Transparency of peer activities confounds cheap talk in joint projects.

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Abstract

Peer activities influence incentives in teams involving incomplete information through the information they transmit. One channel of information transmission is through costless signalling – team bonding, motivational sessions etc. – which in the language of game theory is cheap talk. Another is through the transparency of peers' costly actions. While each of these can be separately beneficial, it is shown that the strategic interplay between them is such that, jointly they convey no further information and may result in negative welfare consequences.

The theoretical core is a thorough analysis of all cheap talk extensions of a standard voluntary contribution game.

Keywords: Cheap talk extensions; Teams; Transparency; Peer influence; Voluntary contributions; Pseudo-mode.

JEL Classification: D29, C79, H41

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1 Introduction

This paper concerns the strategic interplay between two types of information transmission within a team that result from activities of the peers.[1] One form of information transmission occurs simply with team members talking to each other, holding motivational sessions etc. – inherently costless activities that may convey what the success of the team as a whole means to the individuals. I call such activities, in the language of game theory, *cheap talk*. Information transmission also occurs through the transparency of peers' costly actions. Transparency in some cases is the result of the structural organization of the workplace whereby some employees observe what others do before choosing how much effort to exert. Elsewhere, tasks are organized to occur sequentially. The purpose of this paper is to demonstrate that although transparency and cheap talk are separately valuable, together they convey no further information and negative welfare consequences.

The theoretical interest in the phenomenon being addressed here accompanies the marked changes workplace organization and product development during that last two decades. The traditional method of product development involves sequential effort. Sometimes termed the "waterfall method" of product development, each functional area of expertise performs its functions before passing responsibility to the next functional area. Arguably, here it is the second type information that is predominant. A currently more popular development process is integrated product development, sometimes linked to Apple, in which "employees talk incessantly about what they call "deep collaboration" or "cross-pollination" or "concurrent engineering." Essentially it means that products don't pass from team to team. There aren't discrete, sequential development stages. Instead, it's simultaneous and organic." [2] A cross-functional team negotiates a design taking a holistic view of both the engineering constraints and final consumer demands. Arguably again, the communication that occurs between the team members with disparate areas of expertise is cheap talk.

I make my point in the context of a pair of players (which can be generalized to finitely many players) who seek to complete a joint project to which they can each contribute only once, at an unrecoverable cost if the project does not succeed. The value of the project's com-

[1] Empirically it is well documented that activities of peers have an important influence on incentives in teams and partnerships. See Ichino and Maggi [2000], Falk and Ichino [2006] and the vast literature that follows Kandel and Lazear [1992].

[2] The quote is from a Time Magazine article by Lev Grossman's: "How Apple Does It" (See Grossman [2005]). See Krishnan and Ulrich [2001] and the references therein for an overview of the academic literature on the changing nature of product development.

pletion to a player is her private information. Transparent or observable behavior is a game form in which effort contributions are sequential, non-transparency involves simultaneous choices. Broadly speaking, we first show that equilibria under transparency dominate those under non-transparency. However, non-transparent contributions augmented with a round a cheap talk involving only two messages dominates transparent behavior, even allowing for arbitrary (albeit one shot) communication possibilities. It also dominates non-transparent choices without cheap talk.

A broad intuition for limited impact of cheap talk under transparency that transcends the specifics of the model here is as follows. When players choose sequentially, sequential rationality refines the set of equilibria. This may rule out some unattractive equilibria, which one may argue is a "good thing" provided there is no cheap talk. However, when each of these games is augmented with a stage of pre-play communication, for cheap talk to be informative, it may well be that it is precisely these "unattractive equilibria" that act as punishments for deviation from the equilibrium path. Of course, this argument only describes the informational aspects. Establishing the welfare properties requires us to borrow insights that underlie the "revenue/payoff equivalence" principle from auction theory (See Myerson [1981]) and typically apply only to quasi-linear environments like the one considered here.

It is worth emphasizing, that even abstracting from issues concerning cheap talk, our results on transparency vs. non-transparency add another dimension to a large literature on voluntary contributions to a discrete public good or for funding of joint projects. A recurring theme in this considerable literature is whether sequential actions accentuate the free-rider effect and lower total contributions. (See Admati and Perry [1991], Varian [1994], Andreoni [1998], Marx and Matthews [2000] or Compte and Jehiel [2003].)

Typically this literature considers environments of complete information. (See Bag and Roy [2007] however.) With the presence of incomplete information, a leader, when it is her turn to contribute, has to take a bet on whether the follower's valuation is high enough to free-ride on the latter's contribution. Not only do our results suggest that sequencing decisions helps, we identify that it is the player with the highest *range* of valuations that must be preferred as the leader.[3]

There is a considerable body of work on the design of incentives by a principal with multiple agents that in particular involves peer monitoring. Among the more prominent

[3] Hermalin [1998] offers a theory of "Leadership". Our conclusion that the player with the highest range of values must lead is an exercise in comparative statics with respect to different sequences. Hermalin [1998]'s result is about endogenously determined leaders.

early contributions are Ma et al. [1988], Varian [1990] and Stiglitz [1990]. Winter [2006] is a more recent contribution which expands this literature by considering sequential choices. Except for the fact that peer activities influence incentives, this paper does not have much in common with this strand of the literature. For, our interest is *not* the optimal design of incentives by an external agent. In fact, one consideration for modelling the team problem here to be a threshold type where only *total* contributions matter is to minimize the external incentive issues. Rather, the focus is on how peer activities affect incentives within the team for a given external incentive structure.

In the last respect, our paper falls squarely into the research program initiated by the influential piece by Kandel and Lazear [1992]. However, they (and much of the literature that follows it) models the effect of peer activities using a reduced functional form, much like in the literature on reciprocity (See Fehr and Gachter [1998]). On the other hand, I regard peer activities as information transmission mechanisms and divide them based on whether the signalling actions are inherently costly. Our results then have obvious implications for the structural makeup of the workplace.

The role of prep-lay communication in joint projects where contributions are necessarily simultaneous is the subject of Agastya et al. [2007]. See also the references in Farrell and Rabin [1996] and in particular Matthews and Postlewaite [1989]. Baliga and Morris [2002] study games with spill-overs and actions are sequential. They offer sufficient conditions under which cheap talk may be completely uninformative. These conditions are not met for the contribution game being studied here.

In summary, the theoretical core of this paper is a comparison of cheap talk extensions of the simultaneous *and* sequential move games of contributions toward a joint project. Among the related areas of literature described above, I am not aware of any contribution that systematically (or otherwise) compares, in terms of their efficiency, likelihood of project's completion and total contributions, the equilibria of this class of games.

The organization of the rest of the paper and an elaboration of the results is as follows. A model of team given in Section 2 in which a certain project is success only if the total effort/contribution of the two players reaches a threshold, say $k > 0$. The incremental benefit that accrues to a player from the project's completion is her private information, drawn from an interval $[\underline{v}_i, \overline{v}_i]$ and let $r_i = \overline{v}_i - \underline{v}_i$ be the *range* of the player's valuations. Section 3 studies contributions to the joint project without communication. The more economically interesting part is Section 3.1 where we explain why observability in the absence of cheap talk is valuable. In particular, Proposition 3 shows that if priors are concave and ordered as per first order stochastic dominance, making the player with the highest r_i move first leads to – a) the highest probability of project's completion, b) the highest expected total contribution and c) Pareto dominance (relative to non-transparent behavior) in terms of players' ex-ante payoffs. Moreover, this sequence of moves is uniquely optimal whenever k is above a threshold, say $\hat{k}$.

