Emerging Markets and Heavy Tails*

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*Rustam Ibragimov gratefully acknowledges partial support by the National Science Foundation grant SES-0820124, Clark and Warburg Research Funds (Department of Economics, Harvard University) and a Harvard Academy Junior Faculty Development grant. Marat Ibragimov gratefully acknowledges support by a grant R08-1123 from the Economics Education and Research Consortium (EERC), with funds provided by the Global Development Network and the Government of Sweden. The authors thank Xavier Gabaix, Rufat Khamidov, Vladimir Popov and the participants at the 27th EERC Research Workshop at Kyiv School of Economics and the seminars at the University of Cambridge, Risk Metrics Group London and MIT for helpful comments and discussions.

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Abstract

Emerging economic, financial and foreign exchange markets are subject to more extreme external and internal shocks than their developed counter-parts. The higher degree of volatility suffered by these economies leads to the expectation that heavy-tailedness properties for key variables in these markets, including foreign exchange rates, will be more pronounced. We focus on this hypothesis using recently proposed robust tail index estimation methods applied to data on exchange rates for a number of developed and emerging economies. We find that the tail indices indeed tend to be smaller in emerging foreign exchange markets compared to the developed economies. In particular, the tail index estimates obtained indicate that, in contrast to developed economies, variances may be infinite for foreign exchange rates in the most of emerging economies considered. The obtained empirical results have important implications for a number of economic, financial and econometric models and economic policy decisions and forecasting.

JEL Classification: C13, F31, F37

Keywords: Heavy-tailedness, power laws, tail indices, robust estimation, log-log rank-size regression, foreign exchange rates, emerging markets, developing markets, post-Soviet economies, Asian economies, financial crisis

1 Introduction

1.1 Heavy tails in economics and finance

Numerous contributions in economics, finance, risk management and insurance have indicated that distributions of many variables of interest in these fields exhibit deviations from Gaussianity, including those in the form of heavy tails (see, among others, the discussion and reviews in Embrechts, Klüppelberg & Mikosch, 1997; Rachev, Menn & Fabozzi, 2005; Gabaix, 2009; Ibragimov, 2009*a*, and references therein). This stream of literature goes back to Mandelbrot (1963) (see also Fama, 1965, and the papers in Mandelbrot, 1997) who pioneered the study of heavy-tailed distributions in economics and finance.

In models involving a heavy-tailed risk or return r it is usually assumed that the distribution of r has power tails, so that

$$P(r > x) \sim \frac{C_1}{x^{\zeta_1}}, \tag{1}$$

$$P(r < -x) \sim \frac{C_2}{x^{\zeta_2}}, \tag{2}$$

$\zeta_1, \zeta_2 > 0$, $C_1, C_2 > 0$, as $x \to +\infty$, that implies, with $\zeta = \min(\zeta_1, \zeta_2)$,

$$P(|r| > x) \sim \frac{C}{x^{\zeta}}, \tag{3}$$

$C > 0$, as $x \to +\infty$ (here and throughout the article, $f(x) \sim g(x)$ as $x \to +\infty$ means that $\lim_{x\to+\infty} \frac{f(x)}{g(x)} = 1$). The parameter ζ in (1)-(3) is referred to as the tail index, or the tail exponent, of the distribution of r. An important property of risks or returns r satisfying (1)-(3) is that the absolute moments of r are finite if and only if their order is less than $\zeta = \min(\zeta_1, \zeta_2) : E|r|^s < \infty$ if $s < \zeta$ and $E|r|^s = \infty$ if $s \geq \zeta$. In particular, risks or returns r that follow (1)-(3) with $\zeta \leq 2$ have infinite second moments: $Er^2 = \infty$. If (1)-(3) hold with $\zeta \leq 1$, then the first absolute moment of r is infinite: $E|r| = \infty$.

Many recent studies argue that the tail indices ζ in heavy-tailed models (3) typically lie in the interval $\zeta \in (2, 5)$ for financial returns on various stocks and stock indices in developed economies (see, among others, Jansen & de Vries, 1991; Loretan & Phillips, 1994 and Gabaix, Gopikrishnan, Plerou & Stanley, 2006). Among other results, Gabaix et al. (2006) provide empirical estimates that support heavy-tailed distributions (3) with tail indices $\zeta \approx 3$ for financial returns on many stocks and stock indices in developed countries and markets. Tail indices $\zeta \in (2,4)$ imply, in particular, that the returns have finite variances; however, their fourth moments are infinite. Power laws (3) with $\zeta \approx 1$ (Zipf laws) have been found to hold for firm sizes and city sizes (see Gabaix, 1999; Axtell, 2001). As discussed by Nešlehova, Embrechts & Chavez-Demoulin (2006), tail indices less than one are observed for empirical loss distributions of a number of operational risks. Silverberg & Verspagen (2007) report the tail indices ζ to be significantly less than one for financial returns from technological innovations. The analysis in Ibragimov, Jaffee & Walden (2009) indicates that the tail indices may be considerably less than one for economic losses from earthquakes and other natural disasters.

1.2 Determinants of heavy tails and volatility in stock and foreign exchange markets

Gabaix et al. (2006) and several subsequent contributions propose theoretical models that explain the observed heavy-tailed patterns with tail indices $\zeta \approx 3$ in financial returns in terms of trading by large market participants, for example, the largest mutual funds whose size distributions follow power laws (3) with the tail index $\zeta_S \approx 1$ (Zipf laws). The key ingredient in these models is the power functional form for the price impact Δp of a trade of size V :

$$r = \Delta p \sim CV^{\gamma}, \tag{4}$$

$\gamma, C > 0$, for large trades V. The model in Gabaix et al. (2006) implies that the tail index ζ of financial returns relates to the tail indices ζ_V and ζ_S of trading volumes and the sizes of market participants as $\zeta = \zeta_V/\gamma$ and $\zeta_V = (1 + 1/\gamma)\zeta_S$. For the values

$$\zeta_S = 1 \tag{5}$$

and $\gamma = 1/2$ (with the square-root price impact relation in (4)) typically observed in empirical research this implies that $\zeta \approx 3$. The mechanism explaining heavy tails for financial returns and trading volumes and the corresponding tail index values is common to many developed financial as well as foreign exchange markets.

Following Meese & Rogoff (1983), it is widely acknowledged that economic fundamentals do not predict exchange rates, particularly in the short run. Many determinants of exchange rate volatility have been suggested in the literature:

1. State of market development: Gokcan (2000) highlights the differences between emerging markets and developed markets in the speed and reliability of information available to investors, associated with modes of telecommunication, accounting systems, and general market efficiency.

2. Heterogeneous agents: De Grauwe & Grimaldi (2004) present a model of the exchange rate market with (i) fundamentalists, who follow negative feedback rules, comparing the current market exchange rates with a "fundamental" rate and forecasting the future market rate to move towards it; and (ii) chartists, who have adaptive expectations and are pure noise traders who do not take into account information concerning the fundamental exchange rate. The model reproduces empirical patterns of fat tails and volatility clustering.

3. Currency basket versus Single currency link: Wilson & Ren (2008) propose that a peg to a basket of currencies results in less average volatility (fat tails) than a peg to a single alone. However, there can be potentially an increase in volatility against one or more particular currencies.

4. Interventionist exchange rate policy: Wang & Yang (2006) attribute the presence of asymmetric responses in exchange rate volatility to the active participation of the government in the foreign exchange market. Typically, the central bank intervenes more often when there is a depreciation of the domestic currency than when there is an appreciation. Consequently, shocks to the market of equal magnitude will generate different responses depending on whether they are associated with an exchange rate appreciation or depreciation.

For the above reasons, among others, emerging economic, financial and foreign exchange markets are likely to be more volatile than their developed counter-parts and subject to more extreme external and internal shocks. It is natural to expect that heavy-tailedness properties will be more pronounced in these markets for exchange rates.

