What Do Teachers Do? Public Education, Quality-Quantity Tradeoff and Growth

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Abstract

This paper analyses the contribution of teachers in a public education system and its implication for growth. We focus exclusively on teacher-specific factors that determine the overall quality of education under formal schooling process. In this context, two important aspects of the quality of education has been taken into account: (a) teacher quality; (b) teacher-student ratio. We argue that these two factor enter differently in the education technology and therefore have differential impact of the process of human capital formation. In a public education system where teachers' remunerations are paid by the goverment and financed by taxation, for any given amount of government revenue, there exists a trade-off between teacher quality and teacher quantity which impacts on growth. In this context we discuss the optimal aducation policy as well the optimal taxation policy of the government.

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1 Introduction

In this paper we analyse the contribution of teachers in the process of human capital formation and growth. Modern growth theory emphasizes the role of human capital in the process of long run economic growth. Accordingly the focus of development strategy in almost every country has now shifted towards human capital formation - in particular towards education and schooling. While education has always been viewed as a primary mechanism to improve individual welfare, in recent years it has assumed even more importance due to its positive link to the overall growth process. Education, or rather the lack of it, is identified as a prime cause of persistent poverty and underdevelopment. And to the extent that majority of the population in a low income country lack resources to undergo expensive private schooling, free education through a public education system is perceived to be a fundamental instrument to abate poverty, enhance social mobility, and at the same time promote long run economic growth.

The positive relationship between education and growth is well established in the literature - both at the theoretical as well as empirical level. However schooling per se does not necessarily lead to higher growth (Pritchett 2001). Obviously the quality of schooling matters. In fact, there exist significant differences in the quality of schooling in the developed vis-a-vis the developing world, which evidently have impacted upon their respective growth trajectories.

As Hanushek and Woessmann observe: "Most people would, in casual conversation, acknowledge that a year of schooling in a school in a Brazilian Amazon village was not the same as a year of schooling in a school in Belgium. ... The data suggest that the casual conversation may actually tend to understate the magnitude of differences. ..(I)gnoring quality differences significantly distorts the picture about the relationship between education and economic outcome."[1] Yet, there are very few works in growth theory that explicitly take into account the quality factor.

In this paper we explicitly take into account the quality of schooling and analyse how it impacts on growth. To be sure, quality of schooling has many different aspects - some of which are teacher-specific, some of which are related to the school infrastrcture - and each of these plays a distinct role in the learning process of a student. Here we focus on two teacher-specific inputs. First is the teacher-student ratio which signifies how much personal attention a teacher can give to a student. The second input is related to teaching methodology and the ability of the teacher to get across to her students. While this second aspect of schooling

[1] Hanushek and Woessmann (2007), pp.1.

quality is not directly observable, we infer that a better qualified (or better trained) teacher would in general be able to convey the teaching material to the student more lucidly. Thus this second aspect is captured in our model by the average quality of teachers. We provide a theoretical framework relating these two aspects of quality of schooling to growth, when education is publicly provided.

We develop an endogenous growth model where quality of schooling affects the skill level acquired by the future generation. Quality of schooling improves if more teachers are employed and if better quality teachers are employed. But in order to attract better quality teachers from their alternative professions, the government has to pay higher salaries. Thus for any given amount of government revenue, there is a trade-off involved for the government. It has to decide whether to go for quality or quantity. In this context we derive the optimal education policy of the government.

Apart from the teacher quality-teacher quantity trade off, the tax rate itself is a policy varible that the government can use to influence the schooling outcome and the consequent growth process. Any increase in the tax rate eases the revenue constraint of the government and thus improves the overall quality of schooling. But at the same time it lowers the incentives of the students to exert effort (since a part of the product of this effort would be taxed away). In this context we show that the optimal taxation policy of the government may differ depending on whether its objective is to maximize growth or maximize welfare.

