Inequality, Neighbourhoods and Welfare of Poor*

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Abstract

The key idea explored in this paper is that private establishments take both the location and income mix of people into account while making strategic decisions like entry and the price and quality of their products and services. We develop a model that integrates consumers' income distribution with the spatial distribution of their location and look at the consequence of an increase in income inequality on the welfare of the poor. We find an inverted-U shaped relationship between income inequality and the welfare of the poor: if we compare a cross-section of societies, the poor community as a whole is initially better-off living in relatively richer societies, but, beyond a point, the aggregate consumer surplus of the poor starts declining as the society becomes richer. Interestingly the same inverted-U shaped relationship is also observed between income inequality and market access of the poor. There exist multiple equilibria: a bad equilibrium where all the poor are excluded exists simultaneously with a good equilibrium where at least some poor (if not all of them) get served by the market. We have isolated the higher income gap between the rich and poor as the key factor that exposes the poor to this complete exclusion possibility. Finally we compare a mixed-income economy where rich and poor live side by side with a single-income economy inhabited only by a single income group and show that poor are better-off staying in the mixed-income economy as long as the poor income is below a feasibility threshold.

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1 Introduction

The key idea explored in this paper is the following: though being poor in itself is a huge disadvantage, the situation might be influenced considerably by the type of neighbourhood the poor lives in as private establishments like educational institutions, health care facilities or credit institutions take both the location and income mix of people into account while making strategic decisions like whether to enter into the neighbourhood at all, and, upon entry, what price and quality to choose for their products and services. Is staying with the rich, a virtue for the poor or a source of resentment? Would they prefer to stay in poor neighborhoods because living in an affluent one costs too much? Or does living in a poor neighborhood make poor people significantly poorer as they do not even have access to many basic facilities? These are the kinds of questions we are interested in exploring in this paper.

A casual walk across the city streets of any developing country might suffice to illustrate the idea. While moving across poor neighborhoods one comes across many roadside vendors selling tea, providing barber services and so on from shops requiring minimal physical investment and, understandably, the quality on offer is quite basic. As one moves into relatively richer neighbourhoods, one is bound to come across more sophisticated counterparts of the same products and services: road side vendors are replaced by air-conditioned cafes, shining beauty salons and so on. Similar products and services could become heavily capital intensive and highly specialized in nature depending on the income mix of the neighbourhood. Changes in price of course reflect these changes in quality level.

Given these differences in price and quality, which neighbourhood does an individual prefer to be in? Answer to this depends not just on the cost relative to income, but also on the ease of access of the facilities. This is because certain goods and services are required at regular intervals so that distance becomes an important factor. In the less developed countries distance from schools is an important factor leading to high drop-out rates. Similarly distance from the nearby health care facility is a major reason resulting in higher mortality of both mother and child during child birth in rural areas of developing countries. How readily a product or service is available is thus determined by the neighborhood an individual lives in. So it is the interaction of the two, the individual's income and his postcode, that determines his welfare.

There is a substantial body of evidence showing how neighborhood poverty affects poor people's ability to access facilities such as health care and schooling. Consider health care first. Montgomery et al. (2005) find that both household and neighborhood living standards can make a significantly important difference to health. They report striking differentials in health depending on the region: poor city dwellers often face health risks that are nearly as bad as what is seen in the countryside, and sometimes the risks are decidedly worse. For instance, they find that in the slums of Nairobi, rates of child mortality substantially exceed those found elsewhere in Nairobi, and are high enough even to exceed rural rates of mortality. Wilson (1987) argues that concentrated poverty leaves neighborhoods without the middle-class capital to support strong local organizations. Thus there is less investment in the health care resources. Similarly according to the October 31, 2006 news release from the Stanford University School of Medicine, death rates are highest among people of low socioeconomic status who also lived in affluent neighborhoods. In India Das and Hammer (2005) have found that doctors located in the poorest neighborhoods are one full standard deviation worse than doctors located in the richest neighborhoods of Delhi. Also, in India the ratio of hospital beds to population in rural areas is fifteen times lower than that for urban areas, and the ratio of doctors to population in rural areas is almost six times lower than that in the urban population (Deogaonkar, 2004). As per the records of the Ministry of Health and Family Welfare, Government of India, a total of 74% of the graduate doctors live in urban areas, serving only 28% of the national population, while the rural population remains largely unserved. Although the health care facilities are overwhelmingly concentrated in urban areas, the economic distance, which includes cost of health care, prevents access for the urban poor (Deogaonkar, 2004). While the rural poor are underserved, at least they can access the limited number of government-supported medical facilities that are available to them. The urban poor fares even worse because they cannot afford to visit the private facilities that thrive in India's cities (Price Water House Coopers, 2007).

Neighborhood determines not just the quality and price of the service but also its ease of access. In a survey on rural Rajasthan, Banerjee and Duflo (2009) distinguished between three broad categories of health care facilities: public, private and traditional, and even within traditional and private practitioners they find huge variation in the level of qualification. This difference in qualification was replicated in the cost of an average visit to these facilities. The mean and median distance to the traditional healer is much lower than the distance to the closest private facility, implying convenient accessibility of relatively poor and cheap quality services.

Similarly on education a study by the Education Policy and Data Center (2008) has found that the net school attendance rate variation is higher in the rural areas, and, in about half of the countries of the world, the net attendance rate is strongly negatively correlated with the relative poverty rate. This is because higher prevalence of poor households has spillover effects on the sub-national region, leading to, for example, fewer common resources for school. For India Agrawal (2010) reports that educational facilities are distributed unequally between rural and urban areas. In rural areas, students suffer from scarcity and inadequate accessibility of schools (as well as poor quality of education) which also forces them to travel larger distances. Tilak and Sudarshan (2001) has found statistically significant, strong and inverse relationship between levels of educational attainment and levels of poverty in Indian households. Poor households are unable to get good quality education.

Although the evidence is compelling, there seems to be very little analytical research to understand how neighbourhood effects interacting with income inequality affect poor people's ability to access basic facilities like health care services, schooling, and so on. This paper makes an early attempt to model this interaction by integrating consumers' income distribution with the spatial distribution of their location and explores the consequences of an increase in income inequality on the welfare of the poor in general, and their access to market in particular.

We consider a homogeneous product in a competitive framework with free entry and exit. It is very interesting to investigate the interaction of inequality and neighbourhood effect in such an ideal market structure. The preference structure reflects the higher willingness to pay of the richer consumers and the consumers' reluctance to travel farther to access the product or service under consideration. The industrial structure is characterized by the presence of a fixed cost of production. The inequality-neighbourhood interaction is captured by the following spatial structure: we consider a circular city across which the consumers are identically distributed with rich and poor consumers living side by side. The set-up is a two-stage game. In the first stage, firms decide whether to enter or not, and the entering firms locate equidistantly around the circumference of the circular city. In the second stage, firms choose their prices simultaneously. In this set-up we explore the interaction of income inequality with the neighbourhood effect in determining the market outcomes and its consequences on the market access and welfare of the poor.

We find an inverted-U shaped relationship between income inequality and the welfare of the poor: if we compare a cross-section of societies, the poor community as a whole is initially better-off living in relatively richer societies, but, beyond a point, the aggregate consumer surplus of the poor starts declining as the society becomes richer. Interestingly the same inverted-U shaped relationship is also observed between income inequality and market access of the poor. The reason for this inverted-U shaped relationship can be traced to the opposing welfare impacts of income inequality working through equilibrium price and number of firms.

Consumers benefit from the increase in number of firms but lose from the increase in price. In the model when product quality is exogenously given, both price and number of firms increase steadily as the neigbourhood of the poor becomes richer. For the poor community as a whole the number of firms effect dominates initially: the poor located closer to the firms get to consume the product and hence the market access of the poor increases as the number of firms increases. But, beyond a point, the adverse price effect takes over. The study by Li and Zhu (2006) lends strong empirical support to our theoretical results. They find an inverted-U association between self-reported health status and inequality using individual data from the China Health and Nutrition Survey (CHNS).

We find that the nature of equilibrium depends on two income thresholds of the poor. We identify an *upper income threshold* for the poor income such that all poor consumers get served by the market only if the poor income is above this upper income threshold. On the other hand there exists a *lower income threshold* for the poor income such that no poor consumer is served if the poor income is below this lower threshold. When the poor income is in between the upper and lower income thresholds, there are pockets of the city where the poor are left out of the market: only those poor who are located closer to the firms get served, others get excluded. The size of these exclusion pockets increases as the poor income decreases.

We have also identified the possibility of *multiple equilibria*. There exists a whole range of parameter values such that a bad equilibrium and a good equilibrium exist side by side for the same parameter configurations. Under the good equilibrium at least some poor (if not all of them) gets served, those who are located closer to the firms. Whereas under the bad equilibrium all the poor are excluded; the firms completely ignore their presence and choose the price and quality as if there were only rich individuals residing in the city. We have isolated the higher income gap between the rich and poor as the key factor that exposes the poor to the complete exclusion possibility. We have also found that poor are more likely to be completely excluded when they are a minority: firms may completely ignore the poor even when the rich are not ultra rich just because the rich are more in number.

Finally we compare a mixed-income economy where rich and poor live side by side with a single-income economy inhabited only by a single income group. We have identified a *feasibility income threshold* in a single-income economy such that it is not feasible for any firm to operate if the common income is below this feasibility threshold. Comparing mixed versus single-income economies we show that poor are better-off staying in the mixed-income economy as long as the poor income is below this feasibility threshold. At least some poor get to enjoy the product or service in the mixed-income economy as the firms recover their fixed costs due to the higher willingness to pay of the rich. This is not possible in a single-income poor economy.