1.3 Empirical literature

Estimates of the tail indices for various time series are available for a wide range of developed financial and economic markets. Specifically, in the analysis of exchange rates, Loretan & Phillips

(1994) analyze large samples of daily exchange rates for developed markets using Hill's estimator, and obtain estimates in the interval $\zeta \in (2, 4)$ implying finite variances and infinite fourth moments. Cotter (2005) obtains similar results using more recent data sets on exchange rates in developed markets including the Euro. Boothe & Glassman (1987) provide maximum likelihood estimates for Student-t, infinite variance stable and mixtures of normal distributions fitted to exchange rates in developed markets. Their results indicate that the Student-t distributions and mixtures of two normals provide the best fit, with the tail indices (degrees of freedom) ζ in the Student-t case in the range $(1.8, 4.3)$ similar to the empirical results for asset returns discussed above.

To our knowledge, however, there are very few studies that relate to emerging and developing economies. Akgiray, Booth & Seifert (1988) focus on maximum likelihood estimation in parametric stable and Generalized Pareto power law families fitted to monthly observations on a number of Latin American exchange rates (see also Fofack & Nolan, 2001, for maximum likelihood estimates for infinite variance stable distributions fitted to different exchange rates). Koedijk, Stork & de Vries (1992) estimate the tail indices for Latin American exchange rates in Akgiray et al. (1988) using Hill's estimator and obtain wide confidence intervals that indicate that the variance and even first moments of the time series may be infinite. The analysis in Koedijk et al. (1992) further suggests different tail behavior in foreign exchange rate returns under different exchange rate regimes. The analysis in Akgiray et al. (1988) and Koedijk et al. (1992) are based on relatively small samples of monthly observations and therefore have wide confidence intervals. Using the tests for structural breaks in tail indices developed in Quintos, Fan & Phillips (2001), Candelon & Straetmans (2006) and Payaslioğlu (2009) focus on the analysis of changes in the tail exponents of exchange rates of emerging and developed currencies over time. The estimates for (relatively small) samples of quarterly data on exchange rates in Asian, Latin American and European economies in Pozo & Amuedo-Dorantes (2003) produce confidence intervals for tail indices ζ that indicate that the variances of the considered time series may be infinite.

1.4 Robust inference for heavy-tailed emerging vs. developed markets

The main goal of this paper is a robust analysis of heavy-tailedness properties in emerging and developing foreign exchange markets in comparison to developed markets. In particular, we focus on the analysis of the hypothesis that heavy-tailedness properties are more pronounced in emerging markets exchange rates. We use recently proposed robust tail index estimation methods (see Gabaix

& Ibragimov, 2011, and Section 2) applied to large data sets on exchange rates for a number of developed and emerging economies (see Section 3). This is in contrast to previous studies of exchange rates in emerging markets that focus on applications of inference methods based on model-specific parametric maximum likelihood procedures and (semiparametric) Hill's estimators, with a number of contributions providing estimates only for relatively small data sets, with potentially non-robust conclusions (see the discussion in Sections 1.3 and 2).

We employ the robust tail index inference approaches using the bias-corrected log-log rank-size regressions with correct standard errors in Gabaix & Ibragimov (2011). We find that the tail indices for foreign exchange rates in emerging economies are indeed considerably smaller than in the case of developed markets. In particular, the tail index estimates imply that, in contrast to developed markets, the value of the tail index $\zeta = 2$ is not rejected on commonly used statistical significance levels for foreign exchange rates in the most of emerging economies considered (Section 4). Thus, the variances may be infinite for these exchange rates. These results point to the seriousness of the macroeconomic management challenge in the economies.

Our results have implications for a number of economic, financial and econometric models and economic policy decisions and forecasting (see Section 5). In particular, they indicate that the models explaining heavy tails need to be modified in the case of emerging markets. For analysts, our results point to the need for robust inference methods in the analysis of emerging foreign exchange markets (Section 5). Our tail index estimates for foreign exchange rates in emerging markets may be used to forecast the patterns in their future development and convergence to heavy-tailed distributions implied by theoretical results for the equilibrium in the developed case in the literature.

The paper is organized as follows. Section 2 discusses the tail index estimation approaces available in the literature. In particular, it focuses on robust bias-corrected log-log rank-size regressions with correct standard errors in Gabaix & Ibragimov (2011) that provide the main tail index inference methods used in the paper. Section 3 discusses the data on foreign exchange rates in developed and emerging markets used in the analysis. Section 4 presents the tail index estimation results for foreign exchange rates considered. In Section 5, we discuss the implications of the empirical results obtained for several economic, financial and econometric models as well as for economic policy decisions and forecasting. Finally, Section 6 concludes and presents suggestions for further research.

2 Estimation methods

Several approaches to the inference about the tail index ζ of heavy-tailed distributions are available in the literature (see, among others, the reviews in Embrechts et al., 1997, and Beirlant, Goegebeur, Teugels & Segers, 2004). The two most commonly used ones are Hill's estimator and the OLS approach using the log-log rank-size regression.

It was reported in a number of studies that inference on the tail index using Hill's estimator suffers from several problems, including sensitivity to dependence and small sample sizes (see, among others, Embrechts et al. 1997, Chapter 6). Motivated by these problems, several studies have focused on the alternative approaches to the tail index estimation. For instance, Huisman, Koedijk, Kool & Palm (2001) propose a weighted analogue of Hill's estimator that was reported to correct its small sample bias for sample sizes less than 1000. Embrechts et al. (1997), among others, advocate sophisticated non-linear procedures for tail index estimation.

Let $r_1, r_2, ..., r_N$ be a sample from a population satisfying power law (3). Further, let

$$|r|_{(1)} \geq |r|_{(2)} \geq ... \geq |r|_{(n)} \tag{6}$$

be decreasingly ordered largest absolute values of observations in the sample. Despite the availability of more sophisticated methods, a popular way to estimate the tail index ζ is still to run the following OLS log-log rank-size regression with $\gamma = 0$:

$$\log\,(t - \gamma) = a - b \log\,|r|_{(t)}, \tag{7}$$

or, in other words, calling t the rank of an observation, and $|r|_{(t)}$ its size: $\log(\text{Rank} - \gamma) = a - b\log(\text{Size})$ (here and throughout the proposal, $\log(\cdot)$ stands for the natural logarithm). Similar log-log rank-size regressions applied to positive and negative observations r_t in the sample are employed to estimate the tail indices ζ_1 and ζ_2 in (1) and (2). The reason for the popularity of the OLS approaches to tail index estimation is arguably the simplicity and robustness of these methods. In various frameworks, the log-log rank-size regressions of form (7) in the case $\gamma = 0$ and closely related procedures were employed, in particular, in Levy (2003), Levy & Levy (2003), Helpman, Melitz & Yeaple (2004), and many other works (see also the review and references in Gabaix & Ibragimov, 2011).

Unfortunately, tail index estimation procedures based on OLS log-log rank-size regressions (7) with $\gamma = 0$ are strongly biased in small samples. The recent study by Gabaix & Ibragimov (2011) provides a simple practical remedy for this bias, and argues that, if one wants to use an OLS regression, one should use the Rank $-1/2$, and run $\log(\text{Rank} - 1/2) = a - b\log(\text{Size})$. The shift of $1/2$ is optimal, and reduces the bias to a leading order. The standard error on the Pareto exponent ζ is not the OLS standard error, but is asymptotically $(2/n)^{1/2}\zeta$. Numerical results in Gabaix & Ibragimov (2011) further demonstrate the advantage of the proposed approach over the standard OLS estimation procedures (7) with $\gamma = 0$ and indicate that it performs well under deviations from power laws and dependent heavy-tailed processes, including GARCH models. The modifications of the OLS log-log rank-size regressions with the optimal shift $\gamma = 1/2$ and the correct standard errors provided by Gabaix & Ibragimov (2011) were subsequently used in Hinloopen & van Marrewijk (2006), Bosker, Brakman, Garretsen, de Jong & Schramm (2007), Bosker, Brakman, Garretsen, & Schramm (2008), Gabaix & Landier (2008), Ioannides, Overman, Rossi-Hansberg & Schmidheiny (2008), Le Gallo & Chasco (2008), Zhang, Chen & Wang (2009) and several other works. Due to inherent heterogeneity and dependence properties and data availability constraints, foreign exchange rates in emerging and developing markets provide natural areas for applications of robust inference methods discussed in Gabaix & Ibragimov (2011). These methods provide the main tools for the empirical analysis in this paper.