Our work is related to the existing theoretical literature on public education and growth, e.g., Glomm and Ravikumar (1992, 1998, 2001); Eckstein and Zilcha (1994); Benabou (1996); Zhang (1996); Blankenau and Simpson (2004); Boldrin (2005); Viaene, J.M and Zilcha, I. (2009). However, most of these models focus on total public expenditure on education, and do not differentiate between quality and quantity of schooling. In contrast, we focus exclusively on the quality of schooling - in particular and the teacher-student ratio and teacher quality and analyse how these factors together impact on the learning process and the consequent human capital formation. In this sense our model complements this existing body of theoretical work.

The quality-quantity consideration has been explicitly taken in account in Tamura (2001) and our framework is closely related to that of Tamura. In fact we extend Tamura's work to an economy with hetergenous agents and in this context discuss the optimal policy choice of recruiting teachers in the public schools. More specifically, we discuss the conditions under which the highest skilled people vis-a-vis the lowest skilled people get recruited as teachers.

At a broader level our work contributes the literature on public spending and growth (e.g., Barro (1990); (Alesina and Rodrik (1994)). Our model can be interpreted as an extension of this framework by introducing a quality-quantity choice in the provision of the public input.

The structure of the paper is as follows. The next section describes the general framework of the model. Section 3 elaborates upon the education policy. Section 4 describes the human capital dynamics and growth. Section 5 decribes the optimal taxation policy of the government.It also compares various optimal tax rates when objective functions differ. Section 6 concludes the paper.

2 The Model

2.1 General Framework

Time is discrete, represented by $t = 0, 1, 2, ...$ At any point of time the economy is populated by two successive overlapping generations of dynasties. Each generation consists of a continuum of population of measure one.

Each agent is born with some innate ability[2] which varies across agents within a cohort. We assume that innate ability within a cohort is uniformly distributed over an interval of unit lenght, given by $[a, a+1]$. Thus agents belonging to the same generation can be indexed by their inherent ability factor, x , such that $x \in [a, a+1]$. Also, innate ability is i.i.d. accross generations which implies that parental ability does not directly impact on childrens' ability.

The life cycle of a representative agent of any cohort is as follows. The agent lives exactly for two periods - defined for convenience as childhood and adulthood, and has exactly one offspring born to her during adulthood. During her childhood the agent consumes nothing and only exerts effort in acquiring education/skill. Upon adulthood, she works and earns a certain wage income (depending on the skill level acquired by her during childhood) of which a part is taxed away to pay for the education of the next generation. She consumes her entire net income in adulthood. The agent dies at the end of this period.

2.2 Preferences

Agents within a generation and across generations have identical preferences. An agent derives positive utility from own consumption, denoted by c, and from the contribution

[2] The term 'ability' does not necessarily mean intelligence or merit. It could represent factors like patience, tenacity, motivation, ambition or any other individual-specific factor which determines educational achievement - over and above effort.

made towards next generation's education, denoted by b.The latter can be thought as a type of educational bequest - although the entire amount is not spent on the education of her own child. The fact the she derives utility from this contribution reflects "joy of giving" or "warm glow" altruism.[3] The agent also derive negative utility from efforts exerted during childhood in aquiring education, denoted by e.

Consider an representative agent who has an innate ability x.Let e denote the effort level spent by the agent and let $y^x(e)$ denotes the her (expected) adulthood income which, among other things, depends on the effort spent in childhood in acquiring education.The lifetime utility of the agent is represented by the following quasi-linear utility function:

$$U \equiv c^{\epsilon} b^{(1-\epsilon)} - e; \quad 0 < \epsilon < 1. \tag{1}$$

where,

$$\begin{aligned} c &= (1-\tau) y^x(e); \quad (2) \\ b &= \tau y^x(e), \quad (3) \end{aligned}$$

Substituting the values of c and b in the utility function, we get the indirect utility as a function of her effort level:

$$\hat{U}^x(e) = (1-\tau)^{\epsilon} \tau^{1-\epsilon} y^x(e) - e. \tag{4}$$

We assume that the agents are endowed with perfect foresight. Thus a forward-looking agent who can correctly anticipate her adulthood income would maximize (4) to decide about her optimal effort choice. We shall come back to the precise value of the optimal effort level later.

