Gradual repayment with sequential financing in micro-finance

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Abstract

This paper examines two dynamic features associated with many micro-finance schemes, namely *gradual repayment* and *sequential financing*. We argue that a unified explanation of both these aspects can be built around dynamic incentives, in particular the simple idea that the incentive to default should be relatively uniformly distributed across time. We formalize this intuition in a model that allows project returns to accrue over time rather than at a single point, and takes *ex post* moral hazard problems very seriously. We show that schemes with gradual repayment can improve efficiency vis-a-vis schemes that do not, and further, in the presence of social sanctions, sequential lending can help improve project efficiency and may even implement the efficient outcome. Interestingly, if social capital is manifest in the borrowers' ability to make side-payments, that may reduce project efficiency.

Key words: Group-lending, sequential financing, gradual repayment.
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1 Introduction

This paper develops a unified framework that seeks to explain two dynamic features associated with many micro-finance[1] schemes, namely *gradual repayment* and *sequential financing*. Moreover, we do so in a scenario with *dynamic joint liability* whereby default by any borrower not only triggers project liquidation for all active borrowers, but all group members who are yet to obtain a loan are denied loans as well. In so doing we take the viewpoint that dynamic incentives are a key to understanding the success of many micro-finance schemes,[2] thus complementing the existing literature on micro-finance institutions (discussed later).

Gradual repayment is a 'near-universal feature' (Bauer, Chytilova and Morduch, 2008, pp. 2), found across micro-finance institutions (henceforth MFIs) and across countries. It refers to the fact that under many micro-finance schemes, including Grameen I, repayment starts quite early, often before the project has yielded its full benefit. Moreover, the repayments involve quite small, but regular instalments, e.g. equal weekly instalments under Grameen I. Zeller et. al. (1996)[3] find that similar weekly instalments are being used by ASA and RDRS in Bangladesh. Morduch (1999) has reported similar practices for Bank Rakyat Indonesia (BRI) and Badan Kredit Indonesia (BKI), and for FINCA Village banks in a number of countries such as Malawi, Haiti, Kosovo, Democratic Republic of the Congo, and Afghanistan.

Sequential financing is another institutional design widely adopted by many MFIs. In the case of Grameen I (and many of its replicators), for example, each group is constituted of five members. Loans are initially given to only two of the members (to be repaid over a period of one year).

[1]In fact by 1997 there were around 8-10 million households under similar lending programs in the world (see Ghatak, 2000) and practitioners such as the World Bank were pushing the number towards 100 million households by 2005.

[2]Aghion and Morduch (2005, pp. 119) also emphasize the importance of dynamic institutions.

[3]Zeller et. al. (1996) have analyzed 128 micro-finance groups in Bangladesh, associated with three different MFIs, ASA (Association of Social Advancement), BRAC (Bangladesh Rural Advancement Committee) and RDRS (Rangpur Dinajpur Rural Service).

If they manage to repay the early instalments then, after a month or so, the next set of two borrowers receives loans and so on.[4] Zeller et. al. (1996) also find that under all three MFIs studied by them, loans were provided in sequence. Under RDRS, for example, credit is provided in two instalments with only half the members receiving loans at first. The other half receive loans only when the first set of recipients had partly repaid their loans.

Even more interestingly, many MFIs, including Grameen I and Grameen I replicators, RDRS, ASA etc. practice a *combination* of both sequential financing and gradual repayment.[5] However, despite some recent works (discussed later), both gradual repayment and sequential financing, as well as the interaction between the two, remain poorly understood.

Our approach is based on the simple idea that incentives to default are higher in case the amount to be repaid is higher. This leads to the intuition that default incentives must be relatively uniformly distributed across time, so that it is not too large at any single point. We argue that both gradual repayment and sequential lending serve precisely this purpose, augmenting each other when used in unison, thus providing a theory as to why the two often (though not always) go together.

One needs to recognize of course that things are much more subtle than what is portrayed above. Let us begin with individual lending and consider a one shot repayment scheme. Does the incentive to repay necessarily increase in case the amount to be repaid is staggered into two instalments (say)? While this is clearly the case for the second instalment, what about the first? Defaulting on the first instalment, it may be argued, is equivalent to defaulting on the whole loan. If that is the case then staggering of loans should not make a difference. Thus the analytical challenge is to provide a simple framework capable of capturing this intuition.

Before doing so let us however briefly discuss the case of interest, one

[4]See, for example, Morduch (1999).

[5]Other MFIs to practice a combination of both gradual repayment and sequential lending include ASOD (Assistance for Social Organization and Development, ESDO (Eco-Social Development Organization), PMUK (Padakkhep Manobik Unnayan Kendra), POPI (People's Oriented Program Implementation), SSS (Society for Social Service), TMSS (Thengamara Mohila Sabuj Samgha, one of the largest MFIs in Bangladesh), UDDIPON (United Development Initiatives for Programme Action).

where gradual repayment is combined with sequential lending. We shall argue that a similar logic applies whenever there is some minimal form of social capital. Suppose social capital takes the form of social sanctions, where in case of a default, the borrowers who are adversely affected can impose sanctions on the defaulter(s). We however allow for limited collusion among the borrowers, in that no sanction is imposed in case it is beneficial for all borrowers that default takes place. In such a situation, incentive to default may be high if the borrowers are bunched together as group default may be very attractive. This is because the borrowers can make 'group defaults' and not impose any penalties on one another. Sequential lending may be a way of preventing such coordinated default, as in this case the default incentives of the borrowers will be different. Again the idea is to distribute the default incentives relatively uniformly over time via sequential lending.

We then turn to task of formalizing this intuition, arguing that the key to doing so lies in three things (a) formulating project returns more realistically as a stream of income over time,[6] (b) recognizing the pervasiveness of *ex post* moral hazard problems, and (c) limited liability of borrowers. Project size is endogenous with project returns increasing, up to a level, in the level of initial investment. With borrowers being poor, they have to approach some MFI if they want to invest.

The moral hazard problem has two sources. First, project returns are non-verifiable, so that there is an *ex post* moral hazard problem as the borrowers can hide the money. Further, project liquidation is also non-verifiable, and in case of liquidation the borrower obtains an one-shot payment (that depends on the length of time the project has still to run). These two together imply that the MFI can only see whether repayments are being made as per agreement or not, and hence project termination by the MFI can only be conditioned on repayment performance.

We begin by considering the case under individual lending, which allows

[6]Under Grameen, as well as other micro-finance schemes, projects typically involve a steady stream of income over a period of time. One can think of a Grameen woman buying a sewing machine, or a women from Haryana, India buying a cow, or an woman from Andhra Pradesh, India, buying an idli-maker with her micro-finance loan.

us to examine the incentive implications of gradual repayment in a more transparent fashion. Note that in this framework staggering of loans clearly affects incentives. With a one shot repayment scheme, given the dynamic nature of projects and limited liability, time has to pass before the loan can be repaid. At this point however the incentive to default would be high, because of the large instalment, as well as the fact that continuing with the project is no longer that attractive. With staggered repayment however, the early instalments can be asked for at a time when continuation of the project is still very attractive, thus making repayment more attractive. Given this intuition we find that gradual repayment schemes does better compared to one-shot repayment schemes (or any other repayment scheme for that matter), and may even lead to the efficient project size.

We then turn to the more relevant and interesting scenario, one where lending institutions include both sequential lending and gradual repayment. When social capital takes the form of social sanctions, we find that sequential lending necessarily improves efficiency vis-a-vis individual lending. For a two member group, sequential lending can help resolve the problems associated with group default since default by the first recipient may adversely affect the second borrower, thus attracting the social sanctions. Now consider the time when the second borrower obtains her loan. At this point the first borrower may have already repaid a substantial fraction of her loan, and thus be unwilling to default. Thus any default by the second borrower now adversely affects the first borrower, so that the sanction is imposed. It is this subtle interaction of dynamic incentives that ensures that a higher project return can be implemented.