3 Data

The data sets used in the empirical analysis in this paper consist of daily exchange rates to US dollar (USD) for the currencies listed below. The currencies in developed foreign exchange markets considered in the paper are Australian dollar (AUD), Canadian dollar (CAD), Swiss franc (CHF), Danish krone (DKK), Euro (EUR), Great Britain pound (GBP), Japanese yen (JPY) and Norwegian kroner (NOK). The currencies in emerging markets in the data set used in the analysis are Chinese renminbi (CNY), Hong Kong dollar (HKD), Indian rupee (INR), South Korean won (KRW), Malaysian ringgit (MYR), Russian ruble (RUB), Singapore dollar (SGD), Taiwan dollar (TWD), Thai baht (THB) and Phillippine peso (PHP).[1]

[1]The classification of the markets considered as emerging follows the list tracked by the *Economist*; this list includes Hong Kong, Singapore and Saudi Arabia and the following economies in the Morgan Stanley Emerging Markets Index: Brazil, Chile, China (mainland), Colombia, Czech Republic, Egypt, Hungary, India, Indonesia, Iran, Israel, Jordan, Malaysia, Mexico, Morocco, Pakistan, Peru, Philippines, Poland, Russia, South Africa, South Korea, Taiwan, Thailand, Tunisia, Turkey and Vietnam (the Morgan Stanley Capital International classifies the economies

The data on AUD, CAD, CHF, DKK, GBP, HKD, JPY, SGD, THB and PHP exchange rates are available for the period from October 17, 1994 to December 4, 2008; and the data on EUR is available from 4 January, 1998 to November 12, 2008.[2] The data set on INR, KRW, MYR, NOK and TWD exchange rates covers the period from October 28, 1997 to October 28, 1998. The data set for RUB is from January 1, 1999 to October 29, 2008. Figures 1-6 illustrate and contrast the dynamics of foreign exchange rates in developed and emerging markets using the data for EUR, GBP, and RUB.

We also present the results on estimation of the tail indices of GBP, EUR and the base currency used in the empirical analysis, USD, using the data on London Gold and Silver Fix prices in these currencies per Troy ounce from the London Bullion Market Association. The price data set covers the period from January 2, 1968 to March 18, 2009 for London Gold and Silver Fix prices in USD and GBP and from January 2, 2002 to March 18, 2009 for the prices in EUR.[3]

4 Estimation results

Tables 1 and 2 present estimation results for the tail indices in power law models (3) for the rate of growth of exchange rates in emerging and developing markets discussed in Section 3.

A stylized statistical fact which is common to a wide set of financial assets, in addition to heavy-tailedness properties discussed in Section 1.1, is the so-called gain/loss asymmetry (see Cont, 2001). According to this stylized fact, one typically observes large drawdowns in stock prices but not equally large upward movement. At the same time, this property usually does not hold for exchange rates where one observes higher symmetry in up/down moves. Motivated by the stylized facts of gain/loss asymmetry in financial markets and higher symmetry in drawdowns and upward movements in foreign exchange markets, we also present, in Tables 3 and 4, the estimates of the tail indices in up/down moves of the exchange rates in power law models (1) and (2).

The estimation results for the tail indices in models (3) are obtained using log-log rank-size

of Hong Kong and Singapore as developed markets).

[2]The exchange rates of these currencies to USD were converted from their rates to CNY using the exchange rate of USD to CNY. The exchange rate of CNY to USD, in turn, was converted from its rate to HKD using the exchange rate of USD to HKD.

[3]Estimation uses the Gold a.m. Fix prices.

regressions (7) with the optimal shift $\gamma = 1/2$ and the correct standard errors $(2/n)^{1/2}\zeta$, as discussed in Section 2. The empirical analysis of the tail indices in up/down movements in the exchange rates is performed using the analogues of regressions (7) with $\gamma = 1/2$ for positive and negative values of their growth rates (see Section 2). The (correct) 95% confidence intervals reported in Tables 1-4 are constructed using the above standard errors. The estimation results are presented for 5% and 10% truncation levels for extreme observations like in (6) used in estimation by log-log rank-size regressions in form (7) with $\gamma = 1/2 : n \approx 0.05N$ and $n \approx 0.1N$, where N is the total sample size for the exchange rates considered. For illustration, Figures 4-6 provide the log-log rank-size plots that correspond to estimated regressions (7) with the optimal shift $\gamma = 1/2$ for EUR, GBP and RUB.

Throughout the rest of the discussion in this section, for brevity, we refer to the extreme observations on the 5% and 10% truncation levels in (6) used in tail index estimation in Tables 1-4 by $AUD_{5\%}$, $AUD_{10\%}$, $AUD(+)_{5\%}$, $AUD(+)_{10\%}$, $AUD(-)_{5\%}$, $AUD(-)_{10\%}$, etc. Failure to reject the null hypothesis $H_a : \zeta = \zeta_0$ on the tail index ζ refers to the 5% significance level and the two-sided alternative $H_a : \zeta \neq \zeta_0$. Rejection of H_0 refers to the 2.5% significance level and the one-sided alternatives $H_a : \zeta < \zeta_0$ or $H_a : \zeta > \zeta_0$. Similar to the results reported in Tables 1-4, the confidence intervals discussed below are 95% confidence intervals.

The empirical results in Tables 1 and 2 indicate remarkable differences in the degree of heavy-tailedness of foreign exchange rates in developed and emerging markets. The point estimates of the tail indices ζ of foreign exchange rates in developed economies in Table 1 lie between 2.8 and 4.8. This is similar to the empirical results for developed financial markets in the literature that, as discussed in the introduction, report the tail indices in the interval $\zeta \in (2, 5)$ for returns on many stocks and stock indices in developed economies.

The null hypothesis $\zeta = 2$ is rejected in favor of $\zeta > 2$ for exchange rates in all the developed markets in Table 1. In addition, the null hypothesis $\zeta = 3$ is not rejected for $AUD_{5\%}$, $AUD_{10\%}$, $CAD_{5\%}$, $CAD_{10\%}$, $JPY_{5\%}$, $JPY_{10\%}$, $NOK_{5\%}$ and $NOK_{10\%}$. For $AUD_{5\%}$, $AUD_{10\%}$, $CAD_{5\%}$, $CAD_{10\%}$ and $JPY_{10\%}$, the null hypothesis $\zeta = 4$ is rejected in favor of $\zeta < 4$. For $CHF_{5\%}$, $CHF_{10\%}$, $DKK_{5\%}$, $DKK_{10\%}$, $EUR_{5\%}$, $EUR_{10\%}$, $GBP_{5\%}$ and $GBP_{10\%}$, the hypothesis $\zeta = 3$ is rejected in favor of $\zeta > 3$. For the later exchange rate samples, together with $JPY_{5\%}$, $NOK_{5\%}$ and $NOK_{10\%}$, the null hypothesis $\zeta = 4$ is not rejected in favor of $\zeta \neq 4$.

In summary, the above conclusions imply that the exchange rates in all the developed markets in Table 1 have finite variances. In addition, CHF, DKK, EUR and GBP have finite third moments; however, the fourth moments of these exchange rates may be infinite. In contrast, AUD and CAD have infinite third and possibly infinite fourth moments. The third and fourth moments may be infinite for JPY and NOK. All in all, the empirical results in Table 1 imply that CHF, DKK, EUR and GBP exchange rates are less heavy-tailed than AUD, CAD, JPY and NOK exchange rates, with AUD and CAD being the most heavy-tailed exchange rates in Table 1. In particular, it is likely that the former exchange rates are subject to less and smaller extreme external and internal shocks.

The empirical results for exchange rates in emerging markets in Table 2 are summarized as follows. The point estimates of the tail indices of all the exchange rates in the table lie between 1.3 and 3.4. In particular, the point estimates of the tail indices of CNY, HKD, MYR, PHP, SGD and RUB exchange rates are between 1.6 and 2.6.

