On the Growth of the Services Sector

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Abstract

It aims at explaining why/how the services sector may grow faster than manufacturing. It develops a two-sector, closed-economy model, having a manufacturing sector and a services sector. Accumulation of human capital serves as the basis of growth. The analysis focuses on business services, although household services are considered. It is argued that differences in returns to scale between the two sectors and employment frictions in manufacturing underlie how the growth rate of the services sector may be higher. Conditions behind how within the services sector the business services sub-sector may grow faster than household services are identified.

JEL Classification: J24; L60; L80; O41
Keywords: Business services; Household services; Manufacturing; Returns to scale, Employment frictions; Economic growth

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1 Introduction

To understand the growth process, the coarsest disaggregation of a macro economy typically contains the industrial or the manufacturing sector, and, agriculture. The former is viewed as the modern sector upon which hinges the overall growth of the economy, while the latter is considered as the traditional, primary sector catering to the most primary need for existence – food. Starting from Lewis (1954), there are numerous two-sector models, e.g., Uzawa (1961), Matsuyama (1992) and Hayashi and Prescott (2008), among many others.

The experience of many economies in the post WWII era has become increasingly divorced from this traditional depiction of an economy. Over years, it is the services sector that has overtaken manufacturing as the 'leading' or the largest sector of a modern economy. In many countries, it now constitutes more than 50% of the GDP, and, moreover, still growing faster than manufacturing.

Figure 1 depicts the dynamics of sectoral composition of the economies of U.S., U.K. and Japan into manufacturing, services and agriculture. By 1970 the service sector constituted at least 50% of GDP of these countries and over time, its share has been growing – implying that the service sector is growing faster than the other two.

Table 1: Compounded Annual Growth Rates (%): Manufacturing and Services

	1970-90		1990-2006		1970-2006	
Country	Manuf.	Services	Manuf.	Services	Manuf.	Services
US	1.8	3.9	2.2	3.6	2.0	3.6
UK	-0.5	3.1	0.9	3.9	0.2	3.4
Japan	3.9	4.2	0.6	1.6	2.2	2.8

Source: EU KLEMS[1]

Table 1 lists the annual compound growth rates of the manufacturing sector and the services sector in the same three countries over 1970-2006. It is clear that the latter has out-paced the former.

Figure 2 is the analog of Figure 1 for three newly emerging markets, namely, Brazil, China and India. In Brazil a boom in the services sector occurred the mid nineties and since then its (dominant) share has remains somewhat unchanged. In China, both manufacturing and services are growing at a similar pace, while the former remains the dominant sector. In India, the service sector crossed

[1]It is a statistical and analytical research project funded by the European Commission, meant to create two databases (one for each type of research). It contains measures of economic growth, productivity, employment etc. for all EU member countries (except Bulgaria and Romania), U.S., Japan and Canada.

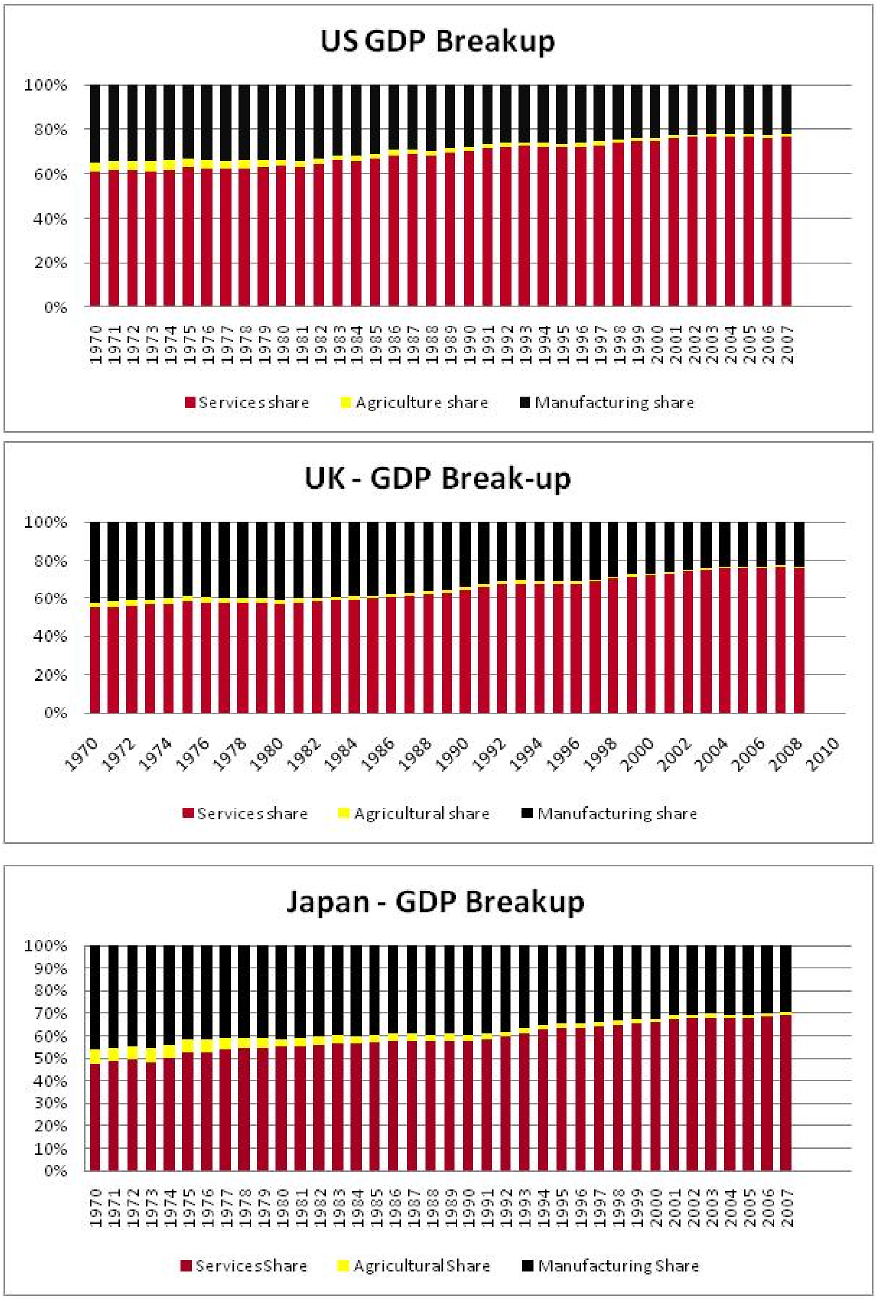

Figure 1: Share of Manufacturing, Services and Agriculture in real GDP: 1970-2006; *Source:* World Development Indicators, World Bank

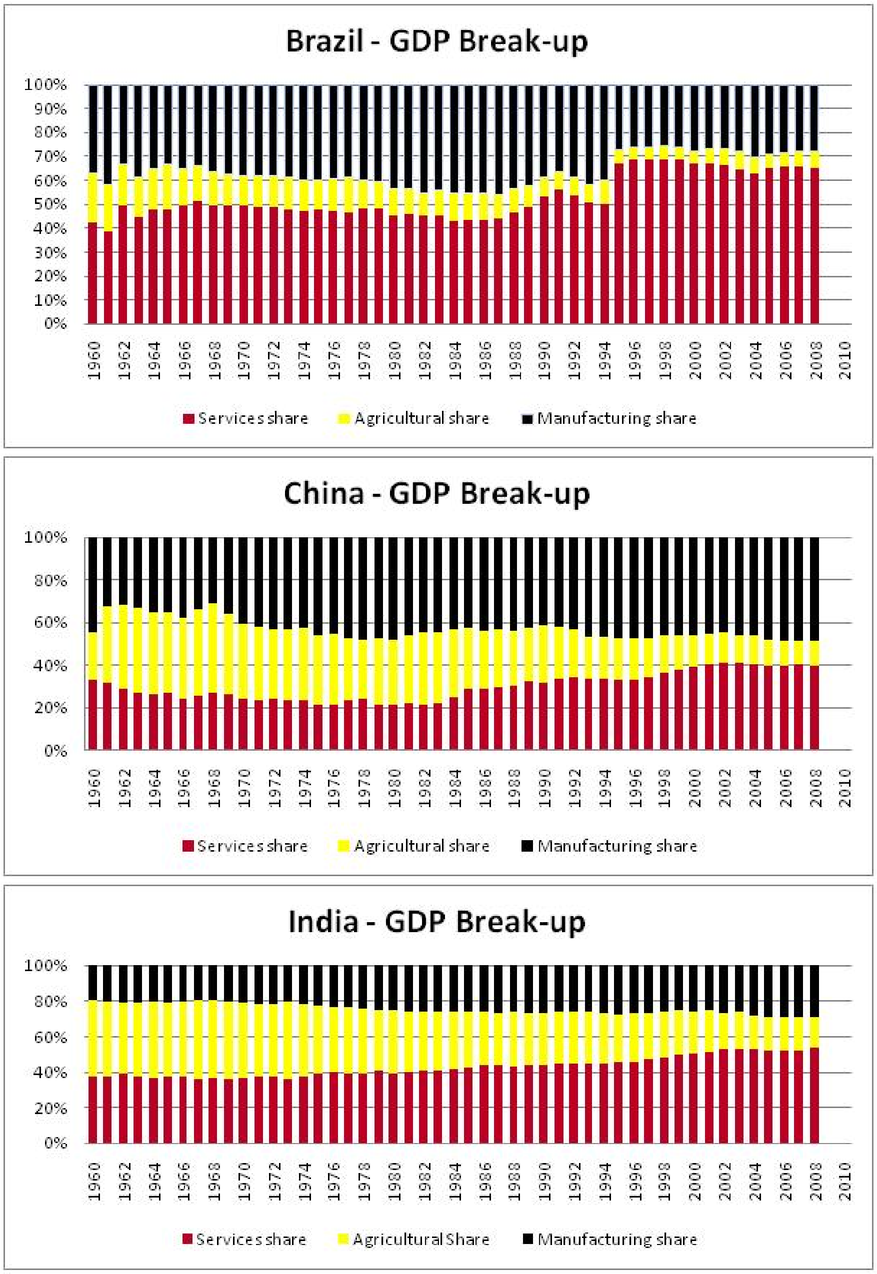

Figure 2: Share of Manufacturing, Services and Agriculture in real GDP: 1970-2006; *Source:* World Development Indicators, World Bank

50%-share around 2000, and, has been growing faster than manufacturing and agriculture.

Buera and Kaboski (2009) emphasize that the growth of services is driven by that of consumer services. In 2000, services formed 80% of household consumption in US while in the same year the services sector's share was just over 45% in the value-added. Services in consumption have shown a slightly higher growth in the period 1950-2000 than services in value added.[2]

Eichengreen and Gupta (2009) present an empiricial study of the growth of services along with per capita income across sixty countries from 1950-2005. They identify two waves of growth in this sector, one in countries with low level of per capita GDP and the second with higher levels of per capita GDP. The second jump in the services is due to growth of services that are receptive to applications of information technology and are increasingly tradable across borders.

Sectoral employment shares in U.S., U.K. and Japan are exhibited in Figure 3. Employment in the services sector is expanding faster than in manufacturing. Indeed, the pattern is stronger than what is revealed in Figure 3: as Table 2 shows, there is indeed a declining trend of employment in manufacturing.

Ngai and Pissarides (2008) note that TFP growth rate is higher in manufacturing than in the services sector. This pushes labor out of the former sector to the latter sector. Employment declines in manufacturing and grows in the services sector.

Table 2: Growth Rate of Sectoral Employment (in %)

	1970-90		1990-06		1970-06	
Country	Manuf.	Services	Manuf.	Services	Manuf.	Services
US	-0.2	2.9	-1.7	1.8	-1.0	2.2
UK	-2.7	1.1	-3.0	1.4	-2.8	1.2
Japan	0.4	2.0	-1.6	1.3	-0.5	1.6

Source: EU KLEMS

The growth-employment patterns shown in Figures 1–3 suggest the following stylized facts:

In developed economies, over the last forty years or so

[2] They build a theoretical model to explain demand shifts to skill-intensive consumption services as income rises. Their model is consistent with a rising trend in skill acquisition and skill premium, a rising relative price of services and product cycles between home and market production of services.

