Intertemporal Pricing with Capacity Constraint and Outside Option

Shubhro Sarkar*†

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Abstract

We consider the intertemporal pricing problem of a good which is finitely durable, where seller(s) face a capacity constraint and buyers get access to an outside option with some probability. The outside option allows buyers to buy the product at the same, or at a lower price from another seller in the case of a duopoly, and fetches the product through a scheme for free, in the case of a monopoly seller. Using a model of incomplete information, we solve for the equilibrium price path, and show that there exists interfirm price dispersion in equilibrium and in certain cases, a last-minute discount.

Keywords: Intertemporal pricing, Perfect Bayesian Equilibrium, Last-Minute Discounts, Interfirm Price Dispersion.

JEL Codes: L10, L93, C72.

1 Introduction

The main focus of this paper is to solve for the price path of a good which is finitely durable, while the seller faces a capacity constraint and where buyers have some probability of getting access to an outside option, in a dynamic model with

*Indira Gandhi Institute of Development Research, Gen. A.K. Vaidya Marg, Goregaon East, Mumbai, 400065, India. Email: shubhro@igidr.ac.in

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incomplete information. We show that the possibility of buyers getting access to an outside option in a subsequent period, limits the price a seller can charge in the initial period(s). The outside option allows buyers to buy the product at the same, or at a lower price from another seller in the case of a duopoly, and fetches the product through a scheme for free, in the case of a monopoly seller. We solve for the conditions under which a seller(s) offers a *last-minute discount* to buyers. We also show that there exists *interfirm price dispersion* in equilibrium, due to the differences in beliefs of buyers being of high valuation across market segments.

Airline tickets, movie-tickets being sold by scalpers, advertisement slots on television are examples of goods which are finitely durable, where the seller faces a capacity constraint along with demand uncertainty. In the case of an airline ticket, the seller while selling a ticket at a certain price to a buyer, effectively signs a futures contract with the buyer promising to render a service under certain terms and conditions. The price at which an agreement is signed at date t need not be offered again at a later date, such that the seller faces the same *time-consistency* issues as a durable goods monopolist. In such cases, the price charged in any period by a seller, depends amongst other things, on the outside options available to the buyer in subsequent periods. Ticket scalpers are often seen peddling tickets in front of movie theatres in several cities in developing countries. If tickets are no longer available from the theatre showing the movie, the scalper typically raises prices as the show time draws closer; if tickets are available, he sells the ticket at the "window price" and covers his costs.

We construct a model where sellers offer a single unit of a product for sale over two periods, after which it is assumed to be lost forever. We consider two market structures, with one or two firms. We assume minimum consumer heterogeneity, such that each buyer is one of two types, "high" or "low", which is private information. Sellers announce prices in each period without commitment, with the objective of maximizing discounted stream of revenue. In the market with a single seller, the demand side consists of two buyers. Buyers are strategic in the sense that they can either buy the product immediately in the first period, or wait either for an outside option or for a lower price from the same seller in the second period. Buyers of different types are assumed to get the outside option with different probabilities.

In the market with two sellers, the market is assumed to consist of two segments or groups, with one seller and two buyers belonging to each group. We assume that the demand side in the first period for each seller consists of buyers from the same group only. However, no such restriction is placed in the second period, such that buyers can buy from either seller provided the seller(s) hasn't sold out his unit. Buyers are assumed to first approach a seller offering the lowest price, and either seller if they are offering the same price. We also assume that the probability with which a buyer is of "high" type varies across the two segments. This implies that the products being offered by the two sellers have certain characteristics, one of which is more likely to attract a high valuation buyer than the other. Certain buyers initially attempt to buy the product with features they value most[1]. If they fail, they then turn to the other seller. All agents are assumed to be risk neutral.

We find that depending on parameter values, the price path is increasing, decreasing or horizontal. The seller offers the product at a price high enough, such that high valuation buyers remain indifferent between buying and waiting, and that this price is decreasing in the probability of getting the outside option. In the case with a single seller, the seller announces a 'sale' in the second period if he is unable to sell the unit in the first. Similarly, a 'sale' is announced by both sellers if neither were able to sell their units in the first period in the duopoly model. This is interpreted as a last-minute discount, as sellers find it unlikely that they will be able to sell their unit at a high price. However, if there is only one seller in the second period, whether or not he announces a 'sale', depends on his updated belief that the remaining buyer from the other group is of high valuation. Prices charged by the two sellers are found to be different in the first period. Assuming that discount factors are the same, these differences are attributed to differences in beliefs of the agents, that the probability that a buyer is of high valuation, varies across the groups.

The rest of the paper is organized as follows. We review some related literature in section 2. In section 3 we analyze the model with a monopoly seller, and with two sellers in section 4. We suggest some extensions in the following section and conclude in section 6.

2 Review of Literature

Yield management (or revenue management) deals with goods which are durable for a finite horizon and where capacity is costly. It uses pricing strategies along with tools like market segmentation and seat inventory control, with the objective of maximizing firm revenue. Highly sophisticated yield management (YM) systems are used by airlines, hotels and other industries selling products which

[1]Examples of such features include weekend layoffs, non-stop flights, flexible departure dates in the case of airline tickets.

expire after a certain date and where capacity is fixed. McAfee and Velde (2007) provide an extensive survey of YM research in operations research journals, and contribute to the literature by testing predictions regarding airline prices[2].

A number of papers in recent years have addressed certain limitations in theYM literature. Most papers primarily treated fares as exogenous, and solved for the seat-inventory control problem[3]. They typically considered a single firm framework and had buyers who are non-strategic, in the sense that they do not have the option of waiting for a cheaper prices. YM Levin et. al. (2009) present a dynamic pricing model where oligopolistic firms sell differentiated perishable goods to a finite population of strategic buyers in a finite number of periods. In a stochastic dynamic game, firms announce prices with the objective of maximizing total expected revenue, while each consumer responds according to a linear random utility choice model, with the added option of delaying his purchase. The model generates a vector of probabilities with which a consumer in a particular segment gets one unit of the good from one firm. They prove the existence of a Markov-Perfect Equilibrium for a special case where consumers express an eagerness to purchase several of the available products. They conclude that firms may benefit more from limiting the information available to buyers than from allowing full information and responding to the resultant strategic behavior in an optimal fashion.

Su (2007) considers a model of intertemporal pricing with strategic buyers, where buyers are partitioned into one of four types, depending on whether they have high or low valuation for the product and whether they are patient or impatient (myopic). Buyers arrive at a fixed rate and the *monopoly seller* solves for the optimal pricing policy and capacity rationing schedule. He shows that the optimal pricing policy is increasing if high-valued patient buyers or low-valued impatient buyers dominate the market and is decreasing otherwise. His paper extends the stream of literature based on the theory of durable goods first developed by Coase (1972). For instance, Conlisk et. al. (1984) consider a model where a monopoly

[2]For a review of dynamic pricing models which deal with revenue management, see Bitran and Caldentey (2003).

[3]For example, Brumelle and McGill (1993) and Wollmer (1992), use a model where lower fare class customers book tickets before their higher fare class counterparts. Airlines solve for a critical number of seats in each fare class, which are reserved for potential future passengers who are willing to pay a higher price. Booking requests for a particular fare class are accepted if and only if the number of empty seats is strictly greater than its critical level and rejected otherwise. Wollmer shows that this critical value is a decreasing function of the fare price and is equal to zero for the highest fare class.

seller offers a durable good for sale over an infinite horizon, with a new cohort of buyers entering the market in each period. They show that the price path is cyclic, involving periodic sales, and that higher valuation buyers are willing to pay a lower price as a sale approaches. Sobel (1984) extends the framework to include multiple sellers. However, these models involve an infinite horizon while the sellers do not face a capacity constraint.

Jerath et. al. (2010) compares the benefits of last-minute sales directly to consumers versus through an opaque intermediary[4]. Buyers make decisions based on expectations regarding future availability, which turn out to be correct in equilibrium. They show that direct last-minute sales are preferred over selling through opaque intermediaries when consumers' valuation for the product is high or product differentiation is low, or both. Ovchinnikov and Milner (2007) study a pricing model with a single firm over multiple seasons, where consumers learn to delay their purchases after observing recurrent last-minute discounts.

There are other related papers which deal with the relationship between capacity constraints, demand uncertainty and pricing schedules. Dana's (1998) paper on advance-purchase discounts has a market with individual and aggregate consumer demand uncertainty. Price-taking firms set prices before demand is known and may offer *advance-purchase discounts*. In this case firms discriminate between buyers who have low willingness to pay but have a better chance of buying the product and buyers who have a higher willingness to pay but have a low probability of making a purchase. Thus firms screen buyers by their demand uncertainty, offering lower prices to consumers with certain demand in order to lower the costs of holding unutilized capacity or excess inventory. Gale and Holmes (1992) show in a model with a monopoly seller offering "peak" and "off-peak" flights, that an advance-purchase discount is offered on "off-peak" (low demand) flights, such that leisure travelers self select to fly in "off-peak" flights, as they have a lower cost of waiting. They then predict that there will be fewer discounted tickets on the "peak" flights.

Our contribution is to extend and to elucidate the results both from the YM and durable goods literature.

[4]While buying the opaque product from an intermediary, a buyer does not know the exact terms of the contract. For example, the buyer cannot predict which firm will provide the product, or at what time the product will be available.

3 Model with Monopoly Seller

A two-period model, with a single seller facing a capacity constraint and two buyers is developed which helps analyze the optimal price path for the monopoly seller in an intertemporal price discrimination problem.

Setting. Time is discrete. There are two periods. The product has a lifetime of two periods, after which it is assumed to be lost forever.