The null hypothesis $\zeta = 1$ is rejected in favor of $\zeta > 1$ for all exchange rates in Table 2. The null hypothesis $H_0 : \zeta = 2$ is not rejected for $CNY_{5\%}$, $CNY_{10\%}$, $HKD_{5\%}$, $HKD_{10\%}$, $MYR_{5\%}$, $MYR_{10\%}$, $PHP_{5\%}$, $PHP_{10\%}$, $SGD_{5\%}$, $SGD_{10\%}$, $THB_{5\%}$, $RUB_{5\%}$, $RUB_{10\%}$ and $TWD_{10\%}$. It is rejected in favor of $H_a : \zeta > 2$ for $INR_{5\%}$, $INR_{10\%}$ and $TWD_{5\%}$, and in favor of $H_a : \zeta < 2$ for $KRW_{5\%}$, $KRW_{10\%}$ and $THB_{10\%}$. The hypothesis $H_0 : \zeta = 3$ is not rejected for $INR_{5\%}$, $INR_{10\%}$, $SGD_{5\%}$ and $TWD_{5\%}$, and is rejected in favor of $H_a : \zeta < 3$ for all other exchange rate sample considered. $H_0 : \zeta = 4$ is not rejected only for $INR_{5\%}$ and $TWD_{5\%}$; for all other samples used in the empirical analysis reported in Table 2, this hypothesis is rejected in favor of $\zeta < 4$.

The empirical results in Table 2 thus imply that the first moments are finite for all exchange rates in the table. The variance is infinite for KRW and is finite for INR. The variances may be infinite for all other exchange rates considered in Table 2 (CNY, HKD, MYR, PHP, SGD, THB, RUB and TWD). The CNY, HKD, KRW, PHP, THB, RUB time series have infinite third moments. The third moments may also be infinite for INR, MYR, SGD and TWD. The fourth moments may be infinite for INR and TWD; they are infinite for all the remaining exchange rates in Table 2.

In summary, according to the estimation results in Tables 1 and 2, the tail indices of exchange rates in emerging markets are typically considerably smaller than those of exchange rates in developed economies. Thus, heavy-tailedness properties of exchange rates in emerging markets are indeed more pronounced than in their developed counter-parts. In particular, one of the key differences of heavy-tailedness properties of developed and emerging markets is that the exchange rates in developed countries have finite variances, in contrast to the exchange rates in emerging economies. Namely, the exchange rates in most of the emerging markets considered in the paper may or do have infinite second moments, according to the tail index estimation results obtained (see Section 5 for further discussion of the implications of these and other empirical results in the paper for a number of economic, financial and econometric models). The differences in the tail index estimates for developed and emerging markets in Tables 1 and 2 are illustrated in Figures 4-6 that present the log-log rank-size plots constructed using estimated regressions (7) with the optimal shift $\gamma = 1/2$ for EUR, GBP and RUB. The slope of the log-log rank-size plot for RUB in Figure 8 is considerably smaller than those of the log-log rank-size plots for EUR and GBP.

Tables 3 and 4 present the tail index estimation results in models (1) and (2) for the signed rates of growth of exchange rates in developed and emerging markets. The comparison of the empirical results in Tables 1 and 3 indicates that the point estimates and confidence intervals for the tail indices are quite similar in models (3)-(2) for developed foreign exchange markets. At the same time, according to the point estimates and confidence intervals in Table 3, the upward moves in CHF, DKK, EUR, GBP and NOK appear to be somewhat more heavy-tailed than the downward moves in these exchange rates. The point estimates of the tail indices for $AUD(+)_{5\%}$, $AUD(+)_{10\%}$, $AUD(-)_{5\%}$, $AUD(-)_{10\%}$, $JPY(+)_{10\%}$ and $JPY(-)_{10\%}$ suggest the same conclusion for AUD and JPY. Interestingly, the results in the table also indicate that heavy-tailedness is somewhat more pronounced in the downward moves of the Canadian dollar exchange rate than in its upward moves.

Importantly, in contrast to the conclusions for symmetric model (3) for developed foreign exchange markets discussed above, the null hypothesis $H_0 : \zeta = 2$ is not rejected for $AUD(+)_{5\%}$ and $AUD(-)_{5\%}$. The null hypothesis $H_0 : \zeta = 3$ that implies possibly infinite third moments is not rejected (using both 5% and 10% levels of truncation for extreme observations) for $AUD(+)$, $AUD(-)$, $CAD(+)$, $CAD(-)$, $CHF(+)$, $DKK(+)$, $GBP(+)$, $GBP(-)$, $JPY(+)$, $JPY(-)$ and $NOK(+)$; it is also not rejected for $EUR(+)_{10\%}$. The comparisons with the conclusions for model (3) implied by the results in Table 1 point out that heavy-tailedness is more pronounced in models (1) and (2) for the signed rate of growth of GBP than in symmetric model (3) for this exchange rate. Similar conclusion hold for model (1) for upward moves in CHF and DKK. In particular, in contrast to the unsigned GBP, CHF and DKK time series dealt with in Table 1, the third moments may be infinite for $GBP(+)$, $GBP(-)$, $CHF(+)$ and $DKK(+)$.

In summary, the empirical results in Table 3 support the conclusion on symmetry in heavy-tailedness properties of upward/downward moves in exchange rates in developed markets, similar to symmetry in the behavior of volatility of foreign exchange markets reported in the literature (see the beginning of the section and the discussion of the contrasting stylized fact of gain/loss asymmetry in financial markets therein). At the same time, the upward moves of exchange rates in all the developed markets considered, except CAD, appear to be somewhat more heavy-tailed than their downward moves. This type of asymmetry is the opposite of the gain/loss asymmetry in financial markets, where the magnitude of large drawdowns typically exceeds that of large upward moves. It maybe due to regulatory interventions during large drawdowns in developed foreign exchange markets. In addition, the results in Table 4 indicate that, for some of the time series in developed markets, such as $AUD(+)$, $AUD(-)$, $GBP(+)$, $GBP(-)$, $CHF(+)$ and $DKK(+)$, the heavy-tailedness properties are more pronounced in models (1) and (2) for signed rates of growth than in symmetric model (3).

The empirical results in Table 4 indicate that the tail index estimates are very similar in models (1) and (2) for upward and downward moves in the foreign exchange rates in all the emerging markets considered. For the exchange rates in all the emerging markets, the tail index estimates in models (1) and (2) in Table 4 are very close to the corresponding tail index estimates in symmetric model (3) reported in Tables 2. Among the exchange rates in Table 4, somewhat more pronounced heavy-tailedness in upward moves than in downward movements is observed for RUB, SGD and THB. The tails of downward moves of INR appear to be somewhat more heavy-tailed than those of upward moves in this exchange rate; in particular, the null hypothesis $\zeta = 2$ implying infinite variances is not rejected for $INR(-)_{10\%}$, in contrast to INR and INR (+) time series. The above conclusions indicate overall high symmetry in up/down moves of exchange rates in emerging markets.

Table 5 presents the estimates of the tail indices of USD, GBP and EUR calculated using the London Gold and Silver Fix prices in these currencies. The results are qualitatively similar to those in Table 1 for developed markets, with the null hypothesis $\zeta = 2$ rejected in favor of $\zeta > 2$ (finite variances) for USD, GBP and EUR in terms of gold and silver prices, both at the 5% and 10% truncation levels. However, in contrast to the estimates in Table 1 for GBP and EUR in terms of gold and silver prices, the 95% confidence intervals in Table 5 for these currencies contain $\zeta = 3$, so that $\zeta = 3$ is not rejected. The null hypothesis $\zeta = 4$ is rejected in favor of $\zeta < 4$ for all the exchange rate time series considered in Table 5, except for the estimates calculated using gold prices and 5% truncation levels. This suggests that the estimates for the exchange rates in terms of silver prices are more robust, due to lower volatility of the latter. The estimates obtained for longer exchange rate time series (from Jan. 1968 to March 2008 for USD and GBP) confirm further robustness of the above conclusions on the tail indices.