The idea that people with higher income generally have higher valuation for services like health, education, or credit, and so have higher willingness to spend on them, and that firms do take this into account while making strategic decisions was first developed by Gabszewicz and Thisse (1979) in the vertical differentiation models introduced by them. This strand of the literature has been developed and extended further by Shaked and Sutton (1982, 1983, 1987).[1] In our model we allow consumers to differ with respect to both their income and location. The basic horizontal product differentiation model was introduced by Hotelling (1929) and was later developed by d'Aspremont et al. (1979) and Salop (1979). The literature on industrial organization that follows these seminal works (for example, Economides, 1989, 1993; Neven and Thisse, 1990; Degryse, 1996) looks at product specifications combining both the vertical and horizontal characteristics. But the industrial organization literature typically does not allow for the possibility of exclusion. Atkinson (1995) is the only work that we are aware of that looks at the possibility of non-consumption arising out of income gap in the context of determination of poverty and capability by firm behaviour and industrial structure. But people even at the same level of income might not consume because of higher distance, and this is especially relevant for services like health, education, or credit. This is the feature we would like to highlight in our work.

The paper is organized as follows. Section 2 outlines the model with the spatial structure capturing the inequality-neighbourhood interaction. Section 3 analyzes the generic scenario where the poor has partial market access while the rich has complete access. The effect of inequality on market access and welfare of the poor is investigated in section 4. In section 5 we characterize all the equilibrium possibilities highlighting the role of inequality in generating the possibility of multiple equilibria. The comparison with the single-income economy is also discussed in this section. Finally we conclude in section 6.

2 The Model

To model the interaction of neighbourhood effects with income inequality in a simple and tractable way, we adapt the framework of Salop (1979). We consider a circular city of circum-

[1] Relatively recently Benassi et al. (2006) analyze the effect of income concentration on product differentiation based on a given income distribution while Yurko (2009) furthers this research by considering more general specification of income distribution function.

ference L units where two types of consumers – rich and poor – are identically distributed across the city. At each point on the circumference of the city there are f_R proportion of rich with income Y_R and f_P proportion of poor with income Y_P. Obviously $Y_R > Y_P$, and $f_R + f_P = 1$.

There are n private establishments in the city providing a homogeneous product or service. Examples of such establishments are private schools, hospitals, banks, and so on. For the sake of brevity let us refer to them as firms. These n firms are located equidistant to each other around the circle so that the distance between adjacent firms is $\frac{L}{n}$. The number of firms is not fixed; it is determined endogenously from free entry and exit condition.

Each consumer buys either one unit of the homogeneous product from his most preferred firm, or does not buy the product at all. Let θY be the *gross* utility a consumer with income Y enjoys from consuming the product. Here $\theta > 1$ is a preference parameter indicating valuation of the product. Since $\theta Y_R > \theta Y_P$, this formulation of gross utility captures the feature that willingness to pay is higher for the rich. Let us use the notations x_j for location of firm j and p_j for the price it charges, $j = 1, 2, ..., n$. A consumer at location z has to travel a distance $|x_j - z|$ to access the product or service from firm j and he incurs a travel or transportation cost of $t\,|x_j - z|$. Of course he has to pay the price p_j. Hence the *net* utility of a consumer at location z with income Y and purchasing from firm j is given by

$$U(z, Y, j) = \theta Y - p_j - t\,|x_j - z|\,.$$

If a consumer does not buy the product, he still has his income Y to spend on other goods and services implying that his reservation utility is Y.

Production requires fixed costs; in order to produce any output at all, each firm must incur a fixed cost F. Further, there is a marginal cost of production, c, which is independent of output. Profit of firm j charging a price p_j is then given by

$$\pi_j = [p_j - c]D_j - F,$$

where D_j denotes demand faced by firm j. Given the spatial structure, we elaborate in the next subsection how D_j depends on firm j's own price, p_j, and on the prices of the two adjacent firms, p_{j-1} and p_{j+1}.

The set-up is a two-stage game. In the first stage, firms decide whether to enter or not, and the entering firms locate equidistantly around the circumference of the circular city. In the second stage, firms choose their prices simultaneously.

2.1 Demand Structure

Consider firm j located between the two adjacent firms $j-1$ and $j+1$. Let $\delta_{j,j+1}$ denote the distance from firm j of the marginal consumer with income Y who is indifferent between firms j and $j+1$, that is, $U\left(x_j+\delta_{j,j+1}, Y, j\right)=U\left(x_j+\delta_{j,j+1}, Y, j+1\right)$. It follows that

$$\delta_{j,j+1}=\frac{1}{2t}\left[\left(p_{j+1}-p_j\right)+t\left(\frac{L}{n}\right)\right].$$

Utility of this marginal consumer is $\frac{1}{2}\left[2\theta Y-\left(p_{j+1}+p_j\right)-t\left(\frac{L}{n}\right)\right]$.

Let $\overline{Y}_{j,j+1}$ denote the income level such that the consumer with income $\overline{Y}_{j,j+1}$ who is indifferent between firms j and $j+1$ at a distance $\delta_{j,j+1}$ is also indifferent between buying and not buying, that is, $U\left(x_j+\delta_{j,j+1}, \overline{Y}_{j,j+1}, j\right)=\overline{Y}_{j,j+1}$. It follows that

$$\overline{Y}_{j,j+1}=\frac{\left(p_{j+1}+p_j\right)+t\left(\frac{L}{n}\right)}{2\left(\theta-1\right)}.$$

Clearly $U\left(x_j+\delta_{j,j+1}, Y, j\right)\geq Y$ for all $Y\geq\overline{Y}_{j,j+1}$, and the marginal consumer with income Y (at a distance $\delta_{j,j+1}$ from firm j) will buy from firm j. But $U\left(x_j+\delta_{j,j+1}, Y, j\right)<Y$ for all $Y<\overline{Y}_{j,j+1}$, and the marginal consumer with income Y (at a distance $\delta_{j,j+1}$ from firm j) will *not* buy from firm j. The implication for demand is that for all $Y\geq\overline{Y}_{j,j+1}$, the measure of consumers located between x_j and x_{j+1} and buying from firm j is $\delta_{j,j+1}$.

Now consider the consumers with income $Y<\overline{Y}_{j,j+1}$. Let $\eta_{j,j+1}\left(Y\right)$ denote the distance from firm j of the consumer with income $Y<\overline{Y}_{j,j+1}$ who is indifferent between buying and not buying from firm j, that is, $U\left(x_j+\eta_{j,j+1}, Y, j\right)=Y$. It follows that

$$\eta_{j,j+1}\left(Y\right)=\frac{1}{t}\left[Y\left(\theta-1\right)-p_j\right].$$

But note that $\eta_{j,j+1}\left(Y\right)<0$ for

$$Y<\frac{p_j}{\theta_j-1}\equiv\underline{Y}_j,$$

that is, consumers with income $Y<\underline{Y}_j$ are not buying from firm j even when they are located at the same location as firm j. The implication for demand is that the measure of consumers located between x_j and x_{j+1} and buying from firm j is $\eta_{j,j+1}\left(Y\right)$ for all $\underline{Y}_j\leq Y<\overline{Y}_{j,j+1}$, and 0 for $Y<\underline{Y}_j$.

Proceeding in the same way we can define $\overline{Y}_{j,j-1}$, $\underline{Y}_j$, $\delta_{j,j-1}$ and $\eta_{j,j-1}\left(Y\right)$ symmetrically replacing $j+1$ with $j-1$ in the corresponding expressions and conclude that the measure of consumers located between x_j and x_{j-1} and buying from firm j is $\delta_{j,j-1}$ for $Y\geq\overline{Y}_{j,j-1}$, $\eta_{j,j-1}\left(Y\right)$ for $\underline{Y}_j\leq Y<\overline{Y}_{j,j-1}$, and 0 for $Y<\underline{Y}_j$.

It is interesting to note the difference in demand patterns arising from the relatively rich and poor. For the relatively rich consumers (with $Y \geq \overline{Y}_{j,j-1}$ or $Y \geq \overline{Y}_{j,j+1}$) firm j has to compete with the two adjacent firms, and the demand reflects that: $\delta_{j,j+1}$ and $\delta_{j,j-1}$ does depend on the strategic choices of the two adjacent firms, p_{j+1} and p_{j-1}, respectively. In contrast, firm j does not compete with its adjacent firms for the relatively poor consumers (with $\underline{Y}_j \leq Y < \overline{Y}_{j,j-1}$ and $\underline{Y}_j \leq Y < \overline{Y}_{j,j+1}$); they form a captive market for firm j over which it exercises some monopoly power.

Difference between the rich and poor gets reflected in the price response to demand also. Price response for the part of demand arising from the rich, $\frac{\partial \delta_{j,j+1}}{\partial p_j} = \frac{\partial \delta_{j,j-1}}{\partial p_j} = -\frac{1}{2t}$, is clearly lower than that arising from the poor, $\frac{\partial \eta_{j,j+1}(Y)}{\partial p_j} = \frac{\partial \eta_{j,j-1}(Y)}{\partial p_j} = -\frac{1}{t}$, because of the presence of competitive pressure.

2.2 The Symmetric Equilibrium

Given the symmetric model structure, in what follows we characterize the symmetric equilibrium where each of the entering n firms chooses the same price in stage 2, that is, $p_j = p$, for all j.

In a symmetric equilibrium the income thresholds relevant to define the demand structure become

$$\overline{Y}_{j,j+1} = \overline{Y}_{j,j-1} = \frac{2p + \frac{tL}{n}}{2(\theta - 1)} = \frac{p}{\theta - 1} + \frac{tL}{2n(\theta - 1)} \equiv \overline{Y}, \tag{1}$$

and

$$\underline{Y}_{j,j+1} = \underline{Y}_{j,j-1} = \frac{p_j}{\theta_j - 1} = \frac{p}{\theta - 1} \equiv \underline{Y}. \tag{2}$$

We always consider the scenario where the rich has complete market coverage, that is, $Y_R \geq \overline{Y}$. Then, depending on whether the poor has complete or partial coverages, that is, depending on the position of Y_P vis-a-vis $\overline{Y}$ and $\underline{Y}$, we have the following cases to consider:

(1) $Y_R > \overline{Y}$ and $Y_P > \overline{Y}$;

(2) $Y_R > \overline{Y}$ and $Y_P = \overline{Y}$;

(3) $Y_R > \overline{Y}$ and $\underline{Y} < Y_P < \overline{Y}$;

(4) $Y_R > \overline{Y}$ and $Y_P < \underline{Y}$;

(5) $Y_R = \overline{Y}$ and $\underline{Y} < Y_P < \overline{Y}$.

