[Preliminary and incomplete. Comments welcome.]

Knowledge Transfer and Partial Equity Ownership

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Abstract

When firms form an alliance, it often involves one firm acquiring an equity stake in its alliance partner. Such an alliance lessens the competition, but induces knowledge transfer within the alliance. This paper explores oligopoly models that capture this important link between knowledge transfer and partial equity ownership (PEO). We consider an industry consisting of $n + 2$ firms, where firm 1 has superior knowledge that other firms in the industry do not have. Firms 1 and 2 have an option of forming an equity strategic alliance in which firm 1 owns a fraction of firm 2's share. The link between knowledge transfer and PEO endogenously determines the equilibrium level of PEO in our model. Previous theoretical models of PEO, in which the levels of PEO are exogenously assumed, have shown that PEO arrangements would decrease welfare by reducing the degree of competition in the industry. We demonstrate that PEO arrangements can increase welfare because they induce knowledge transfer. Our analysis indicates that antitrust authorities should either prohibit, partially permit, or permit PEO to maximize total surplus and/or consumer surplus, and identifies conditions under which one of these three relevant policy options is optimal.

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1 Introduction

A strategic alliance exists when two or more independent organizations cooperate in the development, manufacturing, or sale of products or services (Barney, 2002). In recent years, the incidence and importance of inter-firm collaborations has substantially increased (Caloghirou, Ioannides and Vonortas, 2003). It has been widely recognized in the management literature that one of the most fundamental objectives of strategic alliances is the transfer of knowledge between partner firms.[1]

Modern economies are becoming increasingly knowledge intensive. Drucker (1993), for example, has argued that in the new economy, knowledge is not just another resource alongside the traditional factors of production–labor, capital, and land–but the only meaningful resource today.[2] Knowledge is often classified into tacit and explicit knowledge in the organizational knowledge literature.[3] Explicit (or codified) knowledge is transmittable in formal, systematic language, whereas tacit knowledge is non-verbalizable, intuitive, and unarticulated (Polanyi, 1962, 1966). Because tacit knowledge is difficult to convey, its transfer requires greater effort (Reagans and McEvily, 2003). Tacit knowledge can be transferred through up-close observation, demonstration, hands-on experience, sharing feelings and emotions, and face-to-face contacts (Hamel, 1991; Nonaka, 1994; Nonaka and Takeuchi, 1995; Cavusgil et al., 2003).

Transfer of explicit knowledge can be facilitated through licensing and contracting because it is verifiable. However, since tacit knowledge is not codifiable and hence not verifiable, licensing and contracting can play, at best, limited roles in transferring tacit knowledge (Mowery, 1993; Pisano, 1990). Instead, equity ownership can play a critical role in facilitating the transfer of tacit knowledge (Mowery et al., 1996). Using patent citations as a proxy for knowledge flows, Mowery et al. (1996) and Gomes-Casseres et al. (2006) empirically explored effects of equity ownership between alliance partners on the extent of knowledge flow. Empirical results of both studies supported the hypothesis that equity ownership en-

[1] See Hamel, 1991; Mowery, Oxley and Silverman, 1996; Gomes-Casseres, Hagedoorn and Jaffe, 2006; Oxley and Wada, 2009. Gomes-Casseres et al. (2006), for example, hypothesized that knowledge flows between alliance partners would be greater than flows between pairs of nonallied firms, and found empirical results that are consistent with this hypothesis.

[2] See Quinn (1992) and Toffler (1999) for similar arguments.

[3] See Polanyi (1962, 1966) for a seminal work on tacitness of knowledge. See also Nonaka (1994), Nonaka and Takeuchi (1995), Grant (1996), Inkpen and Dinur (1998), Adler (2001), among others.

hances the extent of knowledge flow between alliance partners.[4] Examples of equity strategic alliances listed below also demonstrates the connection between equity ownership and knowledge transfer.[5]

• In March 2007, Citigroup and Nikko Cordial announced the formation of a strategic alliance. Citigroup, after acquiring a 4.9% stake in Nikko, will help its partner develop a strategic solution through transferring considerable know-how and expertise in principal investments and private equity.

• In April 2004, Harvey World Travel announced its plan to take an initial 11% equity holding in Webjet, an internet travel business specialist. Webjet's Managing Director, David Clarke, said that the arrangement would provide Webjet with a strategic development partner which would enhance Webjet's ability to capitalize on opportunities in a rapidly changing travel market in the Australian region.

• In December 2000, Vodafone announced that it would acquire a 15% stake in Japan Telecom for Yen 249 billion. The Chairman of Japan Telecom said that, under this strategic alliance, his company would benefit from Vodafone's global leadership in mobile communications, access to world-wide technology, content and expertise.

Partial equity ownership (PEO) induces knowledge transfer between alliance partners. This paper explores oligopoly models that capture this important link between PEO and knowledge transfer. We consider an industry consisting of $n + 2$ firms, where firm 1 has superior knowledge that other firms in the industry do not have. The knowledge is tacit and

[4]Both studies used Cooperate Agreements and Technology Indicators (CATI) database developed by the Maastricht Economic Research Institute in Technology (MERIT) to identify alliances of firms. Mowery et al. (1996) focused on bilateral alliances that involved at least one U.S. firm and were established during 1985 and 1986, and the patent data were drawn from the Micropatent data base, which contains all information recorded on the front page of every patent granted in the U.S. since 1975. Gomes-Casseres et al. (2006) matched the firms in the CATI database to the NBER Patent Citations Data File. They used only alliance and patent data from information technology sectors, and analyzed citations on an annual basis from 1975 to 1999.

See also Ono, Nakazato, Davis, and Alley (2004) for an empirical evidence from Japanese automobile industry. They also put forth theoretical framework incorporating POA and technology transfer, but they did not derive the equilibrium of the model by solving it.

[5]Details are available, respectively, from http://www.citigroup.com/citigroup/press/2007/070306a.htm, http://www.webjet.com.au/about/webjet-to-enter-major-commercial-and-equity-alliance-with-harvey-world-travel-limited/,
http://www.vodafone.com/start/media_relations/news/group_press_releases/2000/press_release20_12.html

not contractible. Firms 1 and 2 have an option of forming an equity strategic alliance in which firm 1 owns a fraction $\theta \in [0, 1]$ of firm 2's share, while n other firms are assumed to be independent.

