Optimal Tax and Expenditure Policy in the Presence of Migration - Are Credit Restrictions Important?*

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Abstract

This paper concerns optimal income taxation in the presence of emigration. The basic model is a two-period model where all agents are identical and live in the home country in the first period of life, but where some emigrate at the end of the first period. It is shown that with a binding credit restriction, the government will tax labor income in the first period at a higher rate than otherwise, whereas

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the labor income tax in the second period is unaffected by emigration. With heterogenous agents, the labor income tax in period two will be affected by emigration.

1 Introduction

Emigration is a key issue in many countries. Since the most productive agents are likely to be the ones who leave a country, emigration will erode the tax base and thereby have a detrimental effect on the capacity to provide public goods, or fund publicly provided pensions, in the future. Emigration may also reduce the benefits of public investments in education because parts of the future benefits will leak out of the country. Both these aspects imply that emigration is likely to be an important factor influencing economic policy. Consequently, policy implications of labor mobility have received large attention in the optimal tax literature.[1] One strand of the literature has focused on the role of income redistribution within a fiscal federation.[2] Another has analyzed how governments should tax labor income from mobile agents who divide their time between several jurisdictions.[3]

All above mentioned studies have one thing in common: they analyze economic policy and emigration within a static framework. In a static framework, an agent emigrates if the utility of moving abroad exceeds the utility of staying at home. This implies that if the domestic income increases relative to the income that can be attained abroad, emigration will decrease. This fits

[1] See, for example, Wilson (1982a,b), Wildasin (1991), Wilson (1992), Bjorvatn (1998), Boadway et al (1998), Osmundsen (1999), Aronsson and Blomquist (2003), Sato (2004), and Simula and Trannoy (2006).

[2] See, for example, Wildasin (1991).

[3] See, for example, Osmundsen et al (2000).

the stylized facts concerning emigration from countries with a relatively high per-capita income but it does not fit the stylized facts concerning emigration from poor countries. Rather, for poor countries, the propensity to emigrate seems to *increase* with rising income levels. This empirically observed relationship between emigration and the per capita income has been labelled the *inverted U-curve*.[4] A number of costs, including a purely monetary cost of moving but also various cultural, linguistic and political "costs", have been introduced in order to explain both why people generally are less mobile than the standard theories predict but also why people tend to be less mobile in very poor countries than in slightly richer countries.

We believe that to account for the income-emigration pattern observed in poor countries, it is essential to analyze emigration in an intertemporal framework and recognize that agents are likely to face credit restrictions, i.e. they may not be able to finance the move abroad by borrowing on future income. Rather, they most likely have to finance the move by first working in the home country to save for the "ticket". This means that agents who choose to emigrate will give up consumption today in order to have a higher consumption tomorrow. Consequently, their first period consumption will be lower than for the agents who choose not to emigrate (if both groups have the same income). This implies that an increase in private income in the period when potential emigrants still work in the home country will improve their situation, in utility terms, relatively more than for the agents who choose not to emigrate. This will have a positive effect on emigration which may account for the positive relationship between per-capita income and emigration which we observe in poor countries.

The argument above leads to the following question: if the emigration-

[4]See Fischer et al (1997).

income pattern differs between countries, for the reasons laid out above, what will be the consequences for economic policy? In this paper we analyze how emigration influences the optimal tax and expenditure policy in a small open economy. We use a stylized two-period model where all agents live and work in the home country in the first period of life but in the second period, some agents may emigrate. Emigration gives rise to a negative tax base externality in the second period and the government's objective is to choose optimal linear labor and capital income tax rates to finance the provision of a public good in both time periods. We characterize the optimal tax and expenditure policy and compare it to what it would have looked like if emigration would have been zero. In the final part of the paper, the model is extended to allow agents to differ in terms of labor productivity, and where only high-skilled agents emigrate.

This paper contributes to the literature in primarily three ways. First, we analyze the interaction betwen emigration and economic policy in an intertemporal framework. This makes it possible to see how migration influences both ex ante and ex post tax rates, as well as public expenditure. Second, by introducing a binding credit restriction, we also introduce a feature which has been omitted in the earlier literature on economic policy and emigration, but which may be a potentially very important characteristic influencing many emigration decisions. Finally, we show that the extent to which the optimal tax and expenditure policy is influenced by migration depends on whether agents are homogenous or heterogenous.

The outline of the paper is as follows. In Section 2, we present the basic model with homogenous agents while Section 3 addresses the optimal tax and expenditure policy in the basic model. In Section 4, we extend the model to allow for heterogeneity in labor productivity between agents. Section 5 concludes the paper.

2 The Basic Model

We will use a two-period model to analyze tax policy and emigration in a small open economy, henceforth referred to as the ´home country´. The home country is made up of three types of decision making units; private agents, firms and a government. We start by characterizing the private agents.

2.1 The Private Agents

Private agents live for two time periods and they work and supply labor in both time periods. There is no population growth and at the start of period one, the economy is made up of N_1 (exogenously given) agents. Each agent has the option to emigrate. If it chooses to do so, it leaves the home country at the end of period one and lives abroad in the second time period.

All agents have identical preferences and labor productivity, and the instantaneous utility in period t, $t = 1, 2$, is written

$$\phi\left(c_t, l_t, G_t\right) = u\left(c_t - e\left(l_t\right)\right) + \eta\left(G_t\right) \tag{1}$$

where c_t is private consumption, l_t the hours of work, G_t a public good, and where $e\left(l_t\right)$ can be interpreted as the monetary value attached to the disutility of work.[5] The utility functions satisfy the standard conditions $u', \eta' > 0$ and $u'', \eta'' < 0$. As for the function $e\left(l_t\right)$, we assume that it is constant elastic and satisfies $e', e'' > 0$. For notational convenience, we also

[5] Another example of a study which has used a similar concept is Aronsson and Wehke (2006). The functional form $u\left(c_t - e\left(l_t\right)\right)$ is also found in e.g. the labor market literature regarding asymmetric information (see Blanchard and Fischer (1989)).

define $x_t = c_t - e(l_t)$. An agent's intertemporal utility is represented by the following time separable utility function

$$U = \sum_{t=1}^{2} \left[\beta^{t-1} u(x_t) + \beta^{t-1} \eta(G_t)\right] \tag{2}$$

where β is a constant discount factor.

Turning to the intertemporal budget constraint, it will differ between nonemigrating and migrating agents. Let us, therefore, begin by characterizing the behavior of a nonemigrating agent.

Nonemigrating Agents

The maximization problem of a nonemigrating agent is written

$$\max \sum_{t=1}^{2} \left[\beta^{t-1} u(x_t) + \beta^{t-1} \eta(G_t)\right] \tag{3}$$

subject to

$$c_1 = (1 - \tau_1) w_1 l_1 - s_1 \tag{4}$$

$$c_2 = (1 + \bar{r}_2) s_1 + (1 - \tau_2) w_2 l_2 \tag{5}$$

$$x_t = c_t - e(l_t) \tag{6}$$

where w_t and τ_t are, respectively, the gross wage and the labor income tax rate in period t, s_1 is the saving made in period one and $\bar{r}_2$ is the interest rate. For analytical convenience, we assume that $\bar{r}_2$ is an exogenously given world market interest rate.

