some analytical problems related to the existence of a balanced growth path and the feasibility of the no-specialization condition[7]; 3) our assumption has some empirical support as shown by Ardelean (2007) according to which the value of the love of variety parameter is significantly lower than what assumed in NEG models. Allowing for a larger-than-unity intersectoral elasticity of substitution, requires the introduction of a natural restriction on its value relative to the one of the intrasectoral elasticity of substitution. The introduction of two distinct sectors would in fact be useless if substituting goods from the traditional to the manufacturing sector (and vice-versa) was easier than substituting goods within the differentiated industrial sectors. In other words, in order for the representation in terms of two distinct sectors to be meaningful, we need goods belonging to different sectors to be poorer substitutes than varieties coming from the same differentiated sector. The formal expression of this idea requires that the intersectoral elasticity of substitution $\frac{1}{1-\alpha}$ is lower than the intrasectoral elasticity of substitution σ:

$$\frac{1}{1-\alpha} < \sigma$$

which means that α should be lower than $\frac{\sigma-1}{\sigma}$. This assumption, which will be maintained for the rest of the paper, states that α cannot not be too high. It is worth to note that this assumption is automatically satisfied in the standard Cobb-Douglas approach where $\frac{1}{1-\alpha} = 1$ and $\sigma > 1$.

The infinitely-lived representative consumer's optimization is carried out in three stages. In the first stage the agent intertemporally allocates consumption between expenditure and savings. In the second stage she allocates expenditure between manufacture and traditional goods, while in the last stage she allocates manufacture expenditure across varieties. As a result of the intertemporal optimization program, the path of consumption expenditure E across time is given by the standard Euler equation:

$$\frac{\dot{E}}{E} = r - \rho \tag{7}$$

with the interest rate r satisfying the no-arbitrage-opportunity condition between investment in the safe asset and capital accumulation:

$$r = \frac{\pi}{F} + \frac{\dot{F}}{F} \tag{8}$$

where π is the rental rate of capital and F its asset value which, due to perfect competition in the I-sector, is equal to its marginal cost of production.

In the second stage the agent chooses how to allocate the expenditure between manufacture and the traditional good according to the following optimization program:

$$\begin{aligned} \max_{C_M, C_T} Q_t &= \left[\delta\left(n^{w^{\frac{1}{1-\sigma}}} C_M\right)^{\alpha} + (1-\delta) C_T^{\alpha}\right]^{\frac{1}{\alpha}} \\ s.t. &: \quad P_M C_M + p_T C_T = E \end{aligned} \tag{9}$$

[7]The role of love for variety in our model is explained in details in Cerina and Pigliaru (2007) who introduce and study the analytical implications of the following second-stage utility function

$$Q_t = \left[\delta\left(n^{w^{\frac{1}{1-\sigma}+v}} C_M\right)^{\alpha} + (1-\delta) C_T^{\alpha}\right]^{\frac{1}{\alpha}}$$

By setting $v = 0$ we obtain (5)

As a result of the maximization we obtain the following demand for the manufactured and the traditional goods:

$$P_M C_M = \mu \left(n^w, \frac{P_M}{p_T} \right) E \tag{10}$$

$$p_T C_T = \left(1 - \mu \left(n^w, \frac{P_M}{p_T} \right) \right) E \tag{11}$$

where p_T is the price of the traditional good, $P_M = \left[\int_{i=0}^{K+K^*} p_i^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}$ is the Dixit-Stiglitz perfect price index and $\mu(n^w, \frac{P_M}{p_T})$ is the share of expenditure in manufacture which, unlike the CD case, is not exogenously fixed but it is endogenously determined via the optimization process and it is a function of the total number of varieties and of goods' relative prices. This feature is crucial to our analysis.

The northern share of expenditure in manufacture is given by:

$$\mu \left(n^w, \frac{P_M}{p_T} \right) = \left(\frac{1}{1 + \left(\frac{P_M}{p_T} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1-\delta}{\delta} \right)^{\frac{1}{1-\alpha}} \left(n^{w\,\frac{1}{1-\sigma}} \right)^{-\frac{\alpha}{1-\alpha}}} \right) \tag{12}$$

while the symmetric expression for the south is:

$$\mu \left(n^w, \frac{P_M^*}{p_T^*} \right) = \left(\frac{1}{1 + \left(\frac{P_M^*}{p_T^*} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1-\delta}{\delta} \right)^{\frac{1}{1-\alpha}} \left(n^{w\,\frac{1}{1-\sigma}} \right)^{-\frac{\alpha}{1-\alpha}}} \right) \tag{13}$$

so that northern and southern expenditure shares only differ because of the difference between northern and southern relative prices.

Finally, in the third stage, the amount of $M-$ goods expenditure $\mu \left(n^w, \frac{P_M}{p_T} \right) E$ is allocated across varieties according to the a CES demand function for a typical M-variety $c_j = \frac{p_j^{-\sigma}}{P_M^{1-\sigma}} \mu \left(n^w, \frac{P_M}{p_T} \right) E$, where p_j is variety j's consumer price. Southern optimization conditions are isomorphic.

2.3 Specialization Patterns and Non-Unitary Elasticity of Substitution

Due to perfect competition in the T-sector, the price of the agricultural good must be equal to the wage of the traditional sector's workers: $p_T = w_T$. Moreover, *as long as both regions produce some* T, the assumption of free trade in T implies that not only price, but also wages are equalized across regions. It is therefore convenient to choose home labour as numeraire so that:

$$p_T = p_T^* = w_T = w_T^* = 1$$

As a first consequence, northern and southern expenditure shares are now only function of the respective industrial price indexes and of the total number of varieties so that we can write:

$$\mu \left(n^w, \frac{P_M}{p_T} \right) = \mu \left(n^w, P_M \right)$$
$$\mu \left(n^w, \frac{P_M^*}{p_T^*} \right) = \mu \left(n^w, P_M^* \right)$$

As it is well-known, it's not always the case that both regions produce some T. An assumption is actually needed in order to avoid complete specialization: a single country's labour endowment must be insufficient to meet global demand. Formally, the CES approach version of this condition is the following:

$$L = L^* < \left(\left[1 - \mu(n^w, P_M) \right] s_E + \left[1 - \mu(n^w, P_M^*) \right] (1 - s_E) \right) E^w \tag{14}$$

where $s_E = \frac{E}{E^w}$ is northern expenditure share and $E^w = E + E^*$.

In the standard CD approach, where $\mu(n^w, P_M) = \mu(n^w, P_M^*) = \mu$, this condition collapses to:

$$L = L^* < (1 - \mu) E^w.$$

The purpose of making this assumption, which is standard in most NEGG models[8], is to maintain the M-sector and the I-sector wages fixed at the unit value: since labour is mobile across sector, as long as the T - sector is present in both regions, a simple arbitrage condition suggests that wages of the three sectors cannot differ. Hence, $M-$ sector and I-sector wages are tied to T -sector wages which, in turn, remain fixed at the level of the unit price of a traditional good. Therefore:

$$w_M = w_M^* = w_T = w_T = w = 1 \tag{15}$$

Finally, since wages are uniform and all varieties' demand have the same constant elasticity σ, firms' profit maximization yields local and export prices that are identical for all varieties no matter where they are produced: $p = wa_M \frac{\sigma}{\sigma-1}$. Then, imposing the standard normalization which assigns the value $\frac{\sigma-1}{\sigma}$ to the marginal labor unit requirement and using (**??**), we finally obtain:

$$p = w = 1 \tag{16}$$

As usual, since trade in the $M-$good is impeded by iceberg import barriers, prices for markets abroad are higher:

$$p^* = \tau p; \ \tau \geq 1$$

By labeling as p_M^{ij} the price of a particular variety produced in region i and sold in region j (so that $p^{ij} = \tau p^{ii}$) and by imposing $p = 1$, the $M-$goods price indexes might be expressed as follows:

$$P_M = \left[\int_0^n (p_M^{NN})^{1-\sigma} di + \int_0^{n^*} (p_M^{SN})^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} = (s_K + (1 - s_K)\,\phi)^{\frac{1}{1-\sigma}}\, n^{w\frac{1}{1-\sigma}} \tag{17}$$

$$P_M^* = \left[\int_0^n (p_M^{NS})^{1-\sigma} di + \int_0^{n^*} (p_M^{SS})^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} = (\phi s_K + 1 - s_K)^{\frac{1}{1-\sigma}}\, n^{w\frac{1}{1-\sigma}} \tag{18}$$

where $\phi = \tau^{1-\sigma}$ is the so called "phi-ness of trade" which ranges from 0 (prohibitive trade) to 1 (costless trade).

Substituting the new expressions for the $M-$goods price indexes in the northern and southern $M-$goods expenditure shares, yields:

$$\mu(s_K, \phi) = \left(\frac{1}{1 + \left(\frac{1-\delta}{\delta}\right)^{\frac{1}{1-\alpha}} (s_K + (1 - s_K)\,\phi)^{\frac{\alpha}{(1-\sigma)(1-\alpha)}}} \right) \tag{19}$$

$$\mu^*(s_K, \phi) = \left(\frac{1}{1 + \left(\frac{1-\delta}{\delta}\right)^{\frac{1}{1-\alpha}} (\phi s_K + 1 - s_K)^{\frac{\alpha}{(1-\sigma)(1-\alpha)}}} \right). \tag{20}$$

As we can see the shares of expenditure in manufactures now depends on the localization of firms s_K and the freeness of trade ϕ.

We can make a number of important observations from analysing these two expressions.

[8] See Bellone and Maupertuis (2003) and Andrés (2007) for an analysis of the implications of removing this assumption.

First, when the elasticity of substitution between the two goods is different from 1, (i.e. $\alpha \neq 0$), north and south expenditure shares differ ($\mu(s_K, \phi) \neq \mu^*(s_K, \phi)$) in correspondence to any geographical allocation of the manufacturing industry except for $s_K = 1/2$ (symmetric equilibrium). In particular, we find that[9]

$$\alpha > (<)\, 0 \Leftrightarrow \frac{\partial \mu}{\partial s_K} = \frac{\alpha\,(1-\phi)\,\mu\,(1-\mu)}{(1-\alpha)\,(\sigma-1)\,((s_K + (1-s_K)\phi))} > (<)\, 0 \tag{21}$$

$$\alpha > (<)\, 0 \Leftrightarrow \frac{\partial \mu^*}{\partial s_K} = \frac{\alpha\,(\phi-1)\,\mu^*\,(1-\mu^*)}{(1-\alpha)\,(\sigma-1)\,((s_K + (1-s_K)\phi))} < (>)\, 0 \tag{22}$$

Hence, when $\alpha > 0$, production shifting to the north ($\partial s_K > 0$) leads to a relative increase in the southern price index for the M goods because southern consumers have to buy a larger fraction of M goods from the north, which are more expensive because of trade costs. Unlike the CD case, where this phenomenon had no consequences on the expenditure shares for manufactures which remained constant across time and space, in the CES case expenditure shares on M goods are influenced by the geographical allocation of industries because they depend on relative prices and relative prices change with s_K.

Secondly, the impact of trade costs is the following:

$$\alpha > (<)\, 0 \Rightarrow \frac{\partial \mu}{\partial \phi} = \frac{\alpha(1-s_K)\mu\,(1-\mu)}{(1-\alpha)\,(\sigma-1)\,((s_K + (1-s_K)\phi))} > (<)\, 0 \tag{23}$$

$$\alpha > (<)\, 0 \Rightarrow \frac{\partial \mu^*}{\partial \phi} = \frac{\alpha s_K \mu^*\,(1-\mu^*)}{(1-\alpha)\,(\sigma-1)\,((s_K + (1-s_K)\phi))} > (<)\, 0 \tag{24}$$

so that, when the two kinds of commodities are good substitutes ($\alpha > 0$) economic integration gives rise to an increase in the expenditure share for manufactured goods in both regions: manufactures are now cheaper in both regions and since they are good substitutes of the traditional goods, agents in both regions will not only increase their total consumption, but also their shares of expenditure. Obviously, the smaller the share of manufacturing firms already present in the north (south), the larger the increase in expenditure share for the M good in the north (south). The opposite happens when the two kinds of goods are poor substitutes: in this case, even if manufactures are cheaper, agents cannot easily shift consumption from the traditional to the differentiated good. In this case, even if total consumption on manufactures may increase, the share of expenditure will be reduced.

Third, since s_K is constant in steady-state by definition and ϕ is a parameter, expenditure shares on industrial goods are constant in steady state, allowing for the existence of a balance growth path and for the feasibility of the no-specialization condition. The latter, by using (15) and (16), can be written as follows:

$$L < \left([1-\mu(s_K,\phi)]\, s_E + [1-\mu^*(s_K,\phi)]\,(1-s_E)\right) E^w, \;\; \forall\, (s_K, \phi) \in (0,1) \subset \mathbb{R}^2. \tag{25}$$

Since s_E has to be constant by definition and even[10]:

$$E^w(s_E, s_K, \phi) = \frac{(2L - L_I - L_I^*)\,\sigma}{s_E\,(\sigma - \mu(s_K,\phi)) + (1-s_E)\,(\sigma - \mu^*(s_K,\phi))} \tag{26}$$

is constant in steady state, (**??**) can be accepted without any particular loss of generality. However, it is important to highlight that, in the line of Andrès (2007), our analysis can be developed even without the no-specialisation assumption.

3 Equilibrium and stability analysis

This section analyses the effects of our departures from the standard NEGG literature on the equilibrium dynamics of the allocation of northern and southern firms.

[9] For simplicity's sake we omit the arguments of the functions μ and μ^*.

[10] The expression for E^w can be found by using an appropriate labour market-clearing condition.