In Section 4 we reconsider the observability issue by now allowing for pre-play communication. We find that the above result on the optimal sequence is completely overturned. Indeed, we now find that whenever $r_i > \hat{k}$, it is not optimal for Player i's action to be observable. A corollary is then that with pre play communication, players choices must not be observable. The main building block is Proposition 5. It characterizes several properties of equilibria when cheap talk co-exists with transparency. It shows that for any communication structure augmented to the sequential contribution game, at most one message can be sent with a positive probability following which the project has a positive completion probability in the continuation game. This enables us to conclude that any outcome that can be achieved through a cheap talk extension of the sequential contributions game can also be achieved via an augmentation of the simultaneous move game with a message space containing only *two messages*. This comparison is taken up in Section 4.3. Thus, the conclusions given in Section 3.1 and Section 4.3 taken together give the economic insights related to peer activities described in this Introduction. Section 5 concludes.

Two further points are worth noting before we delve into the formal details. First, it may be of some independent interest that some of the sufficient conditions that we impose in prior beliefs have to do with a new concept called the *pseudo-mode* of a probability distribution that I introduce. The pseudo-mode of a probability distribution is the same as the mode of a uni-modal distribution but differs otherwise.[4] Second, a semi-formal outline of the main argument is given in Section 2.1 for the inefficacy of cheap talk under transparency.

2 Basic setup

We model team activity as contributions to a threshold public good. That is, a joint project between Player 1 and Player 2 requires a total investment of $k > 0$ for completion. The

[4] For instance, in the case of the uniform distribution on $[\underline{v}_i, \overline{v}_i]$, the pseudo-mode is $\underline{v}_i$.

payoff of Player i in the event that the two players contribute $c_1 \geq 0$ and $c_2 \geq 0$ is

$$u_i(c_1, c_2|v_i) = \begin{cases} v_i - c_i & \text{if } c_1 + c_2 \geq k \\ -c_i & \text{otherwise}. \end{cases}$$

v_i, the benefit from completion of the project to Player i is assumed to be her private information. v_1 and v_2 are independent draws of random variables distributed according to respective cumulative probability distributions F_1 and F_2 which are the players' prior beliefs. Assume that F_i admits a continuous density f_i that is positive on a non-trivial interval $[\underline{v}_i, \overline{v}_i]$. The assumption that types are independently distributed is important. Correlated types warrant a separate analysis.

Given a realization (v_1, v_2) of benefits, we say that it is *ex-post efficient* to complete the project if $v_1 + v_2 \geq k$ and *ex-post strictly efficient* if the inequality is strict. Ex-post inefficiency is defined similarly.

The above payoff specification seeks to capture two key aspects of a general team environment. First, often in a team, one member's actions are a substitute for another's actions, thereby creating an incentive to shirk. This aspect of free-riding is clearly well captured by the above payoff specification since it is the sum of players' contributions that determines whether the project succeeds. Second, in a team, different members bring distinct complementary skills which give rise to the need for a team in the first place. To emphasize this latter aspect of teams, we maintain the following restriction throughout our analysis:

$$\bar{v}_i < k \quad \text{for } i = 1, 2. \tag{1}$$

(1) captures the idea that no one player has the ability/skills (or the incentive to acquire them) to unilaterally complete the project.

It is implicit in the above payoff specification that the benefit from not undertaking the project is normalized to zero. Therefore $\underline{v}_i$ is the minimum incremental benefit from completing the project for Player i. We assume that this is non-negative. Finally, to keep the problem interesting, we assume that

$$\underline{v}_1 + \underline{v}_2 < k < \overline{v}_1 + \overline{v}_2. \tag{2}$$

It will be evident that in all the game forms we will consider, if the first inequality fails, it is easy to ensure the project's completion with probability one. If the second inequality fails, it is always ex-post inefficient to complete the project and it will never be undertaken.

To analyze the effects of different types of information, we shall study different procedures by which contributions to the joint project occur and compare the equilibria of the resulting Bayesian games. To begin, we think of transparency of a peer's actions as one player choosing how much to contribute after observing the contribution of the other. Let $\mathcal{S}_i$ denote the situation where Player i leads and chooses her contribution, which is then observed by Player j who then makes her choice. Let $\mathcal{C}$ denote the case where contributions are simultaneous, i.e. the non-transparent case. The notation is chosen to denote that contributions occur *a la* Stackelberg and Cournot respectively. The question of whether transparency of a peer's costly actions helps amounts to comparing equilibria of $\mathcal{C}$ with those of $\mathcal{S}_1$ and $\mathcal{S}_2$.

It is clear that it is not necessary to analyze $\mathcal{S}_1$ and $\mathcal{S}_2$ separately. In what follows we study $\mathcal{S} \equiv \mathcal{S}_1$ and conclusions for $\mathcal{S}_2$ are then drawn in the obvious manner.

Remark 1 (Contribution mecahnisms) *An important feature that is common to all the games that we study is that each of them is a contribution mechanism, by which we mean that a player's contribution cannot be recovered even if the project is not completed. Therefore, with little loss in generality, a player of type v_i is assumed to never contribute more than v_i in any equilibrium of the relevant game. This will be implicit in our arguments.*

Cheap talk extensions

Recall that in this paper, besides the information conveyed through the sequencing of decisions, we are also interested in its interplay with the information conveyed from non-costly actions. To model this, we need to consider cheap talk extensions of the games $\mathcal{C}$ and $\mathcal{S}$. In a cheap talk extension of either $\mathcal{C}$ or $\mathcal{S}$, prior to choosing their contributions to the project, players exchange non-binding messages. What messages are permitted is an exogenous variable. The thrust of our main result is to show that cheap talk has a greater impact under simultaneous contributions than under sequential contributions. In fact, we will be showing that simultaneous exchange of binary messages followed by a play of $\mathcal{C}$, a procedure that we denote by $\mathcal{C}^*$, is enough to dominate exchange of arbitrary messages followed by a play of $\mathcal{S}$. The game $\mathcal{C}^*$ is described below.

$\mathcal{C}^*$ is a multi-stage game of incomplete information with the following two stages:

Stage 0 Each player simultaneously chooses a message from $\{y, n\}$. (The message "y" stands for "Yes, I intend to contribute a positive amount to the project" while n stands for "No, I do not plan to contribute".)

Stage 1 The messages are made public following which both players chooses simultaneously contribute to the project.

Let us now consider introducing pre-play communication to $\mathcal{S}$. The effect of any pre-play communication in general is an (endogenous) revision of beliefs of the players' types after the communication stage. Observe that the choice of Player 2 in the second stage of $\mathcal{S}$ depends only on what Player 1 contributes and not on the latter's type. Player 1's contribution however does depend on her beliefs about Player 2's type. Therefore, in considering the effects of pre-play communication under $\mathcal{S}$, it suffices to endow only Player 2 with a message space.

A cheap talk extension of $\mathcal{S}$ is therefore, without much loss in generality, a multi-stage game $\langle M, \mathcal{S} \rangle$ where $\mathcal{M}$ is an arbitrary message space, with at least <u>two</u> elements such that:

Stage 0 Player 2 sends a message $m \in M$.

Stage 1 Player 1 observes m and chooses how much to contribute.

Stage 2 Player 2 observes Player 1's contribution and chooses how much to contribute.

In other words, under $\langle M, \mathcal{S} \rangle$, each announcement $m \in M$ by Player 1 is followed by a play of $\mathcal{S}$.

Remark 2 (Equilibrium) A play of any of the procedures $\mathcal{C}$, $\mathcal{S}$, $\mathcal{C}^*$ and $\langle M, \mathcal{S} \rangle$, together with the informational assumptions given earlier yield a multi-stage game of incomplete information as described in Chapter 8, Fudenberg and Tirole [1991]. By an equilibrium of these games we will mean "Perfect Bayesian Equilibrium" as given by Definition 8.3 in Fudenberg and Tirole [1991].[5]

2.1 Outline of the main argument

I will now outline the argument for adding a stage of pre-play communication can help in $\mathcal{C}$ but not necessarily so in $\mathcal{S}$.