Substituting equations (4) and (6) into equation (3) and maximizing w.r.t. l_t produces the following first order condition

$$(1 - \tau_t) w_t - e'(l_t) = 0 \quad \forall t = 1, 2 \tag{7}$$

Defining $\omega_t = (1 - \tau_t) w_t$ to be the net wage, equation (7) implicitly defines a labor supply function, $l_t = l(\omega_t)$, which is increasing in ω_t. The assumption that $e'(l_t)$ is constant elastic implies that the labor supply function $l(\omega_t)$ is also constant elastic.

The first-order condition for the saving can be written as

$$\frac{u'(x_1)}{\beta u'(x_2)} = 1 + \bar{r}_2 \tag{8}$$

This is a standard condition for the optimal choice of intertemporal consumption and combined with equation (7), equation (8) implicitly defines savings as a function $s_1 = s(\omega)$, where $\omega = (\omega_1, \omega_2)$. We will also consider the special case when savings are zero. This is motivated by the fact that in many developing countries, poor agents do not have enough means to save for the future. Rather they would like to borrow money but usually, the credit markets are simply not available for these types of agents. In this situation equation (8) is redundant and we will refer to the situation of a nonbinding credit restriction as Case 1, whereas the situation with a binding credit restriction will be referred to as Case 2.

Finally, observe that the indirect utility function associated with optimal behavior, both in the case of a binding and a nonbinding credit restriction, can be written[6] as $V(\omega, G)$, where $G = (G_1, G_2)$.

Emigrating Agents

We now turn to the emigrating agents. It is assumed that all agents live and work in the home country in the first period of life. At the end of period one, the agents who want to emigrate move abroad, which means that it is only

[6]However, for given levels of ω and G, the utility levels will of course differ between the two cases (unless the optimal savings decision features $s_1 = 0$).

in the second period of life that an emigrating agent actually lives abroad. Let p denote the emigration cost facing an agent and assume that this cost must be paid at the end of period one. If we let the superindex "$\circ$" denote variables associated with an emigrating agent, the maximization problem for an emigrating agent can be written as

$$\max \sum_{t=1}^{2} \left[\beta^{t-1} u\left(x_t^{\circ}\right) + \beta^{t-1} \eta\left(G_t\right)\right] \tag{9}$$

subject to

$$c_1^{\circ} = \left(1 - \tau_1\right) w_1 l_1^{\circ} - s_1^{\circ} - p \tag{10}$$

$$c_2^{\circ} = \left(1 + \bar{r}_2\right) s_1^{\circ} + \left(1 - \bar{\tau}_2\right) \bar{w}_2 l_2^{\circ} \tag{11}$$

$$x_t^{\circ} = c_t^{\circ} - e\left(l_t^{\circ}\right) \tag{12}$$

where $\bar{\tau}_2$ is the foreign tax rate and $\bar{w}_2$ the foreign gross wage. Let us define $\bar{\omega}_2$ to be the foreign net wage and $\bar{G}_2$ to be the foreign provision of the public good.

It is straightforward to verify that the first-order condition for labor in period 1 is identical to that of a nonemigrating agent, i.e. $l_1^{\circ} = l_1 = l\left(\omega_1\right)$. The optimal solution implicitly defines the indirect utility function for an emigrating agent as $V^{\circ} = V\left(\omega^{\circ}, G^{\circ}, p\right)$, where $\omega^{\circ} = \left(\omega_1, \bar{\omega}_2\right)$ and $G^{\circ} = \left(G_1, \bar{G}_2\right)$.

2.2 The Emigration Function

The emigration cost, p, is assumed to be distributed among the agents according to a known distribution function $D\left(p\right)$ with support $\left[p_{\min}, p_{\max}\right]$. A agent will emigrate if $V < V^{\circ}$, whereas he/she will not emigrate if $V > V^{\circ}$.

Since the indirect utility function V° is monotonously decreasing in the emigration cost, it follows that if $p_{\min}$ is sufficiently small to guarantee $V(\omega, G) < V(\omega^{\circ}, G^{\circ}, p_{\min})$, and if $p_{\max}$ is sufficiently large to guarantee $V(\omega, G) > V(\omega^{\circ}, G^{\circ}, p_{\max})$, then there must exist a marginal agent with an emigration cost p_m, who is indifferent between emigrating or remaining inside the country. For this marginal agent, the following equality holds

$$V(\omega, G) = V(\omega^{\circ}, G^{\circ}, p_m) \tag{13}$$

Since agents with $p < p_m$ will emigrate, the number of agents who leave the home country at the end of period 1 is given by $M = N_1 D(p_m)$. This equation, in combination with equation (13), implicitly defines an emigration function of the following type

$$M = M\left(\omega, \omega^{\circ}, G_2, \bar{G}_2\right) \tag{14}$$

Observe that this emigration function is independent of first period public expenditure. The reason is that $\eta(G_1)$ appears additively on both sides of equation (13), so that it can be cancelled out. In the Appendix, we show that the emigration function satisfies the following properties[7]

$$\text{sign } \frac{\partial M}{\partial \omega_1} = \text{sign } \left[u'(x_1^m) - u'(x_1)\right] \tag{15}$$

$$\frac{\partial M}{\partial \omega_2} < 0, \qquad \frac{\partial M}{\partial G_2} < 0 \tag{16}$$

where the superindex "m" refers to the marginal agent. The partial derivatives in equations (15) and (16) will play a key role for the results to be derived below, and let us therefore interpret them in some detail.

Equation (15) implies that the sign of $\partial M/\partial \omega_1$ depends on whether x_1^m is larger or smaller than x_1. One can show that $x_1^m > x_1$ can only occur (i) if

[7] We disregard the partial derivatives w.r.t. the exogenous foreign variables ω_2° and G_2°.

the agents can freely borrow money to finance the consumption in period one and (ii) if the net income difference in period two of emigrating or staying at home is large enough to satisfy the following inequality

$$\frac{\bar{\omega}_2 l\left(\bar{\omega}_2\right)-\omega_2 l\left(\omega_2\right)}{\left(1+\bar{r}_2\right)}>p_m \tag{17}$$

Inequality (15) says that if the discounted value of the net income difference in period two is larger than the emigration cost for the marginal agent, then the marginal agent can afford to have a higher consumption than the none-migrating agents. In this situation, an increase in ω_1 will increase the utility for a nonemigrating agent by relatively more than for the marginal agent. This makes it more attractive than before not to emigrate which, in turn, reduces the number of emigrating agents, i.e. then $\partial M/\partial\omega_1 < 0$. This corresponds to the standard view in the migration literature, where a reduction in the income difference between countries reduces emigration from the poor country.