2.3 Production

A single final commodity using human capital/skill (H). Technology for final good production is standard AK type:

$$Y_t = w H_t^Y \tag{5}$$

where H^Y denotes the part of the total stock of human capital that is employed in final goods production and w is a positive constant. The final good sector is characterized by competitive firms who earn zero profit and pay a constant wage rate per unit of skill, given by w (which is the marginal as well as average product of human capital in this AK-technology set up).

[3]This type of indirect bequest has been used by Glomm and Ravikumar (1992) and many others in the public education literature.

2.4 Human Capital Formation

Human capital is acquired through compulsory schooling in public schools. We postulate that human capital acqiured by a child through formal schooling depends on two broad sets of factors: (a) the overall quality of schooling, and (b) the absorbtive capacity of the student. The overall quality of schooling is determined by various inputs provided by the teachers. We focus on two specific teacher-related inputs: (i) how much person attention a student gets from the teacher, and (ii) the average quality of teaching. The former is captured by the teacher-student ratio (θ_t), while the latter is proxied by the average skill level of teachers (h_t^{TA}). These two teacher-related inputs determine the overall quality of schooling through the following education technology:

$$Q_t = (\theta_t - \underline{\theta})^{\alpha} \left(h_t^{TA}\right)^{1-\alpha}; 0 < \alpha < 1, \tag{6}$$

where $\underline{\theta}$ denotes the minimum number (measure) of teachers required to make the education technology viable.[4]

For any given quality of schooling, the skill level acquired by a student also depends on her absorptive capacity. Consider an young agent with inherent ability x, who has an absorptive capacity of A_x. Then the skill level acquired by this young agent (to be employed in next period) is given by:

$$h_{t+1}^{x} = A_x Q_t^{\gamma} \; ; 0 < \gamma \leqq 1. \tag{7}$$

The absobtive capacity of a student of course depends on her innate ability. But innate ability can be complemented by hard work. Accordingly, the abstorbtive capacity of a student of ability x, who puts in an effort level e, is given by:

$$A_x = e^{\beta} x^{1-\beta}; 0 < \beta < 1. \tag{8}$$

There are several features of the education and skill formation technology that require further elucidation. First, notice that the education technology specified by equation (6) is quite similar to Tamura (2001), except for our assumption that a critical minimum mass of teachers is needed to make the formal education system productive. If the proportion of teachers in the adult population falls short of this critical minimum value, given by $\underline{\theta}$, then formal education becomes unviable. The parameter $\underline{\theta}$ can be thought of as some kind of a fixed cost associated with the education technology: a part of the labour force has to be

[4] Recall that total student population (consisting of the entire young generation) is of measure 1. Thus θ measures the proportion of adult population that is engaged in teaching profession as well as the corresponding teacher-student ratio.

employed in setting up the basic school infrastructure before other factors begin to contribute positively to the outcome. In this sense the education technology is IRS with respect to θ.[5]

Secondly, the Cobb-Douglas specification of the absorbtive capacity (equation (8)) inherent ability and effort level are complementary to each other[6]: a person who has higher innate ability *ceteris paribus* also has higher incentive to exert effort.

Finally, we should also emphasize here that the our specification of the human capital formation technology is different from the standard format (e.g. Tamura, 2001; Viaene and Zilcha, 2009) in one important aspect: it completely ignores the role of 'home education' in influencing the level of human capital acquired by the next generation. In other words, we do not consider the impact of parental human capital on that of the children. This is **not** to deny the role of parents in the learning process of a child. While the quality of education is certainly influenced by non-school factors such as parental education, home environment etc., in this paper our focus is exclusively on school-specific factors asociated with the formal learning process. Therefore we deliberately shut off other mechanisms of human capital transmission across generations.[7]

3 Education Policy

We focus exclusively on a public education system where compulsory schooling is provided to every child free of cost and the salary bill of the teachers is financed by the government. We assume that the government offers each teacher her opportunity wage, i.e., the wage that she would be able to earn by employing her skills elsewhere. This implies that if the government wants to employ an agent with human capital h, then it has to pay a salary of wh. (We shall assume that whenver the government pays the market-equivalent salary to any person, that

[5] One could likewise assume some minimum qualification for teachers - which would translate into a critical minimum value for h^{TA}. Tamura. for example, assumes that there exists a minimum hiring standard for teachers, which imposes a lower bound on h^{TA} (although he did not provide any economic justification for this lower bound). We ignore the minimum bound on h^{TA} here, because we are talking about a generic education technology where some degree of schooling can be provided by anybody irrespective of her own level of education. Minimum hiring standards for teachers seems more applicable in the context of higher education.