Definition 1 (0-outcome) The 0-outcome refers to an outcome in which neither player makes a positive contribution regardless of her type and the project is never completed.

[5] In the case of $\mathcal{C}$, this is of course the usual Bayes Nash equilibrium.

Given (1), it is evident that under $\mathcal{C}$, it is a Bayes Nash equilibrium for neither player to contribute regardless of her type. Call this the 0-outcome. When one augments $\mathcal{C}$ with a round of pre-play communication, upon exchanging messages, players will revise their beliefs. Nevertheless, under any Bayes consistent posterior beliefs, since no one player has a willingness to complete the project unilaterally to begin with,

(A) *In $\mathcal{C}^*$, the 0-outcome is a feasible equilibrium outcome in the continuation game that follows any messages.*

Let us now consider $\mathcal{S}$. Assume $\overline{v}_1 + \underline{v}_2 > k$. If Player 1 were to contribute $k - \underline{v}_2$ and sequentially rational play will require that Player 2 complete the project. Thus, Player 1 types $v_1 \approx \overline{v}_1$ *must* contribute a positive amount regardless of their Bayes consistent posterior beliefs after exchanging messages. This allows us to conclude that

(B) *Whenever $k < \overline{v}_1 + \underline{v}_2$, in any cheap talk extension of $\mathcal{S}$, the 0-outcome is infeasible in the continuation game that follows any message.*

(A) and (B) allow us to draw the key insight. Under non-observability, (A) offers a threat that allows one to separate high type players with lower type players. Under observability of Player 1's actions, (B) tells us that such a threat is no longer available. So fewer outcomes can be sustained as a result. To illustrate, suppose that $[\underline{v}_i, \overline{v}_i] = [a, 1]$ for some a such that $1 < k < 1 + a$.

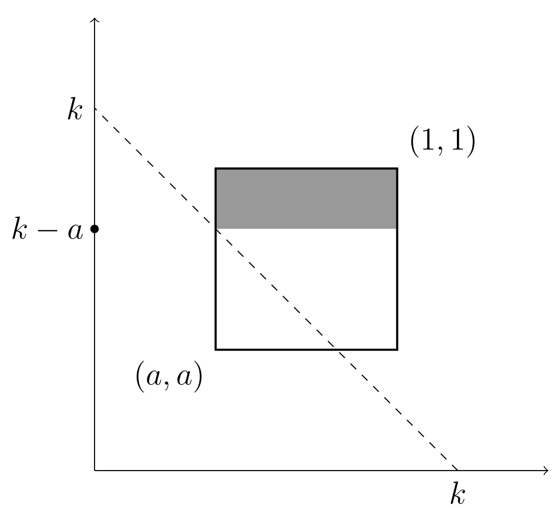

Figure 1: $\mathcal{C}^*$ has an equilibrium in which, ex-post the project is completed only in the shaded region. Not so for $\langle M, S\rangle$

The state-space is depicted as the thick edged rectangle in the Figure 1. Suppose we try to achieve the outcome in which the project is completed if and only if $v_2 \geq k - a$, i.e. in the shaded region. Outcomes where the project is completed only if the two players types lie above some respective thresholds that add up to k will shortly be defined as cost-sharing outcomes and shown to have superior welfare properties.

This can be implemented using $\mathcal{C}^*$ as follows via the following strategy profile: At the communication stage, Player 1 announces y regardless of her type while Player 2 announces y if $v_2 \geq k - a$ and n otherwise. At the contribution stage, unless both players have announced y, neither player contributes regardless of her type. Otherwise, Player 1 contributes a regardless of her type while Player 2 contributes $k - a$ if $v_2 \geq 0$ and zero otherwise. It is not hard to verify that this is a Perfect Bayesian equilibrium of $\mathcal{C}^*$. The key thing to note in this implementation is the ability of the two players coordinate on the 0-outcome when Player 2 announces n.

Can such an outcome be achieved in a the cheap talk extension $\langle S, M \rangle$ with $M = \{y, n\}$? The answer is no. Indeed, consider a candidate equilibrium configuration in which Player 2 sends the message y if and only if her type $v_2 \geq v_2^* = k - a$. When Player 1 observes the message n, her posterior is that Player 2's type lies in $[a, v_2^*]$. In the continuation game following this message, with such a posterior, Player 1 of type $\overline{v}_1$'s payoff from contributing a is $\overline{v}_1 - a > 0$, so the best response must be to contribute some positive amount. Also, given her posterior, she would never contribute less than $k - v_2^*$. Therefore, following an announcement of n by Player 2, the 0-outcome is no longer feasible and moreover, v_2^* receives a *positive* interim payoff. On the other hand, by reporting y, Player 1's posterior is that Player 2's type lies in $[v_2^*, 1]$ and hence she would never contribute more than $k - v_2^*$ — type v_2^* receives *zero* interim payoff. The lack of indifference of the payoff of v_2^* between announcing y and n shows that the posited equilibrium configuration is not possible.

Of course, in the above argument the message game is held fixed and only a particular configuration was considered. One could reasonably ask whether there are other communication schemes with possible equilibrium configurations that can overcome the above difficulties. The crux of the above argument, that there cannot exist a marginal type v_2^* who cannot remain indifferent between between sending a pair of messages both of which lead to a positive probability of the project's completion, will be shown to be valid for arbitrary communication schemes (and even for parameters that do not satisfy the inequality given in (B)).

2.2 Further preliminaries

In this section we shall collect some further definitions and notation that will be used in our analysis.

An *outcome* of $\mathcal{C}, \mathcal{C}^*, \mathcal{S}$ or $\langle M, \mathcal{S} \rangle$ is a tuple $\langle q, t_1, t_2 \rangle$ where $q : [\underline{v}_1, \overline{v}_1] \times [\underline{v}_2, \overline{v}_2] \longrightarrow [0, 1]$ and $t_i : [\underline{v}_1, \overline{v}_1] \times [\underline{v}_2, \overline{v}_2] \longrightarrow \mathbb{R}_+$. Here $q(v_1, v_2)$ is the *ex-post* probability that the project is completed when the true state is (v_1, v_2) and the contribution of Player i is $t_i(v_1, v_2)$. Thus, in the 0-outcome for example, $q(v_1, v_2) \equiv 0$ and $t_i(v_1, v_2) \equiv 0$.

Definition 2 (Cost Sharing Outcome) An outcome (q, t_1, t_2) is a cost sharing outcome if there exists a real number x such that

1. $v_1 \geq x$ and $v_2 \geq k - x$ implies i. $q(v_1, v_2) = 1$, ii. $t_1(v_1, v_2) = x$ and $t_2(v_1, v_2) = k - x$.
2. $q(v_1, v_2) = t_1(v_1, v_2) = t_2(v_1, v_2) \equiv 0$ otherwise.

We let $\mathcal{E}(x)$ denote such an outcome.

In other words, in a cost sharing outcome, whenever Player 1 and Player 2 types are above their respective thresholds of x and $k - x$, the project is completed with probability one and each pay x and $k - x$ (only if the project is completed).

We shall say that $\langle q, t_1, t_2 \rangle$ is *feasible* or is an *equilibrium outcome* for any of these games if there is an equilibrium of that relevant game in which it can be realized.

Throughout, when we say the "probability of the project's completion" we mean the *ex-ante probability* of the project's completion which is

$$E[q(v_1, v_2)] = \int_{\underline{v}_1}^{\overline{v}_1} \int_{\underline{v}_1}^{\overline{v}_1} q(v_1, v_2) \mathrm{d}F_1(v_1) \mathrm{d}F(v_2).$$

Thus, in a cost sharing outcome $\mathcal{E}(x)$, the probability of the project's completion is $(1 - F_1(x))(1 - F_2(k - x))$ and the expected total contribution is $(1 - F_1(x))(1 - F_2(k - x))k$.