Firm 2's profit increases if firm 1 transfers its superior technological knowledge to firm 2. But, a fraction θ of firm 2's profit belongs to firm 1, and this in turn gives firm 1 an incentive to transfer its knowledge. The equilibrium level of PEO, θ^*, is endogenously determined through the following trade-off. On the one hand, a higher θ increases the incentive for firm 1 to transfer its knowledge, and weakens the degree of competition between firms 1 and 2. This works to the alliance partners' advantage. On the other hand, a higher θ reduces the alliance partners' competitive position against other firms outside the alliance. This works to the alliance partners' disadvantage. Given this trade-off, the alliance partners choose the optimal level of θ that maximizes their joint profits.

PEO arrangements among competitors alter their competitive incentives. The competitive effects of PEO have been previously studied in the context of static oligopoly as well as repeated oligopoly models, in which the levels of PEO are exogenously given (see Section 2 for details). To the best of our knowledge, however, no previous papers have explicitly analyzed the process in which PEO induces knowledge transfer between competing firms. The present paper fills this important gap in the literature by exploring a model in which the level of PEO is endogenously determined through the link between PEO and knowledge transfer. Focusing on the competitive effect, existing theoretical models of PEO demonstrate that PEO arrangements could decrease welfare by reducing the degree of competition in the industry. This result suggests that antitrust authorities should consider the trade-off between enhanced production efficiency and reduced competition in cases of PEO. Our analysis indicates that endogenously determined levels of PEO may increase or decrease welfare, and identifies a range of parameterizations under which PEO increases (or decreases) welfare. We then consider three relevant policy interventions (prohibit PEO, partially permit PEO, or permit PEO) for antitrust authorities to maximize total surplus and/or consumer surplus, and identify conditions under which one of these three options is optimal.

Cases of PEO in a competitor have gone mostly unchallenged by antitrust agencies (see Gilo, 2000). However, antitrust agencies have recently begun to pay increasing attention to the possible antitrust harms of PEO. For example, Deborah Platt Majoras, the then Deputy Assistant Attorney General of the Antitrust Division of the U.S. Department of Justice, mentioned in her speech given in April 2002 that PEO can raise antitrust issues when the two companies or their subsidiaries are competitors. Also, several legal scholars have argued

that PEO, even if it is not accompanied by control/influence rights, results in antitrust harms in oligopolistic industries, by reducing quantities and raising prices (Gilo, 2000; O'Brien and Salop, 2000, 2001). Their arguments are consistent with the previous literature on economic theoretical analyses of PEO, in which the level of PEO is exogenously assumed. In contrast, by exploring the link between PEO and knowledge transfer, our analysis yields richer policy implications as detailed in Section 5.

The remainder of the paper is organized as follows. Section 2 discusses this study's relationship to the literature. Section 3 analyzes a model that captures the link between PEO and knowledge transfer under the linear homogeneous demand, and explores its welfare consequences and antitrust implications. Section 4 demonstrates, using a differentiated oligopoly model, that partial permission of PEO can be the optimal antitrust policy. Section 5 summarizes this paper's policy implications, and Section 6 concludes the paper.

2 Relationship to the literature

PEO arrangements among competitors alter their competitive incentives. The competitive effect of PEO have been previously studied in the context of static oligopoly as well as repeated oligopoly models. Reynolds and Snapp (1986), in their seminal contribution to theoretical analyses of PEO, analyzed a modified Cournot oligopoly model consisting of n firms that produce the homogeneous product with the same constant marginal cost c. The n firms are linked by PEO, where each firm i holds ownership interest v_{ik} in firm k. Ownership interests are not accompanied by any decision making rights in the sense that each firm i determines the amount of its own production under any PEO structures. The levels of PEO (the values of v_{ik}, $i = 1, ..., n$, $k = 1, ..., n$, $i \neq k$) are exogenously given in their model.[6] Under the model outlined above, Reynolds and Snapp showed that, if one or more Cournot competitors increase the level of ownership links with rival firms, equilibrium market output will decline. That is, they demonstrated that, in markets where entry is difficult, PEO could result in less output and higher prices because PEO arrangements reduce the degree of competition among participants by linking their profitability.[7]

[6]More precisely, Reynolds and Snapp make a distinction between firms and plants, where firms are profit-maximizing decision-making units that control plants. Each firm i owns the decision-making right of plant i. In addition to its own share (which is $1 - \sum_{k \neq i} v_{ki}$) of plant i's profit, each firm i also receives v_{ik} share of plant k's ($k \neq i$) profit.

[7]Drawing on the work of Reynolds and Snapp (1986), Bresnahan and Salop (1986) devised a Modified Herfindahl-Hirshman Index (MHHI) to quantify the competitive effects of horizontal joint ventures under a

Could firms increase their profitability by linking themselves through PEO? Consider PEO between firms 1 and 2 under the modified Cournot oligopoly model analyzed by Reynolds and Snapp (1986), where other $n-2$ firms have no PEO arrangements. As pointed out by Reitman (1994), the equilibrium joint profit of firms 1 and 2 decline under any levels of PEO for all $n \geq 3$, where the result is similar to the finding of Salant, Switzer and Reynolds (1983) for merger.[8] Reitman (1994) showed, using a conjectural variations model, that with conjectures that lead to more rivalry equilibria than Cournot, there exist individually rational PEO arrangements with any number of firms in the industry.

Farrell and Shapiro (1990) showed that, under the Cournot oligopoly model with homogeneous goods, PEO arrangements between two firms can increase their equilibrium joint profit if their production efficiency is different. Suppose that firm 1 holds a share, α, of the stock of firm 2, and each firm j (=2, ..., n) holds no PEO in other firms. As firm 1 increases its stake α in firm 2, firm 1 reduces its output while all other firms increase their outputs in the equilibrium. This is because, as α increases, firm 1 is increasingly willing to sacrifice profits at its own facility in order to augment profits at firm 2. Farrell and Shapiro found that, if firm 1 is smaller than firm 2 (that is, if firm 1 is less cost efficient than firm 2), then a certain level of PEO α increases the joint equilibrium profit of firms 1 and 2, because a larger fraction of their output is produced under more cost-efficient production facility, firm 2. Note, however, that in practice, we often see the reverse: a big firm buys part of a smaller firm, as acknowledged by Farrell and Shapiro.[9] They also showed that the PEO α may increase total surplus as well when firm 1 is smaller than firm 2.

How would PEO affect the ability of firms to engage in tacit collusion? Malueg (1992) addressed this question by considering a repeated symmetric Cournot duopoly model in which the firms hold identical stakes in one another (cross ownership), and showed that increasing the degree of cross ownership may decrease the ease or likelihood of collusion. Investigating a family of demand functions, Malueg found that the curvature of the demand function can alter the possibilities for collusion. Gilo, Moshe and Spiegel (2006) addressed

number of alternative financial interest and control arrangements. Also, Kwoka (1992) analyzed the output and profit effects of horizontal joint ventures under a conjectural variations model in which a joint venture constitutes a new producer rather than replaces an existing firm.