Following Baldwin, Martin and Ottaviano (2001), we assume that capital is immobile. Indeed, capital mobility can be seen as a special case of capital immobility (a case where profits are always equalized across regions and $\frac{\partial s_E}{\partial s_K} = 0$). Moreover, as we shall see, capital mobility does not provide any significant departure from the standard model from the point of view of the location equilibria: even when the intersectoral elasticity of substitution is allowed to be different from the unit value, still every initial allocation of firms is always stable. However, it should be clear that our analysis can be carried on even in the case of capital mobility. In particular, the results of the growth analysis developed in section 4 holds whatever the assumption on the mobility of capital.

In models with capital immobility the reward of the accumulable factor (in this case firms' profits) is spent locally. Thereby an increase in the share of firms (production shiftings) leads to expenditure shiftings through the permanent income hypothesis. Expenditure shiftings in turn foster further production shiftings because, due to increasing returns, the incentive to invest in new firms is higher in the region where expenditure is higher. This is the *demand-linked circular causality.*

This agglomeration force is counterbalanced by a dispersion force, the *market-crowding force*, according to which, thanks to the less than perfect substitutability between varieties, an increase in the number of firms located in one region will decrease firms' profits and then will give an incentive for firms to move to the other region. The interplay between these two opposite forces will shape the pattern of the equilibrium location of firms as a function of the trade costs. Such pattern is well established in NEGG models (Baldwin, Martin and Ottaviano 2001, Baldwin at al. 2004, Baldwin and Martin 2004): in the absence of localized spillovers, there is only one interior equilibrium, the symmetric allocation where the share of firms is evenly distributed among the two regions. Moreover, since the symmetric equilibrium is stable when trade costs are high and unstable when trade costs are low, catastrophic agglomeration always occurs when trade between the two countries is easy enough. That happens because, even though both forces decreases as trade costs become lower, the demand-linked force is lower than the market crowding force (in absolute value) when trade costs are low, while the opposite happens when trade costs are high.

By adopting the CES approach we are able to question the robustness of such conclusions. First of all, the symmetric equilibrium may not be the only interior equilibrium: while the latter is still a global equilibrium (i.e. for any value of the parameters), two other non-symmetric interior equilibria emerge for some intermediate value of trade costs. It is shown that these equilibria, when they exists, are stable when the intersectoral elasticity of substitution is either higher than 1 or sufficiently low. By contrast, the non-symmetric interior steady states are unstable when the elasticity of substitution is not-too smaller than 1. In particular, not-too-poor substitution between the two kind of goods gives rise to a multiple equilibria pattern with three different steady states (the symmetric and the two core-periphery allocations) stable at the same time. In other words, if the economy starts from a non-symmetric equilibrium and trade costs are neither too low or too high, a very small shock can give rise to a catastrophic agglomeration or to a catastrophic dispersion.

The reasons of these departures can be found in the non-linearity of the no-arbitrage condition and in the associated emergence of a new force, that we call **expenditure share effect**. This force fosters agglomeration or dispersion depending on whether the T and the $M-$commodities are respectively good or poor substitutes. By introducing this new force, which acts through the northern and southern M-goods expenditure shares, we also show that, depending on the different values of the intersectoral elasticity of substitution, the symmetric equilibrium might be unstable for *every* value of trade costs. We will now explore the mechanism in detail.

3.1 Tobin's q and Steady-state Allocations

Before analysing the equilibrium dynamics of firms' allocation, it is worth reviewing the analytical approach according to which such analysis will be carried on. As in standard NEGG models, we will make use of the Tobin's q approach (Baldwin and Forslid 1999 and 2000). We know that the equilibrium level of investment (production in the I sector) is characterized by the equality of the stock market value of a unit of capital (denoted with the symbol V) and the replacement cost of capital, F. With E and E^* constant in steady state, the Euler equation gives us $r = r^* = \rho$. Moreover, in steady state, the growth rate of the world capital stock K^w (or of the number of varieties) will be constant and will either be common ($g = g^*$ in the interior case) or north's g (in the core-periphery case)[11]. In either case, the steady-state values of investing in new units of K are:

$$V = \frac{\pi}{\rho + g}; V^* = \frac{\pi^*}{\rho + g}.$$

Firms' profit maximization and iceberg trade-costs lead to the following expression for northern and southern firms' profits:

$$\pi = B(s_E, s_K, \phi)\frac{E^w}{\sigma K^w} \tag{27}$$

$$\pi^* = B^*(s_E, s_K, \phi)\frac{E^w}{\sigma K^w} \tag{28}$$

where

$$B(s_E, s_K, \phi) = \left[\frac{s_E}{s_K + (1 - s_K)\phi}\mu(s_K, \phi) + \frac{\phi(1 - s_E)}{\phi s_K + (1 - s_K)}\mu^*(s_K, \phi)\right]$$

and

$$B^*(s_E, s_K, \phi) = \left[\frac{s_E \phi}{s_K + (1 - s_K)\phi}\mu(s_K, \phi) + \frac{1 - s_E}{\phi s_K + (1 - s_K)}\mu^*(s_K, \phi)\right]$$

Notice that this expression differs from the standard NEGG in only one respect: it relies on endogenous $M-$good expenditure shares which now depend on s_E, s_K and ϕ.

By using (2), the labour market condition and the expression for northern and southern profits, we obtain the following expression for the northern and southern Tobin's q:

$$q = \frac{V}{F} = B(s_E, s_K, \phi)\frac{E^w}{(\rho + g)\sigma} \tag{29}$$

$$q^* = \frac{V}{F} = B^*(s_E, s_K, \phi)\frac{E^w}{(\rho + g)\sigma} \tag{30}$$

Where will investment in K will take place? Firms will decide to invest in the most-profitable region, i.e., in the region where Tobin's q is higher. Since firms are free to move and to be created in the north or in the south (even though, with capital immobility, firm's owners are forced to spend their profits in the region where their firm is located), a first condition characterizing *any interior equilibria* ($g = g^*$) is the following:

$$q = q^* = 1 \tag{31}$$

The first equality (*no-arbitrage condition*) tells us that, in any interior equilibrium, there will be no incentive for any firm to move to another region. While the second (*optimal investment condition*) tells us that, in equilibrium, firms will decide to invest up to the level at which the expected discounted value of the firm itself is equal to the replacement cost of capital. The latter is crucial in order to find the expression for the rate of growth but it will not help us in finding the steady state level of s_K. Hence, we focus on the former. By using (**??**), (**??**), (**??**) and (**??**) in (**??**) we find the steady-state relation between the northern market size s_E and the northern share of firms s_K which can be written as:

[11]By time-differentiating $s_K = \frac{K}{K^w}$, we obtain that the dynamics of the share of manufacturing firms allocated in the north is

$$\dot{s}_K = s_K(1 - s_K)\left(\frac{\dot{K}}{K} - \frac{\dot{K}^*}{K^*}\right)$$

so that only two kinds of steady-state ($\dot{s}_K = 0$) are possible: 1) a steady-state in which the rate of growth of capital is equalized across countries ($g = g^*$); 2) a steady-state in which the manufacturing industries are allocated and grow in only one region ($s_K = 0$ or $s_K = 1$).

$$s_E^N(s_K,\phi)= \frac{\mu^*(s_K,\phi)\,(s_K+(1-s_K)\,\phi)}{\mu(s_K,\phi)\,(\phi s_K+(1-s_K))+\mu^*(s_K,\phi)\,(s_K+(1-s_K)\,\phi)} \tag{32}$$

The other relevant equilibrium condition is given by the definition of s_E when labour markets clear. This condition, also called *permanent income condition*, gives us a relation between northern market size s_E and the share of firms owned by northern entrepreneurs s_K:

$$s_E^P(s_K) = \frac{E}{E^w} = \frac{L+\rho s_K}{2L+\rho} \tag{33}$$

By subtracting the two functions, we define a new implicit function s_K whose zeros represent the interior steady state allocations of our economy:

$$f\left(s_K,\phi\right) = s_E^N\left(s_K,\phi\right) - s_E^P\left(s_K\right). \tag{34}$$

We define an *interior steady state allocation* as any value of $s_K^* \in (0,1)$ such that $f\left(s_K^*,\phi\right) = 0$. It is easy to see that the symmetric allocation $s_K = \frac{1}{2}$ is always an equilibrium, as in this case $f\left(\frac{1}{2},\phi\right) = \frac{1}{2} - \frac{1}{2} = 0$. In order to fully capture the role of expenditure shares, it is worth concentrating on the properties of $f\left(s_K,\phi\right)$ as it governs all the results related on the number of equilibria and their stability. While the permanent income relation is not affected by endogenous expenditure shares - $s_E^P\left(s_K\right)$ is a straight line increasing in s_K both in the with unitary or non-unitary intersectoral elasticity of substitution - the main source of all the deviations from the standard case can be traced back to the non-linearity of $s_E^N\left(s_K,\phi\right)$ - and then of f - induced by endogenous expenditure shares. In the standard case, where $\mu(s_K,\phi) = \mu^*(s_K,\phi) = \mu$ for any $(s_K,\phi) \in [0,1]^2$, $s_E^N(s_K,\phi)$ reduces to

$$s_E^N(s_K,\phi)= \frac{s_K+(1-s_K)\,\phi}{1+\phi}$$

It is then linear in both s_K and ϕ, being increasing in the former and decreasing in the latter. Such double linearity has two main consequences: first the symmetric steady state is also the *unique* interior steady state; second, trade costs only affect the *slope* of $s_E^N(s_K,\phi)$ with respect to s_K - and accordingly the stability of the symmetric equilibrium - but not its second derivative which is always nil for any values of ϕ. Things are much more complicated - yet readable and insightful, when expenditure shares are endogenous. In this case $s_E^N(s_K,\phi)$ is still increasing in s_K, but it loses its linearity becoming S-shaped or inverted S-shaped according to different values of α and all other relevant parameters. This non-linearity gives rise, for some intermediate values of trade costs, to two new intersections with $s_E^P(s_K)$ on the bi-dimensional plane(s_E,s_K), thereby opening the door to a multiple equilibria pattern. Moreover, as $\frac{\partial s_E^N(s_K,\phi)}{\partial s_K}$ - which has a crucial role on equilibria stability - is now affected by changes in ϕ, trade costs now affect both the slope *and* the curvature of$s_E^N(s_K,\phi)$ so that both uniqueness/multiplicity and stability patterns can be analysed in their changing behaviour as trade costs gets freer. In what follows, we perform such formal analysis in detail. Although closed form solutions for break and sustain points of ϕ are not possible, a rich qualitative analysis of uniqueness/multiplicity and stability patterns - and the linkage between the two - can nevertheless be obtained.

3.2 Interior steady states

In the proposition contained in this section, whose long proof is confined in the appendix, we provide the necessary and sufficient condition for the interior steady state to be unique or threefold. However, before stating it, it will be useful to define a new function. Consider $f\left(s_K,\phi\right)$ as defined in (**??**). By using (**??**) and (**??**), it can also be written as

$$f\left(s_K,\phi\right) = \frac{\left(s_K+(1-s_K)\,\phi\right)+Z\left(s_K+(1-s_K)\,\phi\right)^x}{1+\phi+Z\left(s_K+(1-s_K)\,\phi\right)^x+Z\left(\phi s_K+1-s_K\right)^x} - \frac{L+\rho s_K}{2L+\rho}$$

where $Z = \left(\frac{1-\delta}{\delta}\right)^{\frac{1}{1-\alpha}} \in (0, \infty)$ and $x = \frac{\sigma(1-\alpha)-1}{(\sigma-1)(1-\alpha)} \in (0, \infty)$, so that $\alpha > (<)0 \Leftrightarrow x < (>)1$ (in the standard case, $x = 1$, i.e. $\alpha = 0$).

Also notice that $f(\cdot)$ is symmetric with respect to the point $\left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right)$, meaning that $f(s_K) = -f(1 - s_K)$. This symmetry is very important as it allows us to limit the analysis to the interval $s \in \left[0, \frac{1}{2}\right)$ and then extend it to the rest of the interval $s_K \in \left[\frac{1}{2}, 1\right]$ by simply respecting the symmetry rule.

Now, define the function

$$
\begin{aligned}
h(s_K, \phi) &= f(s_K, \phi)\, k(s_K, \phi) && (35)\\
&= (2s_K - 1)(L - \phi(L + \rho)) + \frac{(L + \rho(1 - s_K))\, Z}{(s_K + (1 - s_K)\phi)^{-x}} - \frac{(L + \rho s_K)\, Z}{(\phi s_K + 1 - s_K)^{-x}} && (36)
\end{aligned}
$$

Where $k(s_K, \phi) = [1 + \phi + Z(s_K + (1 - s_K)\phi)^x + Z(\phi s_K + 1 - s_K)^x](2L + \rho)$ is simply the product of the two denominators in f.

Since $k(s_K, \phi) > 0$ for every $s_K \in [0, 1]$, we have that $f(s_K, \phi) = 0 \Leftrightarrow h(s_K.\phi) = 0$: every zero of $h(\cdot)$ is also an interior steady state and vice-versa. In particular, it is easy to see that $h\left(\frac{1}{2}, \phi\right) = 0$. Also notice that

$$\frac{\partial h(s_K, \phi)}{\partial s_K} = \frac{\partial f(s_K, \phi)}{\partial s_K} k(s_K, \phi) + \frac{\partial k(s_K, \phi)}{\partial s_K} f(s_K, \phi)$$

but since $f\left(\frac{1}{2}, \phi\right) = 0$ we also have

$$\frac{\partial h(\frac{1}{2}, \phi)}{\partial s_K} = \frac{\partial f(\frac{1}{2}, \phi)}{\partial s_K} k\left(\frac{1}{2}, \phi\right) \tag{37}$$

so that $sign\left[\frac{\partial h(\frac{1}{2},\phi)}{\partial s_K}\right] = sign\left[\frac{\partial f(\frac{1}{2},\phi)}{\partial s_K}\right]$. Given, these properties we prefer to concentrate on $h(s_K.\phi)$ as it is much easier to deal with from the mathematical point of view. We are now ready to state our first proposition.

