STRATEGIC INFORMATION REVELATION WHEN EXPERTS COMPETE TO INFLUENCE*

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ABSTRACT. We consider a persuasion game where multiple experts with potentially conflicting self-interests attempt to persuade a decision-maker, or, a judge. The judge prefers to take an action that is most appropriate given the true state of the world but the experts' preferences over the actions are independent of the state. The judge has no commitment power and takes his best action given the experts' reports. Experts have private types: an informed expert observes the state but an uninformed expert does not. An expert cannot lie but an informed expert may conceal information by pretending to be uninformed. We offer a general characterization of the equilibrium. We show that an increase in the quality of an expert (i.e., his prior likelihood of being informed) can reduce the judge's ex-ante expected payoff. Moreover, the judge's expected payoff may be maximized when the experts have identical (but extreme) agenda rather than conflicting self-interests.

1. Introduction

Decision-makers often rely on advice from experts. However, if the experts are themselves interested in the decision, they may attempt to influence the decision-maker by withholding or providing selective information. To counteract such manipulation, decision-makers often solicit advice from experts with conflicting preference, based on the premise that competition between experts will lead to more information being revealed. For example, a judge may invite experts' testimony from both the plaintiff and the defendant, a policy maker listens to advocacy groups representing different interests, and a voter listens to policy stands of different candidates.

While several authors have studied the issue of eliciting private information from competing experts (Milgrom and Roberts, 1986; Shin, 1994, 1998; Kamenica and Gentzkow, 2010; Gentzkow and Kamenica 2010; Gul and Pesendorfer, 2010), the extant literature has not fully explored the link between the *extent of conflict* among the experts and the quality of decision making. This article attempts to bridge this gap.

We develop a simple model of competition for persuasion that allows us to explore certain key questions, such as: Does a greater conflict of interest between experts lead to better decision making? Does the quality of decision making necessarily improve if the competing experts are more informed? Should policy advisors be chosen from those with moderate or extreme policy preferences? In a model with uncertainty over what each expert knows, we find that these questions have nuanced answers that have important implications for design of expert panels. First, employing experts that are more likely to be informed may lead to worse decisions if the judge has limited commitment power. Second, it may be better for the decision-maker to employ experts who have similar interests rather than to promote competition by employing experts with opposing views.

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We adopt a model of persuasion game (Milgrom and Roberts, 1986; Glazer and Rubinstein, 2001, 2004, 2006) with the following features. The decision-maker, or the "judge" wants to take an action matching the "state", $\boldsymbol{\theta}$, and each agent A_i, or "expert," privately observes the state with a certain (commonly known) probability α_i. The judge cannot detect whether an expert has observed the state. We call α_i the "quality" of an expert as it reflects the expert's ability to gather the necessary information. Unlike some of the existing models of persuasion game where the judge chooses between just two actions (Glazer and Rubinstein, 2001; Shin, 1998) we assume a richer, continuous action space. An expert A_i's preference is described by his "ideal action" x_i : irrespective of the underlying state, he prefers that the judge take an action as close to x_i as possible. We assume that the relevant information consists of hard evidence (e.g. legal documents) which can be verified. An expert has the choice to either "report" the state or to feign ignorance and pool with the genuinely uninformed.[1] So, an informed expert reveals information in states he finds favorable and conceals information in states that are unfavorable to him. However, a state that is unfavorable to one expert may be favorable to another, and thus competition may mitigate the problem of information manipulation by experts. We assume that the judge cannot simply commit to an action that would maximize truthtelling. Moreover, the judge cannot write contracts to "buy information" from experts: thus, the experts' incentives are driven only by the judge's action.

As a concrete example of the environment described above, consider a judge listening to the expert reports from both the plaintiff and the defendant to decide on the amount of a monetary damage that the defendant must pay to the plaintiff.[2] The plaintiff's expert prefers a higher damage payment whereas the defendant's expert attempts to lower the damage amount. We assume that both experts have access to the same data or evidence, which is a natural assumption in many judicial systems where both sides of the litigation get equal access to all "discovery documents" of the case. However, the experts vary in terms of their abilities to assess the extent of damage by analyzing the available data. The available data and the experts' analyses are verifiable evidence, so they cannot be fabricated. Thus, if an expert fails to analyze the data effectively, his findings are necessarily uninformative and he cannot produce any assessment of the damage. In contrast, if an expert can successfully analyze the data he has two options: either to reveal his assessment or to withhold it, claiming to have failed to analyze the data. A "better" expert has a higher probability of being able to analyze the data and elicit definite information about the true extent of the harm caused by the defendant.

In this setting, when any expert reveals the state, the judge takes the action matching the state. The equilibrium of this game is completely characterized by the "default action" of the judge—the action, y^* (say), that the judge takes when every expert fails to report the state. An expert's report matters only in the event when no other expert reveals the state.[3] So, he reports if and only if the observed state yields an action closer to his ideal point x_i than the default action y^*. Therefore, an (informed) expert's disclosure strategy is represented by a revelation set: the states which he would report truthfully to the judge. In particular, each expert's revelation set is a set of "favorable states" close to his ideal action and the judge's default action y^* is the best-response to such a disclosure strategy of the experts. This observation leads to a simple characterization of

[1] A similar strategy set for the experts is also assumed by Shavell (1989) in the model of pre-settlement information sharing between the plaintiff and the defendent. Also, in a related article, Dziuda (2010) examines a persuasion game with a single expert who cannot prove that she has reported all dimensions of the state. Moreover, there is an exogenous probability that an expert is honest. The strategic expert takes advantage of the possibility that there is an honest type, much like the informed expert pooling with the uninformed type in our model.

[2] A similar setting is also invoked by considered by Shin (1994). However, as will be apparent below, our environment differs from Shin's in terms of several key features of the underlying information structure.

[3] Several authors (e.g., Wolinsky, 2002; Gerardi, et al 2009) who study the issue of information extraction from experts with divergent agends from a mechanism design approach also make use of the idea that experts condition their report on the fact of being *pivotal*.

the equilibrium. Also, the equilibrium is robust to whether the experts reports simultaneously, or in any pre-specified sequence.[4]

Given this characterization, we explore the link between the extent of conflict (among the experts) and the quality of decision making by the judge. To do so, we confine attention to the case where the state/action space is the unit interval, and there are only two experts.[5] The unit interval allows us unambiguously order the states and helps to capture the extent of conflict between the experts.

The legal example discussed before is a special case of our model where the state and action space is the unit interval, and while one expert prefers as high an action as possible, the other one prefers as low an action as possible (i.e. $x_1 = 0$ and $x_2 = 1$). We call this the case of completely opposed experts and it is similar in spirit with the environment analyzed by Shin (1994). Shin argues that the "burden of proof" should lie more with the expert who is more informed ex-ante. Similarly, in our model, the judge's default action (y^*) favors the expert who has a lower probability of observing the state. Shin uses an information structure where the amount of revelation by the experts is independent of the burden of proof.[6] However, in our case, the extent of information revealed by each expert depends on the judge's default action and on the information revealed by the rival expert. This mutual interdependence of expert strategies allows us to highlight novel ways in which the overall informational content of the debate is affected by the information that each expert has.

A surprising finding of our model is that a better expert (i.e. an expert who is informed with a higher probability) does not always lead to a better decision. When the experts' interests are in conflict, an increase in the quality of A_1 moves the judge's default action away from A_1's ideal point, and consequently, closer to A_2's ideal point. As a result, A_1 now reveals *more* information whereas A_2 reveals *less* (as the default action is now more favorable to A_2). Thus, there are some states which will be revealed with a lower probability when the quality of an expert increases. Now, consider the general case with "moderate" rather than extreme experts, i.e., where $0 \leq x_1 \leq x_2 \leq 1$. Such a situation often arises in a panel of policy advisors. In an equilibrium where the judge chooses a default action $y^* \in (x_1, x_2)$, the revelation set of expert A_1 will be an interval to the left of y^*, $[a_1, y^*]$, say, with x_1 at its center, and that of A_2 will similarly be an interval to the right of y^*, $[y^*, a_2]$, say, with x_2 at its center. Consequent to a change in expert quality, a change in the default action y^* will lead to a shrinkage of one revelation set and expansion of another. In particular, the boundaries of these sets change not just around y^* but away from y^* too (i.e., a_is would change as well). Now, a change in reporting strategies in states close to y^* only has a second-order effect on the judge's ex-ante payoff (as y^* is the best-response to the initial reporting strategies of the experts), a change in reporting strategies for states around a_is far away from y^* will have a first-order effect. Therefore, if the experts are sufficiently moderate, it is possible that an increase in the quality of one expert makes the judge worse off (in the ex-ante sense) due to strategic interaction between experts.[7]

In the general case, there can be multiple equilibria, and two classes of equilibria may emerge: one with $y^* \in (x_1, x_2)$ implying disjoint revelation sets and another with $y^* \notin (x_1, x_2)$, implying

[4]It is also worth mentioning that if we consider an environment where experts can obfuscate by reporting a set containing the actual state (rather than being forced to report the observed state precisely), the same outcome still obtains in equilibrium, provided the judge uses beliefs that are akin to the "skeptical posture" in Milgrom and Roberts (1986).

[5]Indeed, one-dimensional debates are of special interest. As argued by Spector (2000), multidimensional debates have a tendency to be reduced to single-dimensional ones: when preferences of the debaters are similar but beliefs about the consequences of the various decisions diverge, under certain conditions, public communication either resolves the disagreement between beliefs or the debate becomes one-dimensional at the limit.

[6]In Shin's structure, experts observe different signals about the state while in our model, all informed experts observe the same information.

[7]It is important to note that this intuition does not rely on the unidimensional structure of the state/action space and it is simply driven by the fact that the judge's default action must be a best-response to the experts' strategies.

that one expert's revelation set is a subset of the other's. In the former case, we say that we have an equilibrium with *conflict* and in the latter case, we say that we have an equilibrium with *partial congruence*, the terminology arising from whether there is any set of states where the expert's revelation incentives are aligned. Whether we have conflict or congruence in debate is therefore an equilibrium phenomenon. We have already discussed above how in an equilibrium with conflict, an expansion in one expert's revelation set is accompanied by a shrinkage of the other. On the contrary, in an equilibrium with partial congruence, the experts' revelation sets expand or shrink together: if the default action moves further away from (closer to) the interval (x_1, x_2), both experts reveal in more (fewer) states.

Consider the extreme case where both experts want the decision-maker to take as low an action as possible (i.e.$x_1 = x_2 = 0$). To fix ideas, suppose that a panel of anti-war activists are consulted while deciding on foreign policy. It is optimal for the policy-maker to decide on a very hawkish policy unless the activists can present convincing evidence to the contrary. In this case, the default action is like a punishment that the judge can threaten both the experts with, in case they fail to reveal the state. Formally, the revelation set of each expert is $[0, y^*]$: if the state lies in this interval, then it is revealed if either expert observes the state, and if the state is very high (i.e. greater than y^*), it is never revealed to the judge.

A comparison of the two polar cases—one with extreme and opposed experts and the other with extreme but similar experts—demonstrates an important trade-off relevant to the design of expert panels. If the panel consists of two opposed experts, the revelation sets "cover" the state space: conditional on *both* experts having observed the state, it will always be revealed to the judge. However, the judge has limited punishment power: in order to punish one expert, the default action must favor the other. On the other hand, if the panel consists of two extreme but similar experts, the judge has more power to punish both experts, but some states will never be revealed in equilibrium.

Since we can vary the degree of conflict or congruence among experts as a parameter in the model, we can examine the optimal degree of conflict from the point of view of the judge. If the judge were to choose the configuration of ideal points in the expert panel in order to maximize his ex-ante payoff, what would she choose? Dewatripont and Tirole (1999) argues in favor of using competing experts (rather than using one expert to argue the case) because it is easier to address the moral hazard problem involved in costly information acquisition. In the same vein, Shin (1998) shows that even if the judge is as well informed as the experts, on an average it is better to employ completely opposed experts than the judge undertaking his own investigation. Also, in a model where experts engage in costly information acquisition, Gerardi and Yariv (2010) shows that if experts can report in sequence, the optimal mechanism involves using opposing experts. In contrast to this literature that by and large supports the use of competing experts, we show that it is sometimes optimal to use similar but extreme experts (irrespective of the sequence of reporting).[8] However, we should point out that the cost of information acquisition features prominently in these other models, while such concerns do not play any role in our environment.

How would our results change if the decision-maker could commit to a default action? First, a simple envelope theorem-like argument would ensure that increase in expert quality would always make the judge better off. Thus, the perverse fact that a judge can be made worse off by improvement in expert quality is due to a commitment problem. On the other hand, the results on panel design are robust to whether or not the judge can commit to a default action. First, the optimal profile of expert ideal points (given the profile of qualities) always includes extreme experts, either completely opposed or completely identical. Second, with two extreme experts, irrespective of whether they are similar or opposed, the default action of the judge in Nash equilibrium is the also

[8]In fact, we show that whenever the distribution of the states is uniform and the judge's preferences are given by a quadratic loss function, the judge's expected payoff is always maximized when experts have identical but extreme agenda.

the optimal default action under commitment. In other words, the ability to commit to a default action leads to the same optimal profile of expert ideal points (and same payoff) that would be obtained in absence of such ability.