3 Transparency of a peer's actions

In this section, we analyze the effect of transparency of a peer's action which entails a comparison of the equilibria of the procedures $\mathcal{C}$ and $\mathcal{S}$. We begin with a complete analysis of the game $\mathcal{S}$ and take up the actual comparisons in Section 3.1 after describing the equilibria under non-transparency, namely $\mathcal{C}$, by drawing on Agastya et al. [2007].

Recall that in $\mathcal{S}$, Player 2 contributes after she observes the contribution of Player 1. Hence a strategy of Player 1 is a function $S_1 : [\underline{v}_1, \overline{v}_1] \longrightarrow \mathbb{R}$ whereas a strategy of Player 2 is a function $S_2 : [0, k] \times [\underline{v}_2, \overline{v}_2] \longrightarrow [0, k]$ where $S_2(c, v_2)$ is her contribution when she is of type v_2 and the history is that Player 1 has contributed c.

Determination of equilibria of $\mathcal{S}$ is as usual through backward induction. We begin by calculating the reaction function of Player 2. Upon observing a contribution $c_1 \geq 0$, Player 2 of type v_2 need only decide whether to complete the project by contributing $k - c_1$ or not contribute at all. Contribution is a best response only if $v_2 \geq k - c_1$. Therefore,

$$S_2^*(c, v_2) = \begin{cases} k - c_1 & \text{if} \quad v_2 \geq k - c_1 \\ 0 & \text{otherwise,} \end{cases}$$

must be the equilibrium strategy of Player 2 in any equilibrium of $\mathcal{S}$.

Fixing this behavior of Player 2 let us discuss the incentives of Player 1. By contributing an amount c, Player 1 simply ensures that Player 2 types only above $k - c$ will contribute and complete the project. Therefore, Player 1's choice problem, taking the above optimal behavior of Player 2 as a given, can be recast as choosing a threshold type v_2 and contribute an amount, in fact $k - v_2$, so that all types of Player 2 whose value is at least v_2 will finish the project. Doing so will leave Player 1 with a payoff of

$$G(v_1, v_2) = (1 - F_2(v_2)) v_1 + v_2 - k \tag{3}$$

when she is of type v_1. Alternatively, Player 1 can choose not to contribute, which gives her a payoff of zero. Therefore, any type v_1 such that $G(v_1, v_2) > 0$ for some $v_2 \in [\underline{v}_2, \overline{v}_2]$ will contribute a positive amount and others contribute zero. Noting that $G(v_1, v_2)$ is increasing in its first argument, its maximum function

$$g(v_1) = \max_{v_2 \in [\underline{v}_2, \overline{v}_2]} G(v_1, v_2)$$

is increasing and continuous (by the well known Theorem of the Maximum). Therefore, there exists a unique $w_1^* \in [\underline{v}_1, \overline{v}_1]$ with the property that

$$\begin{array}{ll} g(v_1) < 0 & \text{if } v_1 < w_1^* \\ g(v_1) > 0 & \text{if } v_1 > w_1^*. \end{array} \tag{4}$$

Thus, every Player 1 type above w_1^* receives a positive payoff and contributes a positive amount. Those types below w_1^* are better off by not contributing. Also notice that, by continuity of g, $g(w_1^*) = 0$ whenever w_1^* is in the interior of $[\underline{v}_1, \overline{v}_1]$.

Given an equilibrium of $\mathcal{S}$, let $U_i^s(v_i)$ denote the interim payoff of type v_i player in that equilibrium of $\mathcal{S}$.

Proposition 1 *In any equilibrium of $\mathcal{S}$,*

1. *Player 1 types $v_1 < w_1^*$ do not contribute and a type $v_1 \in [w_1^*, \overline{v}_1]$ contributes a positive amount $k - \varphi(v_1)$ where*

$$\varphi(v_1) \in \operatorname{argmax}_{v_2} G(v_1, v_2).$$

Moreover, φ is non-increasing and the interim payoff of Player 1 is

$$U_1^s(v_1) = \begin{cases} g(v_1) & \text{if } v_1 \geq w_1^* \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

2. *Player 2 chooses S_2^*. Let*

$$w_2^* = \varphi(\overline{v}_1). \tag{6}$$

The interim payoff of Player 2 is

$$U_2^s(v_2) = \begin{cases} \displaystyle\int_{w_1^*}^{\overline{v}_1} \max\{v_2 - \varphi(v_1), 0\} \mathrm{d}F_1(v_1) & \text{if } v_2 \geq w_2^* \\ 0 & \text{otherwise.} \end{cases}$$

In particular, $U_2^s(w_2^*) = 0$ and if the probability of completion is positive, $U_2^s(\overline{v}_2) > 0$.

Remark 3 It is worth drawing attention in particular to the interim payoff of Player 2 types w_2^* and $\overline{v}_2$ in an equilibrium. Part 2 shows that the payoff of the infimum (w_2^*) of all Player 2 types that contribute a positive amount is necessarily driven to the individually rational payoff of zero while the highest type necessarily derives a positive rent. This property plays a crucial role in determining the structure of equilibria of cheap talk extensions of $\mathcal{S}$ and holds even if F_2 does not admit a positive density everywhere on $[\underline{v}_2, \overline{v}_2]$.

Although Proposition 1 completely characterizes all equilibria of $\mathcal{S}$, it does not reveal whether an equilibrium with a positive completion probability can occur or whether 0-outcome is the unique equilibrium outcome. Fortunately, it is possible to offer a simple sufficient condition on F_2 to address these issues. To develop such a condition, we introduce the following (new) concept for probability distribution functions.

Definition 3 (Pseudo-mode) The pseudo-mode of the probability distribution F_i, denoted by μ_i, is the smallest x such that its density is non-increasing everywhere to the right of x.

The idea of a pseudo-mode is closely aligned to the notion of the mode of a distribution. For instance when the distribution is uni-modal, the density is increasing everywhere to the left and decreasing everywhere to the right of the unique mode. In this case the mode is the same as the pseudo-mode. That the two concepts are different can be seen by considering the uniform distribution. The mode is no longer unique. Yet, since the density is a constant, the pseudo-mode is uniquely determined as the lower end of the support. Also note that the restriction of F_i to $[\mu_i, \overline{v}_i]$ is (weakly) concave.

Proposition 2 *$\mathcal{S}$ admits an equilibrium in which the probability of completion is positive if and only if*

$$G(\overline{v}_1, v_2) > 0 \textit{ for some } v_2 \in [\underline{v}_2, \mu_2]. \tag{7}$$

Proof of Proposition 2. Suppose (7) holds, then $g(\overline{v}_1) > 0$ which implies $w_1^* < \overline{v}_1$. By Proposition 1, all types $[w_1^*, \overline{v}_1]$ will contribute a positive amount leading to an positive ex-ante probability of the project's completion. Conversely, suppose (7) fails. From the definition of pseudo-mode, we note that in the region $[\mu_2, \overline{v}_2]$, the function $G(\overline{v}_1, \cdot)$ is weakly convex and hence $G(\overline{v}_1, v_2) \leq \max\{G(\overline{v}_1, \mu_2), G(\overline{v}_1, \overline{v}_2)\}$ for all $v_2 \in [\mu_2, \overline{v}_2]$. Now using (1) and the fact that (7) fails, we conclude $G(\overline{v}_1, v_2) \leq 0$ for all $v_2 \in [\underline{v}_2, \overline{v}_2]$, i.e. $g(\overline{v}_1) \leq 0$. This in turn means $w_1^* = \overline{v}_1$ and by Proposition 1, the project is never completed. ■

The following corollaries are two easy consequences of Proposition 2 that offer sufficient conditions concerning the (positive) probability of completion of the project.

