# The Distributional Consequences of Government Spending[†]

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## Abstract

Government provision of public goods is an important mechanism through which wealth can be redistributed across society. This paper develops a model in which public capital is both an engine of growth and a determinant of the distributions of wealth, income, and welfare. Government spending increases wealth inequality over time, regardless of how it is financed. The time paths of both pre- and post-tax income inequality, however, are highly sensitive to financing policies, and in many cases are characterized by sharp intertemporal tradeoffs, where income inequality declines in the short run but increases in the long run. The growth-inequality relationship is shown to depend critically on (i) how externalities impinge on allocation decisions, (ii) financing policies, and (iii) the time period of consideration. Finally, public investment generates sharp trade-offs between average welfare and its distribution, with government spending improving average welfare, but also increasing its dispersion. These results underscore the friction between fiscal policies that target growth but may have consequences for inequality. Robustness checks are conducted for (i) the magnitude of the spillover effect of government spending, and (ii) the intratemporal elasticity of substitution between (a) capital and labor in production and (b) consumption and leisure in utility.

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## 1. Introduction

"*The expressway network (in China) has…helped to promote a sharp increase in private car ownership… roads are sometimes built expressly for the purpose of converting countryside into revenue-generating urban land…For Beijing's airport expansion, 15 villages were flattened and their more than 10,000 residents resettled…but...former farmers…(were) barred from unemployment benefits and other welfare privileges*." The Economist (February 14, 2008)

"*The (Healthcare) bill that President Obama signed…is the federal government's biggest attack on economic inequality*." The New York Times (March 23, 2010)

Over the last three decades, income inequality has steadily risen around the world, both in non-OECD and most OECD countries. As the gap between the rich and the poor has increased, the alleviation of poverty and inequality has emerged as one of the most challenging public policy problems. A central consideration in this discussion is the role of government spending programs – specifically on public infrastructure such as roads, healthcare, education, etc. – in stimulating economic growth and reducing inequality.

Beginning with Arrow and Kurz (1970) and later Barro (1990), the relationship between public investment and growth has been widely studied, with general agreement that government spending on infrastructure can yield significant productivity and growth benefits.[1] At the same time, by affecting factor productivity and therefore relative factor returns, public investment also plays a critical role in the evolution of wealth and income distributions as the economy grows over time. This raises the question of the potential relationship between growth and inequality generated by government investment, although, *a priori*, it is unclear what the nature of such a connection will be. The objective of this paper is to investigate this important relationship in some detail.

But in contrast to the public expenditure-growth relationship, empirical evidence on the link between public investment and inequality is sparse, inconclusive, and largely anecdotal. For instance, Ferranti et al. (2004), Fan and Zhang (2004), and Calderon and Serven (2004) find that public investment in roads, dams, and telecommunications has contributed toward the alleviation of

[1] See Arrow and Kurz (1970), Barro (1990), Futagami, et al. (1993), Glomm and Ravikumar (1994), and Fisher and Turnovsky (1998) for the early theoretical contributions. On the empirical side, see Gramlich (1994) for a survey of the early empirical literature, along with a discussion of the econometric issues. The consensus remains that infrastructure contributes positively and significantly to output, though less than originally suggested by Aschauer (1989). Easterly and Rebelo (1993) and Loayza, Fajnzylber, and Calderon (2005) provide evidence of a positive relationship between infrastructure and long-term growth. See Calderon and Serven (2004) for a summary of some recent empirical studies.

inequality and poverty in China and Latin America. In contrast, Brakman et al. (2002) find that government spending on infrastructure has increased regional disparities within Europe. Banerjee (2004) and Banerjee and Somanathan (2007) report that in India, access to critical infrastructure services and public goods is in general positively correlated with the distribution of income and social status, even though the provision of such goods is intended to benefit the poor. A World Bank (2006) report also finds that the quality and performance of state-provided infrastructure services tend to be the worst in India's poorest states. Further, Khandker and Koolwal (2007) find that access to paved roads has had limited distributional impact in rural Bangladesh. The diversity of these findings is especially significant, given that most high-growth emerging markets such as China, India, and Eastern Europe have substantially increased public infrastructure spending over the last decade. It underscores the need for a well-specified analytical framework within which the link between infrastructure spending, its financing, growth, and inequality can be systematically studied.[2]

This paper, therefore, seeks to synthesize two extensive, but independent, strands of literature into a unified framework. On the one hand, the theoretical literature on growth and inequality has not dealt with issues related to public investment and its financing.[3] On the other hand, the extensive literature on public investment and growth (see footnote 1) has generally ignored distributional questions. Studying the public policy-growth-inequality relationship in the context of a fully specified dynamic model therefore represents an important synthesis of previous work. In doing so, we address the following issues pertaining to government investment and its financing:

(i) The mechanism through which government spending on public goods and accompanying taxation policies affects the distributions of wealth, income (pre- and post-tax), and welfare, in the short run, during transition, and in the long run.

(ii) The dynamics of the growth-inequality relationship along the transitional path.

(iii) Trade-offs between average welfare and its dispersion resulting from fiscal shocks.

[2] Lopez (2004) is one of the few papers to analyze systematically the effect of infrastructure on both growth and income inequality. Using panel data he finds that infrastructure raises the growth rate, while reducing income inequality.

[3] This literature has explored the determinants of the interdependency between growth and inequality, focusing mainly on the productivity of private capital (Bertola, 1993), differences in individual propensities to save (Chatterjee, 1994), structural characteristics such as civil liberty and openness (Lundberg and Squire, 2003), and heterogeneity in initial wealth, skills, or preferences in a representative consumer framework (Caselli and Ventura, 2000, Sorger, 2000, García-Peñalosa and Turnovsky, 2006).

The model we employ has several key elements. First, the underlying source of heterogeneity arises through agents' differential initial endowments of private capital. Combined with an endogenous labor-leisure choice, this yields an endogenous distribution of income.[4] Indeed, recent empirical evidence points to the importance of the return to capital as being a critical determinant of inequality; see Atkinson (2003), and the recent empirical evidence for the OECD provided by Checchi and García-Peñalosa (2010).[5]

Second, we introduce a growing stock of a government-provided capital good (public capital). This interacts with the aggregate stock of private capital to generate composite externalities for both labor productivity in production and the labor-leisure allocation in utility. The government has a range of fiscal instruments available to finance its investment, namely distortionary taxes on capital income, labor income, and consumption, and a non-distortionary lump-sum tax (equivalent to government debt). The accumulation of public capital and the spillovers it generates serves both as an engine of sustained growth, and also as a driver of relative returns to capital and labor, with consequences for the evolution of wealth and income inequality. In equilibrium, both the economy's growth rate and inequality are endogenously determined.

Because of the underlying preference structure assumed, the model satisfies the Gorman (1953) aggregation conditions, enabling the dynamics to be analyzed sequentially. First, the dynamics of the aggregate stock of capital and leisure (labor supply) are jointly determined independently of distributional considerations.[6] The distributions of wealth and income and their dynamics are then characterized in terms of the aggregate magnitudes. But even with this recursive equilibrium structure, the model is too complex to study analytically, and instead, is analyzed numerically. We compare an increase in the rate of government investment on public capital, financed by the use of the alternative fiscal instruments and obtain a number of interesting and empirically testable hypotheses.

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[4] This feature is common to several recent papers, e.g. Sorger (2002), García-Peñalosa and Turnovsky (2006), Kraay and Raddatz (2007), Turnovsky and García-Peñalosa (2008), Carroll and Young (2009) and Barnett et al. (2009).

[5] We are abstracting from other important elements central to the growth-income inequality relationship, These include capital market imperfections, the role of human capital, the (unequal) distribution of natural resources, socio-economic stratification, and technological progress; see Banerjee and Newman (1993), Galor and Zeira (1993), Benabou (1996), Galor and Tsiddon (1997), Aghion and Bolton (1997), Bhattacharya (1998), and Gylfason and Zoega (2003).

[6] Caselli and Ventura (2000) term this property the "representative consumer theory of distribution."

(i) Government spending on public capital leads to a gradual increase in wealth inequality, regardless of how it is financed. In contrast, the time paths of both pre- and post-tax income inequality are highly sensitive to the financing policy adopted, and in many cases are characterized by sharp intertemporal tradeoffs. For example, while government investment financed by a lump-sum or consumption tax leads to a short-run *decline* in income inequality, this is completely reversed over time, leading to an *increase* in the long-run dispersion of income. This is somewhat surprising, since lump-sum taxes are a non-distortionary source of financing and government spending creates a larger stock of a non-excludable and non-rival public good. Public expenditure financed by capital or labor income taxes yields sharp differences between pre-tax and post-tax income inequality, both in the short run and over time. But regardless of the financing, both measures of income inequality increase over time. This is consistent with trends in OECD countries, where both government spending and inequality have risen steadily over time.[7]

(ii) Our results also provide insights into the growth-income inequality relationship, an issue that has been a source of lively debate.[8] We show that this relationship depends critically on (a) how externalities impinge on allocation decisions, (b) the financing policies for government spending, and (c) the time period of consideration – i.e. short run, transition path, or the long run. These results underscore the ambiguity in the growth-inequality relationship that is characteristic of recent empirical studies (see footnote 8).

(iii) While the relationship between an economy's income growth and inequality are important, the relationship between average welfare and its dispersion (welfare inequality) is arguably of greater significance to policymakers. We find that public investment generates sharp trade-offs between average welfare and its distribution, in the sense that while government expenditure improves average welfare, it also increases its inequality. However, spending financed by taxing consumption or labor income is associated with less adverse tradeoffs.

[7] Total government spending in the OECD countries has increased from an average of about 21 percent of GDP in 1972 to about 26 percent in 1999 (source: GFS Database). Income inequality also increased in most of the major OECD countries during this time; see Smeeding (2002).

[8] Empirical studies that have explored the causality between growth and income inequality have generally yielded ambiguous results. For example, while Alesina and Rodrik (1994), Persson and Tabellini (1994), and Perotti (1998) find an inverse relationship, studies by Li and Zou (1998), Barro (2000), and Forbes (2000) have documented a positive link.

To check the robustness of these conclusions, we conduct an extensive sensitivity analysis to some of the economy's structural conditions. We focus on three key aspects: (i) the structure of the composite public-private externality in the utility and production functions, (ii) the intratemporal elasticity of substitution between private capital and effective labor in the production function, and (iii) the intratemporal elasticity of substitution between consumption and effective leisure in the utility function. Overall, our benchmark results remain robust to variations in these parameters.

The rest of the paper is organized as follows. Section 2 lays down the analytical framework and Section 3 derives the macroeconomic equilibrium for the aggregate economy. Section 4 derives the distributional dynamics and characterizes the evolution of the different measures of inequality. Section 5 conducts numerical policy experiments and discusses their predictions. Section 6 summarizes the sensitivity analyses that we have performed, while Section 7 concludes.