[8]Salant, Switzer and Reynolds showed, under a Cournot oligopoly model with a linear demand and symmetric constant marginal costs across all n firms, that horizontal mergers can be profitable only if more than 80 percent of the firms merge. Then, mergers between two firms are not profitable for any $n \geq 3$.

[9]Farrell and Shapiro pointed out that in their framework, such purchases will be profitable only if firm 1 gains control over firm 2's actions.

this question under more general setup. They considered a repeated Bertrand oligopoly model consisting of n firms in which firms and/or their controllers acquire some of their rivals' nonvoting shares. The n firms need not have similar stakes in one another in their model. Gilo et al. established necessary and sufficient conditions for PEO arrangements to facilitate tacit collusion and also examined how tacit collusion is affected when firms' controllers make direct passive investments in rival firms.

In the literature on theoretical analyses of PEO arrangements, several papers have pointed out the link between PEO and knowledge transfer. For example, Roynolds and Snapp (1986) pointed out that PEO offers a means for appropriating the returns to technology transfer. Also, Reitman (1994) argued that PEO arrangements may be beneficial to society if they encourage firms to exchange expertise or assets that would otherwise not be made available. However, to the best of our knowledge, no previous papers have explicitly analyzed the process in which PEO induces knowledge transfer between competing firms. The present paper fills this important gap in the literature by exploring a model in which the level of PEO is endogenously determined through the link between PEO and knowledge transfer.

3 PEO induces knowledge transfer

3.1 The model

Consider an industry consisting of $(n+2)$ firms ($n \geq 1$) that produce a homogenous product. The industry faces a linear inverse demand given by $P(Q) = a - dQ$ ($a > 0$, $d > 0$) where Q denotes the industry output. Let p_i and q_i denote each firm i's ($i = 1, 2, ..., n$) price and quantity, respectively. Each firm i's cost for producing q_i (> 0) units of the product is $c_i q_i$ where c_i (> 0) denotes firm i's constant marginal cost. Compared to all other firms, firm 1 has a cost advantage due to its superior knowledge that other firms do not have.

Firms 1 and 2 have an option to form an equity alliance. In particular, they negotiate and jointly choose the level of firm 1's ownership in firm 2's equity, denoted θ ($0 \leq \theta \leq 1$), and the monetary terms of the equity transfer. Given θ, firm 1 decides whether to transfer its superior knowledge to firm 2. Assume that firms' constant marginal costs are $c_1 = c - x < c_2 = c_3 = ... = c_{n+2} = c$ without knowledge transfer, where $c > x > 0$. Note that x captures firm 1's cost advantage due to its superior knowledge. If firm 1 transfers its knowledge to firm 2, firm 2's marginal cost is reduced to $c_2 = c - x$. Knowledge transfer does not affect the constant marginal costs of firms 1, 3, ..., $n + 2$. They remain at $c_1 = c - x$ and

$c_3 = ... = c_{n+2} = c$. We assume that firms $3, ..., n+2$ are independent and cannot form alliances.

We consider the three-stage game described below:

Stage 1 [Alliance formation]: Firms 1 and 2 negotiate and jointly choose the level of firm 1's ownership in firm 2's equity, denoted θ ($0 \leq \theta \leq 1$), and the monetary terms of the equity transfer through efficient bargaining.[10] The level of θ becomes common knowledge.

Stage 2 [Knowledge transfer]: Firm 1 determines whether or not to transfer its superior knowledge to firm 2. Firms' constant marginal costs are given by $c_1 = c_2 = c - x < c_3 = ... = c_{n+2} = c$ if knowledge is transferred, and by $c_1 = c - x < c_2 = c_3 = ... = c_{n+2} = c$ otherwise, where $c > x > 0$.

Stage 3 [Cournot competition]: Whether or not firm 1 transferred its knowledge to firm 2 becomes common knowledge, and hence every firm knows $(c_1, c_2, c_3, ..., c_{n+2})$. If $\theta \in [0, \frac{1}{2}]$, each firm i simultaneously and non-cooperatively chooses q_i to maximize its profit. If $\theta \in (\frac{1}{2}, 1]$, firm 1 chooses q_1 and q_2 and firm m $(= 3, ..., n+2)$ chooses q_m, simultaneously and non-cooperatively, to maximize their own profits.

3.2 Equilibrium characterization

We derive Subgame Perfect Nash Equilibria (SPNE) in pure strategies of the model described above. Define variable $k \in \{0, 1\}$ by $k = 1$ if firm 1 transferred its knowledge to firm 2 at Stage 2 and $k = 0$ otherwise. Note that proofs of lemmas and propositions are presented in the Appendix.

Every Stage 3 subgame can be represented by (θ, k). Each firm i's $(i = 1, 2, ..., n+2)$ profit, denoted by $\pi_i(\theta, k, q_1, q_2, ..., q_{n+2})$, is given by

$$\begin{aligned} \pi_1(\theta, k, q_1, q_2, ..., q_{n+2}) &= [P(Q) - (c - x)]q_1 + \theta[P(Q) - (c - kx)]q_2, \\ \pi_2(\theta, k, q_1, q_2, ..., q_{n+2}) &= (1 - \theta)[P(Q) - (c - kx)]q_2, \\ \pi_m(\theta, k, q_1, q_2, ..., q_{n+2}) &= [P(Q) - c]q_m, \end{aligned} \tag{1}$$

where $m = 3, ..., n+2$. Throughout the analysis in this section, we make the following assumption.

Assumption 1: $x < \frac{a-c}{2} \equiv \bar{x}$.

[10] We can assume the generalized Nash bargaining. Distribution of bargaining power between firms 1 and 2 do not affect results of the paper. See footnote 13 for a related discussion.

This is the necessary and sufficient condition for each firm i to produce a strictly positive amount of the product in the equilibrium of the Stage 3 subgame for all (θ, k, n) where $\theta \in [0, \frac{1}{2}]$ (see Claim 1 in the Appendix).