Supply Side. There is a single seller of the product. This seller has just one unit of the product to sell. The seller chooses prices in order to maximize the discounted sum of revenue[5], calculated at discount rate δ, with $0 < \delta < 1$. In period 1, the seller cannot make a binding commitment about the price in period 2.

Demand side. There are two buyers. Each buyer might be of one of the following two types, "high" ($\theta_i = \theta_H$) or "low" ($\theta_i = \theta_L$). With probability ρ each buyer can be of "high" type with valuation $V_1 \in \mathbb{R}^+$, and with probability $1 - \rho$, each can be a "low" valuation buyer with valuation $V_2 \in \mathbb{R}^+$, with $V_1 > V_2 > 0$. Each buyer's type is known privately only to himself. Both the buyers and the seller are assumed to be risk-neutral.

Each buyer has access to some outside option only in the period 2, which he gets without any payment, with some probability.[6] We use $\omega_i = \{0, 1\}$ to denote whether or not player i got the outside option, with $\omega_i = 1$ denoting he did. If the buyer is of "high" type, the corresponding probability of getting the outside option is q_1, and if the buyer is of "low" type, the probability of getting the outside option is q_2, with $0 < q_2 < q_1 < 1$. Both the buyers are assumed to have the same rate of discount as the seller. Both the buyers are assumed to be price takers and there are no resales. All agents are risk neutral.

Timing of events. At the beginning of period 1, the seller announces the price for period 1, p_1. Each buyer decides whether to buy the product in period 1 itself or to wait for the outside option available in period 2. If the buyer enters the market in period 1 and buys the product, he leaves the market and the game ends immediately. If the product is not sold in period 1, at the beginning of period 2, seller announces the price for period 2, p_2. If the buyer decided to wait for the outside option in period 2, he realizes whether or not he got the outside option. In case the buyer gets the outside option, he leaves the market; in the event the buyer fails to get the outside option and decides to remain in the market, he de-

[5] Cost is assumed to be zero in this model.

[6] The outside option might be thought of as a frequent flyer program which fetches a free ticket to the interested buyer.

cides whether or not to buy the product based on price in period 2. If a buyer is indifferent between buying in period 1 immediately and waiting for the outside option in the next period, he is assumed to make the purchase immediately. We assume that a *proportional rationing* rule is followed.

Equilibrium Concept. Since buyer's type is private information and the game is sequential in nature, the appropriate equilibrium concept is Perfect Bayesian Equilibrium.

3.1 Perfect Bayesian Equilibrium

The seller announces p_2 only when neither buyer bought the product in the first period. While announcing the price in period 2, the seller must take into account the fact that each buyer might get the outside option in period 2. Since prices in period 2 are announced prior to the buyers' realization of the outside option, the following two cases are considered.

Case 1. Neither buyer gets the outside option. We define this event as NBO. The corresponding probability is[7]

$$\Pi = (\mu[H|p_1])^2(1-q_1)^2 + (\mu[L|p_1])^2(1-q_2)^2 + 2(\mu[H|p_1])(\mu[L|p_1]) \\ (1-q_1)(1-q_2), \tag{1}$$

where $\mu[H|p_1]$ is the updated belief of the seller that a buyer is of "high" type, given that the product remained unsold in period 1 at price p_1, and $\mu[L|p_1]$ is the updated belief of the seller that a buyer is of "low" type, given that the unit product remained unsold in period 1 at price p_1.

Expected revenue from announcing price $p_2 = V_1$ is

$$V_1 \Pr[\text{At least one buyer is "high" type} \cap NBO]$$

where, $\Pr[\text{At least one buyer is "high" type} \cap NBO]$ is given by

$$\Pr[\{(\theta_1 = \theta_H, \theta_2 = \theta_H) \cup (\theta_1 = \theta_H, \theta_2 = \theta_L) \cup (\theta_1 = \theta_L, \theta_2 = \theta_H)\} \\ \cap\{(\omega_1 = 0) \cap (\omega_2 = 0)\}]$$

$$= \Pr[\{(\theta_1 = \theta_H) \cap (\omega_1 = 0)\} \cap \{(\theta_2 = \theta_H) \cap (\omega_2 = 0)\}] \\ + \Pr[\{(\theta_1 = \theta_H) \cap (\omega_1 = 0)\} \cap \{(\theta_2 = \theta_L) \cap (\omega_2 = 0)\}] \\ + \Pr[\{(\theta_1 = \theta_L) \cap (\omega_1 = 0)\} \cap \{(\theta_2 = \theta_H) \cap (\omega_2 = 0)\}]$$

[7] See appendix A for calculations.

$$= \Pr(NBO) \times \{\Pr(\theta_1 = \theta_H|\omega_1 = 0)\Pr(\theta_2 = \theta_H|\omega_2 = 0) +$$
$$\Pr(\theta_1 = \theta_H|\omega_1 = 0)\Pr(\theta_2 = \theta_L|\omega_2 = 0)$$
$$+ \Pr(\theta_1 = \theta_L|\omega_1 = 0)\Pr(\theta_2 = \theta_H|\omega_2 = 0)\}.$$

Using Baye's rule, we get

$$\Pr[\theta_i = \theta_H|\omega_i = 0] = \Lambda = \frac{(1-q_1)\mu[H|p_1]}{(1-q_1)\mu[H|p_1] + (1-q_2)\mu[L|p_1]}, \qquad (\clubsuit)$$

$$and\ \Pr[\theta_i = \theta_L|\omega_i = 0] = \Gamma = \frac{(1-q_2)\mu[L|p_1]}{(1-q_1)\mu[H|p_1] + (1-q_2)\mu[L|p_1]}. \qquad (\spadesuit)$$

Substituting, we get expected revenue from announcing $p_2 = V_1$ is $V_1\Pi(\Lambda^2 + 2\Lambda\Gamma)$.

Case 2. Only one buyer gets the outside option. We define this event as *ONE*. The corresponding probability is[8]

$$\Phi = 2(\mu[H|p_1])^2 q_1(1-q_1) + 2(\mu[L|p_1])^2 q_2(1-q_2) + 2\mu[H|p_1]\mu[L|p_1]q_1(1-q_2)$$
$$+2\mu[L|p_1]\mu[H|p_1]q_2(1-q_1) \qquad (2)$$

Expected revenue from charging $p_2 = V_1$ is[9]

$$V_1 \Pr[\text{Buyer without outside option is high type} \cap ONE]$$
$$= V_1 \Pr[\theta_i = \theta_H|\omega_i = 0] \times \Pr[ONE]$$

where $\Pr[\theta_i = \theta_H|\omega_i = 0]$ is given by ($\clubsuit$).
In the remaining instance, both buyers get the outside option and leave the market. Thus, total expected revenue from charging $p_2 = V_1$ is

$$V_1\Pi(\Lambda^2 + 2\Lambda\Gamma) + V_1\Phi\Lambda.$$

Similarly, expected revenue from charging $p_2 = V_2$ is $V_2(\Pi + \Phi)$. The seller chooses p_2 as follows:

$$p_2 = \begin{cases} V_1 \text{ if } V_1\Pi(\Lambda^2 + 2\Lambda\Gamma) + V_1\Phi\Lambda \geq V_2(\Pi + \Phi) \\ V_2 \text{ otherwise} \end{cases} \qquad (3)$$

[8] See appendix B for calculations.
[9] See appendix C for calculations.

Seller chooses p_1 as follows. The seller could set $p_1 = p_1^H$, the price which would make a "high" type buyer indifferent between buying the product in period 1 and waiting for the outside option in period 2. In the event that the "high" valuation buyer(s) chooses (choose) not to buy the product in period 1, the seller will get the signal that neither of the two buyers are of "high" type. In period 2, the seller then sets the price $p_2 = V_2$. Thus p_1^H is defined by the following function

$$\frac{1}{2}\rho(V_1 - p_1^H) + (1-\rho)(V_1 - p_1^H) = \delta[q_1 V_1 + (1-q_1)\{\frac{1}{2}(1-\rho)(1-q_2)(V_1 - V_2) + (1-\rho)q_2(V_1 - V_2)\}]$$

$$\implies p_1^H = \frac{1}{2-\rho}[V_1(2-\rho-2\delta q_1) - \delta(1-q_1)(1-\rho)(1+q_2)(V_1 - V_2)] \quad (4)$$

such that, $\dfrac{\partial p_1^H}{\partial q_1} < 0 \; \forall q_2, \rho$. Similarly, the seller could choose to set $p_1 = p_1^L$, the price which would make a "low" type buyer indifferent between buying the product in period 1 and waiting for the outside option in period 2. In the event the "low" type buyer decides not to buy the product at this price, the other buyer definitely will, such that no unit will be left for sale in period 2. Thus, p_1^L is defined as $\frac{1}{2}(V_2 - p_1^L) = \delta V_2 q_2$, such that,

$$p_1^L = V_2 - 2\delta V_2 q_2 = V_2(1 - 2\delta q_2). \quad (5)$$

The seller thus updates his beliefs in period 2 as follows. If announced price in period 1 is $p_1 = p_1^H$ and neither buyer buys the product, the seller updates his belief such that $\mu[H|p_1 = p_1^H] = 0$ and $\mu[L|p_1 = p_1^H] = 1$. If announced price in period 1 is $p_1 = p_1^L$ and neither buyer buys the product (which is off the equilibrium path), the seller is unable to update his belief, such that $\mu[H|p_1 = p_1^H] = \rho$ and $\mu[L|p_1 = p_1^H] = 1-\rho$. Thus with $\mu[H|p_1 = p_1^H] = 0$ and $\mu[L|p_1 = p_1^H] = 1$, using the decision rule specified by equation (3), the seller announces $p_2 = V_2$.