In what follows we analyze in detail case (3), the most generic case where all the rich consumers are served, whereas, for the poor, some are served while others are left out. Analysis of the other cases is similar, and we summarize and discuss the relevant results in section 5.

3 Partial Market Access for Poor and Complete Access for Rich

For case (3), $Y_R > \overline{Y}$ and $\underline{Y} < Y_P < \overline{Y}$, let us first derive the expression for demand faced by firm j. It follows from the demand structure discussed in section 2.1 that

$$\begin{aligned} D_j &= f_R \left[\delta_{j,j+1} + \delta_{j,j-1}\right] + f_P \left[\eta_{j,j+1}\left(Y_P\right) + \eta_{j,j-1}\left(Y_P\right)\right] \\ &= f_R \left[\frac{(p_{j-1} + p_{j+1} - 2p_j) + 2t\left(\dfrac{L}{n}\right)}{2t}\right] + 2 f_P \left[\frac{Y_P\left(\theta - 1\right) - p_j}{t}\right], \end{aligned} \tag{3}$$

so that the price response to demand is given by $\dfrac{\partial D_j}{\partial p_j} = -\left[\dfrac{f_R + 2f_P}{t}\right]$.

In stage 2, given the entry decision in stage 1, firm j chooses its price to maximize profit, π_j. The first-order condition with respect to price implies

$$D_j = (p_j - c) \cdot \left[\frac{f_R + 2f_P}{t}\right]. \tag{4}$$

In stage 1, firms' entry decision is determined by the zero-profit condition. Using (4) the expression for profit becomes

$$\pi_j = (p_j - c)\, D_j - F = (p_j - c)^2 \cdot \left[\frac{f_R + 2f_P}{t}\right] - F,$$

so that the zero-profit condition implies

$$(p_j - c)^2 \cdot \left[\frac{f_R + 2f_P}{t}\right] - F = 0. \tag{5}$$

Using (3), (4), and (5) we derive the equilibrium price and number of firms:

$$p = c + \sqrt{\frac{tF}{1 + f_P}}, \tag{6}$$

$$\frac{L}{n} = \frac{1}{t\left(1 - f_P\right)} \left[(1 + 3f_P)\sqrt{\frac{tF}{1 + f_P}} - 2f_P\left[Y_P\left(\theta - 1\right) - c\right]\right]. \tag{7}$$

Note that since the firms are competing for rich consumers, both price and number of firms are independent of the rich income. Price is also independent of the poor income. But, since the poor forms a captive market for the firms the size of which is restricted by their income, number of firms increases with the poor income. As poor income increases, demand size of each firm increases, and, price remaining the same, each firm makes more than normal profit. This super-normal profit attracts fresh entry of firms into the city.

Before we investigate this case any further, it is important to identify parameter values, in particular the income ranges of rich and poor under which this case arises. Recall that this case arises when $Y_R > \overline{Y}$ and $\underline{Y} < Y_P < \overline{Y}$, where the income thresholds $\overline{Y}$ and $\underline{Y}$ are endogenous (as expressed in equations (1) and (2)). Substituting the equilibrium values of price and number of firms into the expressions for $\overline{Y}$ and $\underline{Y}$ we find that $Y_P < \overline{Y}$ implies

$$Y_P\left(\theta-1\right)-c<\frac{3+f_P}{2}\sqrt{\frac{tF}{1+f_P}},$$

whereas $Y_P > \underline{Y}$ implies

$$Y_P\left(\theta-1\right)-c>\sqrt{\frac{tF}{1+f_P}}.$$

Combining the two we get

$$\sqrt{\frac{tF}{1+f_P}}<Y_P\left(\theta-1\right)-c<\frac{3+f_P}{2}\sqrt{\frac{tF}{1+f_P}}.$$

Similarly, $Y_R > \overline{Y}$ implies

$$\left[f_RY_R+f_PY_P\right]\left(\theta-1\right)-c>\frac{3+f_P}{2}\sqrt{\frac{tF}{1+f_P}}.$$

Thus we conclude that case (3) arises when the poor and rich incomes are such that

$$\begin{aligned}\frac{c+\sqrt{\frac{tF}{1+f_P}}}{(\theta-1)} \quad &< \quad Y_P<\frac{c+\frac{3+f_P}{2}\sqrt{\frac{tF}{1+f_P}}}{(\theta-1)}\\ &\text{and}\\ f_RY_R+f_PY_P \quad &> \quad \frac{c+\frac{3+f_P}{2}\sqrt{\frac{tF}{1+f_P}}}{(\theta-1)}.\end{aligned} \tag{8}$$

So we have identified an upper income threshold and a lower income threshold for the poor income such that if the poor income is in between these two thresholds whereas the rich income is high enough so that the average income is higher than the upper income threshold, then the firms do not compete with the adjacent firms for the poor consumers but do so only for the rich consumers. All the rich consumers are served by the market, but some poor are left out – only those poor who are located closer to the firms get served.

4 Income Inequality and Welfare of Poor

Now we use this generic case (3) to analyze the impact of income inequality on the welfare of the poor.

Consider the rich consumers first. Since all the rich consumers are served, the market access of the poor can be thought of as in proportion to that of the rich. To calculate the aggregate consumer surplus of the rich community as a whole we proceed as follows. Surplus to a rich consumer located at a distance x from the firm from which it is buying is $Y_R\theta - p - tx - Y_R$.[2] Since there are n firms each with a market coverage of $\frac{L}{2n}$ on either side of its location, the aggregate consumer surplus of the rich community is

$$CS_R = 2n\int_0^{\frac{L}{2n}} \left[Y_R(\theta-1) - p - tx\right] dx = L\left[Y_R(\theta-1) - p - \frac{t}{4}\left(\frac{L}{n}\right)\right].$$

As expected, consumer surplus increases with income (Y_R) and number of firms (n), and decreases with travel cost (t) and price (p). Since price and number of firms are endogenous, substituting their equilibrium values from equations (6) and (7) we derive the expression for aggregate consumer surplus of the rich community solely in terms of the parameters of the model:

$$CS_R = L\left[Y_R(\theta-1) + \frac{1}{2}\frac{f_P}{f_R}Y_P(\theta-1) - c\left(1 - \frac{1}{2}\frac{f_P}{f_R}\right) - \sqrt{\frac{tF}{1+f_P}}\left(1 + \frac{1+3f_P}{4f_R}\right)\right]. \quad (9)$$

It is interesting to note that consumer surplus of the rich increases even when the income of the poor increases. As noted in the last section, as poor income increases price remains the same but number of firms increases. A higher number of firms implies less travel cost for the rich and hence their consumer surplus increases.

Coming to the poor consumers, consider their market access first. Not all the poor can afford to buy the product: only the poor up to the distance $\frac{Y_P(\theta-1)-p}{t}$ from any firm are buying the product; those in between the distance $\frac{Y_P(\theta-1)-p}{t}$ and $\frac{L}{2n}$ cannot afford

[2] Recall that the reservation utility of the rich is Y_R.

it. Hence the aggregate real consumption of the poor community is

$$C_P = 2n \int_0^{\frac{Y_P(\theta-1)-p}{t}} dx = \frac{2n}{t}\left[Y_P(\theta-1)-p\right].$$

The tension between price and number of firms is clear: an increase in number of firms increases market access while a price increase reduces it. Substituting the equilibrium values of price and number of firms we get

$$C_P = \frac{2L(1-f_P)}{(1+3f_P)\sqrt{\frac{tF}{1+f_P}} - 2f_P\left[Y_P(\theta-1)-c\right]}\left[Y_P(\theta-1)-c-\sqrt{\frac{tF}{1+f_P}}\right]. \qquad (10)$$

Finally consider the aggregate consumer surplus of the poor. Since the poor in between the distance $\frac{Y_P(\theta-1)-p}{t}$ and $\frac{L}{2n}$ from any firm does not buy the product, their consumer surplus is zero. Hence the aggregate consumer surplus of the poor is

$$CS_P = 2n\left[\int_0^{\frac{Y_P(\theta-1)-p}{t}} \left[Y_P(\theta-1)-p-tx\right]dx\right] = \frac{n}{t}\left[Y_P(\theta-1)-p\right]^2.$$

Similar to the aggregate real consumption, an increase in number of firms increases aggregate consumer surplus of the poor while a price increase reduces it. Substituting the equilibrium values we derive

$$CS_P = \frac{L(1-f_P)}{(1+3f_P)\sqrt{\frac{tF}{1+f_P}} - 2f_P\left[Y_P(\theta-1)-c\right]}\left[Y_P(\theta-1)-c-\sqrt{\frac{tF}{1+f_P}}\right]^2. \qquad (11)$$

Note that both the aggregate real consumption and consumer surplus of the poor increases as poor income increases. There are two effects at work. First is the direct 'valuation effect': as income increases value of the product to the consumers increases. Second effect is the indirect effect working through the increase in number of firms as poor income increases. Both the effects work in the same direction reinforcing each other.

Now to see the effect of income inequality on the welfare of the poor we conduct the following comparative static analysis: we vary f_P keeping Y_P and Y_R fixed. That is, we follow the poor with the same income level and compare the aggregate market access and consumer surplus of the poor community as a whole when they live in relatively richer societies (as f_P decreases from 1 to 0).