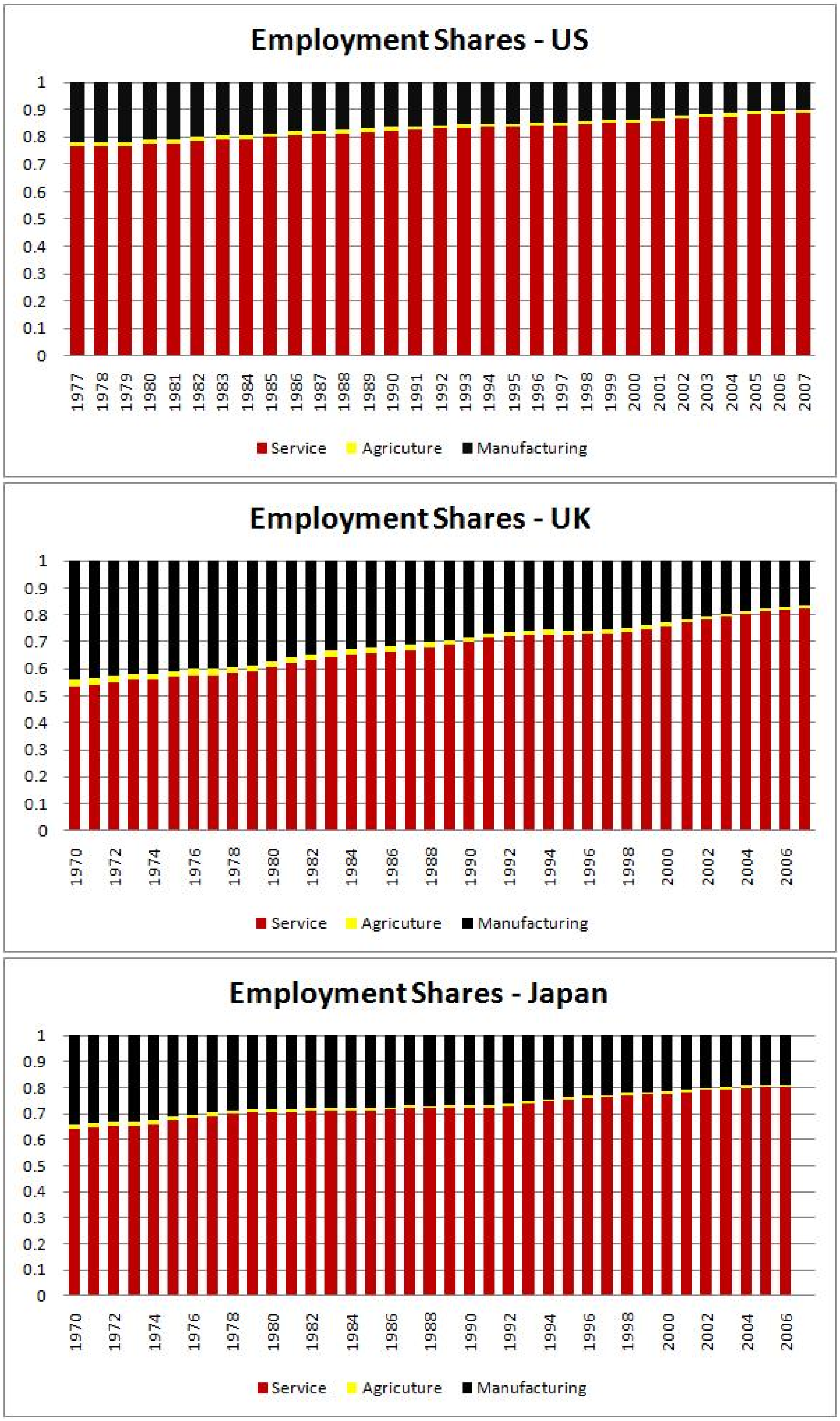

Figure 3: Employment Shares of Manufacturing, Services and Agriculture: 1970-2006; *Source:* EU KLEMS

Fact 1. The services sector has been growing faster than manufacturing.

Fact 2. Employment in the services sector has grown while that in the manufacturing has fallen.

Comparing the sectoral output and employment growth rates from Tables 1 and 2,

Fact 3: Output growth rates exceed employment growth rates in both manufacturing and services.[3]

There are various kinds of services. National accounts of most countries have roughly classified services into: Wholesale and Retail Trade; Hotels and Restaurants; Transport, Storage, Post and Telecommunications; Finance, Insurance, Real Estate and Business Services; Community, Social and Personal Service; Electricity, Gas and Water Supply; Construction. It is not true that all sub-sectors of the service sector have grown uniformly. We can broadly classify various services into two types: business services and non-business (or other) services.

While non-business services constitute the lion's share in the sector, as Figure 4 illustrates, in the U.S., U.K. and Japan it is the business service sub-sector which is growing faster. Table 3 records the share of this sub-sector at three points of time. Over the span of thirty-seven years, its share has nearly or more than doubled in the three countries.

Table 3: Share of Business Services (% of Total Services)

Country	1970	1990	2007
US	6.65	11.27	14.77
UK	8.64	12.96	17.04
Japan	4.50	9.11	13.92

Source: EU KLEMS

From Table 3, we may deduce

[3]That is, output per worker has increased in both sectors.

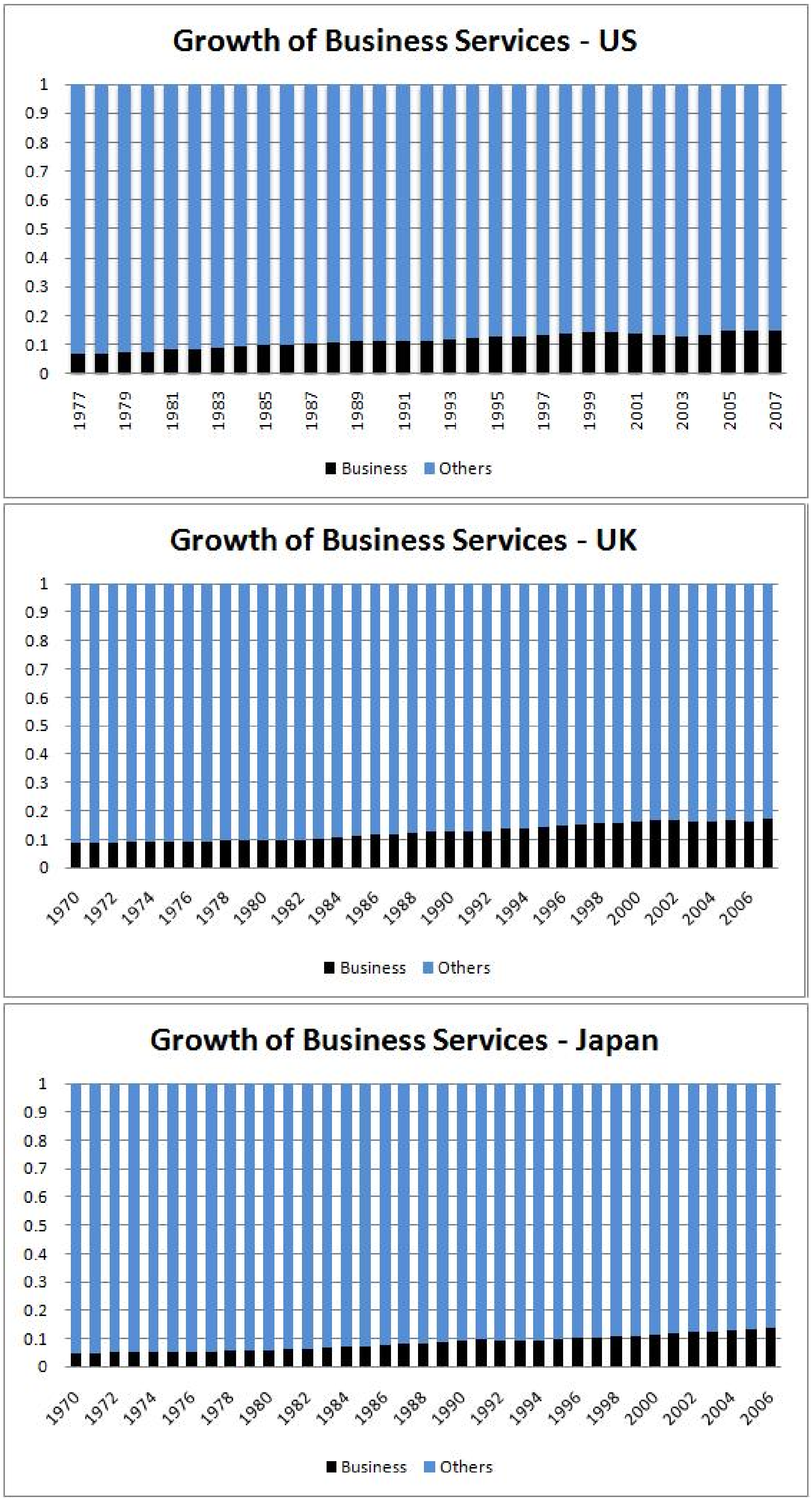

Figure 4: The Share of Business Services in Total Services: 1970-2006; *Source:* EU KLEMS

Fact 4: Business services have grown faster than non-business services.

It is worth-noting that he business services data in Table 3 and Figure 4 includes outsourcing activities. Hence some critics have pointed that the growth of business services might just be an 'accounting' phenomenon. The tasks which were performed in-house by the manufacturing firms are now bought from service firms. However, the growth of business services does not seem to be primarily driven by outsourcing. According to Kox and Rubalcaba (2007), outsourcing can explain only a small part of the growth of business services. There are reasons behind this.

First, the IT revolution in the 1970s led to application of technology in novel ways which itself led to creation of *new* services (such as internet, market research and consultancy). Second, as Beyers and Lindahl (1996) have found the need for specialized knowledge is by far the most important factor behind the demand for producer services.[4] Third, as observed by Kox (2001), the services rendered by the business services suppliers are superior to the prior in-house service activities of the outsourcing firm.[5]

Not only the relative output and employment between manufacturing and services sectors have been changing over time, relative price movements have also occurred. Figure 5 plots the trend in the price of business services and total services relative to manufacturing in the U.S., U.K. and Japan. Thus,

Fact 5: The price of services relative to manufacturing has been rising.

The objective of this paper is to provide a rationale behind *some* of the above stylized facts.

There are a number of studies, attributing the rising share of services in GDP to preferences changes accompanying economic development. In the long run, the argument goes, the rise in real income shifts demand from agricultural goods to manufacturing goods and then to services.[6]

The manufacturing sector outgrowing the agricultural sector is understandable in terms of the preference-shift hypothesis. But the services sector outpacing manufacturing does not seem to be adequately explained by this hypothesis, since

[4]This also explains why the growth of business services began post late 1960s and not before.

[5]Raa and Wolff (2000) find that the use of business services had led to higher total factor productivity growth in manufacturing - clearly indicating the additional benefit of business services over in-house services. Kox and Rubalcaba (2007) find that in Europeon Union business services has generated knowledge and productivity spillovers in other industries.

[6]See, for example, Smith (2001), Fisher (1933).

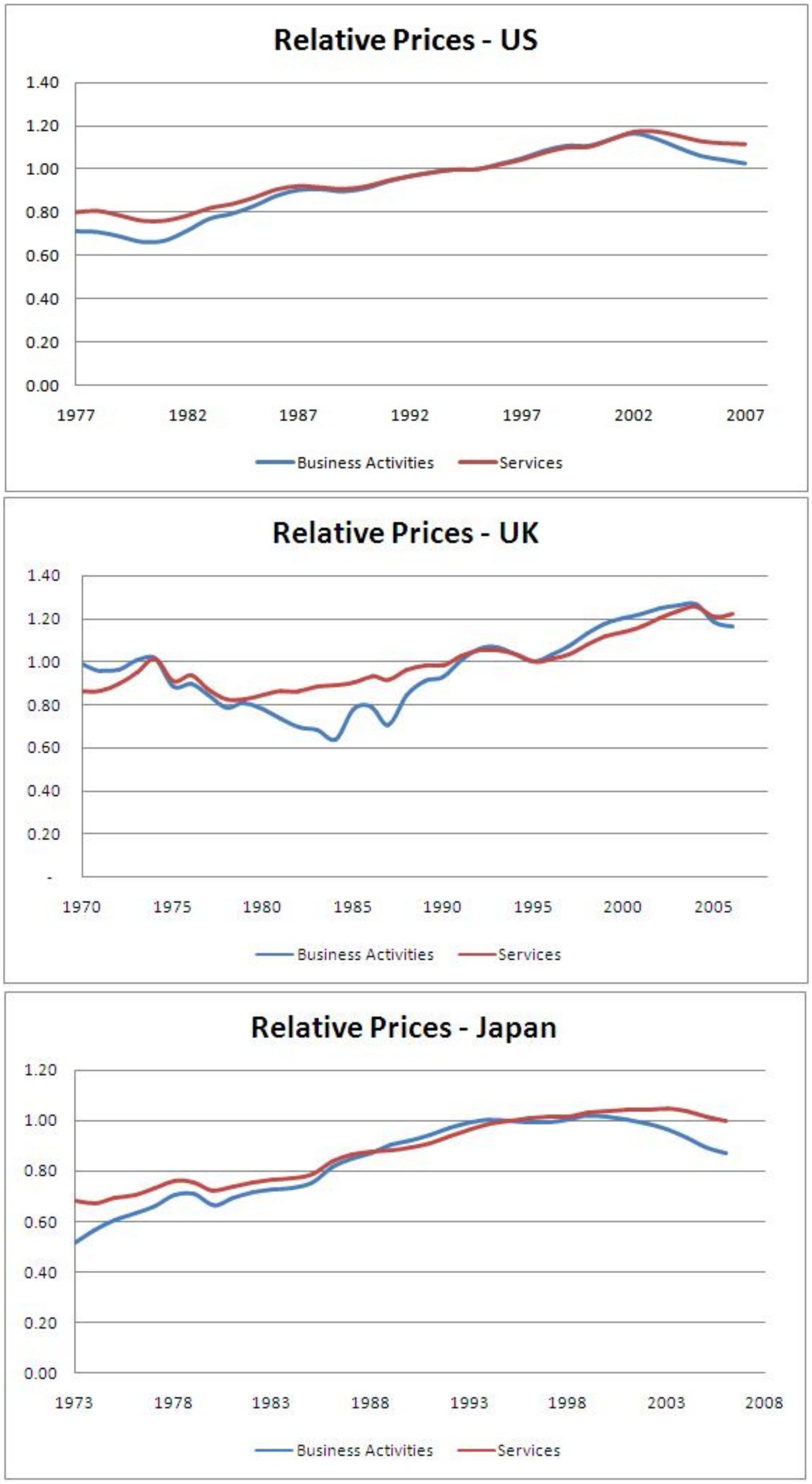

Figure 5: The Rise in the Relative Price of Business and Total Services vis-a-vis Manufacturing: 1970-2006; *Source:* EU KLEMS

the argument is applicable to consumer services, not business services. It is not obvious how a derived sector like business services may grow faster than the 'parent' sector, manufacturing.

