Maximizing Human Development*

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Abstract

The Human Development Index (HDI) is a widely-used measure of a country's overall human development. We examine the allocations implied by the maximization of this index, using a standard growth model – an extended version of Mankiw, Romer, and Weil's (1992) model – and compare these with the allocations implied by the golden rule in that model. We find that maximization of the HDI leads to the over accumulation of both physical and human capital, relative to the golden rule, and consumption is pushed to minimal levels. We then propose an alternative specification of the HDI, which replaces its income component with a consumption component. Maximization of this alternative HDI yields a "human development golden rule" which balances consumption, education and health expenditures. We advocate the method of optimization subject to constraints for revealing the consequences of taking a policy measure seriously.

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"The Human Development Index is a very crude measure, but it is a better crude measure than Gross National Product or Gross Domestic Product."

Amartya Sen[1], 1998 Nobel Laureate in Economics

"The Human Development Index really helped to generate political competition. And, just as competition is very good in markets to make them efficient, political competition is also very good."

Inge Kaul[1], Director, Human Development Reports 1990-1994

1 Introduction

The progress of nations, and their relative standing, has most often been assessed using per capita GDP as a crude measure of wealth. This "income approach" has been criticized as emphasizing means over ends and for being too narrow. Building on the work of Amartya Sen and co-authors, a number of academics and policy analysts have championed the "human development approach".[2] This alternative approach understands development as the expansion of peoples' capabilities "to live better and richer lives, through more freedom and opportunity" (Anand and Sen (2000b) p84). The progress of the growth of such capabilities has been measured by outcomes ("functionings") as documented in the Human Development Reports. These Reports regularly publish country rankings for five indexes designed to evaluate aggregate outcomes. Foremost amongst these indexes is the Human Development Index (HDI), which evaluates overall human development.[3]

[1]Quoted from United Nations Development Programme (2005) video: "People First: The Human Development Reports".

[2]Both approaches have roots in longstanding traditions. Anand and Sen (2000a) trace the human development approach to the philosophies of Aristotle and Kant and describe how it "relates to the more conventional analyses to be found in the standard economics literature – from Adam Smith onwards". They relate the income/wealth approach to the 'old opulence-oriented approach'. See Pritchett and Summers (1996) for recent evidence supporting this approach and Anand and Ravallion (1993) and Stiglitz, Sen and Fitoussi (2009) for evidence in favor of the human development approach.

[3]Other indexes include the Human Poverty Index for developing countries (HPI-1), Human Poverty Index for selected OECD countries (HPI-2), Gender Development Index (GDI), and the Gender Empowerment Measure (GEM). See Technical Note 1 in Human Development Report 2009.

The HDI annual ranking of nations is widely cited and is claimed to have the effect of generating competition between nations (see the above quote, for example). The HDI, arguably, has come to rival per capita GDP as the leading measure for evaluating human well-being. It attempts to capture overall well-being in terms of three primary dimensions: a long and healthy life, knowledge, and a decent standard of living. In formalizing the index, these three dimensions are represented, respectively, by life expectancy, education, and per capita GDP. Thus, the HDI is a more general measure of well-being than per capita income alone. Crucially for the human development approach, health and knowledge are viewed as valued ends in themselves rather than simply means to increase income.

This paper takes the HDI seriously as a basis for planning. We ask the following questions. Is maximization of the HDI a sensible basis for improving human development? How do development plans change when we alter the human development criteria, and how do they compare with income and traditional growth approaches? Consistent with the lead quote from Amartya Sen, we find that the HDI is, indeed, a crude measure on which to base planning – though a better crude measure than income. Maximization of the HDI implies both physical and human capital overaccumulation compared to the standard golden rule and also leads to minimal consumption. We therefore propose a simple change to the HDI which, we argue, sensibly balances the human development ends of consumption, education and health.

Our method is to analyze the long-run implications of using a generalized HDI as the key criterion for development planning in macroeconomies, using the conceptual lens of conventional economic growth theory. To do so, we construct a simple growth model which includes health, education, and income, as endogenous variables. This model is based on the extended Solow model in Mankiw, Romer, and Weil (1992), but further extends the model to include health. We identify the allocations that maximize the generalized HDI in this model, and compare them with those that satisfy the more traditional growth criterion of dynamic efficiency.

Some results are immediate and startling. Maximization of the generalized HDI, if taken literally, leads to the result that consumption should be set to zero – or, more generally, to its minimal sustainable level. This result is obtained because consumption does not enter the objective function and simply represents a cost to the planning program. To avoid this corner result, we modify the model to give consumption an instrumental role in increasing production. Specifically, we consider a model where output is increasing in consumption levels, up to a critical threshold consumption level.[4] Even with this modification, the HDI-maximizing plan exhibits minimal consumption. Intuitively, the resources freed from consumption are allocated among the three types of capital: physical, educational, and health capital. Both educational and health capital are valued ends (i.e., play a direct role in the objective function) and physical capital is valuable because it increases income – another valued end. Relative to the traditional welfare criterion in the Solow model – the golden rule, which maximizes steady state consumption[5] – maximization of the HDI generically implies both physical and human capital overaccumulation.

Of course, the traditional "income approach", where per capita income is the sole criterion for planning, leads to a similar outcome: it implies over-accumulation of capital in general. This approach has come to be known as "capital fundamentalism" in some circles.[6] In the human development approach, the positive weight on education and health leads to accentuation these particular types of capital. As such, both the income and human development approach might be characterized as capital fundamentalist. The fact that the human development approach gives priority to human capital and health capital while still adhering to physical capital fundamentalism, suggests that it may be providing a new impetus towards capital fundamentalism in development planning.

The usage of income in the human development approach is explained by Anand and Sen (p86, 2000b):

> "The use of 'command over resources' in the HDI is strictly as a residual catch-all, to reflect something of the basic capabilities not already

[4] The essential rationale for a production function of this basic form goes back as far as Leibenstein (1957): "The amount of work that the representative laborer can be expected to perform depends on his energy level, his health, his vitality, etc., which in turn depend on his consumption level (which depends on income level) and, most directly, on the nutritive value of his food intake."

[5] Phelps (1966, p.5) called this the golden rule as "... each generation saves (for future generations) that fraction of income which it would have had past generations save for it." Phelps' analysis is in the context of the Solow model in which generations are not explicitly identified.

[6] King and Levine (1994) describe the traditional capital fundamentalism arising out of the income approach as well as the new capital fundamentalism implicit in endogenous growth theory. Mankiw, Romer and Weil (1992)) ascribe much of the differences in income between nations to human capital.

incorporated in the measures of longevity and education."