There is a line of work where the judge is assumed to be able to commit to a mechanism to elicit the truth from multiple experts by exploiting the divergence in interests. This literature includes Wolinsky (2002) and Gerardi et. al (2009) which have been mentioned before. A related set of papers relaxes the extent of verifiability of messages and examines conditions for revelation of truth given diversity of expert preferences (Lipman and Seppi, 1995; Bull and Watson, 2004; Deneckere and Severinov, 2008; Ben-Porath and Lipman, 2009; and Kartik and Tercieux, 2010). While the mechanism design literature emphasizes exploiting differences in expert preferences for eliciting the truth, we show that the judge might optimally want to have experts with similar preferences, even if she could commit to an optimal default action.

There is a strand of literature in cheap talk that has also focussed on the question of full information revelation in the presence of multiple senders of information.[9] Krishna and Morgan (2001) shows the value of competition (opposite expert biases) in improving communication in the unidimensional state/action space. Battaglini (2002) shows that if the state/action space is multidimensional and unbounded, then there is an equilibrium where the state is always revealed. While the papers in the cheap talk literature assume that the experts always know the state precisely, we are interested in a situation where there is uncertainty about what an expert knows. In our model too, when the experts have completely opposed agenda, information is fully revealed to the judge in the event that both experts know the state. This result is similar to the full revelation result with opposed biases in Krishna and Morgan (2001). However, we point out that uncertainty about expert information opens up new channels of strategic manipulation by experts that the judge has to contend with. Competition then limits the ability of the judge to punish experts, and under certain circumstances, the judge may be better off employing experts with extreme but completely identical preferences.

The rest of article is organized as follows. The next section presents a general version of the model described above, followed by a general characterization of the equilibrium in section 3. Section 4 analyzes the role of the experts' quality on information revelation when experts are completely opposed (i.e., $x_1 = 0$ and $x_1 = 1$). Section 5 discusses how the nature of the persuasion game changes with the extent of opposition between the experts. Some extensions and robustness issues, including the value of commitment to the judge are discussed in section 6. A final section concludes.

2. A general model

We consider a model of a persuasion game between a set of n experts, $A_1, ..., A_n$, and a judge, J. The judge needs to choose an action $\mathbf{y} \in Y \subset \mathbb{R}^k$ that is most appropriate given the underlying state of the nature $\boldsymbol{\theta} \in \Theta$ where Θ is a compact and convex subset of $\mathbb{R}^k$. For the sake of analytical convenience, we assume that in state θ, the ideal action for the judge is $\mathbf{y} = \boldsymbol{\theta}$. So, without loss of generality, we assume that $Y = \Theta$.

The judge cannot observe $\boldsymbol{\theta}$ directly but has a (commonly known) prior belief on $\boldsymbol{\theta}$ that is given by the probability distribution function $F(\boldsymbol{\theta})$ (and the associated density function $f(\boldsymbol{\theta})$) that is continuous and has full support on Θ. Before choosing her action, the judge receives a report, m_i, from each of the n experts who may or may not have observed the realized value of $\boldsymbol{\theta}$. Expert A_i's type, a_i, can either be "informed" ($a_i = 1$) or "uninformed" ($a_i = 0$), where $\Pr(a_i = 1) = \alpha_i \in [0, 1)$. An informed expert observes the state whereas an uninformed expert

[9]Gilligan and Krehbiel (1989) and Austen-Smith (1990, 1993) are some early models analyzing informational properties of "debates" between multiple experts with divergent interests in a cheap talk setting. While all of the cheap talk literature treats expert ideal actions as state-dependent, Chakraborty and Harbaugh (2010) have expert preferences similar to our paper in the sense that the experts have state independent ideal actions.

does not.[10] Since α_i represents expert A_i's prior likelihood of being informed, we can interpret α_i as a measure of the "quality" of the expert.

Experts have their own agenda, and therefore, have preferences over the judge's action. We assume that these preferences are independent of the underlying state. Expert A_i's agenda is identified by his most favored action, or, "ideal point," $\mathbf{x}_i \in \Theta$ (independent of the realization of $\boldsymbol{\theta}$). We assume that any report about the state is verifiable. So, upon observing the state $\boldsymbol{\theta}$**,** an informed expert is left with the choice of whether to disclose the state (i.e., $m_i = \boldsymbol{\theta}$) or conceal it (i.e., $m_i = \emptyset$, say) in order to induce the judge to take a favorable action.[11] Also, the reports are assumed to be costless to the expert and affects his payoff only through its impact on the judge's action.

The judge's payoff is $u_J(\mathbf{y}; \boldsymbol{\theta})$, where u_J is continuous and twice differentiable in all its arguments. To capture the fact that the judge wants to take action $\boldsymbol{\theta}$ in state $\boldsymbol{\theta}$, we assume that given $\boldsymbol{\theta}$, $u_J(\mathbf{y}; \boldsymbol{\theta})$ is strictly concave in y and is maximized at $y = \boldsymbol{\theta}$. We normalize the maximal payoff $u_J(\boldsymbol{\theta}; \boldsymbol{\theta})$ to 0.[12]

The payoff of expert A_i, $u_i(\mathbf{y}; \mathbf{x}_i)$, is also assumed to be symmetric and single-peaked around $\mathbf{x}_i$. So, $u_i(\mathbf{y}; \mathbf{x}_i) = v(\|\mathbf{y} - \mathbf{x}_i\|)$ for some strictly decreasing function v. In other words, an expert prefers the judge to take an action that matches his ideal point ($\mathbf{x}_i$) and the further off is the judge's action from the expert's own ideal point, the lower is his payoff.

Since F is atom-less, we can focus on pure strategies without loss of generality. The strategy of an informed expert A_i is $m_i(\boldsymbol{\theta}) \in \{\boldsymbol{\theta}, \emptyset\}$ for all $\boldsymbol{\theta} \in \Theta$ and that of an uninformed expert by $m_i = \phi$ (by assumption). Denote a profile of reports from all experts $\{m_1, m_2, ...m_n\}$ by $\mathbf{m}$. For any state $\boldsymbol{\theta}$, denote by $m_{-i}(\boldsymbol{\theta})$ the profile of report of all experts except that of A_i. Finally, let $\mathbf{y} = y(\mathbf{m})$ be the action taken by the judge upon receiving the report profile $\mathbf{m}$.

We use *perfect Bayesian Nash* equilibrium (PBE) as a solution concept. Let $\mu(\boldsymbol{\theta} \mid \mathbf{m})$ be the posterior belief of the judge upon receiving the experts' reports $\mathbf{m}$. A strategy profile $\langle \mathbf{m}^*, y^*(\mathbf{m}) \rangle$ along with a belief μ^* constitutes a PBE of this game if the following holds:

(i) For all i, if A_i is informed, then for all $\boldsymbol{\theta} \in \Theta$, $m_i(\boldsymbol{\theta}) = \boldsymbol{\theta}$ if and only if,

$$\mathbb{E}u_i\left(y^*(\boldsymbol{\theta}, m^*_{-i}(\boldsymbol{\theta})); \mathbf{x}_i\right) \geq \mathbb{E}u_i\left(y^*(\emptyset, m^*_{-i}(\boldsymbol{\theta})); \mathbf{x}_i\right)$$

where the expectation is taken over the types of all other experts. And if A_i is uninformed, $m_i = \emptyset$.

(ii) The judge's action

$$y^*(\mathbf{m}) = \arg\max_{\mathbf{y} \in \boldsymbol{\Theta}} \int u_J(\mathbf{y}; \boldsymbol{\theta})\, \mu^*(\boldsymbol{\theta} \mid \mathbf{m})$$

for all $\mathbf{m}$.

(iii) The posterior belief of the judge $\mu^*(\boldsymbol{\theta} \mid \mathbf{m})$ is obtained by using Bayes rule given the prior belief $F(\boldsymbol{\theta})$ and the strategy profile of the experts, $\mathbf{m}^*$.

3. Equilibrium characterization

Having presented the general model of the persuasion game, we now discuss its equilibrium characterization. It turns out that an equilibrium in this game is completely characterized by the

[10] That the experts can directly observe the true state is assumed for the sake of expositional clarity. Indeed, one can consider a more general setting where instead of the true state $\boldsymbol{\theta}$, the (informed) experts can only observe a common signal s that is informative of $\boldsymbol{\theta}$. If the density of s conditional on $\boldsymbol{\theta}$ satisfies strict MLRP, the key insights of our model continue to hold in this more general setting.

[11] An alternative and more general way to model the expert's action is to assume that an informed expert can report any subset $S_i \subseteq \Theta$ containing the true state; i.e., $\boldsymbol{\theta} \in S_i$ (Milgrom and Roberts, 1986; Shin, 1994). As we will discuss in the next section, any equilibrium of our model can also be sustained under the more general case where an informed expert can report any S_i containing the ture $\boldsymbol{\theta}$. Thus, for the expositional clarity, we use the simpler framework where the an informed expert either reports the true state or completely conceals it by feigning ignorance.

[12] The substantive import of this normalization is that the judge places equal value on getting the right action in each state.

"default action" of the judge, i.e., the action that the judge would take if all experts fail to reveal the state. The judge's default action also pins down an expert's revelation strategy—an expert reveals the observed state if only if it is more favorable to him compared to the judge's default action.

In order to characterize the equilibrium, first consider the judge's strategy. The best-response of the judge upon receiving the report profile $(\mathbf{m})$ is

$$y^{*}(\mathbf{m})=\begin{cases}\boldsymbol{\theta} & if\ m_i(\boldsymbol{\theta})=\boldsymbol{\theta} \text{ for some } i\\ \mathbf{y} & otherwise\end{cases} \tag{1}$$

where

$$\mathbf{y}=\arg\max_{\mathbf{y}'\in\Theta}\int u_J\left(\mathbf{y}';\boldsymbol{\theta}\right)\mu\left(\boldsymbol{\theta}\mid\mathbf{m}=\emptyset\right)d\theta.$$

In other words, if at least one expert reveals the state, the state is known to the judge with certainty and the judge trivially takes the action that exactly matches the state. But when all experts fail to reveal the state, then the judge takes an action that maximizes her expected payoff taking into account the experts' reporting strategies. Therefore, the judge's strategy choice is equivalent to the choice of a default action $\mathbf{y}$ consequent on receiving no advice from any of the experts.

Now consider the action of an informed expert A_i given the strategy of the judge. By revealing the state, the expert induces an action $\boldsymbol{\theta}$ by the judge. In contrast, if the expert conceals the information, he induces the judge to take the default action $(\mathbf{y})$ only when no other expert reveals the state (otherwise, the judge's action is still $\boldsymbol{\theta}$). Thus, A_i decides on whether to reveal the information conditioning on the event where he is *pivotal* in determining whether the judge will take action $\boldsymbol{\theta}$ or $\mathbf{y}$. So, given any state $\boldsymbol{\theta}$, A_i reveals $\boldsymbol{\theta}$ if and only if he prefers the observed state to the judge's default action, $\mathbf{y}$ (i.e., if and only if $u_i(\boldsymbol{\theta};\mathbf{x}_i)\geq u_i(\mathbf{y};\mathbf{x}_i)$).[13] But since u_i is single peaked and symmetric around x_i, $u_i(\boldsymbol{\theta};\mathbf{x}_i)\geq u_i(\mathbf{y};\mathbf{x}_i)\Leftrightarrow\|\boldsymbol{\theta}-\mathbf{x}_i\|\leq\|\mathbf{y}-\mathbf{x}_i\|$. So, A_i reveals the state if and only if A_i's agenda $(\mathbf{x}_i)$ is closer to the observed state $(\boldsymbol{\theta})$ than the agenda is to the judge's default action $(\mathbf{y})$. In what follows, we call the set $\Theta_i=\{\theta\in\Theta\mid\ \|\boldsymbol{\theta}-\mathbf{x}_i\|\leq\|\mathbf{y}-\mathbf{x}_i\|\}$ the "revelation set" for expert A_i.

The above discussion is summarized in the following proposition that characterizes the equilibrium of the game.

Proposition 1. *There always exists a PBE of this game. Moreover, in any PBE of this game an informed expert's strategy is*

$$m_i^{*}(\boldsymbol{\theta})=\begin{cases}\boldsymbol{\theta} & if\ \boldsymbol{\theta}\in\Theta_i^{*}=\{\boldsymbol{\theta}\in\Theta\mid\ \|\boldsymbol{\theta}-\mathbf{x}_i\|\leq\|\mathbf{y}^{*}-\mathbf{x}_i\|\}\\ \emptyset & otherwise\end{cases}$$

and the judge's strategy is

$$y^{*}(\mathbf{m})=\begin{cases}\boldsymbol{\theta} & if\ m_i(\boldsymbol{\theta})=\boldsymbol{\theta}\ for\ some\ i\\ \mathbf{y}^{*} & otherwise\end{cases},$$

where

$$\mathbf{y}^{*}=\arg\max_{\mathbf{y}'\in\Theta}\int u_J\left(\mathbf{y}';\boldsymbol{\theta}\right)f\left(\theta\mid\mathbf{m}=\emptyset;\ m_1^{*},...,m_n^{*}\right)d\theta.$$

[13] With a slight abuse of language, by referring to an expert's preferences over states we mean his preferences over the judge's action where the judge is known to take an action that exactly matches the underlying state.