Corollary 1 *Suppose Player i's action is transparent, i.e. consider $\mathcal{S}_i$.*

1. *If it is ex-post inefficient to complete the project when Player i is the highest type and Player j's type is the pseudo-mode, i.e. $\overline{v}_i + \mu_j \leq k$, then the unique equilibrium outcome is the 0-outcome.*

2. *If it is ex-post strictly efficient to complete the project when Player i is highest type and Player j is the lowest type, i.e. $\overline{v}_i + \underline{v}_j > k$, then in every equilibrium, there is a positive probability of completion.*

Proof of Corollary 1. Observe that $G(v_1, v_2) \leq v_1 + v_2 - k \leq v_1 + \mu_2 - k$ for all $v_1, v_2 \in [\underline{v}_2, \mu_2]$. Given our hypothesis in Part 1, (7) is violated. Applying Proposition 2 gives the result. To prove Part 2, simply note that $G(v_1, \underline{v}_2) = v_1 + \underline{v}_2 - k$ which is positive given our hypothesis. So (7) holds and we again apply Proposition 2 to complete the proof.l

Observe that if the probability distribution is concave (or uniform), then its density is non-increasing throughout the support. We can then refine the previous corollary as follows. First, let

$$r_i = \overline{v}_i - \underline{v}_i$$

be the range of the original distribution F_i and set

$$\kappa \;=\; k - \underline{v}_1 - \underline{v}_2. \tag{8}$$

κ is the cost of completion of the project net of minimal possible total benefit the two players assign to the project's completion.

Corollary 2 (Concave Priors) *Suppose Player i's action is transparent and F_j is concave. The unique equilibrium outcome is the 0-outcome if $r_i \leq \kappa$ and is otherwise the cost-sharing outcome $\mathcal{E}(k - \underline{v}_2)$ if $i = 1$ and $\mathcal{E}(\underline{v}_1)$ if $i = 2$.*

Note that the condition $r_i > \kappa$ for a positive completion probability can be rewritten as as $\overline{v}_i + \underline{v}_j > k$. In other words, with a concave prior, there is a positive completion probability only if it is ex-post strictly efficient to complete the project when the player whose action is transparent is the highest type and the follower is the lowest type.

Proof of Corllary 2. Let $i = 1$. The claim about the zero outcome is simply the first part of the previous Corollary since with a concave prior $\mu_2 = \underline{v}_2$. Next, for any v_1, $G(v_1, \cdot)$ is convex in its second argument and must achieve a maximum at one of the two extreme points $\overline{v}_2$ or $\underline{v}_2$. Therefore, in an equilibrium of $\mathcal{S}$, Player 1 either contributes $k - \underline{v}_2$ or does not contribute at all, from which we conclude that $w_1^* = k - \underline{v}_2$ resulting in the cost sharing outcome $\mathcal{E}(k - \underline{v}_2)$.

3.1 Whose actions (if any) must be transparent?

We now turn to the question of whether transparency of a peer's action helps and if so which player's action must be made transparent. This question is of course related to whether sequentiality of voluntary contributions to a public good dominate simultaneous choices, the subject of a rich literature since Varian [1994], Admati and Perry [1991]. There, the leader knows that the follower's preferences and the knowledge the follower cannot free-ride lowers the leaders incentive to contribute. The total contributions under sequential behavior is lower. (See Marx and Matthews [2000] however.) Here on the other hand, the leader is unsure of the follower's incentives to complete a project, given her choice. Therefore, whether sequencing decisions benefits or hurts is a priori not clear. Furthermore, if it is indeed the case that sequential contributions dominate, what features of the type distributions F_1 and F_2 determine should be the leader?

The comparison across different formats may be made across several criteria – the interim Pareto optimality, the total expected contributions or the overall probability of completion. To progress further, we borrow a result from Agastya et al. [2007] concerning $\mathcal{C}$. There, it was shown that the existence of an equilibrium with a positive probability of completion in $\mathcal{C}$ is equivalent to the existence of a pair of types (v_1, v_2) such that $H(v_1, v_2) > 0$ where

$$H(v_1, v_2) \;=\; (1 - F_2(v_2))v_1 + (1 - F_1(v_1))v_2 - k \tag{9}$$

When F_1 and F_2 are concave, H is convex and hence its possible maxima include $(\underline{v}_1, \underline{v}_2)$, $(\overline{v}_1, \underline{v}_2)$, $(\underline{v}_1, \overline{v}_2)$ and $(\overline{v}_1, \overline{v}_2)$ with the respective values under H being $\underline{v}_1 + \underline{v}_2 - k$, $\overline{v}_1 - k$, $\overline{v}_2 - k$ and $-k$. This observation, along with (1) and (2)) gives us:

Fact 1 *Suppose F_1 and F_2 are concave. Under non-transparency, i.e. in the game $\mathcal{C}$, the unique equilibrium outcome is the* 0*-outcome.*

Comparing Fact 1 and Corollary 2, it is immediate that, with concave priors, transparency of Player i's action dominates non-transparency across all the above mentioned criteria whenever $r_i > \kappa$. At least under the assumption of concave priors, the issue then is really *which* player's action should be made transparent rather than *whether* a players' actions should be transparent. To answer this question, we introduce the following.

Definition 4 (Normalized distribution) Given a probability distribution F whose support is $[\underline{v}, \overline{v}]$, its normalized probability distribution function is $\hat{F}$ where $\hat{F}(x) = F(\underline{v} + x)$ for all $x \geq 0$.

It is common-knowledge that Player i values the project at least $\underline{v}_i$. $\hat{F}_i(x)$ is therefore the likelihood that she values the project an additional x units. Note that the support of the normalized distribution $\hat{F}_i$ is $[0, r_i]$. Also writing $\underline{\kappa} = \max\{0, r_1 - \underline{v}_2, r_2 - \underline{v}_1\}$, the conditions (1)-(2) may be re-expressed compactly as

$$\underline{\kappa} \; < \; \kappa \; < \; r_1 + r_2 \tag{10}$$

Recall that $\hat{F}_i$ is said to first order stochastically dominate $\hat{F}_j$ if $\hat{F}_i(x) \leq \hat{F}_j(x)$ for all $x \geq 0$, with a strict inequality for some x.

Proposition 3 *Suppose the type distributions are concave and $\hat{F}_i$ first order stochastically dominates $\hat{F}_j$. Then, Player i's choice should be transparent to achieve*

1. *the highest probability of completion and*
2. *the highest expected total contribution.*

Moreover, this sequence is uniquely optimal unless $\kappa \geq \max\{r_1, r_2\}$, in which case the 0-outcome is the unique equilibrium outcome under all transparent and non-transparent behavior.

Remark 4 (Ordering under uniform priors) When F_1 and F_2 are the uniform distributions, then $\hat{F}_i(x) = x/r_i$ and therefore $\hat{F}_i$ stochastically dominates $\hat{F}_j$ if and only if $r_i > r_j$. According to Proposition 3, it is optimal for the player with the largest range to be the leader.

Proposition 3 is summarized in Figure 2. First note that when $\hat{F}_i$ first order stochastically dominates $\hat{F}_j$, then Player i has a larger range of types, i.e. $r_i > r_j$. When $\kappa \geq r_i$ (region **C**), then an application of Corollary 2 tells us that the cost of completion is too high and regardless of which player's action is transparent, the unique outcome is the 0-outcome, just as is it is with non-transparency. In the intermediate range of $r_j \leq \kappa < r_i$ (region **B**), again from Corollary 2, the unique outcome is the 0-outcome unless Player i is the leader. It is only when $\kappa \leq \min\{r_1, r_2\}$ (region **A**), that there is a positive probability of the project's completion regardless of whether Player 1 or Player 2 contributes. It is here that stochastic dominance of $\hat{F}_i$ over $\hat{F}_j$ plays a role in determining that Player i should be the leader.