As such ,we propose a friendly amendment to the HDI, modifying it by replacing income with consumption, defined as income net of education, health and capital expenditures. The optimal planning conditions, the "human development golden rule" (HDGR), that correspond to this criterion are identified and characterized in this paper. The HDGR still exhibits human capital overaccumulation relative to the traditional consumption-based golden rule. The marginal condition for physical capital in the HDGR is the same as the standard golden rule; however, physical capital is overaccumulated when it is a complement with human capital. It should be stressed that the capital accumulation is relative to the traditional golden rule benchmark. The emphasis on capital in the HDGR is not inefficient on its own terms because each capital is efficiently traded-off relative to a valued end, consumption.

An important feature of the actual HDI is that it is specified with upper and lower bound values for the arguments. For completeness, we look at cases when the steady state involves some arguments achieving their upper bounds. We also extend the model to include exogenous technological change, and derive the corresponding golden rule in this case. The human development golden rule conditions are the same except for the inclusion of a parameter for the rate of technological change and variables being expressed in intensive units. With technological change, all the arguments in the objective will eventually hit their upper bounds unless those bounds trend up at the same rate as technological change. Thus, with technological change, human development is either eventually assured given fixed boundaries or, when the boundaries increase, the HDI can be viewed as a way of ranking nations in a relative way.

The remainder of the paper is structured as follows. Section 2 provides an overview and definition of the HDI and introduces a general human development objective function, which nests the income approach and the human development approach as special cases. In Section 3 we present the extended Solow model, analyze its properties, and derive the traditional (consumption-maximizing) golden rule for the model. Section 4 develops the HDI-maximizing rule for the basic model as well as for extensions of the model to include boundaries conditions and exogenous economic growth. In Section 5 we show how the human development golden rule changes when consumption replaces income in the objective function. Finally, in Section 6 we discuss future directions for research and argue that measures that used to inform policy should be evaluated by the policies they imply.

2 The HDI Objective

The HDI was first reported in the Human Development Report 1990. Over the years the index has changed considerably. Currently it has the following specific form:

$$HDI(l,\varepsilon,y)=\frac{1}{3}\left(\frac{l-25}{85-25}\right)+\frac{1}{3}\left(\frac{\varepsilon}{100}\right)+\frac{1}{3}\left(\frac{\ln y-\ln 100}{\ln 40000-\ln 100}\right)$$

where l is life expectancy, ε =(2/3)(Adult literacy index)+ (1/3)(gross enrollment index) is an index of education, and y is GDP per capita (PPP US$). The HDI is linear in education and life expectancy and logarithmic in income. In addition there are bounds on each of the variables. Life expectancy is bounded between 25 and 85 years, and income is bounded between $100 and $40,000. Both of the sub-indexes that make up the education index are bounded between 0 and 100. Currently, there are no countries below the lower bounds and no countries at the upper bound for life expectancy. There are several countries where income exceeds $40,000 and also several countries where education is assessed at 100.[7]

A major advantage of the HDI is that it is straightforward, and built from data that is widely available. However, the form of the index is related to a more general philosophy and methodology. The specific arguments of the HDI, are called "indicators". They are intended to simply represent the three most important dimensions for well being: the dimension "long and healthy life" is represented by the indicator of life expectancy l, the dimension of "knowledge" is represented by the indicator of education ε, and the dimension of decent standard of living is represented by per capita GDP y. As mentioned above, and explored

[7]For an explanation of the index see Technical Note 1 in the Human Development Report 2009. When the index was first formed the lower bounds and upper bounds from the yearly data were used. Thus, the index was used to simply rank countries yearly. The index was changed to have fixed bounds in order to trace the improvement in national achievements over time. Still the fixed bounds have been adjusted periodically to reflect new realities. For example, the lower bound on life expectancy was lowered to 25 years from 35 years following the reduction in life expectancy from AIDs in some sub-Saharan countries.

further below, the underlying philosophy behind the choice of these dimensions is arguably not particularly well served by the choice of indicator. However the consistent message is that these dimensions and their indicators are intended to represent *valued ends*. It is also recognized that these same primary ends of human development are also primary means, but this is not what the index is meant to capture.

There have been a number of critiques of the HDI.[8] The criticisms have been specific to the form of the indicators as well as the form of the overall function. Criticism has been encouraged and has lead to revisions of the index. The absence of the consideration of gender life expectancy differences lead to the Gender Development Index. In response to criticism of the education indictor, a "gross enrolment ratio", has been incorporated into the education index. The per capita GDP index was changed to include the log of income, which reduces the score of higher income countries. Anand and Sen (2000b) criticize the per capita GDP index for not being a direct measure of capabilities and also for not considering intra-country inequality. A further limitation of the HDI relevant to this paper is that it incorporates no intertemporal trade-offs. In this sense it is a static concept.

We represent a general, twice differentiable human development objective function as follows:

$$D(h(t), e(t), y(t)) \tag{1}$$

where $h(t)$ and $e(t)$ are the current per capita stocks of health and education human capital at time t. We denote the upper bounds and lower bounds of the arguments $(h(t), e(t), y(t))$ as $(\underline{h}, \underline{e}, \underline{y})$ and $(\overline{h}, \overline{e}, \overline{y})$ respectively. Using subscripts to denote partial derivatives, we assume that $D_j > 0$ and $D_{jj} < 0$, for $j(t) \in [\underline{j}, \overline{j})$, $D_j = 0$ for $j(t) < \underline{j}$ and $j(t) \geq \overline{j}$, and $D_{ji} = 0$ for $j \neq i$ where $j, i = h, e, y$. For simplicity this index is assumed to be strictly concave and separable in its arguments. The lower and upper bounds on each argument are permitted to change over time.

Stocks of health and education human capital replace the specific indicators of the HDI in our general function for three reasons. First, the implications of the specific indicators themselves have already been examined, in detail, in Engineer, King and Roy (2008) – in a static analysis. Second, our dynamic model has

[8] The criticisms and responses are reviewed by Raworth and Stewart (2005). Also, see Hicks (1997), Noorbakhsh, (1998), Mazumdar (2003), Cahill, (2005), Osberg and Sharpe (2005).

stocks of human capital, so the same variables can represent both means and ends. Third, these stocks are a more general indicators. For example, Engineer, Roy and Fink (2010) criticize the life expectancy indicator for not capturing the health part of the dimension "long and healthy" life. This suggests the inclusion of a morbidity component in that indicator. Here, health human capital is a concept that generally captures the dimension. Whereas the specific indicator life expectancy is linear in the HDI, we make health capital strictly concave. Though this is more convenient for our calculations it is also realistic. Kakwani (1993) points out that it is increasingly expensive to increase life expectancy and it may be prohibitively expensive to achieve the upper bound of 85 years.