We begin by considering an institutional structure that is quite often seen in reality, namely *two stage* lending schemes (e.g. consider the many Grameen I replicators all over the world, as well as the RDRS in Bangladesh). Consider any project size, say k. We show that as long as the moral hazard problem is relatively small (compared to the profitability of the project), one can find a two member scheme such that all borrowers invest at the level k. Interestingly, this condition turns out to be independent of the size of the social sanction. The intuition is very similar to that for the two member case, with two additional points of interest. First, group size may need to be sufficiently large so as to ensure that default by one sub-group of borrowers attracts a sufficiently large penalty from the other group. Second, at the point where the first set of borrowers have already completed their projects, the incentive to default is quite large. At this point though, the second set of borrowers do not default since the project is relatively profitable. This, however, is only half the story. Sequential lending, in addition, plays a critical role in ensuring that by the time the second borrower obtains a loan, the first borrower does not have too much to gain from deviation, since the first borrower's project is nearing completion.

We then argue that *any* given project size, in particular the efficient one, can be supported if one allows for more than two stages, demonstrating the power of sequential lending coupled with gradual repayment. The role of sequentiality is critical for this result, as the multi-stage nature of the program ensures that by the time the penultimate group of borrowers complete their projects, the final group of borrowers would be nearing the end of their own projects, and would have no incentive to default. Moreover, we find that the corresponding scheme need not be too protracted.

Note that the preceding framework implicitly assumes that borrowers cannot make side payments. Given that there may be situations where side payments are feasible,[7] we next turn to analyzing this case. We suppose that the presence of side payments allows the borrowers to take default decisions based on maximizing aggregate group payoff, finding that this may allow the MFI to sustain more efficient projects compared to that under individual lending. The intuition is as follows. In the presence of side-payments default is more costly, as the group takes into consideration what the group as whole is going to lose in case of a deviation.

We begin with two stage lending programs, showing that any loan size can be supported as long as the moral hazard problem is not too severe. The key here is that the net incentive to default for the group is non-monotonic with time, being initially decreasing, and then increasing with time. Thus keeping down default incentives imply that the second set of loans must be provided neither too early, nor too late. Our scheme does precisely that.

[7] In Indian villages, farmers often partially insure each other through gifts and loans (Townsend, 1994). This is discussed in somewhat greater details later.

Interestingly, we find that there is a possible downside to social capital (interpreted as the borrowers maximizing group payoff) in that repayment incentives are lower, and consequently the maximal feasible project size is smaller with side-payments, compared to the case where such side payments are not possible. This effect is essentially because side-payments allow for coordinated defaults, and is brought out even more starkly by by our final result: for loan schemes that are not too protracted, we show that an efficient project size may not be attainable even if one allows for more than two stages.

1.1 Related Literature

One stream of the literature examines the role of static joint liability in harnessing peer monitoring (see, e.g. Banerjee et al. (1994), Conning (1996), Ghatak and Guinnane (1999), Stiglitz (1990), and Varian (1990)), the essential idea being that with joint liability any group member has more to lose in case other group members default, providing her with an incentive to monitor.

Another important branch of the literature examines the role of joint liability in promoting homogenous group-formation (see, e.g. Ghatak (1999, 2000) and Tassel (1999)), especially in the presence of asymmetric information. Here the intuition is that while, with joint liability, both good and bad borrowers would want good borrowers as partners, good borrowers can offer more attractive contracts to their partners, so that group formation necessarily involves positive assortative matching. Another related paper is Besley and Coate (1995), who was the first to allow for social sanctions in a group-lending contest.

Gradual repayment has been analyzed in Jain and Mansuri (2003), Field and Pande (2008) and Field, Pande and Paap (2009). In Jain and Mansuri (2003), the requirement of early repayment forces borrowers to borrow from friends/local moneylenders who have better information about the credit worthiness of the borrowers. Thus early repayment is a device for tapping into the information possessed by these agents. Our framework is complementary to that of Jain and Mansuri (2003), in that we rely on dynamic incentives, rather than on asymmetric information, or the presence of informal lenders, for our analysis.

Recently Field and Pande (2008) and Field et. al. (2009) perform an extremely interesting set of experiments. Their studies show that a shift from weekly to monthly instalment does not effect repayment immediately (Field and Pande, 2008), but may adversely affect repayment rates after a few loan cycles (Field et al., 2009). Thus our results are not inconsistent with the experimental facts observed in these papers.

Turning to sequential lending, Roy Chowdhury (2005) argues that sequential lending is a tool for generating peer monitoring, where the peer monitoring aims at solving an ex ante moral hazard problem. Roy Chowdhury (2007) on the other hand shows that in the presence of contingent renewal, sequential lending leads to homogeneous group formation whereby good and bad borrowers group together. In such a scenario, sequential lending allows the lender to test for the composition of a group relatively cheaply. The present paper thus extends the earlier literature on micro-finance by providing an unified theory of both gradual repayment and sequential financing.

The rest of the paper is organized as follows. The next section describes the institutional setting, before going on to analyze the case of individual lending. Sections 3 and 4 examine a scenario with both gradual repayment, as well as sequential lending. While Section 3 examines a scenario with social sanctions, Section 4 allows for side-payments among borrowers. Section 5 concludes. Some of the proofs can be found in the Appendix.

2 Institutional Setting

The model comprises a lender, namely an MFI, and a set of potential borrowers. Each borrower has a project that requires a start-up capital of k, where k is endogenous and can take any value in $[0, K]$. Further, project returns accrue over time, starting at $t = 0$ (say), so that a project of size k yields a return of $F(k)$ at every $\tau \in [0, 1]$. The borrowers can hide project returns, generating an *ex post* moral hazard problem. We assume that $F(k)$ is increasing and strictly concave with $F(0) = 0$. The borrowers however have no money, and hence must borrow the amount k from the MFI in case they decide to implement $F(k)$.[8] If the MFI lends an amount k to some borrower and a project of size k is funded, they agree on a repayment schedule for the borrower to pay back her loan obligations.

At any date t a borrower can default on her repayment obligation, when the project is liquidated with the borrower obtaining a private benefit of $(1 - t)b$ and the lender obtaining nothing.[9] We assume that $F(k) > bk$ for all k, so that such liquidation is inefficient. The lender cannot observe whether the project has been liquidated or not, hence such liquidation also involves another moral hazard problem, which is second of the two moral hazard problems which are at the heart of this paper. As is standard in this literature, we assume there is limited liability on the part of borrowers. Thus in case of default, the only penalty is that the contract is terminated and the asset liquidated. Further, the lender obtains no payoff. Finally, for exposition we assume that neither the MFI, nor the lenders discount the future.

2.1 Individual Lending

The case of individual lending forms a benchmark for the later analysis. This is also of independent interest since, as discussed in the introduction, many MFIs adopt gradual repayment without sequential repayment (ASA, for example, has some group loans without group guarantees, as well as individual loans, where it adopts gradual repayments, see, Annual report, 2007, ASA).

We visualize the following scenario: at $t = 0$, the MFI selects a borrower and enters into a contract with her that specifies the amount borrowed k, and the repayment scheme $y(t, k), t \in [0, 1]$. If the borrower accepts the contract, she immediately invests k in the project and has to make payments according to the repayment schedule. Throughout we assume that the repayment

[8] In an earlier version of the paper we allowed for borrower income/savings. This does not affect the results qualitatively.