If, on the other hand, (i) the discounted value of the net income difference in period two of emigrating or staying at home is smaller than the cost of migration for the marginal agent[8], i.e. if the inequality in (15) goes in the other direction, or (ii) if the agents face a binding credit restriction, then $x_1 > x_1^m$. In this situation, an increase in ω_1 will increase the utility for the marginal agent by relatively more than for the nonemigrating agent. This will make it more attractive than before to emigrate which increases the number of emigrating agents, i.e. then $\partial M/\partial\omega_1 > 0$. This is in line with the observation made in the introduction, where emigration from poor countries seems to increase with disposable income.

[8]Note, however, that an agent may still want to emigrate in this situation if the provision of the public good abroad in period 2 is sufficiently large compared to the domestic provision of the public good in period 2.

Since these differences in emigration patterns will play a key role for the design of public policy, it is interesting to analyse both the case when $\partial M/\partial\omega_1 < 0$ (i.e. when there is no credit restriction, which corresponds to Case (i) defined above) and when $\partial M/\partial\omega_1 > 0$ (i.e. when the credit restriction is binding, which corresponds to Case (ii) defined above).

2.3 The Firms

The production sector of the economy is made up of competitive firms. They produce a homogenous good which can be traded on the world market. The world market producer price is treated as fixed and is normalized to one, and we also normalize the number of firms to one. In each time period, the firm uses labor and physical capital in the production process, which is described by a constant returns to scale (CRS) production function, $F(K_t, L_t)$, where K_t is capital and $L_t = N_t l_t$. The production function is increasing and concave in both arguments, and capital and labor are complements in production in the sense that the cross derivative in $F(K_t, L_t)$ is positive. Capital is hired on the world capital market while labor is hired on the domestic labor market. The rental cost of capital is given by $R_t = \bar{r}_t + \theta_t$, where $\bar{r}_t$ is the world interest rate and θ_t a domestic tax on capital. The firm´s total cost in period t is given by $R_t K_t + w_t N_t l_t$. Normalizing the production function w.r.t. L_t, we obtain $f(k_t) = F(K_t, L_t)/L_t$, where $k_t = K_t/L_t$ is the capital stock per working hour. We can now write the first-order conditions as

$$\bar{r}_t + \theta_t = \frac{\partial f(k_t)}{\partial k_t} \tag{18}$$

$$w_t = f(k_t) - k_t \frac{\partial f(k_t)}{\partial k_t} \tag{19}$$

which need no further interpretation.

2.4 Equilibrium

Since M agents emigrate at the end of period one, $N_2 = N_1 - M$ agents remain in the home country in period two. Combining $N_2 = N_1 - M$, $l_t = l(\omega_t)$, and first-order conditions (18) and (19), respectively, the equilibrium wage rates and the equilibrium capital stocks in period t can be written as functions of the government´s decision variables

$$w_1 = w(\theta_1), \quad K_1 = K(\theta_1, \tau_1) \tag{20}$$

$$w_2 = w(\theta_2), \quad K_2 = K(\theta_2, \tau_2, M) \tag{21}$$

where we have omitted the notation of the exogenous foreign interest rate and the constant N_1. Note that the equilibrium wage rates are neither influenced by the labor income tax rate, nor the level of migration. This is a consequence of the functional form of the production function: under CRS, equations (18) and (19) uniquely determine w_t as a function of θ_t only.

3 Optimal Policy

Turning to the government, we assume that its objective is to maximize the utility of the nonemigrating agents, subject to a minimum restriction, $\bar{V}^{\circ}$, on the emigrating agents´ utility. However, note that the utility of an emigrating agent is strictly larger than the utility of a nonemigrating agent, i.e. $V < V^{\circ}$ (except for the marginal agent where $V = V_m^{\circ}$). This implies that if the government´s minimum utility restriction for the emigrating agents is not larger than the utility the government wants the nonemigrating agents to achieve, i.e. if $\bar{V}^{\circ} < V$, which we will assume, then the minimum utility restriction will not be binding.

The intertemporal framework is very simple. Since there are only two time periods, the government´s problem is to determine the optimal tax and expenditure policy for the two time periods. It is straightforward to apply this model within a more general overlapping generations framework, but for our purpose it is sufficient to consider only two time periods; before and after emigration takes place, since this captures the essentials of the problem we want to analyze.

There is a potential time inconsistency problem. The reason is that in the second period when emigration has already taken place, there may be an incentive for the government to change the labor income tax rate and the public expenditure announced in the first period. Although this potential problem is recognized, we follow previous studies in optimal taxation, such as Pirttilä and Tuomala (2001) and Aronsson et al (2008), by assuming that the government can credibly commit to the announced tax and expenditure policy.

The government's decision variables are the labor and capital tax rates, τ_t and θ_t, as well as the provision of the public good, G_t, in both periods. The government is also allowed to borrow funds, B_1, on the world market in the first period which must be repaid with interest in the second period. The government´s budget constraint in each time period is written as

$$G_1 = \theta_1 K_1 (\tau_1, \theta_1) + \tau_1 w_1 (\theta_1) l_1 (\omega_1) N_1 + B_1 \tag{22}$$

$$G_2 = \theta_2 K_2 (\tau_2, \theta_2, M) + \tau_2 w_2 (\theta_2) l_2 (\omega_2) (N_1 - M) - (1 + \bar{r}_2) B_1 \tag{23}$$

The government also recognizes that emigration is determined by equation (13) and includes it as an additional restriction in its optimization problem. We can then write the Lagrangian corresponding to the government´s maximization problem as

$$
\begin{aligned}
\mathcal{L} = & V(\omega, G) + \gamma_1 \left[\theta_1 K_1(\tau_1, \theta_1) + \tau_1 w_1(\theta_1) l_1(\omega_1) N_1 + B_1 - G_1\right] \\
& + \gamma_2 \left[\theta_2 K_2(\tau_2, \theta_2, M) + \tau_2 w_2(\theta_2) l_2(\omega_2)(N_1 - M) - (1 + \bar{r}_2) B_1 - G_2\right] \\
& + \kappa \left[M - N_1 D(p_m(\omega_1, \omega_2, G_2))\right] \qquad (24)
\end{aligned}
$$

where γ_1, γ_2 and κ are Lagrange multipliers. The first-order conditions are presented in the Appendix.

3.1 Optimal Policy in the Absence of Emigration

Let us, as a point of reference, briefly characterize the optimal policy in the absence of migration. Define

$$
\alpha_t = \frac{N_t \gamma_t}{\beta^{t-1} u'(x_t)}, \quad MRS_t = \frac{\eta'(G_t)}{u'(x_t)}, \quad \varepsilon = \frac{\partial l_t}{\partial \omega_t} \frac{\omega_t}{l_t} \qquad (25)
$$

where α_t is defined to be the marginal cost of public funds (MCPF) in real terms, MRS_t is the marginal rate of substitution between the public and the private good and ε is the (constant) labor supply elasticity of the net wage. It is now straightforward to show that in the absence of migration, the optimal policy will be characterized by the following equations

$$
\theta_t^* = 0 \qquad (26)
$$

$$
\frac{\tau_t^*}{1 - \tau_t^*} = \left(1 - \frac{1}{\alpha_t}\right) \frac{1}{\varepsilon} \qquad (27)
$$

$$
\frac{1}{\alpha_t} N_t MRS_t = 1 \qquad (28)
$$

where "$*$" indicates an optimal value. One can show that $\alpha_t > 1$ in at least one of the two time periods and we will interpret the policy rules conditional on that $\alpha_t > 1$ in both time periods.