Several issues are worth noting in the context of the proposition above: First, an important implication of the above characterization result is that an (informed) expert's equilibrium strategy can be described only by its associated revelation set Θ_i^*. Note that a revelation set is a sphere in $\mathbb{R}^k$ centered around the expert's agenda $\mathbf{x}_i$. Also, in equilibrium, all revelation sets must share a common boundary point $\mathbf{y}^*$ (see Figure 1), which is the equilibrium default action of the judge.

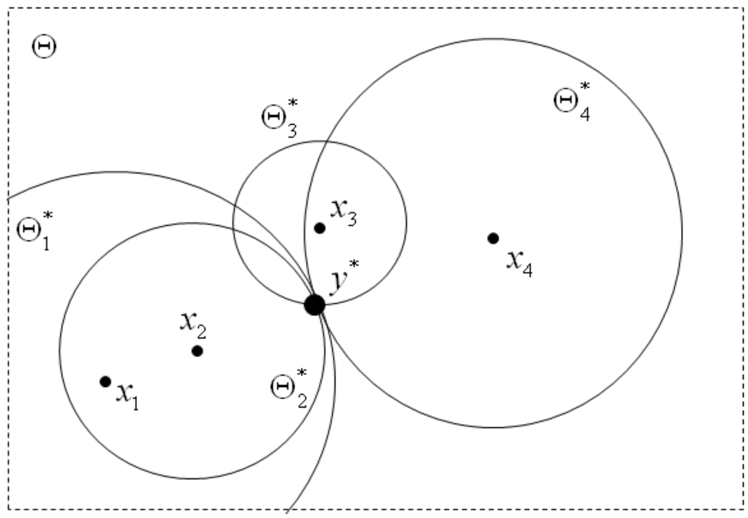

Figure 1. Revelation sets in equilibrium; $n = 4$, $\Theta \subset \mathbb{R}^2$.

Second, the equilibrium characterization does not change even if the experts send their messages sequentially. If the experts are asked to speak in some pre-specified order, or some subset (possibly all) of them may be asked to speak simultaneously, the sequence of reports does not make a difference since each expert's decision is conditioned on the event that he is pivotal. It is also easy to see that the outcome will be the same even if some experts knew the reports of some other experts before they spoke. The fact that all informed experts have the same information is important for this feature of our model.[14] This finding is similar in spirit with Dekel and Piccione (2000) who show, in the context of a voting game, that the symmetric equilibria of the simultaneous voting game are also equilibria in any sequential voting structure.

Finally, such an equilibrium characterization continues to hold if one consider a more general strategy space for the experts a la Milgrom and Roberts (1986) where an informed expert reports a subset of states, say S_i, that contains the true state, i.e., $\boldsymbol{\theta} \in S_i$ (see, e.g., Milgrom and Roberts, 1986). Under the expanded strategy space, the above equilibrium is supported by an off-the-equilibrium belief that is similar in spirit with the "skeptical posture" discussed by Milgrom and Roberts (1986): if no expert reports the state and some expert A_i deviates and reports a strict subset S_i of the state space, then the judge believes that the true state is the one in S_i that is least favorable to A_i.[15]

Since we are primarily interested in using this model to analyze the nature of the game when experts compete to influence the judge, in the rest of this article we focus our attention on a unidimensional state space. Such a state space has a feature that preference ordering over the states for one expert *may* be the complete opposite of the ordering for his rival expert (when the experts' agenda are sufficiently diverse). So, we end this section by revisiting the equilibrium

[14]Ottaviani and Sorensen (2001) show that in presence of reputational concerns, the sequencing of experts does matter. In our case, the experts are concerned not with their reputation but only with the final action, and in this setting, the sequence is immaterial.

[15]If multiple experts report a strict subset of the state space, then the judge picks an expert randomly from the set of deviators and punishes him. The formal proof is available with the authors.

characterization for a special case when $\Theta \subset \mathbb{R}$ and $n = 2$, as given by the corollary (to proposition 1) below.

Corollary 1. *Suppose that $\theta \in [0,1]$ and $n = 2$. In equilibrium, for $i = 1, 2$, A_i reveals the truth if and only if $\theta \in \Theta_i^* = [0,1] \cap [x_i - |y^* - x_i|\,,\ x_i + |y^* - x_i|]$ where y^* is given by the equation*

$$\int \frac{\partial}{\partial y} u_J \left(y^*; \theta\right) dF \left(\theta \mid m_1 = m_2 = \emptyset; m_1^*(\theta), m_2^*(\theta)\right) = 0. \tag{2}$$

Moreover, y^* always lies in $(0, 1)$.

In a persuasion game between two experts when the state space is unidimensional, the experts' equilibrium revelation sets, Θ_i^*, are intervals in $\mathbb{R}$ that share exactly one common boundary point, y^*. In what follows, we use this simple framework to elaborate on the linkage between the nature of the persuasion game and some of the key features of the environment that reflects the extent of conflict between the experts, such as the experts' quality and the relative positions of their personal agenda.

4. Experts with completely opposed agenda

In this section we explore the nature of the persuasion game under the canonical setting of completely opposing experts. To analyze this issue we focus on a special case of the environment highlighted in corollary 1: we consider a game with two experts in a unidimensional state space $\Theta = [0, 1]$ such that $x_1 = 0$ and $x_2 = 1$. That is, the agenda of the two experts are completely opposed—the "higher" is the judge's action the less favorable it is to A_1 and the more favorable it is to A_2 .

By corollary 1, when $x_1 = 0$ and $x_2 = 1$ there is an equilibrium of this game where $\Theta_1^* = [0, y^*]$ and $\Theta_2^* = [y^*, 1]$, where y^* is given by equation (2). Moreover, in this case, the equilibrium is unique and equation (2) can be further simplified. These observations are summarized in the following proposition.

Proposition 2. *When the experts have completely opposed agenda, there is a unique equilibrium of the game where $\Theta_1^* = [0, y^*]\,,\ \Theta_2^* = [y^*, 1]$ and $y^* \in (0, 1)$ solves*

$$\begin{array}{l}(1 - \alpha_1) \Pr\left(\theta \le y^*\right) \int_0^{y^*} u_J' \left(y^*; \theta\right) dF \left(\theta | \theta \le y^*\right) \\ + (1 - \alpha_2) \Pr\left(\theta > y^*\right) \int_{y^*}^1 u_J' \left(y^*; \theta\right) dF \left(\theta | \theta > y^*\right) = 0,\end{array}$$

i.e.,

$$(1 - \alpha_1) \int_0^{y^*} u_J' \left(y^*; \theta\right) dF\left(\theta\right) + (1 - \alpha_2) \int_{y^*}^1 u_J' \left(y^*; \theta\right) dF\left(\theta\right) = 0. \tag{3}$$

The key features of this equilibrium are intuitively obvious. The revelation sets of the two experts constitute a partition of the state space—the state space is broken into two revelation sets, each set containing states deemed favorable by one expert but unfavorable by the other. Thus, even if both experts are informed, each state is revealed by exactly one expert while the other expert chooses to conceal the information. Also note that if both experts are informed, then the true state is necessarily revealed in the equilibrium.

If one interprets concealment of information as sending an uninformative message, this framework justifies why such messages might be employed profitably in equilibrium. An informed expert who finds that the state is unfavorable pools with the uninformed expert by sending the uninformative message, hoping that the other expert has not observed the state.[16]

Having characterized the equilibrium, one might be interested in its comparative statics properties. Using equation (3) above, we can investigate how the equilibrium behavior changes with the experts' quality α_i (i.e., an expert's prior likelihood of being informed).

Proposition 3. *When the experts have completely opposed agenda, an increase in the quality of an expert moves the judge's default action away from the expert's private agenda. In equilibrium, the revelation set Θ_i expands with α_i and shrinks with α_j.*

The proposition says that as an expert A_i's quality increases, the default action of the judge becomes more unfavorable to A_i and more favorable to his rival A_j. In other words, $\partial y^*/\partial \alpha_1 > 0$ and $\partial y^*/\partial \alpha_2 < 0$. Since the default action also completely determines the revelation set of each expert, proposition 3 suggests that each expert reveals more information as his own quality increases and less information as the rival's quality increases.

The intuition behind Proposition 3 is as follows: recall that $\Theta_1^* = [0, y^*]$ and $\Theta_2^* = [y^*, 1]$, and consider an increase in A_1's quality (α_1). With higher α_1, A_1 is more likely to know the state and reveal it if $\theta \in \Theta_1^*$. So, if the judge does not receive any report on the state, her posterior belief shifts probability weight from the event of genuine non-observance of the state by A_1 to event that the state is unfavorable to A_1. In effect, the judge shifts weight from $\theta \in \Theta_1^*$ to $\theta \in \Theta_2^*$ and increases her action (y^*) accordingly. Since each expert reveals information if and only if the state is more favorable than the default action y^*, an increase in y^* induces more revelation from A_1 and less from A_2. Note that in this case the revelation decisions of the two experts can be conceived as *strategic substitutes* in the sense that an expansion of one expert's revelation set leads to contraction of the other's.

It is worth noting that Proposition 3 is similar is spirit with the findings in Shin (1994) who argues that the burden of proof should lie with the more informed expert. That is, unless the state is proven, the judge should favor the less informed expert compared to the benchmark action, say, $\tilde{y}$, that would obtain if she were to use only the information contained in the prior distribution of the states. But this is one of the implications of Proposition 3—in the equilibrium with completely opposed experts, we must have $y^* > \widetilde{y}$ if $\alpha_1 > \alpha_2$ and $y^* < \widetilde{y}$ if $\alpha_1 < \alpha_2$; i.e., the default action favors the less informed expert.[17]

But in contrast with Shin, we primarily focus on the extent of information revealed rather than the burden of proof. Observe that a direct implication of Proposition 3 is that an increase in the quality of one expert can lead to less information being revealed *ex-post,* i.e., given a certain state of the world. Suppose that due to an increase in α_1, the default action by the judge increases from y^* to y^{**}. Now, in the event that A_1 is uninformed and A_2 is informed, a state $\theta \in (y^*, y^{**})$ would not be revealed following the increase in the quality of A_1, while it would have been revealed previously. As a special case, the above observation suggests that having an opposing expert can reduce information revelation *ex-post* compared to the case when there is only one expert.[18]

[16] This observation is in contrast with the so-called "unravelling" argument (see Milgrom 1981; Milgrom and Roberts, 1986) that suggests that non-disclosure of information may signal "bad news" for the sender with certainty and therefore is never useful in equilibrium. But in our framework, such messages are useful since the judge cannot tell whether the expert is indeed informed or not (a similar argument is also presented in Shin, 1998).

[17] This finding is also reminiscent of Che and Kartik (2009) who derive a similar result in a single expert model: they show that the "no-information" action of the decision-maker moves away from the expert's ideal point with an increase in the likelihood that the expert is informed about the true state.

[18] Our model reduces to a persuasion game between the judge and a single expert when the other expert is known to be uninformed with certainty. For example, if $\alpha_1 = 0$, then the game reduces to one where the judge faces a single expert A_2. Now, the introduction of a new expert A_1 can be represented as an increase in α_1, and as we have argued, this may lead to less information being revealed ex post.

An increase in quality of one expert may induce some states to be revealed with lower probability, but it also induces some other states to be revealed with a higher probability. Overall, it is not clear a priori whether more or less information is revealed. In this article, we take the judge's ex-ante welfare as an index of the volume of information revealed in equilibrium. We see later in Proposition 4 that when the experts have completely opposed agenda, the judge's ex-ante equilibrium payoff is increasing in an expert's quality (i.e., $\partial \mathbb{E}_\theta \left[u_J \left(y^*; \theta \right) \mid \mathbf{m}^* \right] / \partial \alpha_i > 0$ for $i = 1, 2$). In other words, even if an increase in an expert quality may reduce the judge's payoff *ex-post*, it always increase the judge's payoff *ex-ante*, provided that the experts have completely opposed agenda.

We conclude this section with two important remarks. First, when the two experts' agenda are not completely opposed, the above analysis still continues to hold as long as the agenda are not "too similar" in the following sense: suppose, given the α_is and the prior distribution over the state space, the judge's default action with extreme experts (i.e., when $x_1 = 0$ and $x_2 = 1$) is $\hat{y}$, which is the solution to equation (3). For a given agenda profile (x_1, x_2) of the experts, we say that the experts are *sufficiently opposed* whenever $x_1 < \hat{y}/2$ and $x_2 > (1 + \hat{y})/2$. Indeed, when the experts are sufficiently opposed, then there exists at least one equilibrium of the game that is isomorphic to the equilibrium of the game with completely opposing agenda. That is, there is an equilibrium of the game where $\Theta_1^* = [0, y^*]$, $\Theta_2^* = [y^*, 1]$ and y^* solves equation (3). Moreover, $\Theta_1^* \cup \Theta_2^* = [0, 1]$ only if the experts are sufficiently opposed.[19]

Second, if the agenda are indeed "too similar," the nature of equilibrium may differ and the comparative statics results as discussed above may be reversed. For example, an increase in the quality of an expert may lead to a decrease in the judge's *ex-ante* payoff. The following section elaborates on this issue.