Consider equilibria of Stage 3 subgames. Let $q_i^*(\theta, k, n)$ and $\pi_i^*(\theta, k, n)$ respectively denote firm i's quantity and profit in the equilibrium of the Stage 3 subgame represented by (θ, k). Suppose $\theta \in [0, \frac{1}{2}]$. Then, at Stage 3, each firm i simultaneously and non-cooperatively chooses q_i to maximize $\pi_i(\theta, k, q_1, q_2, ..., q_{n+2})$. Through a standard analysis of Cournot competition, we find that the equilibrium is unique for any given (θ, k, n), where the equilibrium quantities are given by

$$\begin{aligned} q_1^*(\theta, k, n) &= \frac{(1-\theta)(a-c) + [n+2-(1+(n+1)\theta)k]x}{d(n+3-\theta)}, \\ q_2^*(\theta, k, n) &= \frac{a-c-(1-(n+2)k)x}{d(n+3-\theta)}, \\ q_m^*(\theta, k, n) &= \frac{a-c-(1+(1-\theta)k)x}{d(n+3-\theta)}, \end{aligned} \tag{2}$$

where $m = 3, ..., n+2$, and each firm i's equilibrium profit is given by

$$\pi_i^*(\theta, k, n) = \pi_i(\theta, k, q_1^*(\theta, k, n), q_2^*(\theta, k, n), ..., q_{n+2}^*(\theta, k, n)). \tag{3}$$

Next consider Stage 3 subgames with $\theta \in (\frac{1}{2}, 1]$. In this class of Stage 3 subgames, firm 1 chooses q_1 and q_2 to maximize $\pi_1(\theta, k, q_1, q_2, ..., q_{n+2})$. Note that firm 2 cannot be more cost effective than firm 1. Then, since firms 1 and 2 produce a homogeneous product, firm 1 cannot be strictly better off by producing a strictly positive quantity at firm 2. Given this, for expositional simplicity we make a tie-breaking assumption that, if firm 1 is indifferent between shutting down and not shutting down firm 2, firm 1 chooses to shut down firm 2. We then find that the equilibrium of any Stage 3 subgame with $\theta \in (\frac{1}{2}, 1]$ is unique, where the equilibrium quantities are given by $q_i^*(\theta, k, n) = q_i^*(0, 0, n-1)$ for $i = 1, 3, ..., n+2$ and $q_2^*(\theta, k, n) = 0$, and each firm i's equilibrium profit is given by (3).

We next consider firm 1's optimal decision at Stage 2 in Stage 2 subgames. Given θ, firm 1 decides whether or not to transfer its knowledge to firm 2 (that is, $k = 1$ or 0) in order to maximize its profit in the subsequent Stage 3 subgame $\pi_1^*(\theta, k, n)$. If $\theta \in (\frac{1}{2}, 1]$, firm 1 shuts down firm 2 in the equilibrium of the subsequent Stage 3 subgame as mentioned above, and hence firm 1 is indifferent between transferring its knowledge and not transferring it. For expositional simplicity, we assume that firm 1 chooses not to transfer its knowledge (i.e., $k = 0$) at stage 2. Now suppose $\theta \in [0, \frac{1}{2}]$. Firm 1 transfers its knowledge to firm 2 if and

only if $\pi_1^*(\theta, 1, n) - \pi_1^*(\theta, 0, n) \geq 0$. When does this condition hold? Proposition 1 provides a precise characterization.

Proposition 1 [Knowledge transfer]: Consider firm 1's decision at Stage 2 in the equilibrium of the Stage 2 subgame. There exists a threshold $x_1(n) \equiv \frac{2n+1}{6n+10}(a-c) < \bar{x}$ with the following properties.
(i) Suppose $0 < x \leq x_1(n)$. Then there exists a unique value $\hat{\theta}(x, n) \in (0, \frac{1}{2}]$ such that firm 1 transfers knowledge to firm 2 if and only if $\theta \in [\hat{\theta}(x, n), \frac{1}{2}]$, where $\hat{\theta}(x, n)$ is strictly increasing in x and strictly decreasing in n for all $x \in (0, x_1(n))$ and $n \geq 1$.
(ii) Suppose $x_1(n) < x < \bar{x}$. Then firm 1 does not transfer knowledge to firm 2 for any $\theta \in [0, \frac{1}{2}]$.

(**Figure 1 to be inserted here**)

Firm 1 has cost advantage over $n+1$ other firms. By transferring its knowledge, firm 1 loses its competitive advantage over firm 2, but a fraction θ of firm 2's profit belongs to firm 1. Proposition 1 tells us that firm 1 chooses to transfer its knowledge if θ is sufficiently high so that $\theta \in [\hat{\theta}(x, n), \frac{1}{2}]$ holds, where $\hat{\theta}(x, n)$ is the minimum PEO that induces knowledge transfer. As x increases, firm 1's loss of its competitive advantage through knowledge transfer becomes more substantial, and hence a higher level of PEO is necessary for firm 1 to have an incentive to transfer its knowledge to firm 2. That is, $\hat{\theta}(x, n)$ is increasing in x. And, once x exceeds $x_1(n)$, firm 1 has no incentive to transfer its knowledge even when $\theta = \frac{1}{2}$. Note also that firm 2 is one of the $n+1$ other firms that have cost disadvantage over firm 1. Then, as n increases, firm 1's loss of its competitive advantage by transferring its knowledge to firm 2 becomes less substantial. This means that a lower level of PEO is sufficient to compensate firm 1 for the loss of competitive advantage, implying that $\hat{\theta}(x, n)$ is decreasing in n. This also means that an increase in n increases the maximum amount of competitive advantage firm 1 is willing to lose against firm 2 by transferring its knowledge, implying that $x_1(n)$ $(\equiv \frac{2n+1}{6n+10}(a-c))$ is increasing in n.

At Stage 1, firms 1 and 2 jointly determine the level of θ to maximize their joint profit in the subsequent equilibrium.[11] The following lemma is useful for deriving the equilibrium of

[11]Let η^* denote the amount of money transferred from firm 1 to firm 2 in the equilibrium, and let θ^* and k^* respectively denote equilibrium values of θ and k. If the monetary transfer is determined by the generalized Nash bargaining process in which firm 1's bargaining power is given by $\beta \in (0, 1)$, then η^* is

the entire game. In what follows, let $\pi_{12}^*(\theta, k, n) \equiv \pi_1^*(\theta, k, n) + \pi_2^*(\theta, k, n)$ denote the joint profit of firms 1 and 2 in the equilibrium of the Stage 3 subgame represented by (θ, k).

Lemma 1: For any given k ($\in \{0, 1\}$), $\pi_{12}^*(\theta, k, n)$ is strictly decreasing in θ for all $\theta \in [0, \frac{1}{2}]$, and constant for all $\theta \in (\frac{1}{2}, 1]$. Furthermore, for any given $\theta' \in (\frac{1}{2}, 1]$, (i) $\pi_{12}^*(\frac{1}{2}, 1, n) > \pi_{12}^*(\theta', 1, n)$ holds for all $x \in (0, \bar{x})$, and (ii) $\pi_{12}^*(0, 0, n) > (=, <)\ \pi_{12}^*(\theta', 0, n)$ holds if $x < (=, >)\ x_2(n) \equiv \frac{n^2+2n-1}{3n^2+2n+11}(a - c)$, where $x_2(n) < \bar{x}$ holds.