It has been argued that as a manufacturing firm grows in size it may prefer not to hire employees for menial and conventional tasks and thus outsource these jobs to some service firms; see Goodman and Steadman (2002). The reason behind this has to do with labor problems associated with a large labor force, such as large scale shirking, lack of effective supervision and paperwork. Beep Technologies, for example, quotes in its website a case of a computer chip maker hiring a staffing company to monitor and manage all of its non-exempt hiring. There is another case of a university hiring an information technology company to manage its entire desktop, PCs and computer network. Note that a small firm or organization would have found it more economical to do these tasks in-house rather outsource them.

Even in a scenario where there are no labor 'problems' as such, some believe that it may be simply inefficient to employ workers than to buy the relevant services. Quinn (2000) writes that in current times the only way of staying ahead in business is by outsourcing innovation, as innovation calls for a complex knowledge which only a broad network of specialists can offer. Leading companies have lowered innovation costs and risks by 60 to 90 per cent while similarly decreasing cycle times.[7]

The preceding argument alludes to some notions of labor congestion or friction in the manufacturing sector.

The main purpose of this paper is to build a simple theoretical model that is independent of the service-oriented relative demand shift, although we do consider it. Specifically, two mechanisms are explored, both hinging on the existence of some fundamental differences in technologies of producing manufacturing and services.

The first assumes that returns to scale are less in manufacturing than in the services sector. If so, it is easy to see how the latter may grow faster than the former. Suppose that services are produced by one input, labor, and, industry-level output in the services sector is related one-to-one with labor employment in that sector – that is, there are constant-returns in producing services at the industry level. In contrast, let manufacturing output vary with labor and business services under a decreasing-returns technology. Suppose that in the steady state employment in both sectors grows at the same rate. It follows immediately that labor employment and service output would grow at the same rate, while manufacturing output would grow at a lesser rate. The implication is that a

[7]Cycle times refer to the total time it takes to complete one batch or shift of some specified manufacturing and allied operations.

sector, namely services, whose existence is derived from demand by another, that is, manufacturing, can grow faster.

Our second 'story' allows labor frictions in manufacturing, which leads to the outcome that employment in manufacturing grows slower than that in the services sector and therefore manufacturing grows at the lower rate.

To place this paper in its perspective, the following may be noted.

1. Our analysis abstracts from TFP changes in both manufacturing and services. Thus, the ranking of growth rates of *intra*-sector employment and output (*Fact 3*) is not our focus.

2. Any model to adequately explain the recent surge of the service sector must take into account the role of IT services – particularly in the services sector. But we abstract from that, and, thus, the model is purported to provide some understanding of how and why the services sector grew faster than manufacturing *before* the advent of the IT revolution.

3. There is no denying that the service-oriented relative demand shift has considerable explaining power behind higher growth of the services sector as a whole. As mentioned earlier, we do incorporate it in our analysis.

The paper is organized as follows. Section 2 develops our basic model that features differences in returns to scale. There are two sectors, one producing a manufacturing good with the help of labor and business services and the other business services with the help of labor only. Employment frictions in manufacturing are introduced in Section 3. Section 4 considers a more general technology in the services sector, allowing for manufacturing as an input. Consumption services are introduced in Section 5, where it is assumed that the demand for consumption services has unitary elasticity with respect to income. In Section 6, preferences are defined such that the income elasticity of demand for services exceeds one, and, hence, there is a relative demand shift towards consumption services as income expands are examined. This section examines the ramifications of this toward sectoral growth processes. Section 7 concludes the paper.

2 The Basic Model: Difference in Returns to Scale

The source of growth *per se* is not our central concern. How growth rates may differ across sectors is our focus. In what follows, a simple human-capital-accumulation based growth story will be developed.

We consider a closed economy having two sectors: manufacturing and services. The former produces a homogeneous good – which is the numeraire – in a perfectly competitive market. Following Eswaran and Kotwal (2002) and Matsuyama (2010) the service-sector output is differentiated, produced in a monopolistically competitive market. Services are used as inputs in the manufacturing sector; they are not consumed by households.

The core assumption is that manufacturing is subject to diminishing-returns to scale, while increasing returns prevail at the firm level in the services sector. More generally, higher returns to scale in the services sector – not necessarily increasing returns in that sector and decreasing-returns in manufacturing – would imply higher growth in the services sector.

Such differences in returns to scale have empirical support.

There are numerous empirical studies on returns to scale in various industries, especially in manufacturing. For the U.S. economy, Basu et al. (2006) present returns to scale estimates for twenty-one manufacturing industries, which are updated from the earlier study by Basu and Fernald (1997). For manufacturing as a whole, there is evidence of decreasing returns for gross output (less so for value-added), while there are increasing returns to scale for durable manufacturing and decreasing returns of non-durable manufacturing. For Philippines, evidence of mildly decreasing returns to scale in manufacturing is found by Yamagata (2000). For developing countries in general, Tybout (2000) reports constant or mildly increasing returns.[8]

Relatively fewer estimates that are available on returns to scale in the services sector indicate increasing returns. For the U.S., Basu et al. (2006) report scale elasticities for transportation, communication, trade and a service basket including health, education, legal services, automotive repair, hotel business etc., which generally exceed unity. Scale economies are also found for retail trade in Israel (Ofer (1973)), banking and finance in the U.S. (McAllister and McManus (1993)) and hospital industry in the U.S. (Berry (1967) and Wilson and Carey (2004)).

[8] By using data on trade flows and factor content relations, Antweiler and Trefler (2002) estimate cost price equations and indirectly infer scale elasticities of industries in the manufacturing sector. The methodology permits to test whether returns to scale are constant or increasing, when the same industry is pooled across trading countries. Increasing-returns are found in about one-third of industries in the sample, constant-returns in another one-third and for the remaining it is inconclusive. There are no service industries in their sample.

2.1 The Manufacturing Sector

A manufacturing firm uses two variable inputs, labor and services, and returns to scale are diminishing. Implicitly, a fixed factor is present, which earns profits.[9,10] Normalizing the fixed input to unity, let

$$q_{mt} = L_{mt}^{\alpha} q_{st}^{\beta}, \ \alpha, \ \beta > 0, \ \alpha + \beta < 1, \tag{1}$$

be the production function, where L_{mt} is labor in effective units (to be clear later) and q_{st} is a composite of business services. At any time there are N_t varieties of service inputs and

$$q_{st} = \left(\int_0^{N_t} q_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1,$$

where q_{it} is the amount used of service variety i and σ is the elasticity of substitution between any two service inputs.

Following the adaptation by Ciccone and Matsuyama (1996) of the standard Dixit-Stiglitz consumption demand system to demand for intermediates, we calculate the price of the composite service input - which is the price at which the total expenditure on all individual services equals the expenditure on the composite service input, i.e., $p_{st} q_{st} = \int_0^{N_t} p_{it} q_{it} di$. The composite price has the expression

$$p_{st} = \left(\int_0^{N_t} p_{it}^{-(\sigma-1)} di \right)^{-\frac{1}{\sigma-1}}. \tag{2}$$

Under symmetry,

$$q_{st} p_{st} = N_t q_{it} p_{it}. \tag{3}$$

We can further use (2) to write the demand for each service input as

$$q_{it} = \left(\frac{p_{it}}{p_{st}} \right)^{-\sigma} q_{st}. \tag{4}$$

Profit maximization yields the standard first order conditions:

$$\alpha L_{mt}^{\alpha-1} q_{st}^{\beta} = w_t \tag{5}$$

$$\beta L_{mt}^{\alpha} q_{st}^{\beta-1} = p_{st}, \tag{6}$$

[9] We may interpret the fixed factor as land. Indeed, in recent decades land has become a major issue in manufacturing. Acquiring land for establishing or expanding manufacturing is getting increasingly costly and growing environmental regulations have led to stringent limitations for the use of acquired land towards industrial activities.

[10] In his two-sector growth model Matsuyama (1992) also assumed decreasing returns technology for manufacturing.

where w_t is the wage rate.

Labor is measured in efficiency units and it grows over time. The growth process will be specified later, but, at the moment it is to be noted that w_t is the wage rate per such efficiency unit, *not* earnings per worker per unit of time; see, for instance, Jung and Mercenier (2010).

2.2 The Services Sector

A service provider supplies a unique brand. The production technology of any service is of the simple Dixit-Stiglitz-Krugman kind that is linear and satisfies increasing-returns:

$$q_{it} = L_{it} - 1,$$

where the fixed labor requirement has been normalized to unity. Firm i faces the demand function (4). One obtains the standard constant-mark-up condition:

$$\frac{p_{it}}{w_t} = \frac{\sigma}{\sigma - 1}. \tag{7}$$

Together with the zero-profit condition, the employment and output produced by each firm are fixed by technology and preference parameters:

$$L_{it} = \sigma; \quad q_{it} = q_t = \sigma - 1. \tag{8}$$

It follows that total employment as well as total output in the services sector is one to one related with the number of varieties or firms, N_t. In this sense, this sector exhibits constant-returns in the aggregate.

Further, we have

$$q_{st} = N_t^{\frac{\sigma}{\sigma-1}} q_t = N_t^{\frac{\sigma}{\sigma-1}} (\sigma - 1). \tag{9}$$

Note that $q_{st} > N_t q_t$, implying productivity gains in manufacturing from use of service varieties.[11]

2.3 General Equilibrium

Using the last expression the manufacturing output equals

$$q_{mt} = N_t^{\frac{\beta\sigma}{\sigma-1}} L_{mt}^{\alpha} (\sigma - 1)^{\beta}. \tag{10}$$

We can state the first-order condition (5) as

$$\frac{\alpha q_{mt}}{L_{mt}} = w_t. \tag{11}$$

[11] See Ethier (1982), Romer (1987) and Matsuyama (2010).

Next, divide (5) by (6) and use (3) and (8) to obtain

$$\frac{\alpha(\sigma-1)}{\beta}\frac{N_t}{L_{mt}}=\frac{w_t}{p_{it}}. \tag{12}$$

By substituting the constant mark-up condition in the services sector into the above yields

$$\frac{\alpha\sigma N_t}{\beta L_{mt}}=1. \tag{13}$$

It implies that the ratio of employment in the two sectors is constant over time.

The static general equilibrium of the production side of this economy is solved by (10), (11), (13) and the following full employment condition:

$$L_{mt}+\sigma N_t=\bar{L}_t, \tag{14}$$

in which the expression of L_{it} in (8) has been used and $\bar{L}_t$ is the total supply of the labor input at t. The four equations solve q_{mt}, L_{mt}, N_t and w_t.

By elimination, it is straightforward to obtain

$$w_t=k\bar{L}_t^{-(1-\alpha-\frac{\beta\sigma}{\sigma-1})},\quad \text{where } k\equiv\frac{\alpha^{\alpha}\beta^{\frac{\beta\sigma}{\sigma-1}}(\sigma-1)^{\beta}}{\sigma^{\frac{\beta\sigma}{\sigma-1}}(\alpha+\beta)^{\alpha+\frac{\beta\sigma}{\sigma-1}-1}}>0. \tag{15}$$

Consider the effect of an increase in $\bar{L}$. Notice that the exponent of $\bar{L}_t$ can be positive, and, if so, it would imply $dw_t/d\bar{L}_t>0$, meaning instability in the labor market. The source of the instability lies in increasing returns with respect to labor in the services sector at the firm level. We 'need' to assume the stability condition that α and β be small and σ be large enough relative to each other – such that the exponent of $\bar{L}_t$ in (15) is negative; that is,

$$\alpha+\frac{\beta\sigma}{\sigma-1}<1,^{12} \tag{A1}$$

so that $dw_t/d\bar{L}_t<0$.

2.4 Households

The economy consists of infinitely lived representative households, who can be treated as one unit. At a given point of time, the representative household possesses L_t units of effective labor and one unit of time. It could spend its

[12] This condition also implies stability in terms of labor movement from one sector to the other *a la* Neary (1978).

time in either augmenting its human capital or working in production sectors. Let $H_t \in (0, 1)$ denote time in human capital investment and let

$$L_{t+1} = a_L H_t L_t, \quad a_L > 1. \tag{16}$$

Thus the growth rate of human capital is proportional to the time invested in human capital. Since there are no education sectors, eq. (16) can be seen as a self-learning function. The tradeoff is that the higher the investment in human capital, the greater will be the effective labor and hence the higher will be the total wage earnings in the future, but the less will be the total earnings in the current period.

There are two sources of income: wage income in both sectors and profit income in manufacturing (π_m). In making consumption decisions profit income treated as exogenous by a household.

The household consumes the manufacturing good only, not services. Denoting the discount factor by ρ, amount consumed of the manufacturing good by c_{mt} and assuming the felicity function $\ln c_{mt}$, its problem is to maximize $\sum_{t=0}^{\infty} \rho^t \ln c_{mt}$, subject to (16) and the budget $c_{mt} \leq w_t \bar{L}_t + \pi_{mt}$, where $\bar{L}_t \equiv (1 - H_t)L_t$ is the total effective labor working in the production sectors.

The household chooses $\{c_{mt}\}_0^\infty$, $\{H_t\}_0^\infty$, $\{L_t\}_1^\infty$, given L_0. The following is the Euler equation:

$$\frac{c_{mt+1}/w_{t+1}}{c_{mt}/w_t} = \rho a_L. \tag{17}$$

We assume $\rho a_L > 1$, such that the c_{mt}/w_t ratio grows at a positive rate. A marginal increase in investment entails a marginal loss in terms of current utility equal to w_t/c_{mt} and entitles a marginal gain in terms of future utility equal to $a_L w_{t+1}/c_{mt+1}$. At the optimum, the former is equal to the discounted value of the latter.

2.5 Dynamics

Market clearing is given by the relation, $c_{mt} = q_{mt}$. Substituting it into the Euler equation we observe that the q_{mt}/w_t ratio grows at the (gross) rate ρa_L. In static equilibrium the ratio, q_{mt}/w_t, is a function of $\bar{L}_t$. Rearranging (11), (13) and (14) we get,

$$\frac{q_{mt}}{w_t} = \frac{\bar{L}_t}{\alpha + \beta}, \tag{18}$$

It follows then that $\bar{L}_t$ grows at this rate too. The employment ratio between the two sectors remaining constant, employment in each sector grows at the rate ρa_L. There is no transitional dynamics.