In period 1, if the seller sets $p_1 = p_1^L$, the product is sold for sure either to a "high" type or to a "low" type buyer. On the other hand, if the price charged in period 1 is p_1^H, the unit is sold if there's at least one high type buyer in the market, and the game ends immediately. The corresponding expected revenue is $p_1^H\{1-(1-\rho)^2\}$. In the event the unit is not sold in period 1 at price p_1^H, the seller sets $p_2 = V_2$, assuming that both buyers in the market are of "low" type.

The corresponding probability that the ticket is sold in period 2 and price V_2 is $(1-\rho)^2(1-q_2^2)$. The seller chooses p_1 as follows.

$$p_1 = \begin{cases} p_1^H \text{ if } p_1^H\{1-(1-\rho)^2\} + \delta V_2(1-\rho)^2(1-q_2^2) \geq p_1^L \\ p_1^L \text{ otherwise.} \end{cases} \tag{6}$$

We have to ensure that the solution is incentive compatible. This means that a buyer of "high" type should not refuse to buy the product when the price of p_1^H is offered in period 1 and instead prefer to wait for the outside option in period 2. The required condition for ensuring incentive compatibility is

$$V_1 - p_1^H > \delta[q_1V_1 + (1-q_1)\{\frac{1}{2}(1-\rho)(1-q_2)(V_1-V_2) + (1-\rho)q_2(V_1-V_2)\}]. \tag{7}$$

Since by definition $\frac{1}{2}\rho(V_1-p_1^H) + (1-\rho)(V_1-p_1^H) = \delta[q_1V_1 + (1-q_1)\{\frac{1}{2}(1-\rho)(1-q_2)(V_1-V_2) + (1-\rho)q_2(V_1-V_2)\}]$, the above inequality holds.

Proposition 1. *A Perfect Bayesian Equilibrium of this game consists of the following beliefs and strategies. The single seller chooses prices $(p_1, p_2) \in \mathbb{R}_+^2$, with*

$$p_2 = \begin{cases} V_1 \textit{ if } \Pi V_1(\Lambda^2 + 2\Lambda\Gamma) + V_1\Phi\Lambda \geq V_2(\Pi+\Phi) \\ V_2 \textit{ otherwise} \end{cases}, \ where$$

$$\begin{gathered} \Pi = \rho^2(1-q_1)^2 + (1-\rho)^2(1-q_2)^2 + 2\rho(1-\rho)(1-q_1)(1-q_2), \\ \Phi = 2\rho^2 q_1(1-q_1) + 2(1-\rho)^2 q_2(1-q_2) + 2\rho(1-\rho)q_1(1-q_2) \\ +2(1-\rho)\rho q_2(1-q_1), \\ \Lambda = \frac{(1-q_1)\rho}{(1-q_1)\rho + (1-q_2)(1-\rho)} \textit{ and} \\ \Gamma = \frac{(1-q_2)(1-\rho)}{(1-q_1)\rho + (1-q_2)(1-\rho)}, \end{gathered}$$

when the corresponding updated beliefs are $\mu[H|p_1 = p_1^L] = \rho$ and $\mu[L|p_1 = p_1^L] = 1-\rho$. However, $p_2 = V_2$ when the updated beliefs are $\mu[H|p_1 = p_1^H] = 0$, $\mu[L|p_1 = p_1^H] = 1$.

In period 1, the seller announces

$$p_1 = \begin{cases} p_1^H \textit{ if } p_1^H\{1-(1-\rho)^2\} + \delta V_2(1-\rho)^2(1-q_2^2) > p_1^L \\ p_1^L \textit{ otherwise} \end{cases}, \textit{ where}$$

$$p_1^H = \frac{1}{2-\rho}[V_1(2-\rho-2\delta q_1) - \delta(1-q_1)(1-\rho)(1+q_2)(V_1-V_2)],$$

and $p_1^L = V_2 - 2\delta q_2 V_2$, *when the corresponding prior beliefs are* $\mu[H] = \rho$ *and* $\mu[L] = 1-\rho$.

The "high" type buyer follows the following strategy:

$$In\,period\,1,\ chosen\ action = \begin{cases} \text{Buy in period 1 if } p_1 \leq p_1^H \\ \text{Wait for outside option otherwise} \end{cases}.$$

In the event the "high" type buyer chose to wait for the outside option in period 2, and fails to get it,

$$in\,period\,2,\ chosen\ action = \begin{cases} \text{Buy in period 2 if } p_2 \leq V_1 \\ \text{Not buy otherwise} \end{cases}.$$

The "low" type buyer follows the following strategy:

$$In\,period\,1,\ chosen\ action = \begin{cases} \text{Buy in period 1 if } p_1 \leq p_1^L \\ \text{Wait for outside option otherwise} \end{cases}.$$

In the event the "low" type buyer chose to wait for the outside option in period 2, and fails to get it,

$$in\,period\,2,\ chosen\ action = \begin{cases} \text{Buy in period 2 if } p_2 \leq V_2 \\ \text{Not buy otherwise} \end{cases}.$$

3.2 Comparative Statics

In this section, we discuss cases where certain parameters in the model described above, approach possible limiting values, and analyze the types of possible price paths.

Proposition 2. *(i) With* $q_1, q_2 \to 0$ *the seller sets* $p_1 = p_1^H = V_1$ *when* $\rho \to 1$ *and chooses* $p_1 = p_1^L = V_2$ *with* $\rho \to 0$. *(ii) With* $\rho \to 1$ *if* $V_1(1-2\delta) > V_2$, *the seller charges* $p_1 = p_1^H = V_1(1-2\delta q_1)\ \forall q_1, q_2 \in (0,1)$. *However, if* $V_1(1-2\delta) \leq V_2$, *then the seller charges* $p_1 = p_1^H = V_1(1-2\delta q_1)$ *if* $V_1 > V_2 + 2\delta(q_1V_1 - q_2V_2)$ *and* $p_1 = p_1^L = V_2(1-2\delta q_2)$ *otherwise.*

Proof. (i) We consider the case where both types of buyers are almost sure of entering the market in period 2, given that they chose not to purchase the product in period 1, since the probability with which they get the outside option in the second period approaches zero, i.e., $q_1 \to 0$ (and hence $q_2 \to 0$). From equation (4) we find that with $q_1, q_2 \to 0$, the value of $p_1^H = (1-\theta)V_1 + \theta V_2$, where $\theta = \dfrac{\delta(1-\rho)}{2-\rho} < 1$, such that $V_2 < p_1^H < V_1$. We also get $p_1^L = V_2$. Using the decision rule specified by equation (6), the required condition for $p_1 = p_1^H$ is

$$\left[V_1\left\{1-\frac{\delta(1-\rho)}{2-\rho}\right\}+\frac{\delta(1-\rho)}{2-\rho}V_2\right]\{1-(1-\rho)^2\}+\delta V_2(1-\rho)^2(1-q_2^2) \geq V_2$$

As $\rho \to 1$, this inequality is satisfied, such that $p_1 = p_1^H = V_1$, while as $\rho \to 0$, the above inequality is not satisfied, such that $p_1 = p_1^L = V_2$. Price in period 2 is $p_2 = V_2$ if the product remains unsold in period 1.

(ii) Assuming that the prior probability that a buyer if of "high" type approaches one, i.e., $\rho \to 1$, the seller could either charge $p_1 = p_1^H = V_1(1-2\delta q_1)$ or $p_1 = p_1^L = V_2(1-2\delta q_2)$. Using the decision rule (6), we get $p_1 = p_1^H$ if

$$V_1(1-2\delta q_1) > V_2(1-2\delta q_2) \implies V_1 > V_2 + 2\delta(q_1V_1 - q_2V_2)$$

and $p_1 = p_1^L$ otherwise. Since $q_1, q_2 \in (0,1)$, $V_1(1-2\delta q_1) \in (V_1(1-2\delta), V_1)$ and $V_2(1-2\delta q_2) \in (V_2(1-2\delta), V_2)$, such that if we assume that $V_1(1-2\delta) > V_2$, the above inequality will hold $\forall q_1, q_2 \implies p_1 = p_1^H = V_1(1-2\delta q_1)$. If $V_1(1-2\delta) \leq V_2$, then the seller charges $p_1 = p_1^H = V_1(1-2\delta q_1)$ if $V_1 > V_2 + 2\delta(q_1V_1 - q_2V_2)$ and p_1^L otherwise. Price in period 2 is $p_2 = V_2$ if the product remains unsold in period 1. $\square$

We should note that if $V_2 \geq V_1(1-2\delta q_1) > V_2(1-2\delta q_2) = p_1^L$, then with $p_1 = p_1^H = V_1(1-2\delta q_1)$, even though a low valuation buyer can afford to purchase the product, he chooses to wait for the outside option in period 2. Further, from equation (4) we get

$$\frac{\partial p_1^H}{\partial \rho} = -\frac{2\delta q_1 V_1}{(2-\rho)^2} + \frac{\delta(1-q_1)(1+q_2)(V_1-V_2)}{(2-\rho)^2} \gtreqless 0 \tag{8}$$

if $q_1 \lesseqgtr \dfrac{V_1 - V_2}{3V_1 - V_2}$, and assuming that $q_2 \to 0$.