5 Implications for economic models and policy decisions

Heavy-tailedness has important implications for robustness of many economic models, leading, in a number of settings, to reversals of conclusions of these models to the opposite ones.

The tail indices are greater than one and the first moments are finite for foreign exchange rates in all the emerging and developing markets considered in the paper. These conclusions are important and encouraging because, as discussed in Ibragimov (2009*b*) and Ibragimov (2009*a*), the stylized facts on diversification are robust to heavy-tailedness of risks or returns in value at risk (VaR) models as long as the distributions of these risks or returns are moderately heavy-tailed with tail indices $\zeta > 1$ and finite means. However, the stylized fact that portfolio diversification is preferable is reversed in value at risk models and extremely heavy-tailed risks with tail indices less than one and infinite first moments. According to the results in Ibragimov (2007) (see also the discussion in Ibragimov, 2009*a*), similar robustness vs. reversal conclusions hold for the efficiency properties of the sample mean for heavy-tailed populations. Namely, the sample mean is the best linear unbiased estimator of the population mean in the sense of peakedness properties for moderately heavy-tailed populations with tail indices $\zeta > 1$. In addition, for such populations, the sample mean exhibits the property of monotone consistency and, thus, an increase in the sample size always improves its performance. However, efficiency of the sample mean in the sense of its peakedness decreases with the sample size if the sample mean is used to estimate the population center under extreme heavy-tailedness with tail indices $\zeta < 1$.

The empirical estimates obtained in this paper further indicate that the tail indices may be less than two for foreign exchange rates in a number of emerging and developing markets. Such pronounced heavy-tailedness with tail indices $\zeta \leq 2$ presents a challenge for applications of standard statistical and econometric methods. In particular, as pointed out by Granger & Orr (1972) and in a number of more recent studies (see, among others, Ch. 7 in Embrechts et al., 1997, and references therein) many classical approaches to inference based on variances and (auto)correlations such as regression and spectral analysis, least squares methods and autoregressive models may not apply directly in the case of heavy-tailed observations with infinite second or higher moments.

Our results imply that traditional economic and financial models and econometric and statistical methods should be applied with care in heavy-tailed settings that are exhibited by exchange rates in a wide range of emerging and developing markets. This is especially important in the case of the tail indices close to the value $\zeta = 1$ that, in many cases, provides the critical robustness boundary and the threshold value $\zeta = 2$ that necessitates deviations from the usual inference methods.

The empirical results obtained in the paper also have important implications for economic policy decision making and macroeconomic forecasting. First, they confirm that, as is natural to expect, foreign exchange rates in emerging and developing markets are indeed typically more heavy-tailed and volatile than those in their developed counter-parts and are thus subject to more frequent and extreme external and internal shocks. Second, one should note that the tail index values $\zeta \approx 3$ for financial returns implied by the empirical results in Gabaix et al. (2006) are, essentially, an equilibrium outcome in developed markets. It is characteristic of markets with well-developed mechanisms discussed in Gabaix et al. (2006) where such heavy tails are generated by trading of large market participants with size distributions that have the tail indices $\zeta = 1$. Thus, one can use the actually observed tail indices for key variables in emerging and developing markets, including the foreign exchange rates considered in this paper, to forecast the patterns in their future development and convergence to heavy-tailed distributions with tail index values $\zeta \approx 3$ implied by theoretical results for the developed market case. Third, the presence of extreme heavy-tailedness and volatility suggests that specifications of some models explaining heavy tails (see the discussion at the beginning of Section 1.2) may need to be modified in the case of emerging and developing economies and foreign exchange markets. Such modifications, in particular, may focus on the following hypotheses and tests. Due to government intervention and regulation, it seems likely that the tail indices may be less than $\zeta_S = 1$ in (5) for largest participants in emerging and developing foreign exchange markets (implying the failure of the Zipf's law), with the states being some of the largest key traders. The differences of the tail indices of foreign exchange rates in emerging and developing economies from those in the developed markets may also be due to deviations of the price impact relation from the square root functional form (4) with $\gamma = 1/2$, e.g., with the square root replaced by a different power function. In addition, it may be the case that the power relations like in (4) in emerging and developing economies needs to be replaced by a different functional dependence between these key variables affecting distributional and heavy-tailedness properties of financial returns and exchange rates.

More generally, our results point to the extreme nature of policy challenges faced by emerging economies. The effects of heavy-tailedness of exchange rates can be expected to mirror the effects of exchange rate volatility. These include, among others:

1. Increased inflation volatility: Domestic prices of imported goods tend to fluctuate with exchange rate (Arize & Malindretos, 1997). Further, as suggested by the 'currency substitution' hypothesis, depreciation leads to inflationary pressures, and vice versa (Cuddington, 1983).

2. Trade reduction: If exchange rate movements are not fully anticipated, an increase in exchange rate volatility, which increases risk, will lead risk-averse agents to reduce their import/export activity and to reallocate production toward other markets because hedging foreign exchange risk is costly (Dell'Ariccia, 1998).

3. Reduction of foreign direct investment (FDI)/investment in export-oriented industries: The "risk aversion" argument (Cushman, 1985, 1988) suggests that higher exchange-rate variability lowers the certainty equivalent expected values of investment projects, and FDI is reduced accordingly.

4. Unemployment: Labour unions in export-oriented industries demand a risk premium in their wages, resulting in lower labour demand (Andersen & Sørensen, 1988). Adverse effect on investment, as discussed above, also contributes.

5. Economic growth: Higher exchange rate volatility leads to lower growth due to its adverse effects on investment, trade and macroeconomic stability (Schnabl, 2009).

6 Conclusion and suggestions for further research

This paper provides the results on a robust assessment and wide scale comparisons of heavy-tailedness properties and tail indices for foreign exchange rates in a number of post-Soviet and Asian emerging and developing economies. The estimation results obtained indicate that the exchange rates in such economies are typically much more heavy-tailed than those in developed markets. In particular, the estimates imply that the tail indices for exchange rates in many emerging and developing economies may be less than two thus implying infinite variances. These results highlight the volatile characteristics of the economies in consideration that make them qualitatively different from the well-studied developed markets and call for applications of reliable and robust inference methods. In addition, as discussed in the paper, the empirical results obtained have important implications for several economic and financial models, including value at risk and diversification analysis, econometric and statistical inference methods employed in the study of emerging and developing economies as well as for macroeconomic forecasting and policy decision making.

Important problems for further research on heavy tails in emerging and developing foreign exchange markets include the analysis of tail indices for sizes of largest market participants, trading volume, the volume of export and import and their concentration across different industries and trade partners. These empirical conclusions may be further applied to develop analogues of the existing economic and financial models explaining heavy tails for emerging and developing foreign exchange markets. It is also of interest to complement the analysis in this paper using comparisons with a wide range of other estimation and inference approaches, including Hill's estimates for tail indices, their small sample analogues developed in Huisman et al. (2001) and other robust econometric and statistical procedures under heavy tails such as the $t-$statistic based robust inference methods proposed in Ibragimov & Müller (2010). Cross-country regressions of tail index estimates on such variables as the measures of per capita GDP volatility and those of government intervention in foreign exchange markets may be useful in the analysis of the effects of key macroeconomic indicators on heavy-tailedness properties of exchange rates.

Further research may also focus on the analysis of structural breaks in tail indices and heavy-tailedness properties of exchange rates in developed and emerging markets following financial crises and/or government interventions. In particular, it would be of interest to study and compare the tail indices in these markets and their structural breaks following the on-going world economic crisis of 2008. Similarly, it is of interest to focus on the analysis of structural breaks in the tail index of RUB following the 1998 Russian financial crisis. The analysis of these problems may be conducted similar to the study of structural breaks in tail indices of stock returns in Asian markets during the Asian crisis in Quintos et al. (2001) using the inference methods developed therein. The preliminary estimation results by the authors suggest the change in heavy-tailedness properties of some of exchange rates considered in the on-going economic crisis, but the conclusions need further investigation due to relatively small sample sizes available following its beginning in 2008.

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Table 1. Tail index estimates for exchange rates in developed markets.