## 2. Analytical Framework

The analytical framework we employ is an endogenous growth model in which both private and public capital are accumulated, and hence the evolution of the economy is characterized by transitional dynamics, as in Futagami et al. (1993) and Turnovsky (1997). Since the solution procedure is based on that developed by Turnovsky and García-Peñalosa (2008), where it is discussed at length, details are omitted insofar as possible.[9]

### 2.1 Firms and Technology

All firms are identical and are indexed by $j$. The representative firm produces output in accordance with the CES production function:

$$Y_j = A\left[\alpha\left(X_P L_j\right)^{-\rho} + \left(1-\alpha\right)K_j^{-\rho}\right]^{-1/\rho} \tag{1a}$$

where $K_j$ and $L_j$ represent the individual firm's capital stock and employment of labor,

[9] We should emphasize that the Turnovsky and García-Peñalosa (2008) analysis is very different in that it employs a Ramsey model, rather than an endogenous growth model. It is also addresses very different issues, being concerned with structural changes, such as changes in technology, and indeed abstracts from fiscal issues that we are addressing here.

respectively, and $s \equiv 1/(1+\rho)$ represents the elasticity of substitution in production between capital and effective units of labor. In addition, production is influenced by an aggregate composite externality, $X_P$, which we take to be a geometric weighted average of the economy's aggregate stocks of private and public capital, ($K, K_G$) namely

$$X_P = K^{\varepsilon} K_G^{1-\varepsilon}, \qquad 0 \le \varepsilon \le 1 \tag{1b}$$

That is, "raw" labor interacts with the composite production externality to create labor efficiency units, which in turn interact with private capital to produce output. The production function has constant returns to scale in both the private factors and in the accumulating factors, and accordingly, sustains an equilibrium of endogenous growth. The composite externality represents a combination of the role of private capital as knowledge as in Romer (1986), together with public capital as in Futagami et al (1993) and subsequent authors, and can be justified in two ways. First, as will become evident below, it helps provide a plausible calibration of the aggregate economy, something that is generically problematic in the conventional one-sector endogenous growth model. Second, the notion that an economy's infrastructure contributing to labor efficiency reflects a combination both public and private components is itself entirely reasonable.

All firms are also assumed to face identical competitive production conditions, and hence will choose exactly the same levels of employment of labor and private capital, i.e., $K_j = K$ and $L_j = L$, for all *j*, where $L$ denote the average economy-wide levels of private capital and labor employment, respectively. Thus, equilibrium output of the representative firm, and therefore the economy-wide average output is

$$Y = A\left[\alpha\left(X_P L\right)^{-\rho} + \left(1-\alpha\right)K^{-\rho}\right]^{-1/\rho} \tag{2}$$

Letting $z \equiv K_G/K$ denote the ratio of the economy-wide stock of public capital to private capital, we can write $y \equiv Y/K$, the average product of aggregate private capital, in the form:

$$y \equiv y\left(z,l\right) = A\left[\alpha\left\{\left(1-l\right)z^{1-\varepsilon}\right\}^{-\rho} + \left(1-\alpha\right)\right]^{-1/\rho} \tag{2'}$$

where $l = 1 - L$ denotes the average allocation of time to leisure in the economy. With both factors being paid their respective private marginal products, the economy-wide returns to capital and labor, determined in competitive factor markets, may be expressed as

$$r = r(z,l) \equiv (1-\alpha) A^{-\rho} y(z,l)^{1+\rho} \tag{3a}$$

$$w = \omega(z,l) K; \;\; \omega(z,l) \equiv \alpha A^{-\rho} y(z,l)^{1+\rho} z^{-\rho(1-\varepsilon)} (1-l)^{-(1+\rho)} \tag{3b}$$

Thus, as long as $\varepsilon < 1$, the real wage rate and the return to private capital depend on the ratio of public to private capital and the average allocation of time to work (or leisure).[10]

## 2.2 Consumers

There is a continuum of infinitely-lived consumers, indexed by *i*, who are identical in all respects except for their initial endowments of private capital, $K_{i,0}$. Each consumer is also endowed with one unit of time that can be allocated to either leisure, $l_i$, or work, $L_i = 1 - l_i$ =1. Consumer *i* maximizes utility over an infinite horizon from his flow of consumption, $C_i$, and leisure, using the following CES utility function:

$$U_i = \int_0^\infty \frac{1}{\gamma} \left[ C_i^{-\upsilon} + \theta \left( X_U l_i \right)^{-\upsilon} \right]^{-\gamma/\upsilon} e^{-\beta t} dt \tag{4a}$$

where $q \equiv 1/(1+\upsilon)$ denotes the intra-temporal elasticity of substitution between consumption and leisure in the utility function, and $e \equiv 1/(1-\gamma)$ represents the inter-temporal elasticity of substitution. Each consumer's utility is also affected by an aggregate composite externality, $X_U$, which is a geometric weighted average of the economy's aggregate stocks of public and private capital:

$$X_U = K^{\varphi} K_G^{1-\varphi}, \; 0 \le \varphi \le 1 \tag{4b}$$

This composite externality in (4b) interacts with the time allocated to leisure by consumer *i* to generate utility benefits, which in turn are weighted by $\theta$ in yielding overall utility. There are

[10] If $\varepsilon = 1$ the factor returns depend only on leisure, as public capital does not affect production.

several reasons motivating the preferences specified in (4a) and (4b). The first is that the conventional Cobb-Douglas formulation of utility has the undesirable implication that for plausible values of the intertemporal elasticity of substitution ($0<e<1$), consumption and leisure are Edgeworth "substitutes".[11] Generalizing the utility function to CES allows them to be complements or substitutes depending upon whether $e \gtrless q$.[12] But for the CES function to have the appropriate homogeneity properties to sustain endogenous growth, the externality must interact with leisure as we have specified. Finally, the notion that the utility derived from leisure depends upon amenities due to the provision of both public and private capital is indeed plausible.

Each agent chooses $C_i$, $l_i$, and his rate of capital accumulation, $\dot{K}_i$ to maximize (4a) subject to (4b), their initial endowment of capital, $K_{i,0}$, and the following flow budget constraint

$$\dot{K}_i = \left(1-\tau_k\right) rK_i + \left(1-\tau_w\right) w(1-l_i) - \left(1+\tau_c\right) C_i - T \tag{5}$$

where $\tau_k$, $\tau_w$, and $\tau_c$ are the tax rates levied on the agent's capital income, labor income, and consumption expenditures, respectively and $T$ represents a lump-sum tax levied by the government uniformly on all $i$ individuals. In making these decisions the agent takes the real wage rate and the return on private capital, determined in competitive factor markets, as given, and in addition he also treats all tax and policy variables as exogenous.

Optimizing with respect to $C_i, l_i$, and $K_i$ yields the following standard first-order conditions

$$\left[C_i^{-\upsilon} + \theta\left(X_U l_i\right)^{-\upsilon}\right]^{-\frac{\gamma}{\upsilon}-1} C_i^{-\upsilon-1} = \lambda_i\left(1+\tau_c\right) \tag{6a}$$

$$\theta X_U^{-\upsilon}\left[C_i^{-\upsilon} + \theta\left(X_U l_i\right)^{-\upsilon}\right]^{-\frac{\gamma}{\upsilon}-1} l_i^{-\upsilon-1} = \lambda_i\left(1-\tau_w\right)\omega\left(z,l\right)K \tag{6b}$$

$$\left(1-\tau_k\right) r\left(z,l\right) = \beta - \frac{\dot{\lambda}_i}{\lambda_i} \tag{6c}$$

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[11] The conventional Cobb-Douglas utility function is of the form: $(C_i l_i^{\theta})^{\gamma}/\gamma$. Two goods are said to be Edgeworth complements or substitutes according to whether their cross partial derivatives in the utility function are positive or negative.

[12] Just as the empirical evidence for the intertemporal elasticity of substitution covers much of the range (0,1), estimates of the (intratemporal) elasticity of substitution between consumption and leisure, $q$, as low as 0.4 have been obtained, well below the value of unity implicit in the conventional specification of the form $(C_i l_i^{\theta})^{\gamma}/\gamma$. Stern (1976) provides an early well known example of empirical evidence that yields an estimate of $q$ around 0.4.

where $\lambda_i$ is agent $i$'s shadow value of capital, together with the transversality condition

$$\lim_{t\to\infty}\lambda_i K_i e^{-\beta t} = 0 \tag{6d}$$

Dividing (6b) by (6a), yields

$$\frac{C_i}{l_i} = \Omega(z,l)K \tag{7}$$

where, $\Omega(z,l) \equiv \left[(1-\tau_w)\omega(z,l)z^{\upsilon(1-\varphi)} \big/ \theta(1+\tau_c)\right]^{1/(1+\upsilon)}$. From (7) we see that each agent chooses the same ratio of consumption to leisure, which depends on $\tau_w, \tau_c$, $z$, $l$, and $K$. Using (7), we may write the individual's rate of capital accumulation, (5), as

$$\frac{\dot{K}_i}{K_i} = (1-\tau_k)r(z,l) + \left[(1-\tau_w)\omega(z,l)(1-l_i) - (1+\tau_c)\Omega(z,l)l_i - \frac{T}{K}\right]\frac{K}{K_i} \tag{8}$$

## 2.3 Government

The government provides the stock of public capital, which is assumed to be non-rival and non-excludable, and evolves according to

$$\dot{K}_G = G = gY,\ 0 < g < 1 \tag{9}$$

where $G$ is the flow of new public investment, which is tied to the scale of the economy, given by aggregate output $Y$. Therefore, $g$ represents the fraction of aggregate output allocated to public investment by the government.[13] The government finances its investment by tax revenues, and maintains a balanced budget at all points of time:

$$G = \tau_k rK + \tau_w w(1-l) + \tau_c C + T \tag{10}$$

Dividing (10) by $K$, we can write this in the form

$$gy(z,l) = \tau_k r(z,l) + \tau_w \omega(z,l)(1-l) + \tau_c \Omega(z,l)l + \tau y(z,l) \tag{10'}$$

[13] For simplicity we abstract from depreciation of either form of capital.

where lump-sum tax revenues are expressed as a proportion $\tau$ $(0<\tau<1)$ of aggregate output, namely $T=\tau Y$. It is clear from (10') that if the tax and expenditure rates, $\tau_k, \tau_w, \tau_c$, and $g$ are maintained constant, then as $z$ and $l$ progress along the transitional path the fraction of output levied as lump-sum taxes, $\tau$, will continually vary in order for the government budget to remain in balance.

## 3. Macroeconomic Equilibrium

In general, the economy-wide average of a variable, $X_i$ is represented by $(1/N)\sum_i^N X_i \equiv X$. Because of the homogeneity of the utility function and perfect factor markets, we can show that all individuals choose the same growth rates for consumption and leisure, implying that average consumption, $C$, and leisure, $l$, will also grow at the same rates; i.e.,[14]

$$\frac{\dot{C}_i}{C_i}=\frac{\dot{C}}{C},\ \frac{\dot{l}_i}{l_i}=\frac{\dot{l}}{l}, \qquad \text{for each } i \tag{11}$$

As a result, the system can be aggregated perfectly over agents. Each individual, however, will choose different *levels* of consumption and leisure, depending upon his resources; in particular,

$$l_i=\pi_i l \qquad \frac{1}{N}\sum_i^N \pi_i=1 \tag{12}$$

where the relative leisure, $\pi_i$, chosen by agent $i$ is constant over time and is determined by (22) below. Summing (7) over all agents yields the aggregate consumption-capital ratio

$$\frac{C}{K}=\Omega(z,l)l \tag{7'}$$

Next, summing (8) and invoking $T/K=\tau Y/K=\tau y(k,l)$ yields the growth rate of aggregate capital

$$\frac{\dot{K}}{K}=(1-\tau_k)r+\left[(1-\tau_w)\omega(z,l)(1-l)-(1+\tau_c)\Omega(z,l)l-\tau y(z,l)\right] \tag{13}$$

Combining the latter with (10') yields the aggregate goods market clearing condition

[14] The relationship (11) is critical in facilitating the aggregation. It is obtained by taking the time derivative of equations (6a), (7), and noting (6c); See Turnovsky and García-Peñalosa (2008) for more details.