Equations (26) and (27) basically imply that the optimal tax rates are inversely related to the respective factor price elasticities. Since capital is perfectly mobile (infinitely elastic), whereas the labor supply elasticity is finite, the capital tax rate is zero while the labor income tax rate is positive. Equation in (28) is a modified Samuelson rule. Since the marginal rate of transformation, MRT_t, between the private and the public good is one, the modified Samuelson rule in the presence of a distortionary tax on labor implies $N_1 MRS_t > MRT_t$. All these results are standard and well known in the literature.

Finally, observe that because $\theta_t^* = 0$, we can combine equations (27) and (28) to obtain an overall efficiency condition for the optimal tax and expenditure policy

$$\frac{\tau_t^*}{1-\tau_t^*} = \left(1 - \frac{1}{N_t MRS_t}\right)\frac{1}{\varepsilon} \tag{29}$$

This efficiency condition gives the relationship between τ_t^* and G_t^* (where the latter appears in MRS_t) in the second-best optimum. Equation (29) will serve as a point of reference when we evaluate the optimal policies to be derived below.

3.2 Optimal Policy in the Presence of Emigration

Before we characterize the optimal policy in the presence of emigration, let us ask the following question: if the government determines the optimal tax and expenditure policy without recognizing how the policy instruments influence emigration, what will be the welfare effect of an exogenous increase in M? To answer this question, observe that if the government chooses the optimal policy conditional on M, it maximizes the Lagrangian in (24) without recognizing the last constraint in (24), i.e. the restriction $M =$

$N_1 D\left(p_m\left(\omega_1, \omega_2, G_2\right)\right)$ would be redundant. By using the Envelope Theorem, we can derive the following result;

Proposition 1: *If the government treats the level of emigration as exogenous when it determines the optimal policy, the welfare effect of an increase in M is negative and given by*

$$\frac{\partial \mathcal{L}}{\partial M} = -\gamma_2 \tau_2 w_2 l_2 < 0$$

The explanation for this negative welfare effect is that emigration causes a negative fiscal externality because when a private agent chooses to emigrate, he/she does not take into account that this will erode the future tax base for labor in the home country.

Proposition 1 implies that the government has an incentive to reduce the level of emigration. Let us, therefore, turn to the optimal policy when the government treats emigration as endogenous. If we use the short notation

$$\rho_t = \frac{\kappa}{\alpha_t l_t \beta^{t-1} u_t'} > 0$$

where $\kappa = \gamma_2 \tau_2 w_2 l_2$, one can show that the optimal policy in period t, $t = 1, 2$, is characterized by the following equations

$$\theta_t^* = 0 \tag{30}$$

$$\frac{\tau_t^*}{1 - \tau_t^*} = \left(1 - \frac{1}{\alpha_t}\right) \frac{1}{\varepsilon} + \frac{\rho_t}{\varepsilon} \frac{\partial M}{\partial \omega_t} \tag{31}$$

$$\frac{1}{\alpha_1} N_1 MRS_1 = 1 \tag{32}$$

$$\frac{1}{\alpha_2} N_2 MRS_2 = 1 + \frac{\kappa N_2}{\alpha_2 \beta u_2'} \frac{\partial M}{\partial G_2} \tag{33}$$

where the derivations are presented in the Appendix. Compared with the optimal policy in the absence of emigration, given by equations (26) - (28), we see that (i) the capital tax rates are still zero, (ii) the labor income tax formulas contain an additional term which is explicitly related to emigration, and (iii) emigration influences the provision of the public good in the second period.

We begin by interpreting the optimal labor income tax rate in period one. If we make use of that equation (15) defines $\partial M/\partial \omega_1$, the following results immediately follows;

Proposition 2: *Case(i): If agents can freely borrow funds on the capital market, and if $\bar{\omega}_2$ is sufficiently high to imply $x_1^m > x_1$, then the presence of emigration provides the government with an incentive to tax labor in period one at a <u>lower</u> rate than otherwise.*

Case(ii): If agents face a credit restriction so that the saving is zero, then the presence of emigration provides the government with an incentive to tax labor in period one at a <u>higher</u> rate than otherwise.

To explain the first part of Proposition 2, recall from the discussion in Section 2.2 that if $x_1^m > x_1$ so that $u'(x_1^m) < u'(x_1)$, then a reduction of τ_1 will have a larger effect on the utility of a nonemigrating agent than on the utility of an emigrating agent. As a consequence, the previously marginal agent (who before the tax cut was indifferent between emigrating and staying at home) will, after the tax cut, have a higher utility if he/she does not emigrate than if he/she emigrates. This means that the previously marginal agent will now choose to stay at home rather than to emigrate. Hence, in case (i) there will be a positive relationship between M and τ_1. This provides the government with an incentive to set τ_1 at a lower rate than otherwise.

As for the second part of the Proposition, observe that in the presence of a binding credit restriction, the first period consumption levels for a nonemigrating agent and the marginal agent satisfy

$$c_1 = \omega_1 l_1(\omega_1) > c_1^m = \omega_1 l_1(\omega_1) - p \tag{34}$$

Since $x = c - e(l)$, this inequality implies $x_1^m < x_1$ and $u'(x_1^m) > u'(x_1)$. In this case, an **increase** in τ_1, which decreases the first period net wage, will have a negative effect on the utilities of both emigrating and nonemigrating agents. However, because $u'(x_1^m) > u'(x_1)$, the utility loss will be larger for the emigrating agents. For the previously marginal agent, the alternative not to emigrate will now dominate over the alternative to emigrate, which means that the number of agents who choose to emigrate is reduced. This argument implies a **negative** relationship between M and τ_1 in Case (ii), which provides the government with an incentive to set the labor tax at a higher rate than otherwise.

Finally, observe that emigration does not directly influence the provision of the public good in period one (equation (32) does not contain any term directly linked to emigration). The explanation is that since the utility is additively separable in G, the provision of the public good will not influence the utility difference between emigrating or remaining at home. However, the provision of the public good will be **indirectly** influenced by emigration because the tax rate, and hence the tax revenues, will be influenced by emigration. This can be seen more clearly if we combine equations (31) and (32) and derive the overall efficiency condition for the optimal tax and expenditure policy

$$\frac{\tau_1^*}{1-\tau_1^*} = \left(1 - \frac{1}{N_1 MRS_1}\right)\frac{1}{\varepsilon} + \frac{\kappa}{\varepsilon N_1 MRS_1 l_1 u_1'}\frac{\partial M}{\partial \omega_1} \tag{35}$$

Compared with equation (29), equation (35) implies that emigration will influence the relationship between τ_1^* and G_1^*.