Currency	N	**Truncation, %**	n	$\hat{\zeta}$	**S.e.=**$\sqrt{\frac{2}{n}}\hat{\zeta}$	**95% CI**
AUD	3467	10	347	2.83	0.21	(2.41, 3.25)
		5	173	2.87	0.31	(2.26, 3.47)
CAD	3470	10	347	3.02	0.23	(2.57, 3.47)
		5	174	3.22	0.35	(2.55, 3.90)
CHF	3473	10	347	4.24	0.32	(3.61, 4.87)
		5	174	4.73	0.51	(3.74, 5.73)
DKK	3470	10	347	3.96	0.30	(3.37, 4.54)
		5	174	4.71	0.51	(3.72, 5.71)
EUR	2579	10	258	4.36	0.38	(3.61, 5.12)
		5	129	4.76	0.59	(3.60, 5.92)
GBP	3470	10	347	3.86	0.29	(3.28, 4.43)
		5	174	4.19	0.45	(3.31, 5.07)
JPY	3462	10	346	3.26	0.25	(2.77, 3.74)
		5	173	3.58	0.38	(2.82, 4.33)
NOK	2833	10	283	3.53	0.30	(2.95, 4.12)
		5	142	3.61	0.43	(2.77, 4.45)

Table 2. Tail index estimates for exchange rates in emerging markets.

Currency	N	**Truncation, %**	n	$\hat{\zeta}$	**S.e.**$=\sqrt{\frac{2}{n}}\hat{\zeta}$	**95% CI**
CNY	3239	10	324	2.12	0.17	(1.80, 2.45)
		5	162	2.26	0.25	(1.77, 2.76)
HKD	3316	10	332	2.13	0.17	(1.81, 2.45)
		5	166	2.09	0.23	(1.64, 2.54)
INR	2644	10	264	2.57	0.22	(2.13, 3.01)
		5	132	3.37	0.42	(2.56, 4.19)
KRW	1544	10	154	1.63	0.19	(1.27, 2.00)
		5	77	1.38	0.22	(0.95, 1.82)
MYR	1479	10	148	2.44	0.28	(1.89, 3.00)
		5	74	2.56	0.42	(1.74, 3.39)
PHP	3375	10	338	1.76	0.14	(1.49, 2.02)
		5	169	1.80	0.20	(1.41, 2.18)
SGD	3455	10	346	2.33	0.18	(1.98, 2.67)
		5	173	2.50	0.27	(1.97, 3.03)
THB	3392	10	339	1.67	0.13	(1.42, 1.93)
		5	170	2.01	0.22	(1.58, 2.44)
RUB	2263	10	226	1.90	0.18	(1.55, 2.25)
		5	113	1.93	0.26	(1.43, 2.44)
TWD	1497	10	150	2.34	0.27	(1.81, 2.86)
		5	75	3.24	0.53	(2.20, 4.28)

Table 3. Tail index estimates for upward (+) and downward (−) movements in exchange rates in developed markets.

Currency	N	**Truncation, %**	n	$\hat{\zeta}$	**S.e.**$=\sqrt{\frac{2}{n}}\hat{\zeta}$	**95% CI**
AUD (+)	1789	10	180	2.79	0.29	(2.22, 3.37)
		5	89	2.73	0.41	(1.93, 3.54)
AUD (−)	909	10	91	2.94	0.44	(2.08, 3.79)
		5	45	2.99	0.63	(1.76, 4.23)
CAD (+)	1724	10	172	3.27	0.35	(2.58, 3.96)
		5	86	3.60	0.55	(2.53, 4.68)
CAD (−)	1746	10	175	2.80	0.30	(2.21, 3.38)
		5	87	2.87	0.44	(2.02, 3.73)
CHF (+)	1711	10	171	3.55	0.38	(2.79, 4.30)
		5	86	3.38	0.51	(2.37, 4.38)
CHF (−)	1762	10	176	4.65	0.50	(3.68, 5.62)
		5	88	5.21	0.79	(3.67, 6.75)
DKK (+)	1714	10	171	3.65	0.39	(2.88, 4.42)
		5	86	4.05	0.62	(2.84, 5.26)
DKK (−)	1756	10	176	4.08	0.44	(3.23, 4.94)
		5	88	4.94	0.74	(3.48, 6.40)
EUR (+)	1284	10	128	4.12	0.51	(3.11, 5.13)
		5	64	4.45	0.79	(2.91, 5.99)
EUR (−)	1295	10	130	4.56	0.57	(3.45, 5.66)
		5	65	5.00	0.88	(3.28, 6.72)
GBP (+)	1719	10	172	3.57	0.38	(2.81, 4.32)
		5	66	3.22	0.56	(2.12, 4.32)
GBP (−)	1751	10	175	3.71	0.40	(2.93, 4.48)
		5	88	4.10	0.62	(2.89, 5.31)
JPY (+)	1700	10	170	3.12	0.34	(2.45, 3.78)
		5	85	3.62	0.56	(2.53, 4.71)
JPY (−)	1762	10	176	3.45	0.37	(2.73, 4.17)
		5	88	3.60	0.54	(2.54, 4.66)
NOK (+)	1413	10	141	3.18	0.38	(2.44, 3.93)
		5	71	3.20	0.54	(2.15, 4.25)
NOK (−)	1420	10	142	3.97	0.47	(3.05, 4.90)
		5	71	4.16	0.70	(2.79, 5.53)

Table 4. Tail index estimates for upward (+) and downward (−) movements in exchange rates in emerging markets.

Currency	N	**Truncation, %**	n	$\hat{\zeta}$	**S.e.**$=\sqrt{\frac{2}{n}}\hat{\zeta}$	**95% CI**
CNY (+)	1724	10	172	2.18	0.23	(1.72, 2.64)
		5	86	2.20	0.34	(1.55, 2.86)
CNY (−)	1515	10	152	2.04	0.23	(1.58, 2.50)
		5	76	2.17	0.35	(1.48, 2.86)
HKD (+)	1639	10	164	2.12	0.23	(1.66, 2.58)
		5	82	2.03	0.32	(1.41, 2.65)
HKD (−)	1677	10	168	2.10	0.23	(1.65, 2.55)
		5	84	2.10	0.32	(1.46, 2.73)
INR (+)	1354	10	135	2.73	0.33	(2.08, 3.38)
		5	68	3.47	0.60	(2.31, 4.64)
INR (−)	1290	10	129	2.47	0.31	(1.86, 3.07)
		5	65	3.14	0.55	(2.06, 4.21)
KRW (+)	785	10	79	1.60	0.26	(1.10, 2.11)
		5	39	1.33	0.30	(0.74, 1.92)
KRW (−)	759	10	76	1.63	0.27	(1.12, 2.15)
		5	38	1.40	0.32	(0.77, 2.03)
MYR (+)	757	10	76	2.49	0.40	(1.70, 3.28)
		5	38	2.51	0.58	(1,38, 3.64)
MYR (−)	722	10	72	2.35	0.39	(1.58, 3.12)
		5	36	2.52	0.60	(1.36, 3.69)
PHP (+)	1674	10	167	1.77	0.19	(1.39, 2.15)
		5	84	1.74	0.27	(1.21, 2.26)
PHP (−)	1701	10	170	1.73	0.19	(1.36, 2.10)
		5	85	1.83	0.28	(1.28, 2.38)

Table 4 (ctd.). Tail index estimates for upward (+) and downward (−) movements in exchange rates in emerging markets.

Currency	N	**Truncation, %**	n	$\hat{\zeta}$	**S.e.**$=\sqrt{\frac{2}{n}}\hat{\zeta}$	**95% CI**
RUB (+)	1056	10	106	1.73	0.24	(1.27, 2.20)
		5	53	2.03	0.39	(1.26, 2.81)
RUB (−)	1207	10	121	2.50	0.32	(1.87, 3.13)
		5	60	2.37	0.43	(1.52, 3.22)
SGD (+)	1747	10	175	2.07	0.22	(1.64, 2.50)
		5	87	2.16	0.33	(1.52, 2.80)
SGD (−)	1708	10	171	2.64	0.29	(2.08, 3.20)
		5	85	2.94	0.45	(2.05, 3.82)
THB (+)	1734	10	173	1.57	0.17	(1.24, 1.90)
		5	87	1.93	0.29	(1.36, 2.51)
THB (−)	1658	10	166	1.76	0.19	(1.38, 2.14)
		5	83	1.99	0.31	(1.38, 2.59)

Table 5. Tail index estimates for USD, GBP and EUR in terms of gold and silver prices.