[6] The second order cross partial is positive.

[7] There seems to exist some empirical evidence that supports this assumption. For example, Card and Krueger (1992) write: "Controlling for measures of school quality, however, we find no evidence that returns to education are related to the income or schooling levels of the parents' generation." However, there are other empirical studies which have re-iterated the importance of 'home education' (e.g., Woessmann, 2003).

person is willing to engage in teaching, even though wage-wise she is actually indifferent). And the government pays the salary bill of the teachers by taxing the income of the entire working population (current adults) while maintaining a balanced budget.[8]

The objective of the education policy is to allocate the government tax revenue efficiently so as to maximise the overall quality of schooling, Q_t (as in Tamura (2001)). In doing so, the binding constraint faced by the gorvernment is of course its revenue cosnstraint - as represented by the balanced budget condition. But its choices are also restricted by the distribution of human capital in the economy in the following way. For any given teacher-student ratio (θ), the best quality teachers in the economy consists of θ measure of people who are at the very upper end of the distribution. This generates an upper bound on the possible average quality of teachers. Likewise, the worst quality teachers in the economy consists of θ measure of people who are at the very lower end of the distribution. This generates an lower bound on the possible average quality of teachers. We elaborate below the exact specification of all these constraints.

Let us assume that the government imposes a proportional labour income tax at a time-invariant rate τ which is fixed arbitrarily at this point.[9] This generates a total tax revenue at time t, given by $T_t \equiv \tau \int\limits_a^{a+1} wh_t^x dx = \tau w H_t$, where H_t denotes the aggregate stock of human capital time t (defined as the sum-total of all adult agents' human capital who differ in terms of ability, i.e., $H_t \equiv \int\limits_a^{a+1} h_t^x dx$). On the other hand the total salary bill of the teachers employed by the government is given by wH_t^T,where H_t^T denotes the aggregate human capital of the teachers. Thus at any point of time t, the balanced budget condition of the government

[8]By employing a set of the current adults as teachers, the government shifts a part of the working population from final goods production to teaching. Since salaries of these people have to be paid in terms of the final good, effectively the government has to tax away a part of the final output produed by the rest to pay for the salary bill of the teachers. However, the post-tax wage rates in the teaching and the non-teaching (final goods production) sectors being equal, this is equivalent to taxing the income of the entire working population.

[9]Eventually we shall talk about the optimal choice of τ.The reason for maintaining a time-invariant tax rate is because: (a) we want to focus on a balanced growth path which cannot be attained unless the tax rate is constant; and (b) we want to keep the incentive structure same for different generations, which implies retaining the same policy parameters.

implies the following relationship between H_t and H_t^T:

$$\begin{aligned}\tau w H_t &= w H_t^T \\ &\Rightarrow \; H_t^T = \tau H_t. \end{aligned} \tag{9}$$

Notice that the balanced budget condition itself captures the implied trade-off between teacher quality and teacher quantity. For any given tax rate τ and for any historically given stock of human capital H_t, the RHS of the above equation is fixed. The LHS on the other hand can be mechanically written as $H_t^T = \theta_t h_t^{TA}$. Substituting this in the above balanced-budget condition, one can immediately see the quality-quantity tradeoff involved here:

$$\theta_t h_t^{TA} = \tau H_t. \tag{10}$$

The precise values of θ_t and h_t^{TA} are to be determined by the education policy of the government. However, the RHS of equation (10) being a constant (for a given τ), the equation is represented by a rectangular hyperbola. Thus if the government opts for a higher average quality of teachers then it must compromise in terms of the number of teachers.