Proposition 3. *With $q_1 \to 1$, $p_1^H = V_1\left(1 - \dfrac{2\delta}{2-\rho}\right)$ which is strictly less than $p_1^H = V_1\left[1-\dfrac{\delta(1-\rho)}{2-\rho}\right]+\dfrac{\delta(1-\rho)}{2-\rho}V_2$ when $q_1 \to 0$. With $q_1 \to 1$, seller chooses*

$p_1 = p_1^H = V_1\left(1 - \frac{2\delta}{2-\rho}\right)$ *if* $V_1\left(1 - \frac{2\delta}{2-\rho}\right)[1-(1-\rho)^2] + \delta V_2(1-\rho)^2(1-q_2^2) \geq V_2(1-2\delta q_2)$.

Proof. Assume to the contrary that $V_1\left(1 - \frac{2\delta}{2-\rho}\right) \geq V_1\left[1 - \frac{\delta(1-\rho)}{2-\rho}\right] + \frac{\delta(1-\rho)}{2-\rho}V_2 \implies -V_1(1+\rho) \geq V_2(1-\rho)$ which is a contradiction since $\rho > 0$. Rest of the proof is identical to the proof of proposition 2. □

These cases show that for different values of the parameters in the above model, it is possible to yield a price path which might be falling, rising or is horizontal over the two periods. For example, if $q_1, q_2 \to 0$ and $\rho \to 1$, then p_1^H is close to V_1. In the unlikely event that both buyers are of "low" type, the seller is unable to sell the unit in period 1 and announces a price $p_2 = V_2$. This explains why at times, airlines offer *last-minute discounts*[10]. Prices also fall if $\rho \to 1$ and if $V_1(1-2\delta) > V_2$ such that $p_1 = p_1^H = V_1(1-2\delta q_1)$ and the seller is unable to sell the unit in period 1 and announces $p_2 = V_2$. The price path could be horizontal in case $V_1(1-2\delta q_1) = V_2$ and rising if $V_2 > V_1(1-2\delta q_1)$.

4 Model with Two Sellers

We now develop a two-period model with two sellers, each of whom faces a capacity constraint and four buyers, each of whom might be one of two types. The outside option for each buyer in this case is the availability of the same good from another seller.

We once again assume that there are two periods and that the product has a lifetime of two periods, after which it is no longer available. There are two sellers in the market, $s = 1, 2$ and four buyers, $m = 1, 2, 3, 4$. Each seller has a single unit of the product for sale. We assume that the market consists of two groups, $(g = 1, 2)$ as follows. Group 1 consists of seller $s = 1$ and buyers $m = 1, 2$, while group 2 consists of seller $s = 2$ and buyers $m = 3, 4$. As in section 3, each of these buyers might be one of two types, "high" or "low", such that a buyer of "high" type has valuation V_1 and a buyer of "low" type has valuation V_2, with $V_1 > V_2 > 0$. We further assume that the probability that each buyer in group 1 is of "high" type is $\rho \in (0, 1)$, while the corresponding probability for group 2

[10]Su (2007) has a similar result.

is $\rho' \in (0, 1)$. Firms often use various signals to "identify" buyers with a higher willingness to pay. For example, in the case of airline tickets, buyers who do not choose *flexible dates* or *weekend layoffs* might be construed as having a higher valuation for the product. Thus tickets which come with these restrictions, could be assumed to attract lower valuation buyers. We use this assumption to show that in equilibrium, *price dispersion* exists. These priors are common knowledge. Each buyer's type is private information. All the agents are assumed to be risk-neutral. The agents are assumed to have a common discount factor $\delta \in (0, 1)$.

In period 1, sellers announce prices in period 1, $p_1 = (p_1^1, p_1^2)$ simultaneously. We assume that only buyers in group 1 approach seller $s = 1$ in order to buy the product, while buyers $m = 3, 4$ approach only seller $s = 2$ in period 1 to make a purchase. Buyers choose either to buy the product or to wait. If both buyers are interested in buying the product, the unit is randomly allocated to one buyer. If the unit is sold, the seller leaves the market, while the buyer who failed to purchase the unit, remains in the market. We assume, that if a buyer is indifferent between buying and waiting, he buys the unit in period 1.

If at least one unit remains unsold, we move to the second period, where one of the following two cases arise. In the first scenario, only the unit of seller $s = \{1, 2\}$ remains unsold. In this case, the seller announces a price p_2. Buyers who are "active", are buyers $m = 1, 2$ ($m = 3, 4$) if $s = 1$ ($s = 2$), as well as the remaining buyer from group $g \neq s$. Once again, buyers choose whether or not to buy the product. The *proportional rationing rule* is used in case there is more than one buyer who wants to purchase the good. In the second scenario, units of both sellers $s = 1, 2$ remains unsold. Sellers announce $p_2 = (p_2^1, p_2^2)$ simultaneously. The buyers might then have access to an *outside option* as follows.

(i) If $p_2^s > p_2^j = V_2$ with $j \neq s$, then all buyers first approach seller j, where j might not belong to the same group as buyer m. If the "high" ("low") type buyer fails to get the unit from seller j, she then approaches seller s, provided $p_2^s \leq V_1$ (V_2). (ii) If $p_2^s = V_2$ $\forall s$, then all buyers can buy from either seller. (iii) If $p_2^s \leq V_1$ $\forall s$, all "high" type buyers can buy from either seller. In case there is only one "high" type buyer, she randomly picks one of the two sellers and purchases the good. In all cases, we assume that the proportional rationing rule is used, such that in case (iii), the probability that a "high" type buyer gets the good is given by $2/h$, where $h \geq 2$ is the number of "high" type buyers.

Thus a "high" type buyer from group 1 might have an outside option in the sense that she can purchase the good from $s = 2$ in period 2 if $p_2^2 = p_2^1 \leq V_1$ or $p_2^2 < p_2^1 \leq V_1$. A "low" type buyer from group 2 on the other hand, might have access to an outside option if $p_2^1 = p_2^2 \leq V_2$ or $p_2^1 < p_2^2 \leq V_2$.

4.1 Perfect Bayesian Equilibrium

We begin by analyzing the strategies of the seller(s) and buyers in period 2. Since period 2 is reached if at least one of the two units is unsold in period 1, we consider strategies for a seller for the following two cases: (i) where the rival seller has sold his unit, and (ii) rival's unit remains unsold.

4.1.1 Rival's Unit is Sold

In the event that the competitor managed to sell his unit, there is no outside option available to the buyers remaining in the market. If we assume that $s = 1$ remains in the market in period 2, the seller announces $p_2 = \{V_1, V_2\}$, with the objective of maximizing expected revenue[11]. We use the following notation to denote updated beliefs (if possible) of the seller in the second period. Given that the product remained unsold in period 1 at price p_1^1, $\mu[H|p_1^1]$ denotes the probability that a buyer belonging to the same group as $s = 1$ is of "high" type and given that the rival's unit was sold at price p_1^2, $\mu[H|p_1^2]$ is the probability that the remaining buyer belonging to group 2 is of "high" type. The seller is thus assumed to be able to distinguish between buyers from his own group and those from the other group[12]. However, the seller is unable to update his prior belief about the type of the buyer from group 2 such that

$$\mu[H|p_1^2] = \rho' \; \forall p_1^2. \tag{9}$$

The probability that the seller can sell the unit from announcing price $p_2 = V_1$ is thus

$$(\mu[H|p_1^1])^2 + 2\mu[H|p_1^1]\mu[L|p_1^1] + \mu[H|p_1^2](\mu[L|p_1^1])^2.$$

This is because, the seller can sell the unit at price V_1 if at least one of the two buyers from his own group is of "high" type, and in the event both the buyers from his own group are of "low" type, the seller can still sell the unit to the buyer from the other group, provided he is of "high" type. The seller thus chooses p_2 as

[11] Costs are assumed to be zero.

[12] We make this assumption mainly for technical reasons. In case the seller is unable to distinguish between buyers from the two groups, the seller would have a common updated belief, such that, each of the remaining three buyers would be of "high" type with probability $\mu[H]$. Sellers like Amazon.com have been known to use "cookies", to identify buyers who visit their online sites for the first time, as well buyers who have made prior visits.

follows:

$$p_2 = \begin{cases} V_1 \text{ if } V_1 \left\{(\mu[H|p_1^1])^2 + 2\mu[H|p_1^1]\mu[L|p_1^1] + \mu[H|p_1^2](\mu[L|p_1^1])^2\right\} \geq V_2 \\ V_2 \text{ otherwise.} \end{cases} \tag{10}$$

If we allow for mixed strategies, the seller could be assumed to choose $p_2 = V_1$ with probability $\alpha \in (0, 1)$.