5. The role of diversity of agenda

In the case of "sufficiently opposed" experts, we have noted that the revelation sets of the two experts form a partition of the state space, i.e., the revelation sets are mutually exclusive and exhaustive. That the sets are disjoint implies that there is "conflict" between experts in equilibrium: all the states contained in an expert's revelation are deemed favorable by that expert but unfavorable by the other. Also recall that since the revelation sets are also exhaustive (i.e., $\Theta_1^* \cup \Theta_2^* = [0, 1]$), the state is always revealed when both experts are informed. In this section, we examine the more general case with $0 \leq x_1 \leq x_2 \leq 1$, i.e., when the experts are not necessarily sufficiently opposed in their agenda. To do that, we first study the contrasting case where experts have no conflict of interest.

5.1. Experts with similar agenda.

Consider a polar case where the experts are completely congruent in their preferences, but both prefer as low an action as possible, i.e. $x_1 = x_2 = 0$. To fix ideas, consider the following example: a panel consisting of two anti-war activists are asked to present evidence before a policy maker on foreign policy. Then, if neither of the two activists can provide convincing evidence to the policy maker, she places a high likelihood on a state of the world that requires going to war, and decides on a very hawkish policy. This fact, in turn, forces each of the activists to reveal the state of the world as long as it is not too strongly in favor of going to war. According to corollary 1, the equilibrium in this case is unique where both the experts have

[19] The arguement behind this statement is straightforward. Without loss of generality, assume $0 \leq x_1 < x_2 \leq 1$. First note that from Corollary (1), we know that in any equilibrium, the judge's default action $y^* \in (0, 1)$. So, if in equilibrium, we have $\Theta_1^* \cup \Theta_2^* = [0, 1]$, it must be the case that $x_1 < y^* < x_2$. But, because we have $\Theta_1^* \cup \Theta_2^* = [0, 1]$, we must have the equilibrium revelation sets as $[0, y^*]$ and $[y^*, 1]$. This implies that $x_1 \leq y^*/2$ and $x_2 \geq (1 + y^*)/2$. But then, y^* must be a solution to equation (3), which is known to have a unique solution $\hat{y}$. So we must have $y^* = \hat{y}$. Hence x_1 and x_2 are sufficiently opposed.

the same revelation set: $[0, y^*]$ and the judge's default action is described by the following equation

$$(1-\alpha_1)(1-\alpha_2)\int_0^{y^*} u_J'(y^*,\theta)dF(\theta) + \int_{y^*}^1 u_J'(y^*,\theta)dF(\theta) = 0. \tag{4}$$

Since the two revelation sets are the same, we have "congruence" among experts in equilibrium. The trade-off faced by the judge is that a high default action induces a larger truth-telling set for each expert, but on the other hand, in case neither expert observes the state, the judge runs the risk of making a very bad decision. Notice that if the state is less than y^*, the judge is assured of a correct decision as long as *at least one* expert observes the state, but if the state is larger than y^*, it is never revealed, even if both experts are informed.

Similarly, if each expert wants as high an action as possible (i.e., $x_1 = x_2 = 1$), the equilibrium is unique and both the revelation sets are $[y^*, 1]$, where the default action y^* is given by the unique solution to the following equation

$$\int_0^{y^*} u_J'(y^*,\theta)dF(\theta) + (1-\alpha_1)(1-\alpha_2)\int_{y^*}^1 u_J'(y^*,\theta)dF(\theta) = 0. \tag{5}$$

While the case with extreme but congruent agenda is a polar case, the equilibrium is robust to small changes in expert preferences. In particular, suppose y_0 is the solution to equation (4). Then as long as both x_1 and x_2 are weakly less than $y_0/2$, there is an equilibrium where the default action is y_0 and the revelation sets are both $[0, y_0]$. Similarly, if y_1 is the solution to equation (5), as long as both experts' ideal points are weakly greater than $(1 + y_1)/2$, the equilibrium outcome described by equation (5) still holds. In both these cases, both experts have extreme agenda, but unlike the case with sufficiently opposed experts, the conflict of interest between the experts is low.

Definition 1. *Suppose the (unique) solutions to equations (2), (4) and (5) are $\widehat{y}$, y_0 and y_1 respectively. Given a profile of ideal points $\{x_1, x_2\}$ for the two experts, we say that the experts are "sufficiently extreme" if any one of the following is true: (i) $x_1 \leq \widehat{y}/2$ and $x_2 \geq (1+\widehat{y})/2$, or (ii) $\max\{x_1, x_2\} \leq y_0/2$, or (iii) $\min\{x_1, x_2\} \geq (1+y_1)/2$.*

Under the umbrella definition of sufficiently extreme experts, we bunch together the case of sufficiently opposed experts (case (*i*) in the definition) with sufficiently similar but extreme experts (cases (*ii*) and (*iii*) in the definition). We shall presently see that these cases share certain features that are important for the exposition of this paper.

5.2. **Nature of equilibria.** Having looked at the polar cases, we are now in a position to study the nature of equilibria for the general case where $0 \leq x_1 \leq x_2 \leq 1$. In general, as we let the experts' ideal points vary, we can have two classes of equilibria, one with conflict where the revelation sets are disjoint (but always adjacent), and another with (partial) congruence where one expert's revelation set is a weak subset of the other's.[20] While the equilibrium is unique in the polar cases, in general the game may have more than one equilibrium, where some equilibria exhibit conflict and some exhibit partial congruence. Figure 2 below illustrates these features. The following set of examples illustrates this point when θ is distributed uniformly in $[0, 1]$ and $u_J(y;\theta)$ represents a quadratic loss function.

[20]Note that any equilibrium of this game must fall into one of these two classes. In other words, there cannot be equilibria where the revelation sets are neither disjoint nor with one being a subset of the other. This observation follows from the fact that the two experts' revelation sets must share a common boundary point which is given by the judge's default action y^* and the revelation sets are intervals in $\mathbb{R}$.

Example 1. *Consider the case where F is uniform and $u_J(y;\theta) = -(y-\theta)^2$. Also assume that $\alpha_1 = \alpha_2$. In this case, if $x_1 = 0$ and $x_2 = 1$, equation (2) yields $y^* = 1/2$. So, whenever $x_1 < y^*/2 = 1/4$ and $x_2 > (1+y^*)/2 = 3/4$, the judge's default action remains at $y^* = 1/2$ and the experts' revelation sets are $\Theta_1^* = [0, 1/2]$ and $\Theta_2^* = [1/2, 1]$. Thus, the equilibrium exhibits conflict and all states are revealed if both experts are informed.*

Example 2. *Similarly, in the case where $\alpha_1 = \alpha_2 = 0.9$, if $x_1 = x_2 = 0$, equation (4) yields $y^* = 10/11$. So, whenever $\max\{x_1, x_2\} \leq y^*/2 = 5/11$, the judge's default action remains at $y^* = 10/11$ and the experts' revelation sets are $\Theta_1^* = \Theta_2^* = [0, 10/11]$. Thus, the equilibrium exhibits congruence and all states in $[0, 10/11]$ are revealed if either expert is informed.*

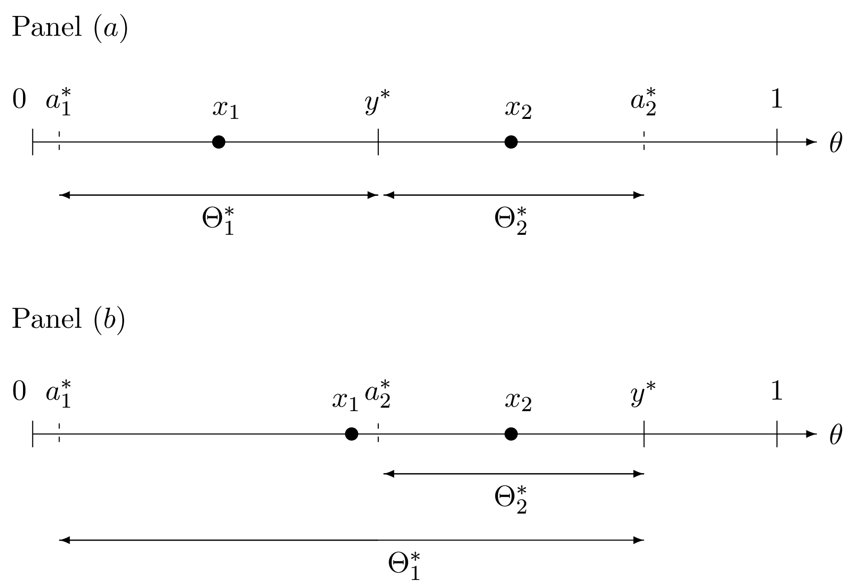

Figure 2. Revelation sets in equilibrium: "conflict" (panel (a)) and "partial congruence" (panel (b))

Example 3. *Now consider a case where the experts' agenda are not so extreme. Suppose $x_1 = 0.3$, $x_2 = 0.7$, and $\alpha_1 = \alpha_2 = 0.9$. In this case, there are three equilibria and both conflict and partial congruence can be observed in equilibrium. There is one equilibrium exhibiting conflict where $y^* = 0.5$, $\Theta_1^* = [0.1, 0.5]$ and $\Theta_2^* = [0.5, 0.9]$. Note that even if the equilibrium exhibits conflict, the revelation sets are such that the states lying in the set $[0, 0.1) \cup (0.9, 1]$ are never revealed in equilibrium. Further note that there are two equilibria exhibiting partial congruence: (i) $y^* = 0.76$, $\Theta_1^* = [0, 0.76]$ and $\Theta_2^* = [0.64, 0.76]$. (ii) $y^* = 0.24$, $\Theta_1^* = [0.24, 0.36]$ and $\Theta_2^* = [0.24, 1]$. Also, in each of these two equilibria, there are states that are never revealed even if both experts are informed.*

In light of the above examples, it is important to note the following: the degree of opposition among the experts' as given by their private agenda need not indicate whether the experts' interests in equilibrium are in conflict or in partial congruence. Because both experts and the judge are playing best-response to each other, for the same underlying parameters there can be multiple equilibria depending on the kind of coordination between the players. Both types of equilibria—conflict and partial congruence—may originate in the persuasion game played by the same set of experts. It is also interesting to note that under a partial congruence equilibrium, the revelation decisions of the experts behave like *strategic complements*: if the judge's default action changes, the revelation sets of *both* experts expands or contracts (depending of the parameter values) in tandem.

At this point, we ask the following question: under what circumstances would the judge benefit from higher expert quality? It turns out that due to strategic manipulation of information, the judge's expected payoff may *decrease* with an increase in the quality of an expert. In fact, the comparative static properties of the outcome critically depend on the nature of the equilibrium played, and in particular, on the properties of the revelation sets. The following definition is useful for the subsequent discussion on this issue.

Definition 2. *An equilibrium of the persuasion game is said to be "locally insensitive" to the experts' agenda if the equilibrium revelation set of each expert is either* $[0, y^*]$ *or* $[y^*, 1]$.

According to the above definition, if each of the equilibrium revelation sets is either $[0, y^*]$ or $[y^*, 1]$, then a small change in expert agenda does not affect the equilibrium outcome of the game. Under what conditions on the primitives is this feature likely to arise in equilibrium? The following lemma says that local insensitivity to expert agenda is intimately linked with the extremeness of expert agenda (but *not* with the extent of conflict). We skip the proof of the lemma as it follows from the previous discussion.

Lemma 1. *An equilibrium of the persuasion game is locally insensitive to the experts' agenda only if the experts are sufficiently extreme. On the other hand, if the experts are sufficiently extreme, then there is at least one equilibrium which is locally insensitive to the experts' agenda.*

The following proposition says that an increase in an expert's quality is guaranteed to induce an ex-ante increase in the judge's payoff *only if* the equilibrium is locally insensitive to the experts' agenda.

Proposition 4. *Suppose* $0 \leq x_1 < x_2 \leq 1$. *If an equilibrium is locally insensitive to the expert agenda, then the expected payoff of the judge is increasing in the experts' quality, i.e.* $\partial \mathbb{E}_\theta \left[u_J \left(y^*; \theta \right) \mid \mathbf{m}^* \right] / \partial \alpha_i > 0$ *for* $i = 1, 2$. *Else, the sign of* $\partial \mathbb{E}_\theta \left[u_J \left(y^*; \theta \right) \mid \mathbf{m}^* \right] / \partial \alpha_i$ *is ambiguous.*

The key implication of the above proposition is that the judge's *ex-ante* payoff need not always increase when the expert's quality improves. When the experts are sufficiently extreme, there is at least one equilibrium where the judge benefits from the experts being more informed (and as we have already seen, sometimes this equilibrium is unique). But if the experts are moderate, then it is possible that a higher expert quality hurts the judge.

We discuss the intuition for the above proposition in the context of a "conflict" equilibrium. The intuition can be best explained with the help of Figure 3 below. Suppose that the local insensitivity condition does not hold, i.e., the revelation sets for A_1 is $[a_1^*, y^*]$ and that of A_2 is $[y^*, a_2^*]$, where y^* is the judge's default action, and $0 < a_1^* < a_2^* < 1$, implying $\Theta_1^* \cup \Theta_2^* \subset [0, 1]$. Now suppose that α_2 increases and the judge's default action moves to the left to y^{**}. Consequently, the revelation set for A_2 expands to $[y^{**}, a_2^{**}]$ and that of A_1 shrinks to $[a_1^{**}, y^{**}]$. Now, the judge is more likely to learn the state if it is in A_2's initial revelation set, i.e., $\theta \in [y^*, a_2^*]$, which is the "direct" effect that increases the judge's payoff (indicated in the figure by the region B).