One odd feature that both the HDI and our more general function share is that they both mix stocks and flows. Income is a flow variable whereas the variables representing education and health are stock variables. However, we can express our function entirely in terms of stock variables by substituting the intensive production function for income. As is standard, the intensive production function can be expressed in per capita levels of stocks of inputs. When health and education capital are a means to enhance production they enter the production function along with physical capital, $y(t) = f(k(t), h(t), e(t))$. Substituting this production function yields an indirect function defined solely in stock variables:

$$D(h(t), e(t), f(k(t), h(t), e(t))) = \Gamma(h(t), e(t), k(t))$$

The function Γ is strictly concave in all its arguments when production is strictly concave. The partial derivatives are related: $\Gamma_h = D_h + D_y f_h > 0$, $\Gamma_e = D_e + D_y f_e > 0$ and $\Gamma_k = D_y f_k > 0$. Observe that physical capital enters the objective only indirectly through income, whereas both health and education capital also enter the objective directly. In this sense, human capital is given priority over physical capital in the reduced form human development objective function.

2.1 An Example

Consider an explicit version of the general function:

$$D(h(t), e(t), y(t); w, W) = wlny(t) + (1 - w)(Wlne(t) + (1 - W)lnh(t))$$

where the weight w is the relative weight on income and $(1-w)$ the weight on the health and education component. Within the health and education component, W is the relative weight on education. Setting $w = 1$ implies that income is the only argument and, as such, in this case, the objective and can be thought of as the special case of the "income approach". At another extreme, if $w = 0$ then income in excluded from the index, as some advocates of the human development approach have essentially argued (as discussed in Engineer, King, and Roy (2008)).

Suppose production is Cobb-Douglas: $y(t) = k(t)^{\alpha}h(t)^{\beta}e(t)^{\gamma}$ (where the coefficients are positive and sum to less than one). Then substitution of this production function into the objective yields:

$$\Gamma(h(t), e(t), k(t)) = [\beta w+(1-w)(1-W)]lnh(t)+[\gamma w+(1-w)W]lne(t)+\alpha wlnk(t)$$

When $w = 1/3$ and $W = 1/2$, we have the following effective weights on each of $h(t), e(t)$, and $k(t)$ respectively:

$$[w\beta + (1-w)(1-W)] = \frac{1}{3}[1+\beta], \qquad [w\gamma + (1-w)W] = \frac{1}{3}[1+\gamma], \qquad w\alpha = \frac{\alpha}{3}$$

Notice that, if education and health did not enter into the production function, then the weights on education and health would be 1/3 and still dominate the weight on physical capital.

3 The Extended Solow Model

In this section, we present an extended Solow model similar to the one given in Mankiw, Romer, and Weil (1992), with education human capital in the production function, but extended further to include both health human capital and consumption in production. The production function is the product of two functions:

$$Y(t) = F(K(t), H(t), E(t), L(t))\Phi(c(t)/c_s(t)) \tag{2}$$

where $K(t), H(t), E(t)$, and $L(t)$ are, respectively, aggregate values for physical capital, health capital, education capital, and labour. The component function F has the standard properties: it is increasing in each of its arguments, strictly concave, and has constant returns to scale.

The Φ function captures the effect of per capita consumption on output. Below a critical "threshold consumption" level, $c_s(t)$, output is increasing in per capita consumption $c(t)$. Above $c_s(t)$, further increments in $c(t)$ have no further effects on output:

$$\Phi(c(t)/c_s(t)) < 1, \quad \Phi' > 0, \quad \Phi'' < 0 \quad \text{for} \quad 0 \leq c(t)/c_s(t) < 1$$

$$\Phi(c(t)/c_s(t)) = 1 \quad \text{for} \quad c(t)/c_s(t) \geq 1$$

$$\lim_{(c/c_s)\to 0} \Phi' \to \infty, \qquad \lim_{(c/c_s)\to 1} \Phi' \to 0$$

As discussed in the introduction, we include the consumption component in the production function to avoid the immediate implication that consumption should be set equal to zero to maximize the HDI value of this economy. Since labour needs to consume, at least a little, to produce, we introduce a limited productive role for consumption. We do this in the simplest possible way with the multipicative term $\Phi(c(t)/c_s(t))$.Though $\Phi(c(t)/c_s(t))$ acts like a multifactor productivity term in production, this specification can be readily derived as the reduced form from a production function where consumption only causally affects the effectiveness of labour.[9]

The equations of motion for each of the inputs $K(t), H(t), E(t)$, and $L(t)$ are, respectively:

$$\dot{K}(t) = I_K(t) - \delta K(t) \tag{3}$$

$$\dot{H}(t) = I_H(t) - \delta H(t) \tag{4}$$

$$\dot{E}(t) = I_E(t) - \delta E(t) \tag{5}$$

$$L(t) = N(t), \qquad \dot{N}(t) = nN(t) \tag{6}$$

where δ is the depreciation rate, which we assume to be common for all types of

[9]For example, consider a Cobb Douglas production function, $Y(t) = K(t)^\alpha H(t)^\beta E(t)^\gamma (\phi(c(t)/c_s(t))L(t))^{(1-\alpha-\beta-\gamma)}$ where the coefficients are positive and sum to less than one, $\phi(c(t)/c_s(t))^{(1-\alpha-\beta-\gamma)} = \Phi(c(t)/c_s(t))$, and we collect the other terms in $F = K(t)^\alpha H(t)^\beta E(t)^\gamma (L(t))^{(1-\alpha-\beta-\gamma)}$. Similarly, consumption could augment the productive effectiveness of health and education capital.

We also considered other alternative methods of introducing consumption into the production technology, indirectly, through its effects on level of health and education capital. This complicates the analysis considerably without sustantially changing most of the qualitative results of the paper.

capital and I_J are the aggregate investments for $J = K, H, E$. Population $N(t)$ grows at exogenous rate n, and the population equals the labour force. Dots over variables denote their time derivatives. The resource constraint is:

$$Y(t) = C(t) + I_K(t) + I_E(t) + I_H(t)$$

This constraint can be expressed in terms of savings rates:

$$C(t) = (1 - s_K(t) + s_E(t) + s_H(t))Y(t) \tag{7}$$

where $s_J(t) \equiv I_J(t)/Y(t)$. We can now express the model in per capita terms. With constant returns to scale in F, we can divide this function by $L(t)$ to find per capita income in terms of the intensive production function f. Accordingly:

$$y(t) = f(k(t), h(t), e(t))\Phi(c(t)/c_s(t)) \tag{2'}$$

$$\dot{k}(t) = s_K(t)y(t) - (n + \delta)k(t) \tag{3'}$$

$$\dot{h}(t) = s_H(t)y(t) - (n + \delta)h(t)) \tag{4'}$$

$$\dot{e}(t) = s_E(t)y(t) - (n + \delta)e(t) \tag{5'}$$

$$c(t) = (1 - s_K(t) - s_H(t) - s_E(t))y(t) \tag{7'}$$

where

$$y(t) \equiv \frac{Y(t)}{L(t)}, \;\; c(t) \equiv \frac{C(t)}{L(t)}, \;\; k(t) \equiv \frac{K(t)}{L(t)}, \;\; e(t) \equiv \frac{E(t)}{L(t)}, \;\; h(t) \equiv \frac{H(t)}{L(t)}$$