The distribution of human capital at any point of time t imposes two additional constraints on the choice of θ_t and h_t^{TA}. First, note that for any given value of θ_t, $H_t^T \leqq \int_{1+a-\theta}^{1+a} h_t^x dx$, where the RHS represents the aggregate human capital of the upper θ-proportion of the population. Noting that $H_t^T = \theta_t h_t^{TA}$, we can write this condition as follows:

$$\begin{aligned} h_t^{TA} &\leq \frac{1}{\theta_t}\int_{1+a-\theta}^{1+a} h_t^x dx \\ &= \frac{1}{\theta_t} H_t \frac{\int_{1+a-\theta}^{1+a} h_t^x dx}{\int_a^{1+a} h_t^x dx} \\ &= \frac{1}{\theta_t} H_t \frac{\int_{1+a-\theta}^{1+a} x dx}{\int_a^{1+a} x dx} \\ &= H_t \frac{(1+a)^2 - (1+a-\theta_t)^2}{(2a+1)\theta_t} \\ &= H_t \frac{2(1+a) - \theta_t}{(2a+1)} \equiv h_t^{TA^{MAX}}(\theta_t). \end{aligned} \tag{11}$$

Similarly, for any given value of θ_t, $H_t^T \geqq \int_{1+a-\theta}^{1+a} h_t^x dx$, where the RHS represents the aggregate human capital of the bottom θ-proportion of the population. Once again, we can write this condition as follows:

$$\begin{aligned} h_t^{TA} &\geq \frac{1}{\theta_t}\int_a^{a+\theta} h_t^x dx \\ &= \frac{1}{\theta_t} H_t \frac{\int_a^{a+\theta} h_t^x dx}{\int_a^{1+a} h_t^x dx} \\ &= H_t \frac{(a+\theta_t)^2 - (a)^2}{(2a+1)\theta_t} \\ &= H_t \frac{2a+\theta_t}{(2a+1)} \equiv h_t^{TA^{MIN}}(\theta_t). \end{aligned} \tag{12}$$

In view of the balanced budget condition and the two feasibility constraints (equations (10), (11) and (12) respectively), the optimal education policy of the government consists of solution to the following constarined optimization exercise:

$$Max_{\{\theta_t, h_t^{TA}\}} Q_t = (\theta_t - \underline{\theta})^{\alpha} \left(h_t^{TA}\right)^{1-\alpha}$$

subject to

$$\begin{aligned} \text{(i) } h_t^{TA} &= \frac{\tau H_t}{\theta_t}; \\ \text{(ii) } h_t^{TA} &\leqq H_t \frac{2(1+a)-\theta_t}{(2a+1)}; \\ \text{(iii) } h_t^{TA} &\geqq H_t \frac{2a+\theta_t}{(2a+1)}. \end{aligned}$$

In solving the above problem, the government takes H_t, the distribution of H_t and τ as given.

It is easy to verify that there exits an interior optima (such that only the balanced budget condition binds) whenever $\frac{2a+\frac{1-\alpha}{1-2\alpha}\underline{\theta}}{(2a+1)}\left(\frac{1-\alpha}{1-2\alpha}\right)\underline{\theta} < \tau < \frac{2(1+a)-\frac{1-\alpha}{1-2\alpha}\underline{\theta}}{(2a+1)}\left(\frac{1-\alpha}{1-2\alpha}\right)\underline{\theta}$ and at this interior optima the optimal teacher-student ratio and the corresponding average quality fo teachers are respectively give by:

$$\begin{aligned} \theta^* &= \frac{1-\alpha}{1-2\alpha}\underline{\theta}; \\ \left(h_t^{TA}\right)^* &= \tau H_t \frac{1-2\alpha}{\underline{\theta}(1-\alpha)}. \end{aligned}$$

Notice that θ^* is independent of τ.