4.1.2 Rival's Unit Remains Unsold

In case the competitor is unable to sell his unit, all four buyers from period 1 remain in the market and become "active" in period 2. The sellers announce prices simultaneously after updating their beliefs. In this case both $p_2^s \in \{V_1, V_2\}, \forall s$ in case sellers use only pure strategies, and choose V_1 with some probability in case of mixed strategies. Given that the units were unsold in both groups, $s = 1$ updates his belief that each buyer from group 1 is of "high" type with probability $\widehat{\mu}[H|p_1^1]$ and that each buyer from group 2 is of "high" type with probability $\widehat{\mu}[H|p_1^2]$. Similarly, $s = 2$ updates his beliefs that each buyer from group 2 (group 1) is of "high" type with probability $\widetilde{\mu}[H|p_1^2]$ ($\widetilde{\mu}[H|p_1^1]$). Allowing for mixed strategies, we assume that seller 2 chooses $p_2^2 = V_1$ with probability β and $p_2^2 = V_2$ with probability $1 - \beta$. If $p_2^2 = V_1$ the probability that $s = 1$ will be able to sell his unit at price $p_2^1 = V_1$ is

$$\begin{aligned} \Omega = \frac{1}{2}\{2\widehat{\mu}[H|p_1^1]\widehat{\mu}[L|p_1^1](\widehat{\mu}[L|p_1^2])^2 + 2\widehat{\mu}[H|p_1^2]\widehat{\mu}[L|p_1^2](\widehat{\mu}[L|p_1^1])^2\} \\ +\{(\widehat{\mu}[H|p_1^1])^2(\widehat{\mu}[L|p_1^2])^2 + (\widehat{\mu}[L|p_1^1])^2(\widehat{\mu}[H|p_1^2])^2 + \\ (2\widehat{\mu}[H|p_1^1]\widehat{\mu}[L|p_1^1])(2\widehat{\mu}[H|p_1^2]\widehat{\mu}[L|p_1^2])\} + \\ \{(\widehat{\mu}[H|p_1^1])^2 2\widehat{\mu}[H|p_1^2]\widehat{\mu}[L|p_1^2] + (\widehat{\mu}[H|p_1^2])^2 2\widehat{\mu}[H|p_1^1]\widehat{\mu}[L|p_1^1]\} \\ +(\widehat{\mu}[H|p_1^1])^2(\widehat{\mu}[H|p_1^2])^2. \end{aligned} \tag{11}$$

The seller $s = 1$ is unable to sell his unit at price $p_2^1 = V_1$ given that $p_2^2 = V_1$ if either all four buyers from the two groups are of "low" type or only one of the four buyers is of "high" type and this buyer chooses to buy the unit from the other seller. In case there is more than one "high" type buyer in the market, the seller is able to sell the unit for sure at that price. If $p_2^2 = V_2$, the probability that the seller

$s = 1$ will be able to sell the unit at price $p_2^1 = V_1$ is

$$\begin{aligned}\Psi = \frac{1}{2}[(\widehat{\mu}[H|p_1^1])(\widehat{\mu}[H|p_1^2])^2 + \{2\widehat{\mu}[H|p_1^2]\widehat{\mu}[L|p_1^2](\widehat{\mu}[H|p_1^1]) \\ +\widehat{\mu}[L|p_1^1](\widehat{\mu}[H|p_1^2])^2\} + \{\widehat{\mu}[H|p_1^1](\widehat{\mu}[L|p_1^2])^2 \\ +2\widehat{\mu}[H|p_1^2]\widehat{\mu}[L|p_1^2](\widehat{\mu}[L|p_1^1])\}]+ \\ \frac{1}{2}[(\widehat{\mu}[H|p_1^1])^2(\widehat{\mu}[H|p_1^2]) + \{2\widehat{\mu}[H|p_1^1]\widehat{\mu}[L|p_1^1](\widehat{\mu}[H|p_1^2]) \\ +(\widehat{\mu}[H|p_1^1])^2(\widehat{\mu}[L|p_1^2])\} + \{2\widehat{\mu}[H|p_1^1]\widehat{\mu}[L|p_1^1](\widehat{\mu}[L|p_1^2]) \\ +(\widehat{\mu}[L|p_1^1])^2(\widehat{\mu}[H|p_1^2])\}].\end{aligned} \tag{12}$$

In case the rival charges price V_2, all the buyers first approach $s = 2$, where each buyer gets the unit with probability $1/4$. Thus, probability that a group 1 buyer gets a unit at a lower price is $1/2$. In that case, the unit is sold by seller $s = 1$ at price V_1 if either, at least one of the two buyers from the other group is of "high" type or, both the buyers from the other group are of "low" type and the remaining buyer from group 1 is of "high" type. In the other case, a buyer from group 2 gets the outside option (with probability $1/2$). The seller $s = 1$ can then sell the unit at a higher price if either, at least one buyer from group 1 is of "high" type or, both the buyers from group 1 are of "low" type and the remaining buyer from the other group is of "high" type. The seller $s = 1$ chooses p_2^1 as follows:

$$p_2^1 = \begin{cases} V_1 \text{ if } V_1[\beta\Omega + (1-\beta)\Psi] > V_2 \\ V_2 \text{ if } V_1[\beta\Omega + (1-\beta)\Psi] < V_2 \end{cases} \tag{13}$$

and chooses a mixed strategy when the expected revenue from announcing price $p_2^1 = V_1$ is equal to V_2. We can similarly solve for the strategy of seller $s = 2$ in case both the units are unsold or only the unit of seller $s = 2$ remains unsold.

4.1.3 Period 1

In period 1, both the sellers choose a price high enough, which would make a "high" type buyer (if there are any in their respective markets), indifferent between buying the product in period 1 and waiting for a lower price in period 2. Thus seller $s = 1$ announces $p_1^1 = p_1^H > V_2$ and seller $s = 2$ announces $p_1^2 = p_1^{H\prime} > V_2$. In the event that both the units remain unsold, both the sellers update their beliefs that none of the buyers are of "high" type. In case only the unit in group 1 remains unsold, the seller $s = 1$ updates the belief that none of the buyers in his group are of "high" type, and uses equation (9) to update his belief about the type of the remaining buyer from group 2. Similarly, if the unit from group 2 is the only one unsold, $s = 2$ updates his belief that both the buyers in group 2 are "low" type and is unable to update his prior, that the remaining buyer from group 1 has high valuation with probability

$$\overline{\mu}[H|p_1^1] = \rho \, \forall p_1^1.$$

Thus $\widehat{\mu}[H|p_1^1] = \widehat{\mu}[H|p_1^2] = \widetilde{\mu}[H|p_1^2] = \widetilde{\mu}[H|p_1^1] = 0$ if both the units remain unsold. This implies that $\Omega = \Psi = 0,$ such that $s = 1$ chooses $p_2^1 = V_2$ in period 2. We can similarly show that $s = 2$ will also choose $p_2^2 = V_2$. If the unit in group 1 is the only one which remains unsold, $s = 1$ uses the updated beliefs in (10) to announce $p_2 = V_1$ if

$$V_1\mu[H|p_1^2] = V_1\rho' \geq V_2 \tag{14}$$

and V_2 otherwise. With the objective of solving for a unique equilibrium, we consider only pure strategies. Depending on whether or not condition (14) holds, the price charged by seller $s = 1$ will vary as follows.

(1) $V_1\mu[H|p_1^2] < V_2$ and $V_1\overline{\mu}[H|p_1^1] < V_2$.In this case, the only seller in period 2 holds a 'sale' in period 2, such that the price charged in period 1, p_1^H is obtained from the following equation:

$$\begin{aligned}\frac{\rho}{2}(V_1 - p_1^H) + (1-\rho)(V_1 - p_1^H) \;\; &= \;\; \delta\Bigg[\{\rho^{'2} + 2\rho^{'}(1-\rho^{'})\}(V_1 - V_2)\frac{1}{3}(1-\rho) \\ &\quad + (1-\rho^{'})^2(V_1 - V_2)\left(\frac{\rho}{3} + (1-\rho)\frac{1}{2}\right)\Bigg]\end{aligned}$$

$$\implies p_1^H = V_1 - \delta(V_1 - V_2)\left[\frac{\dfrac{N}{3}(1-\rho) + M\left(\dfrac{1}{2} - \dfrac{\rho}{6}\right)}{1-\rho/2}\right] = p_1^{H1} \tag{15}$$

where $N = 2\rho^{'} - \rho^{'2}$ denotes the probability that the unit in group 2 will be sold at price $p_1^{H\prime}$, while $M = (1-\rho^{'})^2$ is the probability that it will remain unsold. Thus price charged by $s = 1$ in period 1 depends not only on the prior belief that a buyer in group 1 is of high valuation, but also on the prior belief that a buyer in group 2 is of "high" type. Irrespective of whether or not $s = 2$ was able to sell his unit, the seller $s = 1$ announces price $p_2^1 = V_2$. However, the probability with which a "high" type buyer is able to purchase the unit in period 2 differs, depending on whether or not $s = 2$ was able to sell his unit. If there is no outside option, and the other buyer in group 1 hasn't already purchased the unit in group 1, the probability of getting the remaining unit is $1/3$. With the outside option available, the probability of getting the outside option is $1/3$ $(1/2)$ if the other buyer from group 1 is "high" ("low") type[13]. From (15) it follows that

$$\frac{\partial p_1^{H1}}{\partial \rho'} = -\delta(V_1 - V_2)\frac{1}{(1-\rho/2)}\left[\frac{1}{3}(1-\rho)\frac{\partial N}{\partial \rho'} + \left(\frac{1}{2} - \frac{\rho}{6}\right)\frac{\partial M}{\partial \rho'}\right] > 0$$

$$\begin{aligned} \text{and } \frac{\partial p_1^{H1}}{\partial \rho} &= -\delta(V_1 - V_2)\frac{1}{(1-\rho/2)^2}\left[\frac{1}{12}(1 - 6\rho' + 3\rho'^2)\right] \\ &\Longrightarrow \frac{\partial p_1^{H1}}{\partial \rho}\begin{cases} > 0 \text{ if } \rho' < 0.18 \\ < 0 \text{ if } \rho' > 0.18 \end{cases}. \end{aligned}$$

The seller could instead choose to offer a price which would make "low" type buyers indifferent between buying and waiting. However, in this case, the best price that a low valuation buyer could hope to get in period 2 is V_2, such that the seller should announce $p_1 = p_1^L = V_2$. The decision rule used by the seller in period 1 is as follows:

$$p_1^1 = \begin{cases} p_1^H \text{ if } p_1^H\{1-(1-\rho)^2\} + \delta(1-\rho)^2 V_2 > V_2 \\ p_1^L \text{ otherwise.} \end{cases} \tag{16}$$

while the decision rule of $s = 2$ is:

$$p_1^2 = \begin{cases} p_1^{H\prime} \text{ if } p_1^{H\prime}\{1-(1-\rho')^2\} + \delta(1-\rho')^2 V_2 > V_2 \\ p_1^L \text{ otherwise.} \end{cases} \tag{17}$$

where $p_1^{H\prime}$ is calculated in a similar manner.