We concentrate on the steady state of the model. In the steady state per capita quantities settle down to constants so that aggregate quantities grow at the rate of the population. In the steady state, then, $\dot{k}(t) = \dot{h}(t) = \dot{e}(t) = 0$, and (2′)-(5′) imply:

$$y = f(k, h, e)\Phi(c/c_s) \tag{2''}$$

$$k = \frac{s_K y}{n + \delta} \tag{3''}$$

$$h = \frac{s_H y}{n + \delta} \tag{4''}$$

$$e = \frac{s_E y}{n + \delta} \tag{5''}$$

Here we assume threshold consumption $c_s(t)$ is a constant c_s. Using equations (2″)-(5″), the resource constraint for consumption can be expressed solely in terms of the capital stocks in the steady state:

$$c = f(k, h, e)\Phi(c/c_s) - (k + h + e)(n + \delta) \tag{8}$$

3.1 The (Consumption) Golden Rule

To find the traditional golden rule we find the steady state levels of the capital stocks that maximize steady state consumption, as given in equation (8). The first-order conditions are:

$$\frac{dc}{dk} = \frac{\Phi f_k - (n + \delta)}{1 - f\Phi_c} = 0 \quad \Rightarrow \quad MP_K = \Phi f_k = (n + \delta)$$

$$\frac{dc}{dh} = \frac{\Phi f_h - (n + \delta)}{1 - f\Phi_c} = 0 \quad \Rightarrow \quad MP_H = \Phi f_h = (n + \delta)$$

$$\frac{dc}{de} = \frac{\Phi f_e - (n + \delta)}{1 - f\Phi_c} = 0 \quad \Rightarrow \quad MP_E = \Phi f_e = (n + \delta)$$

All the conditions imply that the marginal products be equated to the breakeven replacement rate: $MP_K = MP_H = MP_E = n + \delta$.[10] This condition identifies the golden rule in our model. There are two special cases. If $c^* \geq c_s$, then $\Phi = 1$ and $\Phi_c = 0$ where the star superscript indicates the golden rule value. In this case, consumption is not productive at the margin and the golden rule condition for physical capital is completely standard, $f_k = n + \delta$. Here the planner should also set, $f_h = f_e = n + \delta$. The other case is where $c^* < c_s$ so that $\Phi < 1$ and $\Phi_c > 0$. It follows that $f_k = f_h = f_e = (n + \delta)/\Phi$. Since $\Phi < 1$, this implies that $f_k = f_h = f_e > n + \delta$. This second case only obtains when the

[10]In writing these conditions we have assumed that the denominator term is positive, which requires $f\Phi_c < 1$. The content of this restriction is simply that the marginal product of consumption $MP_C = f\Phi_c < 1$,which we carry as a maintained assumption. Conversely, if $MP_C > 1$ then this would imply that 1 unit allocated to consumption generates more than one unit of production. This cannot be an optimum because greater consumption could be generated by continuing to allocate resources to consumption (generating perpetually increasing consumption). We assume that at least one capital input is sufficiently productive, so that $MP_C = 1$ is non-optimal.

threshold consumption is sufficiently high: $c_s > c^{**}$, where c^{**} is the golden-rule consumption in the conventional planner's problem where consumption is not a productive input.[11]

The golden rule involves setting the marginal products for all forms of capital to the same breakeven rate. This implies investment per capita of $(h^*+e^*+k^*)(n+\delta)$, where $(h^* + e^* + k^*)$ is the golden rule total capital stock. There will be over investment when $(h + e + k)(n + \delta) > (h^* + e^* + k^*)(n + \delta)$. Over investment implies capital over accummulation.[12]

Definition 1 *There is capital over accumulation when the total capital stock is greater than the golden rule total capital stock:* $h + e + k > h^* + e^* + k^*$.

We now show that if the marginal products are lower than the breakeven rate, then there is capital overaccumulation. If $MP_J \leq n + g$ for all $J(= H, E, K)$ and $MP_J < n + g$ for at least one $J(= H, E, K)$, then by the strict concavity of the production function it follows that output is greater $y > y^*$. As consumption can not be greater than the golden rule level, $c \leq c^*$, it follows that there is over investment, $y - c = (h + e + k)(n + \delta) > (h^* + e^* + k^*)(n + \delta) = y^* - c^*$, and therefore capital over accumulation. The following proposition summarizes.

Proposition 1 *The golden rule equates all the marginal products:* $MP_J = n + g$ *for all* $J(= H, E, K)$*. Capital over accumulation exists if* $MP_J \leq n + g$ *for all* J *and* $MP_J < n + g$ *for at least one* J*.*

3.2 Maximizing the HDI

We now consider the problem choosing steady state values of h, e, y, k, and c to maximize the value of the Human Development Index, as represented in (1), subject to the production ($2''$) and feasibility constraint (8). In per-capita terms,

[11] Let $\{k^{**}, h^{**}, e^{**}\}$ denote the solution to the planner's problem when consumption does not enter production, $Y(t) = F(K(t), H(t), E(t), L(t))$. Then the second case, $c^* < c_s$, prevails iff $c_s > c^{**} = f(k^{**}, h^{**}, e^{**}) - (k^{**} + h^{**} + e^{**})(n + \delta)$.

[12] Capital over accummulation is of concern because it implies dynamic ineffiiciency in this model. See De la Croix and Michel (2002) and King and Ferguson (1993) for details.

the problem becomes:

$$\max_{\{h,e,y,k,c\}} D(h,e,y) \text{ st } \begin{aligned} c &= y-(n+\delta)(k+h+e) \\ y &= f(k,h,e)\Phi(c/c_s) \end{aligned}$$

Here we assume that the bounds on the indicator variables (h, e, y) are non-binding, an assumption that is relaxed later.