Let us now turn to the optimal policy in the second period. Beginning with the tax formula for τ_2^*, note that the last term in equation (31) is proportional to $\partial M/\partial \omega_2$. Since $\partial M/\partial \omega_2 < 0$, it shows that in the presence of emigration, the government has an incentive to tax labor in the second period at a lower rate than in the absence of emigration. The inutition is that, **all else equal**, a lower tax on labor in the second period will improve the utility of a nonemigrating agent by more relative to that of an emigrating agent. This will, in turn, reduce the number of emigrating agents.

As for the provision of the public good in period two, equation (33) shows that since $\partial M/\partial G_2 < 0$, the presence of emigration will, **all else equal**, provide the government with an incentive to overprovide the public good. The intuition is that by providing more of the public good in period two, the relative utility of a nonemigrating agent visavi the utility of an emigrating agent is improved, which has a negative effect on emigration.

Observe that the policy in period two features two conflicting motives. On one hand, emigration produces an incentive to reduce the labor income tax, which reduces the potential to provide the public good, and on the other hand emigration provides an incentive to increase the expenditure on the public good. Which motive will dominate? To answer this question, we combine equations (31) and (33) with the expressions for the comparative static derivatives for $\partial M/\partial \omega_2$ and $\partial M/\partial G_2$, respectively, to derive the overall efficiency condition for the optimal tax and expenditure policy in period two. We can then derive the following result;

Proposition 3: *The presence of emigration will <u>not</u> directly influence the optimal tax and expenditure policy in period two.*

To prove Proposition 3, let us consider the overall efficiency condition for the optimal tax and expenditure policy in period two. One can show that it is given by

$$\frac{\tau_2^*}{1-\tau_2^*} = \left(1 - \frac{1}{N_2 MRS_2}\right)\frac{1}{\varepsilon} \tag{36}$$

By comparing equation (36) with equation (29), we see that the overall efficiency condition is equivalent to that which would follow in the absence of emigration, which implies that the presence of emigration will not influence the policy rule that determines the relationship between the labor tax and the provision of the public good in period two. To see the intuition behind this result, recall that the number of emigrants is implicitly determined by equation (13). One implication of this equation is that the second period utility for nonemigrating agents is negatively related to emigration. Therefore, to minimize emigration, the government should maximize the second period utility of the nonemigrating agents. However, since the government´s objective already features maximizing the utility of the nonemigrating agents, the emigration constraint will not conflict with - or add any new dimension - to the government´s basic objective. In particular, since the basic objective is to choose τ_2 and G_2 in order to maximize the second period utility, regardless of whether the emigration constraint is present or not in the government´s optimization problem, and since there is a unique relationship between τ_2 and G_2 which achieves this (given by the overall efficiency condition in equation (29)), the relationship between τ_2 and G_2 will not be directly influenced by the presence of emigration.

One consequence of the argument above is that the government has an incentive to transfer resources from period one to period two. To see this, note first that in the **absence of emigration**, the optimality condition which determines the amount of government borrowing in the first period, B_1, is given by

$$1+\bar{r}_2=\frac{\gamma_1}{\gamma_2} \tag{37}$$

If we use the definition $MRS_t = \eta'(G_t)/u'(c_t)$ and the government's first-order condition for public good provision in period one and two, respectively, it can be shown that equation (37) implies

$$\frac{MRS_2}{MRS_1}=\frac{u'(x_1)}{(1+\bar{r}_2)\,\beta u'(x_2)} \tag{38}$$

If the credit restriction does not bind, we can also use equation (8) in which case equation (38) reduces to $MRS_2/MRS_1 = 1$. This latter condition shows that (i) in the absence of emigration and (ii) in the absence of a binding credit restriction, the government's net borrowing is such that the marginal rates of substitution between the public and private goods are equalized between the time periods.

However, in the presence of emigration and with a binding credit restriction, equation (38) is modified to read

$$\frac{MRS_2}{MRS_1}=\frac{u'(x_1)}{(1+\bar{r}_2)\,\beta u'(x_2)}+\frac{\kappa}{MRS_1}\frac{u'(x_1)}{\beta u'(x_2)}\frac{\partial M}{\partial G_2} \tag{39}$$

whereas if the credit restriction does not bind, equation (39) reduces to

$$\frac{MRS_2}{MRS_1}=1+\frac{(1+\bar{r}_2)\,\kappa}{MRS_1}\frac{\partial M}{\partial G_2} \tag{40}$$

Since $\partial M/\partial G_2 < 0$, the second term on the right hand side of both equation (39) and equation (40) is negative, which indicates that the presence of emigration provides the government with an incentive to provide relatively more of the public good in period two than in period one. This is achieved by transfering resources from the first period to the second. We can summarize this result in the following Proposition;

Proposition 4: *In the presence of emigration, the government has an incentive to transfer more resources than otherwise to the second period. This means that the net borrowing in period one will be smaller than otherwise.*

To give the intuition for this result, recall that to minimize emigration, the government needs to maximize the utility of the nonemigrating agents. Note, however, that since both emigrating and nonemigrating households live in the home country in the first period, any policy which maximizes the utility of a nonemigrating agent in period one also improves the first period utility of an emigrating agent. This implies that any policy aimed to reduce emigration by improving the first period utility of the nonemigrating agents is partially offset because it simultaneously improves the utility of the emigrating agents. This is not the case in period two because any policy which improves the second period utility of the nonemigrating agents will not spill over to the second period utility of the emigrating households. Hence, to achieve the goal of minimizing emigration by improving the utility of a nonemigrating agent, there is "more bang for the buck" by improving the second period utility rather than by improving the first period utility. On the margin, it will therefore be optimal to transfer government funds from period one to period two which can be used to improve the second period utility of the nonemigrating agent.

4 Heterogenous Agents

Let us now extend the model and assume that the economy is made up of two types of agents, denoted type 1 and type 2, respectively. The agents differ in terms of labor productivity, with type 1 agents being low-skilled and type 2 agents being high-skilled. In line with the bulk of the literature on migration, we assume that the high-skilled agents are mobile across borders whereas the low-skilled are not. This means that equation (13) now defines a marginal agent which is high-skilled and that emigration equation (14) now applies to high-skilled agents.

Since the economy now consists of two agent types, we expand the model to contain two production sectors, denoted 1 and 2. We assume that only high-skilled labor can be used in the production process in sector 2. To keep the model as simple as possible, we also assume that the wage in sector 2 always exceeds the wage in sector 1. This implies that all high-skilled agents will prefer to work in sector 2 whereas all low-skilled agents will work in sector 1. Both sectors produce the same output good and in sector 1, a linear technology is used in the production process, whereas sector 2 uses the production technology described in Section 2.3.