Proposition 1 *(Number of interior steady states) The system displays one or three interior steady state allocations: the symmetric allocation $s_K = \frac{1}{2}$ (which is a "global" interior steady state) and two non-symmetric allocations: $s_K^*(L, x, \rho, \phi, Z)$ and $s_K^{**}(L, x, \rho, \phi, Z) = 1 - s_K^*(L, x, \rho, \phi, Z)$ which may emerge only for some values of the parameters. The interior steady state is unique and equal to $\frac{1}{2}$ when $f(0, \phi)\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K} \leq 0$, while there are 3 interior steady states when $f(0, \phi)\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K} > 0$.*

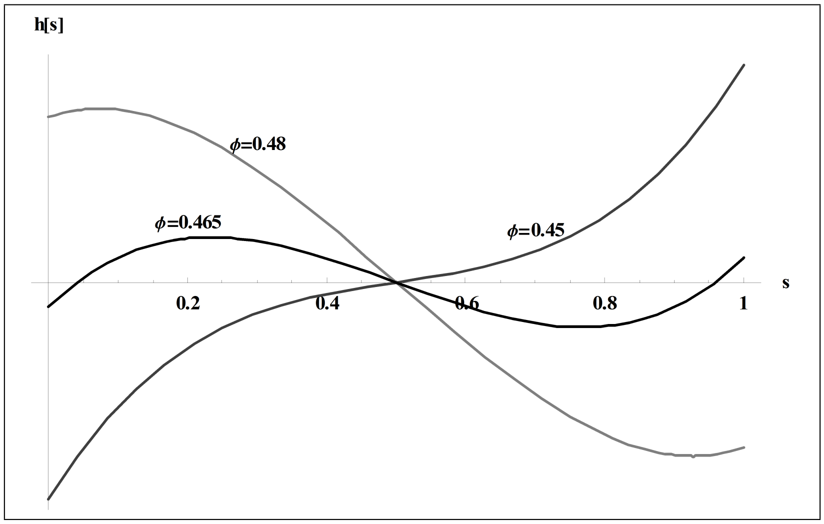
Proof. See the appendix. ■

This proposition provides a necessary and sufficient condition for the uniqueness/multiplicity of steady states. It states that, given the monotonicity of $\frac{\partial f(s_K,\phi)}{\partial s_K}$ in the interval $\left[0, \frac{1}{2}\right)$ and the symmetry of f, uniqueness is guaranteed when $f(0, \phi)$, (i.e. the intercept of f in $s_K = 0$) and $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ (i.e. the slope of f in the symmetric equilibrium) have opposite sign.

Despite its importance, proposition 1 is not particularly informative as long as we don't provide an analysis concerning the way $f(0, \phi)\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ changes sign as trade costs decline. As figure **??** suggests, as trade costs affect both the intercept and the slope of h in terms of s_K, uniqueness/multiplicity patterns are highly sensitive to market integration. Moreover, as we will see, there is always a feasible value of ϕ such that the economy switches from a regime of unique steady state to a regime of multiple steady state, whatever the degree of substitutability between goods.

Because of the crucial linkages with the stability issues, such analysis will be performed in the section 3.4 together with the stability map.

Figure 1: How trade costs affect the number of interior steady states ($x = 0.5$)



3.3 Core-periphery steady states

As for core-periphery equilibria, things are much simpler. As already anticipated, interior steady states are not the only allocations where the regional share of industrial firm is constant: s_K is constant ($\dot{s}_K = 0$) even when it is equal to either 1 or 0, i.e., when the whole industrial sector is located in only one region. Since the two core-periphery allocations are perfectly symmetric, we just focus on the first where the North gets the core. By following Baldwin and Martin (2004), we consider for $s_K = 1$ to be an equilibrium, it must be that $q = V/F = 1$ and $q^* = V^*/F^* < 1$ for this distribution of capital ownership: continuous accumulation is profitable in the north since $V = F$, but $V^* < F$ so no southern agent would choose to setup a new firm. Defining the core-periphery equilibrium this way, it implies that it is stable whenever it exists.

3.4 Stability map of equilibria

In this section we provide a complete stability map for the equilibria of our economy. As we will see, this analysis is intimately linked to the issue of the number of interior steady states. At the end of this section we will be able to state, for any value of the trade costs, the existence and stability of any kind of steady state (symmetric, non-symmetric or core-periphery) Following Baldwin and Martin (2004)[12] we consider the ratio of northern and southern Tobin's q:

$$\frac{q}{q^*} = \frac{B(s_E, s_K, \phi)}{B^*(s_E, s_K, \phi)} = \frac{\left[\frac{s_E}{s_K+(1-s_K)\phi}\mu(s_K,\phi) + \frac{\phi(1-s_E)}{\phi s_K+(1-s_K)}\mu^*(s_K,\phi)\right]}{\left[\frac{s_E\phi}{s_K+(1-s_K)\phi}\mu(s_K,\phi) + \frac{1-s_E}{\phi s_K+(1-s_K)}\mu^*(s_K,\phi)\right]} = \gamma\left(s_E, s_K, \phi\right) \tag{38}$$

Starting from any interior steady-state allocation where $\gamma(s_E, s_K, \phi) = 1$, any increase (decrease) in $\gamma(s_E, s_K, \phi)$ will make investments in the North (South) more profitable and thus will lead to a production shifting to the North (South). Hence any allocation will be stable if a production shifting, say, to the north ($\partial s_K > 0$) will reduce $\gamma(s_E, s_K, \phi)$. By contrast, if $\gamma(s_E, s_K, \phi)$ will increase following an increase in s_K, then the equilibrium is unstable and agglomeration or dispersion processes might be activated.

We remind that this method is the same employed by standard NEGG models. The only and crucial difference is that, in our framework, the northern and southern expenditure shares $\mu(s_K, \phi)$ and $\mu^*(s_K, \phi)$

[12] A more formal stability analysis, involving the study of the sign of the Jacobian associated to dynamic system in E, E^* and s_K, has been carried out and its results are identical to those reported in this section. Such calculations are available at request.

play a crucial role because their value is not fixed but depends on geography and trade costs.

Taking the derivative of $\gamma\left(s_E, s_K, \phi\right)$ with respect s_K and then using the no-arbitrage condition (which must be true in every interior steady state) we find

$$\frac{\partial \gamma\left(s_E\left(s_K\right), s_K, \phi\right)}{\partial s_K} = A\left(s_K, \phi\right) - B\left(s_K, \phi\right) + C\left(s_K, \phi\right) \tag{39}$$

where

$$A\left(s_K, \phi\right) = \left(\frac{d\mu}{ds_K}/\mu - \frac{d\mu^*}{ds_K}/\mu^*\right)\frac{(1-\phi)}{(1+\phi)} \quad : \quad \text{expenditure share effect}$$

$$B\left(s_K, \phi\right) = -\frac{(1-\phi)^2}{\left(s_K + \left(1-s_K\right)\phi\right)\left(\phi s_K + \left(1-s_K\right)\right)} \quad : \quad \text{market crowding effect}$$

$$C\left(s_K, \phi\right) = \frac{(1-\phi)}{(1+\phi)}\frac{ds_E\left(s_K\right)}{ds_K}\frac{\left(\mu\left(\phi s_K + \left(1-s_K\right)\right) + \mu^*\left(s_K + \left(1-s_K\right)\phi\right)\right)^2}{\mu\mu^*\left(s_K + \left(1-s_K\right)\phi\right)\left(\phi s_K + \left(1-s_K\right)\right)} \quad : \quad \text{demand effect}$$

The last two forces are the same we encounter in the standard NEGG model and they are the formal representation of, respectively, the *market-crowding effect* and the *demand-linked effect*. In the standard model, the stability of the equilibrium is the result of the relative strength of just these two forces. The first force represents the novelty of our model. In the standard case, where $\mu^*\left(s_K, \phi\right) = \mu\left(s_K, \phi\right) = \mu$ and then $\frac{\partial \mu}{\partial s_K} = \frac{\partial \mu^*}{\partial s_K} = 0$, this force simply does not exist. We dub this force as the **expenditure share effect** in order to highlight the link between the existence of this force and the fact that the expenditure shares are endogenous (thanks to a non-unitary value of the intersectoral elasticity of substitution). As we will see in detail below, the expenditure share effect might be a stabilizing (when negative) or destabilizing one (when positive) depending on whether the manufactured and the traditional good are respectively poor ($\alpha < 0$) or good ($\alpha > 0$) substitutes.

But what is the economic intuition behind this force? Imagine a firm moving from south to north ($\partial s_K \geq 0$). For a given value of ϕ, this production shifting reduces the manufactured good price index in the North and increases the one in the South. In the standard case, where the manufactured and the traditional goods are neither good nor poor substitutes, this relative change in the price levels has no effect on the respective expenditure shares. By contrast when the intersectoral elasticity of substitution is allowed to vary from the unitary value, the shares of expenditure change with the $M-$price index and hence with s_K. In particular, when the manufactured and the traditional goods are good substitutes ($\alpha > 0$), a reduction in the relative price level in the North leads to an increase $\left(\frac{\partial \mu}{\partial s_K} \geq 0\right)$ in the northern expenditure shares and a decrease $\left(\frac{\partial \mu^*}{\partial s_K} \leq 0\right)$ in the southern expenditure shares, then increasing the relative market size in the north and providing an (additional) incentive to the southern firms to relocate in the north. The opposite ($\frac{\partial \mu}{\partial s_K} \leq 0$ and $\frac{\partial \mu^*}{\partial s_K} \geq 0$) happens when the manufactured and the traditional goods are poor substitutes ($\alpha < 0$): in this case, southern relative market size increases and this gives an incentive for the moving firm to come back home. This is why, when the M and the T goods are good substitutes the expenditure share effect acts as an destabilizing force, while the opposite happens when the M and the T goods are poor substitutes. In the existing literature the expenditure share effect is not activated since $\frac{d\mu}{ds_K}/\mu = \frac{d\mu^*}{ds_K}/\mu^* = 0$.

More formally, **any interior equilibria** is stable (unstable) when

$$\frac{\partial \gamma\left(s_E, s_K, \phi\right)}{\partial s_K} \quad \leq \quad (>)\, 0.$$

By (**??**) and (**??**) that happens when

$$\frac{ds_E^P\left(s_K\right)}{ds_K} = \frac{\rho}{2L+\rho} \leq (>)\, \frac{\mu\mu^*\left(1-\phi^2\right) - \left(\frac{d\mu}{ds_K}\mu^* - \frac{d\mu^*}{ds_K}\mu\right)\left(s_K + \left(1-s_K\right)\phi\right)\left(\phi s_K + \left(1-s_K\right)\right)}{\left(\mu\left(\phi s_K + \left(1-s_K\right)\right) + \mu^*\left(s_K + \left(1-s_K\right)\phi\right)\right)^2}$$

By computation we find that

$$\frac{\partial s_E^N\left(s_K,\phi\right)}{\partial s_K}=\frac{\mu\mu^*\left(1-\phi^2\right)-\left(\frac{d\mu}{ds_K}\mu^*-\frac{d\mu^*}{ds_K}\mu\right)\left(s_K+\left(1-s_K\right)\phi\right)\left(\phi s_K+\left(1-s_K\right)\right)}{\left(\mu\left(\phi s_K+\left(1-s_K\right)\right)+\mu^*\left(s_K+\left(1-s_K\right)\phi\right)\right)^2}$$

This proves the following proposition

Proposition 2 *In any interior equilibrium we have sign*$\frac{\partial\gamma(s_E,s_K^*,\phi)}{\partial s_K}=-$*sign*$\frac{\partial f(s_K^*,\phi)}{\partial s_K}$. *Therefore each interior steady state* s_K^* *allocation is stable (unstable) whenever*

$$\frac{\partial f(s_K^*,\phi)}{\partial s_K}=\frac{\mu\mu^*\left(1-\phi\right)\left[\left(1+\phi\right)-\left(1-x\right)\left(2-\mu-\mu^*\right)\left(\phi s_K+\left(1-s_K\right)\right)\right]}{\left(\mu\left(\phi s_K+\left(1-s_K\right)\right)+\mu^*\left(s_K+\left(1-s_K\right)\phi\right)\right)^2}-\frac{\rho}{2L+\rho}\geq(<)\,0$$

.

In other words, any interior equilibria is stable (unstable) if the graph of f in the plane $(s_K,f(s_K,\phi))$ crosses the horizontal axis with positive (negative) inclination.

Proposition 2 has several very important implications.

The first implication concerns the fact that the particular shape of the function f (and then h) allows us to focus only on the value of this derivative in $s_K=\frac{1}{2}$ in order to deduce the stability properties of each (interior or core-periphery) steady state. It is in fact straightforward, by proposition 1 and by continuity and symmetry of f (and then h) that the sign of $\frac{\partial f(s_K^*,\phi)}{\partial s_K}$ in the symmetric equilibrium must be opposite to the sign of the same derivative in the two interior non-symmetric equilibria

More formally, if $s_K^*\in\left(0,\frac{1}{2}\right)$ is a non-symmetric steady state for some ϕ, then we have

$$\left(\frac{\partial f(s_K^*,\phi)}{\partial s_K}\right)\left(\frac{\partial f(\frac{1}{2},\phi)}{\partial s_K}\right)=\left(\frac{\partial f(1-s_K^*,\phi)}{\partial s_K}\right)\left(\frac{\partial f(\frac{1}{2},\phi)}{\partial s_K}\right)<0. \tag{40}$$

As a consequence, by proposition 2, the non-symmetric equilibria (when they exists) are unstable when the symmetric equilibrium is stable and vice versa. By applying a similar reasoning we can conclude that $s_K=0$ and $s_K=1$ are (local) attractors, and therefore the two core-periphery equilibria exists, only when the non-symmetric interior steady states exist *and* are unstable *or* when the symmetric steady state is unique and unstable.