$$\frac{\dot{K}}{K}=(1-g)y(z,l)-\Omega(z,l)l \tag{13'}$$

Given the homogeneity of the underlying utility function and production function in the capital stocks, the long-run equilibrium of this economy is a balanced growth path along which all aggregate variables grow at a common rate and average leisure is constant. The transitional dynamics of the aggregate economy are driven by the evolution of the ratio of public to private capital, $z$, and leisure, $l$.

$$\frac{\dot{z}}{z}=g\frac{y(z,l)}{z}-\left[(1-g)y(z,l)-\Omega(z,l)l\right] \tag{14a}$$

$$\frac{\dot{l}}{l}=\frac{H(z,l)}{J(z,l)} \tag{14b}$$

where

$$\begin{aligned} H(z,l)\equiv{}&(1-\tau_k)r(z,l)-\beta-(1-\gamma)\frac{\dot{K}}{K}\\ &+\left\{\frac{\theta\left[z^{-(1-\varphi)}\Omega\right]^{v}(\upsilon+\gamma)(1-\varphi)}{1+\theta\left[z^{-(1-\varphi)}\Omega\right]^{v}}-\left[\frac{(1-\gamma)+(1+\upsilon)\theta\left[z^{-(1-\varphi)}\Omega\right]^{v}}{1+\theta\left[z^{-(1-\varphi)}\Omega\right]^{v}}\right]\frac{\Omega_z z}{\Omega}\right\}\frac{\dot{z}}{z}, \end{aligned}$$

$$J(z,l)\equiv 1-\gamma+\left[\frac{(1-\gamma)+(1+\upsilon)\theta\left[z^{-(1-\varphi)}\Omega\right]^{v}}{1+\theta\left[z^{-(1-\varphi)}\Omega\right]^{v}}\right]\frac{\Omega_l l}{\Omega}$$

and $\dot{K}/K$ and $\dot{z}/z$ are given by (13') and (14a), respectively.

Equation (14a) is obtained directly from the definition of $z$ and asserts that the growth of the public to private capital ratio equals the differential growth rates of the two components. Equation (14b) is more involved and is obtained by combining equations (11) with the time derivatives of (6a) and (7).[15] It describes the required adjustment in leisure necessary to ensure that the rate of return on consumption equals the changing rate of return on capital, as the productive capacity of the economy evolves through the accumulation of public and private capital.

[15] Details of these calculations are available from the authors on request.

### 3.1 Steady State and Aggregate Dynamics

Assuming that the system is stable, the aggregate economy will converge to a balanced growth path characterized by a constant public to private capital ratio, $\tilde{z}$, and leisure, $\tilde{l}$. Setting $\dot{z} = \dot{l} = 0$ in (14), the corresponding steady-state conditions can be expressed as:

$$g\frac{y\left(\tilde{z},\tilde{l}\right)}{\tilde{z}} = (1-g)y\left(\tilde{z},\tilde{l}\right) - \Omega\left(\tilde{z},\tilde{l}\right)\tilde{l} \tag{15a}$$

$$\frac{(1-\tau_k)r\left(\tilde{z},\tilde{l}\right)-\beta}{1-\gamma} = g\frac{y\left(\tilde{z},\tilde{l}\right)}{\tilde{z}} \equiv \tilde{\psi} \tag{15b}$$

These two equations determine $\tilde{z}$ and $\tilde{l}$, such that public capital, private capital, and consumption, all grow at the common rate, that we denote by $\tilde{\psi}$. Given $\tilde{z}$ and $\tilde{l}$, (7') then determines the steady-state consumption-private capital ratio, $\tilde{c}$, namely

$$\tilde{c} = \Omega(\tilde{z},\tilde{l})\tilde{l} \tag{13a'}$$

Finally, the transversality condition (6d) together with (6c) implies $\tilde{\psi} + \beta - \tilde{r}(\tilde{z},\tilde{l})(1-\tau_k) - \beta < 0$, i.e., $\tilde{\psi} < \tilde{r}(1-\tau_k)$, which combined with (13) in steady state, yields

$$(1+\tau_c)\Omega(\tilde{z},\tilde{l})\tilde{l} > (1-\tau_w)\omega(\tilde{z},\tilde{l})(1-\tilde{l}) - \tau y(\tilde{z},\tilde{l}) \tag{16}$$

For the long-run growth rate to be sustainable, consumption expenditure inclusive of tax must exceed after-tax labor income inclusive of lump-sum taxes, so that some (net) capital income is allocated to consumption. This viability condition imposes a restriction on leisure that is necessary to constrain the growth rate and is important in characterizing the distributional dynamics.

To consider the aggregate transitional dynamics, we linearize (14) around the steady state $\tilde{z}$ and $\tilde{l}$, expressing the system in the form

$$\begin{bmatrix}\dot{z}\\ \dot{l}\end{bmatrix} = \begin{bmatrix}\overset{(-)}{a_{11}} & \overset{(+)}{a_{12}}\\ \overset{(+)}{a_{21}} & \overset{(+)}{a_{22}}\end{bmatrix}\begin{bmatrix}z-\tilde{z}\\ l-\tilde{l}\end{bmatrix} \tag{18}$$

where $a_{ij}$ are defined in the Appendix, and the signs in parentheses indicate the likely signs of the corresponding element, implying that the dynamics exhibits saddle-point stability.[16] The stable transition path of the aggregate economy can be described by

$$z(t) = \tilde{z} + \left[ z(0) - \tilde{z} \right] e^{\mu t} \tag{19a}$$

$$l(t) = \tilde{l} + \frac{a_{21}}{(\mu - a_{22})} \left[ z(t) - \tilde{z} \right] \tag{19b}$$

where $\mu$ is the stable (negative) eigenvalue corresponding to the linearized system in (18). We can further show that for plausible range of parameter values, the slope of the saddle path is negative, implying that along the transition path, the evolution of leisure is inversely related to that of the public-private capital ratio. Intuitively, an increase in public to private capital raises the productivity of private capital, raising the wage rate and inducing agents to increase their labor supply and to reduce their leisure. The exception is if $\varepsilon = \varphi = 1$, when the externality is fully private, in which case $a_{21} = 0$ [see Appendix] and $l(t)$ immediately jumps to its steady state level $\tilde{l}$. Finally, the consumption-private capital ratio evolves according to

$$c(t) - \tilde{c} = \left[ \Omega_z\left(\tilde{z}, \tilde{l}\right)\tilde{l} + \left\{ \Omega_l\left(\tilde{z}, \tilde{l}\right)\tilde{l} + \Omega\left(\tilde{z}, \tilde{l}\right) \right\} \left( \frac{a_{21}}{\mu - a_{22}} \right) \right] \left[ z(t) - \tilde{z} \right] \tag{19c}$$

The dynamic time paths described in (19a)-(19c) represent the average (mean) behavior of this heterogeneous agent economy. Since both infrastructure and private capital represent stocks that are being accumulated, we rule out instantaneous jumps in $z$. However, leisure, the consumption-capital ratio, and the various growth rates can respond instantaneously to new information.

## 4. Distributional Dynamics

The characterization of the aggregate economy in Section 3 represents the behavior of the *averages* of the key economic variables. The fact that this is independent of any distributional

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[16] We have conducted extensive numerical simulations for a plausible range of parameters to confirm these signs and also the saddle-point property of the model.

aspects is a consequence of the homogeneity of the utility function and the perfect aggregation that this permits. The next step is to characterize the behavior of a cross-section of agents, and to determine the evolution of that cross-section relative to that of the average. Specifically, we focus on the distributional dynamics of private capital (wealth), pre-tax, and post-tax income, and welfare.

### 4.1 Distribution of Private Capital (Wealth)

To derive the dynamics of the relative capital stock of individual *i*, $k_i \equiv K_i/K$ (the agent's relative wealth) we combine (8) and (13). To facilitatethe derivation, it is convenient to define:

$$\Delta(z,l) \equiv (1-\tau_w)\omega(z,l) - \tau y(\tilde{z},l); \;\; \Gamma(z,l) \equiv \left[(1+\tau_c)\Omega(z,l) + (1-\tau_w)\omega(z,l)\right] > 0$$

enabling us to express the evolution of relative wealth (capital) in the form

$$\dot{k}_i(t) = -\Gamma(z,l)(l_i - l) + \left[\Gamma(z,l)l - \Delta(z,l)\right](k_i(t)-1) \tag{20}$$

Using this notation, the viability condition (16) can be expressed as $\Gamma(\tilde{z},\tilde{l})\tilde{l} > \Delta(\tilde{z},\tilde{l})$ implying that the dynamic equation (20) is locally unstable near the steady state. A key element of a stable (bounded) solution includes the steady-state to (20), which implies the positive relationship between the agent's steady state share of the private capital stock and leisure:

$$\tilde{l}_i - \tilde{l} = \left[\tilde{l} - \frac{\Delta(\tilde{z},\tilde{l})}{\Gamma(\tilde{z},\tilde{l})}\right](\tilde{k}_i - 1) \tag{21}$$

Thus the transversality condition implies that an individual who in the long run has above-average private capital, given by $\tilde{k}_i - 1$, enjoys above-average leisure at the steady-state, i.e., $(\tilde{l}_i - \tilde{l}) > 0$.[17] Using (12), this equation also yields agent *i's* (constant) allocation of leisure time:

$$\pi_i - 1 = \left(1 - \frac{\Delta(\tilde{z},\tilde{l})}{\Gamma(\tilde{z},\tilde{l})\tilde{l}}\right)(\tilde{k}_i - 1) \tag{22}$$

Again, using (12) and substituting (22) into (20), yields

[17] This is consistent with various sources of empirical evidence that finds a negative relationship between wealth and relative labor supply; see for example, Holtz-Eakin et al. (1993), Cheng and French (2000), and Algan et al. (2003).

$$\dot{k}_i = -\Gamma(z,l)l\left[1-\frac{\Delta(\tilde{z},\tilde{l})}{\Gamma(\tilde{z},\tilde{l})\tilde{l}}\right](\tilde{k}_i-1)+\left[\Gamma(z,l)l-\Delta(z,l)\right](k_i-1) \qquad (20')$$

linearizing (20') around the steady-state levels $\tilde{z},\tilde{l}$, and $\tilde{k}_i$, while noting (19a) and (19b), implies the following equation of motion for relative wealth

$$\dot{k}_i = \delta_1(\tilde{z},\tilde{l})(\tilde{k}_i-1)\left[z(t)-\tilde{z}\right]+\delta_2(\tilde{z},\tilde{l})\left[k_i(t)-\tilde{k}_i\right] \qquad (23)$$

where

$$\delta_1(\tilde{z},\tilde{l}) \equiv \frac{1}{\Gamma(\tilde{z},\tilde{l})}\left(\Delta(\tilde{z},\tilde{l})\Gamma_z(\tilde{z},\tilde{l})-\Delta_z(\tilde{z},\tilde{l})\Gamma(\tilde{z},\tilde{l})\right)+\left(\frac{\Delta(\tilde{z},\tilde{l})}{\Gamma(\tilde{z},\tilde{l})\tilde{l}}\left(\Gamma(\tilde{z},\tilde{l})+\Gamma_l(\tilde{z},\tilde{l})\tilde{l}\right)-\Delta_l(\tilde{z},\tilde{l})\right)\left(\frac{a_{21}}{\mu-a_{22}}\right)$$

$$\delta_2(\tilde{z},\tilde{l}) \equiv \Gamma(\tilde{z},\tilde{l})\tilde{l}-\Delta(\tilde{z},\tilde{l})>0$$