Currency	N	**Truncation, %**	n	$\hat{\zeta}$	**S.e.**$=\sqrt{\frac{2}{n}}\hat{\zeta}$	**95% CI**
USD/Gold	2320	10	232	3.21	0.30	(2.63, 3.79)
		5	116	3.44	0.45	(2.56, 4.33)
USD/Silver	2320	10	227	2.75	0.26	(2.25, 3.26)
		5	114	3.16	0.42	(2.34, 3.99)
GBP/Gold	2330	10	233	3.02	0.28	(2.47, 3.57)
		5	117	3.21	0.42	(2.39, 4.04)
GBP/Silver	2326	10	233	2.83	0.26	(2.32, 3.35)
		5	116	3.08	0.40	(2.29, 3.87)
EUR/Gold	2330	10	233	3.07	0.28	(2.51, 3.63)
		5	117	3.39	0.44	(2.52, 4.25)
EUR/Silver	2572	10	257	2.89	0.25	(2.39, 3.38)
		5	129	3.14	0.39	(2.37, 3.91)

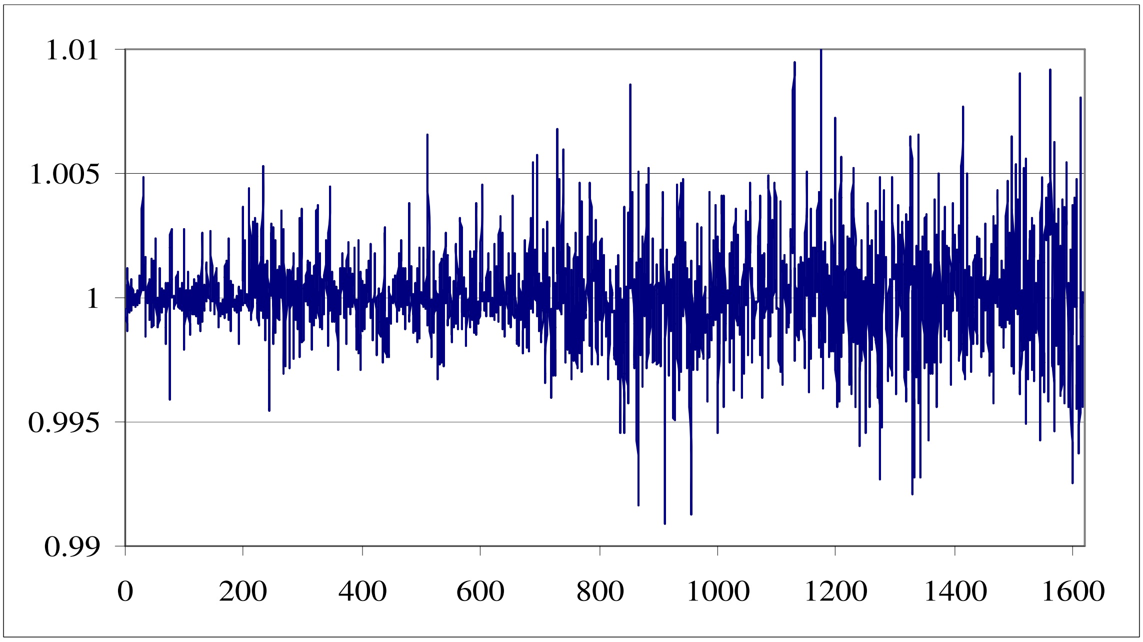

Figure 1. The rate of growth of the EURO exchange rate.

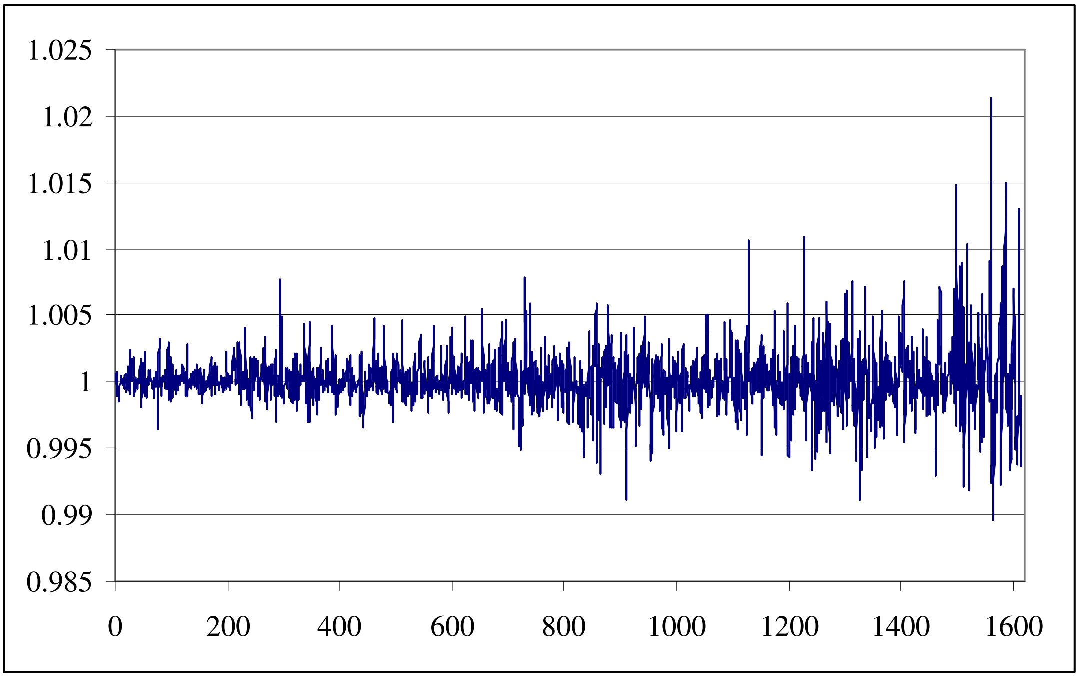

Figure 2. The rate of growth of the GBP exchange rate.

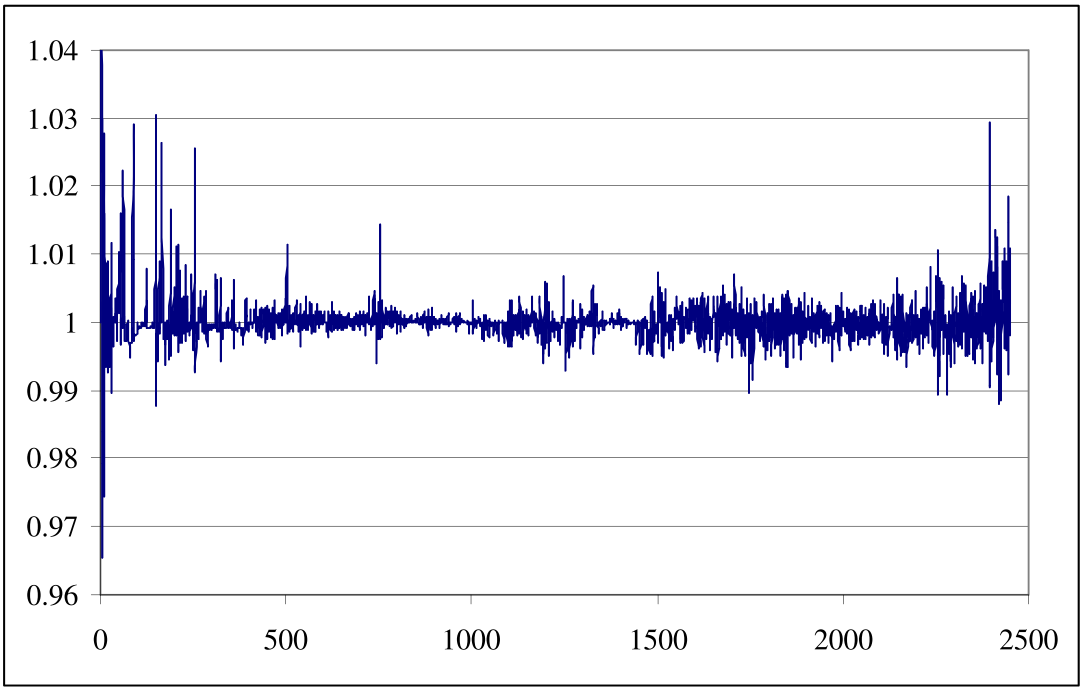

Figure 3. The rate of growth of the RUB exchange rate.