The second implication is that the sign of $\frac{\partial f(\frac{1}{2},\phi)}{\partial s_K}$ is not only informative on the stability of any kind of equilibria, but it is also a determinant of the uniqueness or multiplicity regime. It is therefore necessary to study how the sign of this derivative changes with the trade costs in order to gain simultaneous informations on the number of equilibria and on their stability as trade costs decline.

3.4.1 The sign of $\frac{\partial f(\frac{1}{2},\phi)}{\partial s_K}$

Such derivative can be written as

$$\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}=\underbrace{(x-1)\left(1-\mu\left(\frac{1}{2},\phi\right)\right)\frac{(1-\phi)}{(1+\phi)}}_{\text{Expenditure share}}+\underbrace{\frac{(1-\phi)}{(1+\phi)}}_{\text{Market-crowding}}-\underbrace{\frac{\rho}{2L+\rho}}_{\text{Demand-linked}}$$

where, we remind, $x=\frac{\sigma(1-\alpha)-1}{(\sigma-1)(1-\alpha)}\in(0,\infty)$, so that $\alpha>(<)0\Leftrightarrow x<(>)1$. Again, it is easy to see that when $x<1$ the expenditure share effect is an agglomeration force. In this case, in fact, it will contribute to reduce the value $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ being negative as the demand-linked force. The opposite happens when $x>1$: in this case the expenditure share effect has the same sign of the market-crowding effect and it acts as a dispersion force. Needless to say, when $x=1$, the expenditure share effect just vanishes and the model collapses to the standard one.

A second implication that can be drawn from this expression is that, as long as $\frac{1}{1-\alpha} < \sigma$, and therefore $x > 0$, we always have

$$\underbrace{(x-1)\left(1-\mu\left(\frac{1}{2},\phi\right)\right)\frac{(1-\phi)}{(1+\phi)}}_{\text{Expenditure share effect}} + \underbrace{\frac{(1-\phi)}{(1+\phi)}}_{\text{Market-crowding effect}} \geq 0 \text{ for any } \phi \in [0,1] \tag{41}$$

so that the expenditure share effect will never offset the market-crowding effect. From this result, we can derive a corollary for the capital mobility case. In this case, s_n should not equal s_K, profits are equalized among regions (so that f is always zero) and, above all, there is no permanent income condition so that $\frac{\partial s_E}{\partial s_K} = 0$. Hence the stability condition reduces to (**??**) and, just as in the standard case, the symmetric steady-state is always stable when capital is mobile.

But our main interest is to find the set of values of the freeness of trade such that $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ is positive (negative) and then the symmetric steady state is stable (unstable). In other words, we aim to investigate the existence of a *break-point*, that is the value of ϕ above which the stability of the interior equilibria is broken, and then an infinitesimal production shifting in the North (South) will trigger a self-reinforcing mechanism which will lead to a non-symmetric outcome. In the standard CD case, since $\alpha = 0$, we have that:

$$\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K} < 0 \Leftrightarrow \phi > \phi_B^{CD}$$

where $\phi_B^{CD} = \frac{L}{L+\rho}$ is the break-point level of the trade costs. Since $\phi_B^{CD} \in (0,1)$, there is always a feasible value of the trade costs above which the interior equilibrium turns from stable to unstable and then agglomeration will take place. Moreover, such value is always unique as both forces (market crowding and demand-linked) are decreasing in ϕ in absolute value. In our model, it is not possible to calculate an explicit value for the break-point. That's because ϕ enters the expression for $\mu(1/2,\phi)$ as a non-integer power. Nonetheless, we can perform a qualitative analysis and draw several implications from the existence of the expenditure share effect. Actually, the presence of our additional force will introduce the possibility of some additional outcomes which was excluded from the standard CD case.

First notice that, when $\phi = 1$ we surely have

$$\frac{\partial f\left(\frac{1}{2},1\right)}{\partial s_K} < 0$$

so that, by continuity of $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ with respect to ϕ, there is always an interval of the kind $\left(\phi_B^{'},1\right]$ such that the symmetric equilibrium is always unstable for any $\phi \in \left(\phi_B^{'},1\right]$. From this perspective, the prediction of the standard model are robust: when trade costs are low enough the symmetric equilibrium is always unstable and (as we will see) the core-periphery equilibrium is stable as no other interior equilibria exist.

What happens when trade costs are very high? In this case, for $\phi = 0$, we have

$$\frac{\partial f\left(\frac{1}{2},0\right)}{\partial s_K} \geq (<)\, 0 \Longleftrightarrow x-1 \geq (<) -\frac{2L}{2L+\rho}\frac{1}{1-\mu\left(\frac{1}{2},0\right)}$$

.

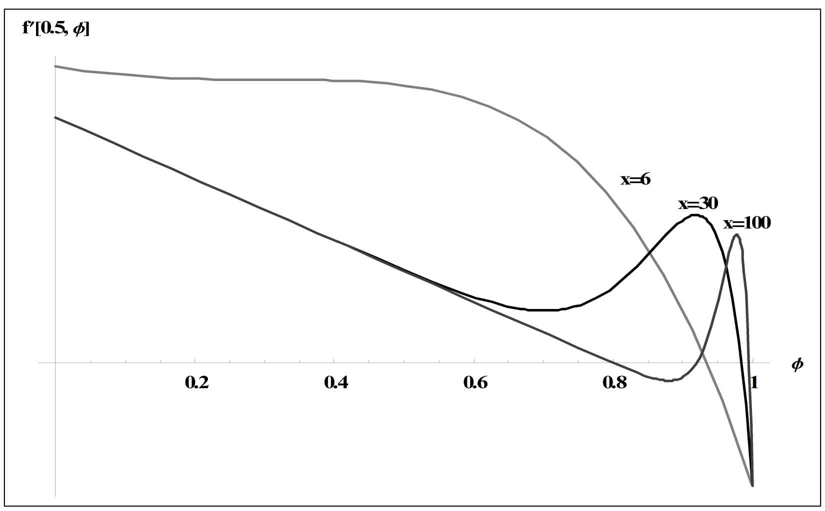
While this inequality always holds in the CD case (since $x = 1$ and the RHS is negative) - meaning that the symmetric equilibrium is always stable when trade costs are high enough, it might not hold in our general approach when x is sufficiently lower than 1 and the RHS is sufficiently small in absolute value. As a consequence, when $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ is always decreasing in ϕ, and we'll see this is always the case when $x < 1$, we have that the break point ϕ_B is unique and negative, meaning that agglomeration occur for any level of trade costs and the symmetric equilibrium is never stable. By contrast, when x is not too low, and then the agglomeration force induced by the expenditure share effect is not too strong or it is actually a dispersion force, then $\frac{\partial f\left(\frac{1}{2},0\right)}{\partial s_K} > 0$ and, by continuity arguments, there is always an interval of the kind $\left[0, \phi_B^{''}\right]$ such that the symmetric steady state is always stable for any $\phi \in \left[0, \phi_B^{''}\right]$.

Is it always the case that $\phi_B^{'} = \phi_B^{''}$? Or, in words, is there always a unique value of ϕ above which the symmetric equilibrium switch from stable to unstable and then the break point is unique with non-unitary elasticity of substitution as well? Unfortunately we cannot give a positive answer to this question. Indeed the answer would have been positive if we could guarantee that $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ is always decreasing in ϕ The latter is in fact a sufficient (but not necessary) condition for a single break-point to exist. However, when x is very high, the expenditure share effect may not be monotonically decreasing in ϕ and, in some cases, this non-linearity in ϕ might give rise to a double break-point! A careful look at the partial derivative of $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ with respect to ϕ will convince us

$$\frac{\partial^2 f\left(\frac{1}{2},\phi\right)}{\partial s_K \partial \phi} < 0 \Leftrightarrow (1-x)^2 (1-\phi)^2 \mu (1-\mu) < x(1-\mu) + \mu \tag{42}$$

Even though $\frac{\partial^2 f\left(\frac{1}{2},\phi\right)}{\partial s_K \partial \phi}$ is always negative when ϕ is sufficiently close to 1, the term $(1-x)^2$ on the LHS can be very large when x is big and it can prevent condition (**??**) to be satisfied. Hence, as figure **??** illustrates

Figure 2: The possibility of double break point (L=2, $\rho = 0.5$, Z=1)



there can be an interval of ϕ, call it $\left(\phi_S, \phi_B^{'}\right)$, where the symmetric equilibrium gains stability back before losing it once again when ϕ reaches $\left(\phi_B^{'}, 1\right]$. Such highly complex behaviour, which is nevertheless a feasible outcome of the model, is ruled out when x is low enough. Unfortunately it is not possible to express the maximum value of x, call it $\widehat{x}$, as a function of the remaining parameters. In order to avoid any additional complexity, the equilibrium analysis is intended to be limited to a range of x belonging to the interval $(0, \widehat{x})$. We believe this is not a significantly loss of generality as $\widehat{x}$ is surely larger than 2 and tends to infinity as ϕ tends to 1.[13] In any case, the appendix will provide a sketch of the highly complex behaviour of the model in case $x > \widehat{x}$.

[13] By exploting the relationship between f and h and the fact that their partial derivative with respect to s_K evaluated in $s_K = \frac{1}{2}$ must have the same sign, it is possible to show that

$$\widehat{x} > 1 + \frac{2(L+\rho)(1+\phi)}{(2L+\rho)(1-\phi)} > 2$$

When $x \in (0, \widehat{x})$ then it is guaranteed that $\phi_B^{'} = \phi_B^{''} = \phi_B$ and ϕ_B is the unique break-point of our model

$$\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K} \leq (>) 0 \Longleftrightarrow \phi \leq (>) \phi^B. \tag{43}$$

Once we have ruled out the possibility of double break-points, we can compare ϕ^B with the break-point of the CD case. By straightforward computation, we find that $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K} = 0$ implies

$$(1-x) = \frac{2\,(L+\rho)}{(1-\mu\,(1/2,\phi))\,(1-\phi)\,(2L+\rho)}\left(\phi_B^{CD} - \phi\right) \tag{44}$$

Since $\frac{2(L+\rho)}{(1-\mu(1/2,\phi))(1-\phi)(2L+\rho)}$ is always positive, $(1-x)$ and $\left(\phi_B^{CD} - \phi\right)$ must have the same sign, meaning that the break-point in our model may be higher or lower than ϕ_B^{CD} depending on whether the intersectoral elasticity of substitution is larger or smaller than 1. Formally:

$$\begin{aligned} \phi_B &< \phi_B^{CD} \Leftrightarrow \alpha > 0 \\ \phi_B &> \phi_B^{CD} \Leftrightarrow \alpha < 0 \end{aligned}$$

In other words, and quite intuitively, the presence of an additional agglomeration force (the expenditure share effect when $\alpha > 0$), shifts the break-point to a lower level so that catastrophic agglomeration is more likely and it occurs for a larger set of values of ϕ. By contrast, when the expenditure share effect acts as a dispersion force ($\alpha < 0$), the break-point shifts to an upper level so that catastrophic agglomeration is less likely as it occurs for a smaller set of values of ϕ.

Summing up, we have shown that, when $x < \widehat{x}$, there is always a value of freeness of trade, $\phi_B < 1$, above which the symmetric steady state looses stability. This is the break-point of our economy. ϕ_B can be larger or smaller than ϕ_B^{CD} according to whether the traditional and the industrial goods are respectively good or poor substitutes. Finally, when the two commodities are very good substitutes, it might be that $\phi_B < 0$: if this is the case the symmetric equilibrium is always unstable.

As already anticipated, the way $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ changes sign with ϕ does not only matter for stability but, as proposition 1 shows, it is also a determinant for the number of interior steady state of the model. In order to see how the number of equilibria changes with ϕ we also need to know the behaviour of $h\,(0,\phi)$. This will be the topic of the next section.

3.4.2 The sign of $f\,(0,\phi)$ and the way trade costs affect the number of equilibria

As for $f\,(0,\phi)$, things are much much easier:

$$f\,(0,\phi) \leq (>) 0 \Longleftrightarrow \frac{\phi + Z\phi^x}{1+Z} \leq (>) \frac{L}{L+\rho}$$

as $\phi + Z\phi^x$ is always decreasing in ϕ, and $f\,(0,0)\,f\,(0,1) < 0$, there is always a unique and positive value of ϕ, call it $\hat{\phi}$, such that $f\,(0,\phi) = 0$:

$$f\,(0,\phi) \leq (>) 0 \Longleftrightarrow \phi \leq (>) \hat{\phi}. \tag{45}$$

Once we are sure that there $f\,(0,\phi)$ and $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ change sign for only one value of ϕ, respectively $\hat{\phi}$ and ϕ^B, we are ready to state the following proposition, which provide the necessary and sufficient conditions for the interior steady state to be unique or threefold in terms of ϕ.