Equation (23) highlights how the evolution of the economy-wide ratio of public to private capital impacts the evolution of relative wealth, both directly, and indirectly through $l(t)$. With $\delta_2>0$, the bounded solution to (23) is of the form

$$k_i(t)-1=(\tilde{k}_i-1)\left[1+\frac{\delta_1}{\mu-\delta_2}\left[z(t)-\tilde{z}\right]\right]=(\tilde{k}_i-1)\left[1+\frac{\delta_1}{\mu-\delta_2}\left(z_0-\tilde{z}\right)e^{\mu t}\right] \qquad (24)$$

Setting $t$ = 0 in (26) gives

$$k_i(0)-1\equiv k_{i,0}-1=(\tilde{k}_i-1)\left[1+\frac{\delta_1}{\mu-\delta_2}\left(z_0-\tilde{z}\right)\right] \qquad (24')$$

Thus the evolution of agent *i*'s relative capital stock is determined as follows. First, given the steady state of the aggregate economy, and his initial endowment, $k_{i,0}$, (24') determines the agent's steady-state relative stock of capital, $(\tilde{k}_i-1)$, which together with (24) then yields its entire time path, $k_i(t)$, and together with (22) determines the agent's (constant) relative leisure, $\pi_i$.[18]

Because of the linearity of (24) and (24') in terms involving $k_i$, we can immediately transform these equations into corresponding relationships for the standard deviation of the

[18] The ranking of agents according to their wealth remains unchanged throughout the transition.

distribution of capital, which serves as a convenient measure of wealth inequality. Therefore,

$$\sigma_k\left(t\right)=\left[1+\frac{\delta_1}{\mu-\delta_2}\left(z(t)-\tilde{z}\right)\right]\tilde{\sigma}_k, \tag{25a}$$

$\sigma_k\left(t\right)$ denotes the standard deviation of relative wealth at time *t*. Setting $t=0$, the relationship between the initial distribution of wealth and its steady-state distribution is given by

$$\tilde{\sigma}_k=\left[1+\frac{\delta_1}{\mu-\delta_2}\left(z_0-\tilde{z}\right)\right]^{-1}\sigma_{k,0} \tag{25b}$$

Thus (25a) and (25b) completely characterize the evolution of the standard deviation of relative capital, given its initial distribution, $\sigma_{k,0}$, and the time path of the economy-wide infrastructure to private capital ratio. From (25b), we see that the steady-state distribution of relative wealth is therefore determined by (i) its initial distribution, $\sigma_{k,0}$, and (ii) the long-run change in the public-private capital ratio.[19]

Further insight is obtained by considering the conventional case of Cobb-Douglas production function and constant elasticity utility function, $\rho=\upsilon=0$, when $\delta_1\leq 0$ enabling us to establish[20]

$$\operatorname{sgn}\left[\delta_1(\tilde{z},\tilde{l})(z_0-\tilde{z})\right]=\operatorname{sgn}\left[l(0)-\tilde{l}\right] \text{ and hence } \operatorname{sgn}\left[k_i-\tilde{k}_i\right]=\operatorname{sgn}\left[(\tilde{k}_i-1)(z_0-\tilde{z})\right]$$

Assuming the plausible case of a negatively sloped aggregate saddle path, (19b), if the expansion in government investment raises the long-run ratio of public to private capital, then $l(0)>\tilde{l}$. For people who end up above the mean level of wealth, their relative wealth will have increased during the transition $\tilde{k}_i>k_{i,0}$, while for people who end up below the mean level of wealth, their relative wealth will have decreased, $\tilde{k}_i<k_{i,0}$, implying a widening of the wealth distribution. This equation further implies that the closer $l(0)$ jumps toward its steady state, $\tilde{l}$, so the smaller the subsequent adjustment in $l(t)$, the smaller is the overall change in the distribution of wealth. This is because if the economy and therefore all individuals immediately fully adjust their respective leisure times

[19] The fact that the long-run distribution depends upon the initial distribution reflects a hysteresis property resulting from the "zero root" associated with (11). This turns out to have important implications for wealth and income inequality that are explored in another context by Atolia, Chatterjee, and Turnovsky (2010).

[20] We are also imposing the weak restriction $\left(\alpha(1-\tau_w)-\tau\right)>0$

instantaneously, they will all accumulate wealth at the same rate, causing the wealth distribution to remain unchanged. This occurs if $\varepsilon = \varphi = 1$.

### 4.2 Distribution of Pre-tax Income

Gross income of individual $i$ is defined as $Y_i = r(z,l)K_i + \omega(z,l)(1-l_i)K$ while average income is $Y = \left[r(z,l) + \omega(z,l)(1-l)\right]K$. Using (21), the relative pre-tax income of agent $i$ can be expressed in the form

$$y_i(t) - 1 = s_k(t)(k_i(t) - 1) - [1 - s_k(t)]\frac{l(t)}{1-l(t)}\left[1 - \frac{\Delta(\tilde{z},\tilde{l})}{\Gamma(\tilde{z},\tilde{l})\tilde{l}}\right]\left(\tilde{k}_i - 1\right) \tag{26}$$

where $s_k(t) \equiv r(z,l)/y(z,l) = (1-\alpha)[Ay(z,l)]^{-\rho}$ is the equilibrium share of output received by capital. Thus the distribution of pre-tax income can be written in the following equivalent forms

$$\sigma_y(t) = s_k(t)\sigma_k(t) - [1 - s_k(t)]\frac{l(t)}{1-l(t)}\left[1 - \frac{\Delta(\tilde{z},\tilde{l})}{\Gamma(\tilde{z},\tilde{l})\tilde{l}}\right]\tilde{\sigma}_k \equiv \zeta(t))\sigma_k(t) \tag{27a}$$

where
$$\zeta(t) \equiv s_k(t) - [1 - s_k(t)]\frac{l(t)}{1-l(t)}\left[1 - \frac{\Delta(\tilde{z},\tilde{l})}{\Gamma(\tilde{z},\tilde{l})\tilde{l}}\right]\left[1 + \frac{\delta_1}{\mu - \delta_2}\left(z(t) - \tilde{z}\right)\right]^{-1} \tag{27b}$$

Thus pre-tax income inequality is driven by two factors. The first is the dynamics of wealth inequality, $\sigma_k(t)$, discussed in Section 4.1. The second is the evolution of factor returns as reflected in factor shares and the supply of labor.[21] While wealth inequality, $\sigma_k(t)$, evolves gradually, the initial jump in leisure, $l(0)$, which impacts on $\sigma_y(0)$, means that any structural or policy shock, causes an initial jump in income inequality, after which it too evolves continuously. As a result, short-run pre-tax income inequality, $\sigma_y(0)$, may over-shoot its long-run equilibrium, $\tilde{\sigma}_y$.

### 4.2 Distribution of Post-tax Income

A key function of income taxes is their redistributive property, necessitating the distinction between pre-tax and post-tax income equality, the latter being arguably of greater significance. We

[21] From (27) we see that the ratio of income inequality to wealth inequality is less than capital's share of output.

define after-tax relative income as[22]

$$y_i^N(t) = \frac{(1-\tau_k)rK_i + (1-\tau_w)wK(1\text{-}l_i(t))}{(1-\tau_k)rK + (1-\tau_w)wK(1\text{-}l)} \tag{28}$$

Recalling the expression for the before-tax relative income and the definition of $\zeta$, we can express after-tax income inequality as

$$\sigma_y^N(t) = \zeta^N(t)\sigma_k(t) \tag{29a}$$

where

$$\zeta^N(t) = \frac{(1-\tau_w)\zeta(t) + s_k(t)(\tau_w - \tau_k)}{(1-s_k(t))(1-\tau_w) + s_k(t)(1-\tau_k)} = \zeta(t) + \left(1-\zeta(t)\right)\frac{s_k(t)(\tau_w - \tau_k)}{(1-s_k(t))(1-\tau_w) + s_k(t)(1-\tau_k)} \tag{29b}$$

Thus, after-tax income distribution will be more (less) equal than the before-tax income distribution, according to whether $\tau_k > (<)\tau_w$. Equations (29) also indicate that the income tax rates, $\tau_k$ and $\tau_w$, exert two effects on the after-tax income inequality. First, by influencing $\zeta(t)$, they influence *gross* factor returns, and therefore the before-tax distribution of income. But in addition, they have direct redistributive effects that are captured by the second term on the right hand side of (29b).

### 4.3 Distribution of Welfare

Economic welfare is another key indicator of the impact of government policies on national well-being, and given the unequal distribution of private wealth and income in the economy, it is important to study its distribution. Recalling the utility function (4a), the instantaneous level of welfare for individual *i* at time *t* is

$$W_i = \frac{1}{\gamma}\left[C_i^{-\upsilon} + \theta\left(z^{1-\varphi}l_iK\right)^{-\upsilon}\right]^{-\frac{\gamma}{\upsilon}} = \frac{1}{\gamma}\left[\Omega\left(z,l\right)^{-\upsilon} + \theta z^{\upsilon(\varphi-1)}\right]^{-\frac{\gamma}{\upsilon}}\left(l_iK\right)^{\gamma} \tag{30}$$

while the average level of instantaneous welfare at that time is given by

[22] Note that this measure ignores the direct distributional impacts of lump-sum transfers, which are arbitrary.

$$W = \frac{1}{\gamma}\left[\Omega(z,l)^{-\upsilon} + \theta z^{\upsilon(\varphi-1)}\right]^{-\frac{\gamma}{\upsilon}} (lK)^{\gamma} \tag{30'}$$

Since this holds for all points of time, we obtain an analogous relationship for the relative intertemporal welfare evaluated along the equilibrium growth path

$$\frac{U_i}{U} = \frac{W_i}{W} = \left(\frac{l_i}{l}\right)^{\gamma} = \pi_i^{\gamma} \tag{31}$$

At each instant of time, agent $i's$ relative welfare remains constant, so that his intertemporal relative welfare is constant as well. Using (22) we can express relative welfare in the form

$$w_i \equiv \frac{U_i}{U} = \frac{W_i}{W} = \left(\frac{l_i}{l}\right)^{\gamma} = \left[1 + \left(1 - \frac{\Delta(\tilde{z},\tilde{l})}{\Gamma(\tilde{z},\tilde{l})\tilde{l}}\right)(\tilde{k}_i - 1)\right]^{\gamma} \tag{32}$$

By applying the monotonic transformation $(w_i)^{1/\gamma} \equiv u(v_i)$, we obtain an expression for the relative welfare of individual *i* expressed in terms of equivalent units of wealth. The standard deviation of welfare across agents is then given by:[23]

$$\sigma_u = \left(1 - \frac{\Delta(\tilde{z},\tilde{l})}{\Gamma(\tilde{z},\tilde{l})\tilde{l}}\right)\tilde{\sigma}_k \tag{33}$$

## 5. Fiscal Policy, Growth, and Inequality: A Numerical Analysis

Given the complexity of the model, we analyze it using numerical simulations. The objective is to determine the effect of an increase in government investment on the economy's growth performance, together with the various distributional measures described above. In doing so, we compare the dynamic adjustment of the economy under four alternative financing schemes, namely where the long-run increase in government investment is fully financed by a (i) lump-sum tax ($\tau$), (ii) capital income tax ($\tau_k$), (iii) labor income tax ($\tau_w$), or (iv) consumption tax ($\tau_c$).

We begin with the following parameterization of a benchmark economy.

[23] Equation (33) also measures the dispersion of consumption and leisure across agents.