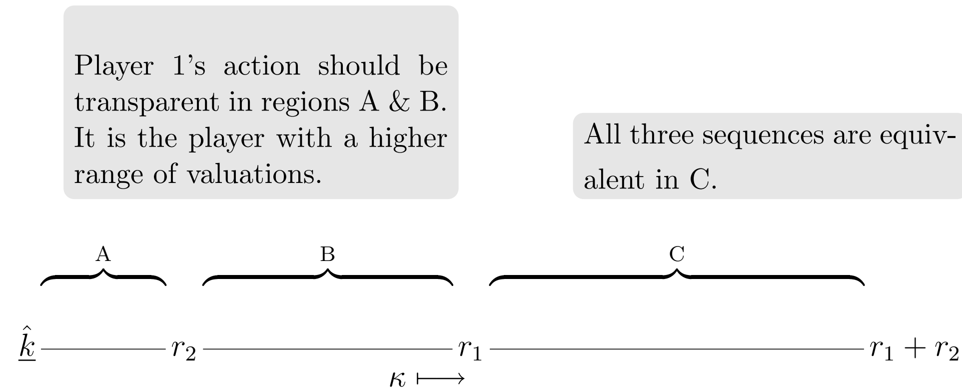

Figure 2: Ranking of different sequences without pre-play communication

The ranking of the different procedures is not as clear cut across the different criteria when we drop the assumption of concavity of the priors. The following Proposition however suggests that non-transparency cannot be an unequivocally superior choice.

Proposition 4 ($\mathcal{C}$ vs. $\mathcal{S}$) *Relative to non-transparency (the game $\mathcal{C}$), under transparency of Player i's action, every type of Player i who contributes a positive amount is better off. (The remaining types of Player i are indifferent.)*

The proposition is intuitive. In $\mathcal{C}$, if at all a Player 2 type contributes a positive amount, her contribution is strictly below her valuation. Therefore, to induce type v_2 and above to contribute, Player 1 would have to be contributing more than $k - v_2$. On the other hand in $\mathcal{S}$, the leader, namely Player 1, need only contribute $k - v_2$ to induce the same outcome. This insight leads to the conclusion that the leader is unequivocally better off. Unfortunately, it is not possible to use a similar argument to compare either the probability of completion or the total expected effort. For, in $\mathcal{C}$, since the contributions of one player cannot conditioned on those of another, the total ex-post contribution at some state (v_1, v_2) may be more or less than k. Under $\mathcal{S}$ on the other hand, the total contribution is either k or less than k at all realizations of v_1 and v_2. Therefore, the comparison between the expected contributions remains ambiguous.

4 Pre-play Communication

In the previous section we studied the implications of information transmission through costly actions. In this section, we shall study how the introduction of a channel for costless information transmission influences those conclusions. For this we need to study the equilibria of $\mathcal{C}^*$ and $\langle M, \mathcal{S} \rangle$ and compare them with the results obtained in the previous section. We begin by collecting some of the results on $\mathcal{C}^*$ from Agastya et al. [2007] in Section 4.1. Then we analyze $\langle M, S \rangle$ in Section 4.2. The comparisons with and without pre-play communication are then taken up in Section 4.3.

4.1 Cheap talk & non-transparent actions

Consider the following set of cost sharing outcomes

$$\mathbb{E} \;=\; \big\{\mathcal{E}(x) \,:\, k - \overline{v}_2 \leq x \leq \overline{v}_1\big\}. \tag{11}$$

The probability of completion in a cost sharing outcome is $(1 - F_1(x))(1 - F_2(k - x))$. $\mathbb{E}$ contains all the cost sharing outcomes that lead to a positive probability of completion of the project.

The following Fact is the combination of Proposition 15, 16 and 18 in Agastya et al. [2007].

Fact 2 *Consider the simultaneous contributions game $\mathcal{C}$ and its binary message cheap talk extension $\mathcal{C}^*$.*

1. *For every equilibrium of $\mathcal{C}$, there exists a cost sharing outcome in $\mathbb{E}$ that a). is Pareto-dominant in the ex-ante and interim sense, b). leads to a higher probability of completion and c). yields a expected higher total contribution.*
2. *Every cost sharing outcome in $\mathbb{E}$ is an equilibrium outcome of $\mathcal{C}^*$. Moreover, if F_1 and F_2 are concave, these are its only equilibrium outcomes.*

The two statements taken together show that introduction of a round of pre-play communication to non-transparent behavior can lead to superior outcomes. In a cost sharing outcome, the payment of one player depends on the type of the other player. As such, it cannot be supported as an equilibrium outcome of $\mathcal{C}$ with the choices being simultaneous. With the addition of a round of pre-play communication, it is possible to achieve such a coordination. That every element of $\mathbb{E}$ is feasible under $\mathcal{C}^*$ is easily verified. (Alternatively see Agastya et al. [2007].) Of greater interest is the Pareto superiority of these outcomes over the equilibria of $\mathcal{C}$, i.e. Part 1 above. It turns out that the main reason for this is that in any equilibrium of $\mathcal{C}$, the infimum of Player i types that contributes a positive amount, denote by $\hat{v}_i$, is such that $\hat{v}_1 + \hat{v}_2 > k$. Using the fact (well known since Myerson [1981]) that payoff depends primarily on the ex-post completion probabilities (see Fact 3 given in the Appendix), the assertion follows.

4.2 Cheap talk & transparent actions

Let us now consider $\langle M, \mathcal{S} \rangle$. When it is Player 2's turn to make a contribution, the history of the game is a pair $h = (m, c)$, which represents that the message m has been sent in the communication stage and Player 1 has contributed c. Player 2's behavior after any such history is clear: contribute $k - c$ and complete the project if $v_2 \geq k - c$ and do not contribute otherwise. Therefore, in what follows, the description of Player 2's strategy in the game will be confined to her behavior at the communication stage.

A strategy of Player 2 is then a function $\sigma : [\underline{v}_2, \overline{v}_2] \longrightarrow M$ where $\sigma(v_2)$ is the message she sends when she is of type v_2. Most generally, one would allow any measurable σ. Instead, to avoid technical complications that would shift the focus of this paper (and those being beyond my ability to resolve), I will assume the following: a σ is permitted as a strategy only if the set of types that send any given message m, namely $\sigma^{-1}(m)$, is the union of at most finitely many intervals. This allows us to exclude pathological situations where Player 2 sends one message if her type is a rational number and another message otherwise. The above restriction on permitted strategies is however general enough to allow behavior that ranges from fully revealing strategies to cases where a pair of types $v_2 < v_2'$ send the same message while other types intermediate to them (v_2, v_2') send a different message.

Proposition 5 *Consider an arbitrary cheap talk extension $\langle M, \mathcal{S} \rangle$ of $\mathcal{S}$.*

1. *In any equilibrium, there can be at most one message (sent with a positive probability) following which the project is completed with a positive probability.*

2. *If $r_1 > \kappa$, all types of Player 2 send the same message – the game $\langle M, \mathcal{S} \rangle$ and $\mathcal{S}$ are outcome equivalent.*

3. *Suppose $\overline{v}_1 + \mu_2 \leq k$. The set of equilibrium outcomes is $\mathbb{E}$.*

The reasoning behind Part 1 may be laid bare by considering the possibility of a situation where a set of types $[a, b] \subseteq [\underline{v}_2, \overline{v}_2]$ send one message, say m_1, and the set of types $[b, c] \subseteq [\underline{v}_2, \overline{v}_2]$ choose another message say m_2. Assume, by way of contradiction, that in the continuation game following both the messages, the project has a positive completion probability. Observe that the marginal type m_1 must be indifferent to reporting either m_1 or m_2. However note that if she should report m_1, in the continuation game, as per Player 1's posterior, she is the highest of all possible types. By Part 2 of Proposition 1, her interim payoff is positive. On the other hand, if she reports m_2, she would be regarded as the smallest possible type in Player 1's posterior. Using Part 2, Proposition 1 again, her interim payoff would be zero and provides the desired contradiction. The argument in the formal proof has to account for the fact that the set of types that send a given message need not be an interval.