Note that

1. Essentially, the statics and the dynamics of the macro economy are disjoint. The static effects of an exogenous increase in $\bar{L}$ map one to one into what happens over time.
2. Neither of the production sectors contributes to the *basis* of long-run growth – which is driven by the technology of human capital accumulation and the discount factor.

As total employment and total output in the service industry are one-to-one related in equilibrium, the service sector output grows at the rate ρa_L. We have already seen that an increase in $\bar{L}_t$ leads to a decline in w_t. Thus w_t decreases over time.[13] In view of (11), the manufacturing output grows at a rate less than ρa_L. Slower growth rate of manufacturing accords with *Fact 1*.

Proposition 1 *The employment levels in both sectors grow at the same rate, but output growth is faster in the services sector.*

Remarks

1. As the wage rate in terms of manufacturing falls over time, the constant mark-up condition (7) implies that p_{it} declines monotonically. That is, the relative price of the service sector (in terms of manufacturing) falls over time. Seen differently, because the services sector grows faster, its relative price in terms of manufacturing declines over time.[14] Our model thus does not accord with *Fact 5*, which says that the relative prices of business as well as total services in terms of manufacturing have been falling.
2. Note that while employment in manufacturing grows at the rate ρa_L, the output in the same sector grows at a lesser rate. This is not consistent with *Fact 3*. But it goes to highlight that it is the TFP growth in both sectors which may be critical in explaining the difference between employment and output within a sector, which our analysis abstracts from.

Dynamics of Learning

By definition, $\bar{L}_t \equiv (1 - H_t)L_t$. Hence,

$$\frac{(1 - H_{t+1})L_{t+1}}{(1 - H_t)L_t} = \frac{\bar{L}_{t+1}}{\bar{L}_t} = \rho a_L.$$

[13] This does *not* imply that households are worse off. Their total earnings, $w_t\bar{L}_t$, increase over time.

[14] A similar result holds in Matsuyama (1992). In his closed-economy model, the manufacturing sector grows faster than the agriculture sector. (The growth rate of the latter is zero in the model.) As a result, the price of the manufacturing relative to agriculture falls perpetually over time.

Using the learning function (16), the above equation reduces to

$$H_{t+1} - H_t = \frac{(1 - H_t)(H_t - \rho)}{H_t}. \tag{19}$$

Clearly and intuitively, along the steady state H_t is equal to ρ, the discount factor. Furthermore, if at any t, $H_t > \rho$ (or respectively $< \rho$), H_t increases (respectively decreases) towards unity (respectively zero). In other words, the steady state is unstable. Thus, under perfect foresight, H_t equals its steady state value ρ for all t. There is no transitional dynamics.

3 Employment Frictions in Manufacturing

Business cycle studies indicate that employment adjustment is sluggish over time. Bewley (1999) argued that during recessionary periods firms are unwilling to drastically reduce employment lest it should lower morale (and hence induce low productivity) among retained workers. According to Jeon and Shapiro (2007), limiting downsizing of employment during downturns signals a sense of turnaround in the near future and keeps workers' efforts away from alternative job search. As an economy begins to ride on a path of recovery, uncertainty about its 'permanency' hinders firms from a hiring binge (Bloom et al. (2009)). Thus, upward adjustment in employment is also typically slow.

Such employment frictions manifest in dynamic labor adjustment costs (of varying employment from one period to the next), and, there are empirical studies supportive of the presence of such costs, e.g., Nickell (1986), Burgess (1988) and Hamermesh and Pfann (1996). At a more basic level, there is considerable evidence of labor turnover – hiring and firing – costs; see, Bentolila and Bertola (1990) and Kulger and Saint-Paul (2000). Such costs imply some degree of inflexibility in employment changes.

Most empirical works on labor turnover pertains to industries in manufacturing. While we could not locate sector specific studies on labor turnover which compare it between manufacturing and services, there is a presumption in the literature that these costs are significantly less in the services sector. Many firms in this sector are informal and small in size and hence more flexible in adjusting employment.[15] To paraphrase Bertola (1992) who analyzes labor turnover costs, "employment is typically quite flexible for small firms and firms in the service sector." In their two-sector open economy model with a traded sector which is manufacturing and a non-traded sector which is services, Cosar et al.

[15]Lotti (2007) reports that for Italy, the average size of firms in the services sector is much smaller than that in manufacturing.

(2010) assume positive turnover costs in manufacturing, while the services sector is assumed to be frictionless.

The objective of this section is show that employment adjustment problems in manufacturing (relative to services) would imply higher growth rate of output *and* employment in the services sector compared to manufacturing. This goes to explain *Fact 1*, and, (partly) *Fact 2*.

However, for modeling convenience, instead of allowing for turnover costs explicitly, we modify the technology of the manufacturing sector which would imply inflexibility in employment variation. That is, as output expands there is proportionately less employment of labor and as output declines there is proportionately more employment of labor.

In other words, the output expansion path in the input space (at given factor-price ratio) must look like the one depicted in Figure 6. For a given wage rate and price of the business service input, as manufacturing output expands from A to B, the employment (L_m) increases proportionately less than the business services (q_s). Similarly when manufacturing output falls from A to C, the fall in labor is proportionately less than that in business services. Hence the adjustment in employment is less flexible whether output expands or contracts.

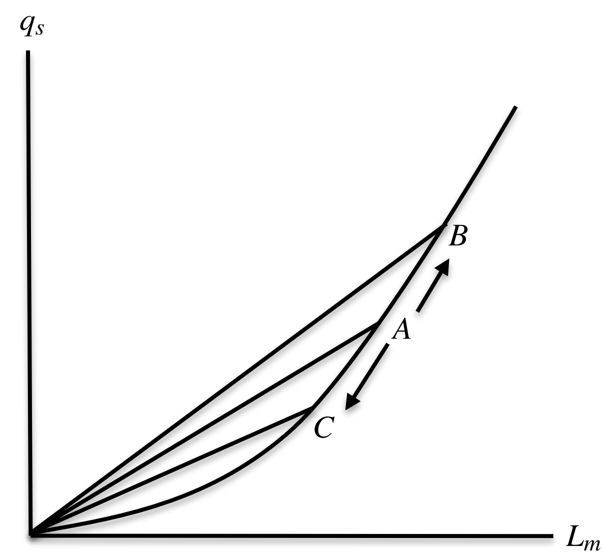

Figure 6: Output Expansion Path in the presence of Employment Frictions

The following production function for the manufacturing sector captures this feature.

$$q_{mt} = L_{mt}^{\alpha} q_{st}^{\beta} - \gamma L_{mt}, \; 0 < \alpha, \; \beta, \; \gamma > 0. \tag{20}$$

The term, $L_{mt}^{\alpha} q_{st}^{\beta}$, may be interpreted as gross output, whereas γL_{mt} can be thought of as a penalty or loss of output because of employment frictions. It means that the output loss does not result only from loss of labor time due to frictions (for example, there may be a loss of material property).

Furthermore, as we shall see, for the purpose of stability in the labor market, we would 'need' to continue with our assumption of decreasing returns to scale in manufacturing, i.e., $\alpha + \beta < 1$.

Notice that the production function (20) is non-homothetic, while the returns to scale are less than unity. Hence, the elasticity of input substitution is variable. Particularly, cost minimization would imply that in response to a proportionate increase in labor and composite service input costs, the proportional reduction in labor employment is less than that in the employment of the composite service input. Likewise, in the face of a proportional decrease in input prices, labor employment is increased less than proportionately compared to the service input. In this sense, γ is the measure of labor employment friction or inflexibility in manufacturing.

Remarks

1. Specification (20) allows for negative marginal product for labor in manufacturing – which can be interpreted as a *strong* congestion effect (whereas diminishing but positive returns for any level of employment may be seen as a situation of weak congestion effect). But, profit maximization would imply that in equilibrium the marginal returns to labor must be positive.[16] However, the *possibility* of negative returns has implications for equilibrium where the returns are positive.
2. Bruno (1968) had advocated such a production function (with the restriction of α and β summing to unity) as belonging to a class of variable elasticity of substitution production functions. It was called the constant marginal share production function.

The first-order condition with respect to labor now is $\alpha L_{mt}^{\alpha-1} q_{st}^{\beta} = w_t + \gamma$. The l.h.s. is equal to the marginal product of labor in producing the gross output, while the r.h.s. can be interpreted as the *effective* marginal cost of labor. The first-order condition with respect to the service input remains as in the basic model. Dividing the two first-order conditions,

$$\frac{\alpha q_{st}}{\beta L_{mt}} = \frac{w_t + \gamma}{p_{st}}.$$

Unlike in the basic model, if w_t and p_{st} decline proportionately, the ratio of q_{st}

[16] Interestingly, for a large public-sector conglomerate in India – SAIL (Steel Authority of India Limited) – in the steel industry, Das and Sengupta (2004) found evidence of negative marginal product of the managerial workforce.

to L_{mt} rises. This underlies why the growth rate of employment in manufacturing will be less than that in the services sector.

Using (8), the analogs of (10), (11) and (13) are:

$$q_{mt} = N_t^{\frac{\beta\sigma}{\sigma-1}} L_{mt}^{\alpha}(\sigma-1)^{\beta} - \gamma L_{mt} \tag{21}$$

$$\frac{\alpha q_{mt}}{L_{mt}} = w_t + (1-\alpha)\gamma \tag{22}$$

$$\frac{\alpha\sigma}{\beta}\frac{N_t}{L_{mt}} = \frac{w_t+\gamma}{w_t}. \tag{23}$$

The very last equation reflects that the ratio of employment between the two sectors is not time-invariant. We may express it as

$$\frac{N_t L_{it}}{L_{mt}} = \frac{\beta}{\alpha} \cdot \frac{w_t+\gamma}{w_t}, \tag{23$'$}$$

which says that ratio of employment in the two sectors is proportional to the ratio of effective marginal costs of hiring labor in the two sectors.

These equations along with the full-employment condition (14) characterize the static equilibrium of the economy. By appropriate substitutions, the following solution equation for the wage rate is obtained:

$$\frac{w_t+\gamma}{\left(\frac{w_t+\gamma}{w_t}\right)^{\frac{\beta\sigma}{\sigma-1}}} = \alpha(\sigma-1)^{\beta}\left(\frac{\beta}{\alpha\sigma}\right)^{\frac{\beta\sigma}{\sigma-1}}\left(\frac{\bar{L}}{1+\frac{\beta}{\alpha}\frac{w_t+\gamma}{w_t}}\right)^{-(1-\alpha-\frac{\beta\sigma}{\sigma-1})}. \tag{24}$$

Implicitly, $w_t = w(\bar{L}_t)$. It is easily verified that the condition (A1) ensures stability in the labor market, i.e., $dw(\bar{L}_t)/d\bar{L}_t < 0$.

The household's problem remains unchanged qualitatively. The Euler equation is the same. Using the manufacturing market clearing condition $q_{mt} = c_{mt}$, the ratio, q_{mt}/w_t grows at the rate ρa_L.

If we substitute (22) and the first-order condition (23) into the full-employment equation (14), we have

$$\frac{q_{mt}}{w_t} = \frac{\bar{L}_t}{\alpha+\beta}\left[1 + \frac{\gamma\alpha(1-\alpha-\beta)}{\alpha w(\bar{L}_t) + \beta(w(\bar{L}_t)+\gamma)}\right]. \tag{25}$$

The r.h.s. is monotonically increasing in $\bar{L}_t$. Hence $\bar{L}_t$ increases with q_{mt}/w_t. As the latter increases over time, $\bar{L}_t$ rises (without bound) and w_t falls over time.

It is easy to establish from (21) - (23) that, under the stability condition (A1), both q_{mt} and L_{mt} are negatively related to w_t. Hence both grow over time. In particular, in view of (23), as w_t decreases with time, N_t grows faster than L_{mt}.

That is, employment growth is higher in the services sector. This is main point of this section.

Dividing (22) by (23) gives

$$\frac{q_{mt}}{N_t} = \frac{\sigma}{\beta} \cdot \frac{w_t[w_t + (1-\alpha)\gamma]}{w_t + \gamma}.$$

The r.h.s. declines over time as the wage rate falls. Hence, q_{mt}/N_t ratio falls, implying that the growth rate of q_{mt} is less than that of N_t. Since the output of the service sector is one to one related to N_t,

Proposition 2 *In the presence of employment frictions in manufacturing, the growth rates of output and employment in the services sector are respectively higher than those of output and employment in the manufacturing sector.*

Finally, we note that the dynamics of human capital investment, H_t, is different from the basic model. It is not equal to ρ for all t. In Appendix 1 it is proved that H_t declines monotonically towards ρ.

4 Manufacturing as an Input in the Production of Services

Production of services typically uses products, tools and equipment from manufacturing – both in the form of durable capital and intermediates. For instance, transportation services use capital goods like vehicles. Financial services extensively require computers and modern tools of information technology. Almost all services use a variety of "consumables" produced in the manufacturing sector. However, physical capital accumulation is beyond the scope our analysis. In what follows, we assume that service production requires labor, and, manufacturing as an intermediate good.

The central implication of the dependence of technology of producing services on manufacturing goods as inputs is a 'locomotive' effect: it tends to slow down the growth rate of the services sector.