[9] It is simple to extend the model to the case where in case of liquidation at t, the lender also obtains a positive payoff. As long as liquidation is still inefficient, allowing for this possibility does not affect the analysis in any way.

schedule satisfies the budget constraint for the borrower, which ensures that the borrower is able to pay her dues from the current gross proceeds of the project. Thus we have that $F(k) \geq y(t,k) \geq 0$ for $t \in [0,1]$. If the borrower fails to meet her payment obligations at any date t, the lender will liquidate the project. Of course, in case of such a default, the borrower will decide to overuse the project and obtain a benefit of $bk(1-t)$ for himself.

A repayment schedule $y(t,k)$ is said to satisfy the *no default (ND)* condition if, for every $t \in [0,1]$,

$$F(k)(1-t) - \int_t^1 y(\tau,k)d\tau \geq bk(1-t). \tag{1}$$

Given k, the lender must break even under the repayment schedule $y(t,k)$ and thus

$$\int_0^1 y(t,k)d(t) \geq k. \tag{2}$$

We will say that a lending scheme $< k, y(t,k) >$ is *feasible* if it satisfies the no default condition and the lender breaks even.

Our focus is on analyzing repayment schemes in which the lender is repaid the loan k in the shortest possible time.

Definition. The *fastest gradual repayment* scheme (FGR in short) $y(t,k)$ is simply given by

$$y(t,k) = \begin{cases} F(k), & \text{if } t \leq \frac{k}{F(k)}, \\ 0, & \text{otherwise.} \end{cases}$$

Finally, let k^* denote the 'efficient' project size, in that k^* maximizes $F(k) - k$.

To understand the incentive implications of an FGR given a loan amount k, let us first consider any $t \geq \frac{k}{F(k)}$. The continuation payoff to the borrower from defaulting is $bk(1-t)$, whereas that from continuing with the scheme is $F(k)(1-t)$ since at this point the borrower has already repaid her loan. Clearly, there is no incentive to default as $F(k) > bk$.

Next let $t < \frac{k}{F(k)}$. Since $\int_0^1 y(t,k)dt = k$, at any date $t < F(k)/k$, $\int_t^1 y(\tau,k)d\tau = k - F(k)t$. Thus, using equation (1), the no default constraint under an FGR is easily shown to equal

$$F(k) - k \geq bk(1-t). \tag{3}$$

Clearly, under an FGR, the ND constraints are satisfied for all t if and only if the ND constraint is satisfied at $t = 0$. Using this constraint at $t = 0$, it is easy to check that an FGR can support the efficient project size of k^* if and only if $F(k^*) - k^* \geq bk^*$.

When $F(k^*) - k^* < bk^*$ however, the *ex post* moral hazard problem has serious efficiency implications in terms of smaller project sizes. The maximum project size $\hat{k}$ is clearly the maximal k that satisfies the no default condition at $t = 0$, i.e.

$$b = \frac{F(\hat{k})}{\hat{k}} - 1. \tag{4}$$

For any $k > \hat{k}$, there is default. Note that $\hat{k}$ is unique (because of the strict concavity of $F(k)$) and moreover, $\hat{k} > 0$ if and only if $F'(0) > b + 1$ (this will be assumed in the sequel). Clearly, $\hat{k}$ is less than the efficient project size k^* in case $F(k^*) - k^* < bk^*$, so that there is inefficiency.

Our interest in fastest gradual schemes (FGRs) is motivated by Proposition 1 below which shows that no other schemes can do strictly better compared to FGR schemes.

Proposition 1 *Consider an individual lending arrangement.*

(a) If a lending scheme $< k, y(t, k) >$ is feasible, then the fastest gradual repayment scheme (FGR) corresponding to the project size k is also feasible.

(b) The efficient project size of k^* can be supported if and only if the ex post moral hazard problem is not very large in the sense that $bk^* \leq F(k^*) - k^*$.

Proof. (a) Given that $< k, y(t, k) >$ is feasible, it satisfies the no default condition at $t = 0$. Now consider an alternative loan scheme with the same level of k, but involving an FGR scheme. By construction, the FGR scheme satisfies the no default condition at $t = 0$, and hence, from our earlier argument, the ND constraints for all t.

(b) See the discussion preceding Proposition 1.■

The intuition as to why an FGR (or more generally a gradual repayment scheme) does well under such an environment is simple. With a gradual repayment scheme, the repayments are staggered, so that default incentives at any one point are not too large. While default incentives are largest at the very start of the project, i.e. at $t = 0$, at this point continuation payoffs are also correspondingly higher. With any other repayment scheme, given the nature of the technology, time has to pass before the MFI can ask for repayment. At such a point however, the borrower has potentially less to gain from continuing with the project, so that default becomes more attractive.

In fact Proposition 1 is consistent with Field et. al. (2009) and Kurosaki and Khan (2009). Recall that Field et. al. (2009) found that switching to a less gradual form of repayment, i.e. from a weekly to a monthly scheme, increases default incentives after some time. Kurosaki and Khan (2009) examine micro-finance schemes in Pakistan. They find that while several such schemes failed in the late 1990s (even though they adopted a group lending design), there was a drastic decrease in default rates from early 2005, when contract designs were changed with more frequent repayment instalments (and improved enforcement of contingent renewal).

Given Proposition 1, it is easy to see that there can be situations where the efficient loan size is feasible under gradual repayment (in particular FGR), but not under an one-shot repayment scheme. Consider a loan scheme involving the efficient loan size of k^* and an one shot repayment scheme. Given limited liability, repayment can only be asked at date $k^*/F(k^*)$. Thus there will be default if $ak(1-k^*/F(k^*))-k^* < bk^*(1-k^*/F(k^*))$. An FGR scheme would, however, still implement k^* provided $bk^* \leq F(k^*) - k^*$.

It is clear from Proposition 1(b) that if b is large, i.e. $F(k^*) - k^* < bk^*$, then individual lending will fail to implement the efficient outcome. Given this, can group contracts help to implement more efficient, i.e. larger projects in such a situation? To this question, we now turn.

3 Group Lending with Social Sanctions

We next examine micro-finance institutions where gradual repayment and group-lending go together, institutions that are often found in reality. While the Grameen example is well known, other MFIs like BRAC, ASA and RDRS in Bangladesh also employ similar schemes (see Zeller et. al. (1996)). We argue that in the presence of some minimal form of social capital, a combination of gradual repayment and sequential financing can help restore efficiency by alleviating the ex post moral hazard problem intrinsic to this setup. Further, given dynamic joint liability the entire group is held responsible (and penalized) in case of default. We capture dynamic joint liability in a very simple way: first, in case of a default, all existing projects are necessarily dissolved and second, any group members who are yet to receive a loan will be denied any future loans.

We examine such schemes in a setup that also involves social sanctions against the defaulting borrower(s) that may be imposed by the non-defaulting group members. Such social sanctions may involve exclusion from public goods, e.g. access to communal assets, informal insurance networks, etc.[10] Further, in this section we examine a scenario where there is no is side-payments among borrowers.[11]

More specifically, consider a group consisting of n members and suppose at date t, m of the borrowers default. With dynamic joint liability, some of the non-defaulting members, numbering (say) k, will be *adversely* affected as a result of this default. These may include borrowers who are yet to obtain a loan, or else may include borrowers who have obtained a loan but have already repaid substantially, so that they would prefer not to default. We assume that each such affected member can invoke a penalty of f on each of the deviating borrowers. Consequently, every defaulting borrower is subjected to an aggregate penalty of kf.

Note that this formulation implicitly allows for limited collusion among the borrowers so that they can jointly decide to default as a group. In case of such 'group default' however, note that social penalties will not be invoked. It seems natural to allow for such limited collusion among borrowers since

[10]We can think of the present formulation being a reduced form approximation of a larger model where such penalties are imposed as part of optimal threat strategies. Alternatively, such penalties can be attributed to social preferences, in particular the presence of altruistic punishers who are willing to penalize a violation of social norms if they feel that such violation hurts the society as a whole. See, e.g. Fehr and Fischbacher (2004), Gintis et. al. (2005), and the references cited therein.