Proposition 3 *The interior steady state is unique when* $\phi \in \left[0, min\left(\hat{\phi}, \phi^B\right)\right] \cap \left[max\left(\hat{\phi}, \phi^B\right), 1\right]$ *while there are three interior steady states when* $\phi \in \left(min\left(\hat{\phi}, \phi^B\right), max\left(\hat{\phi}, \phi^B\right)\right)$

Proof. Using (**??**) and (**??**) and recalling that $sign\,[h\,(0,\phi)] = sign\,[f\,(0,\phi)]$ and $sign\left[\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K}\right] = sign\left[\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}\right]$ we conclude that

$$h\,(0,\phi)\,\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K} \leq 0 \Longleftrightarrow \phi \in \left[0, min\left(\hat{\phi},\phi^B\right)\right] \cap \left[max\left(\hat{\phi},\phi^B\right),1\right] \tag{46}$$

while

$$h\,(0,\phi)\,\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K} > 0 \Longleftrightarrow \phi \in \left(min\left(\hat{\phi},\phi^B\right), max\left(\hat{\phi},\phi^B\right)\right) \tag{47}$$

and the proposition is proven ■

This proposition basically states that multiple interior steady states *always* appear for some (intermediate) values of trade costs. When ϕ is lower than the minimum between $\hat{\phi}$ and ϕ^B, $f\,(0,\phi)\,\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ is not positive and the same happens when ϕ is larger than the maximum between $\hat{\phi}$ and ϕ^B. As a consequence of the previous analysis on the sign of both $f\,(0,\phi)$ and $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ the symmetric equilibrium is the unique interior equilibrium. By contrast, when ϕ is between $\hat{\phi}$ and ϕ^B, being the former larger than the latter or vice versa, then $f\,(0,\phi)\,\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ is positive and two additional non-symmetric interior equilibria appear. It is worth noting that the condition $\phi \in \left(min\left(\hat{\phi},\phi^B\right), max\left(\hat{\phi},\phi^B\right)\right)$ also encompasses the case when $\phi_B < 0$.

We are finally ready to join the uniqueness and the stability analysis and to see how the stability and the number of equilibria are simultaneously affected by trade costs.

3.4.3 Trade costs, the number of equilibria and their stability

Since the number of interior equilibria is decided by the sign of $f\,(0,\phi)\,\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$, when ϕ becomes larger than ϕ^B we have simultaneous consequences on both the stability pattern and on the number of interior equilibria. What is really crucial in this respect is the comparison between ϕ_B and $\widehat{\phi}$. We can then distinguish among three different cases:

- $\phi_B < 0 < \widehat{\phi}$: in this case we distinguish between two regions within the set of feasible values of ϕ: $\left[0,\widehat{\phi}\right)$and $\left[\widehat{\phi},1\right)$. In both regions the symmetric equilibrium is always unstable (we are in the case where x is very low and the two commodities are very close substitutes). Hence, we always have $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K} < 0$. When ϕ belongs to the region $\left[0,\widehat{\phi}\right)$, $f\,(0,\phi)$ is negative so that $f\,(0,\phi)\,\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ is positive. As a consequence, when this is the case, *for high trade costs there are two stable non-symmetric interior equilibria.* As ϕ increases and reaches the region $\left[\widehat{\phi},1\right)$, $f\,(0,\phi)$ becomes positive so that $f\,(0,\phi)\,\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ switches to negative and the two non-symmetric interior equilibria collapse to the core-periphery equilibria while the symmetric equilibrium remains unstable (see fig. **??**).

- $0 < \phi_B < \widehat{\phi}$: In this case we distinguish the following regions within the set of feasible values of ϕ: $[0,\phi_B)$, $\left[\phi_B,\widehat{\phi}\right)$ and $\left[\widehat{\phi},1\right)$. In the first region, when $\phi \in [0,\phi_B)$, $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ is positive and $f\,(0,\phi)$ is negative so that $f\,(0,\phi)\,\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ is negative. As a consequence, *the symmetric equilibrium is stable and unique.* As ϕ increases and reaches the second region $\left[\phi_B,\widehat{\phi}\right)$, $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ becomes negative when $f\,(0,\phi)$ is still negative. Hence $f\,(0,\phi)\,\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ turns from negative to positive. Therefore in this region of intermediate trade costs the symmetric equilibrium looses its stability and two new stable non-symmetric interior equilibria emerge. When ϕ reaches the third region and becomes larger than $\widehat{\phi}$, then $f\,(0,\phi)$ becomes positive and, while $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ remains negative, $f\,(0,\phi)\,\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$ turns negative again. As a consequence, in this third region of trade costs the symmetric equilibrium turns to be unique still being unstable and two core-periphery equilibria emerge (see fig. **??**). [14]

Figure 3: Stability map when $\phi_B < 0 < \widehat{\phi}$

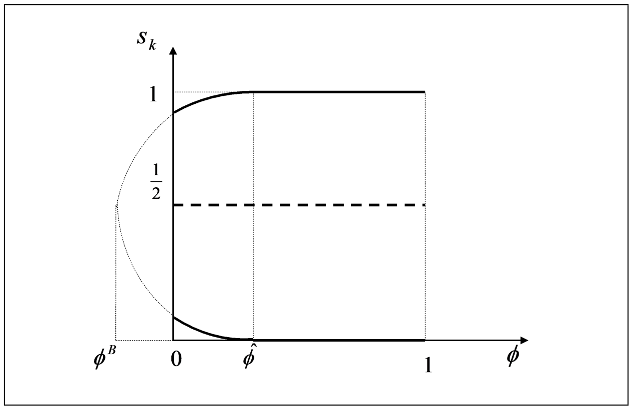
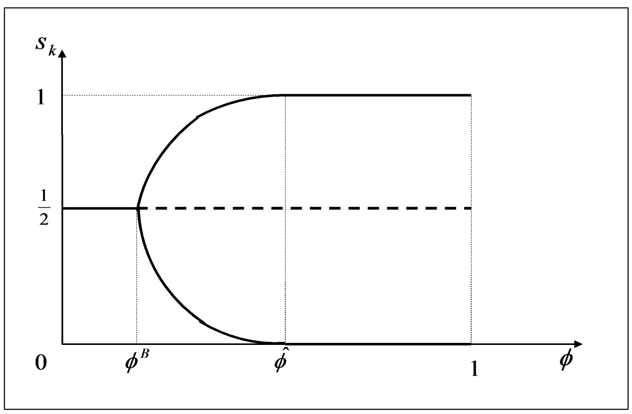


Figure 4: Stability map when $0 < \phi_B < \widehat{\phi}$

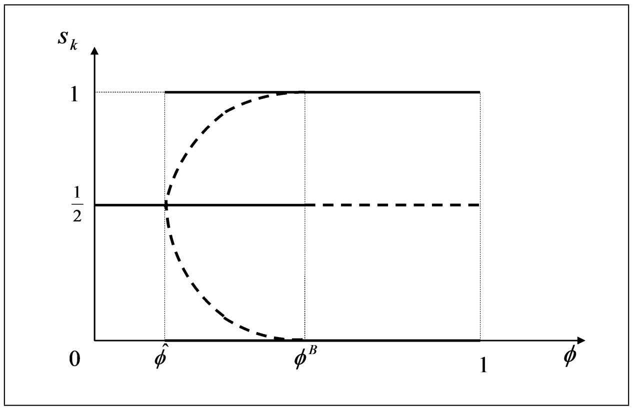


- $0 < \widehat{\phi} < \phi_B$: in this case the regions of trade costs are the following $\left[0, \widehat{\phi}\right)$, $\left[\widehat{\phi}, \phi_B\right)$ and $[\phi_B, 1)$. In the first region, when $\phi \in \left[0, \widehat{\phi}\right)$, things are identical to the previous case: $\frac{\partial f\left(\frac{1}{2}, \phi\right)}{\partial s_K}$ is positive and $f(0, \phi)$ is negative so that $f(0, \phi) \frac{\partial f\left(\frac{1}{2}, \phi\right)}{\partial s_K}$ is negative. As a consequence, again, *the symmetric equilibrium is stable and unique*. As ϕ increases and reaches the second region $\left[\widehat{\phi}, \phi_B\right)$, $f(0, \phi)$ becomes positive when $\frac{\partial f\left(\frac{1}{2}, \phi\right)}{\partial s_K}$ is still positive. Again $f(0, \phi) \frac{\partial f\left(\frac{1}{2}, \phi\right)}{\partial s_K}$ turns from negative to positive, leading to the emergence of multiple interior equilibria but now the new emerging non-symmetric interior steady states are unstable because the symmetric equilibrium is still stable being $\phi \in \left[\widehat{\phi}, \phi_B\right)$. As a consequence, in this region of intermediate trade costs we have a new and very interesting multiple equilibria regime with the symmetric equilibrium and two core-periphery equilibria which are stable at the same time. That means that, when $\phi \in \left[\widehat{\phi}, \phi_B\right)$, starting from an (unstable) interior non-symmetric equilibrium, a small shock in either direction may lead to catastrophic agglomeration or to catastrophic dispersion. When ϕ reaches the third region and becomes larger than ϕ_B, then $\frac{\partial f\left(\frac{1}{2}, \phi\right)}{\partial s_K}$ becomes positive and $f(0, \phi) \frac{\partial f\left(\frac{1}{2}, \phi\right)}{\partial s_K}$, while $f(0, \phi)$ remains negative, turns negative again. As a consequence, exactly as in

[14] From this viewpoint $\widehat{\phi}$ can be assimilated to the sustain point introduced by Baldwin et al. (2001), i.e. the value of trade costs such that the core-periphery equilibria emerge. Even in their model, in fact, the symmetric equilibrium looses its stability before the emergence of the core-periphery equilibria. As a consequence, the break-point smaller than the sustain point and catastrophic agglomeration is ruled out.

the previous case, in this third region of trade costs the symmetric equilibrium turns to be unique and unstable and two core-periphery equilibria emerge (see fig. **??**).

Figure 5: Stability map when $0 < \widehat{\phi} < \phi_B$

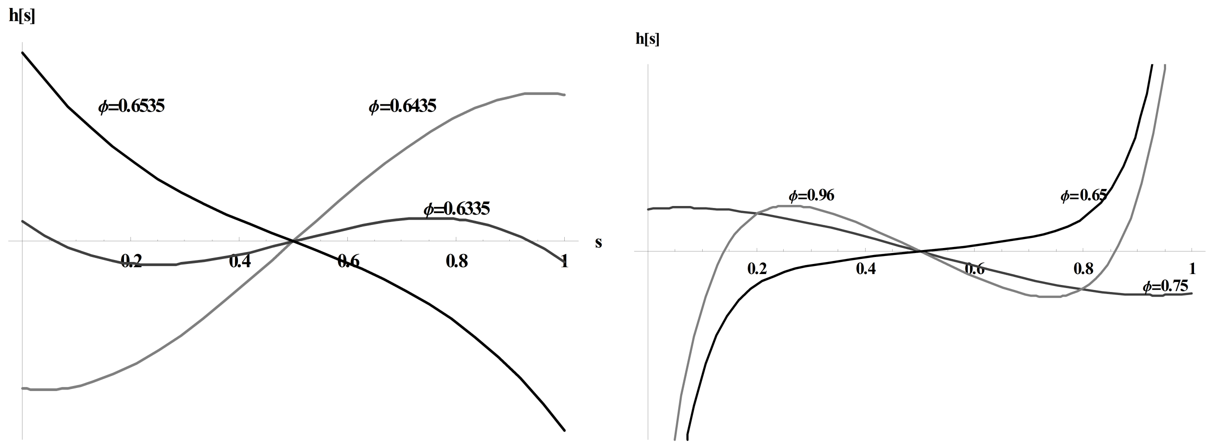


Leaving aside the first case (which might be considered as a sub-case of the second one), from the viewpoint of the qualitative dynamics, the last two cases only differs for the dynamic behaviour in the region of intermediate trade costs. In both cases, when trade costs are high - $\phi \in \left[0, min\left(\hat{\phi}, \phi^B\right)\right]$- the symmetric equilibrium is the unique interior equilibrium and it is stable, while when trade costs are low - $\phi \in \left[max\left(\hat{\phi}, \phi^B\right), 1\right]$, the symmetric equilibrium is still the unique interior equilibrium but it is now unstable and two core-periphery equilibria emerge. Things are substantially different for intermediate values of trade costs: in both cases two new non-symmetric interior equilibria emerge but while they are stable in the second case -$\phi_B < \widehat{\phi}$ - they are unstable in the third - $\phi_B < \widehat{\phi}$ - leading to the possibility of both catastrophic agglomeration and catastrophic dispersion. This result is similar to the one obtained in NEGG models with labour mobility and forward looking expectations (Baldwin and Forslid 2000) but, to the best of our knowledge, it is the first time this result is an outcome of footloose capital model with labour and capital immobility.

Is there any chance to distinguish between the three cases? In other words, can we, by looking at the value of the parameters, say something more about which regime apply? Unfortunately, the quite complicated mathematical form of the function f prevents us from finding a closed-form solution for both ϕ_B and $\widehat{\phi}$. However, by looking at the curvature of the function h we are able to perform some kind of qualitative comparison between ϕ_B and $\widehat{\phi}$ as the following proposition shows.