Let the production function in the services sector be extended as

$$q_{it} = L_{it}^{\eta} {q'}_{mt}^{1-\eta} - 1, \quad 0 < \eta < 1,$$

where q'_{mt} is the manufacturing input. The following two equations are the cost minimizing and the mark-up conditions (equivalently the two profit-maximizing conditions):

$$w_t = \frac{\eta}{1-\eta} \frac{q'_{mt}}{L_{it}}, \tag{26}$$

$$\frac{p_{it}}{w_t} = \frac{\sigma}{\eta(\sigma-1)} \left(\frac{q'_{mt}}{L_{it}} \right)^{-(1-\eta)}. \tag{27}$$

The former implies that the higher the magnitude of η, the smaller is the share of manufacturing in the services sector. The latter implies that the mark-up is not constant.

The zero-profit condition, together with the production function and the last two equations, yields

$$L_{it}^{\eta} {q'}_{mt}^{1-\eta} = \sigma. \tag{28}$$

Thus the firm-level output is time-invariant, equal to $\sigma - 1$ (as before). The last three equations imply

$$L_{it} = \frac{\sigma \eta^{1-\eta}}{(1-\eta)^{1-\eta}} w_t^{-(1-\eta)} \tag{29}$$

$$\frac{p_{it}}{w_t} = \frac{L_{it}}{\eta(\sigma - 1)}. \tag{30}$$

A manufacturing firm's problem is same as in the earlier models. For simplicity, we abstract from labor friction in manufacturing and thus take (1) as the manufacturing production function. Relations (10), (11) and (12) continue to hold. Substituting (30) into (12) gives

$$\frac{\alpha}{\beta\eta} \frac{N_t L_{it}}{L_{mt}} = 1. \tag{31}$$

An immediate implication is that employment in both sectors grows at the same rate.

The static general equilibrium is essentially characterized by (10), (11), (29), (31) and the full employment condition

$$L_{mt} + N_t L_{it} = \bar{L}. \tag{32}$$

Given $\bar{L}_t$, these five equations determine q_{mt}, L_{mt}, N_t, L_{it} and w_t. Appendix 2 proves that that condition (A1) guarantees stability in the labor market.

Once these variables are solved, eqs. (28) and (30) respectively solve q'_{mt} and the relative price p_{it}. The manufacturing market clearing condition is

$$q_{mt} = c_{mt} + N_t q'_{mt}, \tag{33}$$

which essentially determines c_{mt}.

The household optimization problem remains same. The same Euler equation results. The ratio c_{mt}/w_t grows at the rate of ρa_L.

We also show in Appendix 2 that effective labor supply, $\bar{L}_t$, also grows at the rate of ρa_L. As $\bar{L}_t$ increases, (a) the wage rate falls over time, and, (b) in view of (31), in both sectors employment grows at the same rate.[17]

[17]What is different from the case where manufacturing products are not used input in service production is that the service sector output is not one-to-one related with employment in that sector.

As in the previous models, the service sector output grows faster than that of manufacturing sector. To see this, we substitute (29) in (31) and eliminate L_{it}, and then use the resulting equation to substitute for L_{mt} in (11). The resultant relation is:

$$\frac{q_{mt}}{(\sigma-1)N_t}=\frac{\sigma}{(\sigma-1)\beta\eta^{\eta}(1-\eta)^{1-\eta}}w_t^{\eta}.$$

As w_t falls, the ratio q_{mt}/N_t rises, implying that the services sector grows faster. The intuition behind this finding is that falling wages make the service sector more labor intensive over time, as seen from (26). In Appendix 2 it is derived that the individual sectoral growth rates are:

$$\begin{aligned}\frac{q_{mt+1}}{q_{mt}}&=(\rho a_L)^{1-\frac{1-\alpha-\frac{\beta\sigma}{\sigma-1}}{1-(1-\eta)\frac{\beta\sigma}{\sigma-1}}}\\ \frac{N_{t+1}}{N_t}&=(\rho a_L)^{1-\frac{(1-\eta)\left(1-\alpha-\frac{\beta\sigma}{\sigma-1}\right)}{1-(1-\eta)\frac{\beta\sigma}{\sigma-1}}}.\end{aligned}\tag{34}$$

Observe that the growth rates of both sectors are increasing η. Hence, the smaller the magnitude of η, i.e., the larger the share of manufacturing in the services output, the smaller are the growth rates. Intuitively, as a slower growing sector's output is used as input in the faster growing sector, the growth rate of the latter is pulled down, which, in turn, pulls down the growth rate in the former sector.

We also see that the difference between the growth rates is increasing in η. A smaller η means a narrower gap between the growth rates.

Proposition 3 *The higher the share of manufacturing in the services sector, the slower are the growth rates of both sectors and the less is the difference between them.*

In what follows, we revert back to the earlier scenario where labor friction is present in manufacturing and the manufacturing good is not used in the production of services.

5 Services for Households

While business services have been the faster growing component within the basket of services, services consumed by households hold a larger share. In this section, we introduce household or consumer services and examine the growth of the manufacturing sector vis-a-vis business services and household services.

We first consider the case of pure business and and pure consumer services – that is, services that are demanded mostly by businesses and those demanded predominantly by households (such as education, personal care and health). In other words, business and household services are different. Next we analyze the case where same services are demanded by both businesses and households (like retail trade, transport and communication and financial intermediation).

Unlike Buera and Kaboski (2009) however, we abstract from the trade-off between home and market production of consumption services and assume that all such services are provided by the market.

5.1 Pure Household and Pure Business Services

We assume that firms in the services sector specialize in either business or household services. In other words, there are two sub-sectors. The behavior of the business service providers is the same as before. Let the household service providers face similar increasing-returns linear technology. The fixed-cost component or the variable cost coefficient (or both) may differ from those providing business services. For algebraic simplicity however, we use the same production function: $q_{it}^h = L_{it}^h - 1$.

Households derive utility from the manufacturing good as well as consumption services. Let the felicity function be $U_t = \lambda \ln c_{mt} + (1-\lambda) \ln c_{st}^h, \lambda \in (0,1)$. Here c_{st}^h is a composite of services demanded by the representative household and has the expression

$$c_{st}^h = \left(\int_0^{N_t^h} c_{it}^{h\,\frac{\sigma^h-1}{\sigma^h}} di \right)^{\frac{\sigma^h}{\sigma^h-1}}, \quad \sigma^h > 1, \tag{35}$$

where c_{it} is the consumption of any particular service i.

The household's problem is to maximize $\sum_{t=0}^{\infty} \rho^t U_t$, subject to the learning function (16) and the budget $c_{mt} + p_{st}^h c_{st} \leq w_t \bar{L}_t + \pi_{mt}$, where p_{st}^h is the composite price of consumer services.

The relation between the composite and the individual components of consumer services is:

$$p_{st}^h = \left(\int_0^{N_t^h} p_{it}^{h\,-(\sigma^h-1)} di \right)^{-\frac{1}{\sigma^h-1}}. \tag{36}$$

The dichotomy between the static and the dynamic components of the household optimization problem is obvious. The static part yields

$$\frac{\lambda}{1-\lambda} \frac{c_{st}^h}{c_{mt}} = \frac{1}{p_{st}^h}, \tag{37}$$

$$\text{where } c_{it}^h = c_{st}^h \left(\frac{p_{it}^h}{p_{st}^h} \right)^{-\sigma_h}. \tag{38}$$

A consumer service provider faces the demand function (38) and treats c_{st}^h and p_{st}^h parametrically. In turn, it implies a constant mark-up first-order condition of profit maximization:

$$\frac{p_{it}^h}{w_t} = \frac{\sigma^h}{\sigma^h - 1}. \tag{39}$$

This implies symmetry, and, together with the zero-profit condition, leads to a time-invariant level of employment and output at the firm level:

$$L_{it}^h = \sigma^h; \quad q_t^h \equiv q_{it}^h = \sigma^h - 1. \tag{40}$$

Using symmetry, the mark-up condition (39), the expressions in (40) and that $c_{it}^h = q_t^h$ in equilibrium, the service basket demanded by households and its price have the following expressions:

$$c_{st}^h = N_t^{h\frac{\sigma^h}{\sigma^h-1}} c_{it} = N_t^{h\frac{\sigma^h}{\sigma^h-1}} (\sigma^h - 1) \tag{41}$$

$$p_{st}^h = N_t^{h^{-\frac{1}{\sigma^h-1}}} p_{it} = \frac{\sigma^h}{\sigma^h - 1} N_t^{h^{-\frac{1}{\sigma^h-1}}} w_t. \tag{42}$$

The situation of the manufacturing sector is same as in the previous model. Eqs. (21)-(23) continue to hold. Substituting (41), (42) and the manufacturing good market clearing condition $c_{mt} = q_{mt}$ into the first-order condition (37) leads to the analog of (23) for the household sector:

$$\frac{\lambda}{1-\lambda} \frac{\sigma^h N_t^h}{q_{mt}} = \frac{1}{w_t}. \tag{43}$$

Finally, we have the full-employment condition:

$$L_{mt} + \sigma N_t + \sigma^h N_t^h = \bar{L}_t. \tag{44}$$

It includes employment in the sub-sector producing household services.

Eqs. (21)-(23) together with (43) and (44) constitute the static production system of the economy. They determine five variables: wage, employment and output in manufacturing and the number of firms in the two service (sub) sectors. Appendix 3 shows that the labor market is stable under our regularity assumption (A1).

The dynamic part of the household optimization is essentially same as in the basic model. The ratio of total household expenditure to the wage rate grows at ρa_L. Since the expenditure on manufacturing constitutes a constant fraction (λ) of total household expenditure, the Euler equation (17) continues to hold.

It is proved in Appendix 3 that as the ratio q_{mt}/w_t rises over time, $\bar{L}_t$ grows, w_t declines and manufacturing output rises over time.

Ranking of Growth Rates

How do the growth rates of the two service sub-sectors compare with that of manufacturing and with each other?

It will be useful to understand the ranking in the absence of employment frictions in manufacturing. It is clear that employment would grow at the same rate in all the three 'sectors.' Because the technology is similar between the two sub-sectors, their outputs will grow at the same rate. This common rate would exceed the growth rate of manufacturing, since returns to scale are lower in manufacturing.

Consider now the presence of labor frictions in manufacturing. Since business services are used in manufacturing (by definition) along with labor, and, employment of labor in manufacturing is subject to frictions, there is a relatively higher demand for business services and less demand for labor as manufacturing output expands. Therefore, compared to the case of no labor friction in manufacturing, the growth rate of employment in manufacturing is less and that in the business-service sector is higher. This leads to the following ranking of employment growth:

Growth rate of employment in the business-service sub-sector
> that of employment in the consumption-service sub-sector
> that of employment in the manufacturing sector.

The same ranking translates to output growth rates.

Formally, it can be inferred from (22) and (43) that N_t^h/L_{mt} grows over time. Hence employment in the consumer services grows faster than that in manufacturing. Next, we divide (23) with (43) and substitute (22) into the resultant relation to obtain

$$\frac{N_t}{N_t^h} = \frac{\beta\sigma^h\lambda}{\sigma(1-\lambda)} \cdot \frac{w_t+\gamma}{w_t+(1-\alpha)\gamma}. \tag{45}$$

As w_t falls, the r.h.s. increases over time. Thus the business-service employment (respectively output) grows faster than consumer-service employment (respectively output). We see from (43) that the N_t^h/q_{mt} ratio rises. Thus the output of the consumer-service sector rises faster than manufacturing output.

5.2 Services Shared by Businesses and Households

Here, we consider the scenario where the same service is provided both firms in the manufacturing sector and households.

We assume that any particular service producer sells its product to the two segments and can price discriminate. That is, the producer of each brand in the services sector has a single production function and acts like a discriminating monopolist, while the market is monopolistically competitive.

The mark-up equations (7) and (39) continue to hold. Let q_t and q_t^h denote the amount sold to manufacturing firms and households by any individual service provider. The mark-up equations and the zero-profit condition imply

$$\frac{1}{\sigma-1}q_t + \frac{1}{\sigma^h-1}q_t^h = 1. \tag{46}$$

Hence, the equilibrium firm-level output is not time-invariant.

Relations pertaining to the manufacturing sector and households are unchanged. We bring in the variable q_t into (21)-(23) and variable q_t^h into (43), and, write them as

$$q_{mt} = L_{mt}^{\alpha}(N_t q_t)^{\beta} N_t^{\frac{1}{\sigma-1}} - \gamma L_{mt} \tag{47}$$

$$\frac{\alpha\sigma}{\beta(\sigma-1)} N_t q_t = \frac{L_{mt}(w_t+\gamma)}{w_t} \tag{48}$$

$$\frac{\lambda\sigma^h}{(1-\lambda)(\sigma^h-1)} N_t q_t^h = \frac{q_{mt}}{w_t}. \tag{49}$$

The labor market clearing condition now reads as:

$$N_t(q_t + q_t^h + 1) + L_{mt} = \bar{L}_t. \tag{50}$$

The production side of the static general equilibrium is given by (22), the first-order condition with respect to labor in manufacturing, and the last five equations. They determine six variables: q_t, q_t^h, q_{mt}, N_t, L_{mt} and w_t. Appendix 4 shows that under the condition (A1) the labor market is stable.

Since the household optimization problem is unchanged, the Euler equation remains same and thus q_{mt}/w_t grows at the gross rate of ρa_L. Appendix 4 also proves that as this ratio grows, $\bar{L}$ grows, the wage rate falls, and manufacturing output expands over time.

From (49) it follows that that consumer services grow faster than the manufacturing output. Eqs. (22), (48) and (49) yield the following relation on intra-firm allocation of output:

$$\frac{q_t}{q_t^h} = \frac{\beta\lambda(\sigma-1)\sigma^h}{(1-\lambda)\sigma(\sigma^h-1)} \cdot \frac{w_t+\gamma}{w_t+\gamma(1-\alpha)}. \tag{51}$$

The wage rate falling over time has two implications. First, there is a substitution away from service provision to households towards businesses. But firms do *not* tend towards completely specializing in providing service to businesses.