The government maximizes the following Pareto objective function

$$V\left(\omega^1, G\right) + \phi V\left(\omega^2, G\right) \tag{41}$$

where the superindex denotes ability type and ϕ is the relative weight of the type 2 agent in the welfare function. We assume that the government chooses a linear income tax rate in each time period which applies to both ability types. Since capital is perfectly mobile across borders, we know from the analysis above that it will be set to zero and therefore we do not include a capital tax in this part of the analysis. The budget constraints for the two time periods can then be written as

$$G_1 = \tau_1\left[w_1^1 l_1\left(\omega_1^1\right) N_1^1 + w_1^2 l_1\left(\omega_1^2\right) N_1^2\right] + B_1 \tag{42}$$

$$G_2 = \tau_2\left[w_2^1 l_2\left(\omega_2^1\right) N_2^1 + w_2^2 l_2\left(\omega_2^2\right)\left(N_1^2 - M\right)\right] - \left(1 + \bar{r}_2\right) B_1 \tag{43}$$

where $\omega_t^i = \left(1 - \tau_t\right) w_t^i$ for $i = 1, 2$.

The Lagrangian corresponding to this optimization problem, as well as the first-order conditions, are presented in the Appendix and we begin by characterizing the optimal policy in period one. By combining the first-order conditions for τ_1 and G_1 to obtain the overall efficiency condition for the optimal tax and expenditure policy in period one, we can derive the following result;

Proposition 5: *With heterogenous agents, and in the presence of emigration, the optimal tax and expenditure policy in period one is characterized by*

$$\frac{\tau_1^*}{1-\tau_1^*} = \left[1 - \frac{(1+\varphi_1)}{\Lambda_1}\right]\frac{1}{\varepsilon} + \frac{\kappa w_1^2}{\varepsilon \Lambda_1 w_1^1 l_1^1 u_1^{1,\prime}} \frac{\partial M}{\partial \omega_1^2}$$

where

$$\Lambda_1 = \left(\frac{w_1^1 l_1^1 N_1^1 + w_1^2 l_1^2 N_1^2}{w_1^1 l_1^1 N_1^1}\right)\left(N_1^1 MRS_1^1 + \frac{w_1^1 l_1^1 N_1^1}{w_1^2 l_1^2 N_1^2}\varphi_1 N_1^2 MRS_1^2\right) > 0$$

$$\varphi_1 = \frac{\phi u_1^{2,\prime}}{u_1^{1,\prime}} \frac{w_1^2 l_1^2}{w_1^1 l_1^1} > 0$$

To interpret this tax formula, observe that the last term in the tax formula in Proposition 5 reflects emigration, and that the sign of this term depends on the sign of $\partial M/\partial \omega_1^2$. It can be shown that

$$\text{sign}\, \frac{\partial M}{\partial \omega_1^2} = \text{sign}\, \left[u^{2,\prime}\left(x_1^{2,m}\right) - u^{2,\prime}\left(x_1^2\right)\right] \tag{44}$$

If we compare equation (44) with equation (15) in Section 2.2, we see that the former is identical to the latter, except that the marginal utility difference on the right hand side of (44) now refers to type 2 agents. This, in turn, means that we can interpret the sign of $\partial M/\partial \omega_1^2$ along the same lines as we interpreted the corresponding term in equation (15). Furthermore, since $\partial M/\partial \omega_1^2$ enters the tax formula in Proposition 5 in a similar way as the corresponding emigration term enters the tax formula in equation (31) in Section 3.2, we can interpret it in the same way as we interpreted the corresponding term in the tax formula for the labor income tax rate when agents are homogenous, summarized in Proposition 2.

Let us proceed to characterize the optimal policy in the second period. In this case, we can derive the following result;

Proposition 6: *With heterogenous agents, and in the presence of emigration, the optimal tax and expenditure policy in period two is characterized by the following overall efficiency condition*

$$\frac{\tau_2^*}{1-\tau_2^*} = \left[1 - \frac{(1+\varphi_2+\Psi_2)}{\Lambda_2}\right]\frac{1}{\varepsilon}$$

where

$$\Lambda_2 = \left(\frac{w_2^1 l_2^1 N_2^1 + w_2^2 l_2^2 N_2^2}{w_2^1 l_2^1 N_2^1}\right)\left[N_2^1 MRS_2^1 + (\varphi_2+\Psi_2)\left(\frac{w_2^1 l_2^1 N_2^1}{w_2^2 l_2^2 N_2^2}\right) N_2^2 MRS_2^2\right] > 0$$

$$\varphi_2 = \frac{\phi u_2^{2,\prime}}{u_2^{1,\prime}}\frac{w_2^2 l_2^2}{w_2^1 l_2^1} > 0, \quad \Psi_2 = \frac{\kappa N_1 D'}{u_2^{1,\prime} u_2^{\circ,\prime}}\frac{w_2^2 l_2^2}{w_2^1 l_2^1} u_2^{2,\prime} > 0$$

To interpret the tax formula in Proposition 6, observe first that in the absence of emigration, the term Ψ_2 would be zero (because κ would be zero), whereas Ψ_2 will be positive in the presence of emgiration. Hence, when agents are heterogenous, the optimal policy will be influenced by the presence of emigration. This differs from the result in the previous section, where we showed that when agents are homogenous, the economic policy in period two is invariant to emigration.

To explain why the economic policy is not invariant to emigration when agents are heterogenous, recall that with **homogenous** agents, there is no conflict between the government´s objective function (which is to maximize the utility of the nonemigrating agents) and the objective of minimizing emigration (which is achieved by maximizing the second period utility of the nonemigrating agents). On the other hand, when the agents are **heterogenous**, the overall objective to maximize the weighted sum of utilities over both ability types (equation (41)) no longer coincides with the objective function that needs to be maximized in order to minimize emigration. Therefore, the emigration constraint that now appears in the government´s problem effectively serves to attach a higher weight to the utility of the nonemigrating type 2 agent relative the nonemigrating type 1 agent. This additional weight is the term Ψ_2 in the tax formula in Proposition 6 and it appears in two places: in the numerator and the denominator in the quotient inside the square bracket. The appearance of Ψ_2 in the numerator induces the government to set the labor tax at a lower rate than otherwise, whereas the appearance of Ψ_2 in the denominator provides the government with an incentive to tax labor at a higher rate than otherwise in order to provide more of the public good. The net effect is, in general, ambiguous which reflects that when the government maximizes the sum of utilities in equation (41), the optimal policy will be a trade-off between the (marginal) utility of the nonemigrating type 1 agent vis-a-vis the (marginal) utility of the nonemigrating type 2 agent. This trade-off means that from the nonemigrating type 2 agent´s point of view, τ_2^* may either be set "too high" or "too low" in the second-best optimum. If the labor income tax rate is "too high", the nonemigrating type 2 agent´s utility would increase if τ_2^* was reduced, and in this case the weight Ψ_2 in the numerator will dominate over the weight Ψ_2 in the denominator, so that the net effect of the emigration constraint is to reduce τ_2^*. If, on the other hand, the labor income tax rate is "too low", the nonemigrating type 2 agent´s utility would increase if τ_2^* was set at a higher level. In this case the weight Ψ_2 in the denominator will dominate over the weight Ψ_2 in the numerator.