Proposition 4 *The two non-symmetric interior equilibria are stable ($\phi_B < \widehat{\phi}$) when $x < 1$ and when $x > \max\left(2, 1 + \frac{2\rho}{(L+\rho s_K)(1-\phi)}\right)$. They are unstable ($\phi_B > \widehat{\phi}$) when $1 \leq x \leq \left(2, 1 + \frac{2\rho}{(L+\rho s_K)(1-\phi)}\right)$.*

Proof. We know that, by definition, $f\left(0, \widehat{\phi}\right) = h\left(0, \widehat{\phi}\right) = 0$ and $f\left(\frac{1}{2}, \phi\right) = h\left(\frac{1}{2}, \phi\right) = h\left(\frac{1}{2}, \widehat{\phi}\right) = 0$. Hence, when for any $s_K \in \left[0, \frac{1}{2}\right)$ we have $\frac{\partial^2 h(s_K,\phi)}{\partial s_k^2} < 0$, that means that $\frac{\partial h\left(\frac{1}{2},\widehat{\phi}\right)}{\partial s_k} < 0 < \frac{\partial h\left(0,\widehat{\phi}\right)}{\partial s_k}$ and then $\frac{\partial f\left(\frac{1}{2},\widehat{\phi}\right)}{\partial s_k} < 0 < \frac{\partial f\left(0,\widehat{\phi}\right)}{\partial s_k}$. Since, by definition $\frac{\partial f\left(\frac{1}{2},\phi_B\right)}{\partial s_k} = 0$ and $\frac{\partial f\left(\frac{1}{2},\phi\right)}{\partial s_K}$is decreasing in ϕ (provided that $x < \widehat{x}$), then it must be $\phi_B < \widehat{\phi}$. Applying the same reasoning to the case when $\frac{\partial^2 h(s_K,\phi)}{\partial s_k^2} > 0$ and then $\frac{\partial f\left(0,\widehat{\phi}\right)}{\partial s_k} < 0 < \frac{\partial f\left(\frac{1}{2},\widehat{\phi}\right)}{\partial s_k}$, we conclude that $\frac{\partial^2 h(s_K,\phi)}{\partial s_k^2} > 0$ implies $\phi_B > \widehat{\phi}$. To conclude the proof it is sufficient



(a) $x = 1.5$ (Refer to fig. **??** for stability map) (b) $x = 50$ (Refer to fig. **??** for stability map)

Figure 6: Trade costs, number of interior steady states and their stability

to recall, from the proof of proposition 1, that, for any $s_K \in \left[0, \frac{1}{2}\right)$

$$
\begin{aligned}
\frac{\partial^2 h\left(s_K, \phi\right)}{\partial s_k^2} &< 0 \Leftrightarrow x \in (0,1) \\
\frac{\partial^2 h\left(s_K, \phi\right)}{\partial s_k^2} &> 0 \Leftrightarrow x \in \left(1, \max\left(2, 1 + \frac{2\rho}{\left(L + \rho s_K\right)(1-\phi)}\right)\right] \\
\frac{\partial^2 h\left(s_K, \phi\right)}{\partial s_k^2} &< 0 \Leftrightarrow x \in \left(\max\left(2, 1 + \frac{2\rho}{\left(L + \rho s_K\right)(1-\phi)}\right), \infty\right)
\end{aligned}
$$

where, by the symmetry rule, the signs of the second derivative is opposite for $s_K \in \left(\frac{1}{2}, 1\right]$. ■

Proposition 4, together with proposition 3, provides the complete characterization of the qualitative dynamics of our economy as trade costs declines and for different degree of substitutability between the two kinds of goods. As we can see, the impact of non-unitary intersectoral elasticity of substitution is quite dramatic on the dynamic behaviour of the economy (as illustrated in figures **??**, **??** and **??**).

The non-linearity of the optimal investment relation is the main responsible for this complex behaviour. Such non-linearity simply disappears with unitary elasticity so that the well-behaved dynamics of the standard NEGG model is just a knife-edge case. Our model shows that things are much more complicated, but still readable and useful for policy purposes, when a more general approach is adopted.

4 Geography and Integration *always* matter for Growth

A well-established result in the NEGG literature (Baldwin Martin and Ottaviano 2001, Baldwin and Martin 2004, Baldwin et al. 2004) is that geography matters for growth *only* when spillovers are localized. In particular, with localized spillovers, the cost of innovation is minimized when the whole manufacturing sector is located in only one region. If this is the case, innovating firms have a higher incentive to invest in new units of knowledge capital with respect to a situation in which manufacturing firms are dispersed in the two regions. Thereby the rate of growth of new units of knowledge capital g is maximized in the core-periphery equilibrium and "agglomeration is good for growth". When spillovers are global, this is not the case: innovation costs are unaffected by the geographical allocation of firms and the aggregate rate of growth is identical in the two equilibria being common in the symmetric one ($g = g^*$) or north's g in the core-periphery one. Moreover, in the standard case, market integration have no direct influence on the rate of growth which is not dependent on ϕ. When spillovers are localized, trade costs may have an indirect influence on the rate of growth by affecting the geographical allocation of firms: when trade costs are reduced below the break point level, the symmetric equilibrium becomes unstable and the resulting agglomeration process, by lowering the innovation cost, is growth-enhancing. But even this indirect influence will not exist when spillovers are global.

In what follows, we will question these conclusions. We will show that in our more general context (i.e. when the intersectoral elasticity of substitution is not necessarily unitary), *geography and integration always matters for growth*, even in the case when spillovers are global. In particular we show that

1. *Market integration has always a direct effect on growth*: when the intersectoral elasticity of substitution is larger than 1, then market integration (by increasing the share of expenditures in manufactures) is always good for growth. Otherwise, when goods are poor substitutes, integration is bad for growth.

2. *The geographical allocation of firms always matters for growth*: the rate of growth in the symmetric equilibrium differs from the rate of growth in the core-periphery one. In particular, growth is faster (slower) in symmetry if the share of global expenditure dedicated to manufactures is higher (lower) in symmetry than in the core-periphery. If this is the case, then agglomeration is bad (good) for growth

4.1 Growth and economic integration

We now look for the general expression of the growth rate in both the interior and the core-periphery equilibria. Labour market-clearing condition requires that

$$2L = E\left(\frac{\sigma - \mu(s_K, \phi)}{\sigma}\right) + E^*\left(\frac{\sigma - \mu^*(s_K, \phi)}{\sigma}\right) + g s_K + g^*(1 - s_K)$$

It is easy to see that, both in the interior or in the core periphery equilibria (with core in the North), we have $g s_K + g^*(1 - s_K) = g$. In the first case this equality holds because $g = g^*$, while in the second it holds because $s_K = 1$. Hence, by recalling that $E = L + \rho s_K$ and $E^* = L + \rho(1 - s_K)$ we find that

$$g(s_K, \phi) = \frac{L(\mu(s_K, \phi) + \mu^*(s_K, \phi)) - \rho(\sigma - s_K \mu(s_K, \phi) - (1 - s_K)\mu^*(s_K, \phi))}{\sigma}. \tag{48}$$

This expression is then valid for *any* steady state allocation, included the core-periphery one. A simple derivative then will tell us the way growth is affected by trade costs

$$\frac{\partial g}{\partial \phi} = \frac{1}{\sigma}\left(L\left(\frac{\partial \mu}{\partial \phi} + \frac{\partial \mu^*}{\partial \phi}\right) + \rho\left(\frac{\partial \mu}{\partial \phi} s_K + \frac{\partial \mu^*}{\partial \phi}(1 - s_K)\right)\right) \tag{49}$$

and by (**??**) and (**??**) we conclude that:

$$\begin{aligned} \frac{\partial g}{\partial \phi} &> 0 \Leftrightarrow \alpha > 0 \\ \frac{\partial g}{\partial \phi} &< 0 \Leftrightarrow \alpha < 0 \\ \frac{\partial g}{\partial \phi} &= 0 \Leftrightarrow \alpha = 0 \end{aligned}$$

so that integration is good for growth if and only if the traditional and the manufacturing goods are good substitutes. In the standard approach, the special case when $\alpha = 0$, integration has no effect on growth. From the policy perspective, when α is positive, so that the intersectoral elasticity of substitution is larger than unity, the policy maker should promote policies towards market integration in order to maximize the (common) growth rate. By contrast, if we accept that the two kinds of goods are poor substitutes, then policies favoring economic integration are growth-detrimental and if the policy-maker is growth oriented then he should avoid them. What is the economic intuition behind this result? We should first consider that, as (**??**) clearly highlights, growth is positively affected by both northern and southern expenditure share in manufacturing goods: an increase in this variable would increase manufacturing profits, raising Tobin's q and then incentives to invest. As a result, growth would be higher. Then, any policy instrument able to increase total expenditure on manufacturing goods at the world level will accelerate economic growth. The issue is then: what are the determinants of the total expenditure share on manufactures at the world level? From our previous analysis we know that, with CES intermediate utility function, northern and southern expenditure shares depend on the geographical location of firms (s_K) and on the degree of economic integration ϕ. We leave the first determinant aside for a moment and we concentrate on the second. A reduction in the cost of trade will always bring to a reduction in the price index for the manufacturing goods in both regions. However, this reduction will have opposite effect on $\mu(\cdot)+\mu^*(\cdot)$ depending on whether the intersectoral elasticity of substitution is larger or smaller than 1. In the first case, since the traditional good (which is now relatively more expensive) can be easily replaced by the industrial goods, the expenditure shares on the latter will increase in both regions, and this will also increase the growth rate. By contrast, when the traditional good cannot be easily replaced by the industrial goods, a reduction in the price index of industrial goods may increase total expenditure but it will decrease their *share* of expenditure in both regions. As a result, any integration-oriented policy will also reduce growth.

To conclude, in order to better appreciate our results, it is worth comparing them with the impacts that endogenous expenditure shares have in static NEG models. In Murata (2008), where growth is inhibited, trade costs have *positive level effects* since the mass of varieties depends on trade costs via endogenous expenditure share generated by a Stone-Geary non-homothetic utility function. Hence, the degree of industrialization rises side by side with the expenditure share of manufactured goods according to a decline in transport costs. By adding the possibility of growth, our model allows to uncover the emergence of an additional *growth* effects of trade costs as they also affect the rate of growth via the endogenous expenditure share. Such effect is negative when freer trade reduces the expenditure share in manufacturing goods while it is positive when, as in Murata (2008) expenditure shares in manufacturing goods are positively affected by a reduction of transport costs.

4.2 Growth and agglomeration

Using (**??**) we can also compute the effect of an increase in the degree of agglomeration on the growth rate. As we already know, this effect is nil in the standard model with unitary intersectoral elasticity of substitution and global technological spillovers. By contrast, when intersectoral elasticity of substitution is allowed to vary we have

$$\frac{\partial g(s_K,\phi)}{\partial s_K}=\frac{1}{\sigma}\left(\rho(\mu-\mu^*)+\frac{(1-x)(1-\phi)}{(s_K+(1-s_K)\phi)}\left[(L+\rho s_K)\mu(1-\mu)-(L+\rho(1-s_K))\mu^*(1-\mu^*)\right]\right) \quad (50)$$

where, again, $x=\frac{\sigma(1-\alpha)-1}{(\sigma-1)(1-\alpha)}$, so that $\alpha>(<)0 \Leftrightarrow x<(>)1$ and, for simplicity, we have omitted the arguments for the functions μ^* and μ. We observe that this derivative is generally different from zero, meaning that, with non-unitary intersectoral elasticity of substitution, the growth rate is always affected by firms' location, even in the absence of localized spillovers. As expected, the growth effect of agglomeration disappears when $\alpha=0$ as in the standard case. But is it agglomeration good or bad for growth when expenditure shares are endogenous because of non-unitary intersectoral elasticity of substitution? By further analysing equation (**??**), we find that

$$\frac{\partial g\left(\frac{1}{2},\phi\right)}{\partial s_K}=0$$

so that the symmetric equilibrium always represents an maximum or a minimum for the growth rate according to whether the second derivative of g with respect to s_K, computed in the symmetric equilibrium, is respectively negative or positive. By inspection we find that the growth rate is maximized in the symmetric equilibrium when

$$g\left(\frac{1}{2},\phi\right)\geq g(s_K,\phi),\forall s_K\in[0,1]\Longleftrightarrow>(1-x)^2\frac{(1-\phi)}{(1+\phi)}(2L+\rho)(1-2\mu)+(1-x)2\rho<0 \quad (51)$$

Hence, the expenditure shares on industrial goods should be large enough in the symmetric equilibrium in order for growth to be maximized in the symmetric equilibrium (and then for agglomeration to be bad for growth). This conclusion is quite intuitive, given the positive impact that the expenditure share on industrial goods has on growth. In particular, when $x < 1$, and then the two kinds of commodity are good substitutes, μ needs to be sufficiently larger than $\frac{1}{2}$, while when the industrial and the traditional goods are poor substitutes, then a μ lower but sufficiently close to $\frac{1}{2}$ might be enough for growth to be maximized when industry is dispersed among regions. Condition (**??**) has two main merits: the first, as already anticipated, is to shed light on a new trasmission mechanism between agglomeration and growth, given that such mechanism was hidden in the CD approach by the knife-edge case $x = 1$. The second merit is that, unlike standard NEGG models with localized knowledge spillovers, it states that the sign of the relationship between agglomeration and growth can be either positive (i.e. agglomeration is good for growth, as in Baldwin et al. (2001)), or negative (i.e. dispersion is good for growth). From this viewpoint, our model provides a mechanism (alternative to the one proposed by Cerina and Mureddu (2009) [15]) to reconcile theory with the recent empirical evidence according to which the positive relationship between agglomeration and growth is limited to early stages of development (Bruhlart and Sbergami, 2009)

Such mechanism can be more appreciated if we consider that, when $\frac{\partial g(s_K,\phi)}{\partial s_K}$ is monotone in s_K, condition (**??**) implies that $g\left(\frac{1}{2},\phi\right)$ is bigger than $g(1,\phi)$. This latter condition is slightly more interesting from the point of view of the economic intuition:

$$g\left(\frac{1}{2},\phi\right) > g(1,\phi) \Leftrightarrow \mu\left(\frac{1}{2},\phi\right) > s_E^{cp}\mu(1,\phi) + \left(1 - s_E^{cp}\right)\mu^*(1,\phi) \tag{52}$$

where $s_E^{cp} = \frac{L+\rho}{2L+\rho}$ is the market size of the north when the whole industry is concentrated in this region ($s_K = 1$). In other words, growth in the symmetric equilibrium will be faster than in the core-periphery equilibrium if and only if the industrial-goods' expenditure share in manufactures in the symmetric equilibrium (which is common in the two regions), is larger than a weighted average of the industrial goods' expenditure share in the core-periphery equilibrium in the two regions, where the weights are given by the reciprocal regional market sizes. What is significant in this case is then the relative importance of the industrial goods in the consumption bundle at the world level. If at the world level the industrial good is relatively more important in the symmetric equilibrium than in the core-periphery one, then agglomeration is bad for growth and a growth-oriented policy-maker should promote policies which favor dispersion of economic activities. It is worth noting that this condition is not trivial at all since we have:

$$\begin{aligned}
\alpha &> 0 \Leftrightarrow \mu(1,\phi) > \mu\left(\frac{1}{2},\phi\right) > \mu^*(1,\phi) \\
\alpha &< 0 \Leftrightarrow \mu^*(1,\phi) > \mu\left(\frac{1}{2},\phi\right) > \mu(1,\phi)
\end{aligned}$$

A further analysis of condition (**??**) will not provide any significant insight. The validity of condition (**??**) is highly dependent on the curvature of $\mu(\cdot)$ and $\mu^*(\cdot)$ with respect to s_K.