(2) $V_1\mu[H|p_1^2] < V_2$ and $V_1\overline{\mu}[H|p_1^1] \geq V_2$. In this case, while $s = 1$ holds a 'sale' in period 2 if he is the lone seller, $s = 2$ announces $p_2 = V_1$ if he is the only seller in period 2. The price p_1^H is thus obtained from the following equation:

$$\begin{aligned} \frac{\rho}{2}(V_1 - p_1^H) + (1-\rho)(V_1 - p_1^H) &= \delta\Big[\{\rho'^2 + 2\rho'(1-\rho')\}(V_1 - V_2)\frac{1}{3}(1-\rho) \\ &\quad + (1-\rho')^2(V_1 - V_2)(1-\rho)\frac{1}{2}\Big] \end{aligned} \tag{18}$$

[13]This high valuation buyer purchases the good from seller $s = 1$, leaving only the outside option of buying from seller $s = 2$ in period 2.

This implies

$$p_1^H = V_1 - \delta(V_1 - V_2)\left[\frac{(1-\rho)\left(\dfrac{N}{3}+\dfrac{M}{2}\right)}{1-\rho/2}\right] = p_1^{H2} \tag{19}$$

It follows that

$$\frac{\partial p_1^{H2}}{\partial \rho'} = -\delta(V_1 - V_2)\frac{(1-\rho)}{(1-\rho/2)}\left[\frac{1}{3}\frac{\partial N}{\partial \rho'} + \frac{1}{2}\frac{\partial M}{\partial \rho'}\right] > 0$$
$$\text{and } \frac{\partial p_1^{H2}}{\partial \rho} = \delta(V_1 - V_2)\frac{1}{(1-\rho/2)^2}\left(\frac{N}{6}+\frac{M}{4}\right) > 0.$$

The decision rule used by $s = 1$ is the same as in (16), while that of $s = 2$ is:

$$p_1^2 = \begin{cases} p_1^{H\prime} \text{ if } p_1^{H\prime}\{1-(1-\rho^{'})^2\} + \delta(1-\rho^{'})^2[\{1-(1-\rho)^2\}V_1 + (1-\rho)^2V_2] > V_2 \\ p_1^L \text{ otherwise.} \end{cases} \tag{20}$$

(3) $V_1\mu[H|p_1^2] \geq V_2$ and $V_1\overline{\mu}[H|p_1^1] \geq V_2$. In this case, both sellers announce price V_1 if they are the only seller in period 2. The following equation is used to solve for p_1^H :

$$\frac{\rho}{2}(V_1 - p_1^H) + (1-\rho)(V_1 - p_1^H) = \delta(1-\rho^{'})^2\left[(V_1 - V_2)(1-\rho)\frac{1}{2}\right] \tag{21}$$

such that the only case where a high valuation buyer could get a unit at price V_2 in period 2 is if the seller in the other group was unable to sell his unit and the other buyer in the same group is of "low" type. All four buyers in that case, compete for the two units. This implies that

$$p_1^H = V_1 - \delta(V_1 - V_2)\frac{M}{2}\left[\frac{1-\rho}{1-\rho/2}\right] = p_1^{H3} \tag{22}$$

We get,

$$\frac{\partial p_1^{H3}}{\partial \rho'} = -\delta(V_1 - V_2)\frac{1/2(1-\rho)}{(1-\rho/2)}\left[\frac{\partial M}{\partial \rho'}\right] > 0$$
$$\text{and } \frac{\partial p_1^{H3}}{\partial \rho} = \delta(V_1 - V_2)\frac{M}{4\,(1-\rho/2)^2} > 0.$$

The decision rule used by $s = 2$ is the same as (20), while that of $s = 1$ is

$$p_1^1 = \begin{cases} p_1^H \text{ if } p_1^H\{1-(1-\rho)^2\} + \delta(1-\rho)^2[\{1-(1-\rho^{'})^2\}V_1 + (1-\rho^{'})^2 V_2] > V_2 \\ p_1^L \text{ otherwise.} \end{cases} \tag{23}$$

(4) $V_1\mu[H|p_1^2] \geq V_2$ and $V_1\overline{\mu}[H|p_1^1] < V_2$. If $s = 1$ $(s = 2)$ is the lone seller in period 2, he charges $p_2 = V_1$ $(p_2 = V_2)$. p_1^H can be solved as:

$$\frac{\rho}{2}(V_1 - p_1^H) + (1-\rho)(V_1 - p_1^H) = \delta(1-\rho^{'})^2 \left[(V_1 - V_2)\left\{\frac{\rho}{3} + (1-\rho)\frac{1}{2}\right\}\right]$$
$$\implies p_1^H = V_1 - \delta(V_1 - V_2)\frac{M}{2}\left[\frac{1-\rho/3}{1-\rho/2}\right] = p_1^{H4} \tag{24}$$

Thus, $\dfrac{\partial p_1^{H4}}{\partial \rho^{'}} > 0$ and $\dfrac{\partial p_1^{H4}}{\partial \rho} < 0$. The decision rule for $s = 1$ is the same as (23), while that of $s = 2$ is given by (17). An inspection reveals that p_1^H obtained from equation (24) is lower than that obtained from (22). This is due to the higher probability of finding a 'sale' (or outside option) in period 2 by buyers in group 1. Similarly, the p_1^H obtained from equation (15), where $p_2 = V_2$ irrespective of the number of sellers, is smaller than the p_1^H derived from equation (19), which in turn is lower than that from (22). This leads us to the following result.

Result 1. *The price in period 1, p_1^H is highest for the case where $V_1\mu[H|p_1^2] \geq V_2$ and $V_1\overline{\mu}[H|p_1^1] \geq V_2$. Further $p_1^{H3} > p_1^{H2} > p_1^{H1}$ and $p_1^{H3} > p_1^{H4} > p_1^{H1}$.*

Proposition 4. *A* Perfect Bayesian Equilibrium *of the game with two sellers thus consists of the following strategies and beliefs. The seller in group 1 follows decision rule (16) or (23), depending on which of the four cases is relevant, to set $p_1^1 = \{p_1^{H\tau}, p_1^L\}$, $\tau \in \{1, 2, 3, 4\}$ while $s = 2$ uses rule (17) or (20) to announce $p_1^2 = \{p_1^{H\prime}, p_1^L\}$. If both sellers are unable to sell their units in period 1, seller 1 uses decision rule (13), and uses rule (10) if he is the lone seller in the market in period 2. We can define the strategy of $s = 2$ in a similar manner.*

On the equilibrium path, both sellers are able to update their beliefs, such that $\widehat{\mu}[H|p_1^1] = \widehat{\mu}[H|p_1^2] = \widetilde{\mu}[H|p_1^2] = \widetilde{\mu}[H|p_1^1] = 0$. If only $s = 1$ is unable to sell his unit, he updates his belief that both the buyers from his group are of low valuation, and that the probability that the remaining buyer from the other group is of high valuation is given by $\mu[H|p_1^2]$. Similarly, if $s = 2$ is the lone seller in *period 2, he updates his belief that buyers from his own group are of "low" type and that the buyer from group 1 is of high valuation with probability $\overline{\mu}[H|p_1^1]$. However, off the equilibrium path sellers are unable to update their beliefs, such that $\widehat{\mu}[H|p_1^1] = \widetilde{\mu}[H|p_1^1] = \overline{\mu}[H|p_1^1] = \rho$ and $\widehat{\mu}[H|p_1^2] = \widetilde{\mu}[H|p_1^2] = \mu[H|p_1^2] = \rho^{'}$. The "high" type buyers in group 1 (group 2) follow the strategy:*

$$\textit{in period 1, chosen action} = \begin{cases} \textit{Buy in period 1 if } p_1^1 \leq p_1^H \ (p_1^{H\prime}) \\ \textit{Wait otherwise} \end{cases}.$$

and in period 2,

$$\textit{chosen action} = \begin{cases} \textit{Buy in period 1 if } p_1^1 \leq V_1 \\ \textit{Don't Buy otherwise} \end{cases}.$$

Low valuation buyers on the other hand, attempt to purchase a unit as long as price in either period is less than or equal to V_2.

We summarize the possible price paths in the above game for the four different cases in table 1, assuming that it is not the case that both ρ and $\rho^{'} \to 0$, in which case both sellers would announce $p_1^1 = p_1^2 = V_2$, sell their units and leave the market.

Cases	Period 1		Period 2			
			Singe Seller		Multiple Sellers	
	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$
(1)	$p_1^1 = p_1^{H1}$	$p_1^2 = p_1^{H1\prime}$	V_2	V_2	V_2	V_2
(2)	$p_1^1 = p_1^{H2}$	$p_1^2 = p_1^{H2\prime}$	V_2	V_1	V_2	V_2
(3)	$p_1^1 = p_1^{H3}$	$p_1^2 = p_1^{H3\prime}$	V_1	V_1	V_2	V_2
(4)	$p_1^1 = p_1^{H4}$	$p_1^2 = p_1^{H4\prime}$	V_1	V_2	V_2	V_2

Table 1: Prices chosen by the sellers in the two periods on the equilibrium path for the four possible cases. p_1^{H3} is the highest of the four prices and corresponds to the least probability of finding a discount in the second period.