It is convenient to form the Lagrangian for the analysis by substituting the production function for y in to both the objective function and the feasibility constraint:

$$L = D(h,e,f(k,h,e)\Phi(c/c_s)) - \sigma(c - f(k,h,e)\Phi(c/c_s) + (n+\delta)(k+h+e))$$

where $\sigma > 0$ is the marginal value of an exogenous increase in income. The first-order conditions with respect to $c, k, h,$ and e are, respectively

$$D_y f\Phi_c - \sigma(1 - f\Phi_c) = 0 \quad \Rightarrow \quad f\Phi_c = \frac{\sigma}{D_y+\sigma}$$

$$D_y f_k\Phi - \sigma(-f_k\Phi + (n+\delta)) = 0 \quad \Rightarrow \quad f_k\Phi = \frac{\sigma}{D_y+\sigma}(n+\delta)$$

$$D_h + D_y f_h\Phi - \sigma(-f_h\Phi + (n+\delta)) = 0 \quad \Rightarrow \quad f_h\Phi = \frac{\sigma(n+\delta) - D_h}{D_y+\sigma}$$

$$D_e + D_y f_e\Phi - \sigma(-f_e\Phi + (n+\delta)) = 0 \quad \Rightarrow \quad f_e\Phi = \frac{\sigma(n+\delta) - D_e}{D_y+\sigma}$$

These four first-order conditions, together with the feasibility constraint, constitute a system of five equations in five unknowns ($c, k, h, e,$ and σ) that describe the HDI maximizing allocations in the steady state. The Inada conditions on Φ assure that some consumption is needed, but consumption at or beyond the threshold is suboptimal because it is unproductive: $0 < c < c_s$.

The conditions for the the HDI maximizing allocations can be rewritten in terms of marginal products ($MP_C = f\Phi_c$, $MP_K = f_k\Phi$, $MP_H = f_h\Phi$, and $MP_E = f_e\Phi$) as follows:

$$MP_C = \frac{\sigma}{D_y+\sigma} \quad \Rightarrow \quad 0 < MP_C < 1$$

$$MP_K = MP_C(n+\delta) \quad \Rightarrow \quad MP_K < n+\delta$$

$$MP_H = MP_K - \frac{D_h}{D_y}(1 - MP_C) \qquad \Rightarrow \qquad MP_H < MP_K < n + \delta$$

$$MP_E = MP_K - \frac{D_e}{D_y}(1 - MP_C) \qquad \Rightarrow \qquad MP_E < MP_K < n + \delta$$

The MP_C is driven below 1 to the extent that output which is valued in the objective function, $D_y > 0$. The conditions for the capital stocks imply: MP_H, $MP_E < MP_K < n + \delta$. As $MP_J < n + g$ for $J = H, E, K$, we have capital overaccumulation as described in Section 3.1. The following proposition describes this steady state relative to the golden rule.

Proposition 2 *Maximizing the HDI implies minimal consumption (i.e., $c < \min[c_s, c^*]$) and capital over accumulation.*

Notice, also, that maximizing the HDI gives priority to human capital (both for education and health) over physical capital. That is, both h and e are accumulated so that their marginal products are driven below the marginal product of physical capital. Inspection of the conditions reveals this is because both types of human capital, unlike physical capital, are valued not only indirectly through production, but also directly in the objective function: h and e are both means and ends.

The relative values of the marginal products education and health, themselves, depend on their direct weights in the HDI:

$$\frac{D_e}{D_h} = \frac{MP_K - MP_E}{MP_K - MP_H} \qquad \Rightarrow \qquad D_e \gtreqless D_h \qquad \Leftrightarrow \qquad MP_E \lesseqgtr MP_H$$

3.2.1 An Aside on the Income Approach

At this point, it is worthwhile to compare the results with the "income approach", described above, where only income enters the objective function. With first-order conditions take the same form as above with $D_h = D_e = 0$. Thus, the condition for consumption is unchanged requiring $0 < c < c_s$ for the same reasons as before. The optimality conditions imply that the marginal products be equated across types of capital: $MP_J = MP_C(n + \delta) < n + \delta$ for $J = H, E, K$, consistent with capital overaccumulation. Thus, we get the same general outcome as with maximizing the HDI.

Proposition 3 *Maximizing per capita income implies minimal consumption ($c < \min[c_s, c^*]$) and capital over accumulation.*

Using the language discussed above, the income approach is capital fundamentalist. The human development approach is also capital fundamentalist but with an emphasis on human capital.

3.2.2 Bounds on the Indicator Variables in the HDI

As discussed in Section 2, the indicator variables only effect the HDI when they are between their lower and upper bounds (i.e. $D_j > 0$ for $j(t) \in [\underline{j}, \overline{j})$, and $D_j = 0$ for $j(t) < \underline{j}$ and $j(t) \geq \overline{j}$, where $j = h, e, y$). In recent years the lower bounds have been exceeded by all countries and some upper bounds as been met by a few countries. Here, we consider the possibility that the planner may choose indicator variables at or above the upper bounds, but assume that it is infeasible to achieve all of the upper bounds simultaneously.[13] We continue to assume that the planner can and will choose the indicator variables above their lower bounds.

First consider when it is optimal to choose education at, or above, the bound, $e \geq \overline{e}$, but other indicator variables are below their upper bounds. Now the direct marginal benefit for education is $D_e = 0$ and, at the margin, education will be valued like physical capital: $MP_E = MP_K < n+\delta$. Similarly, when health $h \geq \overline{h}$,is the only variable chosen at or above the upper bound, $MP_H = MP_K < n + \delta$. In either case, there is capital overaccumulation. When both health and education are at, or above, their upper bounds both conditions apply and there is capital overaccumulation.

Now consider when it is optimal to choose income $y \geq \overline{y}$. Then $D_y = 0$ and $MP_C = 1$ so there is even less reason to provide consumption. As before $c < c_s$. Though $MP_K = f_k = n + \delta$, there is still capital overaccumulation as either $MP_H = n + \delta - D_h/\sigma < n + \delta$ or $MP_E = n + \delta - D_e/\sigma < n + \delta$. Here capital over accumulation is due to at least one human capital being valued in the objective function, $D_h > 0$ and/or $D_e > 0$, at the margin.[14] Summarizing, the qualitative

[13] If, alternatively, it was feasible to achieve all the upper bounds, then a country would simply choose the variables at or above these bounds. Such a country is sufficiently productive that $\{k, h, e\} \geq \{\overline{k}, \overline{h}, \overline{e}\}$ and $c > 0$ satisfies the resource constraint (8) for a steady state.

[14] Though the marginal product of capital is at the break even rate, we can not assert there is no physical capital overaccumulation in the sense that $k \leq k^*$. This is because k is determined by

results in Proposition 2 generalize to the empirically relevant case where countries choose to exceed all the lower bounds, but can not achieve all the upper bounds.