5 Summary and Discussion

This paper incorporates emigration in a dynamic framework into the theory of optimal linear income taxation. We highlight the importance of credit restrictions and focus the analysis around two special cases: when the agents face a binding credit restriction and when there is no restriction to borrow funds. A binding credit restriction influences emigration because agents who want to emigrate need to forego consumption in the first period of life in order to save for the "ticket". If, on the other hand, the credit restriction does not bind, agents can simply finance the emigration by borrowing on future income.

Since the future tax base is eroded if productive agents leave the home country, emigration gives rise to a fiscal externality. As such, it may either contribute to increase or decrease the ex ante labor income tax rate in comparison with the outcome when emigration is absent. When the credit restriction does not bind, the presence of emigration tends to reduce the ex ante labor income tax but if the agents face a binding credit restriction, the presence of emigration induces the government to tax labor at a higher rate than otherwise. Turning to the ex post labor income tax rate, the question whether it is affected by the presence of emigration or not depends on whether the agents are homogenous or heterogenous. If agents are homogenous, the optimal tax rule for the ex post labor income tax rate is unaffected by emigration, whereas if agents are heterogenous, emigration may either increase or decrease the ex post labor income tax rate.

Future research in this area may take several directions, and we shall point out two of them. First, the assumption that agents who emigrate do not transfer resources back to the home country is a simplification; another alternative is that agents transfer resources back to the home country in the second period. Another extension is to also model the country which is the net receiver of immigrants. Then, it would be interesting to analyze, for example, the simultaneous determination of policies in the country from which there is net emigration and in the country which is a net receiver of immigrants within the context of a Nash game.

6 Appendix

The Migration Function

To derive the properties of the migration function, equation (14), observe that the marginal agent is defined by equation (13), where

$$\begin{aligned} V(\omega, \mathbf{G}) = & u\left[\omega_1 l_1(\omega_1) - e(l_1(\omega_1)) - s_1(\omega_1, \omega_2)\right] + \eta(G_1) \\ & + \beta u\left[\omega_2 l_2(\omega_2) + s_1(\omega_1, \omega_2) - e(l_2(\omega_2))\right] + \beta\eta(G_2) \end{aligned} \tag{A.1}$$

$$\begin{aligned} V(\omega^\circ, \mathbf{G}^\circ, p_m) = & u\left[\omega_1 l_1(\omega_1) - e(l_1(\omega_1)) - s^\circ{}_1(\omega_1, \bar{\omega}_2) - p_m\right] + \eta(G_1) \\ & + \beta u\left[\bar{\omega}_2 l_2(\bar{\omega}_2) + s_1(\omega_1, \bar{\omega}_2) - e(l_2(\bar{\omega}_2))\right] + \beta\eta\left(\bar{G}\right) \end{aligned} \tag{A.2}$$

Equation (13) implicitly defines p_m as a function $p_m(\omega, \omega^\circ, G_2)$ and by differentiating equation (13), we obtain the following comparative statics results

$$\frac{\partial p_m}{\partial \omega_1} = -\frac{l_1\left[u'(x_1) - u'(x_1{}^\circ)\right]}{u'(x_1{}^\circ)} \tag{A.3}$$

$$\frac{\partial p_m}{\partial \omega_2} = -\frac{l_2 \beta u'(x_2)}{u'(x_1{}^\circ)} < 0 \tag{A.4}$$

$$\frac{\partial p_m}{\partial G_2} = -\frac{\beta\eta'(G_2)}{u'(x_1{}^\circ)} < 0 \tag{A.5}$$

Since $M = N_1 D(p_m)$, this implies

$$\frac{\partial M}{\partial \omega_1} = -\frac{l_1 \left[u'(x_1) - u'(x_1{}^{\circ})\right]}{u'(x_1{}^{\circ})} N_1 D'(p_m) \tag{A.6}$$

$$\frac{\partial M}{\partial \omega_2} = -\frac{l_2 \beta u'(x_2)}{u'(x_1{}^{\circ})} N_1 D'(p_m) < 0 \tag{A.7}$$

$$\frac{\partial M}{\partial G_2} = -\frac{\beta \eta'(G_2)}{u'(x_1{}^{\circ})} N_1 D'(p_m) < 0 \tag{A.8}$$

The Government's Problem

By differentiating the Lagrangian in equation (24), we obtain the following first-order conditions

$$\frac{\partial\mathcal{L}}{\partial\tau_1} = 0 = u_1' l_1 \frac{d\omega_1}{d\tau_1} - \kappa N_1 D' \frac{\partial p_m}{\partial\omega_1}\frac{d\omega_1}{d\tau_1}$$
$$+ \gamma_1 \left[\theta_1 \frac{\partial K_1}{\partial\tau_1} + w_1 l_1 N_1 + \tau_1 w_1 N_1 \frac{dl_1}{d\omega_1}\frac{d\omega_1}{d\tau_1}\right] \tag{A.9}$$
$$\frac{\partial\mathcal{L}}{\partial\theta_1} = 0 = u_1' l_1 \frac{d\omega_1}{dw_1}\frac{\partial w_1}{\partial\theta_1} - \kappa N_1 D' \frac{\partial p_m}{\partial\omega_1}\frac{d\omega_1}{dw_1}\frac{\partial w_1}{\partial\theta_1}$$
$$+ \gamma_1 \left[K_1 + \theta_1 \frac{\partial K_1}{\partial\theta_1} + \tau_1 l_1 N_1 \frac{\partial w_1}{\partial\theta_1} + \tau_1 w_1 N_1 \frac{dl_1}{d\omega_1}\frac{d\omega_1}{dw_1}\frac{\partial w_1}{\partial\theta_1}\right] \tag{A.10}$$
$$\frac{\partial\mathcal{L}}{\partial G_1} = \eta_1' - \gamma_1 = 0 \tag{A.11}$$
$$\frac{\partial\mathcal{L}}{\partial\tau_2} = 0 = \beta u_2' l_2 \frac{d\omega_2}{d\tau_2} - \kappa N_1 D' \frac{\partial p_m}{\partial\omega_2}\frac{d\omega_2}{d\tau_2}$$
$$+ \gamma_2 \left[\theta_2 \frac{\partial K_2}{\partial\tau_2} + w_2 l_2 N_2 + \tau_2 w_2 N_2 \frac{dl_2}{d\omega_2}\frac{d\omega_2}{d\tau_2}\right] \tag{A.12}$$
$$\frac{\partial\mathcal{L}}{\partial\theta_2} = 0 = \beta u_2' l_2 \frac{d\omega_2}{dw_2}\frac{\partial w_2}{\partial\theta_2} - \kappa N_1 D' \frac{\partial p_m}{\partial\omega_2}\frac{d\omega_2}{dw_2}\frac{\partial w_2}{\partial\theta_2}$$
$$+ \gamma_2 \left[K_2 + \theta_2 \frac{\partial K_2}{\partial\theta_2} + \tau_2 l_2 N_2 \frac{\partial w_2}{\partial\theta_2} + \tau_2 w_2 N_2 \frac{dl_2}{d\omega_2}\frac{d\omega_2}{dw_2}\frac{\partial w_2}{\partial\theta_2}\right] \tag{A.13}$$
$$\frac{\partial\mathcal{L}}{\partial G_2} = \beta\eta_2' - \gamma_2 - \kappa N_1 D' \frac{\partial p_m}{\partial G_2} = 0 \tag{A.14}$$
$$\frac{\partial\mathcal{L}}{\partial B_1} = \gamma_1 - (1 + \bar{r}_2)\,\gamma_2 = 0 \tag{A.15}$$
$$\frac{\partial\mathcal{L}}{\partial M} = \kappa + \gamma_2 \left[\theta_2 \frac{\partial K_2}{\partial M} - \tau_2 w_2 l_2\right] = 0 \tag{A.16}$$