5 Conclusions

This paper is a first attempt to introduce endogenous expenditure shares in a New Economic Geography and Growth model. We do this by allowing the intersectoral elasticity of substitution to be different from the unit value and we show how this slight generalization of the model leads to different and unexpected outcomes in

[15] In this paper, the authors proposes a NEGG model with non-tradable goods and intersectoral spillovers between the cumulative output of the innovating sector and the non-tradable sector to show that agglomeration might be bad for aggregate real growth when: 1) the spatial range of the intrasectoral technological spillovers within the innovating sector; 2) the intensity of the positive externality from the innovating sector to the non-tradable sector; 3) the expenditure share on non-tradable goods are all large enough.

terms of dynamics of the allocation of economic activities, the equilibrium growth prospect and the policy insights.

Concerning the dynamics of the allocation of economic activities, the main result is the emergence of multiple interior steady states. We have shown that two additional non-symmetric steady states always emerge sooner or later during the process of economic integration - i.e. - for some feasible values of the freeness of trade. Moreover, we have shown that these additional non-symmetric interior steady states are stable - thereby leading to a stability map similar to that of Baldwin e al. (2001) - when the traditional and the industrial goods are either close or very poor substitutes. By contrast, the two additional steady states are unstable for intermediate values of the elasticity of substitution, but in any case lower than unity. In the latter case, the model displays multiple stable steady states (the symmetric and the two core-periphery allocations) so that if the economy starts from a non-symmetric steady states, a small shock in either direction may lead to catastrophic agglomeration or catastrophic dispersion. This result, which is similar to that obtained in model with labor mobility and forward-looking expectations (Baldwin and Forslid 2000), is to the best our knowledge new when obtained by a footlose capital model where capital is immobile. This complex and unexpected behaviour is due to the non-linearity of the optimal investment relation and to the associated emergence of a new force which we call "expenditure share effect". This force acts as a dispersion force, so that the agglomeration process is activated for level of trade openness which are higher than the standard case, when the modern and the traditional goods are poor substitutes. By contrast, it acts as an agglomeration force - and agglomeration is reached for lower degrees of market openness - when the traditional and the industrial goods are good substitutes. In the latter case, when the expenditure share effect is strong enough and the two kinds of commodities are very close substitutes, agglomeration processes are activated whatever the degree of market openness since the symmetric equilibrium is unstable for any level of trade costs.

From the growth perspective, results are not less relevant: 1) unlike the standard NEGG models, the growth rate is influenced by the allocation of economic activities even in absence of localized knowledge spillovers and 2) the degree of economic integration always affects the rate of growth, being growth-detrimental if the intersectoral elasticity of substitution is lower than unity and being growth-enhancing in the opposite case. We are then able to provide a rationale for the rather counterintuitive conclusion according to which an integration-oriented policy rule is bad for growth: this could happen when the two kinds of commodities are poor substitutes and trade integration, by reducing the price of manufacturing goods, induce a reduction in their expenditure shares thereby leading to slower growth because of the diminished size of the increasing return sector.

Albeit more complex than the existing literature, the outcomes of our model are still quite readable and relevant policy implications can be drawn from our conclusions. In particular, our model suggests that a better empirical investigation of the magnitude of the intersectoral elasticity of substitution in the context of a NEG model is strongly needed in order to implement the right policy recommendations. A typical example is the well-established result stating that policy makers should not try to avoid the agglomeration of economic activities because the concentration of the innovative and the increasing returns sectors will increase growth at a global level when spillovers are localized. This conclusion does not take into account the fact that the incentive to invest in new units of capital (and thereby the growth rate) depends on the Dixit-Stiglitz operating profits of manufacturing firms, that in our model are influenced by the share of expenditure in the modern goods. If the average regional expenditure share in this sector is higher in the symmetric equilibrium than in the case of agglomeration, then firms' profits are higher when the economic activities are dispersed among the two regions and concentrating them in only one region will reduce economic growth.

A second message of our paper is that policies should take into account the crucial role of the intersectoral elasticity of substitution. To our knowledge, there are no empirical studies assessing the value of this parameter in the context of a NEG model. An empirical analysis of the intersectoral elasticity of substitution would be an expected follow-up of our analysis and would be highly needed in order to assess the relative empirical relevance of the theoretical results we have obtained.

References

[1] Andres, F. 2006. "Divergence, Wage-Gap and Geography", *Economie Internationale* 4Q, 83-112.

[2] Ardelean, A. 2007 How Strong is Love for Variety. *Working Paper,* Leavey School of Business, University of Santa Clara, California.

[3] Baldwin, R. 1999, "Agglomeration and endogenous capital", *European Economic Review* 43, 253-280,

[4] Baldwin, R., Forslid R. 1999. "Incremental trade policy and endogenous growth: A q-theory approach", *Journal of Economic Dynamics and Control* 23, 797-822.

[5] Baldwin, R., Forslid, R., 2000. "The Core-Periphery Model and Endogenous Growth: Stabilizing and De-Stabilizing Integration", *Economica* 67, 307-324.

[6] Baldwin, R., Martin, P., 2004. "Agglomeration and Regional Growth", in J. V. Henderson and J. F. Thisse (eds.). *Handbook of Regional and Urban Economics*, Vol. 4. Amsterdam: North-Holland, 2671-2711.

[7] Baldwin, R., Forslid, R., Martin, P., Ottaviano, G., Robert-Nicoud, F., 2004. *Economic Geography and Public Policy*, Princeton: Princeton University Press.

[8] Baldwin, R., Martin, P., Ottaviano, G., 2001. "Global Income Divergence, Trade, and Industrialization: The Geography of Growth Take-Offs", *Journal of Economic Growth* 6, 5-37.

[9] Baldwin, R., Robert-Nicoud, F., 2008. "Trade and growth with heterogeneous firms", *Journal of International Economics* 74, 21-34.

[10] Baldwin, R., Okubo, T., 2006. "Heterogeneous firms, agglomeration and economic geography: spatial selection and sorting", *Journal of Economic Geography* 6, 323-346.

[11] Bellone, F., Maupertuis, M., 2003. "Economic Integration and Regional Income Inequalities: Competing Dynamics of Regional Wages and Innovative Capabilities", *Review of International Economics* 11, 512-526.

[12] Benassy, J., 1996. "Taste for variety and optimum production patterns in monopolistic competition", *Economic Letters* 52, 41-47.

[13] Blanchard, O., Kiyotaki, N., 1987. "Monopolistic Competition and the Effects of Aggregate Demand", *American Economic Review* 77, 647-66

[14] Bruhlart M., Sbergami F., 2009. "Agglomeration and growth: Cross-country evidence", *Journal of Urban Economics*, 65: 48-63.

[15] Cerina F., Mureddu, F., 2009. "Is Agglomeration really good for Growth? Global Efficiency and Interregional Equity", Working Paper CRENoS 200913, Centre for North South Economic Research, University of Cagliari and Sassari, Sardinia.

[16] Cerina F., Pigliaru, F., 2007. "Agglomeration and Growth: a critical assessment". In Fingleton (ed.). *New Directions in Economic Geography.* Chelthenam: Edward Elgar, 130-167.

[17] Dixit, A.K., Stiglitz, J.E., 1975. "Monopolistic Competition and Optimum Product Diversity", The Warwick Economics Research Paper Series (TWERPS) 64, University of Warwick.

[18] Dixit, A.K., Stiglitz, J.E., 1977. "Monopolistic Competition and optimum product diversity", *American Economic Review* 67, 297-308.

[19] Eaton, J., Kortum, S., 2002. "Technology, Geography and Trade", *Econometrica* 70, 1741-1779.

[20] Krugman, P., 1991. "Increasing Returns and Economic Geography", *Journal of Political Economy* 99, 483-499.

[21] Martin, P., Ottaviano, G., 1999. "Growing Locations: Industry Location in a Model of Endogenous Growth". *European Economic Review* 43, 281-302.

[22] Martin, P., Rogers, C., 1995. "Industrial location and public infrastructure", *Journal of International Economics* 39, 335-351.

[23] Murata, Y., 2008. "Engel's law, Petty's law, and agglomeration", *Journal of Development Economics* 87, 161-177.

[24] Romer, P., 1990. "Endogenous Technological Change", *Journal of Political Economy* 98, S71-S102.

[25] Samuelson, P.A., 1954. "The transfer problem and transport costs, II: Analysis of trade impediments", *Economic Journal* 64, 264-289.

Appendix

Proof of proposition 1

(Number of interior steady states) The system displays one or three interior steady state allocations: the symmetric allocation $s_K = \frac{1}{2}$ (which is a "global" interior steady state) and two non-symmetric allocations: $s_K^* (L, x, \rho, \phi, Z)$ and $s_K^{**} (L, x, \rho, \phi, Z) = 1 - s_K^* (L, x, \rho, \phi, Z)$ which may emerge only for some values of the parameters. The interior steady state is unique and equal to $\frac{1}{2}$ when $f(0, \phi) \frac{\partial f\left(\frac{1}{2}, \phi\right)}{\partial s_K} \leq 0$, while there are 3 interior steady states when $f(0, \phi) \frac{\partial f\left(\frac{1}{2}, \phi\right)}{\partial s_K} > 0$

Proof.

The first step is to limit the number of the interior steady states - i.e. - the zeros of the function f. As the latter, also h is symmetric so that

$$h(s_K, \phi) = -h(1 - s_K, \phi) \tag{53}$$

As a consequence, since $s_K = \frac{1}{2}$is a global equilibrium and for any possible interior steady state $s_K^* \in \left(0, \frac{1}{2}\right)$ there is another interior steady state $s_K^{**} = 1 - s_K^* \in \left(\frac{1}{2}, 1\right)$, the total number of interior steady states is odd. Moreover, by applying the symmetry rule, we can conclude that $\frac{\partial^2 h\left(\frac{1}{2}, \phi\right)}{\partial s_k^2} = \frac{\partial^2 f\left(\frac{1}{2}, \phi\right)}{\partial s_k^2} = 0$. In fact, by twice-differentiating (**??**) we find

$$\begin{aligned} \frac{\partial h(s_K, \phi)}{\partial s_K} &= \frac{\partial h(1 - s_K, \phi)}{\partial s_K} \\ \frac{\partial^2 h(s_K, \phi)}{\partial s_k^2} &= -\frac{\partial^2 h(1 - s_K, \phi)}{\partial s_k^2} \end{aligned}$$

when $s_K = \frac{1}{2}$, the latter can be true only if

$$\frac{\partial^2 h\left(\frac{1}{2}, \phi\right)}{\partial s_k^2} = -\frac{\partial^2 h\left(\frac{1}{2}, \phi\right)}{\partial s_k^2} = 0$$

Now assume $\frac{\partial^2 h(s_K,\phi)}{\partial s_k^2} \neq 0$ for every $s_K \in \left[0, \frac{1}{2}\right)$. If this is the case, then $\frac{\partial h(s_K,\phi)}{\partial s_K}$ is monotone (increasing or decreasing) in the two intervals $\left[0, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, 1\right]$. Then, by the Bolzano theorem, we know that $\frac{\partial h(s_K,\phi)}{\partial s_K}$ can have *at most one zero in* $s_K \in \left[0, \frac{1}{2}\right)$. In other words

$$\text{if} \frac{\partial^2 h(s_K,\phi)}{\partial s_k^2} \neq 0,\ \forall s_K \in \left[0, \frac{1}{2}\right) \Rightarrow \exists! \bar{s}_K \in \left[0, \frac{1}{2}\right) \ : \ \frac{\partial h(\bar{s}_K,\phi)}{\partial s_K} = 0$$

But if this is the case, then h can have at most 1 maximum (if $\frac{\partial^2 h(s_K,\phi)}{\partial s_k^2} < 0 =$) or one minimum (if $\frac{\partial^2 h(s_K,\phi)}{\partial s_k^2} > 0 =$) in $\left[0, \frac{1}{2}\right)$. Together with the fact that $h\left(\frac{1}{2}, \phi\right) = 0$, this avoids that h can cross the horizontal axis *more than once in* $\left[0, \frac{1}{2}\right)$.

$$\text{if} \frac{\partial^2 h(s_K,\phi)}{\partial s_k^2} \neq 0,\ \forall s_K \in \left[0, \frac{1}{2}\right) \Rightarrow \exists! s_K^* \in \left[0, \frac{1}{2}\right) \ : \ h(s_K^*.\phi) = 0 =$$