This comparative static exercise is worked out in Appendix A.1 and the results are illustrated in the following four figures depicting aggregate real consumption of the poor (Figure

1), aggregate expenditure of the poor (Figure 2), aggregate consumer surplus of the poor (Figure 3) and aggregate consumer surplus of the rich (Figure 4) for a specific set of parameter values under case (3).

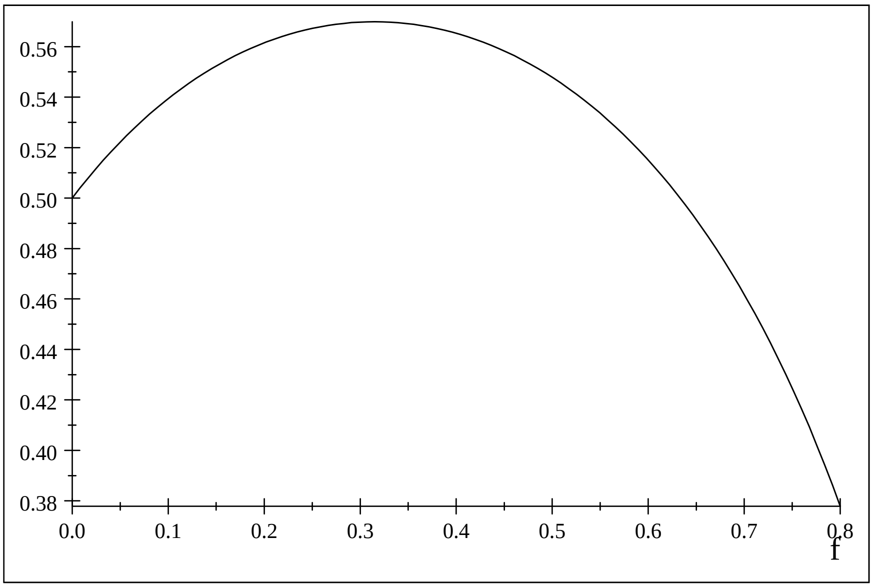

Figure 1: Aggregate Real Consumption of the Poor

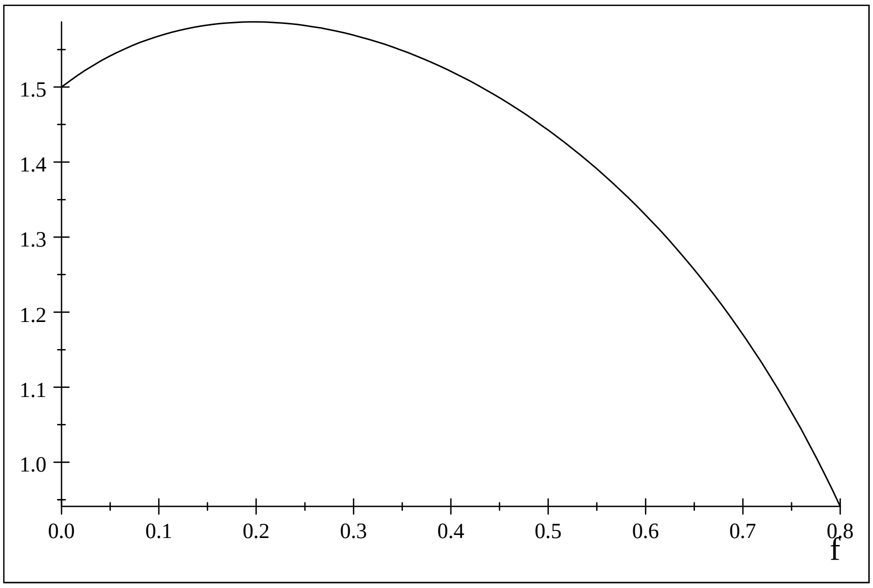

Figure 2: Aggregate Expenditure of the Poor

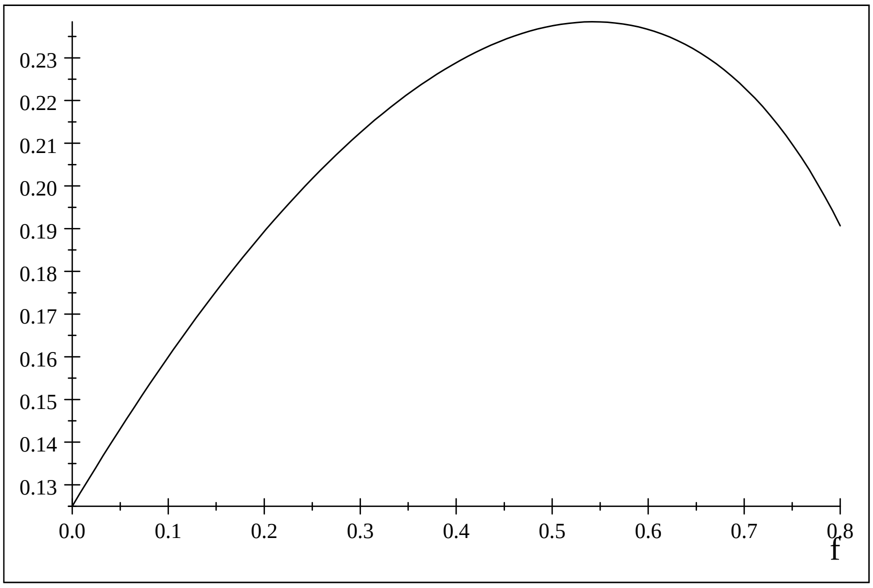

Figure 3: Aggregate Consumer Surplus of the Poor

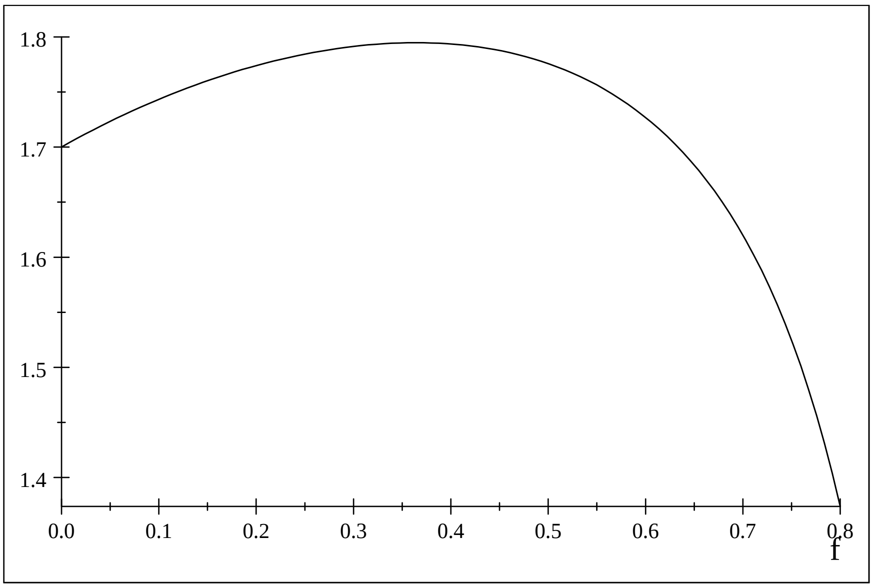

Figure 4: Aggregate Consumer Surplus of the Rich

It is very interesting to observe the "inverted-U" shaped relationship between f_P and aggregate real consumption of poor, f_P and aggregate expenditure of poor, and f_P and aggregate consumer surplus of poor.[3] That is, if we compare a cross-section of societies, the poor community as a whole is initially better-off living in relatively richer societies (as f_P decreases from 1). But, beyond a point, the aggregate consumer surplus of poor starts declining as the society becomes richer.

The reason for this inverted-U shaped relationship can be traced to the equilibrium price and number of firms. It can be checked from their equilibrium values given in equations (6) and (7) that both price and number of firms increases steadily as f_P decreases from 1 to 0.[4] Consumers benefit from the increase in number of firms but lose from the increase in price. For the poor community as a whole the number of firms effect dominates initially: the poor located closer to the firms get to consume the product and the number of poor served increases as the number of firms increases. But, beyond a point, the adverse price effect takes over.

It is important to highlight the role of the spatial structure, in particular to point out that we are getting the inverted-U shape in both real consumption and consumer surplus because the number of firms are also changing endogenously. When we conduct the same analysis with number of firms fixed, both real consumption and consumer surplus of the poor decreases steadily as f_P decreases; that is, we do not see any invetred-U shape in the relationships. The reason is that price increase steadily without any compensating increase in the number of firms.

[3] Establishing the exact "inverted-U" shaped relationship is algebraically tedious. In Appendix A.1 we show the following:

$$\left.\frac{\partial C_P}{\partial f_P}\right|_{f_P=0} > 0 \text{ and } \left.\frac{\partial C_P}{\partial f_P}\right|_{f_P=1} < 0,$$

and

$$\left.\frac{\partial CS_P}{\partial f_P}\right|_{f_P=0} > 0 \text{ and } \left.\frac{\partial CS_P}{\partial f_P}\right|_{f_P=1} < 0.$$

This clearly indicates the "inverted-U" shaped relationship.

[4] While it is obvious from equation (6) that $\frac{\partial p}{\partial f_P} < 0$, from equation (7) we derive

$$\frac{\partial}{\partial f_P}\left(\frac{L}{n}\right) = \frac{\left(3f_P^2 + 6f_P + 7\right)\sqrt{\frac{tF(\theta)}{1+f_P}} - 4(1+f_P)\left[Y_P(\theta-1) - c(\theta)\right]}{2t(1-f_P)^2(1+f_P)} < 0$$

since, under case (3), $\frac{3+f_P}{2}\sqrt{\frac{tF(\theta)}{1+f_P}} > Y_P(\theta-1) - c(\theta)$ implies

$$\left(3f_P^2 + 6f_P + 7\right)\sqrt{\frac{tF(\theta)}{1+f_P}} - 4(1+f_P)\left[Y_P(\theta-1) - c(\theta)\right] > (1-f_P)^2\sqrt{\frac{tF(\theta)}{1+f_P}} \geq 0.$$

5 Characterizing the Equilibrium

In the last two sections we have analyzed in detail the generic case (3) where all the rich consumers are served but only some of the poor consumers are served, others are left out of the market. Analyses of the other four cases are similar and, for the sake of brevity, we do not repeat the detailed analyses in the text and relegate it to Appendix A.2. Instead, in this section we summarize the income ranges of rich and poor under which different cases arise and discuss the implications of income inequality in characterizing the nature of equilibrium.