(**Figure 2-1 and 2-2 to be inserted here**)

First consider $\pi_{12}^*(\theta, 1, n)$, where $k = 1$ (knowledge is transferred) so that $c_1 = c_2 = c - x$ (see Figure 2-1). PEO reduces the degree of competition between firms 1 and 2 and eliminates the competition when $\theta \in (\frac{1}{2}, 1]$. This effect works in the direction of increasing the joint profit of firms 1 and 2 as θ increases. At the same time, weaker competition between firms 1 and 2 induces other n firms to take more aggressive strategy, and this effect works in the direction of decreasing the joint profit of firms 1 and 2 as θ increases. Lemma 1 tells us that the latter effect dominates the former effect for all $\theta \in (0, 1]$, so that $\pi_{12}^*(\theta, 1, n)$ is decreasing in θ as depicted in Figure 2-1.

Next consider $\pi_{12}^*(\theta, 0, n)$, where $k = 0$ (knowledge is not transferred) so that $c_1 = c - x < c_2 = c$ (see Figure 2-2). In addition to the two effects mentioned above, there is another effect at work in this case. That is, an increase in θ within the interval $[0, \frac{1}{2}]$ shifts outputs from cost-efficient firm 1 to cost-inefficient firm 2. This effect works in the direction of reducing the joint profit, and, combined with the two effects mentioned above, yields that $\pi_{12}^*(\theta, 0, n)$ is strictly decreasing in θ for all $\theta \in [0, \frac{1}{2}]$. When $\theta \in (\frac{1}{2}, 1]$, firm 1 shuts down the operation of firm 2. Since all output is shifted to the cost efficient firm (firm 1), the joint profit $\pi_{12}^*(\theta, 0, n)$ discontinuously increases when θ increases from $\frac{1}{2}$ to $\theta \in (\frac{1}{2}, 1]$, as depicted in Figure 2-2. The improvement of cost efficiency becomes more substantial as x increases, and we find that $\pi_{12}^*(\theta', 0, n) > \pi_{12}^*(0, 0, n)$ where $\theta' \in (\frac{1}{2}, 1]$ if and only if $x > x_2(n)$.

We are now ready to derive the equilibrium of the entire game. Consider the joint-profit maximizing level of θ chosen by firms 1 and 2 at Stage 1. Recall that $\pi_{12}^*(\theta, k, n)$ takes a constant value for all $\theta \in (\frac{1}{2}, 1]$ (Lemma 1). In what follows, for expositional simplicity we

determined by $\pi_1^*(\theta^*, k^*, n) - \eta^* = \pi_1^*(0, 0, n) + \beta[\pi_{12}^*(\theta^*, k^*, n) - \pi_{12}^*(0, 0, n)]$. It can be shown that the value of η^* is negative under a certain range of parameterizations. The case of negative η^* can be interpreted as the case in which a part of firm 2's equity is transferred to firm 1 at a price below the on-going market price.

assume that firms 1 and 2 choose $\theta = 1$ if any $\theta \in (\frac{1}{2}, 1]$ is their optimal choice, and interpret this as their merger.

First suppose $x \leq x_1(n)$. In this case, firms 1 and 2 can induce knowledge transfer by choosing $\theta \in [\hat{\theta}(x,n), \frac{1}{2}]$ at Stage 1 (by Proposition 1), and $\pi_{12}^*(\theta, 1, n)$ is strictly decreasing in θ for all $\theta \in [\hat{\theta}(x,n), \frac{1}{2}]$ (by Lemma 1). Hence firms 1 and 2 choose $\theta = \hat{\theta}(x,n)$, the minimum PEO for knowledge transfer, if they intend to induce knowledge transfer from firm 1 to firm 2. Under this option, their joint profit is $\pi_{12}^*(\hat{\theta}(x,n), 1, n)$. At the same time, they have an option of not inducing knowledge transfer. Under the no-knowledge-transfer option, firms 1 and 2 choose $\theta = 0$ if $x \in (0, x_2(n)]$ and chooses $\theta = 1$ otherwise. Comparison between these three options ($\theta = 0, \hat{\theta}(x,n)$, or 1) leads us to Proposition 2 (i) and (ii).

Next suppose $x > x_1(n)$. Proposition 1 tells us that firm 1 does not transfer its knowledge to firm 2 at Stage 2 for any given $\theta \in [0, \frac{1}{2}]$ in this case. Anticipating this, at Stage 1 firms 1 and 2 jointly choose θ that maximizes their joint profit without knowledge transfer, $\pi_{12}^*(\theta, 0, n)$. And, since $x_1(n) > x_2(n)$ holds for all $n \geq 1$, Lemma 1 tells us that firms 1 and 2 choose $\theta = 1$ (merger) for any $x > x_1(n)$, yielding Proposition 2 (iii).

Proposition 2 [Equilibrium characterization]: For any given $x \in (0, \bar{x})$ and $n \geq 1$, there exists a unique value $\theta^*(x,n)$ and a threshold $x_3(n) \in (0, x_1(n))$ such that actions taken by firms 1 and 2 in the unique equilibrium of the game are described by (i) - (iii) below. Furthermore, $x_3(n)$ is strictly decreasing in n for all $n \geq 1$.
(i) Suppose $0 < x \leq x_3(n)$. Then firms 1 and 2 choose $\theta = \theta^*(x,n) \equiv 0$ at Stage 1 and firm 1 does not transfer its knowledge to firm 2 at Stage 2.
(ii) Suppose $x_3(n) < x \leq x_1(n)$. Then firms 1 and 2 choose $\theta = \theta^*(x,n) \equiv \hat{\theta}(x,n)$ at Stage 1 and firm 1 transfers its knowledge to firm 2 at Stage 2, where $\theta^*(x,n)$ is strictly positive and strictly increasing in x for all $x \in (x_3(n), x_1(n)]$ with $\theta^*(x_1(n), n) = \frac{1}{2}$.
(iii) Suppose $x_1(n) < x < \bar{x}$. Then firms 1 and 2 choose $\theta = \theta^*(x,n) \equiv 1$ (merger) at Stage 1.