4 Human Capital Dynamics & Growth

Before we get down to the precise dynamic equation for human capital formation and the associated growth path of the economy, recall that the term H_t^T/H_t denotes the proportion of the total human capital stock that is employed in the teaching professon. Equation (9) tells us that this proportion is directly measured by the tax rate τ. Now output in this economy depends linearly on the part of aggregate stock of human capital that is employed in final good production (H_t^Y). Since τ proportion of the total human capital stock H_t is employed in the teaching professon, it follows that $H_t^Y = H_t - H_t^T = (1-\tau) H_t$, i.e., H_t^Y is also proportional to H_t. This tells us that the rate of growth of output in this economy can be measured by the rate of growth of the aggregate stock of human capital. In words,

$$\frac{Y_{t+1}}{Y_t} - 1 \equiv g_t = \frac{H_{t+1}}{H_t} - 1. \tag{13}$$

Now aggregate human capital stock in the next period, $H_{t+1} \equiv \int_a^{a+1} h_{t+1}^x dx$, is determined by the governement's education policy (h_t^{TA} and θ_t) as well as by the effort spent today by a child in acquiring education (e_t). The latter in turn is chosen optimally by forward-looking agents whose indirect utility is given by (**??**). It is straightforward to see that a child with innate ability x, who is endowed with perfect foresight and correctly anticipates a future income of $y^x(e_t) = wh_{t+1}^x$, will choose her optimal effort level such that

$$(1-\tau)^{\epsilon}\tau^{1-\epsilon} w Q_t^{\gamma} \beta \left(e_t\right)^{\beta-1} x^{1-\beta} - 1 = 0 \tag{14}$$

Solving, we can derive the optimal value of effort (e_t) as:

$$\begin{aligned} e_t &= \left[(1-\tau)^{\epsilon}\tau^{1-\epsilon} w\beta\right]^{1/(1-\beta)} \left[Q_t\right]^{\gamma/(1-\beta)} x \\ &= \left[(1-\tau)^{\epsilon}\tau^{1-\epsilon} w\beta\right]^{1/(1-\beta)} \left[(\theta_t - \underline{\theta})^{\alpha} \left(h_t^{TA}\right)^{1-\alpha}\right]^{\gamma/(1-\beta)} x \end{aligned} \tag{15}$$

Putting this optimal value back in equation (7), we get,

$$h_{t+1}^x = [w\beta]^{\beta/(1-\beta)} \left[(1-\tau)^{\epsilon}\tau^{1-\epsilon}\right]^{\beta/(1-\beta)} \left[(\theta_t - \underline{\theta})^{\alpha} \left(h_t^{TA}\right)\right]^{\gamma\beta/(1-\beta)} x. \tag{16}$$

Aggregating over all agents who differ in terms of innate abilities, the next period's human capital is given by:

$$\begin{aligned} H_{t+1} &= [w\beta]^{\beta/(1-\beta)} \left[(1-\tau)^{\epsilon}\tau^{1-\epsilon}\right]^{\beta/(1-\beta)} \left[(\theta_t - \underline{\theta})^{\alpha} \left(h_t^{TA}\right)\right]^{\gamma\beta/(1-\beta)} \int_a^{a+1} x dx \\ &= \frac{1+2a}{2} [w\beta]^{\beta/(1-\beta)} \left[(1-\tau)^{\epsilon}\tau^{1-\epsilon}\right]^{\beta/(1-\beta)} \left[(\theta_t - \underline{\theta})^{\alpha} \left(h_t^{TA}\right)\right]^{\gamma\beta/(1-\beta)}. \end{aligned} \tag{17}$$

Using equation (10) to substitute for h_t^{TA}, we get the following equation determining the dynamics of the aggregate human capital in this economy:

$$H_{t+1} = \tilde{A}\left[(1-\tau)^{\epsilon}\tau^{1-\epsilon}\right]^{\beta/(1-\beta)}(\tau)^{(1-\alpha)\gamma\beta/(1-\beta)}\left[(\theta_t-\underline{\theta})^{\alpha}\left(\frac{1}{\theta_t}\right)^{1-\alpha}\right]^{\gamma\beta/(1-\beta)}(H_t)^{(1-\alpha)\gamma\beta/(1-\beta)}, \tag{18}$$

where $\tilde{A} \equiv \frac{1+2a}{2}[w\beta]^{\beta/(1-\beta)}$,a constant.