**Benchmark Specification of Structural Parameters**

| | |
|---|---|
| Preferences | $\gamma$ = -1.5, $\beta$ = 0.04, $\theta$ = 1.75, $\upsilon$ = 0 |
| Production | $A$ = 0.6, $\alpha$ = 0.6, $\rho$ = 0 |
| Externalities | $\varphi = \varepsilon$ = 0.6 |
| Fiscal | $g$ = 0.05 $\tau = 0.05$ |

The preference and production functions are non-controversial. Setting $\upsilon$ = 0 and $\rho$ = 0 yields the conventional case, where utility is of the constant elasticity form and production is Cobb-Douglas, so that the respective elasticities of substitution are both unity. The rate of time preference $\beta = 0.04$ is standard, while setting $\gamma = -1.5$ yields an intertemporal elasticity of substitution of 0.4, which is consistent with the bulk of the empirical estimates; see Guvenen (2006). The relative weight of leisure in utility, $\theta = 1.75$ accords with the standard value in the real business cycle literature and is the critical determinant of the allocation of time devoted to leisure. It implies an equilibrium value of $\tilde{l} = 0.714$, consistent with empirical evidence; see e.g. Cooley (1995). Finally, the scale parameter $A$ is key in determining the equilibrium growth rate and $A = 0.6$ yields a plausible equilibrium growth rate of 2.29%.

The less familiar aspects of our parameterization concern the specification of the composite externalities in production and utility and are guided by the following considerations. For the Cobb-Douglas specification the representative firm's production function is of the form $Y_i = A(K_i)^{1-\alpha}(L_i)^{\alpha} K^{\alpha\varepsilon}(K_G)^{\alpha(1-\varepsilon)}$. The conventional Romer (1986) model corresponds to $\varepsilon = 1$, and for $\alpha = 0.6$ implies that the external effect of the aggregate capital stock $(K)$ is significantly more productive than is the firm's own capital $(K_i)$. The stock version of the Barro (1990) model [e.g. Futagami et al, 1993] is obtained when $\varepsilon = 0$, which, with $\alpha = 0.6$ yields an implausibly large productive elasticity for public capital. Neither of these parameterizations is realistic. Setting $\varepsilon = 0.6$, however, helps substantially in resolving both problems. First, the firm's own capital stock is now more productive than the externality (0.40 vs. 0.36), while the productive elasticity of government capital is reduced to 0.24, thus placing it close to the plausible range reported by Gramlich (1994). With the production externality constrained in this way, we set $\varphi = \varepsilon = 0.6$, since we find no compelling reason to assume that there should be any systematic difference in the construction of the two externalities in the benchmark economy. We do, however, test the robustness of this assumption by performing sensitivity analysis with respect to these parameters.

The benchmark government spending ratio, *g*, is assumed to be 5% of GDP, which is roughly consistent with evidence on public infrastructure spending for most OECD countries. Table 1.A. summarizes the key equilibrium characteristics. In addition to those aspects mentioned, it yields an equilibrium ratio of public-private capital of 0.53 and output-private capital ratio of around 0.24.

### 5.1. Increase in Government Spending

We consider the effect of an unanticipated and permanent increase in the rate of government investment from its benchmark rate of 5% of GDP to 8% of GDP. We compare the responses under the four financing schemes noted, namely (i) lump-sum tax, (ii) capital income tax, (iii) labor income tax, and (iv) consumption tax. In all cases we assume that the economy starts from an initial benchmark equilibrium in which government expenditure is fully financed by lump-sum taxes, and all distortionary tax rates are zero, i.e. $\tau_c = \tau_w = \tau_k = 0$, so that $g_0 = \tau_0 = 0.05$ in equation (10'). For the distortionary taxes, we assume that the corresponding tax rate is set such that it fully finances the long-run change in government expenditure. Thus starting from $\tau_c = \tau_w = \tau_k = 0$, the corresponding required changes in the tax rates (given the underlying tax base) are respectively $d\tau = 0.03$, $d\tau_k = 0.075$, , $d\tau_w = 0.05$, $d\tau_c = 0.096$ [see Table 1.B]. This means that during the transition as the tax base is changing, residual lump-sum tax financing must also be employed to ensure that the budget remains balanced at all times.

#### 5.1.1 Aggregate Effects

Table 1B(i) shows the effect of an increase in government spending on the steady-state of the aggregate economy. In all cases, the direct stimulus to public investment causes the equilibrium ratio of public to private capital, $\tilde{z}$, to increase. Except when spending is financed by a tax on labor income, leisure falls in the long run, as the higher spending raises the marginal product of labor through the composite externality in the production function. In contrast, when *g* is financed by a tax on labor income the time allocated to leisure increases, as the higher tax rate reduces the after-tax return on labor. But in all cases the effects are small. For all forms of financing, the productive benefits of public capital spending and the consequent private capital accumulation ensure that the equilibrium growth rate increases. They dominate any negative tax effects, although in the case of capital income tax financing with its direct adverse impact on the return to capital, the positive growth effects are small. Overall, the differential impacts on growth, leisure and the ratio of public to private capital reflect the varying degrees of distortions associated with the different tax rates.[24]

### 5.1.2 Distributional Effects

Table 1B(ii) reports the short-run (instantaneous) and long-run effects on wealth, pre-tax, and post-tax income inequality. All these effects are calculated as percentage changes in the standard deviation relative to the pre-shock steady-state standard deviation.

Row 1 reports the case where the increase in government spending is financed by a lump-sum tax. Being non-distortionary, this policy isolates the pure effect of a government spending increase on the distributional measures. Since the stock of private capital, its initial distribution, and the stock of public capital are initially given, wealth inequality does not change on impact. It does so only gradually, increasing by about 2.7% in the long run. Since the lump-sum tax is non-distortionary, the pre-tax and post-tax distributions of income are identical. In the short run, income inequality declines by 2.6% relative to its pre-shock level. However, over time this decline is reversed, and in the long run income inequality increases by about 5%, thus highlighting how government investment generates a sharp intertemporal trade-off for the distribution of income.

Figure 1A illustrates the dynamic responses of the distributions of wealth and income to this pure government spending shock. During the transition, the increasing stock of public capital raises the marginal product of private capital and encourages private capital accumulation. Since private capital is unequally distributed in the economy, capital-rich agents experience a larger increase in their return on private capital investment than do capital-poor agents. Wealth inequality therefore

[24] We do not discuss the transitional adjustment paths for the aggregate economy, as these are well-known from the public investment-growth literature; see Turnovsky (1997) for an early example. The results are available upon request.

increases in transition to the long-run. By raising the expected long-run return to capital and labor, the higher government spending has a productivity impact on labor supply, causing the real wage to rise and labor supply to increase (not shown). In the short run, since capital-poor agents supply more labor relative to the capital-rich, their higher wage income compresses the dispersion of labor supply, thereby leading to an instantaneous decline in income inequality on impact of the shock. In transition, however, this trend is reversed. The wealth effect of the higher stock of infrastructure eventually causes the increasing dispersion of relative capital to dominate the labor productivity effect, so that income inequality gradually increases. As noted, in the long-run, this leads to an overall increase in income inequality relative to its initial benchmark.

Table 1B(ii) (Rows 2-4) and Figure 1B report the distributional responses to the government spending shock when financed by the three distortionary taxes (capital income, labor income, and consumption). These results depend on the interaction between two counter-acting effects along the transitional path. On the one hand, the higher public spending tends to increase the productivity of both capital and labor, thereby affecting the labor-leisure choice and raising average factor incomes. On the other hand, each distortionary tax permanently reduces the after-tax return on the variable on which it impinges, and this in turn has a dampening effect on productivity and consequently, the labor-leisure allocation decision. Since the financing instruments are distortionary, the response of pre-tax and post-tax income inequality will now be distinct, except for the consumption tax, as it does not impinge directly on factor incomes.

Long-run wealth inequality increases in all three cases, with the largest increase of 3.5% arising when the spending is financed by taxing capital income, one effect of which is to reduce the after-tax return on capital and the average capital stock. This, combined with the higher spending on the public good, leads to a large increase in the ratio of public to private capital, which more than offsets the decline in after-tax return on capital. Again, capital-rich agents experience higher long-run returns on capital than do the capital poor, and wealth inequality increases. In the case of the labor tax, the same effect now operates through the after-tax return on labor. The effects of the consumption tax are qualitatively similar to that of the lump-sum tax-financing case.

Pre-tax and post-tax income inequality move in opposite directions in response to the capital and labor tax financing policies, while for the consumption tax their dynamics are identical. Capital tax-financing raises pre-tax income inequality both in the short run and the long run, while it has exactly the opposite effect on post-tax income inequality. For spending financed by a labor income tax, pre-tax income inequality falls both in the short run and long run, while after a small initial decline, post-tax inequality increases. The long-run decline in post-tax income inequality under capital tax-financing reflects the redistributive effects of the financing policy, since wealth is the primary source of inequality in this economy. Labor tax-financing increases long-run post-tax income inequality by reducing after-tax labor income. Since the capital-poor supply more labor, this increases the dispersion of labor supply which, when combined with the higher wealth inequality, increases long-run post-tax income inequality.

### 5.2. The Growth-Income Inequality Relationship

The relationship between growth and income inequality has been the source of a long-standing debate. Table 2 reports the short-run and long-run relationships between growth and post-tax income inequality resulting from the alternative modes of expenditure financing considered in Section 5.1. Whether this relationship is positive or negative is indicated by the signs in the table. We also examine the sensitivity of this relationship to the magnitude of the two sources of externalities in our model, namely the composite externality in the utility and production functions. In addition to the benchmark case, ($\varphi = \varepsilon = 0.6$), we consider two polar cases: (i) the only externality is a public good in the utility function ($\varphi = 0$, $\varepsilon = 1$), and (ii) the only externality is a public good in the production function. The time paths followed by the GDP growth rate and post-tax inequality are illustrated in Fig. 2.

Overall, our findings underscore the ambiguity in the direction of the growth-inequality relationship that is characteristic of recent empirical studies. The results in Tables 2A-C indicate that the qualitative nature of this relationship depends critically on (i) the composition of the different externalities in terms of their private capital-public capital mix, (ii) the tax policy used to finance government investment, and (iii) the time horizon, namely short run or long run.

### 5.3. Average Welfare and its Dispersion

While evaluating the effects of public policies, its consequences for welfare are of critical importance. With heterogeneous agents, we consider two elements, namely average welfare (essentially the welfare of the mean or representative agent) and its dispersion (welfare inequality) as measured by (33). The effects of increasing government investment from 5% to 8% of GDP for average welfare and its dispersion across agents are summarized in Table 3 for the four modes of financing and for three alternative compositions of the externalities.

In all cases we find that increasing government investment raises average welfare, but also increases welfare inequality. In the benchmark case $(\varepsilon = \varphi = 0.6)$ we see that while lump-sum tax financing yields the largest increase in average utility (4.01%), it also generates the largest increase in welfare inequality (5.42%). To the extent that the policymaker is concerned with this tradeoff he may evaluate the financing options in terms of increased inequality per unit of average welfare gain. On this basis consumption tax-financing would be the preferred option, followed by labor income tax-financing, with capital income tax-financing being the worst. This ranking continues to apply in the polar cases as well. Consumption tax financing continues to be optimal if the public good externality appears only in utility $(\varphi = 0, \varepsilon = 1)$, though in this case lump-sum tax financing yields more inequality per unit of welfare gain than does capital income tax financing.