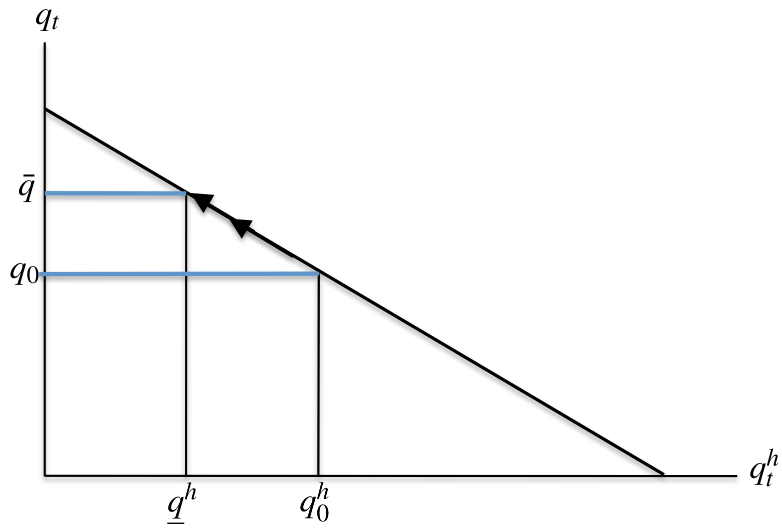

Figure 7: Dynamics of Allocation of Services Provided to Business and Households by a Firm

In the limit, as $q_{mt}/w_t \to \infty$, $\bar{L}_t \to \infty$ and $w_t \to 0$, the ratio q_t/q_t^h approaches a finite real number. The dynamics of a service firm's allocation of output to the business and household sectors is illustrated in Figure 7. The downward sloping straight line represents the allocation equation (46). The initial values of output allocations to the business and household 'sectors' are denoted by q_0 and q_0^h respectively. The dynamics is indicated by the direction of the arrows. The limit values are respectively $\bar{q}$ and $\underline{q}^h$.[18]

Second, the increase in q_t/q_t^h ratio over time implies that business services grow faster than consumer services. Hence the same sectoral growth ranking as in case of pure business and pure household services holds.

Using (22) and (46) – (49), the ratio of employment in the services sector to that in manufacturing has the expression

$$\frac{N_t(q_t + q_t^h + 1)}{L_{mt}} = \frac{\beta}{\alpha} + \frac{1-\lambda}{\lambda} + \frac{\gamma}{w_t}\left[\frac{\beta}{\alpha} + \frac{(1-\lambda)(1-\alpha)}{\lambda}\right],$$

which is a decreasing function of the wage rate. As the wage rate declines over time, it is implied that employment in the services sector grows faster than its

[18] One can compute that

$$\bar{q} = \frac{\beta\lambda\sigma^h(\sigma-1)}{\lambda\beta\sigma^h + (1-\lambda)(1-\alpha)\sigma}; \quad \underline{q}^h = \frac{(1-\alpha)(1-\lambda)(\sigma^h-1)\sigma}{\lambda\beta\sigma^h + (1-\lambda)(1-\alpha)\sigma}.$$

counterpart in manufacturing.

Combining scenarios analyzed in this section and the previous section,

Proposition 4 *In cases of both pure-business-cum-pure-consumption services and same services shared by business and households, the output of the business services grows faster than that of the consumer services, which, in turn, grows faster than the manufacturing output. The growth of employment in the services sector is higher than that in manufacturing.*

Note that Proposition 4 accords with *Fact 4*, to the extent that consumption services represents non-business services.

6 Service-Oriented Relative Demand Shift

The relative rise of the service sector in the post-WWII era has been largely attributed to the hypothesis that as real income rises the consumer demand for services rises more than proportionately, i.e., the income elasticity of demand for household services exceeds one; see, for example Eichengreen and Gupta (2009), among others.

In the presence of such a preference structure, the main implication of the ensuing analysis is that, by itself, such preference shift implies not only a higher growth rate of output in the services sector compared to manufacturing, but – less obviously so – a higher growth rate of employment in the consumer services sector as well.

Let the household's felicity function be

$$U_t = \lambda \ln c_{mt} + (1-\lambda)\ln(c^h_{st} + \delta), \lambda \in (0,1),\ \delta > 0.$$

The presence of the parameter δ, an index of 'non-essentiality' of the services basket in consumption, implies income elasticity of the consumer services basket to be greater than unity. Static optimization has the first-order condition

$$\frac{\lambda}{1-\lambda}\frac{c^h_{st}+\delta}{c_{mt}} = \frac{1}{p^h_{st}}. \tag{52}$$

In the production side, we assume that business and household services are different and provided by different service providers; hence the production side is the same as in Section 5.1.

Substituting the market-clearing condition $c_{mt} = q_{mt}$ as well as c^h_{st} and p^h_{st} from (41) and (42) respectively into (52) gives the analog of (43):

$$\frac{\lambda}{1-\lambda} \cdot \frac{\sigma^h N^h_t + \frac{\delta\sigma^h}{\sigma^h-1} N_t^{h^{-\frac{1}{\sigma^h-1}}}}{q_{mt}} = \frac{1}{w_t}. \tag{53}$$

Earlier equations pertaining to the manufacturing sector, namely, (21) – (23), together with the full-employment condition (44), and eq. (53) solve the production side of the static general equilibrium. However, given q_{mt} and w_t, (53) implies multiple solutions of N_t^h.

If preferences were homothetic, i.e. δ were zero, in view of the log-linear utility function, the marginal utility of purchasing power (MUPP) from consuming the services basket (the ratio of marginal utility to p_{st}^h) is inversely related to total expenditure on it. Under symmetry, total expenditure varies directly with the number of varieties, N_t^h. Hence an increase in N_t^h would lead to a monotonic decline in MUPP of services. However, with $\delta > 0$ as an indicator of non-essentiality of services, in value terms, δp_{st}^h measures how "inessential" the services basket is. The MUPP from service consumption now decreases with total expenditure on services *as well as* δp_{st}^h . Under symmetry, as N_t^h increases, the former increases linearly and MUPP tends to fall. But, since p_{st}^h tends to decline with N_t^h, an increase in N_t^h makes the services basket less inessential and MUPP tends to rise. Overall, an increase in the number of varieties has a non-monotonic effect on MUPP from consuming services. This is the source of multiple solutions of N_t^h from eq. (53).

We express (53) as

$$\frac{q_{mt}}{w_t} = G(N_t^h) \equiv \frac{\lambda}{1-\lambda}\left[\sigma^h N_t^h + \frac{\delta\sigma^h}{(\sigma^h - 1){N_t^h}^{\frac{1}{\sigma^h-1}}}\right]. \tag{54}$$

This function is depicted in quadrant I of Figure 8. We see that for any q_{mt}/w_t, there are at most two solutions of N_t^h [in view of (53) or (54)].

It is shown in Appendix 5 that if condition (A1) is satisfied, then for any given w_t and $\bar{L}_t$, an exogenous increase in N_t^h would tend to increase (respectively decrease) an individual firm's profit according as the initial value of N_t^h lies in the falling arm (respectively rising arm) of the $G(N_t^h)$ function. Thus, given w_t and $\bar{L}_t$, the solution of N_t^h along the rising arm of the $G(N_t^h)$ function is consistent with stability in terms of free entry and exit in the consumer services sub-sector. We assume that this is the (market) solution of N_t^h.

Appendix 5 also shows that $\bar{L}_t$ and N_t^h are negatively or positively related to each other as N_t^h varies along the falling or rising arm of the $G(N_t^h)$ function. This is graphed in quadrant IV of Figure 8. As long as N_t^h is determined along the rising part of the $G(N_t^h)$ function, a higher $\bar{L}_t$ implies a higher value of N_t^h. Furthermore, given condition (A1), eq. (A.12) in the Appendix 5 implies that q_{mt}/w_t is inversely related to w_t, as shown in quadrant II of the same figure. Combining the relations in quadrants I, II and IV, it follows that $dw_t/d\bar{L}_t < 0$, i.e., labor market is stable if (A1) is satisfied. The inverse relationship between $\bar{L}_t$ and w_t is shown in quadrant III of Figure 8 for the sake of completeness.

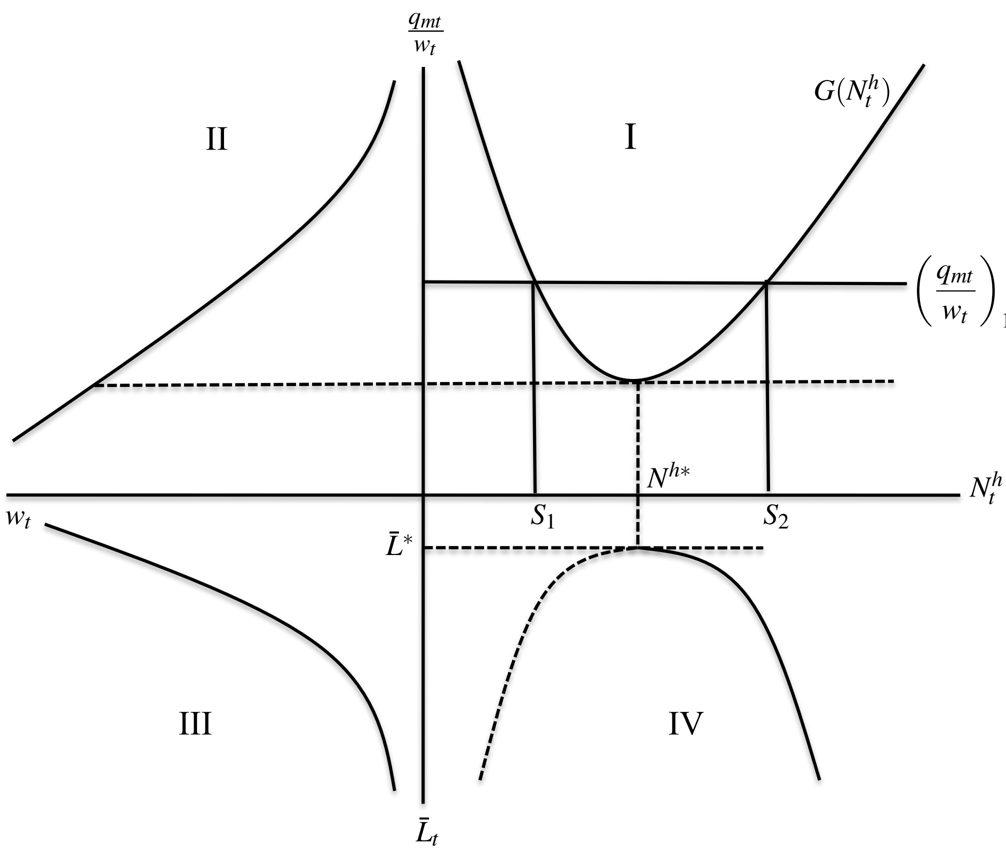

Figure 8: Solution of N_t^h and Negative Relationship Between w_t and $\bar{L}_t$

6.1 Dynamics

The nature of dynamic tradeoff for the household is the same. By substituting the first-order condition (52) into the budget constraint and eliminating c^h_{st}, it can be derived that the ratio c_{mt}/w_t grows at the rate ρa_L. Hence q_{mt}/w_t also grows at this rate. From Figure 8, we observe that N^h_t rises and w_t falls over time.

Notice from quadrant II of Figure 8 that if w_t is very high, q_{mt}/w_t is very small and it may not intersect $G(N^h_t)$ in quadrant I. So for an equilibrium to exist, the wage rate should not be too high. For this to hold, we see from quadrant III that $\bar{L}_t$ must exceed $\bar{L}^*$. We presume that L_0 is sufficiently large such that $\bar{L}_0 > L^*$, and, thus $\bar{L}_t > L^*$ for all t. The dynamics of other variables rests on this assumption.

Output Growth Rates

The relation between the manufacturing sector and the business service firms has not changed from the model in section 5.1. So, just as in that model the business services output expands more rapidly than manufacturing output (from (22) and (23)). We observe from (53) that as long as w_t falls and N^h_t rises over time, the ratio N^h_t/q_{mt} increases. That is, consumer services also rise faster than manufacturing output. Thus, both sub-sectors in the services sector grow faster than manufacturing. However, growth rates cannot be unambiguously ranked between the two sub-sectors, because, on one hand, business services tend to grow faster than consumption services due labor frictions in manufacturing, and, on the other hand, because of the relative demand shift towards consumption services, consumption-service production would tend to grow faster than business services.[19]

Employment Growth Rates

Because of labor frictions in manufacturing, it is obvious that employment in business services grows faster than manufacturing employment. To compare con-

[19] Algebraically, if we divide (23) with (53) and substitute (22) into the resultant relation, we obtain

$$\frac{N_t}{N^h_t} = \frac{\beta\sigma^h\lambda}{\sigma(1-\lambda)} \cdot \frac{w_t+\gamma}{w_t+(1-\alpha)\gamma}\left[1+\frac{\delta}{(\sigma^h-1)N_t^{h\,\frac{\sigma^h}{\sigma^h-1}}}\right].$$

The ranking depends on the magnitude of δ, which measures the shift in relative demand towards consumer services. The ratio N_t/N^h_t increases or decreases and thus the growth rate of the business sub-sector exceeds or falls short of that of the consumer services sub-sector as δ is below or above a threshold.

sumer service sub-sector employment with manufacturing employment, we rearrange (22), (23) and (53) to get

$$\frac{N_t^h}{L_{mt}}\left(\sigma^h + \frac{\sigma^h\delta}{\sigma^h-1}\frac{1}{N_t^{h\,\frac{\sigma^h}{\sigma^h-1}}}\right) = \frac{1-\lambda}{\lambda\alpha}\left[1+\frac{(1-\alpha)\gamma}{w_t}\right]. \tag{55}$$

Falling wages and rising N_t^h imply that employment in consumer services also grows faster than in manufacturing sector. Notice that this ranking holds even when $\gamma = 0$. The reasoning is as follows. As w_t falls, it tends to lower the price of the composite service basket. In the presence of relative demand shift preference, the ratio of spending on consumer services to manufacturing increases, which is a demand shift effect. In both sectors, the respective household spending, equal to total revenues, is proportional to labor costs. Hence the ratio of total labor cost in the consumer services sub-sector to that in manufacturing, equal to ratio of respective employment levels, increases. Thus, over time as w_t falls, employment in the consumer services sub-sector grows faster than that in manufacturing.