But the changes in the revelation sets would also lead to several "indirect" effects. First, the region $[y^{**}, y^*]$ moves from A_1's revelation set to A_2's revelation set, and whether that is good or bad for the judge will depend on the relative magnitudes of α_1 and α_2. Second, for $\theta \in [a_2^*, a_2^{**}]$ (i.e., region C), now there is a positive probability that the judge will learn the state whereas in the initial equilibrium, such a value of θ would have never been revealed. Finally, any $\theta \in [a_1^*, a_1^{**}]$

(as indicated in the figure by region A) will not be revealed even though A_1 would have revealed such a θ in the initial equilibrium.

Now, due to the single-peakedness of the judge's payoff function, we have $u'_J(y^*, \theta) = 0$ at $\theta = y^*$. Thus, small mistakes in states around y^* do not have a first order effect on the judge's payoff. Therefore, the first of the three indirect effects is negligible. The overall effect of an increase in α_2 on the judge's expected payoff would therefore be determined by whether the loss of information from shrinkage of A_1's revelation set is larger than the gain from expansion of A_2's revelation set and the direct effect put together. If there is a large enough probability mass in the interval $[a_1^*, a_1^{**}]$, then an increase in α_2 might well hurt the judge.[21]

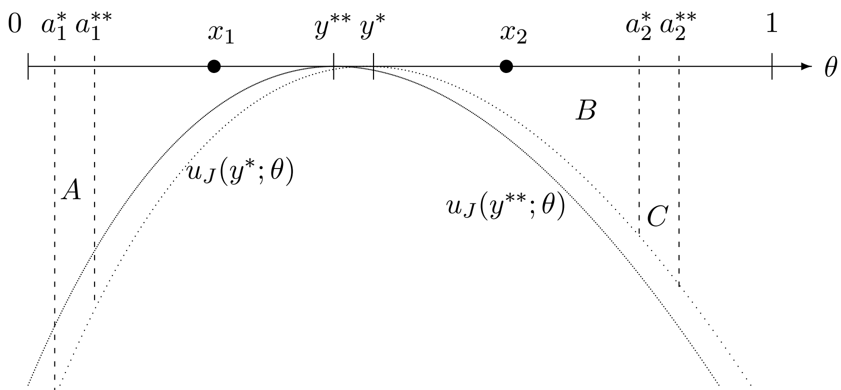

Figure 3. Change in judge's payoff following an increase in α_2 is given by the expected payoff over the regions $B + C - A$

The following example highlights this issue:

Example 4. *Similar to the earlier examples, assume that θ is distributed uniformly on $[0, 1]$ and $u_J(y; \theta) = -(y - \theta)^2$. Now, consider a case where $x_1 = 0.35$, $x_2 = 0.61$ and A_1 is almost surely "uninformed" with $\alpha_1 = 0.01$ and A_2 is almost surely "informed" with $\alpha_2 = 0.95$. Here, there is only one equilibrium exhibiting conflict where $y^* = 0.4539$, $\Theta_1^* = [0.2461, 0.4539]$ and $\Theta_2^* = [0.4539, 0.7661]$. The associated payoff to the judge is -0.07578. Now suppose that the quality of A_1 improves where $\alpha_1 = 0.4$. A new equilibrium (with conflict) is $y^* = 0.4847$, $\Theta_1^* = [0.2153, 0.4847]$ and $\Theta_2^* = [0.4847, 0.7353]$. But the corresponding payoff to the judge reduces by 0.0002 to -0.07598.*

A few issues are worth noting in this context: First, that the argument above does not hold if $\Theta_1^* \cup \Theta_2^* = [0, 1]$. In this case, the regions A and C no longer exist. Thus, the only indirect effect stems from mistakes around y^*, which has no first-order impact. Therefore, the direct effect dominates and the judge is unambiguously better off. This brings out the role of the local insensitivity condition. In equilibria with conflict, local insensitivity is equivalent to having $\Theta_1^* \cup \Theta_2^* = [0, 1]$. In general,

[21] The case of the equilibrium with partial congruence is analogous. Consider an equilibrium with $x_1 < x_2 < y^*$. Suppose the two revelation sets are $[a_1, y^*]$ and $[a_2, y^*]$ respectively, and assume that $a_1 > 0$. Any increase in y^* (to $y^* + \epsilon$) due to an increase in α_1 will lead to an expansion of both revelation sets: the new revelation sets will be $[a_1 - \epsilon, y^* + \epsilon]$ and $[a_2 - \epsilon, y^* + \epsilon]$. These imply two opposite indirect effects: an expansion of revelation sets at a_1 and a_2 will lead to more information being revealed to the judge, but conditional on the state not being revealed to the judge, for the states $\theta < y^*$, the error will increase. While the direct effect of an improvement in expert quality is positive, the net effect may have either sign depending on the strength of the indirect effects.

local insensitivity stipulates that a change in y^* does not affect the revelation set of either expert except around y^*. Therefore, the indirect effect never has a first-order impact, and the direct effect dominates. This explains why, with local insensitivity, the judge is always better off as an expert's quality improves.

Second, the presence of experts is necessary for the finding that an increase in the expert's quality may make the judge worse off. Indeed, if there is a single expert trying to persuade the judge, then an increase in his quality will always move the judge's default action y^* away from the expert's ideal point.[22] Thus, a higher quality only has the direct positive effect of eliciting more information from the expert. The indirect negative effect that we have highlighted above—one that emanates from the strategic interaction between the two experts' revelation sets—disappears.

Finally, we conclude this section with a different implication of proposition 4. Suppose the quality of A_1 increases from α_1 to α_1'. This change can be reinterpreted as the introduction of a third expert A_3 whose ideal point is same as that of A_1, and the probability of observing the state is $\alpha_3 = (\alpha_1' - \alpha_1) / (1 - \alpha_1)$. Now, since the proposition says that an increase in an expert's quality may hurt the judge ex-ante, it can be reinterpreted as saying that the introduction of an additional expert may reduce the quality of decision making. The marginal value of the information brought in by the expert may be negative for the judge.

6. Discussion

In the above analysis, we have studied how the degree of opposition between experts affect information revelation in a persuasion game. We have seen that some perverse results arise when the judge has no ability to commit to any action. To be able to better compare with the literature, in this section, we first look into a situation when the judge is endowed with a minimal commitment power: she can commit to a default action. In particular, we show that such commitment is valuable to the judge only when at least one of the experts is "moderate", i.e. has a strictly interior ideal action.

Next, we examine a design question: what agenda profile $\{x_1, x_2\}$ is most conducive to information revelation? Equivalently, if the judge were to choose the ideal points of the experts in his panel, what would he choose, given their ability levels? Would she induce competition by having completely opposed experts? It turns out that the optimal expert panel always consists of sufficiently extreme experts. In particular, either completely opposed or completely similar but extreme experts are the most conducive to information revelation. Interestingly, this result holds true irrespective of whether or not the judge can commit to an optimal default action. Moreover, it is possible that the optimal agenda profile from the judge's point of view consists of perfectly identical but extreme experts (rather than completely opposed experts). There are two reasons why such a possibility is surprising. First, it stands in stark contrast to the position taken in Dewatripont and Tirole (1999) that advocacy of opposing positions always improves outcomes. Second, while the mechanism design literature emphasizes exploiting differences in expert preferences for eliciting the truth, we show that the judge might optimally want to have experts with identical preferences.

6.1. **The Role of the judge's commitment power.** In the model described above, the judge chooses her default action as a part of her best-response to the experts' revelation strategies. But what happens if the judge could commit to her default action? Clearly, such a commitment power cannot make the judge any worse off since the judge can always replicate an equilibrium outcome by committing to the default action associated with the given equilibrium. So, a more relevant question is whether the judge can *always* improve her payoff if she could commit to a default action up front. As the following proposition shows, it need not be so.

[22]See Appendix B for proof. However, note that in the presence of multiple experts it is *not* guaranteed that an increase in an expert's quality moves the judge's default action away from the expert's ideal point.

Proposition 5. *Suppose both experts have completely extreme agenda (i.e., either $x_1 = 0$ and $x_2 = 1$ or $x_1 = x_2 = 0$ or $x_1 = x_2 = 1$). The judge's ability to commit to an optimal default action leaves her expected payoff and the default action unchanged compared to the case where she does not have such commitment power.*

The above proposition suggests that the judge's commitment power has value only when at least one of the two experts is moderate (i.e., $0 < x_i < 1$). In contrast, when experts are completely opposed ($x_1 = 0$ and $x_2 = 1$) or both experts share a common extreme position ($x_1 = x_2 = 0$ or 1), the judge cannot improve her payoff even if she can commit to her default action.[23]

In fact, the exact condition under which the commitment makes no difference is that there is a unique equilibrium where each revelation set locally insensitive to the experts' private agenda, (which happens only if the experts are sufficiently extreme). The intuition behind this finding is related to the argument behind Proposition 4 and can again be traced from Figure 3 (in section 5). Reconsider the equilibrium described in Figure 3 where the default action y^* and the associated revelation sets are $[a_1^*, y^*] = [2x_1 - y^*, y^*]$ and $[y^*, a_2^*] = [y^*, 2x_2 - y^*]$. Now, if the judge has commitment power, she can choose a potentially different default action and, in the process, manipulate the revelation set. When does such manipulation increase the judge's payoff? Note that a marginal change in the default action y^*, say from y^* to y^{**}, entails a marginal change in the availability of information at the two end points of an expert's revelation set, y^* and a_i^*. As we have noted earlier, changing the common end point y^* has little marginal impact on the judge's payoff as it already maximizes the judge's payoff *given* the experts' revelation set. But the distinct end points (a_1^* and a_2^*) also change (to a_1^{**} and a_2^{**}) with a change in the judge's default action. While playing the best-response to the experts, the judge ignores this effect whereas she can internalize this effect if she can commit to her default action up front. Thus, commitment may have value to the judge. But in the same vain, if in all equilibria of the game, the revelation sets are either $[0, y^*]$ or $[y^*, 1]$, the judge's default action only affects the common end point y^*. As argued above, in this case, committing to a different default action cannot make the judge any better off since y^* already maximizes the judge's payoff given the revelation strategies of the two experts.

An interesting implication of the above argument is that if the judge is able to commit to a default action, her payoff will always increase with the quality of either expert (α_i). The argument relies on envelope theorem.[24] Note that the default action that the judge commits to, say, y^*, maximizes her payoff after accounting for its impact on the experts' revelation sets, Θ_i^*. So, an increase in α_i only has a direct effect on the judge's payoff through increasing the probability of revelation of the state when $\theta \in \Theta_i^*$. The indirect impact of α_i on the revelation set Θ_i^* through the choice of the judge's default action y^* becomes a second order effect since y^* is already the optimal action given the interdependency between y^* and Θ_i^*.

6.2. **Does the judge prefer experts with diverse opinions?** Expert panels may have many different compositions based on the experts' preferences. There are at least two dimensions on the choice of the set of experts that the judge may consult: (i) how extreme the experts are, and (ii) how different their preferences are. What type of panel is the most conducive to information revelation? Our model offers an answer when the state/action space is unidimensional. That is, we can use our model to answer the following question: if the judge can select (or "commission") two experts from a continuum of experts with their ideal points distributed over the range $[0, 1]$, what should she choose?

[23] If the judge could commit to a *distribution over actions* rather than a deterministic action, then too, the proposition would go through as long as the experts have utility functions that are concave in y.

[24] We omit the formal proof as it exactly follows the logic discussed below.

Proposition 6. *Suppose the state space is the unit interval. If the judge could select the agenda profile* $\{x_1, x_2\}$ *of the experts (given the other parameters), then the judge would always choose sufficiently extreme experts. In particular, one of the profiles with extreme ideal points, i.e.* $\{0, 1\}$, $\{0, 0\}$ *or* $\{1, 1\}$ *is always optimal. The optimal profile of ideal points is the same irrespective of whether the judge can or cannot commit to a default action.*

The proposition says that the judge should employ either completely opposed experts or completely similar *but extreme* experts. Notice that if the experts are sufficiently opposed, then there is one equilibrium that has the same strategies and outcomes as that with completely opposed experts, i.e. the profile $\{0, 1\}$. Similarly, if the experts are sufficiently similar but extreme, then there is one equilibrium that is completely identical in terms of strategies and outcome as that with completely identical but extreme experts (i.e., the experts with profile $\{0, 0\}$ or $\{1, 1\}$). Therefore, according to the above proposition, the real choice the judge faces is between employing completely opposed experts or completely identical but extreme experts.