Proposition 4 *Maximizing the HDI implies minimal consumption ($c < \min[c_s, c^*]$) and capital over accumulation when it is infeasible to choose all indicator variables at their upper bounds.*

3.3 Exogenous Technological Change

Including exogenous labour-augmenting technological change in the model does not change the qualitative features of the HDI-maximizing rule, if we assume that $c_s(t)$ grows at the same rate as other per capita variables. In this case, the aggregate production function becomes:

$$Y(t) = F(K(t), H(t), E(t), A(t), L(t))\Phi(c(t)/c_s(t))$$

where $\dot{A} = gA(t)$ and g is the exogenous rate of technological change. The model can be rewritten in terms of efficiency units where the intensive production is:

$$\hat{y}(t) = f(\hat{k}(t), \hat{h}(t), \hat{e}(t))\Phi(\hat{c}(t)/\hat{c}_s(t))$$

The feasibility constraint has the same form with the breakeven capital term including g:

$$\hat{c}(t) = f(\hat{k}(t), \hat{h}(t), \hat{e}(t))\Phi(\hat{c}(t)/\hat{c}_s(t)) - (n + g + \delta)(\hat{k}(t) + \hat{h}(t) + \hat{e}(t))$$

where $\widehat{j}(t) = j(t)/A(t)$ for $j = y, k, h, e, c$, and c_s.

In the steady state, Φ is constant, the hat variables $j = y, k, h, e$, and c are constant, and per capita quantities grow at the constant rate g. If $\widehat{c}(t) < \widehat{c}_s(t)$ then the term Φ is constant when the ratio $\widehat{c}(t)/\ \widehat{c}_s(t)$ is constant. In this case, the threshold level, $c_s(t)$ grows at rate g. Alternatively, if $\widehat{c}(t) \geq \widehat{c}_s(t)$ then Φ is constant and equal to 1 This can be maintained in a steady state as long as

$f_k(k, h, e) = n + \delta$ and hence depends on the chosen levels of h and e. Capital over accumulation and $k \leq k^*$ together require $h + e > h^* + e^*$. This combination of capital levels requires ruling out that all human capital inputs complementing physical capital in production.

$c_s(t)$ grows at rate less than or equal to g. For example, if the threshold value is constant, $c_s(t) = c_s$, the ratio grows at rate g so that $\widehat{c}/\widehat{c}_s(t) \geq 1$ is maintained.

For the analysis of the HDI golden rule, we assume that the general objective function has the same properties as the HDI; for example, it is homogenous of degree one.[15] In this case, the exogenous technology term $A(t)$ can be taken out as a multiplicative factor which does not affect the optimization:

$$D(A(t)\hat{h}(t), A(t)\hat{e}(t), A(t)\hat{y}(t)) = A(t)D(\hat{h}(t), \hat{e}(t), \hat{y}(t))$$

The marginal productivity conditions that describe the (consumption) golden rule are the same as before, except that the breakeven capital accumulation term includes g: $MP_H = MP_E = MP_K = n + g + \delta$. Consider, first, the case where consumption is below the threshold. The term Φ is constant and $0 < \Phi < 1$. (This is only consistent with $c_s(t)$ growing at rate g as described above.) In this case, a comparison of the traditional golden rule with the HDI-maximizing conditions has the precisely same qualitative features.

Staying with the case of $c_s(t)$ growing at rate g, let us now consider the effects of bounds. If the bounds also grow exogenously at rate g, there exists a steady state as above. Homogeneity of degree one of the objective function ensures that the planning problem can be formulated as above. If bounds change at a different rate than g, then there is no steady state. In particular, if the bounds grow at a rate less than g then eventually, it will be feasible for the planner to achieve all the bounds.

When $c_s(t)$ grows at a rate other than g there is no steady state.[16] However, we can resurrect a steady state when $c_s(t)$ grows at a rate less than g by imposing the additional threshold requirement: $c(t)/y(t) \geq s > 0$, where s is a small positive fraction. With this constraint binding $c(t)$ must grow at rate g such that $\widehat{c}/\widehat{c}_s(t) \geq$

[15] The HDI is homogenous of degree one when the same multiplicative factor is applied to the variable as well as the bounds. Anand and Sen (1994) advocate this as desirable feature of a development index.

[16] That there is no steady state when $c_s(t)$ grows at a rate less than g is perhaps surprising. The steady state optimality criteria require that the planner choose margins such that the marginal product of consumption be positive (to equate margins) which implies $\Phi < 1$ and that $\widehat{c}(t)/\widehat{c}_s(t) <$ 1 and constant. Then $c(t)$ would also have to grow at a rate less than g which means it can not be a steady state. Out of the steady state, productivity would be growing at least at rate g while consumption would be growing at a rate smaller than g. Over time the ratio of consumption to output would decline and, in the limit, $c(t)/y(t) \to 0$.

1 and $\Phi = 1$.

4 Alternative Human Development Criteria

As we have seen, the planning criterion of HDI maximization leads to problematic outcomes, at least in the steady state. In particular, it implies that consumption would be set to minimal levels. Moreover, it involves the maximization of income for its own sake – over other valued ends like health and education – something that, arguably, goes against the spirit of the human development approach. As such, income does not have the intended consequence of capturing the dimension 'decent standard of living'.

In this section, we examine what happens when we replace income in the HDI with two different alternatives: disposable income and consumption. We find that using disposable income does not change the problematic outcomes; whereas using consumption in the index yields a "human development golden rule" which efficiently balances consumption versus education and health.

4.1 Maximizing the HDI modified with Disposable Income Replacing Income in the Index

We consider disposable income as an alternative for several reasons. First, Anand and Sen (2000b, p86) argued that this indicator was meant to "reflect something of basic capabilities not already incorporated in measures of longevity and education". Second, Engineer, King and Roy (2008) make the case for modifying the index with disposible income in their static model. In their empirical work they derive disposable income by subtracting public expenditures on health and education from from income. This avoids the double counting of these components in the modified index.

Here we define disposable income by subtracting all expenditures on education and health. Implicitly, this assumes that expenditures on these variables is in the public sector whereas physical capital is in the private sector. Expressed in terms of the intensive variables, per capita disposable income in the steady state is:

$$d(t) \equiv c(t) + (n + \delta)k(t) = y(t) - (n + \delta)(e(t) + h(t))$$

The objective function, now with disposable income replacing income, is denoted:

$$D^d(h(t), e(t), d(t))$$

where the superscript d indicates that the index D^d might a different functional form that D. However, as before, the objective function is strictly concave and, outside of the indicator bounds, the marginal values D_j^d are zero, for $j = h, e, d$. Below we assume for simplicity that the indicators are chosen within the indicator bounds.