Therefore, recalling the symmetry rule and the fact that $f\left(\frac{1}{2}, \phi\right) = h\left(\frac{1}{2}, \phi\right) = 0$, by showing that $h''(s_K, \phi) \neq 0, \forall s_K \in \left[0, \frac{1}{2}\right)$, we are also showing that there can be at most **3** interior steady state allocations in $s_K \in (0, 1)$. We now show that this is the case. By computation we find that

$$\begin{aligned}
\frac{\partial h(s_K,\phi)}{\partial s_K} &= 2(L(1-\phi) - \rho\phi) - \rho Z\left[(s_K + (1-s_K)\phi)^x + (\phi s_K + 1 - s_K)^x\right] \\
&\quad + xZ(1-\phi)\left[(L + \rho(1-s_K))(s_K + (1-s_K)\phi)^{x-1} + (L + \rho s_K)(\phi s_K + 1 - s_K)^{x-1}\right] \\
\frac{\partial^2 h(s_K,\phi)}{\partial s_k^2} &= 2x(1-\phi)\rho Z\left[(\phi s_K + 1 - s_K)^{x-1} - (s_K + (1-s_K)\phi)^{x-1}\right] \\
&\quad + x(x-1)(1-\phi)^2 Z\left[(L + \rho(1-s_K))(s_K + (1-s_K)\phi)^{x-2} - (L + \rho s_K)(\phi s_K + 1 - s_K)^{x-2}\right]
\end{aligned}$$

So that $\frac{\partial^2 h(s_K,\phi)}{\partial s_k^2}$ can be written as the sum of two members:
$2x(1-\phi)\rho Z\left[(\phi s_K + 1 - s_K)^{x-1} - (s_K + (1-s_K)\phi)^{x-1}\right]$
and $x(x-1)(1-\phi)^2 Z\left[(L + \rho(1-s_K))(s_K + (1-s_K)\phi)^{x-2} - (L + \rho s_K)(\phi s_K + 1 - s_K)^{x-2}\right]$ By neglecting the knife-edge case $x = 1$ (where the equilibrium analysis collapses to the standard case and therefore the steady state is always unique and equal to $s_K = \frac{1}{2}$ being $h(s_K)$ a straight line), we can distinguish several cases:

1. $x \in (0, 1)$: in this case, for $s_K \in [0, \frac{1}{2})$, $2x(1-\phi)\rho Z\left[(\phi s_K + 1 - s_K)^{x-1} - (s_K + (1-s_K)\phi)^{x-1}\right] < 0$ because $(\phi s_K + 1 - s_K) - (s_K + (1-s_K)\phi) > 0$. and $\phi \in (0, 1)$ and $x, \rho, Z > 0$. Moreover $x(x-1)(1-\phi)^2 Z\left[(L + \rho(1-s_K))(s_K + (1-s_K)\phi)^{x-2} - (L + \rho s_K)(\phi s_K + 1 - s_K)^{x-2}\right] < 0$ because $\left[(L + \rho(1-s_K))(s_K + (1-s_K)\phi)^{x-2} - (L + \rho s_K)(\phi s_K + 1 - s_K)^{x-2}\right] > 0$ for $x < 1$ and $s_K \in \left[0, \frac{1}{2}\right)$ while $x(x-1)(1-\phi)^2 Z < 0$. As a consequence, when $x < 1$, $\frac{\partial^2 h(s_K,\phi)}{\partial s_k^2} =$ is given by the sum of two negative function and therefore $\frac{\partial^2 h(s_K,\phi)}{\partial s_k^2} < 0 = \forall s_K \in \left[0, \frac{1}{2}\right)$

2. $x \in (1, 2]$: in this case, for $s_K \in \left[0, \frac{1}{2}\right)$, $2x(1-\phi)\rho Z\left[(\phi s_K + 1 - s_K)^{x-1} - (s_K + (1-s_K)\phi)^{x-1}\right] > 0$ because $(\phi s_K + 1 - s_K) - (s_K + (1-s_K)\phi) > 0$ and the common exponent is positive. Analogously $x(x-1)(1-\phi)^2 Z\left[(L + \rho(1-s_K))(s_K + (1-s_K)\phi)^{x-2} - (L + \rho s_K)(\phi s_K + 1 - s_K)^{x-2}\right] > 0$ because $x > 1$ and $\left[(L + \rho(1-s_K))(s_K + (1-s_K)\phi)^{x-2} - (L + \rho s_K)(\phi s_K + 1 - s_K)^{x-2}\right]$ is still positive. As a consequence $\frac{\partial^2 h(s_K,\phi)}{\partial s_k^2} > 0 = \forall s_K \in \left[0, \frac{1}{2}\right)$

3. $x > 2$: in this case the two members might have different sign so that we need to write $\frac{\partial^2 h(s_K,\phi)}{\partial s_k^2} =$ in a different way

$$\frac{\partial^2 h(s_K,\phi)}{\partial s_k^2} = x(1-\phi)Z\left\{(\phi s_K + 1 - s_K)^{x-2} M(s_K) - (s_K + (1-s_K)\phi)^{x-2} N(s_K)\right\} =$$

where $M(s_K) = \rho(2 + s_K(1-\phi)) - L(x-1)(1-\phi) - x\rho s_K(1-\phi)$ and $N(s_K) = \rho(1 + \phi + s_K(1-\phi)) - L(x-1)(1-\phi) - x(1-\phi)\rho(1-s_K)$ are two linear functions in s_K Notice that

$$\begin{aligned} M(s_K) &\geq 0 \Leftrightarrow x \leq 1 + \frac{2\rho}{(L+\rho s_K)(1-\phi)} \\ N(s_K) &\geq 0 \Leftrightarrow x \leq \frac{(1+\phi)\rho + (L+\rho s_K)(1-\phi)}{(1-\phi)(L+\rho(1-s_K))} \end{aligned}$$

we also have

$$M(s_K) > N(s_K), \forall s_K \in \left[0, \frac{1}{2}\right)$$

By assuming - without any loss of generality - that $2 < \frac{(1+\phi)\rho+(L+\rho s_K)(1-\phi)}{(1-\phi)(L+\rho(1-s_K))} < 1 + \frac{2\rho}{(L+\rho s_K)(1-\phi)}$ (if this is not the case, we can just refer to the previous three cases) we can distinguish three different sub cases:

(a) $2 < x < \frac{(1+\phi)\rho+(L+\rho s_K)(1-\phi)}{(1-\phi)(L+\rho(1-s_K))}$: in this case both $M(s_K) > 0$ and $N(s_K) > 0$. Hence we have

$$\frac{\partial^2 h(s_K, \phi)}{\partial s_k^2} > 0 \Leftrightarrow \frac{(\phi s_K + 1 - s_K)^{x-2}}{(s_K + (1-s_K)\phi)^{x-2}} > \frac{N(s_K)}{M(s_K)} =$$

which is always true for any $s_K \in \left[0, \frac{1}{2}\right)$ because $\frac{(\phi s_K+1-s_K)^{x-2}}{(s_K+(1-s_K)\phi)^{x-2}} > 1$ while $\frac{N(s_K)}{M(s_K)} < 1$. Hence, for $2 < x < \frac{(1+\phi)\rho+(L+\rho s_K)(1-\phi)}{(1-\phi)(L+\rho(1-s_K))}$, $h''(s_K)$ is always strictly positive.

(b) $2 < \frac{(1+\phi)\rho+(L+\rho s_K)(1-\phi)}{(1-\phi)(L+\rho(1-s_K))} < x < 1 + \frac{2\rho}{(L+\rho s_K)(1-\phi)}$: in this case $M(s_K) > 0$ while $N(s_K) < 0$. But then $h''(s_K)$ is simply the sum of two strictly positive functions. Therefore, even in this case, $h''(s_K)$ is strictly positive for any value of s_K belonging to $\left[0, \frac{1}{2}\right)$

(c) $2 < 1 + \frac{2\rho}{(L+\rho s_K)(1-\phi)} < x$: in this case both $M(s_K)$ and $N(s_K)$ are negative. Hence, in this case

$$\frac{\partial^2 h(s_K, \phi)}{\partial s_k^2} > 0 \Leftrightarrow \frac{(\phi s_K + 1 - s_K)^{x-2}}{(s_K + (1-s_K)\phi)^{x-2}} < \frac{N(s_K)}{M(s_K)} =$$

which is never true because, as seen before, for every $s_K \in \left[0, \frac{1}{2}\right)$, we have $\frac{(\phi s_K+1-s_K)^{x-2}}{(s_K+(1-s_K)\phi)^{x-2}} > 1$ and $\frac{N(s_K)}{M(s_K)} < 1$. Hence, when $x > 1 + \frac{2\rho}{(L+\rho s_K)(1-\phi)}$, $h''(s) < 0$ for every $s_K \in \left[0, \frac{1}{2}\right)$.

Summing up, we have that, for any $s_K \in \left[0, \frac{1}{2}\right)$

$$\begin{aligned} \frac{\partial^2 h(s_K, \phi)}{\partial s_k^2} &=< 0 \Leftrightarrow x \in (0, 1) \\ \frac{\partial^2 h(s_K, \phi)}{\partial s_k^2} &=> 0 \Leftrightarrow x \in \left(1, \max\left(2, 1 + \frac{2\rho}{(L+\rho s_K)(1-\phi)}\right)\right] \\ \frac{\partial^2 h(s_K, \phi)}{\partial s_k^2} &=< 0 \Leftrightarrow x \in \left(\max\left(2, 1 + \frac{2\rho}{(L+\rho s_K)(1-\phi)}\right), \infty\right) \end{aligned}$$

where, by the symmetry rule, the signs of the second derivative is opposite for $s_K \in \left(\frac{1}{2}, 1\right]$. Hence, for a given value of x, $\frac{\partial h(s_K,\phi)}{\partial s_k}$ is always monotone in $\left[0, \frac{1}{2}\right)$ .Therefore h can have at most 1 zero in $\left[0, \frac{1}{2}\right)$ and, by the symmetry rule and since $h\left(\frac{1}{2}, \phi\right) = 0$, h can have at most 3 zeros in $[0, 1]$. And since $h(s_K^*, \phi) = 0 \Leftrightarrow f(s_K^*, \phi) = 0$, we have shown that the interior steady state allocations (i.e. the values of $s_K \in (0, 1)$ such that $f(s_K, \phi) = 0$) can be *1 or at most* **3**.

Once we have limited the number of equilibria, we are almost ready to provide the necessary and sufficient condition for uniqueness and multiplicity. A necessary and sufficient condition for the existence of three distinct interior steady states is the following

$$h(0, \phi) \frac{\partial h\left(\frac{1}{2}, \phi\right)}{\partial s_K} > 0$$

We first show that the condition is sufficient. If $h(0,\phi)\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K}>0$ then $h(0,\phi)>0\,(<0)$ when $\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K}>0\,(<0)$. Since $h\left(\frac{1}{2},\phi\right)=0$ and h is continuous, h must cross the horizontal axis at least one and thus there must be *at least* one $s_K^*\in\left[0,\frac{1}{2}\right)$

such that $h(s_K^*,\phi)=0$. By proposition 1 we know that such value is unique. Hence, by the symmetry rule (**??**) and by (**??**), $h(0,\phi)\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K}>0$ is a sufficient condition for the existence of **three** interior steady state allocations in the whole interval $[0,1]$. As for necessity, assume that there are three points $s_K^*\in\left[0,\frac{1}{2}\right)$,

$\bar{s}_K=\frac{1}{2}$ and $s_K^{**}=1-s_K^*$ such that $h(s_K^*,\phi)=h\left(\frac{1}{2},\phi\right)=f(1-s_K^*,\phi)=0$. When $\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K}>0\ (<0)$, since $h\left(\frac{1}{2},\phi\right)=0$ and h crosses the horizontal axis only once in $\left[0,\frac{1}{2}\right)$, then it must be $h(0,\phi)>0\,(<0)$. By contrast, when $h(0,\phi)\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K}\le 0$, the *interior steady state allocation is unique and equal to the symmetric allocation* $s_K=\frac{1}{2}$. As for $h(0,\phi)\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K}<0$, it is sufficient to notice that, since the necessary condition for 3 interior steady states does not apply and since there cannot be more than 3 interior steady states, hence there are only one interior steady state, the symmetric allocation $\bar{s}_K=\frac{1}{2}$. As for the knife-edge case $h(0,\phi)\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K}=0$, again the interior steady-state is unique because of the following reasoning. We have three possible cases:

1. $h(0,\phi)=0$ and $\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K}\neq 0$. In this case, since we already know there there is only one $s_K^*\in\left[0,\frac{1}{2}\right)$ such that $h(s_K^*,\phi)=0$, then it should be $s_K^*=0$ which does not satisfy the definition of interior steady state [16].

2. $h(0,\phi)\neq 0$ and $\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K}=0$. Since $\frac{\partial h(s_K,\phi)}{\partial s_K}$ is monotone in $s_K\in\left[0,\frac{1}{2}\right]$ there cannot be other $s_K\in\left(0,\frac{1}{2}\right)$ such that $\frac{\partial h(s_K,\phi)}{\partial s_K}=0$. Hence h cannot cross the horizontal axis in $\left(0,\frac{1}{2}\right)$ and the symmetric equilibrium is unique in $s_K\in[0,1]$.

3. $h(0,\phi)=0$ and $\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K}=0$: this case is ruled out by the strict monotonicity of $\frac{\partial h\left(\frac{1}{2},\phi\right)}{\partial s_K}$.

■

[16] Actually in this case $s_K^*=0$ is a core-periphery equilibrium which also satisfies the interior equilibrium property, i.e., it is such that $f\left(s_K^*\right)=0$. By contrast, the core-periphery outcome need not satisfy this condition.