To see the intuition behind the proposition, first consider the case with commitment. Suppose for some profile $\{x_1, x_2\}$, the optimal default action for the decision-maker is $\overline{y} \in (x_1, x_2)$. Further assume that one of the revelation sets is locally sensitive to the expert's agenda. For example, suppose we have the revelation set of expert A_1 as $[a, \overline{y}]$ where $a > 0$. Now, for the same default action $\overline{y}$, if the profile were $\{0, x_2\}$, then the revelation set of A_2 would be unchanged but that of A_1 would be $[0, \overline{y}]$ which is a superset of the revelation set $[a, \overline{y}]$. Therefore, expert A_1 will reveal the information in more states than before, and this will improve the ex-ante payoff of the judge. Clearly, the optimal choice of default action given the profile $\{0, x_2\}$ leads to an even higher ex-ante expected payoff for the judge. Notice that such a strict improvement would not be possible if the agenda profile was such that both revelation sets were locally insensitive to the expert agenda. Therefore, by lemma 1, the set of optimal profiles consists only of sufficiently extreme experts. Moreover, according to proposition 5, even if the judge were not able to commit to an optimal default action, her equilibrium choice of action and consequent payoff would be the same in case of sufficiently extreme experts. Therefore, irrespective of whether the judge can commit to a default action or not, her expected payoff is maximized by a panel of sufficiently extreme experts.

A more interesting finding is that contrary to the conventional wisdom, completely opposed experts *may* be dominated by identical but extreme experts. Observe that the judge's choice of the default action y^* serves two roles: first, it minimizes the judge's expected loss from taking an action that may not match the underlying state. And second, the further away y^* is from an expert's private agenda, the more incentive he has to reveal the state (i.e., his revelation set expands with y^*). When the experts stand in opposite extremes, an expansion of expert A_1's revelation set (i.e., $\Theta_1^* = [0, y^*]$) necessarily dampens expert A_2's incentives for disclosure (i.e., $\Theta_2^* = [y^*, 1]$ shrinks). Such a countervailing effect disappears when both experts stand at the same extreme. In this case, a default action sufficiently away from their common agenda gives both experts strong incentives to reveal the state if they are indeed informed. Consequently, the judge's expected equilibrium payoff increases as the disclosure of the true state becomes more likely. In other words, the ability of the judge to punish the experts is maximum when the experts have congruent but extreme preferences, and minimum when the experts have completely opposed agenda. This punishment power may trump the role that competition between experts plays in improving information revelation. The following example further illustrates this point. It shows that when the distribution of the states is uniform and the judge's payoff function is represented by quadratic loss function, then the judge's expected payoff is *always* maximized when experts have extreme but identical agenda.

Example 5. *Suppose* $u_J(y, \theta) = -(y - \theta)^2$, F *is uniform and* $x_1 = 0$ *and* $x_2 = 1$. *Now, using equation* (3), *the judge's default action is* $y^* = \sqrt{1 - \alpha_2}/(\sqrt{1 - \alpha_1} + \sqrt{1 - \alpha_2})$, *and the associated* *expected payoff of the judge is*

$$u_J^{01} := \mathbb{E}_\theta \left[u_J \left(y^*; \theta \right) \mid x_1 = 0, x_2 = 1 \right] = - \left(1 - \alpha_1 \right) \left(1 - \alpha_2 \right) / 3 (\sqrt{1 - \alpha_1} + \sqrt{1 - \alpha_2})^2,$$

Similarly, if $x_1 = x_2 = 0$, *judge's default action* $y^* = 1/(\sqrt{(1 - \alpha_1)(1 - \alpha_2)} + 1)$, *and the judge's expected payoff in equilibrium can be computed as*

$$u_J^{00} := \mathbb{E}_\theta \left[u_J \left(y^*; \theta \right) \mid x_1 = x_2 = 0 \right] = - \left(1 - \alpha_1 \right) \left(1 - \alpha_2 \right) / 3 (\sqrt{(1 - \alpha_1)(1 - \alpha_2)} + 1)^2.$$

It is routine the check that the judge's payoff is the same as u_J^{00} *even if* $x_1 = x_2 = 1$. *Now, since for any* α_1, α_2, $\sqrt{1 - \alpha_1} + \sqrt{1 - \alpha_2} < \sqrt{(1 - \alpha_1)(1 - \alpha_2)} + 1$, *we have* $u_J^{00} > u_J^{01}$.

Note that the optimality of choosing identical experts is in sharp contrast with the some of the existing models of persuasion games such as (Dewatripont and Tirole, 1999) that argue for the optimality of using opposing experts. Dewatripont and Tirole (1999) uses a different information structure where each expert can only look for an evidence that is favorable to his agenda, and the expert needs to (privately) exert effort to find the evidence. Consequently, the judge faces a moral hazard problem that a "competition" between opposing expert (to persuade the judge) can alleviate. This effect is absent in our model as the observation of the state is assumed to be costless for an informed expert.[25]

Another way to interpret this finding is that the judge may have a higher expected payoff if there is one sufficiently able expert with extreme preference rather than two experts competing to influence the judge. This finding offers a novel justification for decisions making based on the information provided by a single committee or a single expert, rather than through competitive advocacy by experts with opposed preferences.

7. Conclusion

Reliance on the experts' advice (or "report") is a common practice in a variety of decision making processes. The decision-maker herself may lack the expertise to find or analyze the relevant information needed for effective decision making and may rely on the experts' opinion to reach a conclusion. But experts can be biased. They may have their personal agenda and manipulate the information they provide to the decision-maker so as to induce her to take an action that better serves their own self-interests, rather than empowering the decision-maker with the relevant information so that she can take an appropriate action. However, in many such environments there are also constraints on an expert's ability to manipulate his report. Once revealed, often the information offered by the experts can be verified, and concerns for reputation or threat of penalty for fabrication of evidence (or both) may act as a deterrent for an expert who may consider misrepresent the facts. Moreover, the presence of competing experts with potentially opposed self-interests may undo each others' attempt to conceal unfavorable information.

We consider such a "persuasion game" where two experts with potentially conflicting agenda attempt to persuade a decision-maker, or the "judge", to take a favorable action. The experts have private types: an informed expert observes the true state of the world but an uninformed expert does not. An expert cannot fabricate his report on the state, but an informed expert can conceal the information on the state by pooling with the uninformed ones. In such a setting we ask the following question: how does the *extent of conflict* between the experts affect the extent of information revealed in equilibrium? We focus on two different measures of conflict: (i) how diverse the agenda of the two experts are and (ii) the quality of the opposing experts as reflected by the prior likelihood of an expert being informed.

[25] Note that by the virtue of the continuity of the payoff functions, even if we assume that information acquisition is costly for the experts, it would still be the case that the judge has higher payoff from congruent experts as compared to competing experts as long as the cost of information acquisition is sufficiently low.

We argue that an increase in an expert's own quality can lead to more information revelation by the expert, but an increase in the rival expert's quality may reduce the extent of information disclosure. We highlight two important implications of this observation: an increase in an expert's quality may lead to less information being disclosed *ex post*. And perhaps more interestingly, when the experts' have moderate agenda, an increase in an expert's quality may reduce the decision maker's payoff *ex ante*. This finding runs contrary to the intuitive argument that having better quality experts should lead to better decision making.

Given that the degree of opposition between experts plays such a salient role in the nature of the game, we also ask the following question: if the judge could choose the two experts based on their own agenda, how would she choose the two experts? It turns out that it is always optimal for the judge to engage experts that either completely opposed or completely similar but extreme. And surprisingly, it may be optimal to employ two experts with the same extreme agenda rather than two experts with completely opposed agenda. This finding, again, runs contrary to the common intuition that conflicting experts always reveal more information.

Note that our findings are based on two key assumptions: (i) both experts, if informed, observe the same information about the state, and (ii) conditional on being the "informed" type, the information acquisition by an expert is automatic—that is, the expert does not have to incur any cost or exert any effort to observe the state. The latter assumption rules out any moral hazard issue in the persuasion game. While the key economic effects that we highlight in this article continue to hold even if we relax these assumptions, to what extent our key findings are robust to these assumptions remains an interesting area for future research.

Appendix

Appendix A. This appendix contains the proofs omitted in the text. As most of our proofs rely on nature of the equation (2), it is instructive to further expand this equation as:

$$(A1)\qquad \begin{array}{l} \int_0^{l_1} u_J'\left(y^*;\theta\right)dF\left(\theta\right)+\int_{h_2}^{1} u_J'\left(y^*;\theta\right)dF\left(\theta\right)+\left(1-\alpha_1\right)\int_{l_1}^{l_2} u_J'\left(y^*;\theta\right)dF\left(\theta\right)+ \\ \left(1-\alpha_2\right)\int_{h_1}^{h_2} u_J'\left(y^*;\theta\right)dF\left(\theta\right)+\left(1-\alpha_2\right)\left(1-\alpha_2\right)\int_{l_2}^{h_1} u_J'\left(y^*;\theta\right)dF\left(\theta\right)=0, \end{array}$$

where l_i and h_i are the lower and upper bounds of the revelation interval for expert A_i. That is, $l_1 = \max\{0, x_1 - |y^* - x_1|\}$, $l_2 = \max\{0, x_2 - |y^* - x_2|\}$, $h_1 = \min\{x_1 + |y^* - x_1|, 1\}$, and $h_2 = \min\{x_2 + |y^* - x_2|, 1\}$. We now present the proofs below.

Proof of Proposition 1. That the proposed strategies constitutes an equilibrium is already argued in section 3. So only need to prove existence of the equilibrium. Consider some $y \in \Theta$, which is the judge's action consequent of receiving an all-null report. Each expert's best response (strategy) is only a function of y, given by $m_i(\cdot,\ y) : \Theta \to \Theta \cup \{\emptyset\}$. For every A_i, the function is unique and well-defined, given by the above proposition. Call the experts' best response profile of strategies $m(y) = \{m_i(\cdot,\ y),\ i = 1, 2, ...n\}$. Note again that y^*, the best response of the judge, is simply a function of $m(y)$, and not of the actual profile of reports. That it is well-defined and unique follows from concavity of u_J in y. We write y^* as $G(m(y))$. Therefore, we have a function $G : \Theta \to \Theta$, i.e. from the space of y to itself. Notice that Θ is compact and convex. Also, $m_i(\cdot,\ y)$ is continuous in y for all i. Since f is a continuous pdf, by the theorem of maximum, $G(m(y))$ is continuous in the distribution induced by $m(y)$, and since the induced distribution is continuous in y, $G(m(y))$ is continuous in y. Thus, by Brouwer's fixed point theorem, there exists a fixed point for the function G. It is easy to see that the fixed point y^* is a Nash equilibrium of the game. ■

Proof of Corollary 1. The revelation sets Θ_i^* follows directly from its characterization as given in Proposition 1. Also, equation (2) is simply the first-order condition associated with the maximization problem given in Proposition 1 that y^* is a solution to. The first-order condition is both necessary and sufficient to characterize y^* since the assumption that $u'' < 0$ implies that the second-order condition is always satisfied. Thus, it only remains to show that in equilibrium, $y^* \in (0,1)$.

Suppose that the revelation sets of the two experts are Θ_1 and Θ_2 respectively. Now, the judge's payoff from any default action y is

$$\begin{aligned} U_J(y) := \; & \Pr(\theta \in \Theta \backslash \Theta_1 \cup \Theta_2) \textstyle\int_{\Theta \backslash \Theta_1 \cup \Theta_2} u_J(y,\theta) f(\theta | \theta \in \Theta \backslash \Theta_1 \cup \Theta_2) d\theta \\ & +(1-\alpha_1) \Pr(\theta \in \Theta_1 \backslash \Theta_2) \textstyle\int_{\Theta_1 \backslash \Theta_2} u_J(y,\theta) f(\theta | \theta \in \Theta_1 \backslash \Theta_2) d\theta \\ & +(1-\alpha_2) \Pr(\theta \in \Theta_2 \backslash \Theta_1) \textstyle\int_{\Theta_2 \backslash \Theta_1} u_J(y,\theta) f(\theta | \theta \in \Theta_2 \backslash \Theta_1) d\theta \\ & +(1-\alpha_1)(1-\alpha_2) \Pr(\theta \in \Theta_1 \cap \Theta_2) \textstyle\int_{\Theta_1 \cap \Theta_2} u_J(y,\theta) f(\theta | \theta \in \Theta_1 \cap \Theta_2) d\theta \\ \\ = \; & \textstyle\int_{\Theta \backslash \Theta_1 \cup \Theta_2} u_J(y,\theta) f(\theta) d\theta + (1-\alpha_1) \int_{\Theta_1 \backslash \Theta_2} u_J(y,\theta) f(\theta) d\theta \\ & +(1-\alpha_2) \textstyle\int_{\Theta_2 \backslash \Theta_1} u_J(y,\theta) f(\theta) d\theta + (1-\alpha_1)(1-\alpha_2) \int_{\Theta_1 \cap \Theta_2} u_J(y,\theta) f(\theta) d\theta. \end{aligned}$$

(Here we use the fact that for any set $E \subset \Theta$, $f(\theta|E) = f(\theta) / \Pr(E)$.) Taking derivative with respect to y and setting $y = 0$, we have

$$\begin{aligned} U_J'(0) = \; & \textstyle\int_{\Theta \backslash \Theta_1 \cup \Theta_2} u_J'(0,\theta) f(\theta) d\theta + (1-\alpha_1) \int_{\Theta_1 \backslash \Theta_2} u_J'(0,\theta) f(\theta) d\theta \\ & +(1-\alpha_2) \textstyle\int_{\Theta_2 \backslash \Theta_1} u_J'(0,\theta) f(\theta) d\theta + (1-\alpha_1)(1-\alpha_2) \int_{\Theta_1 \cap \Theta_2} u_J'(0,\theta) f(\theta) d\theta \end{aligned}$$