5.1 Summary of Different Equilibrium Possibilities

Case (1): $\mathbf{Y}_R > \overline{Y}$ and $\mathbf{Y}_P > \overline{Y}$:

This case arises when

$$Y_R > Y_P > \frac{c + \frac{3}{2}\sqrt{tF}}{(\theta - 1)}.$$

In this case firms compete for both consumer types – rich and poor, and all consumers of each type are served.

Case (2): $\mathbf{Y}_R > \overline{Y}$ and $\mathbf{Y}_P = \overline{Y}$:

This case arises when

$$\frac{c + \frac{3 + f_P}{2}\sqrt{\frac{tF}{1 + f_P}}}{(\theta - 1)} < Y_P < \frac{c + \frac{3}{2}\sqrt{tF}}{(\theta - 1)}.$$

Firms compete for the rich, but the marginal poor who is indifferent between two adjacent firms is also indifferent between buying and not buying; all consumers of each type are served though.

Case (3): $\mathbf{Y}_R > \overline{Y}$ and $\underline{Y} < \mathbf{Y}_P < \overline{Y}$:

This case arises when

$$\frac{c + \sqrt{\frac{tF}{1 + f_P}}}{(\theta - 1)} < Y_P < \frac{c + \frac{3 + f_P}{2}\sqrt{\frac{tF}{1 + f_P}}}{(\theta - 1)}$$

and

$$f_R Y_R + f_P Y_P > \frac{c + \frac{3 + f_P}{2}\sqrt{\frac{tF}{1 + f_P}}}{(\theta - 1)}.$$

Firms compete only for the rich, but are monopolist with respect to the poor. All the rich consumers are served, but some poor are left out of the reach of market.

Case (4): $\boldsymbol{Y}_R > \overline{Y}$ **and** $\boldsymbol{Y}_P < \underline{Y}$:

This case arises when

$$
\begin{aligned}
Y_P &< \frac{c + \sqrt{\dfrac{tF}{1 - f_P}}}{(\theta - 1)} \\
&\text{and} \\
Y_R &> \frac{c + \dfrac{3}{2}\sqrt{\dfrac{tF}{1 - f_P}}}{(\theta - 1)}.
\end{aligned}
$$

Firms are competing only for the rich and all the rich consumers are served. But, unfortunately, all the poor consumers are left out.

Case (5): $\boldsymbol{Y}_R = \overline{Y}$ **and** $\underline{Y} < \boldsymbol{Y}_P < \overline{Y}$:

This case arises when

$$
\begin{aligned}
\frac{c + \sqrt{\dfrac{tF}{2}}}{(\theta - 1)} &< Y_P < \frac{c + \sqrt{\dfrac{tF}{1 + f_P}}}{(\theta - 1)} \\
&\text{and} \\
\frac{c + \sqrt{2tF}}{(\theta - 1)} &< f_R Y_R + f_P Y_P < \frac{c + \dfrac{3 + f_P}{2}\sqrt{\dfrac{tF}{1 + f_P}}}{(\theta - 1)}.
\end{aligned}
$$

In this case firms are monopolists with respect to the poor, but even the marginal rich is also indifferent between buying and not buying. All the rich consumers are served. Some poor consumers are served too.

Figure 5 summarizes all these cases plotting the lower and upper bounds of incomes for different values of f_P, the proportion of poor people.

5.2 Implications of Income Inequality

Our analysis of the different cases summarized in the last section has a number of implications of income inequality.

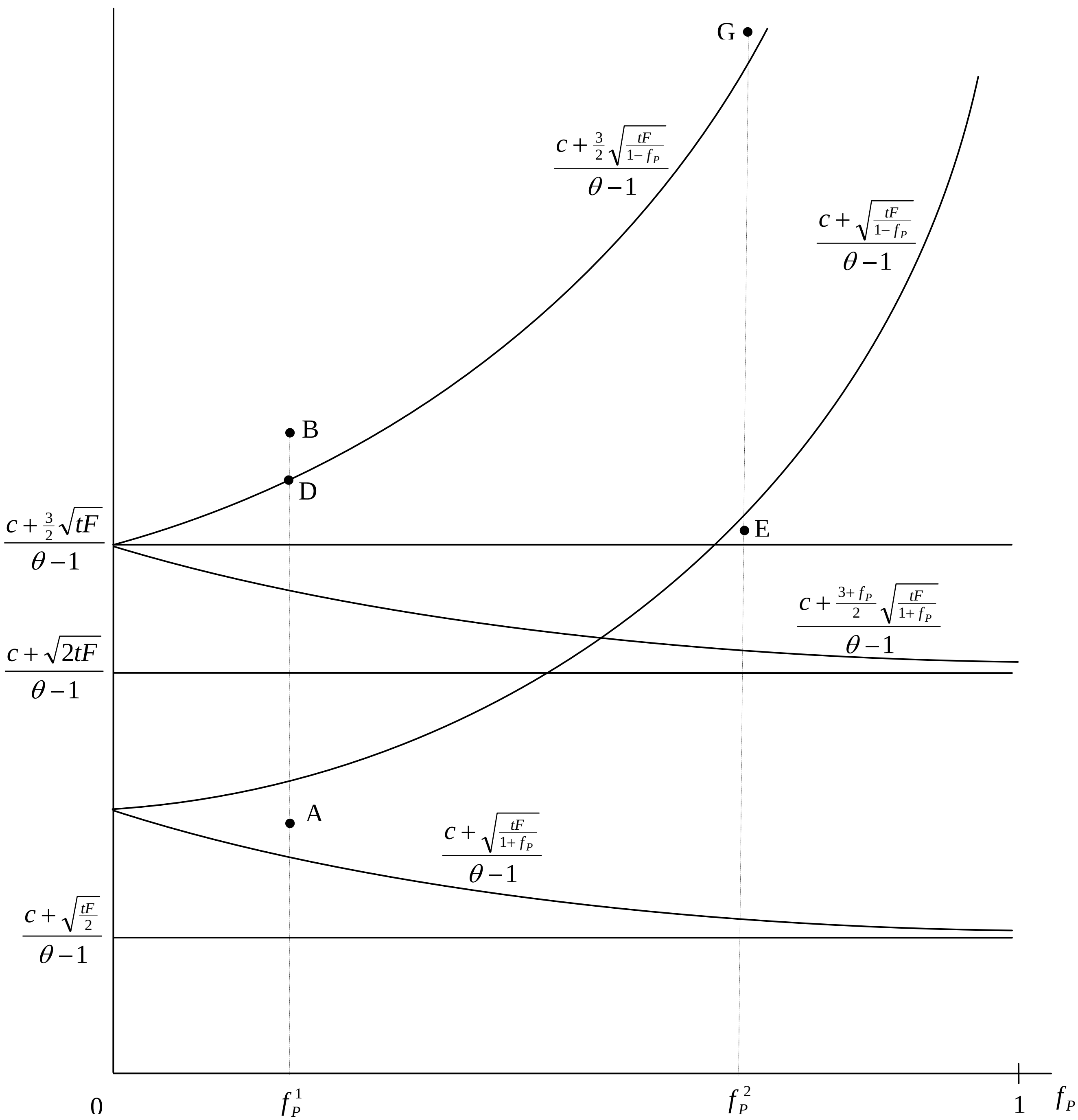

Figure 5: Different Equilibrium Possibilities

5.2.1 Upper threshold for Y_P

From cases (1), (2) and (3) it is clear that there exists an upper income threshold for Y_P, call it $\overline{Y}_P$, defined by

$$\overline{Y}_P \equiv \frac{c + \dfrac{3+f_P}{2}\sqrt{\dfrac{tF}{1+f_P}}}{(\theta - 1)}$$

such that all poor consumers are served only if $Y_P \geq \overline{Y}_P$.

Case (1) shows the existence of another income threshold, $\dfrac{c + \frac{3}{2}\sqrt{tF}}{(\theta - 1)} > \overline{Y}_P$, such that if the income of the poor is above this threshold, then not only all poor consumers are served but, in addition, each firm has to compete with its adjacent firms for *both* poor and rich customers. Equilibrium price and number of firms reflect this competition.

5.2.2 Lower thresholds for Y_P

There are two lower income thresholds for the poor, $\underline{Y}_P$, such that no poor consumer is served if $Y_P < \underline{Y}_P$. Interestingly which threshold is relevant depends on the income of the rich.

When the rich income is high enough so that firms are competing for the rich (cases (1), (2) and (3)), the lower income thresholds for the poor is given by

$$\underline{Y}_P^1 \equiv \frac{c + \sqrt{\dfrac{tF}{1+f_P}}}{(\theta - 1)}.$$

But when the rich income is reasonably low in the sense that the marginal rich is indifferent between buying and not buying (case (5)), then this lower income threshold becomes

$$\underline{Y}_P^2 \equiv \frac{c + \sqrt{\dfrac{tF}{2}}}{(\theta - 1)}.$$

Implication of Income Gap between Rich and Poor:

Notice that $\underline{Y}_P^2 < \underline{Y}_P^1$, that is, the lower income threshold of the poor is lower when the rich income is reasonably low. Thus poor are better off when the income gap between the rich and poor is low.