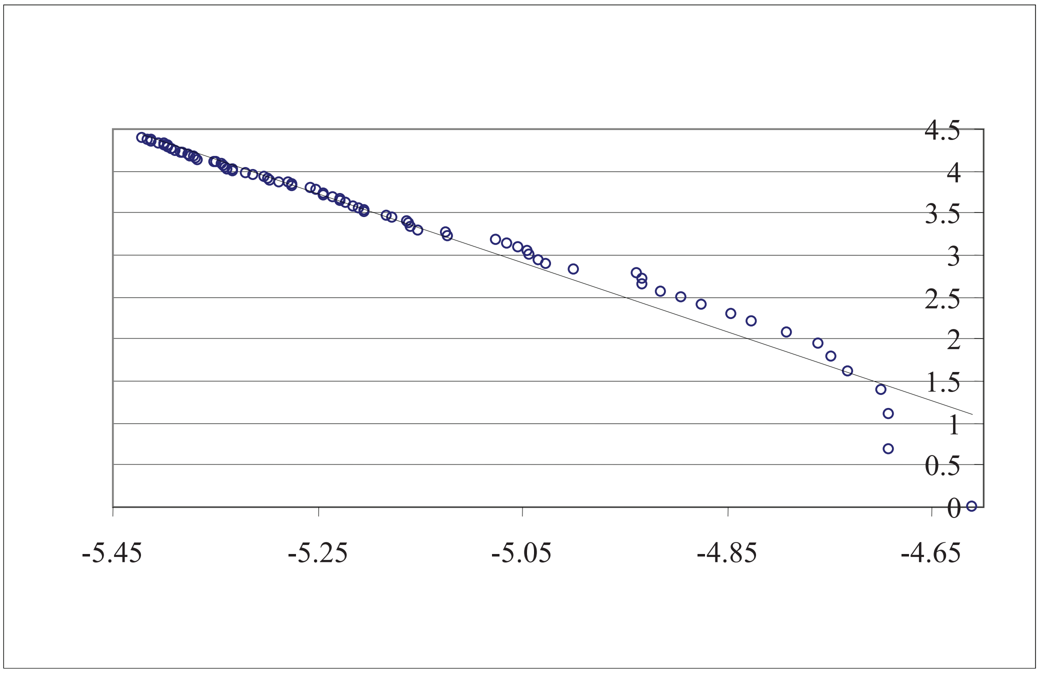

Figure 4. Log-log rank-size plot for the EURO exchange rate using regression (7) with the optimal shift γ=1/2.

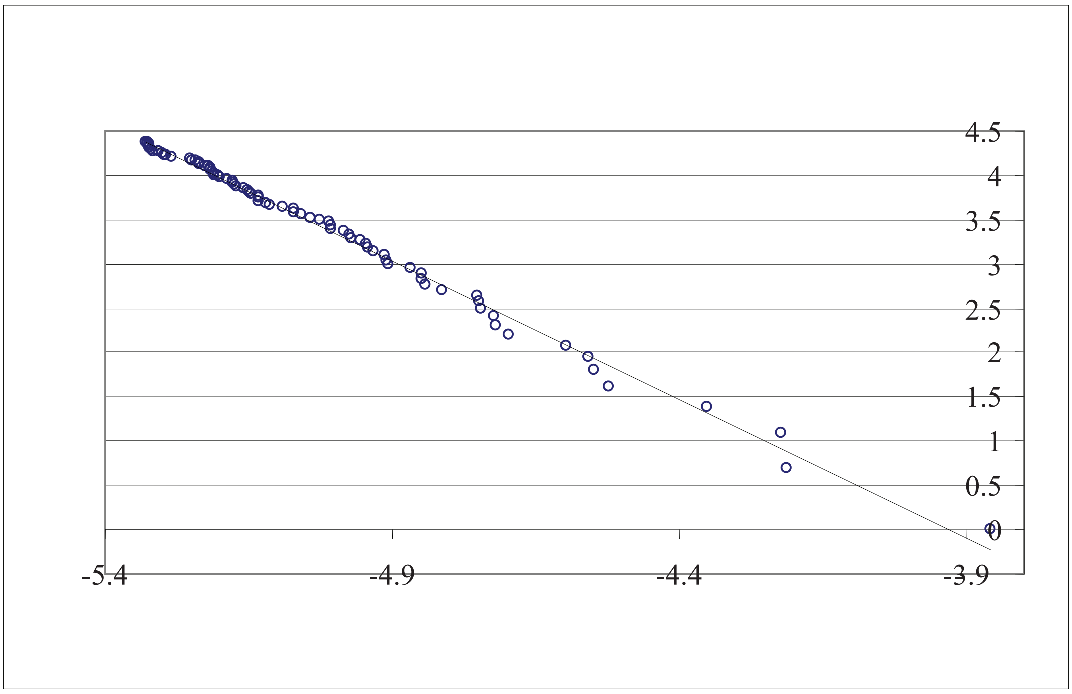

Figure 5. Log-log rank-size plot for the GBP exchange rate using regression (7) with the optimal shift γ=1/2.

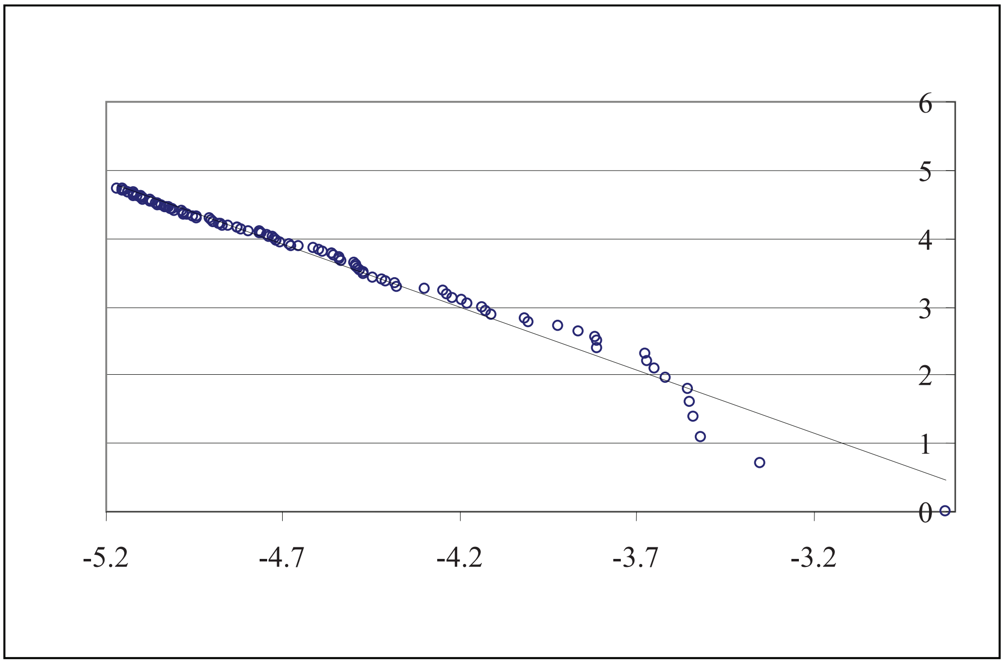

Figure 6. Log-log rank-size plot for the RUB exchange rate using regression (7) with the optimal shift γ=1/2.