Part 2 and Part 3 correspond to situations we considered in Corollary 1. Part 3 is the case where, without pre-play communication the unique equilibrium is for no player to contribute. The feasibility of the 0-outcome is the feature that enables one to divide Player 2 types into two sets, one that sends a high message following which there is a positive completion probability and the other, which coordinates on the now feasible 0-outcome. Indeed, any cost sharing outcome in $\mathbb{E}$ may now be implemented. Part 2 represents a situation where, absent pre-play communication, there is a positive probability of completion. Interestingly, no matter however small this probability of completion is, it completely undermines all information transmission through pre-play communication.

4.3 Transparency vs. non-transparency under cheap talk

We can now ask whether transparency continues to dominate non-transparent behavior when pre-play communication is also a possibility. Virtually all the information that is needed to make the comparison is already available from the several propositions and corollaries we have proved thus far. The conclusions will therefore be given as a series of remarks rather than presenting them as further results. The punchline can be seen from Figure 3. Assume throughout that F_1 and F_2 are concave so that Corollary 2 remains applicable.

Let us begin with the case where $r_i > \kappa$. An application of Part 2, Proposition 5 shows that allowing pre-play communication has no impact if Player i's action is transparent. The unique equilibrium outcome is the $\mathcal{E}(k - \underline{v}_2)$ or $\mathcal{E}(\underline{v}_1)$ depending on whether $i = 1$ or $i = 2$. On the other hand, *any* cost sharing outcome $\mathcal{E}(x) \in \mathbb{E}$ may be achieved through a play of $\mathcal{C}^*$. Recalling that the probability of completion in a cost-sharing outcome is $(1 - F_1(x))(1 - F_2(k - x))$, we conclude that except in a non-generic situation, introduction of cheap talk under non-transparent behavior strictly dominates transparency of Player i's action even if cheap talk is allowed as far as completion probability is concerned.

When $r_i < \kappa$, appealing to Part 3, Proposition 5, we note that the introduction of cheap talk when Player i's actions are transparent, enlarges the set of equilibrium outcomes to $\mathbb{E}$ just as in the case with $\mathcal{C}^*$. We collect these observations in the following figure.

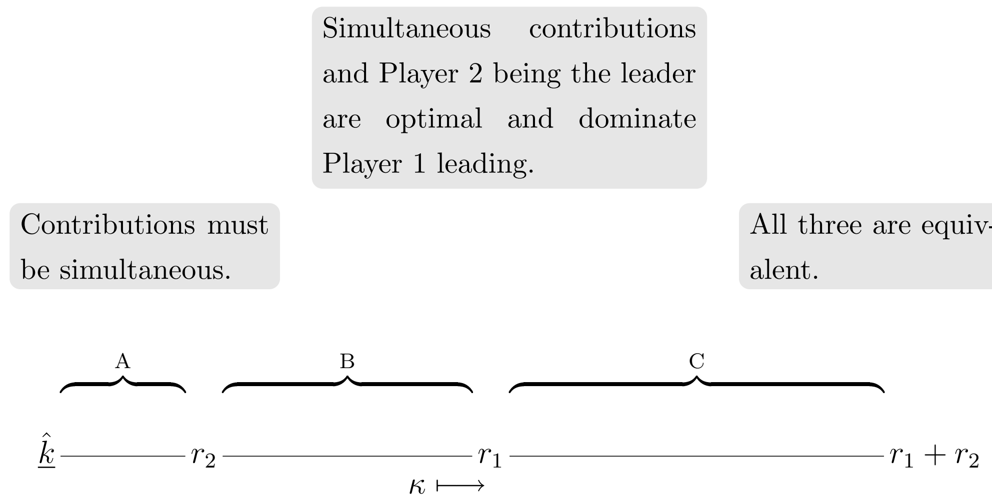

Figure 3: Ranking of different sequences with pre-play communication and concave priors

5 Conclusion

In environments where individual contributions to a project are compensated only if the total contribution reaches a threshold, we have addressed the impact of two channels of information transmission on the incentive to contribute. We have shown that information conveyed through transparency of a players' (costly) action is often beneficial in two ways. First, it improves the incentives of the later movers to contribute which in turn improves the incentive of the leader to contribute as well. Second, sequential rationality required under transparency refines the set of equilibria. We have shown that this latter attractive feature however, is precisely responsible for limiting the effectiveness of transparency in the presence of a second channel of information transmission, namely cheap talk. These we believe, are important insights for organization of teams for undertaking joint projects and given the formal equivalence of the underlying games, to provision of voluntary contributions to joint projects.

There are three aspects of the above analysis which may seem to restrict the applicability of the above analysis. First, the role of concave or uniform priors. Our intention here is to emphasize as setting in which communication has a significant impact when there is no observability but is very limited if there is observability of players actions. Concave or uniform priors provide a simple sufficient (and sometimes necessary) condition for the equilibrium outcome to be the 0-outcome absent communication or observability. Economically these assumptions mean that higher types are relatively sparse (since strictly concave prior would mean that that its mode is the lower end of the support). Although the qualitative nature of our ranking can be rescued for somewhat more general assumptions (since for example, the key elements in such a comparison namely Proposition 2 and Proposition 4 do not rely on concavity), the increased complexity is unlikely to deliver different insights.

Second, here the payoff of a player depends only on her type. So, the only uncertainty that is being resolved for the second mover is whether the former will contribute. One might consider a more general model in which a player learns more about her own type upon observing the actions of the other player. Third, this paper considers arbitrary one-shot communication schemes. A natural question to consider is whether both players can be made better off by allowing further rounds of communication. What is the impact of continuing conversations (a la Aumann and Hart [2003]) with possible interim contributions? These are substantial generalizations beyond the scope of the present work. They will be the subject of future investigations.

Appendix

Given an outcome (q, t_1, t_2), the *interim* probability that the project is completed conditional on Player i being of type v_i is

$$q_1(v_1) = \int_{\underline{v}_1}^{\overline{v}_1} q(v_1, s)\,\mathrm{d}F(s) \quad \text{and} \quad q_2(v_1) = \int_{\underline{v}_2}^{\overline{v}_2} q(s, v_2)\,\mathrm{d}F(s).$$

The following fact is a well-known (implication of incentive compatibility for direct mechanisms) since Myerson (1984).

Fact 3 *If $\langle q, t_1, t_2 \rangle$ is an equilibrium outcome of $\mathcal{C}, \mathcal{C}^*, \mathcal{S}$ or $\langle M, \mathcal{S} \rangle$, the corresponding interim payoff of Player i is*

$$\begin{aligned} U_i(v_i) &= \int_{\underline{v}_i}^{v_i} q_i(\hat{v}_i)\,\mathrm{d}\hat{v}_i + U_i(\underline{v}_i) \quad \forall\, v_i \\ \textit{and} \qquad q_i(v_i) &\geq q_1(v_i') \qquad \textit{whenever } v_i > v_i'. \end{aligned}$$

In particular, $U_i(v_i)$ is non-decreasing.