When the poor income is in between the upper and lower income thresholds, there are pockets of the city where the poor are left out of the market: only those poor who are located closer to the firms get served, others get excluded. The size of these exclusion pockets increases as the poor income decreases.

5.2.3 Possibility of Multiple Equilibria

Case (4) generates the possibility of multiple equilibria. Consider, for example, the income distribution depicted by points A and B in Figure 5: there are f_P^1 proportion of poor with income given by the height of A and $(1 - f_P^1)$ proportion of rich with income B. The income distribution is such that parameter configurations for both cases (3) and (4) are satisfied, generating the multiple equilibria. The equilibrium under case (3) is a good equilibrium where at least some poor get served. The equilibrium outcome under case (4) is a bad outcome: all the poor are excluded; the firms completely ignore their presence and choose the price as if there were only rich individuals residing in the city.

Implication of Income Gap:

Note once again the implication of higher income gap between rich and poor. If the rich income were below the height of D, then this complete exclusion possibility of the poor would not have arisen. It is the higher income gap that exposes the poor to this vulnerable situation.

The implication of income gap could be even more damaging for a multiple equilibria situation like the one depicted by the other income distribution shown in Figure 5: there are f_P^2 proportion of poor with income given by the height of E and $(1 - f_P^2)$ proportion of rich with income G. Here the multiplicity occurs with cases (4) and (1). Recall that case (1) is the best possible outcome that can happen to the poor – income of the poor is high enough so that all the poor are served, and, at the same time, the firms are forced to compete for them. But even then a higher income gap exposes them to the possibility of complete exclusion.

The Case of Minority Poor:

Poor are more likely to be completely excluded when they are a minority, that is, when f_P is low: firms may completely ignore the poor even when the rich are not ultra rich just because the rich are more in number. For instance, in Figure 5, with the same income levels A for poor and B for rich, the complete exclusion possibility does not arise when the proportion of poor is f_P^2; but this possibility does arise when the proportion of poor is f_P^1.

5.3 Comparison with a Single-Income Economy

In section 4 we have identified scenarios where the poor could be better-off living in relatively richer societies. To see how the possibility arises in the simplest possible way it is interesting to compare our model economy with two income groups with a single-income economy. A single-income economy refers to a city inhabited by a single income group; that is, at each point of the city there is a measure 1 of consumers with the same income Y. The single-income economy model is analyzed in Appendix A.3 and the relevant comparison is highlighted below.

The Feasibility Income Threshold in a Single-Income Economy:

In a single-income economy it is not feasible for any firm to operate unless the common income is at least $\frac{c+\sqrt{2tF}}{(\theta-1)}$. If the income is below this feasibility threshold, the willingness to pay is so low that it is not possible for the firms to recover the fixed cost of production. The implication for a single-income poor economy with common income Y_P is that nobody gets to enjoy the product or service when $Y_P < \frac{c+\sqrt{2tF}}{(\theta-1)}$.

Comparing a Single-Income Economy with a Mixed-Income Economy:

With reference to a single-income economy, a mixed-income economy is the one that we are considering so far where at each point of the city there are f_R proportion of rich with income Y_R and f_P proportion of poor with income Y_P. Since both the lower income thresholds of the poor, $\underline{Y}_P^1$ and $\underline{Y}_P^2$, are strictly less than the feasibility threshold, $\frac{c+\sqrt{2tF}}{(\theta-1)}$, it is clear that poor are better-off staying in the mixed-income community as long as the poor income is below this feasibility threshold. At least some poor get to enjoy the product or service in the mixed-income economy as the firms recover their fixed costs due to the higher willingness to pay of the rich. This is not possible in a single-income poor economy.

6 Conclusion

In this paper we develop a model that integrates consumer's income distribution with spatial distribution, and look at the consequence of an increase in income inequality on the welfare of the poor in general, and their access to market in particular. We find an inverted-U shaped relationship between inequality and welfare of the poor. The poor is initially better-off living in relatively richer societies by having access to a wider varieties of products and services. But, beyond a point, the aggregate consumer surplus of the poor starts declining as the society becomes richer: the welfare gain from increase in access to wider varieties of products and services is not enough to offset the corresponding rise in price. Interestingly, the same inverted-U shaped relationship is also observed between income inequality and market access of the poor.

We identify an *upper income threshold* for the poor income such that all poor consumers get served by the market only if the poor income is above this upper income threshold. On the other hand there exists a *lower income threshold* for the poor income such that no poor consumer is served if the poor income is below this lower threshold. When the poor income is in between these two thresholds, there exist pockets where the poor are left out of the market: only those poor who are located closer to the firms get served, others get excluded. The size of these exclusion pockets increases as the poor income decreases.

There exist multiple equilibria: a bad equilibrium where all the poor are excluded can exist simultaneously with a good equilibrium where at least some poor (if not all of them) get served by the market. We have isolated the higher income gap between the rich and poor as the key factor that exposes the poor to this complete exclusion possibility. Finally we compare a mixed-income economy where rich and poor live side by side with a single-income economy inhabited only by a single income group and show that poor are better-off staying in the mixed-income economy as long as the poor income is below a feasibility threshold.

The model has two important limitations. First is the assumption on the spatial structure that rich and poor live side by side in the same location. It would be interesting to investigate how the nature of the equilibrium changes when the rich and poor are geographically segregated – poor residing in the poor ghettos whereas rich live in the rich neighbourhoods. The other limitation is concerned with the homogeneous nature of the product. Although it is very interesting to obtain quite rich results on inequality-neighbourhood interactions even with a homogeneous product, in the presence of differential income groups it is natural for the firms to separate the rich from the poor consumers by offering higher quality products. The current analysis can be interpreted as portraying, in some sense, the average picture. We plan to address both these limitations in our future research.

7 Appendix

A.1 Income Inequality and Welfare of the Poor: Comparative Statics with Respect to f_P

Recall that

$$C_P = \frac{2L(1-f_P)}{(1+3f_P)\sqrt{\frac{tF}{1+f_P}} - 2f_P\left[Y_P(\theta-1)-c\right]}\left[Y_P(\theta-1)-c-\sqrt{\frac{tF}{1+f_P}}\right].$$

After some simplifications we derive

$$\frac{\partial C_P}{\partial f_P}\cdot\frac{1}{2L} = \frac{(1-f_P)\left[\frac{1}{2(1+f_P)}\sqrt{\frac{tF}{1+f_P}}\right]}{(1+3f_P)\sqrt{\frac{tF}{1+f_P}} - 2f_P\left[Y_P(\theta-1)-c\right]}$$

$$-\frac{\left[(3f_P^2+6f_P+7)\sqrt{\frac{tF}{1+f_P}} - 4(1+f_P)\left[Y_P(\theta-1)-c\right]\right]\left[Y_P(\theta-1)-c-\sqrt{\frac{tF}{1+f_P}}\right]}{2(1+f_P)\left[(1+3f_P)\sqrt{\frac{tF}{1+f_P}} - 2f_P\left[Y_P(\theta-1)-c\right]\right]^2}.$$

Note that when $f_P = 1$,

$$\left.\frac{\partial C_P}{\partial f_P}\right|_{f_P=1}\cdot\frac{1}{2L} < 0, \text{ that is, } \left.\frac{\partial C_P}{\partial f_P}\right|_{f_P=1} < 0$$

since the first term vanishes and the second term is strictly negative under case (3) as $Y_P(\theta-1)-c > \sqrt{\frac{tF}{1+f_P}}$ and $\frac{3+f_P}{2}\sqrt{\frac{tF}{1+f_P}} > Y_P(\theta-1)-c$ implies $(3f_P^2+6f_P+7)\sqrt{\frac{tF}{1+f_P}} - 4(1+f_P)\left[Y_P(\theta-1)-c\right] > (1-f_P)^2\sqrt{\frac{tF}{1+f_P}} \geq 0.$

We argue next that when $f_P = 0$,

$$\left.\frac{\partial C_P}{\partial f_P}\right|_{f_P=0} > 0.$$

Note that

$$\begin{aligned}\left.\frac{\partial C_P}{\partial f_P}\right|_{f_P=0}\cdot\frac{1}{2L} &= \frac{tF - \left[7\sqrt{tF} - 4\left[Y_P(\theta-1)-c\right]\right]\left[Y_P(\theta-1)-c-\sqrt{tF}\right]}{2tF} \\ &= \frac{4\left[Y_P(\theta-1)-c\right]^2 + 8tF - 11\left[Y_P(\theta-1)-c\right]\sqrt{tF}}{2tF}.\end{aligned}$$

Denoting $A \equiv [Y_P(\theta-1)-c]$ and $B \equiv \sqrt{tF}$, the numerator of the above expression becomes $4A^2+8B^2-11AB$. Recall that under case (3) we have (when $f_P=0$) $B<A<\frac{3}{2}B$. Hence the problem boils down to determining the sign of $4A^2+8B^2-11AB$ when $B<A<\frac{3}{2}B$. Note that

$$\frac{\partial}{\partial A}\left[4A^2+8B^2-11AB\right]=8A-11B,$$

and

$$\frac{\partial^2}{\partial A^2}\left[4A^2+8B^2-11AB\right]=8>0.$$

That is, treating B as a parameter, $4A^2+8B^2-11AB$ is minimized at $A^*=\frac{11}{8}B$, and the minimum value is $\frac{7}{16}B^2>0$. Then we can conclude that $\left.\frac{\partial C_P}{\partial f_P}\right|_{f_P=0}>0$.