[11]In the next section we consider a scenario where allows for this possibility.

these borrowers are very likely to communicate with each other and if there is a situation where 'default' benefits all borrowers at the expense of the lender, they will do so without invoking the social penalty.

3.1 Two-stage Group Lending Schemes

To see how *social sanctions* and the possibility of limited collusion may alter defaulting incentives in a group context, we shall focus on simple *two-stage group lending* arrangements, in which some members of the group obtain the loan earlier than the rest of the group. Interestingly, the RDRS in Bangladesh follows a scheme where half the members of a group receive the loan initially, followed by the rest at a later date. Some Grameen I replicators also follow such two stage schemes.

We show that group-lending leads to more efficient projects compared to individual lending. We characterize the condition under which a given project size can be supported, finding that the condition essentially ensures that the incentive to default is not too large once some of the borrowers who receive the loan early complete their projects. *Inter alia* we also provide a design for such schemes.

Consider a scheme with n borrowers, where these borrowers are divided into two sub-groups. It is simple to extend the logic in Proposition 1(a) to argue that it is sufficient to consider FGR schemes alone, which is what we do. In what follows, let $< n, m, t_2, k >$ represent a two stage group lending arrangement in which $(n - m)$ borrowers receive loans of k each at $t = 0$, the remaining m members receive loans of k each at t_2, and the repayment schedule is the FGR corresponding to k.

We say that a loan size k is *supported* in a two stage group arrangement if there exists n, m and t_2 such that the scheme $< n, m, t_2, k >$ is feasible.

We first observe that for such a group arrangement to dominate an individual lending scheme, it must be that $t_2 > 0$, i.e. not all members in the group can receive their loans at the same point of time. To see this, consider the arrangement in which all borrowers are advanced a loan k, where $k > \hat{k}$, at $t = 0$. From Proposition 1 it is clear that, in the absence of the social penalty, each borrower is better off defaulting on her loan immediately. Given the presence of limited collusion however, it is also in the interest of the group to default jointly and not impose any social sanctions. Sequential financing can help break this impasse since default by the first set of recipients would result in the second set of borrowers not getting the loan and will thus attract the social penalty.

We then discuss the various *no default* conditions in some detail as this helps bring out the interplay between the various dynamic incentives in operation here. Moreover, since the default payoffs are decreasing in t, for feasibility it is sufficient to check the default incentives at exactly three dates: $t = \{0, t_2, 1\}$.

We first consider the default incentives at $t = 0$. Since all $n-m$ members who receive this loan at $t = 0$ can jointly decide to default, the social penalty will thus be imposed by the remaining m borrowers who will not get the loan and will thus be adversely affected. A defaulting borrower at date 0 thus has a net default pay off of $bk - mf$. Since the lender must break even on her loan, the no default continuation payoff for a borrower at any date is no more than $F(k) - k$. For default to be unprofitable, the following condition then must hold at $t = 0$,

$$bk - mf \leq F(k) - k. \tag{5}$$

Now consider the date t_2 at which the remaining m borrowers receive the loan and suppose some members of this group plans to default. Irrespective of whether a social sanction will be imposed on them by the $(n-m)$ members who receive their loans at $t = 0$ or not, for default to be unprofitable, we must have

$$bk - (n - m)f \leq F(k) - k. \tag{6}$$

Given our assumption of limited collusion however, this sanction will only be imposed if first set of members to receive loans are adversely affected. Now following a default at t_2, the lender will liquidate all the existing projects. Thus a group member receiving her loan at $t = 0$ will have a net liquidation payoff of $bk(1 - t_2)$. A social sanction will be imposed by such a member only if this default payoff is strictly less than her continuation payoff without default. Since the maximum continuation payoff of any member at any t is at most $F(k) - k$, a necessary condition for sanction to be imposed is given by

$$bk(1 - t_2) < F(k) - k. \tag{7}$$

Finally, at date $t = 1$, since the $n - m$ borrowers would have completed their projects, no further sanctions will be forthcoming from this group. Consequently, for default to be unprofitable for the second set of borrowers, it must also be the case that the default payoff of any such borrower is less than her continuation no default payoff. Now the default payoff for the second set of borrowers at $t = 1$ is simply bkt_2, while the continuation payoff is at most $F(k) - k$. Thus, at $t = 1$, we must have

$$bkt_2 \leq F(k) - k. \tag{8}$$

We then characterize conditions such that a loan of size k can be supported, i.e. there is a scheme where all borrowers invest at the level k. From the preceding discussion, adding equations (7) and (8) we have the necessary condition that $bk < 2[F(k) - k]$. In fact, as shown in Proposition 2 below, this condition is sufficient as well. Note that this condition does not depend on the magnitude of the social sanction, f.

While the formal proof of Proposition 2 is given later in the Appendix, the idea is quite intuitive. Let us consider a scheme where there are $2m$ borrowers, with all of them being provided a loan of size k. While half of them get their loans at $t = 0$, the rest do so at a later date, say t_2. Further, m is large enough such that if any borrower deviates, and all m borrowers from the other set of borrowers impose social sanctions, then such deviation is unprofitable. The idea is to provide the second set of loans at a time t_2, such that at this point the first set of borrowers do not want to deviate. Now there is no deviation at $t = 0$ as then the second set of borrowers will impose the sanction, whereas there is no default at t_2 since then the first set of borrowers will impose the sanctions. At $t = 1$ however, the first set of borrowers have already repaid and have no incentive to impose any sanctions. Thus at this point there must be enough incentive for the second set of borrowers to continue with the scheme. This happens if the project technology is profitable enough, which is guaranteed by the condition that $bk < 2[F(k) - k]$.

Proposition 2 *Fix the size of the social sanctions $f > 0$. Then a two-stage sequential group lending scheme can support a loan of k if and only if the moral hazard problem is not too large, i.e. $bk < 2[F(k) - k]$.*

Proposition 2 shows the importance of sequential financing in achieving efficiency. Suppose, for example, that $F(k) - k < bk < 2[F(k) - k]$. Then we know from Proposition 1 that a project size of k cannot be supported under individual lending, even with an FGR scheme. Proposition 2 shows however that even then a combination of sequential lending and an FGR scheme can sustain this project size.[12]

3.2 Multi-stage Lending Schemes

Given Proposition 2, it is natural to ask as to what happens if the efficient project is not very profitable, in the sense that $bk^* > 2[F(k^*) - k^*]$. While we know that in this case efficiency cannot be achieved with two stage schemes, is it possible to do so with more complex schemes? We examine this question for loan schemes that are not too protracted (in a sense defined below). We show that the answer is indeed in the affirmative, thus demonstrating the power of sequentiality and gradual repayment in this context.

Formally, we define a loan scheme to be *one-cyclical*, i.e. not too protracted, if the last set of borrowers receive their loans by $t = 1$. Note that this restriction is naturally satisfied for two member groups.

Proposition 3 *For any level of social sanctions $f > 0$, there is a feasible one-cyclical sequential group lending scheme in which all group members receive the efficient loan of k^*.*

The intuition is as follows. Under the two stage scheme note that the incentive to default is quite large at $t = 1$, when the first set of borrowers have already completed their schemes. Further, with two stage schemes this

[12]To demonstrate the importance of gradual repayment for the result in Proposition 2, suppose that $bk^* < 2[F(k^*) - k^*]$ so that in the presence of FGR, a two stage group scheme can implement the first best. It is straightforward to demonstrate however that if $k^*/F(k^*) > 1/2$, then no group scheme with one shot repayment can implement the efficient outcome.

may happen quite early in the loan cycle of the second set of borrowers, so that they may have a large incentive to default. In case the loan is staggered over more than two stages however, by the time the penultimate group of borrowers finish their own schemes, the last group of borrowers have already repaid their loans, and therefore have no incentive to default.