The other point to observe is that the increase in welfare inequality per unit of welfare gain is highly sensitive to the structure of the externality, with the tradeoff between average welfare and its dispersion being much worse when $\varphi = 1, \varepsilon = 0$. This is because with the externality being in utility, this generates the biggest dispersion in leisure, which was seen in (32) to be the ultimate driving force behind welfare inequality. In any event, the main conclusion to draw from this table is that in terms of welfare and its inequality, consumption tax financing may well be the preferred option.

## 6. Sensitivity Analysis

Given the complex nature of the interactions in our model, it is important to examine the robustness of the results discussed in Section 5 to changes in the specification of the key parameters.

In this section, we briefly summarize the sensitivity analysis we have conducted of our main policy experiments, by focusing on three key aspects of the model's structure: (i) relative magnitude of the composite externality parameters, $\varphi$ and $\varepsilon$, (ii) the intratemporal elasticity of substitution between private capital and effective labor in the production function, $s = 1/(1+\rho)$, and (iii) the intratemporal elasticity of substitution between consumption and leisure in the utility function, $q = 1/(1+\upsilon)$. The results are reported in Table 4, Figure 3, and Figure 4, respectively.[25]

In Table 4 summarizes two polar specifications of the externalities. The first is where the public good externality affects only the utility function ($\varphi$ = 0, $\varepsilon$ = 1), and the second is where it enters only the production function ($\varphi$ = 1, $\varepsilon$ = 0).[26] Figure 3 examines the robustness of our benchmark results to changes in the intratemporal elasticity of substitution between capital and effective labor in the production function. We consider three cases (i) $s$ = 0.4, (ii) $s$ = 0.8, and (iii) $s$ = 1.2. Figure 4 examines the robustness of our benchmark results to changes in the intratemporal elasticity of substitution between consumption and leisure in the utility function. We consider three cases (i) $q$ = 0.4, (ii) $q$ = 0.8, and (iii) $q$ = 1.2. The main message we derive from the sensitivity analysis is that overall is that the results obtained from the benchmark parameterization are generally robust with respect to these structural changes. The one exception is the mild decline in wealth inequality obtained when $s$ = 0.4. This is because when the elasticity of substitution in production is small, a large increase in any tax rate is required to balance the government's budget.

## 7. Conclusions

This paper has examined an important, but neglected policy issue, namely the nature of the growth-inequality relationship arising from government investment policies. Two broad sets of questions have been analyzed:

(i) What are the effects of pro-growth policies, such as government investment in infrastructure, on an economy's wealth and income inequality, and how these are affected by the method of financing?

[25] To keep the discussion brief, we do not report tables corresponding to Figs 3 and 4; they are available upon request.

[26] We have considered all possible comparisons between $\varepsilon$ and $\varphi$, i.e., $\varepsilon > \varphi$, $\varepsilon < \varphi$, and $\varepsilon = \varphi$. The qualitative results from Table 1 remain robust.

(ii) Do government spending policies generate trade-offs between average welfare and its dispersion across agents?

These questions have been studied using a general equilibrium endogenous growth model with heterogeneous agents, where the heterogeneity is due to the initial endowments of private capital (wealth). A key feature of the model is the homogeneity of the underlying preferences which implies that, although growth and inequality are joint equilibrium outcomes, they are nevertheless sequentially determined; aggregate behavior determines distributions, but not vice versa. While this specification of preferences renders the analysis tractable, the dynamic structure remains sufficiently complex to require the use of numerical simulations.

In general, our results suggest that government spending on public capital will increase wealth inequality gradually, irrespective of how it is financed. The mechanism is straightforward. Government investment tends to enhance the productivity of private capital, thereby stimulating its accumulation, and with private capital being more unequally distributed among agents than is labor (the productivity of which is also enhanced), this tends to increase wealth inequality.

In contrast, the consequences for income inequality are sensitive to how the public investment is financed and may be characterized by sharp intertemporal tradeoffs. This is because the short-run response of income in equality is dominated by the initial response of the labor-leisure choice and its impact on factor returns, while over time it is more influenced by the evolution of the economy's private and public wealth. The behavior of pre-tax and post-tax income inequality contrasts sharply, depending upon whether the expenditure is financed by a tax on labor or capital. This underscores the point that pro-growth policies may not always be pro-poor, with sharply contrasting outcomes for income inequality over time. These results are generally robust to variations in the economy's key structural parameters.

The policy experiments we consider also enable us to examine the much-debated growth-inequality relationship. We show that whether this relationship is positive or negative depends critically on the relative magnitude of externalities, underlying financing policies, and the time period of consideration. Finally, our framework provides a natural setting for examining the trade-offs generated by fiscal policy between the average level of welfare and its dispersion. We show that all forms of financing are associated with sharp tradeoffs, although these are less adverse under a labor or a consumption tax.

In summary, this paper makes several contributions to the literature on fiscal policy, economic growth, and inequality, characterizing in some detail the mechanism through which growth-enhancing public policies affect the sources of inequality in a dynamic economy. The rich array of results we derive can be tested empirically, which in turn will open the door for future research. Moreover, the analytical framework in this paper can be readily extended to examine the distributional consequences of other important public policy issues, such as the distributional effects of privatization and pricing of infrastructure goods, and the composition of government spending between different types of public investment, such as education, healthcare, etc. in a multi-sector setting. Finally, while initial private capital is the source of inequality we have addressed, other sources of heterogeneity, such as skill differentials and differential endowments of human capital are also important and represent interesting areas of research that can be analyzed using the methods of this paper.

**TABLE 1**

**A. Benchmark Steady-State Equilibrium**

$g$ = 0.05

$\varepsilon = \varphi$ = 0.6 (composite externality), $q = s = 1$

| **Financing Policy** | | $\tilde{z}$ | $\tilde{l}$ | $\tilde{y}$ | $\tilde{\psi}$ **(%)** |
|---|---|---|---|---|---|
| Lump-sum tax-financing | $\tau = 0.05$ | 0.531 | 0.714 | 0.243 | 2.29 |

**B. Increase in Government Spending: Aggregate and Distributional Effects**
**Benchmark Specification**

$g$ = 0.05 to 0.08

i. Steady-State Aggregate Effects

| **Policy Change** | | $d\tilde{z}$ | $d\tilde{l}$ | $d\tilde{\psi}$ |
|---|---|---|---|---|
| Lump-sum tax-financed increase in g | $d\tau = 0.030$ | 0.259 | -0.01 | 0.206 |
| Capital income tax-financed increase in g | $d\tau_k = 0.075$ | 0.353 | -0.006 | 0.101 |
| Labor income tax-financed increase in g | $d\tau_w = 0.05$ | 0.268 | 0.002 | 0.168 |
| Consumption tax-financed increase in g | $d\tau_c = 0.096$ | 0.265 | -0.001 | 0.179 |

ii. Distributional Effects*

| **Policy Change** | **Wealth Inequality** | | **Pre-tax Income Inequality** | | **Post-tax Income Inequality** | |
|---|---|---|---|---|---|---|
| | **Short-run** | **Long-run** | **Short-run** | **Long-run** | **Short-run** | **Long-run** |
| Lump-sum tax-financed increase in g | 0 | 2.736 | - 2.602 | 4.996 | -2.602 | 4.996 |
| Capital income tax-financed increase in g | 0 | 3.527 | 2.707 | 11.976 | -9.174 | -0.149 |
| Labor income tax-financed increase in g | 0 | 2.805 | -8.256 | -0.331 | -0.110 | 7.933 |
| Consumption tax-financed increase in g | 0 | 2.952 | -3.117 | 4.955 | -3.117 | 4.955 |

*All distributional effects are reported as percentage changes relative to their pre-shock levels: $\left[(\sigma_j(t)-\tilde{\sigma}_{j,0})/\tilde{\sigma}_{j,0}\right]\times 100,\ j=k,y,u$

**TABLE 2**
**Increase in Government Spending: The Growth-Inequality Relationship***
$g$ = 0.05 to 0.08
$q = s = 1$

A. Composite Externality in Utility and Production, $\varepsilon = \varphi = 0.6$ (Benchmark Case)

| Policy Change | Short Run Change | | | Long Run Change | | |
|---|---|---|---|---|---|---|
| | Growth | Post-tax Income Ineq. | Relation | Growth | Post-tax Income Ineq. | Relation |
| Lump-sum tax-financed increase in g | 0.129 | -2.602 | − | 0.206 | 4.996 | + |
| Capital income tax-financed increase in g | 0.044 | -9.174 | − | 0.101 | -0.149 | − |
| Labor income tax-financed increase in g | 0.096 | -0.110 | − | 0.168 | 7.933 | + |
| Consumption tax-financed increase in g | 0.106 | -3.117 | − | 0.179 | 4.955 | + |

B. Public Good Externality in Utility Function: $\varphi = 0$, $\varepsilon = 1$

| Policy Change | Short Run Change | | | Long Run Change | | |
|---|---|---|---|---|---|---|
| | Growth | Post-tax Income Ineq. | Relation | Growth | Post-tax Income Ineq. | Relation |
| Lump-sum tax-financed increase in g | -0.107 | -4.964 | + | 0.025 | 3.373 | + |
| Capital income tax-financed increase in g | -0.215 | -11.631 | + | -0.102 | -2.199 | + |
| Labor income tax-financed increase in g | -0.136 | -2.511 | + | -0.010 | 6.210 | − |
| Consumption tax-financed increase in g | -0.128 | -5.468 | + | -0.0002 | 3.315 | − |

C. Public Good Externality in Production Function: $\varphi = 1$, $\varepsilon = 0$

| Policy Change | Short Run Change | | | Long Run Change | | |
|---|---|---|---|---|---|---|
| | Growth | Post-tax Income Ineq. | Relation | Growth | Post-tax Income Ineq. | Relation |
| Lump-sum tax-financed increase in g | 0.409 | -2.287 | − | 0.446 | 8.392 | + |
| Capital income tax-financed increase in g | 0.377 | -9.087 | − | 0.386 | 4.060 | + |
| Labor income tax-financed increase in g | 0.375 | 0.113 | + | 0.408 | 11.531 | + |
| Consumption tax-financed increase in g | 0.385 | -2.938 | − | 0.419 | 8.479 | + |

*All distributional effects are reported as percentage changes relative to their pre-shock levels: $\left[(\sigma_j(t)-\tilde{\sigma}_{j,0})/\tilde{\sigma}_{j,0}\right]\times 100,\ j = k, y, u$

**TABLE 3**
**Increase in Government Spending:**
**Trade-off between Aggregate Welfare and its Dispersion***
$g$ = 0.05 to 0.080.6
$q = s = 1$

A. Composite Externality in Utility and Production, $\varepsilon = \varphi = 0.6$ (Benchmark Case)

| **Policy Change** | $d\tilde{W}$ (%) | $d\tilde{\sigma}_u$ (%) |
|---|---|---|
| Lump-sum tax-financed increase in g | 4.012 | 5.415 |
| Capital income tax-financed increase in g | 1.790 | 3.620 |
| Labor income tax-financed increase in g | 3.139 | 2.996 |
| Consumption tax-financed increase in g | 3.398 | 2.946 |

B. Public Good Externality in Utility Function: $\varphi = 0$, $\varepsilon = 1$

| **Policy Change** | $d\tilde{W}$ (%) | $d\tilde{\sigma}_u$ (%) |
|---|---|---|
| Lump-sum tax-financed increase in g | 6.830 | 5.773 |
| Capital income tax-financed increase in g | 5.041 | 3.872 |
| Labor income tax-financed increase in g | 5.930 | 3.312 |
| Consumption tax-financed increase in g | 6.198 | 3.299 |