Consider next the aggregate consumer surplus of the poor:

$$CS_P=\frac{L(1-f_P)}{(1+3f_P)\sqrt{\frac{tF}{1+f_P}}-2f_P[Y_P(\theta-1)-c]}\left[Y_P(\theta-1)-c-\sqrt{\frac{tF}{1+f_P}}\right]^2.$$

We derive

$$\frac{\partial CS_P}{\partial f_P}\cdot\frac{1}{L}=\frac{(1-f_P)\left[Y_P(\theta-1)-c-\sqrt{\frac{tF}{1+f_P}}\right]\left[\frac{1}{(1+f_P)}\sqrt{\frac{tF}{1+f_P}}\right]}{(1+3f_P)\sqrt{\frac{tF}{1+f_P}}-2f_P[Y_P(\theta-1)-c]}$$

$$-\frac{\left[(3f_P^2+6f_P+7)\sqrt{\frac{tF}{1+f_P}}-4(1+f_P)[Y_P(\theta-1)-c]\right]\left[Y_P(\theta-1)-c-\sqrt{\frac{tF}{1+f_P}}\right]^2}{2(1+f_P)\left[(1+3f_P)\sqrt{\frac{tF}{1+f_P}}-2f_P[Y_P(\theta-1)-c]\right]^2}.$$

Again when $f_P=1$,

$$\left.\frac{\partial CS_P}{\partial f_P}\right|_{f_P=1}\cdot\frac{1}{L}<0,\ \text{ that is, }\ \left.\frac{\partial CS_P}{\partial f_P}\right|_{f_P=1}<0$$

since the first term vanishes and the second term is strictly negative for the same logic given above.

When $f_P=0$,

$$\begin{aligned}\left.\frac{\partial CS_P}{\partial f_P}\right|_{f_P=0}\cdot\frac{1}{L} &= \left[Y_P(\theta-1)-c-\sqrt{tF}\right]\\ &\quad -\frac{\left[7\sqrt{tF}-4[Y_P(\theta-1)-c]\right]\left[Y_P(\theta-1)-c-\sqrt{tF}\right]^2}{2tF}.\end{aligned}$$

Using the same notation as above we rewrite

$$\left.\frac{\partial CS_P}{\partial f_P}\right|_{f_P=0} \cdot \frac{1}{L} = [A-B] - \frac{[7B-4A]\,[A-B]^2}{2B^2} = \frac{(A-B)}{2B^2}\left[4A^2+9B^2-11AB\right] > 0$$

by the same logic given above. Hence we conclude that $\left.\frac{\partial CS_P}{\partial f_P}\right|_{f_P=0} > 0.$

A.2 Details of the Different Cases

Case (1): $\boldsymbol{Y}_R > \overline{Y}$ and $\boldsymbol{Y}_P > \overline{Y}$:

From the demand structure discussed in section 2.1 it follows that:

$$\begin{aligned} D_j &= f_R\left[\delta_{j,j+1}+\delta_{j,j-1}\right] + f_P\left[\delta_{j,j+1}+\delta_{j,j-1}\right] \\ &= \left[\frac{(p_{j-1}+p_{j+1}-2p_j)+2t\left(\frac{L}{n}\right)}{2t}\right]. \end{aligned} \tag{A.1}$$

This implies that $\frac{\partial D_j}{\partial p_j} = -\frac{1}{t}$. In stage 2, given the entry decision in stage 1, firm j chooses its price to maximize profit, π_j. The first-order condition with respect to price implies

$$D_j = (p_j - c)\cdot\frac{1}{t}. \tag{A.2}$$

In stage 1, firms' entry decision is determined by the zero-profit condition. Using (A.2) the expression for profit becomes

$$\pi_j = (p_j - c)\,D_j - F = (p_j-c)^2\cdot\frac{1}{t} - F,$$

so that the zero-profit condition implies

$$(p_j-c)^2\cdot\frac{1}{t} - F = 0. \tag{A.3}$$

Using (A.1), (A.2) and (A.3) we derive the equilibrium price and number of firms:

$$\begin{aligned} p &= c+\sqrt{tF}, \\ \frac{L}{n} &= \sqrt{\frac{F}{t}}. \end{aligned}$$

Now we identify the income ranges of rich and poor under which this case arises. Recall that this case arises when $Y_R > \overline{Y}$ and $Y_P > \overline{Y}$, where the upper income threshold $\overline{Y}$ is given by

$$\overline{Y} = \frac{p}{\theta - 1} + \frac{tL}{2n(\theta - 1)}.$$

Substituting the equilibrium values of price and number of firms into the expressions for $\overline{Y}$ implies:

$$Y_P\,(\theta - 1) - c > \frac{3}{2}\sqrt{tF}.$$

Thus we conclude that case (1) arises when

$$Y_R > Y_P > \frac{c + \frac{3}{2}\sqrt{tF}}{(\theta - 1)}.$$

Case (2): $\mathbf{Y}_R > \overline{Y}$ and $\mathbf{Y}_P = \overline{Y}$:

This is the case of a 'kinked equilibrium' as in Salop (1979). One extreme of the kink is case (1) described above where the price response to demand is given by $\dfrac{\partial D_j}{\partial p_j} = -\dfrac{1}{t}$. The other extreme is case (3) discussed in section 3 where the price response to demand is given by $\dfrac{\partial D_j}{\partial p_j} = -\left[\dfrac{f_R + 2f_P}{t}\right]$.

Note that since $Y_P = \overline{Y} = \dfrac{p}{\theta - 1} + \dfrac{tL}{2n(\theta - 1)}$, we have

$$p = Y_P(\theta - 1) - \frac{tL}{2n}.$$

For the first extreme, since the price response to demand is $\dfrac{\partial D_j}{\partial p_j} = -\dfrac{1}{t}$, proceeding as in case (1) we can derive the equilibrium price and number of firms as

$$p = c + \sqrt{tF}, \text{ and } \frac{L}{n} = \sqrt{\frac{F}{t}}.$$

Since $p = c + \sqrt{tF}$ and, at the same time, $p = Y_P(\theta - 1) - \dfrac{tL}{2n}$, this implies

$$\frac{L}{n} = \frac{2}{t}\left[Y_P\,(\theta - 1) - c\right] - 2\sqrt{\frac{F}{t}}.$$

But we have $\dfrac{L}{n} = \sqrt{\dfrac{F}{t}}$. It follows that this extreme case arises under the special circumstance when

$$Y_P\,(\theta - 1) - c = \frac{3}{2}\sqrt{tF}. \tag{A.4}$$

For the other extreme, since the price response to demand is $\dfrac{\partial D_j}{\partial p_j} = -\left[\dfrac{f_R + 2f_P}{t}\right]$, proceeding as in case (3) we have

$$p = c + \sqrt{\frac{tF}{1+f_P}}, \text{ and } \frac{L}{n} = \frac{1}{t\,(1-f_P)}\left[(1+3f_P)\sqrt{\frac{tF}{1+f_P}} - 2f_P\left(Y_P\left(\theta-1\right)-c\right)\right].$$

Proceeding as above it now follows that this extreme case arises under the specific parameter values where

$$Y_P\left(\theta-1\right) - c = \frac{3+f_P}{2}\sqrt{\frac{tF}{1+f_P}}. \tag{A.5}$$

Combining these two extremes it follows from (A.4) and (A.5) that case (2) arises when

$$\frac{3+f_P}{2}\sqrt{\frac{tF}{1+f_P}} < Y_P\left(\theta-1\right) - c < \frac{3}{2}\sqrt{tF},$$

that is, when

$$\frac{c + \dfrac{3+f_P}{2}\sqrt{\dfrac{tF}{1+f_P}}}{(\theta-1)} < Y_P < \frac{c+\frac{3}{2}\sqrt{tF}}{(\theta-1)}.$$

Case (4): $\mathbf{Y}_R > \overline{Y}$ and $\mathbf{Y}_P < \underline{Y}$:

In this case the demand for firm j is given by

$$\begin{aligned} D_j &= f_R\left[\delta_{j,j+1} + \delta_{j,j-1}\right] \\ &= f_R\left[\frac{(p_{j-1}+p_{j+1}-2p_j) + 2t\left(\dfrac{L}{n}\right)}{2t}\right]. \end{aligned} \tag{A.6}$$

This implies that $\dfrac{\partial D_j}{\partial p_j} = -\dfrac{f_R}{t}$. In stage 2, given the entry decision in stage 1, firm j chooses its price to maximize profit, π_j. The first-order condition with respect to price implies

$$D_j = (p_j - c)\cdot\frac{f_R}{t}. \tag{A.7}$$

In stage 1, firms' entry decision is determined by the zero-profit condition. Using (A.7) the expression for profit becomes

$$\pi_j = (p_j - c)\,D_j - F = (p_j-c)^2\cdot\frac{f_R}{t} - F,$$

so that the zero-profit condition implies

$$(p_j - c)^2 \cdot \frac{f_R}{t} - F = 0. \tag{A.8}$$

Using (A.6), (A.7) and (A.8) we derive the equilibrium price and number of firms:

$$\begin{aligned} p &= c + \sqrt{\frac{tF}{1-f_P}}, \\ \frac{L}{n} &= \sqrt{\frac{F}{t\,(1-f_P)}}. \end{aligned}$$

Now we identify the income ranges of rich and poor under which this case arises. Recall that this case arises when $Y_R > \overline{Y}$ and $Y_P < \underline{Y}$, where the upper and lower income thresholds are given by

$$\overline{Y} = \frac{p}{\theta - 1} + \frac{tL}{2n(\theta-1)} \text{ and } \underline{Y} = \frac{p}{\theta-1}.$$

Substituting the equilibrium values of price and number of firms we find that $Y_P < \underline{Y}$ implies:

$$Y_P < \frac{c + \sqrt{\frac{tF}{1-f_P}}}{\theta - 1},$$

whereas $Y_R > \overline{Y}$ implies

$$Y_R > \frac{c + \frac{3}{2}\sqrt{\frac{tF}{1-f_P}}}{(\theta-1)}.$$

Case (5): $\mathbf{Y}_R = \overline{Y}$ and $\underline{Y} < \mathbf{Y}_P < \overline{Y}$:

Since $Y_R = \overline{Y}$, this exemplifies another case of 'kinked equilibrium'. One extreme of the kink is case (3) described above where the price response to demand is given by $\frac{\partial D_j}{\partial p_j} = -\left[\frac{f_R + 2f_P}{t}\right]$. For the other extreme the demand from the rich is such that total demand is given by

$$\begin{aligned} D_j &= f_R\left[\eta_{j,j+1}(Y_R) + \eta_{j,j-1}(Y_R)\right] + f_P\left[\eta_{j,j+1}(Y_P) + \eta_{j,j-1}(Y_P)\right] \\ &= 2f_R\left[\frac{Y_R(\theta-1) - p_j}{t}\right] + 2f_P\left[\frac{Y_P(\theta-1) - p_j}{t}\right], \end{aligned}$$

so that the price response to demand is given by $\frac{\partial D_j}{\partial p_j} = -\left[\frac{2f_R + 2f_P}{t}\right] = -\frac{2}{t}$.