Similar to output growth, employment growth rates in the consumer service and business service sub-sectors cannot be ranked however.

In summary

Proposition 5 *In the presence of income-induced relative demand shift towards consumer services, the output growth rates as well as employment growth rates in business and consumer services sub-sectors cannot be ranked, but both growth rates in each sub-sector exceeds those in manufacturing.*

7 Concluding Remarks

In the post WWII world economy the services sector has grown consistently faster than manufacturing. In many countries the share of the services sector in GDP now stands well above 50%. This phenomenon has been mainly attributed to a relative demand shift towards consuming services as real income rises. We have taken the position that while this may be very well true it seems inadequate to explain the growth of *business* services in particular.

Our analysis began with business services and consumption services were introduced later. We believe this has enabled us to uncover other factors (than a relative demand shift towards consumption services) behind the rise of the services sector relative to manufacturing. One is higher returns to scale in the services sector compared to manufacturing and the other is the prevalence of employment frictions in manufacturing (relative to services).

From the perspective of growth theory, our analysis is an example of *unbalanced* growth, which has not been formally examined as intensively as balanced growth.

There are several stylized facts on employment and growth patterns as well as relative price movements in the two industries. By abstracting from TFP growth, the general goal of our analysis is to understand *inter*-sectoral – rather than *intra*-sectoral – differences in the growth rates of employment and output. Factoring in TFP growth would surely enhance our understanding of growth processes across the two sectors. Major productivity improvements have been recorded not just in manufacturing but also in the services sector. The so-called Baumol's disease (see Baumol (1967)) has been "cured" or has not struck. (see Triplett and Bosworth (2003)).

Also, our models are unable to explain in particular the increase in the price of services relative to manufacturing. A prevalent explanation lies in the productivity increase in manufacturing compared to services; see, for example, Baumol (1967) and Alcala and Ciccone (2004) among others. Perhaps a dynamic model incorporating some features of Matsuyama (2009), a static model which considers the impact of productivity gains in manufacturing on sectoral employment and outputs, will be useful. However, Triplett and Bosworth (2003) has noted that the TFP growth in the services sector is no less than that in manufacturing. This seems to weaken the productivity differential argument behind the relative price increase of service goods. There are other explanations as well. Similar to the current paper, Klyuev (2004) has developed a two-sector model of growth. However, the basis of growth in his model is capital accumulation. There are two mobile factors: capital and labor. The critical assumption is that manufacturing is more capital-intensive that the services sector. The Rybczinski effect implies an increase in the relative output of the manufacturing sector at given prices. In a closed economy it translates into an increase in the relative price of the services sector (but a *lower* growth rate of the services sector). Buera and Kaboski (2009) argue that as the services sector becomes more skill intensive, the unit cost of providing services rises, pushing up the relative price of services. A more realistic growth analysis must accommodate some mechanism behind such shifts in the relative price of services.

We have incorporated a very simple source of growth, in which there is no specific role played by either of the production sectors. The static implications of an increase in overall resources available to an economy map directly to growth rates. It is worth exploring the implications of accumulation of physical capital, which consists of manufacturing good. To understand the so-called second wave of burgeoning share of the services sector in an aggregate economy would require featuring computer capital and IT infrastructure.

Last but not least, whereas our analysis is confined to a closed economy, it is important to introduce international trade – in both goods *and* services – which would permit to analyze the growth of the services sector in the context of the global economy.

Appendix 1

It refers to Section 3. We analyze the dynamics of H_t. In the presence of employment frictions in manufacturing, the dynamics of H_t, human capital investment, is different from the basic model. It is not t equal to ρ for all t. It will be shown that under further restrictions H_t monotonically increases over time and approach toward ρ.

Log differentiating (25) gives

$$\widehat{\frac{q_{mt}}{w_t}} = \widehat{\bar{L}}_t[1 + \Psi(\bar{L}_t)], \text{ where } \Psi(\bar{L}_t) \equiv -\frac{\gamma\alpha(1-\alpha-\beta)\bar{L}_t w'(\bar{L}_t)}{((\alpha+\beta)w(\bar{L}_t) + \beta\gamma)(w(\bar{L}_t) + (1-\alpha)\gamma)},$$

where the hat represents proportional change.

Using $\widehat{\frac{q_{mt}}{w_t}} = \rho a_L - 1$, the above equation can be expressed as

$$g_{\bar{L}_t} \equiv \frac{\bar{L}_{t+1}}{\bar{L}_t} = 1 + \frac{\rho a_L - 1}{1 + \Psi(\bar{L}_t)}. \tag{A.1}$$

In the basic model, $\gamma = 0$ and thus $\Psi(\cdot)$ was equal to zero for all t (since $w'(\bar{L}_t) < 0$). Here, it is positive for all t. However, as $t \to \infty$, $\bar{L}_t \to \infty$. In view of (24), both $w(\bar{L}_t)$ and $\bar{L}_t w'(\bar{L}_t)$ approach zero. Therefore, $\Psi(\bar{L}_t) \to 0$ and the growth rate of $\bar{L}_t$ becomes asymptotic to ρa_L.

Lemma 1: Ψ is hump-shaped in $\bar{L}_t$.

Proof: From (24) we get the inverse relationship $\bar{L}_t = \bar{L}(w_t)$, where $\bar{L}'(w_t) < 0$. We use it to get $\Psi(\bar{L}_t) \equiv G(w_t)$. Define $\Omega(w) \equiv 1/G(w_t)$. It may be checked that $\Omega(\cdot) \to \infty$, as $w_t \to 0$ or ∞. Further, $\Omega''(w_t) > 0$. This implies that $\Omega(w_t)$ is a U-shaped function in w_t. Let $w_t^\star$ be the critical w_t which minimizes $\Omega(\cdot)$. It follows that $G(w_t)$ attains maximum at w_t^*. Since $\Psi = G$, $\Psi(\bar{L}_t)$ attains maximum at $\bar{L}(w_t^*)$. Thus $\Psi(\bar{L}_t)$ is hump-shaped in $\bar{L}_t$. ■

We use $\bar{L}_t \equiv (1 - H_t)L_t$, the learning function (16) and (A.1) to get

$$\begin{aligned} \Delta H_t \equiv H_{t+1} - H_t &= (1 - H_t)\left(1 - \frac{g_{\bar{L}_t}/a_L}{H_t}\right) \\ &= (1 - H_t)\left[1 - \frac{1}{H_t a_L} \cdot \left(1 + \frac{\rho a_L - 1}{1 + \Psi(\bar{L}_t)}\right)\right]. \end{aligned} \tag{A.2}$$

Figure 9 depicts the relation $\Delta H_t = 0$ in the $(\bar{L}_t, H_t)$ space where $H_t < 1$ and $\bar{L}_t \geq 0$. In view of (A.2), $\Delta H_t = 0$ yields

$$H_t = \frac{1}{a_L}\left[1 + \frac{\rho a_L - 1}{1 + \Psi(\bar{L}_t)}\right].$$

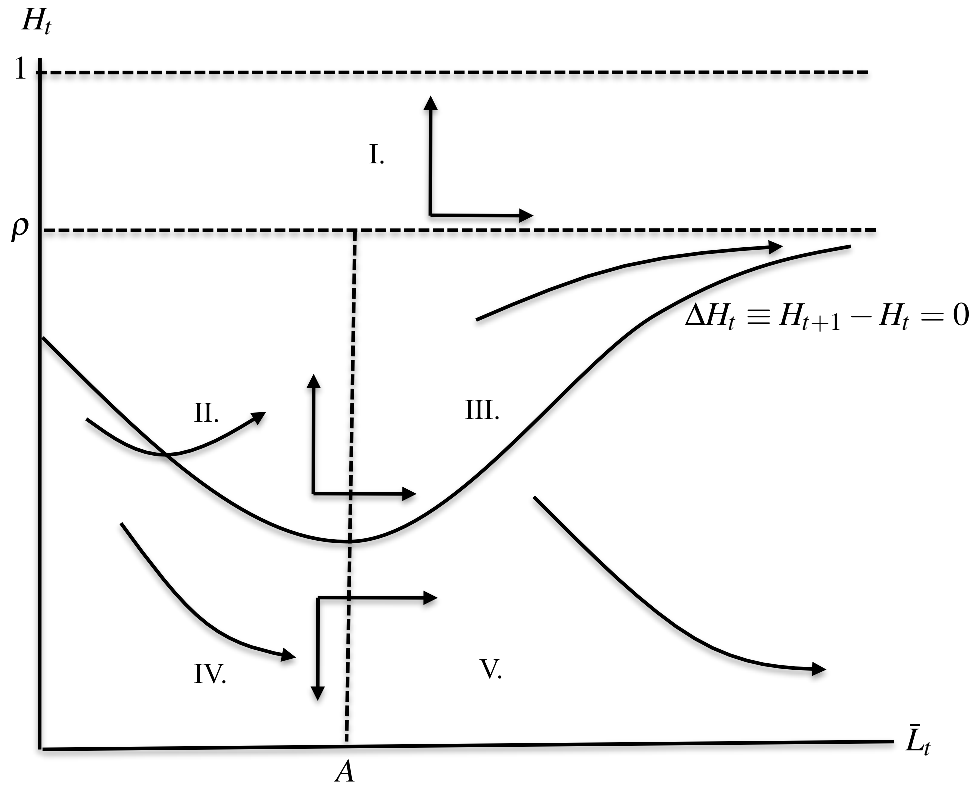

Figure 9: Dynamics of H_t

Given Lemma 1, this implies a valley-shaped locus between $\bar{L}_t$ and H_t, since the Ψ function is hump shaped. The Ψ function is maximized at $\bar{L}_t = A$. We have $\Delta H_t \gtreqless 0$ according as $(\bar{L}_t, H_t)$ lies above or below this curve. This implies the directions of vertical arrows as shown in Figure 9. Because $\bar{L}_t$ increases over time monotonically, the horizontal arrows always point to the right.

It is clear that under perfect foresight the dynamic path cannot originate from or enter into regions I and V. If $\bar{L}_0 < A$, multiple paths towards ρ are possible: H_0 may belong to region II, and then transit to region III, monotonically increasing towards ρ or it may belong to region IV and it is possible that H_t initially falls and then enters regions II and III and finally approach ρ. However, if initially, L_0 is high enough such that $\bar{L}_0 > A$, there is a unique perfect foresight path inside region III, and over time H_t increases monotonically and becomes asymptotic to ρ.

We assume that

$$L_0 > \frac{A}{1-\rho}. \tag{A.3}$$

This implies $\bar{L}_0 \equiv L_0(1 - H_0) > L_0(1 - \rho) > A$.

Under this assumption (and perfect foresight), H_t and $\bar{L}_t$ are positively related, i.e., $\bar{L}_t \equiv \Lambda(H_t)$, where $\Lambda' > 0$; thus

$$L_t = \frac{\Lambda(H_t)}{1 - H_t}.$$

The r.h.s. is monotonic with respect to H_t. Hence, given L_0, H_0 is determined uniquely.

In summary, under the assumption (A.3), H_t increases over time approaching ρ, while L_t and $\bar{L}_t$ grow without bound.

Appendix 2

It refers to Section 4, which analyzes the case where the manufacturing good is used in producing services.

Stability in the Labor Market

We solve the system of equations (10), (11), (29), (31) and (32) to obtain the following relationship between wage rate and labor supply

$$w_t^{1-(1-\eta)\frac{\beta\sigma}{\sigma-1}} = a_0 \bar{L}_t^{-(1-\alpha-\frac{\beta\sigma}{\sigma-1})} \tag{A.4}$$

$$\text{where } a_0 \equiv \alpha \left(\frac{\alpha}{\alpha + \beta\eta} \right)^{-(1-\alpha)} \left(\frac{\beta\eta^\eta (1-\eta)^{1-\eta}}{(\alpha+\beta\eta)\sigma} \right)^{\frac{\beta\sigma}{\sigma-1}} > 0.$$

It is readily seen in (A.4) that if the condition (A1) holds, $dw_t/d\bar{L}_t < 0$, so that the labor market is stable.

Dynamics of $\mathbf{\bar{L}_t, w_t, q_{mt}}$ and $\mathbf{N_t}$

We derive the closed form expressions of the growth rates of these variables, which are constant over time.