In the steady state, the Lagrangian for the planner's problem is:

$$L^d = D^d(h, e, c + (n+\delta)k) - \sigma^d(c - f(k, h, e)\Phi(c/c_s) + (n+\delta)(k + h + e))$$

Again, we can express the first-order conditions in terms of marginal products and marginal rates of substitution:

$$\begin{aligned}
MP_C &= 1 - \frac{D_d^d}{\sigma^d} \quad \Rightarrow \quad 0 \leq MP_C < 1 \quad \Rightarrow \; 0 < \; c < c_s \\
MP_K &= MP_C(n+\delta) \quad \Rightarrow \quad MP_K < n + \delta \\
MP_H &= n + \delta - \frac{D_h^d}{D_d^d}(1 - MP_C) \quad \Rightarrow \quad MP_H < MP_K \\
MP_E &= MP_K - \frac{D_e^d}{D_d^d}(1 - MP_C) \quad \Rightarrow \quad MP_E < MP_K
\end{aligned}$$

Since $MP_K > 0$,it follows that $MP_C = MP_K/(n+\delta) > 0$. Thus, $\Phi'(c/c_s) > 0$ implying $0 < c < c_s$. As $MP_C < 1$, the marginal products are below the break even rate $MP_J < n + \delta$ for all $J = H, E, K$. Thus, we have the following proposition.

Proposition 5 *Maximizing the HDI modified with disposable income implies minimal consumption ($c < c_s$) and capital overaccummulation.*

Modifying the HDI in this way does not alter the qualitative results from those found in *Proposition 2*.[17] The planner prefers to increase disposable income $d = c + (n+\delta)k$ by increasing physical capital k rather than by consumption c.

[17] Like Proposition 2, we can generalize Proposition 5 to consider the upper bounds. The same results obtain when it is infeasible for the planner to achieve all the upper bounds simultaneously.

This modified HDI still emphasizes capital, although disposable income nets out education and health expenditures, $d = y - (n + \delta)(e + h)$ which are otherwise double counted in the objective function through the income term. The next alternative, replacing output with consumption, further avoids double counting by eliminating the accumulation of physical capital for its own sake.

4.2 Maximizing the HDI when Consumption Replaces Income in the Index

Here, we examine what happens when we replace income, in the HDI, with consumption. By consumption we mean output less expenditures on all capital investments. Consumption is arguably is a better proxy for a "decent standard of living" than disposable income, because it also excludes physical capital investment. Per capita consumption, in the steady state, is given by:

$$c(t) = y(t) - (n + \delta)\left(k(t) + h(t) + e(t)\right)$$

The objective function, now with consumption, is denoted:

$$D^c(h(t), e(t), c(t))$$

where the superscript c indicates that the index D^c might have a different functional form than D. However, as before, the objective function is strictly concave and, outside of the upper bounds, the marginal values D^c_j are zero. In the following analysis we assume for simplicity that the indicator variables are interior to their bounds so that $D^c_j > 0$ for $j = h, e, c$.

The corresponding Lagrangian is

$$L^c = D^c(h, e, c) - \sigma^c(c - f(k, h, e)\Phi(c/c_s) + (n + \delta)(k + h + e))$$

The first-order conditions with respect to c, k, h, and e, respectively, are:

$$D^c_c - \sigma^c(1 - f\Phi_c) = 0 \qquad \Rightarrow \qquad D^c_c = \sigma^c(1 - f\Phi_c)$$

$$-\sigma^c(-f_k\Phi + (n + \delta)) = 0 \qquad \Rightarrow \qquad f_k\Phi = (n + \delta)$$

$$D_h^c - \sigma^c(-f_h\Phi + (n+\delta)) = 0 \qquad \Rightarrow \qquad D_h^c = \sigma^c((n+\delta) - f_h\Phi)$$

$$D_e^c - \sigma^c(-f\Phi_e + (n+\delta)) = 0 \qquad \Rightarrow \qquad D_e^c = \sigma^c((n+\delta) - f\Phi_e)$$

These four first-order conditions and the feasibility constraint constitute a system of five equations in five unknowns (c, k,h, e, and σ^c) that describe the optimal policy.

The first-order conditions can be rewritten in terms of marginal products in what we refer to as the *human development golden rule.* This rule with some implications is as follows:

$$MP_C = 1 - \frac{D_c^c}{\sigma^c} \quad \Rightarrow \quad 0 \leq MP_C < 1 \quad \Rightarrow \quad c > 0 \tag{9}$$

$$MP_K = (n+\delta) \tag{10}$$

$$MP_H = MP_K - \frac{D_h^c}{D_c^c}(1 - MP_C) \quad \Rightarrow \quad MP_H < MP_K = n+\delta \tag{11}$$

$$MP_E = MP_K - \frac{D_e^c}{D_c^c}(1 - MP_C) \quad \Rightarrow \quad MP_E < MP_K = n+\delta \tag{12}$$

For consumption there are two cases to consider. If $0 < c < c_s$, then $0 < MP_C < 1$ and $D_c^c < \sigma^c$. Conversely, if $c \geq c_s$ then $MP_C = 0$ and $D_c^c = \sigma^c$. This latter case prevails when c_s is relatively small and consumption is sufficiently valued in the objective function. This is the standard case in economics where production is not affected by consumption on the margin and, hence, might be thought to be the more reasonable case in the steady state.

The condition for the physical capital stock is now: $MP_K = n + \delta$. As before human capital is given priority over physical capital, $MP_H, MP_E < MP_K = n+\delta$. The division of of human capital is described by the marginal rate of substitution between health and education:

$$\frac{D_e^c}{D_h^c} = \frac{MP_K - MP_E}{MP_K - MP_H} \qquad \Rightarrow \qquad D_e^c \gtreqless D_h^c \qquad \Leftrightarrow \qquad MP_E \lesseqgtr MP_H$$

This condition takes the same form as for the corresponding HDI-maximizing rule except that $MP_K = n+g$. The marginal rates of substitution between consumption and health and education human capital are:

$$\frac{D_h^c}{D_c^c} = \frac{(n+\delta) - MP_H}{1 - MP_C}, \qquad \frac{D_e^c}{D_c^c} = \frac{(n+\delta) - MP_E}{1 - MP_C}$$

This condition has no analog in the HDI; it describes the trade off between consumption and the human capital indicator variables. The following proposition summaries:

Proposition 6 *The "human development golden rule" is described by equations (9)-(12). This rule efficiently trades off consumption with human capital indicator variables. Consumption may or may not exceed the threshold $c \gtreqless c_s$, depending on productivity and preferences. Relative to the (consumption) golden rule there is capital over accumulation and $c < c^*$.*

4.2.1 Bounds and Technological Change

There is a novel possibility that it might be optimal to choose consumption at or above its upper bound. The only reason to exceed the upper bound on consumption would be if the bound were less that threshold $\overline{c} < c_s$ so that $MP_C > 1$ evaluated at $c = c_s$. Here we assume that any extra resources not allocated to consumption are allocated to either health or education capital. When at least one of education or health are optimally chosen below their upper bound, then $MP_H, MP_E \leq MP_K = n + \delta$ and either $MP_H < MP_K$ or $MP_E < MP_K$ so there is capital over accumulation. Thus, *Proposition 6* generalizes to whenever at least one of education or health are chosen below their upper bounds.