The logic behind Proposition 2 can be explained as follows. Suppose firms 1 and 2 choose $\theta = 0$ ($< \hat{\theta}(x,n)$) at Stage 1. Then, firm 1's knowledge is not transferred to firm 2 (that is, $k = 0$) at Stage 2, and their joint profit is $\pi_{12}^*(0, 0, n)$ in the subsequent equilibrium. In order to induce knowledge transfer, the level of PEO should be increased from 0 to $\hat{\theta}(x,n)$. Firms 1 and 2 prefer $\theta = \hat{\theta}(x,n)$ to $\theta = 0$ if $\pi_{12}^*(\hat{\theta}(x,n), 1, n) - \pi_{12}^*(0, 0, n) \geq 0$, where $\pi_{12}^*(\hat{\theta}(x,n), 1, n) - \pi_{12}^*(0, 0, n)$ can be decomposed into knowledge transfer effect and PEO

effect as follows:

$$\pi_{12}^*(\hat{\theta}(x,n),1,n)-\pi_{12}^*(0,0,n)=\underbrace{[\pi_{12}^*(\hat{\theta}(x,n),1,n)-\pi_{12}^*(\hat{\theta}(x,n),0,n)]}_{knowledge\ transfer\ effect}+\underbrace{[\pi_{12}^*(\hat{\theta}(x,n),0,n)-\pi_{12}^*(0,0,n)]}_{PEO\ effect}.$$

The knowledge transfer effect is positive. That is, holding the level of PEO fixed at $\theta=\hat{\theta}(x,n)$, knowledge transfer increases the joint profit of firms 1 and 2. The PEO effect can be regarded as the cost that firms 1 and 2 jointly incur in order to induce knowledge transfer, since an increase of θ from 0 to $\hat{\theta}(x,n)$ holding $k=0$ fixed decreases their joint profit. That is, the PEO effect is negative. We find that $\lim_{x\to 0}\hat{\theta}(x,n)>0$. This implies that the PEO effect does not approach zero as x approaches zero, where $\lim_{x\to 0}[\pi_{12}^*(\hat{\theta}(x,n),0,n)-\pi_{12}^*(0,0,n)]<0$. In contrast, the knowledge transfer effect decreases as x decreases, and approaches zero as x approaches zero. Hence, when x is small enough, we have $\pi_{12}^*(\hat{\theta}(x,n),1,n)-\pi_{12}^*(0,0,n)<0$ and so firms 1 and 2 prefer $\theta=0$ to $\theta=\hat{\theta}(x,n)$.

An increase in x increases the knowledge transfer effect, and it also increases the magnitude of the PEO effect because $\hat{\theta}(x,n)$ is increasing in x. We find that the knowledge transfer effect dominates the PEO effect if x is large enough to satisfy $x\in(x_3(n),x_1(n)]$, where firms 1 and 2 choose $\theta=\hat{\theta}(x,n)$ over $\theta=0$. Once x exceeds $x_1(n)$, no PEO $\theta\in[0,\frac{1}{2}]$ can induce knowledge transfer, leaving $\theta=0$ or 1 as relevant options for firms 1 and 2. They choose $\theta=1$ in this case.

Recall that $\hat{\theta}(x,n)$, the minimum PEO that induces knowledge transfer for given (x,n), is decreasing in n by Proposition 1. This works in the direction of reducing the magnitude of PEO effect as n increases. As a consequence we find that $x_3(n)$ is decreasing in n. That is, the minimum level of x at which knowledge transfer through PEO occurs in the equilibrium is decreasing in n. Then, given that $x_1(n)$ is increasing in n, Proposition 2 tells us that knowledge transfer through PEO arrangement is more likely to occur as n increases in the sense that the interval $(x_3(n),x_1(n)]$ gets larger as n increases.

Robustness of the results

We end this subsection by making two points regarding robustness of our results. First, in our model, firm 1 decides whether or not to transfer its knowledge to firm 2, but cannot transfer a part of its knowledge. Our results would remain unchanged in a variant of our model that allows partial transfer of knowledge. To see this, suppose that at Stage 2, firm 1 chooses the extent of knowledge transfer, denoted $k\in[0,1]$, which reduces firm 2's marginal cost from $c_2=c$ to $c_2=c-kx$. That is, firm 1 can choose to transfer its entire knowledge ($k=1$), a part of it ($k\in(0,1)$), or none of it ($k=0$). Given θ, firm 1 chooses k to maximize

its profit in the subsequent Stage 3 subgame $\pi_1^*(\theta, k, n)$. Suppose $\theta \in [0, \frac{1}{2}]$ and let k^* denote the value of k that maximizes $\pi_1^*(\theta, k, n)$.[12] In Appendix we show that $\pi_1^*(\theta, k, n)$ is strictly convex in k, which implies that $k^* \notin (0, 1)$. That is, partial knowledge transfer is never optimal. Firm 1 either transfers its knowledge in full ($k^* = 1$) or transfers no knowledge ($k^* = 0$).[13] Hence our results remain unchanged in this variant of the model.

Second, in our model, firm 1 can hold ownership in firm 2's equity, but firm 2 cannot hold ownership in firm 1's equity. We have considered a variant of the model in which firm 2 can also hold ownership in firm 1's equity. Suppose that at Stage 1, firms 1 and 2 choose θ and ψ, where ψ ($0 \leq \psi \leq 1$) denotes the level of firm 2's ownership in firm 1's equity, and the monetary terms of the equity transfer through efficient bargaining. Everything else is same as in the original model. We have found that the qualitative nature of the results remains mostly unchanged in this variant of the model.[14] As in the original model we find that there exists a threshold $\tilde{x}_3(n) \in (0, x_1(n))$ with the following property. If $0 < x \leq \tilde{x}_3(n)$, knowledge is not transferred at Stage 2 in the equilibrium. At Stage 1 firms 1 and 2 choose $\theta = 0$ and $\psi = max(\frac{-(n-1)(a-c)+(3n+5)x}{a-c+(2n+3)x}, 0) \equiv h(x, n)$, where $h(x, n) < \frac{1}{2}$ holds for all $x \in (0, \tilde{x}_3(n)]$ and $n \geq 1$. Here $\psi = h(x, n)$ plays the role of increasing firms 1 and 2's joint profit by shifting some outputs from the cost-inefficient firm (firm 2) to the cost-efficient firm (firm 1) when $h(x, n) > 0$. If $\tilde{x}_3(n) < x \leq x_1(n)$, knowledge is transferred at Stage 2 in the equilibrium. At Stage 1 firms 1 and 2 choose $\theta = \hat{\theta}(x, n)$ (where $\hat{\theta}(x, n)$ is as defined in the original model) and $\psi = 0$. That is, firm 2's ownership in firm 1's equity plays no roles in facilitating knowledge transfer from firm 1 to firm 2. If $x_1(n) < x < \bar{x}$, firms 1 and 2 choose to merge as in the original model. The qualitative nature of welfare consequences of PEO also remains unchanged in this variant.