C. Public Good Externality in Production Function: $\varphi = 1$, $\varepsilon = 0$

| **Policy Change** | $d\tilde{W}$ (%) | $d\tilde{\sigma}_u$ (%) |
|---|---|---|
| Lump-sum tax-financed increase in g | 3.384 | 6.300 |
| Capital income tax-financed increase in g | 1.227 | 4.929 |
| Labor income tax-financed increase in g | 2.554 | 3.926 |
| Consumption tax-financed increase in g | 2.801 | 3.902 |

*The dispersion of welfare is reported as percentage change relative to its pre-shock level: $\left[(\sigma_u(t) - \tilde{\sigma}_{u,0})/\tilde{\sigma}_{j,0}\right] \times 100$

**TABLE 4**
**Increase in Government Spending: Distributional Effects***
**Sensitivity to Externality Parameters**
$g$ = 0.05 to 0.08
$q = s = 1$

A. Public Good Externality in Utility Function: $\varphi = 0$, $\varepsilon = 1$

| **Policy Change** | **Wealth Inequality** | | **Pre-tax Income Inequality** | | **Post-tax Income Inequality** | |
|---|---|---|---|---|---|---|
| | **Short-run** | **Long-run** | **Short-run** | **Long-run** | **Short-run** | **Long-run** |
| Lump-sum tax-financed increase in g | 0 | 3.098 | -4.964 | 3.373 | -4.964 | 3.373 |
| Capital income tax-financed increase in g | 0 | 3.805 | -0.097 | 9.592 | -11.631 | -2.199 |
| Labor income tax-financed increase in g | 0 | 3.141 | -10.413 | -1.821 | -2.511 | 6.210 |
| Consumption tax-financed increase in g | 0 | 3.317 | -5.468 | 3.315 | -5.468 | 3.315 |

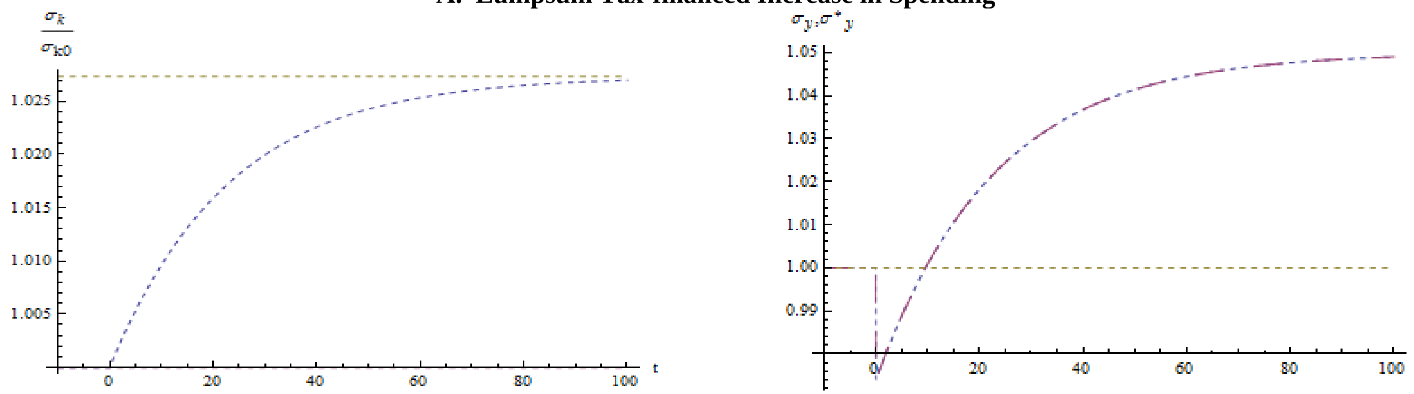
B. Public Good Externality in Production Function: $\varphi = 1$, $\varepsilon = 0$

| **Policy Change** | **Wealth Inequality** | | **Pre-tax Income Inequality** | | **Post-tax Income Inequality** | |
|---|---|---|---|---|---|---|
| | **Short-run** | **Long-run** | **Short-run** | **Long-run** | **Short-run** | **Long-run** |
| Lump-sum tax-financed increase in g | 0 | 3.584 | -2.287 | 8.392 | -2.287 | 8.392 |
| Capital income tax-financed increase in g | 0 | 4.802 | 3.317 | 16.796 | - 9.087 | 4.060 |
| Labor income tax-financed increase in g | 0 | 3.709 | -8.398 | 2.867 | 0.113 | 11.531 |
| Consumption tax-financed increase in g | 0 | 3.895 | -2.938 | 8.479 | -2.938 | 8.479 |

*All distributional effects are reported as percentage changes relative to their pre-shock levels: $\left[(\sigma_j(t)-\tilde{\sigma}_{j,0})/\tilde{\sigma}_{j,0}\right]\times 100,\ j=k,y,u$

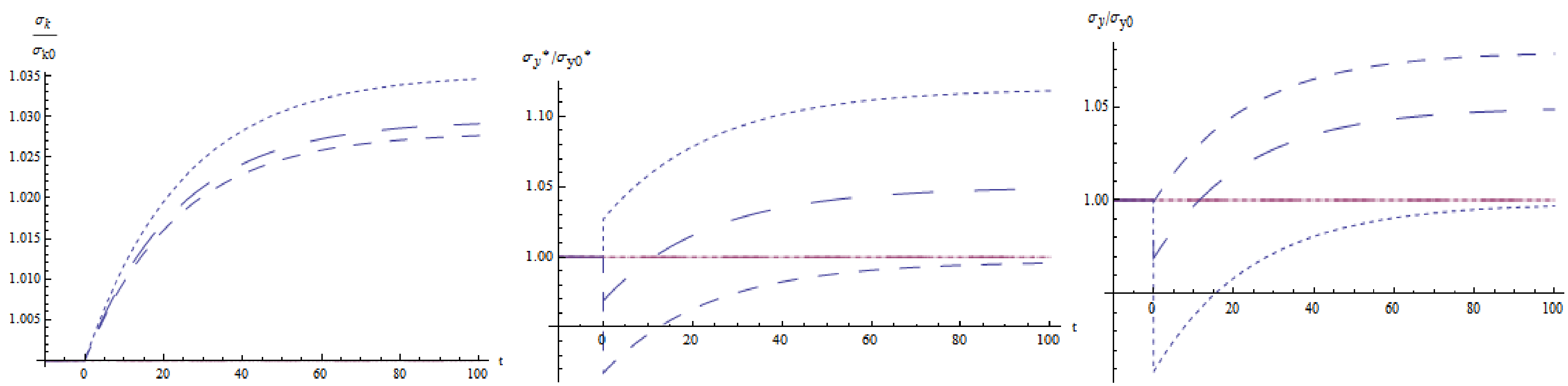
**Figure 1. Increase in Government Spending, g = 0.05 to 0.08**
**Benchmark Case: $s = 1$, $q = 1$, φ = ε= 0.6**

**A. Lumpsum Tax-financed Increase in Spending**




i. Wealth Inequality ii. Pre- and Post-tax Income Inequality

**B. Distortionary Tax-financed Increase in Spending**

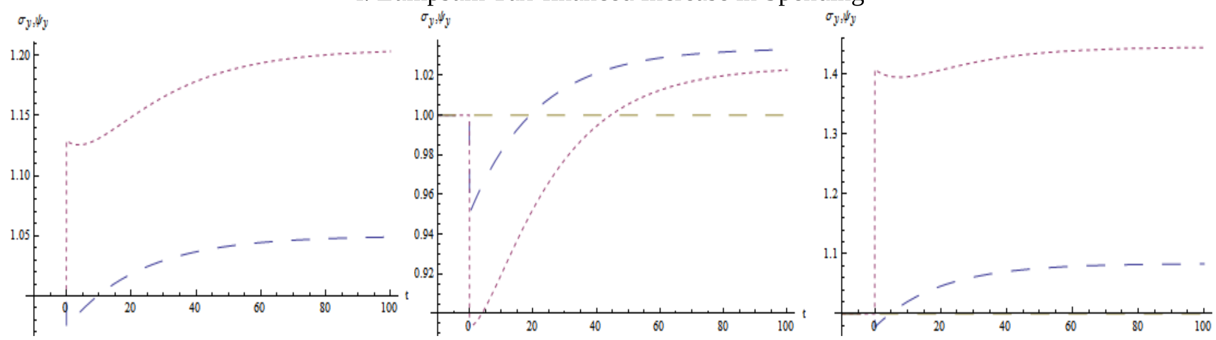



i. Wealth Inequality ii. Pre-tax Income Inequality iii. Post-tax Income Inequality

**………………. Capital income tax-financed _ _ _ _ _ _ _ Labor income tax-financed __ __ __ __ __ Consumption tax-financed**

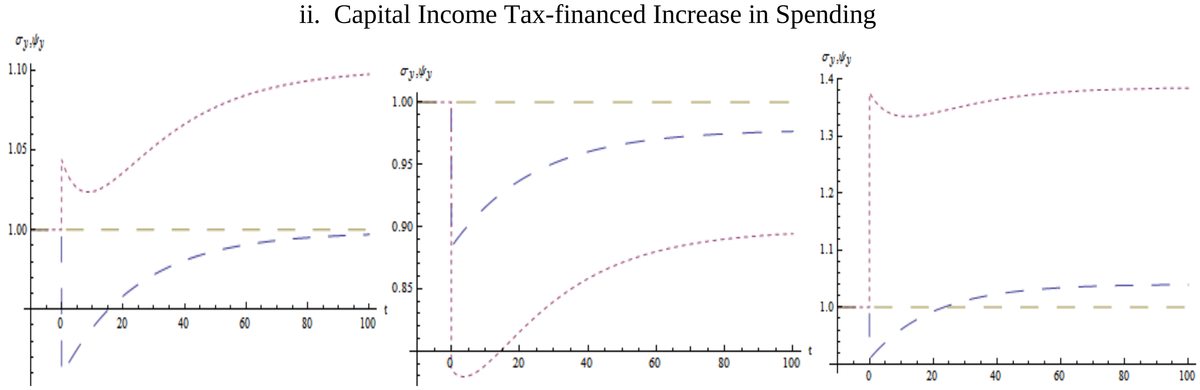
**Figure 2. The Growth-Income Inequality (post-tax) Relationship**
**Sensitivity to Externality Parameters ($\phi$, $\varepsilon$)**