Due to strict single peakedness, $u_J'(0,0) = 0$ and $u_J'(0,\theta) < 0$ for $\theta > 0$. Since $f(\theta)$ is assumed to have full support, $f(\theta) \geq k$, for some finite $k > 0$. So, we can write

$$\begin{aligned} U_J'(0) = \; & \textstyle\int_{\Theta \backslash \Theta_1 \cup \Theta_2 \cup \{0\}} u_J'(0,\theta) f(\theta) d\theta + (1-\alpha_1) \int_{\Theta_1 \backslash \Theta_2 \cup \{0\}} u_J'(0,\theta) f(\theta) d\theta \\ & +(1-\alpha_2) \textstyle\int_{\Theta_2 \backslash \Theta_1 \cup \{0\}} u_J'(0,\theta) f(\theta) d\theta + (1-\alpha_1)(1-\alpha_2) \int_{\Theta_1 \cap \Theta_2 \cup \{0\}} u_J'(0,\theta) f(\theta) d\theta \end{aligned}$$

Now, $\alpha_i \in [0,1)$ implies $(1-\alpha_1)(1-\alpha_2) \in (0,1]$. Also, $(1-\alpha_1)(1-\alpha_2) \leq \max\{1-\alpha_1, 1-\alpha_2\}$. Therefore,

$$\begin{aligned} U_J'(0) < \; & (1-\alpha_1)(1-\alpha_2) \Big[\textstyle\int_{\Theta \backslash \Theta_1 \cup \Theta_2 \cup \{0\}} u_J'(0,\theta) f(\theta) d\theta + \int_{\Theta_1 \backslash \Theta_2 \cup \{0\}} u_J'(0,\theta) f(\theta) d\theta \\ & \qquad + \textstyle\int_{\Theta_2 \backslash \Theta_1 \cup \{0\}} u_J'(0,\theta) f(\theta) d\theta + \int_{\Theta_1 \cap \Theta_2 \cup \{0\}} u_J'(0,\theta) f(\theta) d\theta \Big] \\ \leq \; & (1-\alpha_1)(1-\alpha_2) k \Big[\textstyle\int_{\Theta \backslash \Theta_1 \cup \Theta_2 \cup \{0\}} u_J'(0,\theta) d\theta + \int_{\Theta_1 \backslash \Theta_2 \cup \{0\}} u_J'(0,\theta) d\theta \\ & \qquad + \textstyle\int_{\Theta_2 \backslash \Theta_1 \cup \{0\}} u_J'(0,\theta) d\theta + \int_{\Theta_1 \cap \Theta_2 \cup \{0\}} u_J'(0,\theta) d\theta \Big] \\ \leq \; & (1-\alpha_1)(1-\alpha_2) k \textstyle\int_{\Theta \backslash \{0\}} u_J'(0,\theta) d\theta < 0. \end{aligned}$$

Similarly, we can show that $U_J'(1)$ is strictly greater than a positive number. Therefore, the best response of the judge is always an interior action. ■

Proof of Proposition 2. The proof follows directly from the proof of Proposition 1 by plugging $n = 2$, $\theta \in [0,1]$ and $x_1 = 0$, $x_2 = 1$. The only additional claim that needs to be proved is that the equilibrium is unique. To see this, denote:

$$Z(y) := (1-\alpha_1) \int_0^y u_J'(y;\theta)\, dF(\theta) + (1-\alpha_2) \int_y^1 u_J'(y;\theta)\, dF(\theta).$$

Note that Z is continuous, $Z(0) = (1-\alpha_2) \int_0^1 u_J'(0;\theta)\, dF(\theta) > 0$, $Z(1) = (1-\alpha_1) \int_0^1 u_J'(1;\theta)\, dF(\theta)$ < 0, and

$$Z'(y) = (1-\alpha_1) \int_0^y u_J''(y;\theta)\, dF(\theta) + (1-\alpha_2) \int_y^1 u_J''(y;\theta)\, dF(\theta) < 0.$$

So, by Mean Value Theorem, there exists a value of $y \in [0,1]$ such that $Z(y) = 0$. Moreover, this value must be unique since $Z'(y) < 0$. This observation completes the proof. ■

Proof of Proposition 3. Since y^* must solve equation $(A1)$, by taking the total derivative of $(A1)$ with respect to α_1 (and using the fact that $u_J'(y;y)=0$), one obtains:

$$\left[(1-\alpha_1)\int_0^{y^*} u_J''(y^*;\theta)\,dF(\theta)+(1-\alpha_2)\int_{y^*}^1 u_J''(y^*;\theta)\,dF(\theta)\right]\frac{\partial y^*}{\partial\alpha_1}=\int_0^{y^*} u_J'(y^*;\theta)\,dF(\theta).$$

Now, as $u_J''<0$ and $\int_0^{y^*} u_J'(y^*;\theta)\,dF(\theta)<0$ (since $u_J'(y^*;\theta)<0$ for all $\theta<y^*$) we must have $\partial y^*/\partial\alpha_1>0$. The proof for $\partial y^*/\partial\alpha_2<0$ is analogous, and hence, omitted here. ■

Proof of Proposition 4. **Step 1.** First, note that if the equilibrium is locally insensitive to the experts' agenda, then it must take one of two forms: (i) conflict equilibrium where $\Theta_1^*=[0,y^*]$ and $\Theta_2^*=[y^*,1]$, or (ii) complete congruence equilibrium where $\Theta_1^*=\Theta_2^*=[0,y^*]$ or $[y^*,1]$. Consider the case of conflict equilibrium. Here, in equilibrium,

$$\mathbb{E}\left[u_J(y^*;\theta)\ \mid \mathbf{m}^*\right]=(1-\alpha_1)\int_0^{y^*} u_J(y^*;\theta)\,dF(\theta)+(1-\alpha_2)\int_{y^*}^1 u_J(y^*;\theta)\,dF(\theta)$$

Now, consider an increase in α_1. Here,

$$\begin{aligned}&\tfrac{\partial}{\partial\alpha_1}\mathbb{E}\left[u_J(y^*;\theta)\ \mid \mathbf{m}^*\right]=\\&(1-\alpha_1)\textstyle\int_0^{y^*} u_J'(y^*;\theta)\,dF(\theta)+(1-\alpha_2)\int_{y^*}^1 u_J'(y^*;\theta)\,dF(\theta)-\int_0^{y^*} u_J(y^*)\,dF(\theta)\end{aligned}$$

Using equation (3), the above equation boils down to:

$$\frac{\partial}{\partial\alpha_1}\mathbb{E}\left[u_J(y^*;\theta)\ \mid \mathbf{m}^*\right]=-\int_{\max\{0,2x_1-y^*\}}^{y^*} u_J(y^*)\,dF(\theta)>0.$$

The proof for the case of congruence equilibrium is similar and, hence, omitted here.

Step 2. It remains to show if the equilibrium revelations sets are indeed locally sensitive to the experts' agenda, then $\partial\mathbb{E}\left[u_J(y^*;\theta)\ \mid \mathbf{m}^*\right]/\partial\alpha_1$ cannot be signed. To see this, consider a case where the equilibrium exhibits conflict, that is, $x_1<y^*<x_2$, and $\Theta_1^*\cup\Theta_2^*\subset[0,1]$. That is, we must either have $\Theta_1^*=[2x_1-y^*,y^*]$ or $\Theta_2^*=[y^*,2x_2-y^*]$, or both. So, equation $(A1)$ suggests that in equilibrium,

$$\begin{aligned}&\tfrac{\partial}{\partial\alpha_1}\mathbb{E}\left[u_J(y^*;\theta)\ \mid \mathbf{m}^*\right]=\\&u_J(y^*;l_1)\tfrac{\partial l_1}{\partial y^*}\tfrac{\partial y^*}{\partial\alpha_1}+\textstyle\int_0^{l_1} u_J' dF\tfrac{\partial y^*}{\partial\alpha_1}-u_J(y^*;h_2)\tfrac{\partial h_2}{\partial y^*}\tfrac{\partial y^*}{\partial\alpha_1}+\int_{h_2}^1 u_J'(y^*)\,dF\tfrac{\partial y^*}{\partial\alpha_1}-\\&\textstyle\int_{l_1}^{y^*} u_J(y^*)\,dF+(1-\alpha_1)\left[-u_J(y^*;l_1)\tfrac{\partial l_1}{\partial y^*}\tfrac{\partial y^*}{\partial\alpha_1}+\int_{l_1}^{y^*} u_J'(y^*)\,dF\tfrac{\partial y^*}{\partial\alpha_1}\right]+\\&(1-\alpha_2)\left[u_J(y^*;h_2)\tfrac{\partial h_2}{\partial y^*}\tfrac{\partial y^*}{\partial\alpha_1}+\textstyle\int_{y^*}^{h_2} u_J'(y^*)\,dF\tfrac{\partial y^*}{\partial\alpha_1}\right]\end{aligned}$$

where $l_1=\max\{0,2x_1-y^*\}$ and $h_2=\min\{2x_2-y^*,1\}$. Now, using equation $(A1)$, the above equation boils down to:

$$\begin{aligned}&\tfrac{\partial}{\partial\alpha_1}\mathbb{E}\left[u_J(y^*;\theta)\ \mid \mathbf{m}^*\right]=\\&\left[\alpha_1 u_J(y^*;l_1)\tfrac{\partial l_1}{\partial y^*}-\alpha_2 u_J(y^*;h_2)\tfrac{\partial h_2}{\partial y^*}\right]\tfrac{\partial y^*}{\partial\alpha_1}-\textstyle\int_{\max\{0,2x_1-y^*\}}^{y^*} u_J(y^*)\,dF(\theta)\end{aligned}$$

Now, $(-1)\int_{\max\{0,2x_1-y^*\}}^{y^*} u_J(y^*)\,dF(\theta)>0$, but the term $\alpha_1 u_J(y^*;l_1)\frac{\partial l_1}{\partial y^*}-\alpha_2 u_J(y^*;h_2)\frac{\partial h_2}{\partial y^*}$ cannot be signed a priori unless $\Theta_1^*=[0,y^*]$ and $\Theta_2^*=[y^*,1]$ (in that case this term is 0). So $\partial\mathbb{E}\left[u_J(y^*;\theta)\ \mid \mathbf{m}^*\right]/\partial\alpha_1$ cannot be signed. Similar argument applies for the case of partial congruence equilibrium and for the case of α_2. Hence the proof. ■

Proof of Proposition 5. If $x_1=0$ and $x_2=1$, then by definition, $y^*\in[x_1,x_2]$. So, $\Theta_1^*=\{\theta\mid\theta\le y^*\}=[0,y^*]$ and $\Theta_2^*=\{\theta\mid\theta\ge y^*\}=[y^*,1]$. Hence, $\Theta_1^*\backslash\Theta_2^*=\Theta_1^*$, $\Theta_2^*\backslash\Theta_1^*=\Theta_2^*$, $\Theta_1^*\cap\Theta_2^*=y^*$, and $(\Theta_2^*\cup\Theta_1^*)^c=\varnothing$. So, $(A1)$ boils down to

$$(1-\alpha_1)\int_0^{y^*} u_J'(y^*;\theta)\,dF(\theta)+(1-\alpha_2)\int_{y^*}^1 u_J'(y^*;\theta)\,dF(\theta)=0.$$

Suppose the judge commits to a status quo action $\hat{y}$. So $\hat{y}$ must solve

$$\max_y\ (1-\alpha_1)\int_0^{y} u_J(y;\theta)\,dF(\theta)+(1-\alpha_2)\int_{y}^1 u_J(y;\theta)\,dF(\theta)$$

The first-order condition is

$$\begin{gathered}(1-\alpha_1)\left[u_J(\hat{y};\hat{y})-u_J(\hat{y};0)\,.0+\textstyle\int_0^{\hat{y}} u_J'(\hat{y};\theta)\,dF(\theta)\right]\\ +(1-\alpha_1)\left[u_J(\hat{y};1)\,.0-u_J(\hat{y};\hat{y})+\textstyle\int_{\hat{y}}^1 u_J'(\hat{y};\theta)\,dF(\theta)\right]=0\end{gathered}$$

or

$$(1-\alpha_1)\int_0^{\hat{y}} u_J'(\hat{y};\theta)\,dF(\theta)+(1-\alpha_2)\int_{\hat{y}}^1 u_J'(\hat{y};\theta)\,dF(\theta)=0.$$

But this is the same condition as in equation (3). Hence the judge's choice of the default action (under commitment power) coincides with her default action in the original game. Hence, the payoffs are also identical in the two case. In other words, commitment power has no value to the judge in this case. ■

Proof of Proposition 6. Fix $F,\ \alpha_1,\ \alpha_2$. Now, consider any (x_1,x_2), and suppose the expected utility in equilibrium of the judge is $u_J^*(x_1,x_2)$. We show that

$$\max_{\{x_1,x_2\}\in[0,1]^2} u_J^*(x_1,x_2)\le\max\{u_J^*(0,0),u_J^*(1,1),u_J^*(0,1)\}$$

By Proposition 5, $u_J^*(0,0),u_J^*(1,1)$ and $u_J^*(0,1)$ are the same under commitment or in absence of commitment. To prove the above inequality, we proceed along the following steps.