The corresponding growth rate is given by:

$$\begin{aligned} g_t &= \frac{H_{t+1}}{H_t} - 1 \\ &= \tilde{A}\left[(1-\tau)^{\epsilon}\tau^{1-\epsilon}\right]^{\beta/(1-\beta)}(\tau)^{(1-\alpha)\gamma\beta/(1-\beta)}\left[(\theta_t-\underline{\theta})^{\alpha}\left(\frac{1}{\theta_t}\right)^{1-\alpha}\right]^{\gamma\beta/(1-\beta)}(H_t)^{(1-\alpha)\gamma\beta/(1-\beta)-1} \end{aligned} \tag{19}$$

Since θ_t is chosen optimally through the education policy of the government, and is constant as long a the tax rate is constant, for any time-invariant tax rate τ, the economy will exhibit a balanced growth path if and only if

$$(1-\alpha)\gamma = \frac{1-\beta}{\beta}. \tag{20}$$

Henceforth we shall assume that this parametric configuartion is satisfied.

5 Taxation Policy: Optimal Tax Rate

In our discussion so far we have assumed that the tax rate τ is fixed arbitrarily. However, the tax rate itself is another policy instrument which the government can use to influence the educational outcome and therefore the growth of the economy and/or the welfare of agents. We consider this case here.

In analysing the optimal choice of tax rate, we consider two alternative objective functions of the government: (a) maximization of growth; (b) maximization of welfare. As we already know from the literature on public expenditure and growth (e.g., Barro (1990); Alesina-Rodrik (1994)), the growth maximising policy need not always be welfare maximising. We examine this issue in the context of a balanced growth path.

5.1 Growth Maximizing Tax Rate

Suppose the objective of the government is to maximise growth rate of output. From equation (19), we already know that at the interior optima and under the parametric specifications given by (20), the balanced growth rate is given by :

$$g_t = \tilde{A}(1-\tau)^{(\epsilon\beta)/(1-\beta)}\tau^{(1-\epsilon\beta)/(1-\beta)}\left[(\theta^* - \underline{\theta})^{\alpha}\left(\frac{1}{\theta^*}\right)^{1-\alpha}\right]^{\gamma\beta/(1-\beta)} - 1$$

The corresponding growth maximizing value of τ is:

$$\tau^* = 1 - \epsilon\beta. \tag{21}$$

5.2 Welfare Maximizing Tax Rate

Suppose now that the objective of the government is to maximise welfare. Notice that in our formulation, the income of the current adults is taxed to pay for the education of the younger generation. Since the adult themselves value how much they contribute towards the education of the younger generation, the current tax rate enters the utility function of an adult directly. However the income of a current adult also depends on the previous period's tax rate of the previous period through the quality of schooling factor. As long as the tax rate τ is time invariant, it therefore has a direct and an indirect effect on the welfare of an agent. Thus the welfare on an agent born in period t with an innate ability x, and becomes adult in period $t+1$, is given by:

$$\begin{aligned} W^x &= (1-\tau_{t+1})^{\epsilon}(\tau_{t+1})^{1-\epsilon} y^x(e_t) - e_t \\ &= (1-\tau_{t+1})^{\epsilon}(\tau_{t+1})^{1-\epsilon} w e_t^{\beta} x^{1-\beta}\left[(\theta_t - \underline{\theta})^{\alpha}(h_t^{TA})^{1-\alpha}\right]^{\gamma} - e_t \\ &= (1-\tau_{t+1})^{\epsilon}(\tau_{t+1})^{1-\epsilon} w e_t^{\beta} x^{1-\beta}\left[(\theta_t - \underline{\theta})^{\alpha}\left(\frac{\tau_t H_t}{\theta_t}\right)^{1-\alpha}\right]^{\gamma} - e_t \end{aligned} \tag{22}$$

Substituting for optimal effort level e_t (from equation (15)), and using the assumption of time-invariant tax rate such that $\tau_{t+1} = \tau_t = \tau$, we can write the welfare of the agent as a function of τ in the following way:

$$\begin{aligned} W_t^x &= e_t\left[(1-\tau)^{\epsilon}(\tau)^{1-\epsilon} w e_t^{\beta-1} x^{1-\beta}\left[(\theta_t - \underline{\theta})^{\alpha}\left(\frac{\tau H_t}{\theta_t}\right)^{1-\alpha}\right]^{\gamma} - 1\right] \\ &= e_t\left(\frac{1}{\beta} - 1\right) \\ &= \left(\frac{1-\beta}{\beta}\right)\left[(1-\tau)^{\epsilon}\tau^{1-\epsilon} w\beta\right]^{1/(1-\beta)}\left[(\theta_t - \underline{\theta})^{\alpha}\left(\frac{\tau H_t}{\theta_t}\right)^{1-\alpha}\right]^{\gamma/(1-\beta)} x. \end{aligned} \tag{23}$$