4 Group-lending with Side Payments

An important assumption in the preceding analysis is that agents are not allowed to make side payments. While this is clearly a plausible assumption in many contexts, there are scenarios where it may not be. Following Ghatak (2000), we can appeal to non-pecuniary forms of transfers, e.g. providing free labor services and the use of agricultural implements, to justify side-payments. Pecuniary transfers may involve promises to pay their partners out of the future returns from the project. Following the literature such side payments can be sustained using penalties that involve exclusion from public goods, e.g. access to communal assets, informal insurance networks, etc. Such behavior is even more plausible in the context of rural economies where any deviation is likely to be observable, and inter-linked markets abound.

When such side transfers are possible, it is natural to assume that that repayment/default decisions are made keeping the aggregate payoff of the group in mind. It may be argued that maximizing group income can be interpreted as a form of social capital. In the Grameen, for example, there seems to be some effort at fostering a group identity. At least three of the resolutions, (12, 13 and 14), among 16 resolutions that Grameen members pledge emphasize on group payoff and joint welfare maximization.[13] This is the assumption that we make to analyze the nature of incentive compatible contracts in this setup.[14]

[13]For example, resolution 12 states, "We shall not inflict any injustice on anyone, neither shall we allow anyone to do so", resolution 13 states "We shall collectively undertake bigger investments for higher incomes" and resolution 14 states "We shall always be ready to help each other. If anyone is in difficulty, we shall all help him or her." Source: http://www.grameen-info.org, accessed on May 7, 2009.

[14]Given our assumption that the group maximizes the aggregate payoff, it is of course

4.1 Two Stage Group-lending with Side Payments

As before, we begin our discussion with the analysis of two stage group lending schemes. We characterize the necessary and "sufficient" conditions under which the efficient outcome can be implemented under a two stage scheme. Interestingly, we show that the incentive to default behaves non-monotonically with time, implying that the second set of loans cannot be either too early, or too late. Further, group-lending with side payments leads to greater project efficiency vis-a-vis individual lending, thus demonstrating the importance of sequential lending even in the presence of side-payments. Interestingly however, one finding is that the ability to make side-payments may harm project efficiency, compared to a situation where such payments are not possible.

Consider a two-stage scheme $< n, m, k, t_2 >$. As in the previous section, it is sufficient to examine lending schemes with FGR, as well as to check the default incentives at exactly three dates: $t = \{0, t_2, 1\}$.

First, at $t = 0$, when $n - m$ borrowers receive their loans, default will give a net payoff of $(n - m)bk$ for the entire group. Since the continuation payoff for each of the n members is $F(k) - k$, the group will not default at $t = 0$ if

$$b(n - m)k \leq n[F(k) - k]. \tag{9}$$

Consider now the date t_2 at which the remaining m borrowers receive their loans. If the group plans to default at this date, the net payoff is given by $(n - m)bk(1 - t_2) + mbk$. Incentive compatibility then requires that

$$(n - m)bk(1 - t_2) + mbk \leq n[F(k) - k]. \tag{10}$$

Finally, at date $t = 1$, since the $n - m$ borrowers would have completed their projects by then, if the group defaults at this date, its net payoff is simply $mbkt_2$. For default to be non profitable at date $t = 1$, we than have the necessary condition

$$mbkt_2 \leq m[F(k) - k]. \tag{11}$$

clear that the value of f has no impact on the nature of feasible contracts as the social penalty will never be invoked.

To characterize the set of loan sizes that can be supported in a two stage group lending arrangement, for any $k > 0$, we define the average gross payoff $a(k) = \frac{F(k)}{k}$, and the average net payoff, $x(k) = a(k) - 1$.

We then turn to the task of deriving necessary conditions such that k can be supported. This implies that there exists n, m and t_2 such that equation (9) holds. Dividing both sides of equation (9) by nbk and noting that $x(k) = a(k) - 1 = \frac{F(k)}{k} - 1$, we get

$$1 - \frac{m}{n} \leq \frac{x(k)}{b}. \tag{9.1}$$

Rearranging equation (10) and dividing by nk, one gets

$$[b(1 - m/n) - x(k)] \leq (1 - \frac{m}{n})bt_2. \tag{10.1}$$

By equation (11), we have $bt_2 \leq x(k)$. Using this information in (10.1) and then dividing both sides of (10.1) by $x(k)$, we get

$$\frac{b - x(k)}{x(k)} \leq 1 - \frac{m}{n}. \tag{11.1}$$

Equations (9.1) and (11.1) gives us $\frac{b-x(k)}{x(k)} \leq 1 - \frac{m}{n} \leq \frac{x(k)}{b}$. Cross multiplication in the preceding equation gives us condition (C.1):

$$(C.1): \quad b^2 \leq bx(k) + (x(k))^2.$$

Note that this condition implies that the moral hazard problem is not too large vis-a-vis the net profitability of the project. We next derive another necessary condition with a similar interpretation that we call (C.2):[15]

$$(C.2): \quad b \leq x(k)[1 + \frac{1}{1 + x(k)}].$$

It may be instructive to examine why condition (C.2) is necessary. Let us consider the net incentive to deviate for the group as a whole at the instant t when the rest of the members receive their loans. Let $t(k)$ denote the instant when the initial set of borrowers complete repayment under an FGR, so that

[15] When $b \leq 1$, it is possible to check that if condition (C.1) holds, then condition (C.2) holds as well. For $b = 1$, both conditions in fact simplify to $1 \leq x(k) + (x(k))^2$. Thus, for $b \leq 1$, we shall argue that condition (C.1) is both necessary, as well as almost sufficient to support a loan of size k in a two stage arrangement.

$t(k) = \frac{k}{F(k)} = \frac{1}{a(k)}$. Interestingly, the net incentive to default for a group is decreasing till $t(k)$, and increasing after that. Thus it is necessary that the group does not default if the second set of loans if provided at $t(k)$. Condition (C.2) essentially comes out of these considerations.

Proposition 4 (Necessity) *If a loan size k can be supported in a two-stage sequential group arrangement with side-payments, then conditions (C.1) and (C.2) must hold.*

Proof. Consider the net payoff from default for the group as a whole if the group defaults at t when the remaining m borrowers receive their loans. We denote the net aggregate payoff from default at $t < t(k)$ by $A(t)$, where

$$A(t) = (n-m)bk(1-t) + mbk - n[F(k) - k].$$

We begin by showing that $A(t(k)) \leq 0$. If not, then $A(t(k)) > 0$. Now note that $A(t)$ is decreasing in t, therefore, for all $t < t(k)$, $A(t) > 0$. Since $< n, m, t_2, k >$ is *feasible* at t_2, default should not be profitable for the group. Since $A(t(k)) > 0$, it must be that $t_2 > t(k)$. Now, the continuation payoff of the group at t_2 from not defaulting is $(n-m)F(k)(1-t) + m[F(k) - k]$. Thus, the net payoff of the group from defaulting at $t_2 > t(k)$ is given by

$$B(t) = (n-m)bk(1-t) + mbk - (n-m)F(k)(1-t) - m[F(k) - k].$$

Observe that $A(t) = B(t)$ at $t = t(k)$. Since $B(t)$ is increasing in t, it then follows that $B(t_2) > B(t(k)) = A(t(k)) > 0$. Thus, the group is better off defaulting at t_2, contradicting the fact that $< n, m, t_2, k >$ is feasible. Thus $A(t) \leq 0$ at $t(k) = \frac{k}{F(k)}$. At $t = t(k)$, using equation (10), we then have

$$b - x(k) \leq (b(n-m) - x(k)) \leq [1 - \frac{m}{n}]bt(k) = [1 - \frac{m}{n}]b\frac{1}{a(k)}.$$

The preceding equation and (9.1) then yield

$$\frac{a(k)(b - x(k))}{b} \leq \frac{x(k)}{b}.$$

Since $1 + x(k) = a(k)$, cross multiplication in the preceding equation gives us condition (C.2).■

We now show that satisfying conditions (C.1) and (C.2) is *almost* sufficient as well to support loan size k. The caveat arises because of an integer problem, so that these conditions are in fact sufficient if both these inequalities are strict. The idea is to fix the size of the groups and sub-groups in such a way that there is no incentive to default at $t = 0$. Given this, the critical issue is to fix the time of the second loan appropriately such that there is no incentive to default at the instant when the second borrowers receive their loans. Following from the discussion of (C.2), this cannot be either too early, or too late, otherwise the incentive to default would be very large.