i. Lumpsum Tax-financed Increase in Spending



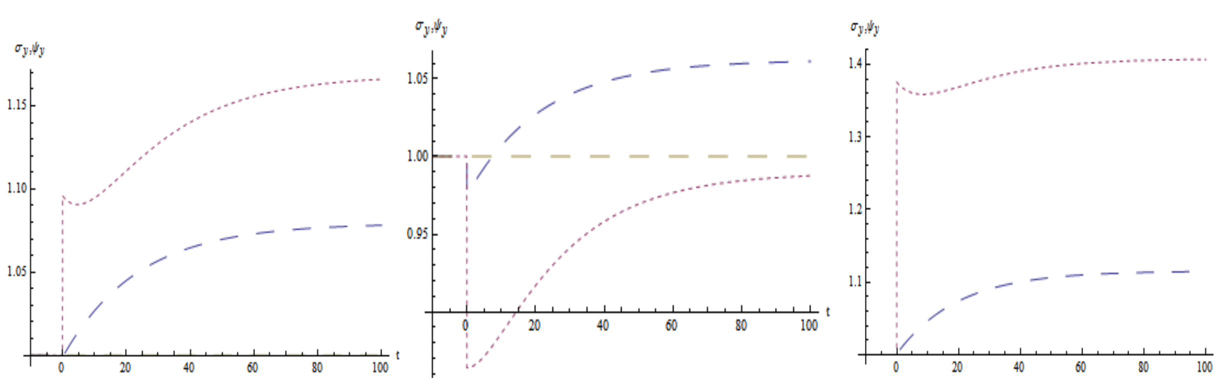

**ϕ = 0.6, ε = 0.6** **ϕ = 0, ε = 1** **ϕ = 1, ε = 0**

ii. Capital Income Tax-financed Increase in Spending




**ϕ = 0.6, ε = 0.6** **ϕ = 0, ε = 1** **ϕ = 1, ε = 0**

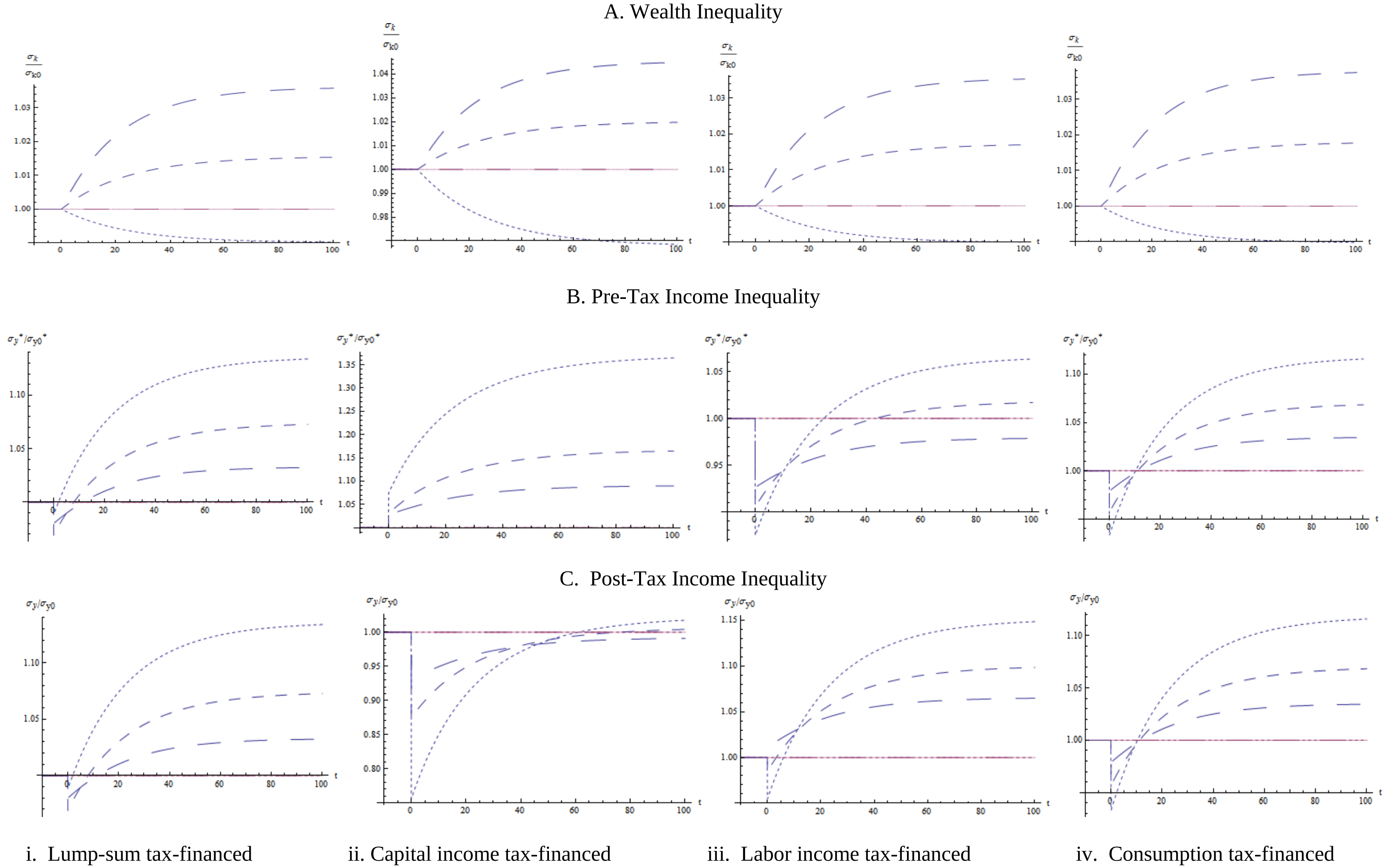
iii. Labor Income Tax-financed Increase in Spending




**ϕ = 0.6, ε = 0.6** **ϕ = 0, ε = 1** **ϕ = 1, ε = 0**

............... GDP Growth _ _ _ _ _ _ _ Post-tax Income Inequality

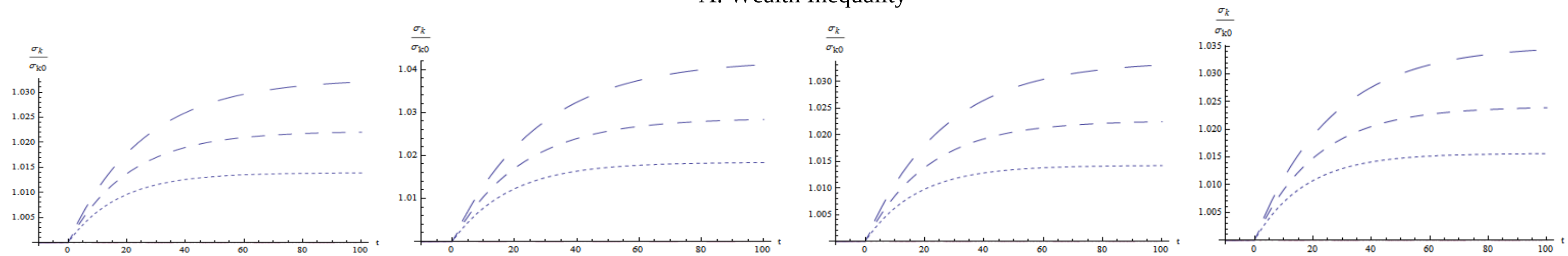
**Figure 3. Increase in Government Spending: Sensitivity to the Elasticity of Substitution in Production**
**g = 0.05 to 0.08, $q = 1$, φ = ε= 0.6**



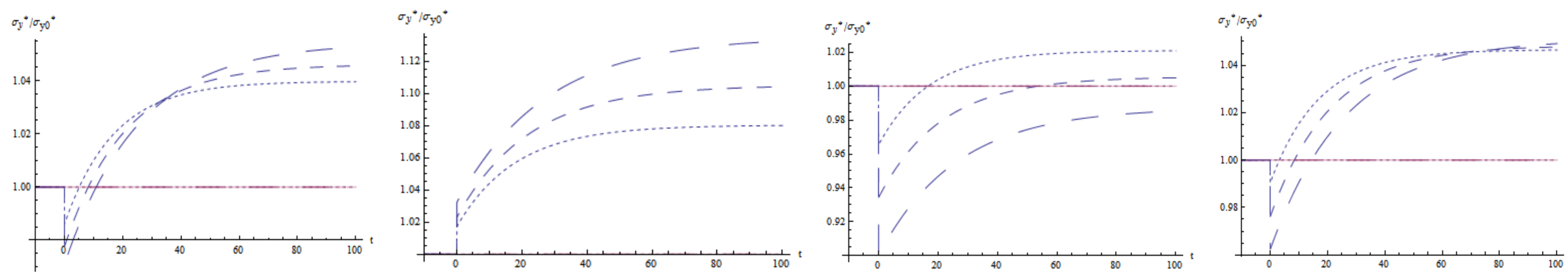

**…..……… s = 0.4 _ _ _ _ _ _ s = 0.8 __ __ __ __ __ s = 1.2**

**Figure 4. Increase in Government Spending: Sensitivity to the Intratemporal Elasticity of Substitution in the Utility Function**
**g = 0.05 to 0.08, $s = 1$, φ = ε= 0.6**

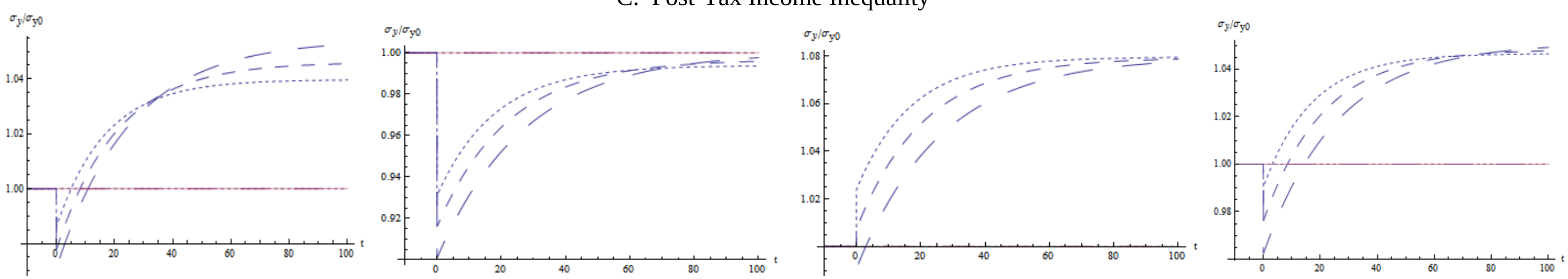
A. Wealth Inequality



B. Pre-Tax Income Inequality



C. Post-Tax Income Inequality



i. Lump-sum tax-financed ii. Capital income tax-financed iii. Labor income tax-financed iv. Consumption tax-financed

**…..……… q = 0.4 _ _ _ _ _ _ q = 0.8 __ __ __ __ __ q = 1.2**

## Appendix: Linearization of the aggregate dynamic system

Linearizing (14) around the steady state:

$$\begin{pmatrix} \dot{z} \\ \dot{l} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} z-\tilde{z} \\ l-\tilde{l} \end{pmatrix} \tag{A.1}$$

where

$$a_{11} \equiv g y_z - \left[(1-g)y_z - \Omega_z l\right]z - \left[(1-g)y - \Omega l\right] \quad a_{12} \equiv g y_l - \left[(1-g)y_l - \Omega_l l - \Omega\right]z$$

$$a_{21} \equiv \left\{(1-\tau_k)r_z - (1-\gamma)\left[(1-g)y_z - \Omega_z l\right] + \left(\frac{\Phi(\upsilon+\gamma)(1-\varphi)}{1+\Phi} + \left[\frac{(1-\gamma)+(1+\upsilon)\Phi}{1+\Phi}\right]\frac{\Omega_z z}{\Omega}\right)\frac{a_{11}}{z}\right\}\frac{l}{J}$$

$$a_{22} \equiv \left\{(1-\tau_k)r_l - (1-\gamma)\left[(1-g)y_l - \Omega_l l - \Omega\right] + \left(\frac{\Phi(\upsilon+\gamma)(1-\varphi)}{1+\Phi} + \left[\frac{(1-\gamma)+(1+\upsilon)\Phi}{1+\Phi}\right]\frac{\Omega_z z}{\Omega}\right)\frac{a_{12}}{z}\right\}\frac{l}{J}$$

and $\Phi \equiv \theta\left[z^{-(1-\varphi)}\Omega\right]^{\nu}$

The partial derivatives, $r_z, y_z$, etc. are obtained from (2'), (3). Note that if $\varepsilon = 1$, that $a_{21} = 0$. The system (A.1) will be locally saddlepoint stable if and only if $a_{11}a_{22} - a_{12}a_{21} < 0$.

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