Introducing exogenous technological change into the analysis does not generically change the results when a steady state exists. As with the HDI-maximizing rule, the steady state conditions are expressed in the intensive hat variables and g enters as a new term. When $c_s(t)$ grows at rate g such that $\widehat{c}_s(t)$ is constant, the analysis is parallel to the one given above: there are two cases – one where $0 < \widehat{c} < \widehat{c}_s$ and the other where $\widehat{c} \geq \widehat{c}_s$.

Unlike under the HDI-maximizing rule, if $c_s(t)$ grows at rate less than g, there may be a steady state. The steady state has the property that $\Phi = 1$, $MP_C = 0$, and the marginal value of consumption is $D_{\widehat{c}} = \widehat{\lambda}$. There will be no steady state with $0 < \widehat{c} < \widehat{c}_s$ for the same reasons as before. As before, in the presence of technological growth, the bounds must also grow at rate g for a steady state to exist away from the bounds. If all bounds grow at a smaller rate, it eventually becomes feasible to achieve all of the bounds.

5 Conclusion

In this paper we have taken the unusual methodological approach of evaluating a well-known overall achievement index, the human development index (HDI), by examining the optimal dynamic plans it implies. Our motivation for maximizing the HDI is both positive and normative, as encapsulated in the opening quotations. We believe this method has been quite revealing in uncovering unintended consequences of using the index as a guide for development planning. The optimal plans for the HDI imply minimal consumption, and physical and human capital over accumulation. This led us to modify the index in a way that better fulfilled the original motivation for the HDI. Adopting the modified index, where per capita consumption replaces per capita GDP, yields a "human development golden rule" which better balances the ends of health, education and a 'decent standard of living'.

Our policy critique, favoring consumption over income, also applies to optimal plans from maximizing per capita GDP. There has been a great deal of work critiquing the use of GDP from a measurement perspective that comes to similar conclusions. For example, the first recommendation in Stiglitz, Sen, and Fitoussi (2009), "Report of the Commission on the Measurement of Economic Performance and Social Progress", is to replace GDP with measures like consumption and net disposable income. We would argue that disposable income per capita is a poor replacement for GDP per capita – both lead to minimal consumption and capital over accumulation – and consumption per capita is a more appropriate replacement.

The method that we have used here, evaluating a criterion by examining its implied economic outcomes, can be applied to other indexes and issues. For example, Engineer, King and Roy (2008) compare the HDI and Gender Development Index (GDI) in a static model.[18] They find plausible assumptions under which

[18] As far as we are aware, the only other paper to take this methodological approach is Bourguignon and Fields (1990). They minimize various poverty indices subject to redistribution

maximizing both indexes yield the same optimal plan. This is despite GDI treating the sexes asymmetrically and being sensitive to inequality. This reveals that HDI tends toward equitable outcomes even though that principle is not built into the index. They also show that with optimal planning the ranking of nations by GDP per capita and HDI score will coincide when only total factor productivity differentiates nations. In that model, differences in ranking are evidence that countries are not pursuing optimal plans. Other questions that might be addressed are: How far is a poverty index reduced by maximizing the HDI or GDI? Does adding other dimensions or indexes to the HDI substantially change development plans? What new dimensions and principles (in the Stiglitz, Sen, and Fitoussi (2009)) should be considered to make the HDI a better measure of well-being?

The analysis of intertemporal planning with multi-dimensional objectives is inherently complex. This is particularly so when there are state variables in the objective function which feedback to production. We have explored the implications of the optimal dynamic plan in an extension of the simplest well-known dynamic model, the Solow model, to get a feel for the issues. Our intertemporal analysis has exclusively concentrated on the steady state. Steady state analysis is straightforward and provides relatively simple and unambiguous conditions which can be compared with a classic benchmark, the golden rule. Arguably, steady state analysis is an appropriate counterpart for an index which has no intertemporal dimension because variables are constant over time. Also, Anand and Sen (2000a) argue that the sustainability of human development should be a primary value, rather than having it developed from welfarist criteria.[19] In this light, the indicators in the HDI should be modified to capture long run averages that can be sustained.

Nevertheless, in growth analysis there is perhaps an overreliance on interpreting steady states as analogues of the long run and ignoring incentive effects. Particu-

constraints. The implied policies can differ dramatically among indexes.

[19] As in the analysis of the golden rule in the Solow model, our analysis is not welfarist in the sense of maximizing a welfare function derived from individual agents utility functions. Such models may not have an optimal path that is a steady state. Here we concentrate on steady states and so our analysis is of sustainable plans. Anand and Sen (2000a) show that sustainable plans are not necessarily optimal in terms of a welfarist criterion. They argue for the normative primacy of sustainable plans. Our paper can be thought of as extending their work to optimal sustainable human development plans. See Pessy (1992) for an evaluation of sustainable development concepts.

larly, in modelling development issues, analysis of transition paths seems more appropriate. Indeed, it might be reasonably argued that human development should be thought of as a process of transition growth to a developed state. Our analysis is more consistent with the view that there is no end to human development – where human development is an ongoing expansion of peoples' abilities to make choices. One relatively straightforward extension would be to examine the transition under fixed savings rates where the rates are set at the implied human development golden rule savings rates. Optimal transition analysis could include a more sophisticated objective with discounting. We would be surprised if the general results of our steady state analysis – minimal consumption, and physical and human capital over accumulation – did not obtain on the transition path. Still transition analysis would generate new results related to other dynamic issues, such as speed of adjustment. Second-best considerations and individual incentive and participation constraints are likely more threatening to the results in this paper. They suggest a new field of dynamic public finance for human development.

Another issue, which we believe would be interesting to explore, would be the tournament aspect implied by the quotation from Inge Kaul, in the beginning of this paper. In particular, the analysis of tournaments in, for example, Lazear and Rosen (1981) and Green and Stokey (1983) could shed light on the relationship between maximizing a country's rank in the HDI and the actual value of the HDI itself.

The method advanced in this paper tries to logically connect policy to measures that are used to inform policy. Indeed, we believe the usefulness of a measure should be assessed against its policy implications. The rigor of maximization subject to feasibility constraints is a check for evaluating multi-dimensional (non-welfarist) indexes. This methodology makes policy trade-offs explicit and reveals the effective goals implicit in taking an index, perhaps too, seriously. Also, it may yield comparisons with traditional bench marks, like the golden rule. Though we have provided a critique of the current version of the HDI, we believe that the critique is easily remedied and that the HDI, and similar measures, can be enhanced. Making explicit connections from measurement to desireable policy outcomes should give policy makers more confidence in pursuing policies towards maximizing human development.

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