Proposition 5 (Sufficiency) *For a given k, let conditions (C.1) and (C.2) hold with strict inequalities. Then the loan size k can be supported in a two-stage sequential group-lending arrangement with side-payments.*

Proof. Given k, choose n, m, t_2^* and $\epsilon > 0$ such that

$$1 - \frac{m}{n} = \frac{x(k)}{b} - \epsilon, \tag{9.2}$$

$$b - x(k) = (1 - \frac{m}{n})bt_2^*. \tag{10.2}$$

Given (C.1) holds with a strict inequality, such a choice of n, m and ϵ, where ϵ can be made to satisfy $\frac{b-x(k)}{x(k)/b-\epsilon} \leq x(k)$, exists in (9.2). Note that if $n - m$ members are given the loan of k at $t = 0$, the no default condition (9) is satisfied at $t = 0$. Now because of (10.2), if the remaining m members of the group are given the loan at $t = t_2^*$, the no default condition (10) is satisfied at $t = t_2^*$. To check that $bt_2^* \leq x(k)$ (the no default condition at $t = 1$), use equation (9.2) and the condition (C.1). Finally, to check t_2^* (as defined in equation (10.2)) is no more than $t(k) = \frac{k}{F(k)}$, use the fact that (C.2) holds strictly.[16] Thus, $< n, m, t_2^*, k >$ is feasible and supports the loan size k.■

The next two remarks together establish two important properties of such schemes. First, there are situations where group-lending schemes do strictly better compared to individual lending. Second, the possibility of

[16]Thus we have to show that $b/a(k) > bt_2^* = (b - x(k))(1 - m/n) = b(b - x(k))/x(k)$, i.e. $x(k)/a(k) > b - x(k)$. This however is equivalent to (C.2).

side-payments may actually harm project efficiency, thus formalizing the intuition that social capital may not always be an unmixed blessing. Despite this caveat though, group-lending outperforms individual lending, again establishing the power of sequential lending in our framework.

Remark 1. It is easy to check that there exists $k > \hat{k}$, such that conditions (C.1) and (C.2) are satisfied. For such parameter values individual lending cannot implement the efficient outcome (Proposition 1), whereas the two stage group lending arrangements can do so.

Remark 2. From Proposition (2), we know that a loan size of k can be supported in the absence of side payments if and only if $b \leq 2x(k)$. It is easy to check that at $b = 2x(k)$, condition (C.1) fails and thus when side payments are possible, the maximum loan size that can be supported is in general lower than that can be supported when group members can not make side payments.

4.2 Multi-stage Lending Schemes

Given Propositions 4 and 5, we then examine if, in the presence of side payments, group lending can necessarily lead to the efficient outcome. As in the previous section we restrict attention to one-cyclical schemes.

Proposition 6 below shows that the answer is in the negative. Recall however from Proposition 3, that in the absence of side payments, the efficient outcome is feasible with an appropriately designed loan scheme. Thus Proposition 6 underlines the fact that social capital need not always be an unmixed blessing, a point that was made in Remark 2 also.

Proposition 6 *Suppose $bk^* > 3[f(k^*) - k^*]$. Then no one cyclical scheme with side payments can implement the efficient outcome.*

The proof, which is somewhat involved, is available from the authors on request. Here we just provide a sketch of the argument. We begin by showing that it is enough to consider lending schemes that involve no bunching, except possibly at $t = 0$. We then derive a set of necessary conditions for such schemes to be default-proof. We manipulate such conditions to obtain a necessary condition of the form that $\frac{F(k^*)-k^*}{bk^*} \geq \frac{n+2}{3n}$. This shows that for $bk^* > 3[F(k^*) - k^*]$, efficient outcomes cannot be sustained for any n.

Remark 3. Finally, note that Proposition 6 restricts attention to one-cyclical schemes. Does this result extend to general schemes? We find that it does so for schemes such that all loans are given by by the time the second set of borrowers complete their projects.[17]

4.3 Gestation Lags in Project Maturity

So far, we have assumed that project returns start immediately. The possibility of a lag in project maturity can be easily incorporated by assuming that if investment is made at time t, then, the returns accrue only after a lag of time interval δ. For concreteness, assume that if a project of size k is chosen at $t = 0$, an instantaneous return of $\tilde{F}(k) = \frac{F(k)}{1-\delta}$ arrives for the time period $[\delta, 1]$. Note that this formulation implies that the aggregate return from the project is independent of δ, thus allowing for comparisons with our earlier results.

It is easy to check that our analysis of the optimal individual loan arrangements is completely unaffected by the introduction of gestation lags in project return. The maximum loan size that can be supported is still given by $\hat{k}$ as defined in Proposition 1.

We finally consider the case of two stage group-lending with side-payments.[18] Let us consider a two-stage lending scheme $< n, m, k, t_2 >$, and examine if a loan of $k > \hat{k}$ can be supported. Clearly, the second set of loans cannot be given before δ, otherwise there will be default (from Proposition 1). Hence let the second set of loans be given at $t_2 > \delta$. We note that in this case there is an additional dimension to the moral hazard problem, in that the first set of borrowers may opt to liquidate their projects at date 0 itself, and then, after δ, use the liquidation payoff to repay the lender till t_2. Once the second set of borrowers obtain their loans at t_2, the group as a whole

[17] We also have some partial results, which have a similar negative connotation, for completely general schemes. These are available from the authors on request.

[18] It is easy to see that the earlier results go through in case the bank can see whether the borrowers are liquidating their projects or not.

defaults. The loan scheme must be designed to guard against this possibility (call it strategy X), which makes efficiency harder to attain.

Suppose that if the first set of borrowers liquidate their project at 0, then they would prefer to repay from δ till t_2 and then default, rather than default at δ, or even earlier.[19] We next examine if at date 0 the group has an incentive to follow the strategy sketched above. A necessary condition for default not to occur is that

$$(n-m)[b - \frac{(t_2-\delta)a(k)}{1-\delta}] + bm \leq n(a(k)-1),$$

where note that the LHS is the aggregate average payoff for the group if it follows strategy X. After some manipulation we have the following necessary condition:[20]

$$b - (a(k)-1) \leq (1-m/n)[2a(k) - b - 1 - \frac{\delta a(k)}{1-\delta}].$$

Note that as the gestation lag, i.e. δ, increases, it becomes harder to satisfy this condition. This establishes that as gestation lag increases, attaining the efficient project size may become more difficult.

In future work we plan to allow for informal lenders, as well as gestation lags, two aspects of reality that have been abstracted from in this framework. We would like to examine if the presence of informal lenders can alleviate the inefficiency associated with gestation lags discussed above, since this would allow the MFIs to ask for repayment even before the project starts yielding any returns, thus forcing the borrowers to take bridge loans from informal lenders. This would provide a unified framework that integrates the intuition developed in Jain and Mansuri (2003) into our framework, thus providing a unified explanation of not only sequential lending and gradual repayment, but also the persistence of informal lenders.