Under the balanced growoth condition and for the interrior solution for θ^*, the welafre of an agent can be written as:

$$\begin{aligned} W_t^x &= \left(\frac{1-\beta}{\beta}\right)[w\beta]^{1/(1-\beta)}\left[(1-\tau)^{\epsilon}\tau^{1-\epsilon}\right]^{1/(1-\beta)}\tau\left[(\theta^*-\underline{\theta})^{\alpha}\left(\frac{1}{\theta^*}\right)^{1-\alpha}\right]^{\gamma/(1-\beta)} H_t x \\ &= \hat{A}(1-\tau)^{\epsilon/(1-\beta)}(\tau)^{(1-\epsilon)/(1-\beta)}\left[(\theta^*-\underline{\theta})^{\alpha}\left(\frac{1}{\theta^*}\right)^{1-\alpha}\right]^{\gamma/(1-\beta)} H_t x, \end{aligned}$$

where $\hat{A} = \left(\frac{1-\beta}{\beta}\right)[w\beta]^{1/(1-\beta)}$, a constant.

For any given value of H_t, the corresponding welfare-maximizing τ is:

$$\tau^{**} = 1-\epsilon. \tag{24}$$

There are two important implication of this result. First, note that the welfare-maximizing tax rate is independent of x. That is all egnts, irrespective of their innate ability would attain maximum welfare at τ^{**}. Secondly, it is obvious that $\tau^* > \tau^{**}$, that is, the growth maximizing tax rate is higher than the welfare maximizing tax rate. Therefore were tax rate to be chosen by majority voting so that each adult chooses votes for her most-preferred tax rate and the one selected by majority is implemented, the corresponding growth rate would be lower than the best that the economy could attain.

6 Conclusion

In this paper we have analyzed the impact of overall quality of education on growth in a framework where better quality of schooling results from the presence of better quality teachers as well as presence of higher number of teachers (reletive to the class size). We show that overall quality of schooling is an important factor contributing to growth. However in an economy where education is publicly provided and is financed by taxation, the requirement of maintaining a balanced budget on the part of the government imposes a trade off in terms of quality and quantity of teachers. For any given amount of tax revenue, in order to attract better quality individuals to teaching profession requires paying a higher salary bill, which in term implies that the less number of teacher can be employed. Since the quality and quantity of teachers enter differently in the education technology, the government has to optimally allocate of its limited resources between teacher quality and teacher quantity in order to ensure efficiency.

Apart from the implied trade off between teacher quality and teacher quantity (which is to be resolved by an efficient education policy), there is another trade off implicit here. This second trade off arises in terms of the taxation policy of the government. Higher taxation eases the balancec budget constraint of the government, which enable it to improve the overall quality of schooling. This has a positive impact on the human capital formation process and therefore on growth. On the other hand, there is a negative impact of higher taxation on the incentive to invest effort in skill formation, which undermines the impetus to growth. This trade off between quality of schooling and effort exerted by the students generates a hump-shaped relationship between tax rate and growth. Since effort has a negative utility cost, this also explains why the growth maximizing tax rate cannot be welfare-maximizing. The welfare maximising tax rate is always lower that the growth maximizing one.

In the context of the trade off between schooling quality and students' effort, it is important to note that in our model we have assumed that individual ability is observable and is known to an agent when she decides how much effort to invest in acquiring human capital. This assumption however is not crucial for our results. One can easily show that the same conclusion will prevail even when agents do not know their exact ability but optimize on the basis of the expected value of ability. In this latter case, the optimal effort chosen will be the same across all agents; however the actual level of human capital will differ across agents depending on their actual levels of ability. Since the nature of the quality-effort trade off in individual skill formation is the same irrespective of the assumed level of ability, the basic results of the model will remain unchanged.

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