We have found that partial cross ownership does not occur in the equilibrium even though it is allowed in this variant of the model. Our conjecture is that partial cross ownership would occur if firm 2 also has unique knowledge that can be transferred to firm 1, because ψ would then play a role of facilitating knowledge transfer from firm 2 to firm 1. Enrichment of the model in this direction is beyond the scope of this paper and left to future research.

[12] Existence of k^* follows immediately from observing that $\pi_1^*(\theta, k, n)$ is continuous in k, and k lies in a compact interval $[0, 1]$.

[13] Similar results exist in the licensing literature. In a duopoly model with fixed fee as payment mechanism, Kabiraj and Marjit (1993) and Ghosh and Saha (2008) have shown that either the best technology is licensed or no technology is licensed at all. The analysis in those papers however do not involve third firm or PEO.

[14] See Supplementary Note for more details of the analysis.

3.3 Welfare consequences and antitrust implications

We now turn to welfare consequences and antitrust implications of PEO. Let $CS(\theta, n)$ and $TS(\theta, n)$ respectively denote consumer surplus and total surplus in the equilibrium of the Stage 2 subgame represented by θ. We first compare $CS(\theta^*(x, n), n)$ and $CS(0, n)$. That is, we compare consumer surplus at the endogenously determined level of PEO $\theta = \theta^*(x, n)$ with consumer surplus at $\theta = 0$. We then compare $TS(\theta^*(x, n), n)$ and $TS(0, n)$. We have that $CS(\theta^*(x, n), n) = CS(0, n)$ and $TS(\theta^*(x, n), n) = TS(0, n)$ for all $x \in (0, x_3(n)]$, because $\theta^*(x, n) = 0$ and knowledge is not transferred in the equilibrium for all $x \in (0, x_3(n)]$. Given this, we focus on the case of $x \in (x_3(n), \bar{x})$ in what follows.

Regarding consumer surplus, we find that $CS(\theta^*(x, 1), 1) < CS(0, 1)$ for all $x \in (x_3(1), \bar{x})$. That is, when $n = 1$, the endogenously determined level of PEO decreases consumer surplus for all relevant x. However, when $n \geq 2$, PEO can increase consumer surplus as Proposition 3 tells us.

Proposition 3 [Consumer surplus]:
(A) For $n = 1$, $CS(\theta^*(x, 1), 1) < CS(0, 1)$ holds for all $x \in (x_3(1), \bar{x})$. That is, the endogenously determined level of PEO decreases consumer surplus for all relevant x when $n = 1$.

(B) For any given $n \geq 2$, there exist a threshold value $x_4(n) \in (x_3(n), x_1(n))$ with the properties (i) - (iii) below. Furthermore, $x_4(n)$ is strictly decreasing in n for all $n \geq 2$.
(i) Suppose $x_3(n) < x \leq x_4(n)$. Then $CS(\theta^*(x, n), n) \leq CS(0, n)$ holds, where equality holds if and only if $x = x_4(n)$.
(ii) Suppose $x_4(n) < x \leq x_1(n)$. Then $CS(\theta^*(x, n), n) > CS(0, n)$ holds.
(iii) Suppose $x_1(n) < x < \bar{x}$. Then $CS(\theta^*(x, n), n) < CS(0, n)$ holds, where $\theta^*(x, n) = 1$.

When $x \in (x_3(n), x_1(n)]$, firms 1 and 2 choose $\theta = \theta^*(x, n)$ to induce knowledge transfer, which reduces firm 2's cost. The cost reduction works in the direction of increasing the industry output, but the chosen PEO $\theta = \theta^*(x, n)$ weakens competition in the industry, working in the direction of reducing the industry output. Hence the net impact on consumers is not immediately obvious. We find that, as x increases, the positive impact associated with the cost reduction becomes increasingly more significant than the negative impact associated with the PEO, and the net impact on consumers becomes positive once x exceeds a threshold $x_4(n)$. Consequently, the endogenously determined level of PEO, $\theta^*(x, n)$, increases consumer

surplus if $x \in (x_4(n), x_1(n)]$. At the same time, however, when x exceeds $x_1(n)$, two firms merge and the merger reduces consumer surplus.

Proposition 3 (B) also tells us that PEO is more likely to benefit consumers as n increases in the sense that the interval $(x_4(n), x_1(n)]$ gets larger as n increases. To understand the logic, recall again that $\hat{\theta}(x, n)$ is decreasing in n. This implies that, holding x fixed, knowledge transfer is induced at a lower level of PEO in the equilibrium as n increases, indicating that consumers are more likely to benefit from PEO as n increases.

Proposition 4 [Total surplus]:
(A) When $n = 1$, there exist values x'_{TS} and x''_{TS}, $x_3(1) < x'_{TS} < x_1(1) < x''_{TS} < \bar{x}$, with the following properties:
(i) Suppose $x_3(1) < x \leq x'_{TS}$. Then $TS(\theta^*(x, 1), 1) \leq TS(0, 1)$ holds, where equality holds if and only if $x = x'_{TS}$.
(ii) Suppose $x'_{TS} < x \leq x_1(1)$. Then $TS(\theta^*(x, 1), 1) > TS(0, 1)$ holds.
(iii) Suppose $x_1(1) < x \leq x''_{TS}$. Then $TS(\theta^*(x, 1), 1) \leq TS(0, 1)$ holds where $\theta^*(x, 1) = 1$, where equality holds if and only if $x = x''_{TS}$.
(iv) Suppose $x''_{TS} < x < \bar{x}$. Then $TS(\theta^*(x, 1), 1) > TS(0, 1)$ holds where $\theta^*(x, 1) = 1$.

(B) For any given $n \geq 2$, $TS(\theta^*(x, n), n) > TS(0, n)$ holds for all $x \in (x_3(n), \bar{x})$.

Whenever PEO increases consumer surplus, PEO also increases total surplus. To see this, pick any $x \in (x_4(n), x_1(n)]$ so that the endogenously determined level of PEO $\theta^*(x, n)$ increases consumer surplus. This means that the industry output is higher under $\theta = \theta^*(x, n)$ than under $\theta = 0$. Also, when $\theta = \theta^*(x, n)$, knowledge is transferred from firm 1 to firm 2 and hence a larger fraction of industry output is produced at the lower cost $c - x$ in the equilibrium. This implies that total surplus is also larger under $\theta = \theta^*(x, n)$ than under $\theta = 0$. Proposition 4 tells us that PEO increases total surplus under a range of parameterizations when $n = 1$, and for all relevant values of x when $n \geq 2$.