Note that since $Y_R = \overline{Y} = \dfrac{p}{\theta - 1} + \dfrac{tL}{2n(\theta - 1)}$, we have

$$p = Y_R(\theta - 1) - \frac{tL}{2n}.$$

For the first extreme, since the price response to demand is $\dfrac{\partial D_j}{\partial p_j} = -\dfrac{1}{t}$, proceeding as in case (3) we can derive the equilibrium price and number of firms as

$$p = c + \sqrt{\frac{tF}{1 + f_P}}, \text{ and } \frac{L}{n} = \frac{1}{t(1 - f_P)}\left[(1 + 3f_P)\sqrt{\frac{tF}{1 + f_P}} - 2f_P\left(Y_P(\theta - 1) - c\right)\right].$$

Since $p = c + \sqrt{\dfrac{tF}{1 + f_P}}$ and, at the same time, $p = Y_R(\theta - 1) - \dfrac{tL}{2n}$, this implies

$$\frac{L}{n} = \frac{2}{t}\left[Y_R(\theta - 1) - c\right] - \frac{2}{t}\sqrt{\frac{tF}{1 + f_P}}.$$

But we have $\dfrac{L}{n} = \dfrac{1}{t(1 - f_P)}\left[(1 + 3f_P)\sqrt{\dfrac{tF}{1 + f_P}} - 2f_P\left[Y_P(\theta - 1) - c\right]\right]$. It follows that this extreme case arises under the special circumstance when

$$\left[f_R Y_R + f_P Y_P\right](\theta - 1) - c = \frac{3 + f_P}{2}\sqrt{\frac{tF}{1 + f_P}}. \tag{A.9}$$

For the other extreme, since the price response to demand is $\dfrac{\partial D_j}{\partial p_j} = -\dfrac{2}{t}$, using the first-order condition, demand structure and the zero-profit condition we derive

$$p = c + \sqrt{\frac{tF}{2}}, \text{ and } \frac{L}{n} = \sqrt{\frac{2F}{t}} + \frac{2f_P\left(Y_R - Y_P\right)(\theta - 1)}{t}.$$

Proceeding as above it now follows that this extreme case arises under the specific parameter values where

$$\left[f_R Y_R + f_P Y_P\right](\theta - 1) - c = \sqrt{2tF}. \tag{A.10}$$

At the same time, $Y_P > \underline{Y}$ implies, for this extreme case,

$$\frac{c + \sqrt{\frac{tF}{2}}}{(\theta - 1)} < Y_P. \tag{A.11}$$

Combining (A.9), (A.10) and (A.11) and the fact that the lower bound for Y_P is $\dfrac{c + \sqrt{\frac{tF}{1 + f_P}}}{(\theta - 1)}$ under case (3) which is just the other extreme for case (5), we conclude that case (5) arises when

$$
\begin{array}{rcl}
\dfrac{c+\sqrt{\dfrac{tF}{2}}}{(\theta-1)} & < & Y_P < \dfrac{c+\sqrt{\dfrac{tF}{1+f_P}}}{(\theta-1)} \\
& \text{and} & \\
\dfrac{c+\sqrt{2tF}}{(\theta-1)} & < & f_R Y_R + f_P Y_P < \dfrac{c+\dfrac{3+f_P}{2}\sqrt{\dfrac{tF}{1+f_P}}}{(\theta-1)}.
\end{array}
$$

A.3 Single-Income Economy

Consider a circular city where at each point of the city there is a measure 1 of consumers with the same income Y. Since there is only one income, we have the following three cases to consider:

(1) $Y > \overline{Y}$;

(2) $Y = \overline{Y}$;

(3) $\underline{Y} < Y < \overline{Y}$.

Case (1): $\boldsymbol{Y} > \overline{Y}$:

From the demand structure discussed in section 2.1 it follows that:

$$
\begin{aligned}
D_j &= \delta_{j,j+1} + \delta_{j,j-1} \\
&= \frac{(p_{j-1} + p_{j+1} - 2p_j) + 2t\left(\dfrac{L}{n}\right)}{2t}.
\end{aligned}
$$

This implies that $\dfrac{\partial D_j}{\partial p_j} = -\dfrac{1}{t}$. Now, similar to the analysis of the two-income groups, using the first-order condition, demand structure and the zero-profit condition we derive

$$
\begin{aligned}
p &= c + \sqrt{tF}, \\
\frac{L}{n} &= \sqrt{\frac{F}{t}}.
\end{aligned}
$$

Substituting these equilibrium values of price and number of firms into the expressions for $\overline{Y}$ we conclude that case (1) arises when

$$Y > \frac{c + \frac{3}{2}\sqrt{tF}}{(\theta - 1)}.$$

In this case firms compete for the consumers and all consumers are served.

Case (2): $\mathbf{Y} = \overline{Y}$:

This, once again, is a case of a 'kinked equilibrium'. One extreme of the kink is case (1) described above where the price response to demand is given by $\frac{\partial D_j}{\partial p_j} = -\frac{1}{t}$. For the other extreme, demand is given by

$$\begin{aligned} D_j &= \eta_{j,j+1}(Y) + \eta_{j,j-1}(Y) \\ &= 2\left[\frac{Y(\theta - 1) - p_j}{t}\right], \end{aligned}$$

so that the price response to demand is $\frac{\partial D_j}{\partial p_j} = -\frac{2}{t}$.

Note that since $Y = \overline{Y} = \frac{p}{\theta - 1} + \frac{tL}{2n(\theta - 1)}$, we have

$$p = Y(\theta - 1) - \frac{tL}{2n}.$$

For the first extreme, since the price response to demand is $\frac{\partial D_j}{\partial p_j} = -\frac{1}{t}$, proceeding as in case (1) we can derive the equilibrium price and number of firms as

$$p = c + \sqrt{tF}, \text{ and } \frac{L}{n} = \sqrt{\frac{F}{t}}.$$

Since $p = c + \sqrt{tF}$ and, at the same time, $p = Y(\theta - 1) - \frac{tL}{2n}$, this implies

$$\frac{L}{n} = \frac{2}{t}\left[Y(\theta - 1) - c\right] - 2\sqrt{\frac{F}{t}}.$$

But we have $\frac{L}{n} = \sqrt{\frac{F}{t}}$. It follows that this extreme case arises under the special circumstance when

$$Y(\theta - 1) - c = \frac{3}{2}\sqrt{tF}. \tag{A.12}$$

For the other extreme, since the price response to demand is $\frac{\partial D_j}{\partial p_j} = -\frac{2}{t}$, using the first-order condition, demand structure and the zero-profit condition we derive

$$p = c + \sqrt{\frac{tF}{2}}, \text{ and } \frac{L}{n} = \sqrt{\frac{2F}{t}}.$$

Proceeding as above it now follows that this extreme case arises under the specific parameter values where

$$Y(\theta - 1) - c = \sqrt{2tF}. \tag{A.13}$$

Combining (A.12) and (A.12) we conclude that case (2) arises when

$$\sqrt{2tF} < Y(\theta - 1) - c < \frac{3}{2}\sqrt{tF}.$$

In this case also all the consumers are served, but the marginal consumer who is indifferent between two adjacent firms is also indifferent between buying and not buying.

Case (3): $\underline{Y} < \mathbf{Y} < \overline{Y}$ **:**

This is the second extreme of case (2) discussed above where demand is given by

$$\begin{aligned} D_j &= \eta_{j,j+1}(Y) + \eta_{j,j-1}(Y) \\ &= 2\left[\frac{Y(\theta - 1) - p_j}{t}\right], \end{aligned}$$

so that the price response to demand is $\frac{\partial D_j}{\partial p_j} = -\frac{2}{t}$. As above using the first-order condition, demand structure and the zero-profit condition we derive

$$p = c + \sqrt{\frac{tF}{2}}, \text{ and } \frac{L}{n} = \sqrt{\frac{2F}{t}}.$$

In equilibrium $D_j = \frac{L}{n}$. Then $D_j = 2\left[\frac{Y(\theta-1) - p_j}{t}\right]$ and $p = c + \sqrt{\frac{tF}{2}}$ give

$$\frac{L}{n} = \frac{2}{t}\left[Y(\theta - 1) - c - \sqrt{\frac{tF}{2}}\right].$$

Since $\frac{L}{n} = \sqrt{\frac{2F}{t}}$, and, at the same time, $\frac{L}{n} = \frac{2}{t}\left[Y(\theta - 1) - c - \sqrt{\frac{tF}{2}}\right]$, it follows that

$$Y(\theta - 1) - c = \sqrt{2tF}.$$

So we conclude that case (3) can occur under this limiting case where $Y(\theta - 1) - c = \sqrt{2tF}$. Following Salop (1979) we can ignore this limiting case.

- From the analysis of the three cases under the single-income economy it is clear that the minimum income required for any firm to operate is such that $Y(\theta - 1) - c = \sqrt{2tF}$. That is, the feasibility income threshold is $\dfrac{c + \sqrt{2tF}}{(\theta - 1)}$.

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