The ratio of employment in the two sectors being constant (from (31)), the labor market clearing condition (32) implies

$$L_{mt} = \frac{\alpha}{\alpha + \beta\eta} \bar{L}_t. \tag{A.5}$$

Substituting (11), (26), (31) and (A.5) into the manufacturing market clearing condition (33) yields

$$\frac{c_{mt}}{w_t} = \frac{1 - \beta + \beta\eta}{\alpha + \beta\eta} \bar{L}_t. \tag{A.6}$$

We know from the Euler equation that c_{mt}/w_t grows at ρa_L. Hence in view of (A.4) and (A.6),

$$\frac{\bar{L}_{t+1}}{\bar{L}_t} = \rho a_L; \qquad \frac{w_{t+1}}{w_t} = (\rho a_L)^{-\frac{1-\alpha-\frac{\beta\sigma}{\sigma-1}}{1-(1-\eta)\frac{\beta\sigma}{\sigma-1}}}.$$

Next, by substituting (A.6) into (A.5) and (A.4), we express L_{mt} and w_t in terms of the ratio c_{mt}/w_t. In turn, substituting those expressions into (11), we get

$$q_{mt} = b_0 \left(\frac{c_{mt}}{w_t}\right)^{1-\frac{1-\alpha-\frac{\beta\sigma}{\sigma-1}}{1-(1-\eta)\frac{\beta\sigma}{\sigma-1}}}, \text{ where} \tag{A.7}$$

$$b_0 \equiv \frac{1}{1-\beta+\beta\eta}\left[d_0\left(\frac{\alpha+\beta\eta}{1-\beta+\beta\eta}\right)^{-\left(1-\alpha-\frac{\beta\sigma}{\sigma-1}\right)}\right]^{\frac{1}{1-(1-\eta)\frac{\beta\sigma}{\sigma-1}}} > 0.$$

Noting again that c_{mt}/w_t grows at ρa_L, (A.7) implies

$$\frac{q_{mt+1}}{q_{mt}} = (\rho a_L)^{1-\frac{1-\alpha-\frac{\beta\sigma}{\sigma-1}}{1-\frac{(1-\eta)\beta\sigma}{\sigma-1}}}.$$

Just as the growth rate of manufacturing output was calculated, we substitute (29), (A.4) and (A.6) into (31) to get,

$$N_t = b_1 \left(\frac{c_{mt}}{w_t}\right)^{1-\frac{(1-\eta)\left(1-\alpha-\frac{\beta\sigma}{\sigma-1}\right)}{1-\frac{(1-\eta)\beta\sigma}{\sigma-1}}}, \text{ where} \tag{A.8}$$

$$b_1 \equiv \frac{\beta\eta^\eta(1-\eta)^{1-\eta}c_0^{1-\eta}}{\sigma}\cdot\frac{(1-\beta+\beta\eta)^{1-\eta}}{\alpha+\beta\eta} > 0.$$

As output per firm is constant, the services-sector output grows at the same rate as its number of firms. (A.8) implies

$$\frac{N_{t+1}}{N_t} = (\rho a_L)^{1-\frac{(1-\eta)\left(1-\alpha-\frac{\beta\sigma}{\sigma-1}\right)}{1-\frac{(1-\eta)\beta\sigma}{\sigma-1}}}.$$

Appendix 3

It refers to Section 5.1, which analyzes the case where business and consumption services are produced by different firms.

Stability in the Labor Market

Solving the static system of equations ((21)-(23), (43) and (44)) we obtain

$$h(w_t) = c_4 \bar{L}_t^{-\left(1-\alpha-\frac{\beta\sigma}{\sigma-1}\right)}, \quad \text{where} \tag{A.9}$$

$$h(w_t) \equiv \left(c_2 + \frac{\gamma c_3}{w_t}\right)^{-\left(1-\alpha-\frac{\beta\sigma}{\sigma-1}\right)} w_t^{\frac{\beta\sigma}{\sigma-1}} (w_t+\gamma)^{1-\frac{\beta\sigma}{\sigma-1}} \quad \text{and}$$

$$c_2 \equiv 1+\frac{\beta}{\alpha}+\frac{1-\lambda}{\lambda\alpha} > 0;\ c_3 \equiv \frac{\beta}{\alpha}+\frac{(1-\lambda)(1-\alpha)}{\lambda\alpha} > 0;\ c_4 \equiv \alpha(\sigma-1)^{\beta}\left(\frac{\beta}{\alpha\sigma}\right)^{\frac{\beta\sigma}{\sigma-1}} > 0.$$

Log-differentiating (A.9) implies $dw_t/d\bar{L}_t < 0$ if (A1) is met.

Dynamics of $\mathbf{\bar{L}_t, w_t, q_{mt}}$ and $\mathbf{N_t}$

Closed form solutions of the growth rates of these variables do not exist, but we characterise how these variables change over time.

Eqs. (22)-(23), (43)-(44) together yield

$$\frac{q_{mt}}{w_t} = \frac{\bar{L}_t}{\frac{(\alpha+\beta)w(\bar{L}_t)+\beta\gamma}{w(\bar{L}_t)+(1-\alpha)\gamma} + \frac{1-\lambda}{\lambda}}, \tag{A.10}$$

where, in view of (A.9), the wage rate is an implicit function of $\bar{L}_t$. The r.h.s. of (A.10) is monotonically increasing in $\bar{L}_t$. Hence q_{mt}/w_t is an increasing function of $\bar{L}_t$. The Euler equation implies that q_{mt}/w_t grows over time; hence $\bar{L}_t$ also rises, and it follows from the stability of labor markets that w_t declines with time.

Substituting (A.9) into (A.10) we obtain the following relationship between manufacturing output and wages:

$$q_{mt} = \frac{c_4}{\alpha}[w_t + \gamma(1-\alpha)] w_t^{-\frac{\frac{\beta\sigma}{\sigma-1}}{1-\alpha-\frac{\beta\sigma}{\sigma-1}}} (w_t+\gamma)^{-\frac{1-\frac{\beta\sigma}{\sigma-1}}{1-\alpha-\frac{\beta\sigma}{\sigma-1}}}.$$

Log-differentiating the above, it is straightforward to derive that $dq_{mt}/dw_t < 0$. As the wage rate falls, the manufacturing output rises over time. Further the manufacturing firm's optimization conditions (22) and (23) imply that N_t grows and its growth rate is greater than that of q_{mt}.

Appendix 4

This refers to Section 5.2, which examines the case where the same service provider sells its services to households and businesses in the manufacturing sector.

Stability in the Labor Market

The static system (Eqs. (22), (47)-(50)) of this economy is solved to yield

$$g(w_t) = d_3 \bar{L}_t^{-\left(1-\alpha-\frac{\beta\sigma}{\sigma-1}\right)}, \quad \text{where} \tag{A.11}$$

$$g(w_t) \equiv \frac{w_t^{\beta}(w_t+\gamma)^{1-\beta}}{\left(d_1 + \frac{\gamma d_2}{w_t}\right)^{\frac{\beta}{\sigma-1}} \left(c_2 + \frac{\gamma c_3}{w_t}\right)^{1-\alpha-\frac{\beta\sigma}{\sigma-1}}}$$

$$d_1 \equiv \frac{\beta}{\alpha\sigma} + \frac{1-\lambda}{\alpha\sigma^h\lambda} > 0;\ d_2 \equiv \frac{\beta}{\alpha\sigma} + \frac{(1-\lambda)(1-\alpha)}{\alpha\sigma^h\lambda} > 0;\ d_3 \equiv \alpha\left(\frac{\beta(\sigma-1)}{\alpha\sigma}\right)^{\beta}$$

It can be derived that $g'(w_t) > 0$. Hence $dw_t/d\bar{L}_t < 0$ and the labor market is stable if condition (A1) is met.

Dynamics of $\mathbf{\bar{L}_t, w_t, q_{mt}}$ and $\mathbf{N_t}$

We follow the same steps as in Appendix 3. From (A.11), implicitly, $w_t = w(\bar{L}_t)$. By substituting (22) and (47)-(49) into (50) we obtain,

$$\frac{q_{mt}}{w_t} = \frac{\bar{L}_t}{\frac{(\alpha+\beta)w(\bar{L}_t)+\beta\gamma}{w_t(\bar{L}_t)+(1-\alpha)\gamma} + \frac{1-\lambda}{\lambda}}.$$

The above expression is the exact relation which was derived in the case of pure business and household services and is stated in (A.10). Hence as before, $\bar{L}_t$ grows over time with q_{mt}/w_t. In view of (A.11), w_t falls with time.

Next, we substitute (A.11) into (A.10). It gives

$$q_{mt} = w_t^{-\frac{\beta}{1-\alpha-\frac{\beta\sigma}{\sigma-1}}}(w_t+\gamma)^{-\frac{1-\beta}{1-\alpha-\frac{\beta\sigma}{\sigma-1}}}\left(d_1 + \frac{d_2}{w_t}\right)^{\frac{\frac{\beta}{\sigma-1}}{1-\alpha-\frac{\beta\sigma}{\sigma-1}}}\left[\frac{w_t+(1-\alpha)\gamma}{\alpha d_3^{1-\alpha-\frac{\beta\sigma}{\sigma-1}}}\right].$$

The r.h.s. decreases with w_t. Hence q_{mt} increases over time. Moreover, from (22) and (23) we get that N_t grows and it grows faster than q_{mt}.

Appendix 5

It refers to Section 6.

Free Entry and Exit Stability analysis in Consumer Services Sub-Sector

Substituting (22) and (23) into the manufacturing production function (21) gives q_{mt} as a function of w_t:

$$q_m(w_t) = \alpha^\alpha(\sigma-1)^\beta \left(\frac{\beta}{\sigma}\right)^{\frac{\beta\sigma}{\sigma-1}} w_t^{-\frac{\frac{\beta\sigma}{\sigma-1}}{1-\alpha-\frac{\beta\sigma}{\sigma-1}}} (w_t+\gamma)^{-\frac{1-\frac{\beta\sigma}{\sigma-1}}{1-\alpha-\frac{\beta\sigma}{\sigma-1}}} [w_t+(1-\alpha)\gamma]. \quad \text{(A.12)}$$

It can be checked that if condition (A1) holds, $dq_{mt}/dw_t < 0$. Hence q_{mt}/w_t decreases with w_t, as shown in quadrant II of Figure 8.

Next we substitute (41) and (42) into (52) and obtain,

$$q_{it}^h = q_i^h(\underset{-}{w_t}, \underset{?}{N_t^h}) \equiv \frac{1-\lambda}{\lambda} \cdot \left[\frac{\sigma^h - 1}{\sigma^h N_t^h} \cdot \frac{\underset{-}{q_{mt}(w_t)}}{w_t} - \frac{\delta}{N_t^{h\,\frac{\sigma^h}{\sigma^h-1}}}\right].$$

Using the above expression and the price-markup condition for consumer services, the profit of a consumer service firm i can be expressed as

$$\pi_{it}^h \equiv \pi_{it}^h(\underset{-}{w_t}, \underset{?}{N_t^h}) = \frac{w_t q_i^h(w_t, N_t^h)}{\sigma^h - 1} - w_t,$$

where we have used that $q_{it} = L_{it} - 1$.

Stability of entry and exit processes requires $\partial\pi_{it}^h/\partial N_t^h < 0$. By using Samuelson's correspondence principle, we shall prove that stability is ensured if and only if in Figure 8, the solution of N_t^h lies in the rising part of the $G(N_t^h)$ function.

In equilibrium, $\pi_{it}^h(w_t, N_t^h) = 0$. Differentiating it,

$$\frac{dN_t^h}{dw_t} = -\frac{\partial\pi_{it}^h/\partial w_t}{\partial\pi_{it}^h/\partial N_t^h}.$$

We know $\partial\pi_{it}/\partial w_t < 0$. Hence the signs of dN_t^h/dw_t and $\partial\pi_{it}^h/\partial N_t^h$ must be the same. Now turn to Figure 8. If the solution of N_t^h is at a point such as S_1 (respectively S_2), $dN_t^h/dw_t >$ $(<)$ 0. It implies $\partial\pi_{it}^h/\partial N_t^h >$ $(<)$ 0 and thus free entry-exit equilibrium is unstable (respectively stable).

Therefore, stability-consistent solution of N_t^h must lie on the rising arm of $G(N_t^h)$ in Figure 8.

Relationship between $\bar{L}_t$ and N_t^h

The system of equations describing the economy at a given point in time are (21)-(23), (44) and (53). We solve this system to get the following relationship between N_t^h and $\bar{L}_t$:

$$\gamma\left[\frac{\kappa_2\bar{L}_t - f(N_t^h)}{k(N_t^h) - \bar{L}_t} + 1\right] = \kappa_3\left[\kappa_0\bar{L}_t - \kappa_1 f(N_t^h)\right]^{-(1-\alpha)}\left[h(N_t^h) - \kappa_1\bar{L}_t\right]^{\frac{\beta\sigma}{\sigma-1}}$$

where

$$f(N_t^h) \equiv \frac{\sigma^h}{1-\lambda}\left[\left(\lambda\beta + (1-\lambda)(1-\alpha)\right) N_t^h + \frac{\beta\lambda\delta}{(\sigma^h - 1){N_t^h}^{\frac{1}{\sigma^h - 1}}}\right]$$

$$h(N_t^h) \equiv \frac{\sigma^h}{(1-\lambda)(1-\alpha-\beta)}\left[N_t^h + \frac{\lambda\delta}{(\sigma^h - 1){N_t^h}^{\frac{1}{\sigma^h - 1}}}\right]$$

$$k(N_t^h) \equiv \frac{\sigma^h\lambda}{1-\lambda}\left[N_t^h + \frac{\delta(\alpha+\beta)(1-\alpha)}{(\sigma^h - 1){N_t^h}^{\frac{1}{\sigma^h - 1}}}\right]$$

$$\kappa_0 \equiv \frac{1-\alpha}{1-\alpha-\beta}; \quad \kappa_1 \equiv \frac{1}{1-\alpha-\beta}; \quad \kappa_2 \equiv 1-\alpha; \quad \kappa_3 \equiv \alpha\left(\frac{\beta}{\sigma}\right)^{\frac{\beta\sigma}{\sigma-1}}(\sigma-1)^{\beta}$$

and κ_0, κ_1, κ_2 and κ_3 are all positive. N_t^{h*} is the critical point at which $G(N_t^h)$ attains minimum.

It is easy to check that when $N_t^h \gtrless N_t^{h*}$, $f'(N_t^h) \gtrless 0$, $h'(N_t^h) \gtrless 0$ and $k'(N_t^h) \gtrless 0$. Hence, an increase in $\bar{L}_t$ is associated with decrease (or increase) in N_t^h according as $N_t^h < N^{h*}$ (or $(N_t^h > N^{h*})$. This relation between $\bar{L}_t$ and N_t^h is shown in quadrant IV of Figure 8.

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