Using the assumption that $\rho \neq \rho^{'}$, we thus show that there exists price dispersion in equilibrium, since $p_1^{H\tau} \neq p_1^{H\tau\prime}$ $\forall \tau$. We also solve for the conditions under which a seller announces a "last-minute discount" in period 2. In case none of the sellers are able to sell their units in period 1, then on the equilibrium path, sellers update their beliefs that none of the buyers have high valuation and both announce price V_2. It is only *off the equilibrium path*, that sellers are unable to update their beliefs and might opt for a *mixed strategy*. On the other hand, if only $s = 2$ is able to sell his unit, whether or not $s = 1$ chooses to announce a 'sale' in period 2, depends on his prior belief $\rho^{'}$. The seller announces $p_2 = V_1$ if $V_1\mu[H|p_1^2] \geq V_2 \Rightarrow \rho^{'} \geq \frac{V_2}{V_1} = \widehat{\rho}$. This cutoff remains the same if $s = 2$ is the only seller in period 2, such that he announces $p_2 = V_1$, if $\rho \geq \widehat{\rho}$. As $(V_1 - V_2) \to 0$, $\widehat{\rho} \to 1$ while as $(V_1 - V_2) \to \infty$, the cutoff $\widehat{\rho} \to 0$.

5 Extensions

While our two-period durable good model with outside option predicts that the price path is either increasing or decreasing or remains horizontal for different parameter values, it would be interesting to see how these results change when a similar model with more than two periods is constructed. We would, in that case, need to allow entry of "new" buyers in each period, for the model to be of significance.

One elementary way to extend our results would be to consider a model with a single seller selling a single unit over T periods, would be to have $2M$ buyers in the market, of which $2N$ are high valuation buyers, where $N < M$. The buyers would be split equally into two groups, such that, each group would have M buyers, with N of them of "high" type. We would then make the following assumptions: (i) two buyers, one from each group would enter the market in each period, till period $T-1$, with $T-1 < N$. (ii) For the sake of simplicity, only "high" type buyers would have access to an outside option in period 2, with probability q_1. The seller would announce a price sequence $\{p_1, p_2, ..., p_T\}$ without commitment, with $p_t = p_t^H$, such that a "high" type buyer entering the market in period t would be indifferent between buying and waiting, and would buy immediately. If the unit is yet to be sold at the beginning of period $t + 1$, the seller would update his belief that t buyers in each group are of "low" type, and that the probability that a buyer from each group is of "high" type is $\mu_{t+1}[H|h_{t+1}] = \frac{N}{M-t}$, with $h_{t+1} = \{p_1, p_2, ..., p_t\}$. If the product remains unsold at the end of $T - 1$ periods, the seller would announce $p_T = V_2$. The price charged by the seller in period t,

p_t^H would be

$$\frac{1}{2}\mu_t(V_1 - p_t^H) + (1-\mu_t)(V_1 - p_t^H) = \delta^{T-t}\left[q_1 V_1 + \right.$$
$$\left.(1-q_1)\left\{\frac{1}{2(T-1)}(1-\mu_t)(V_1-V_2)\right\}\right]$$
$$\Rightarrow p_t^H = \frac{1}{2-\mu_t}\left[V_1(2-\mu_t-2\delta^{T-t}q_1) - \frac{\delta^{T-t}}{T-1}(1-q_1)(1-\mu_t)(V_1-V_2)\right].$$

In the last period, all $2(T-1)$ buyers would compete to purchase the single unit at the 'sale' price. With $\delta \to 1$,

$$p_t^H = V_1 - \frac{1}{2-\mu_t}\left[V_1 q_1 + \frac{1}{T-1}(1-q_1)(1-\mu_t)(V_1-V_2)\right]$$

such that with values of q_1 close to zero (one), the price path would be increasing (decreasing) over time, since μ_t increases as t increases. If price path is increasing over the first $T-1$ periods, a "high" type buyer who enters the market in period t would proceed to purchase the unit immediately, as he would have no incentive to wait till period $T-1$, and would be indifferent between buying in period t and waiting till the last period. A 'sale' in the last period, would once again be interpreted as a "last-minute discount"[14].

6 Conclusion

This paper presents a two-period durable good model in which the seller(s) face a capacity constraint, while buyers get access to an outside option with some probability in the second period. Buyers are assumed to be one of two types, and in the case with a single seller, the buyer with the higher valuation gets the outside option with a higher probability, which provides the object for free. In the duopoly model, buyers get the option of buying the product from a second seller in the second period, provided the second seller was unable to sell his unit in the first period. We solved for the *Perfect Bayesian Equilibrium* in each case, and depending on parameter values found the price path to be increasing, decreasing or horizontal. The price charged in the first period was found to be decreasing as the probability of finding the outside option increased. In case the product remained

[14]While μ_t is endogenously determined in the above example, we could also think of another framework, where the μ_t is exogenously determined and is decreasing or increasing over time.

unsold, the monopoly seller offered a 'sale' in the second period. In the duopoly model, both the sellers offered a 'sale' in case both failed to sell their product in the first period, and for certain parameter values, if only one seller remains in the second period. These 'sales' are interpreted as *last-minute discounts,* as the seller realized that it is unlikely that he would be able to sell the product at a high price in the last period. The result that at times, both the sellers reduce their prices at the same time and to the same level, is similar to that obtained by Sobel (1984). However, unlike Sobel, our model assumes incomplete information and a finite horizon, and that sellers face a capacity constraint.

In the duopoly model, buyers were assumed to belong to two different groups, with the probability that a buyer is of "high" type varying across the groups. We show that there exists *price dispersion* in equilibrium. Assuming that buyers across segments have the same discount factor, we attribute this price dispersion to the difference in probabilities of buyers being of "high" type. In the case of the airline industry, tickets with restrictions such as weekend layoffs or stopovers are designed to attract lower valuation buyers, and could be cited as an example of a product which attracts buyers with lower probability of being "high" type. The conclusion that price dispersion is driven by such differences, finds empirical support in the work of Puller et. al. (2009), who conclude that ticket characteristics explain the bulk of price variation.

While Su (2007) solves the intertemporal pricing problem of a seller facing a capacity constraint over a finite horizon, his model comprises a monopoly seller, and is thus unable to explain price dispersion. Levin et. al. (2009) show that firms lose revenue due to the strategic behavior of buyers in a model which assumes full information. This includes information on remaining capacities of the firms, the number of buyers in the different market segments, and all their distributional and parametric characteristics. This is a fairly strong assumption and is unlikely to hold in most real-world situations.

Buyers in our duopoly model are assumed to be segregated into segments, which are independent in the sense that ticket restrictions attract customers of a certain type and that these buyers are not allowed to cross "fences" and search for lower fares. However, if these buyers are unable to purchase the product in the first period, either due to the capacity constraint or due to high prices, they are free to approach a second seller in the second period. This is a distinguishing feature of our model. While our model is overly simplistic, in the sense that it considers a two-period framework and the simplest possible market structure, it provides intuition for different price paths and for phenomena such as price dispersion and last-minute discounts. We also suggest possible extensions to our model, which we intend to pursue as part of future research.

References

[1] Bitran, G. and Caldentey, R. “An Overview of Pricing Models for Revenue Management”, Manufacturing and Service Operations Management, Summer 2003, 203-229.

[2] Brumelle, S.L. and McGill, J.I. “Airline Seat Allocation with Multiple Nested Fare Classes”, Operations Research, Vol. 41, No. 1, Special Issue on Stochastic and Dynamic Models in Transportation (Jan - Feb, 1993), 127-137.

[3] Coase, Ronald H. “Durability and Monopoly”, Journal of Law and Economics, Vol. 15 (April 1972), 143-149.

[4] Conlisk, John, Gerstner, Eitan and Sobel, Joel “Cyclic Pricing by a Durable Goods Monopolist”, Quarterly Journal of Economics, Vol. 99, Issue 3 (August 1984), 489-505.

[5] Dana, James D. Jr. “Advance-Purchase Discounts and Price Discrimination in Competitive Markets”, Journal of Political Economy, Vol. 106, Issue 2 (April 1998), 345-421.

[6] Gale, Ian and Holmes, Thomas “The Efficiency of Advance-Purchase Discounts in the Presence of Aggregate Demand Uncertainty”, International Journal of Industrial Organization, Vol. 10, Issue 3, September 1992, 413-437.

[7] Jerath, Kinshuk, Netessine, Serguei and Veeraraghavan, Senthil K. “Revenue Management with Strategic Consumers: Last-Minute Selling and Opaque Selling”, Vol. 56, No. 3, March 2010, 430-448.

[8] Levin, Yuri, McGill, Jeff and Nediak, Mikhail “Dynamic Pricing in the Presence of Strategic Consumers and Oligopolistic Competition”, Management Science, Vol. 55, No. 1, January 2009, 32-46.

[9] McAfee, Preston R. and Velde, te Vera “Dynamic Pricing in the Airline Industry”, Handbook of Economics and Information Systems, edited by T.J. Hendershott, Elsevier Handbooks in Information Systems, Volume 1.

[10] Ovchinnikov, Anton and Milner, Joseph M. "Revenue Management with End-of-Period Discounts in the Presence of Customer Learning", (January, 30 2009). Available at SSRN: http://ssrn.com/abstract=1335478

[11] Puller, Steven L., Sengupta, Anirban and Wiggins, Steven N. "Testing Theories of Scarcity Pricing In the Airline Industry", NBER Working Paper No. 15555, December 2009.

[12] Sobel, Joel "The Timing of Sales", Review of Economic Studies, Vol. 51, Issue 3 (July 1984), 353-368.

[13] Stavins, Joanna "Price Discrimination in the Airline Market: The Effect of Market Concentration", Review of Economics and Statistics, Vol. 83, No. 1 (February 2001).