where

$$\frac{d\omega_1}{dw_1} = (1-\tau_1)\,, \quad \frac{d\omega_1}{d\tau_1} = -w_1, \quad \frac{d\omega_1}{d\theta_1} = (1-\tau_1)\,\frac{dw_1}{d\theta_1}$$
$$\frac{d\omega_2}{dw_2} = (1-\tau_2)\,, \quad \frac{d\omega_2}{d\tau_2} = -w_2, \quad \frac{d\omega_2}{d\theta_2} = (1-\tau_2)\,\frac{dw_2}{d\theta_2}$$

To derive the optimal labor and capital tax rates in period 1, we can use these definitions to rewrite the first-order conditions for τ_1 and θ_1 to read

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} \tau_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \tag{A.17}$$

where

$$\begin{aligned}
a_{11} &= -\alpha_1 w_1^2 \frac{dl_1}{d\omega_1}, \quad a_{12} = \frac{\alpha_1}{N_1}\frac{\partial K_1}{\partial \tau_1} \\
a_{21} &= \left[(\alpha_1 - 1)\, l_1 + \alpha_1 \omega_1 \frac{dl_1}{d\omega_1} \right] \frac{\partial w_1}{\partial \theta_1} \\
a_{22} &= \frac{\alpha_1}{N_1}\frac{\partial K_1}{\partial \theta_1} \\
b_1 &= (1 - \alpha_1)\, w_1 l_1 - \frac{\kappa}{u_1'} w_1 \frac{\partial M}{\partial \omega_1} \\
b_2 &= -\left(l_1 \frac{\partial w_1}{\partial \theta_1} + \alpha_1 k_1 l_1 \right) + \frac{\omega_1}{w_1}\frac{\kappa}{u_1'}\frac{\partial M}{\partial \omega_1}\frac{\partial w_1}{\partial \theta_1}
\end{aligned}$$

To derive the tax formulas for τ_1 and θ_1, we use Cramer´s rule to obtain

$$\tau_1 = \frac{|H_\tau|}{|H|}, \quad \theta_1 = \frac{|H_\theta|}{|H|} \tag{A.18}$$

where

$$\begin{aligned}
|H_\tau| &= b_1 a_{22} - b_2 a_{12} \\
|H_\theta| &= a_{11} b_2 - b_1 a_{21} \\
|H| &= a_{11} a_{22} - a_{12} a_{21}
\end{aligned}$$

By using the definitions of the terms in $|H_\tau|$, $|H_\theta|$ and $|H|$, we, after some manipulations obtain the tax formulas in equations (30) and (31). In a similar way, we can derive the tax formulas for τ_2 and θ_2.

To derive equation (33), we multiply first-order condition (A.11) by $N_1/u'(x_1)$, and then use the defintions of MRS_1 and α_1. In a similar way, we can derive equation (**??**) by multiplying first-order condition (A.14) by $N_2/u'(x_2)$, and then use the defintions of MRS_2 and α_2.

Proof of Proposition 5

With heterogenous agents, the government´s Lagrange function is written

$$\begin{aligned}\mathcal{L} &= V\left(\omega^1, \mathbf{G}\right) + \phi V\left(\omega^2, \mathbf{G}\right) \\ &+ \gamma_1\left[\tau_1 w_1^1 l_1^1\left(\omega_1^1\right) N_1 + \tau_1 w_1^2 l_1^2\left(\omega_1^2\right) N_1^2 + B_1 - G_1\right] \\ &+ \gamma_2\left[\tau_2 w_2^1 l_2^1\left(\omega_2^1\right) N_1 + \tau_2 w_2^2 l_2^2\left(\omega_2^2\right)\left(N_1^2 - M\right) - \left(1+\bar{r}_2\right) B_1 - G_2\right] \\ &+ \kappa\left[M - N_1^2 D\left(p_m\left(\omega_1, \omega_2, G_2\right)\right)\right] \end{aligned} \tag{A.19}$$

The first-order conditions become:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \tau_1} &= 0 = u_1^{1,\prime} l_1^1 \frac{d\omega_1^1}{d\tau_1} + \phi u_1^{2,\prime} l_1^2 \frac{d\omega_1^2}{d\tau_1} - \kappa N_1^2 D' \frac{\partial p_m}{\partial \omega_1^2}\frac{d\omega_1^2}{d\tau_1} \\ &+ \gamma_1\left[w_1^1 l_1^1 N_1^1 + w_1^2 l_1^2 N_1^2 + \tau_1 w_1^1 N_1^1 \frac{dl_1^1}{d\omega_1^1}\frac{d\omega_1^1}{d\tau_1} + \tau_1 w_1^2 N_1^2 \frac{dl_1^2}{d\omega_1^2}\frac{d\omega_1^2}{d\tau_1}\right] \end{aligned} \tag{A.20}$$

$$\frac{\partial \mathcal{L}}{\partial G_1} = \eta_1^{1,\prime} + \phi \eta_1^{2,\prime} - \gamma_1 = 0 \tag{A.21}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \tau_2} &= 0 = u_2^{1,\prime} l_2^1 \frac{d\omega_2^1}{d\tau_2} + \phi u_2^{2,\prime} l_2^2 \frac{d\omega_2^2}{d\tau_2} - \kappa N_1^2 D' \frac{\partial p_m}{\partial \omega_2^2}\frac{d\omega_2^2}{d\tau_2} \\ &+ \gamma_2\left[w_2^1 l_2^1 N_2^1 + w_2^2 l_2^2 N_2^2 + \tau_2 w_2^1 N_2^1 \frac{dl_2^1}{d\omega_2^1}\frac{d\omega_2^1}{d\tau_1} + \tau_2 w_2^2 N_2^2 \frac{dl_2^2}{d\omega_2^2}\frac{d\omega_2^2}{d\tau_2}\right] \end{aligned} \tag{A.22}$$

$$\frac{\partial \mathcal{L}}{\partial G_2} = \eta_2^{1,\prime} + \phi \eta_2^{2,\prime} - \gamma_2 - \kappa N_1^2 D' \frac{\partial p_m}{\partial G_2} = 0 \tag{A.23}$$

$$\frac{\partial \mathcal{L}}{\partial B_1} = \gamma_1 - \left(1+\bar{r}_2\right)\gamma_2 = 0 \tag{A.24}$$

$$\frac{\partial \mathcal{L}}{\partial M} = \kappa - \gamma_2 \tau_2 w_2^2 l_2^2 = 0 \tag{A.25}$$

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