5 Conclusion

Micro-finance, in particular group-lending, has made significant progress in resolving some of the underlying problems associated with lending to

[19]This holds whenever $\frac{mb}{(n-m)a(k)} \geq 1 - \frac{1-t}{1-\delta}$.

[20]A proof of the following necessary condition is available on request.

the poor, in particular the lack of information about borrowers and their inadequate collateral. While group-lending schemes are not a recent phenomena,[21] given the recent success of the Grameen Bank in Bangladesh, in particular the high rates of repayment,[22] there is a natural interest in examining whether the innovative institutional features used by many MFIs play a role in their success.

This paper examines two dynamic features associated with many microfinance schemes, namely *gradual repayment* and *sequential financing*. We argue that a unified explanation of both these aspects can be built around dynamic incentives, in particular the simple idea that the incentive to default should be relatively uniformly distributed across time. We formalize this intuition in a model that allows project returns to accrue over time rather than at a single point, and takes *ex post* moral hazard problems very seriously. We show that schemes with gradual repayment can improve efficiency vis-a-vis schemes that do not, and further, in the presence of social sanctions, sequential lending can help improve project efficiency and may even implement the efficient outcome. Interestingly, if social capital is manifest in the borrowers' ability to make side-payments, that may reduce project efficiency.

6 Appendix

Proof of Proposition 2. To prove sufficiency, let m be the smallest integer for which we have

$$bk - mf \leq F(k) - k. \tag{12}$$

Let $\triangle = 2[F(k) - k] - bk^*$ and let $t^* \in (0, 1)$ satisfy

$$bk(1 - t^*) = F(k) - k - \triangle. \tag{13}$$

Such a t^* clearly exists.

[21] See Ghatak and Guinnane (1999) for a discussion of an earlier group-lending scheme in Germany.

[22] Hossein (1988), as well as Morduch (1999) and Christen, Rhyne and Vogel (1994), argues that the Grameen Bank has a repayment rate in excess of 90 percent.

Construct now the following scheme: take the group size $n = 2m$. At $t = 0$, m borrowers are given loans of k, while the remaining m borrowers are given loans of k at $t_2 = t^*$. Every borrower has a repayment obligation given by the FGR corresponding to the loan size k. Finally, there is dynamic joint liability.

Because of our choice of m, it follows from equation (12) that the first set of borrowers will not default at $t = 0$.

Now consider $t = t^*$. If $t^* > \frac{k}{F(k)}$, then the first set of borrowers have already repaid their loans, and therefore, they strictly prefer not to default. Hence let $t < \frac{k}{F(k)}$. Note that the continuation no default payoff to any such borrower is exactly $F(k) - k$. Given the condition that $bk < 2[f(k) - k]$, note that $\triangle > 0$, so that from equation (13), the default payoff $bk(1-t^*) = F(k) - k - \triangle < f(k) - k$. This ensures that the first set of borrowers will be adversely affected if there is any default and consequently each of them will impose the social penalty f on the set of deviating borrowers. Since the first group has m borrowers, a defaulting borrower from the second group will have a payoff of $bt^* - mf$ which by equation (12) is less than $F(k) - k$.

Finally, consider $t = 1$. If at this this date, the second group of borrowers also have repaid their loans, then, they have no incentive to default. Therefore, assume that at $t = 1$, the second set of borrowers are yet to pay back their loans. Thus, the continuation no default payoff to a borrower in this group is then exactly $F(k) - k$, while by defaulting she will get bkt^*. If $bkt^* > F(k) - k$, then adding this with equation (12), we get that $bk > bk$ which is a contradiction and thus $bkt^* \leq F(k) - k$. Thus, we have proved that the scheme is *incentive compatible* and since the repayment schedule is the FGR corresponding to k, the lender also breaks even on each of her loans. Hence the scheme is *feasible*.■

Proof of Proposition 3. Let $a = \frac{F(k^*)}{k^*}$. Note that $a > 1$. Let n be the smallest integer for which we have $n(a-1) > 1$. Since $\hat{k} < k^*$, it follows from Proposition 1(b) that $n \geq 2$. Given $f > 0$, let m be the smallest integer for which we have

$$bk^* - mf < k^*(a-1). \tag{14}$$

Consider now a group of N members where $N = (n+1)m$. Let $t_i, i =$

$0, 1, \ldots, n$ be given by

$$t_0 = 0, t_1 = \frac{1}{na}, t_2 = \frac{2}{na} \ldots, t_n = \frac{n}{na}. \tag{15}$$

The sequential lending scheme operates as follows, at every t_i, m members of the group are advanced a loan of k^*. Moreover, if at any date $t \in [0, 1+t_n]$, any borrower defaults on her repayment obligation, the lender liquidates all projects. Moreover, if the default date t is less than t_n, the lender makes no further loan to the members yet to receive their loans. Finally, the repayment scheme for any borrower is just the FGR corresponding to the investment level k^*.

We now show that at any date $t \in [0, 1+t_n]$, no groups of borrowers have an incentive to default. To show this, it is sufficient to consider the default incentives at dates $t = \{0, t_1, \ldots, t_n\}$ and $t = \{1, 1+t_1, 1+t_2 \ldots, 1+t_{n-1}\}$.

The result is clearly true for all $t \in \{0, t_1, \ldots, t_{n-1}\}$. Since default at any such date by any group of borrowers would mean that at least m borrowers will not be granted a loan. Thus, at the minimum, a defaulting borrower will attract a social penalty of mf. Because of (14), a defaulting borrower must be strictly worse off.

Consider now the date $t_n = \frac{1}{a}$ in which the last set of m borrowers receive their loans. Since $t_n = \frac{1}{a}$, the borrowers who received their loans at $t = 0$ have already repaid their loans and thus the continuation payoff of any such borrower is exactly $F(k^*)(1 - t_n)$. The default payoff for such a borrower at this date, however, is $bk^*(1 - t_n)$. Since $F(k^*) > bk^*$, clearly all such borrowers will be adversely affected by any default decision in the group. Consequently, this group of borrowers will necessarily invoke the social sanction on any defaulting borrower. As a result, the maximum payoff to any defaulting borrower at such a date is at most $bk^* - mf$ which by (14) is strictly less than $F(k^*) - k^*$.

Consider now date $t = 1 + t_k$, at this date, some of the borrowers have already completed their project and thus will not invoke the social sanction. We now show that the set of borrowers who got the loan at $t = t_{k+1}$ must strictly prefer not to default and will be adversely affected by the default decision of any other borrower in the group. To see this note that $(1 - t_k) - (t_{k+1} - t_k) = \frac{na-1}{na} + k/na > \frac{na-1}{na}$. This is strictly greater than $\frac{1}{a}$

since our choice of n satisfies $n(a-1) > 1$. This implies that the group of borrowers receiving their loans at $t = t_k$ have already repaid their loans and thus will be adversely affected by the defaulting decision of any other borrower. Consequently, this set of borrowers will impose a sanction of f on any defaulting borrower at $t = 1 - t_k$. Thus, the net payoff of any defaulting borrower is at most $bk^* - mf$ which by (14) is strictly less than $F(k^*) - k^*$.■

7 Reference

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8 For the Referee: Proof of Proposition 6

Let there be n borrowers labeled $1, 2, \cdots, n$, where we adopt the convention that if $i < j$, then borrower i obtains a loan at least as early as borrower j.

At any instant t, let $n'(t)$ denote the number of borrowers who have obtained a loan strictly before t and are still *active*, i.e. yet to complete their projects. Further, let $n(t)$ denote the number of *new* borrowers who obtain a loan at t.