[14] Su, Xuanming "Intertemporal Pricing with Strategic Consumer Behavior", Management Science, Vol. 53, No. 5, May 2007, 726-741.

[15] Wollmer, Richard D. "An Airline Seat Management Model for a Single Leg Route When Lower Fare Classes Book First", Operations Research, Vol. 40, No. 1, (Jan - Feb, 1992), 26-37.

Appendix A.

Probability of event *NBO*.

$$
\begin{aligned}
\Pi = \Pr\left[\bigcap_i \omega_i = 0\right] &= \prod_i \Pr[\{(\theta_i = \theta_H) \cup (\theta_i = \theta_L)\} \cap (\omega_i = 0)] \\
&= \prod_i \Pr[(\theta_i = \theta_H) \cap (\omega_i = 0) \cup (\theta_i = \theta_L) \cap (\omega_i = 0)] \\
= \prod_i [\Pr(\omega_i = 0|\theta_i = \theta_H) &\Pr(\theta_i = \theta_H) + \Pr(\omega_i = 0|\theta_i = \theta_L) \Pr(\theta_i = \theta_L)] \\
\Rightarrow \Pi = (\mu[H|p_1])^2(1-q_1)^2 &+ (\mu[L|p_1])^2(1-q_2)^2 + 2(\mu[H|p_1])(\mu[L|p_1]) \\
&(1-q_1)(1-q_2). \qquad (1)
\end{aligned}
$$

Appendix B.

Probability of event *ONE*.

Case 2. Only one buyer gets the outside option. We define this event as *ONE*. The corresponding probability is

$$
\begin{aligned}
\Phi &= \Pr[(\omega_i = 0 \cap \omega_j = 1) \cup (\omega_i = 1 \cap \omega_j = 0)] \\
&= \Pr(\omega_i = 0 \cap \omega_j = 1) + \Pr(\omega_i = 1 \cap \omega_j = 0) \\
&= \Pr(\omega_i = 0) \Pr(\omega_j = 1) + \Pr(\omega_i = 1) \Pr(\omega_j = 0).
\end{aligned}
$$

Now,

$$
\begin{aligned}
\Pr(\omega_i = 0) &= \Pr[\{(\theta_i = \theta_H) \cup (\theta_i = \theta_L)\} \cap (\omega_i = 0)] \\
&= \Pr[\{(\theta_i = \theta_H) \cap (\omega_i = 0)\} \cup \{(\theta_i = \theta_L) \cap (\omega_i = 0)\}] \\
&= \Pr[(\theta_i = \theta_H) \cap (\omega_i = 0)] + \Pr[(\theta_i = \theta_L) \cap (\omega_i = 0)] \\
&= \Pr[\omega_i = 0|\theta_i = \theta_H] \Pr(\theta_i = \theta_H) + \Pr[\omega_i = 0|\theta_i = \theta_L] \Pr(\theta_i = \theta_L) \\
&= (1-q_1)\mu(H|p_1) + (1-q_2)\mu(L|p_1).
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
\Pr(\omega_j = 1) &= \Pr[\{(\theta_j = \theta_H) \cup (\theta_j = \theta_L)\} \cap (\omega_j = 1)] \\
&= q_1\mu(H|p_1) + q_2\mu(L|p_1).
\end{aligned}
$$

Substituting,
$\Pr(\omega_i = 0)\Pr(\omega_j = 1) = (1-q_1)q_1(\mu(H|p_1))^2 + (1-q_2)q_2(\mu(L|p_1))^2$
$+(1-q_2)q_1\mu(H|p_1)\mu(L|p_1) + (1-q_1)q_2\mu(H|p_1)\mu(L|p_1).$
Thus,

$$\begin{aligned}\Phi &= 2(1-q_1)q_1(\mu(H|p_1))^2 + 2(1-q_2)q_2(\mu(L|p_1))^2 \\ &\quad + 2(1-q_2)q_1\mu(H|p_1)\mu(L|p_1) + 2(1-q_1)q_2\mu(H|p_1)\mu(L|p_1).\end{aligned} \tag{2}$$

Appendix C.

The seller manages to sell the unit at price V_1 if the buyer without the outside option is of "high" type. Probability that the seller sells the unit at price $p_2 = V_1$ is

$$\begin{aligned}&\Pr[\{(\theta_i = \theta_H \cap \theta_j = \theta_H) \cup (\theta_i = \theta_H \cap \theta_j = \theta_L)\} \cap (\omega_i = 0 \cap \omega_j = 1)] \\ &+ \Pr[\{(\theta_i = \theta_H \cap \theta_j = \theta_H) \cup (\theta_i = \theta_L \cap \theta_j = \theta_H)\} \cap (\omega_i = 1 \cap \omega_j = 0)]\end{aligned}$$

$= \Pr[\{(\theta_i = \theta_H \cap \theta_j = \theta_H) \cap (\omega_i = 0 \cap \omega_j = 1)\} \cup \{(\theta_i = \theta_H \cap \theta_j = \theta_L) \cap (\omega_i = 0 \cap \omega_j = 1)\}] + \Pr[\{(\theta_i = \theta_H \cap \theta_j = \theta_H) \cap (\omega_i = 1 \cap \omega_j = 0)\} \cup \{(\theta_i = \theta_L \cap \theta_j = \theta_H) \cap (\omega_i = 1 \cap \omega_j = 0)\}]$

$$\begin{aligned}&= \Pr[(\theta_i = \theta_H \cap \omega_i = 0) \cap (\theta_j = \theta_H \cap \omega_j = 1)] \\ &\qquad + \Pr[(\theta_i = \theta_H \cap \omega_i = 0) \cap (\theta_j = \theta_L \cap \omega_j = 1)] \\ &+ \Pr[(\theta_i = \theta_H \cap \omega_i = 1) \cap (\theta_j = \theta_H \cap \omega_j = 0)] \\ &\qquad + \Pr[(\theta_i = \theta_L \cap \omega_i = 1) \cap (\theta_j = \theta_H \cap \omega_j = 0)]\end{aligned}$$

$$\begin{aligned}&= \Pr[\theta_i = \theta_H|\omega_i = 0]\Pr(\omega_i = 0)\Pr[\theta_j = \theta_H|\omega_j = 1]\Pr(\omega_j = 1) \\ &\qquad + \Pr[\theta_i = \theta_H|\omega_i = 0]\Pr(\omega_i = 0)\Pr[\theta_j = \theta_L|\omega_j = 1]\Pr(\omega_j = 1) \\ &+ \Pr[\theta_i = \theta_H|\omega_i = 1]\Pr(\omega_i = 1)\Pr[\theta_j = \theta_H|\omega_j = 0]\Pr(\omega_j = 0) \\ &\qquad + \Pr[\theta_i = \theta_L|\omega_i = 1]\Pr(\omega_i = 1)\Pr[\theta_j = \theta_H|\omega_j = 0]\Pr(\omega_j = 0)\end{aligned}$$

$= \Pr(\theta_i = \theta_H|\omega_i = 0)\{\Pr(\theta_j = \theta_H|\omega_j = 1) + \Pr(\theta_j = \theta_L|\omega_j = 1)\}\Pr(\omega_i = 0)\Pr(\omega_j = 1) + \Pr(\theta_j = \theta_H|\omega_j = 0)\{\Pr(\theta_i = \theta_H|\omega_i = 1) + \Pr(\theta_i = \theta_L|\omega_i = 1)\}\Pr(\omega_i = 1)\Pr(\omega_j = 0)$

$$\begin{aligned}&= \Pr(\theta_i = \theta_H|\omega_i = 0)\{\Pr(\omega_i = 0)\Pr(\omega_j = 1) + \Pr(\omega_i = 1)\Pr(\omega_j = 0)\} \\ &= \Pr[\theta_i = \theta_H|\omega_i = 0]\Phi.\end{aligned}$$

Appendix D.

In case the seller is unable to distinguish as to which group a particular buyer came from in period 2, he would be able to sell the unit at price V_1 if there is one buyer who is of "high" type on the equilibrium path, and at least one buyer of "high" type off the equilibrium path. Of the three buyers $h = 1, 2, 3$, $\Pr ob(h \in g = 2) = 1/3 \; \forall h$ such that on the equilibrium path $s = 1$ updates his belief that a buyer h is of "high" type with probability

$$\begin{gathered}\varphi = \Pr ob[\{(h \in g = 1) \cup (h \in g = 2)\} \cap (\theta_h = \theta_H)] \\ = \Pr ob[\{(h \in g = 1) \cap (\theta_h = \theta_H)\} \cup \{(h \in g = 2)) \cap (\theta_h = \theta_H)\}] \\ = \Pr ob[(h \in g = 2) \cap (\theta_h = \theta_H)] = \mu[H|p_1^2]/3\end{gathered}$$

where $\theta_h = \theta_H$ denotes that buyer h is of high valuation. This is because $\Pr ob[\Pr ob[(h \in g = 2) \cap (\theta_h = \theta_H)] = 0$. Given that on the equilibrium path there can be at most one "high" type buyer, probability that there is one such buyer is

$$\begin{aligned}\chi &= \Pr ob[\theta_1 = \theta_H \cup \theta_2 = \theta_H \cup \theta_3 = \theta_H] \\ &= (\mu[H|p_1^2]/3) \times 1 \times 1 + (2/3)(\mu[H|p_1^2]/2)1 + (2/3)(1/2)(\mu[H|p_1^2]) \\ &= \mu[H|p_1^2]\end{aligned}$$

Thus, on the equilibrium path $s = 1$ announces $p_2 = V_1$ if $V_1\mu[H|p_1^2] \geq V_2$ and V_2 otherwise, which is the same decision rule as the case where the seller is able to identify which buyer came from which group.