Step 1: We argue that for any equilibrium with $x_1\le y^*\le x_2$, $u_J^*(x_1,x_2)\le u_J^*(0,1)$. Suppose $x_1\le y^*\le x_2$. Now, the revelation sets of the two experts are $[l_1,y^*]$ and $[y^*,u_2]$ respectively, where $l_1\in[0,y^*]$ and $u_2\in[y^*,1]$. Now,

$$\begin{aligned}&u_J^*(x_1,x_2)=\\ &\textstyle\int_0^{l_1}u_J(y^*,\theta)dF+(1-\alpha_1)\int_{l_1}^{y^*}u_J(y^*,\theta)dF+(1-\alpha_2)\int_{y^*}^{u_2}u_J(y^*,\theta)dF+\int_{u_2}^1 u_J(y^*,\theta)dF\end{aligned}$$

We have already noted in the proof of Proposition 5, $u_J^*(0,1)=\max\limits_y(1-\alpha_1)\int_0^y u_J(y,\theta)dF+(1-\alpha_2)\int_y^1 u_J(y,\theta)dF$. Therefore,

$$\begin{aligned}&u_J^*(0,1)\\ &\ge(1-\alpha_1)\textstyle\int_0^{y^*}u_J(y^*,\theta)dF+(1-\alpha_2)\int_{y^*}^1 u_J(y^*,\theta)dF\\ &=(1-\alpha_1)\left[\textstyle\int_0^{l_1}u_J(y^*,\theta)dF+\int_{l_1}^{y^*}u_J(y^*,\theta)dF\right]+(1-\alpha_2)\left[\textstyle\int_{y^*}^{u_2}u_J(y^*,\theta)dF+\int_{u_2}^1 u_J(y^*,\theta)dF\right]\\ &\ge\textstyle\int_0^{l_1}u_J(y^*,\theta)dF+(1-\alpha_1)\int_{l_1}^{y^*}u_J(y^*,\theta)dF+(1-\alpha_2)\int_{y^*}^{u_2}u_J(y^*,\theta)dF+\int_{u_2}^1 u_J(y^*,\theta)dF\\ &=u_J^*(x_1,x_2),\end{aligned}$$

where the last inequality follows from the fact that the utilities are all non-positive, and $(1-\alpha_i)\in(0,1]$ for $i=1,2$.

Step 2: Next, we argue that for any equilibrium with $x_1\le x_2<y^*$, $u_J^*(x_1,x_2)\le u_J^*(0,0)$ (and similarly, for any equilibrium with $x_2>x_1>y^*$, we have $u_J^*(x_1,x_2)\le u_J^*(1,1)$). Suppose $x_1\le x_2<y^*$. Now, the revelation sets of the two experts are $[l_1,y^*]$ and $[l_2,y^*]$ respectively, where $l_1\in[0,y^*]$ and $l_2\in[l_1,y^*]$. Now,

$$u_J^*(x_1,x_2)=\int_0^{l_1}u_J(y^*,\theta)dF+(1-\alpha_1)\int_{l_1}^{l_2}u_J(y^*,\theta)dF+(1-\alpha_2)(1-\alpha_1)\int_{l_2}^{y^*}u_J(y^*,\theta)dF+\int_{y^*}^1 u_J(y^*,\theta)dF$$

We have already noted in the proof of Proposition 5 that $u_J^*(0,0) = \max_y (1-\alpha_1)(1-\alpha_2)\int_0^y u_J(y,\theta)dF + \int_y^1 u_J(y,\theta)dF$. Therefore,

$$\begin{aligned}
&u_J^*(0,0)\\
&\geq (1-\alpha_1)(1-\alpha_2)\int_0^{y^*} u_J(y^*,\theta)dF + \int_{y^*}^1 u_J(y^*,\theta)dF\\
&= (1-\alpha_1)(1-\alpha_2)\left[\int_0^{l_1} u_J(y^*,\theta)dF + \int_{l_1}^{l_2} u_J(y^*,\theta)dF + \int_{l_2}^{y^*} u_J(y^*,\theta)dF\right] + \int_{y^*}^1 u_J(y^*,\theta)dF\\
&\geq \int_0^{l_1} u_J(y^*,\theta)dF + (1-\alpha_1)\int_{l_1}^{l_2} u_J(y^*,\theta)dF + (1-\alpha_1)(1-\alpha_2)\int_{l_2}^{y^*} u_J(y^*,\theta)dF + \int_{y^*}^1 u_J(y^*,\theta)dF\\
&= u_J^*(x_1,x_2),
\end{aligned}$$

where the last inequality follows from the fact that the utilities are all non-positive, and $(1-\alpha_i) \in (0,1]$ for $i = 1,2$.

By the same logic, we can show that if $y^* < x_1 \leq x_2$, then $u_J^*(1,1) \geq u_J^*(x_1,x_2)$. Therefore, we obtain that

$$\max_{x_i \in [0,1]} u_J^*(x_1,x_2) \leq \max\{u_J^*(0,0), u_J^*(1,1), u_J^*(0,1)\}.$$

Lastly, note that whenever experts are sufficiently opposed, there is an equilibrium where the outcome is the same as that of the profile $\{0,1\}$ independent of whether commitment is allowed or not..Similarly, for sufficiently similar but extreme experts, we have an equilibrium where the outcome is the same as that of completely identical but extreme experts. Hence the proof. ■

Appendix B. This appendix contains the proof of the claim that if there is a single expert, the sign of derivative of the default action y^* with respect to α is positive even if the local insensitivity condition does not hold.

Suppose that the experts ideal point is x and his (interior) revelation set in a given equilibrium is $[a,b]$. So, the payoff of the judge from action y is

$$\begin{aligned}
U_J(y) &= \int_0^a u_J(y;\theta)dF + (1-\alpha)\int_a^b u_J(y;\theta)dF + \int_b^1 u_J(y;\theta)dF\\
&= \int_0^1 u_J(y,\theta)dF - \alpha\int_a^b u_J(y,\theta)dF
\end{aligned}$$

The judge sets $y = y^*$ to maximize $U_J(y)$. Therefore, at y^*, we have the first order condition: $\int_0^1 u_J'(y^*;\theta)dF - \alpha\int_a^b u_J'(y^*;\theta)dF = 0$, and second order condition that follow from concavity: $\int_0^1 u_J''(y^*;\theta)dF - \alpha\int_a^b u_J''(y^*;\theta)dF < 0$. Now, in equilibrium, we must have either $b = y^*$ and $a = 2x - y^*$ or $a = y^*$ and $b = 2x - y^*$. We consider the first case (the second case is identical), and first-order condition boils down to:

$$\int_0^1 u_J'(y^*;\theta)dF - \alpha\int_{2x-y^*}^{y^*} u_J'(y^*;\theta)dF = 0$$

Taking derivatives with respect to α, one obtains:

$$\begin{aligned}
&\tfrac{dy^*}{d\alpha}\left[\int_0^1 u_J''(y^*;\theta)dF - \alpha\left(\int_{2x-y^*}^{y^*} u_J''(y^*;\theta)dF + u_J'(y^*;y^*)f(y^*) - u_J'(y^*;2x-y^*)f(2x-y^*)\right)\right]\\
&= \tfrac{dy^*}{d\alpha}\left[\int_0^1 u_J''(y^*;\theta)dF - \alpha\left(\int_{2x-y^*}^{y^*} u_J''(y^*;\theta)dF - u_J'(y^*;2x-y^*)f(2x-y^*)\right)\right]\\
&= \int_{2x-y^*}^{y^*} u_J'(y^*;\theta)dF
\end{aligned}$$

Therefore,

$$\frac{dy^*}{d\alpha} = \frac{\int_{2x-y^*}^{y^*} u_J'(y^*;\theta)dF}{\left[\int_0^1 u_J''(y^*;\theta)dF - \alpha\int_{2x-y^*}^{y^*} u_J''(y^*;\theta)dF\right] + \alpha u_J'(y^*;2x-y^*)f(2x-y^*)}$$

Now, $u_J'(y^*;\theta) \leq 0$ for all $\theta \in [2x-y^*, y^*]$. Therefore the numerator is negative. Also, the second-order condition along with the fact that $u_J'(y^*;2x-y^*) < 0$ imply that denominator is also negative. Hence $dy^*/d\alpha > 0$.

References

[1] Austen-Smith, D. (1990) "Information transmission in debate," *American Journal of Political Science*, Vol. 34 (1), pp. 124–152.

[2] —–. (1993) "Interested experts and policy advice: Multiple referrals under open rule," *Games and Economic Behavior,* Vol 5, pp. 3-43.

[3] Battaglini, M. (2002) "Multiple referrals and multidimensional cheap talk," *Econometrica*, Vol. 70 (4), pp. 1379–1401.

[4] Ben-Porath, E. and L. Lipman. (2009) "Implementation and partial provability," mimeo, Boston University.

[5] Bull, J. and J. Watson. (2004) "Evidence Disclosure and Verifiability," *Journal of Economic Theory,* Vol 118, pp. 1-31.

[6] Chakraborty, A. and R. Harbaugh. (2010) "Persuasion by cheap talk," *American Economic Review,* forthcoming.

[7] Che, Yeon-Koo, and N. Kartik (2009) "Opinions as Incentives," *Journal of Political Economy* 117(5), pp. 815-860.

[8] Deneckere, R. and S. Severinov. (2008) "Mechanism design with partial state verifiability," *Games and Economic Behavior,* Vol 65, pp. 487-513.

[9] Dekel, E., and M. Piccione. (2000) "Sequential Voting Procedures in Symmetric Binary Elections," *Journal of Political Economy*, Vol 108, pp 34-55.

[10] Dewatripont, M., and J. Tirole. (1999) "Advocates," *The Journal of Political Economy*, Vol. 107(1), pp. 1-39.

[11] Dzuida, W. (2010) "Strategic Argumentation," mimeo, Kellogg School of Management, Northwestern University.

[12] Gerardi, D., R. McLean and A. Postlewaite. (2009) "Aggregation of expert opinions," *Games and Economic Behavior,* Vol 65, pp. 339-371.

[13] —–, and Yariv. (2008) "Costly Expertise," *American Economic Review Papers and Proceedings*, Vol 98, pp. 187-193.

[14] Gilligan, T., Krehbiel, K. (1989) "Asymmetric information and legislative rules with a heterogenous committee," *American Journal of Political Science* Vol. 33, pp. 459-490.

[15] Glazer, J., and A. Rubinstein. (2001) "Debates and decisions: On a rationale of argumentation rules," *Games and Economic Behavior*, Vol. 36, pp. 158–173.

[16] —–, and —–. (2004) "On the Optimal Rules of Persuasion," *Econometrica*, Vol. 72, pp. 1715–36.

[17] —–, and —–. (2006) "A Study in the Pragmatics of Persuasion: A Game Theoretical Approach," *Theoretical Economics*, Vol. 1, pp. 395–410.

[18] Gul, F. and W. Pesendorfer (2010) "The War of Information", mimeo, Princeton University.

[19] Kartik, N. and O. Tercieux. (2010) "Implementation with evidence: Complete information," mimeo, Columbia University.

[20] Kamenica, E. and M. Gentzkow (2010) "Bayesian Persuasion," *American Economic Review,* forthcoming.

[21] Gentzkow, M., and E. Kamenica. (2010) "Competition in Persuasion", mimeo, University of Chicago.

[22] Krishna, V., and J. Morgan. (2001) "A model of expertise," *Quarterly Journal of Economics*, Vol. 116(2), pp. 747-775.

[23] Lipman, B., and D. Seppi. (1995) "Robust inference in communication games with partial provability," *Journal of Economic Theory*, Vol. 66, pp. 370–405.

[24] Milgrom, P. (1981) "Good news and bad news: Representation theorems and applications," *Bell Journal of Economics*, Vol 12, pp. 380–91.

[25] —–, and J. Roberts. (1986) "Relying on the information of interested parties," *Rand Journal of Economics*, Vol. 17, pp. 18–32.

[26] Ottaviani, M., and P. Sorensen. (2001) "Information aggregation in debate: Who should speak first?," *Journal of Public Economics,* Vol 81, pp. 393–421.

[27] Shavell, S. (1989) "Sharing of information prior to settlement or litigation," *RAND Journal of Economics*, Vol. 20, pp. 183–195.

[28] Shin, H. (1994) "The burden of proof in a game of persuasion," *Journal of Economic Theory*, Vol. 64, pp. 253–264.

[29] —–. (1998) "Adversarial and inquisitorial procedures in arbitration," *RAND Journal of Economics,* Vol. 29 (2), pp. 378–405.

[30] Spector, D. (2000) "Rational debate and one-dimensional conflict," *The Quarterly Journal of Economics,* Vol 115, pp. 181–200.

[31] Wolinsky, A. (2002) "Eliciting information from multiple experts," *Games and Economic Behavior,* Vol. 41, pp. 141–160.