Proof of Proposition 1. Part 1 is evident from the discussion that preceded the Proposition, apart from the assertion that φ is non-increasing. To see this, we note that in this equilibrium, the interim probability of project's completion for any type $v_1 \geq w_1^*$ is $q_1(v_1) = 1 - F(\varphi(v_1))$, which we know from Fact 1, is non-decreasing. Part 2, the statement regarding U_2^s is also immediate. It is also clear from that equation that $U_2^s(\overline{v}_2) > 0$ provided the project has a positive completion probability since in this case $w_1^* < \overline{v}_1$. It remains to show that $U_2^s(w_2^*) = 0$. Since φ is non-increasing, $k - w_2^*$ is the highest possible contribution of Player 1. By (1), all Player 2 types $v_2 \leq w_2^*$ must therefore receive a zero payoff. ■

Proof of Proposition 4. In $\mathcal{C}$, a strategy profile is a pair (C_1, C_2) where $C_i : [\underline{v}_i, \overline{v}_i] \longrightarrow [0, k]$. In an equilibrium, C_i can be shown to be non-decreasing and right continuous[6]. Given an equilibrium and a v_1 such that $C_1(v_1) > 0$, let $\phi(v_1) = \inf\{v_2 \mid C_1(v_1) + C_2(v_2) \geq k\}$. Then, the payoff of this type is non-negative and equals

$$\begin{aligned} v_1 (1 - F_2(\phi(v_1))) - C_1(v_1) &\leq v_1 (1 - F_2(\phi(v_1))) + C_2(\phi(v_1)) - k \\ &< v_1 (1 - F_2(\phi(v_1))) + \phi(v_1) - k \\ &\leq g(v_1). \end{aligned}$$

The first of the inequalities is due to the fact that $C_1(v_1) + C_2(\phi(v_2)) \geq k$ while the second inequality follows from the fact that $\phi(v_1)$ type of Player 2 must contribute less than her value for the project. Further, $g(v_1)$, as noted in Part 2, Proposition 1, is the equilibrium payoff of a type v_1 player who makes a contributes positive amount under $\mathcal{S}$. ■

Proof of Proposition 3. From Fact 1, for the relevant values of $\hat{k}$ simultaneous contributions necessarily result in the 0-outcome. On the other hand, if Player 1 moves first, by Corollary 2, the probability of completion is $(1 - F_1(k - \underline{v}_2)) = (1 - \hat{F}_1(\kappa))$. The probability is $(1 - F_2(k - \underline{v}_1)) = (1 - \hat{F}_2(\hat{k}))$ if Player 2 moves first. First order stochastic dominance of $\hat{F}_1$ over $\hat{F}_2$ shows that the former sequence has a higher probability of completion. Since in the equilibria of the sequential contributions, exactly k is contributed whenever the leader contributes a positive amount, the claim regarding total expected contribution also follows. $\kappa \geq r_1$ being the same as $k \geq \overline{v}_1 + \underline{v}_2$ (which also means $k \geq \overline{v}_2 + \underline{v}_1$ since $r_1 > r_2$) only the 0-outcome entails for all sequences of moves. ■

[6] That it should be non-decreasing follows from incentive compatibility and Fact 3. A monotonic function on a compact set can have only finitely discontinuities. Changing the function C at these points is a change on a set of measure zero and has no impact on the equilibrium. Therefore, there is no loss in generality in assuming that C_2 is right continuous.

Proof of Proposition 5. Part 1 Fix an equilibrium (S_1, σ) of a cheap talk extension $\langle M, \mathcal{S} \rangle$ in which there is a positive probability of completion of the project. Let $U_2(v_2)$ be the interim payoff of a type v_2 player in this equilibrium and set $w = \inf\{v_2 \ : \ U_2(v_2) > 0\}$ and $\bar{m} = \sigma(w)$.

By way of contradiction, let $m' \neq \sigma(w)$ be a message such that in the continuation game following m', denoted by $\mathcal{S}(m')$, the project is completed with a positive probability. $S(m')$ is the contribution procedure $\mathcal{S}$ but Player 1's beliefs are given by the posterior distribution, denote it by G, whose support is a union of finitely intervals:[7]

$$[a_1, b_1] \cup \cdots \cup [a_i, b_i] \cup \cdots \cup [a_L, b_L],$$

where $a_i < b_i < a_{i+1}$ for all i. Let $[a_\ell, b_\ell]$ be the first of these intervals that lies to the right of w. Now I claim that the equilibrium payoff of type a_ℓ must be zero. For, letting $C(v_1)$ denote the equilibrium contribution of a type v_1 player in $\mathcal{S}(m')$, we note that for a generic v_1, we must have $C(v_1) \leq k - w'$ — otherwise there would be types $v_2 < w'$ such that $U_2(v_2) > 0$. But given that $[w', a_\ell)$ is not in the support of G, by contributing any amount in $[k - a_\ell, k - w']$, Player 1 can ensure that the project is completed with a probability $(1 - G(a_\ell))$. Therefore, we must in fact have $C(v_1) \leq k - a_\ell$, which in turn makes the interim payoff of a_ℓ equal zero. If $a_\ell > w$, we contradict the definition of w.

On the other hand if $a_\ell = w$, it means type w is indifferent between reporting $\bar{m}$ and m'. Therefore we can repeat the above arguments by setting $\bar{m} \equiv = m' = \sigma(w)$ and arrive at a similar contradiction.

Part 2. $r_1 > k$ is equivalent to $\overline{v}_1 + \underline{v}_2 > k$. Let $\sigma(\overline{v}_2) = \bar{m}$. Since this message is being sent by the highest type, and there is an overall positive probability of completion, probability of project's completion conditional on the message $\bar{m}$ being sent is positive. Let if possible m' be another message that is sent by a positive mass of Player 2 types. Following announcement of m', let $\underline{v}_2'$ be the lower end of its support of Player 1's posterior. In the continuation game after m', by contributing $k - v_2'$, Player 1 types $v_1 \approx \overline{v}_1$ get an approximate (and positive) payoff of $\overline{v}_1 + v_2' - k$. Therefore, following the announcement m' too, there is a positive probability of the project's completion, in contradiction to Part 1.

Part 3. Choose any $\mathcal{E}(x) \in \mathbb{E}$ with $x \neq \overline{v}_1$ and set $d = k - x$ and hence $d \geq \mu_2$. M contains at least two messages, say $\bar{m} \neq m'$, by assumption. Consider the strategy where Player 2 plays $\sigma(v_2) = \bar{m}$ and $\sigma(v_2) = m'$ if $v_2 \geq d$ and $v_2 < d$ respectively. Upon hearing

[7] I am writing the support as the union of *closed* intervals. This is no loss in generality as a type at an edge will be indifferent between sending a pair of messages.

the message $\hat{m}$, the Bayes rational posterior beliefs of Player 1 is the truncation of F_2 to $[d, \overline{v}_2]$, denoted by $\hat{G}_2$. From the definition of μ_2, $\hat{G}_2$ is concave and by Part 2, Corollary 2, the sequentially rational play yields $\mathcal{E}(x)$. Following the message $\hat{m}$, the posterior is the truncation F_2 to $[0, d]$. Using Part 1, Corollary 1, in this case the 0-outcome must result. Putting these together, we conclude that the equilibrium outcome is indeed $\mathcal{E}(x)$.

To prove the converse, we use Part 1 of this Proposition. Accordingly, there is a d such that $[d, \overline{v}_2]$ who send the message $\bar{m}$ and the project has a positive completion probability in the continuation game only when $\bar{m}$ is announced. If $d \leq \mu_2$, then applying Part 1, Corollary 1, the unique outcome is the 0-outcome, contrary to the definition of $\bar{m}$. If $d \geq \mu_2$, repeat the argument in the previous paragraph to conclude that $\mathcal{E}(k - d)$ is the outcome following the message $\bar{m}$. Since the probability of completion is zero after all other messages, the overall outcome is $\mathcal{E}(k - d)$. The proof is complete on recalling that Player 1 would never contribute more than $\overline{v}_1$ and hence $d \geq k - \overline{v}_1$. ■

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