Next let t^i denote some time point where at least one borrower obtains a loan, with the first set of loans being given at t^0, and the last one at t^m. Clearly, $\sum_0^m n(t^i) = n$. For $i > 1$, we define $t_i = t^i - t^{i-1}$.

An FGR lending scheme $L(n)$ involving n borrowers is characterized by (a) $< n(t^j) >$, i.e. how many borrowers obtain a loan at t^j, and (b) what is the magnitude of loan x_i obtained by borrower i. Thus

$$L(n) =< t^0, t^1, \ldots, t^m, n(t^0), \ldots, n(t^m), x_1, \cdots, x_n > .$$

We normalize $t^0 = 0$.

We next turn to the task of defining the no default conditions for a loan scheme $L(n)$. For this we need to define the continuation payoffs from defaulting and not defaulting at any given t.

For any $t \geq 0$, let $\tilde{t}_j(t)$ denote the time elapsed at t since borrower j obtained her loan.

Letting $E_j(t)$ denote the *continuation payoff* of agent j at t in case there is no default after t, $E_j(t)$ equals $a(x_j + s) - x_j$ in case $\tilde{t}_j(t) \leq \frac{x_j}{a(x_j+s)}$, and $a(x_j + s)(1 - \tilde{t}_j(t))$ otherwise, where recall that $a(k) = F(k)/k$.

Thus the *aggregate continuation payoff* at t to all the agents who are still active

$$E(t) = \sum_{j \in n'(t) \cup n(t)} E_j(t).$$

The payoff of an active borrower j at t in case there is any default at t, is $b(x_j + s)(1 - \tilde{t}_j(t))$. Letting $D(t)$ denote the aggregate payoff at t from defaulting for the borrowers who *receive a loan at t itself*,

$$D(t) = \sum_{k \in n(t)} b(x_k + s).$$

Similarly, letting $D'(t)$ denote the aggregate payoff from defaulting at t for the agents who have already received a loan earlier, but are yet to complete their projects

$$D'(t) = \sum_{j \in n'(t)} b(x_j + s)(1 - \tilde{t}_j(t)).$$

Definition. A lending scheme $L(n)$ satisfies the no default (ND) constraint at t iff

$$D(t) + D'(t) \leq E(t). \tag{16}$$

A loan scheme $L(n)$ is said to be *default-proof* if it satisfies the ND conditions for all t.

We consider lending schemes that are not too protracted in that the last borrower to obtain a loan does so at a time when the first set of borrower(s) are yet to complete their projects.

Definition. A lending scheme $L(n)$ is said to be *one-cyclical* if $t^m < 1$.

We first argue that it is sufficient to consider lending schemes that have no bunching except possibly at $t = 0$. For ease of exposition we prove the result for efficient schemes, though our argument applies to other schemes as well.

Lemma 7 *Consider a feasible and efficient n-member group-lending scheme $L(n)$ with FGR and bunching at $t^i > 0$. Then there exists another feasible and efficient scheme $L'(n)$ involving these borrowers where there is no bunching (except possibly at $t = 0$), and where everyone receives their loans either at the same point as under $L(n)$, or earlier.*

Proof. W.l.o.g. let there be two borrowers bunched at $t^i > 0$. Consider an alternative scheme that is identical to this scheme, except for the fact that (a) one of borrowers bunched at t^i, now receives her loan at $t^i - \epsilon$, where $t^{i-1} < t^i - \epsilon < t^i$, and (b) all the agents *active* at t^i (i.e. yet to complete their projects), are also active at $t^i - \epsilon$.

Let us consider the ND conditions under this alternative loan scheme. From equation (16) above note that

(i) The ND condition at any t, $t \leq t^{i-1}$, remains unchanged,

(ii) The ND constraint at any t, $t > t^i$, is either relaxed, or remains unchanged.

Next consider the ND conditions at t^i and $t^i - \epsilon$. Recall that under the original scheme the ND at t^i is of the form

$$D(t^i) + D'(t^i) = 2b + D'(t^i) \leq E(t^i). \tag{17}$$

Note that under the new scheme the ND at t^i, for ϵ small, is

$$b + (1-\epsilon)b + D'(t^i) \leq E(t^i). \tag{18}$$

Given (17), this is clearly satisfied since $\epsilon > 0$. Next consider the ND constraint at $t^i - \epsilon$,

$$b + D'(t^i - \epsilon) \leq E(t^i - \epsilon). \tag{19}$$

From continuity and equation (17), equation (19) above is satisfied for ϵ small.■

Given Lemma 7, in what follows we restrict attention to loan schemes with no bunching except possibly at $t = 0$. Thus we consider schemes $L(n, m)$ such that $n - m$ borrowers obtain their loans at $t = 0$, $0 < m < n$, and the remaining m borrowers are un-bunched and receive loans later on.

Lemma 8 *A necessary condition for $L(n, m)$ to be feasible is that*

$$\frac{F(k^*) - k^*}{bk^*} \geq \frac{2n - m + 1}{4n - m - 1}.$$

Proof. We then derive a set of necessary conditions for $L(n, m)$ to be default-proof. Recall that $t_i = t^i - t^{i-1}$, and let $x(k^*) = \frac{F(k^*)-k^*}{k^*}$.

$$\begin{aligned}
t = t^1: \ b + (1-t_1)b(n-m) &\leq nx(k^*), \\
t = t^2: \ b + b(1-t_2) + b(n-m)(1-t_1-t_2) &\leq nx(k^*), \\
\cdots\cdots\cdots\cdots &\quad \cdots\cdots\cdots\cdots \\
t = t^m: \ b + b(1-t_m) + \cdots b(n-m)(1-t_1 \ldots - t_m) &\leq nx(k^*), \\
t = 1: \ t_1 b + \ldots + (t_1 + \ldots + t_m)b &\leq mx(k^*), \\
\cdots\cdots &\quad \cdots\cdots\cdots\cdots \\
t = 1 + t^{m-1}: \ t_m b &\leq x(k^*).
\end{aligned}$$

Note that the above conditions arise out of the no default conditions at t^1, $t^2, ..., t^m$, 1, $1+t^1, \cdots, 1+t^{m-1}$. At $t = t^1$, for example, this necessary condition coincides with the ND constraint at t^1 if the first $n-m$ borrowers are still repaying their loans at t^1. Otherwise, the corresponding ND yields $b + b(n-m)(1-t_1) \leq m[a(k^*)-1] + (n-m)a(k^*)(1-t_1)$, which yields the same necessary condition (since $a(k^*)(1-t_1) \leq a(k^*)-1$, given that the borrower has already repaid). The other necessary conditions follow a similar logic.

Multiplying the inequality at $t = 1$ by $(n-m)$ and summing this with all the other inequalities yields:

$$b + 2b + \cdots + bm + (n-m)bm \leq x(k^*)[nm + 1 + \cdots + (m-1) + m(n-m)]$$

i.e.

$$\frac{bm(2n-m+1)}{2} \leq \frac{x(k^*))m(4n-m-1)}{2}. \blacksquare$$

We next turn to a

Proof of Proposition 6. We begin by showing that a necessary condition for a one-cyclical scheme $L(n,m)$ to be feasible is that $\frac{F(k^*)-k^*}{bk^*} \geq \frac{n+2}{3n}$. Note that $\frac{d}{dm}[\frac{2n-m+1}{4n-m+1}] < 0$. Thus if there is a lending scheme $L(n,m)$ that satisfies the necessary conditions of Lemma 8, then there is fully sequential scheme which satisfies the necessary condition $\frac{F(k^*)-k^*}{bk^*} \geq \frac{n+2}{3n}$. The result now follows since $\frac{n+2}{3n}$ is decreasing in n, and $\lim_{n\to\infty} \frac{n+2}{3n} = \frac{1}{3}$.$\blacksquare$