Antitrust implications

We now turn to antitrust implications of PEO arrangements. We have demonstrated that PEO can increase consumer surplus and/or total surplus by inducing knowledge transfer, suggesting that antitrust authorities might allow, rather than prohibit, PEO arrangements between competitors to maximize welfare. To make this idea precise, let us consider an antitrust authority whose objective is to maximize total surplus under the following simple extension of the model, in which everything is the same as in the original model except for

the following. At Stage 0, the antitrust authority can announce a maximum permissible level of PEO, denoted $\tilde{\theta} \in [0,1]$. Then, when firms 1 and 2 jointly choose θ at Stage 1, where $\theta \in [0, \tilde{\theta}]$ must be satisfied. Assume, for expositional simplicity, that the antitrust authority announces $\tilde{\theta}$ only if the authority can strictly increase the equilibrium total surplus by doing so.

Knowledge transfer and PEO work in opposite directions from the welfare standpoint. That is, knowledge transfer increases the equilibrium total surplus by reducing firm 2's production cost, while the PEO decreases the equilibrium total surplus by reducing the overall degree of competition in the industry. Then, from the antitrust's standpoint, knowledge should be transferred at $\theta = \hat{\theta}(x,n)$, the minimum PEO that induces knowledge transfer. Under the linear homogeneous demand, the antitrust's preference matches with the choice made by firms 1 and 2, since they choose $\theta = \hat{\theta}(x,n)$ whenever knowledge is transferred in the equilibrium. Hence the antitrust authority's relevant option is either to impose no restrictions on PEO or to prohibit PEO. Proposition 4 tells us that when $n = 1$, the authority prohibits PEO (i.e., announces $\tilde{\theta} = 0$) if $x \in (x_3(1), x'_{TS})$ or $x \in (x_1(1), x''_{TS})$ and allows any levels of PEO (i.e., does not announce $\tilde{\theta}$) otherwise, and when $n \geq 2$, the authority allows any levels of PEO for all x.

What would happen when the antitrust authority's objective is to maximize consumer surplus instead of total surplus? In this case, Proposition 3 tells us that the authority prohibits PEO for all x when $n = 1$, and when $n \geq 2$, the authority allows any levels of PEO if $x \in (x_4(n), x_1(n)]$ and prohibits PEO otherwise. PEO is more likely to hurt consumers as n decreases. This suggests that the authority should put more careful eyes on PEO in industries that have a smaller number of competitors.

4 Partial permission of PEO as an antitrust policy

In the previous section, we have shown that the antitrust authority would either impose no restrictions on PEO or completely prohibit PEO to maximize consumer surplus and/or total surplus under the liner homogeneous demand. In this section, we demonstrate that partial permission of PEO can also be an optimal action of the authority. We illustrate this possibility by extending our model to differentiated oligopoly.

4.1 An extension to differentiated oligopoly

In this extension, everything is the same as in the original model except for the demand structure and the focus on the case of three firms. Consider an economy consisting of an imperfectly competitive sector with three firms, each producing a symmetrically differentiated product, and a competitive numeraire sector whose output is denoted by q_0. Each firm i (=1,2,3) produces product i, and let p_i and q_i denote respectively the price and quantity of product i.

There is a continuum of consumers of the same type, and the representative consumer's preferences are described by the utility function $U(q_1, q_2, q_3) + q_0$, where

$$U(q_1, q_2, q_3) \equiv a(q_1 + q_2 + q_3) - \frac{q_1^2 + q_2^2 + q_3^2}{2} - b(q_1 q_2 + q_2 q_3 + q_3 q_1),$$

$a > 0$ and $b \in (0, 1]$. This yields linear inverse demands:

$$p_i = a - q_i - b(q_j + q_k), \qquad i, j, k \in \{1, 2, 3\}; i \neq j \neq k. \tag{4}$$

This is a standard specification of the representative consumer model, where the consumer prefers product variety (see, for example, Vives, 1999). The term b captures the degree of product differentiation in the market. As b increases, the degree of product differentiation decreases. Note that the case of the linear homogenous demand is nested as a special case of $b = 1$.

As in the previous section, we derive Subgame Perfect Nash Equilibria (SPNE) in pure strategies. Analysis of the extended model is similar to the one presented in the previous section, and so we present just the outline of the analysis in the text with some more analytical details presented in the Appendix. Analogous to Assumption 1, we make Assumption 2.[15]

Assumption 2: $x < \frac{2-b}{2b}(a - c) \equiv \bar{x}$.

Let $q_i^*(\theta, k)$ and $\pi_i^*(\theta, k)$ respectively denote firm i's quantity and profit in the equilibrium of the Stage 3 subgame represented by (θ, k), where $\pi_i^*(\theta, k) = \pi_i(\theta, k, q_1^*(\theta, k), q_2^*(\theta, k), q_3^*(\theta, k))$. Also, let $\pi_{12}^*(\theta, k) \equiv \pi_1^*(\theta, k) + \pi_2^*(\theta, k)$ as in the previous section.

[15] Each firm i produces a strictly positive amount in the equilibrium of the Stage 3 subgame for all (θ, k) where $\theta \in [0, \frac{1}{2}] \Leftrightarrow x < \frac{2-b}{2b}(a - c)$, where the proof is analogous to the proof of Claim 1 presented in the Appendix.

In what follows, we proceed our analysis under a specific value of parameter b, $b = 0.6$. And at the end of the section we discuss the case of $b = 0.4$. We believe that the model can be fully analyzed under any given values of b. However, we have been unable to fully analyze the model without specifying the value of b because of algebraic complexity.

Let $\theta \in [0, \frac{1}{2}]$ be given, and consider firm 1's incentive to transfer its knowledge to firm 2. We obtain Proposition 5, which is qualitatively similar to Proposition 1 in the previous section.

Proposition 5 [Knowledge transfer]: Let $\theta \in [0, \frac{1}{2}]$ be given and consider firm 1's decision at Stage 2 in the equilibrium of the Stage 2 subgame. There exists a unique value $\hat{\theta}(x) \in (0, \frac{1}{2}]$ such that firm 1 transfers its knowledge to firm 2 if and only if $\theta \in [\hat{\theta}(x), \frac{1}{2}]$, where $\hat{\theta}(x)$ is strictly increasing in x with $\lim_{x\to 0} \hat{\theta}(x) \approx 0.236$ and $\lim_{x\to \bar{x}} \hat{\theta}(x) \approx 0.446$.

Next we consider the level of θ that firms 1 and 2 choose to maximize their joint profit in the equilibrium.

Lemma 2: There exists values $\bar{\theta} \approx 0.384$ and $x_1 \approx 0.707(a - c)$ such that (i) $\pi^*_{12}(\theta, 1)$ is strictly increasing in θ for all $\theta \in [0, \bar{\theta})$ and strictly decreasing in θ for all $\theta \in (\bar{\theta}, \frac{1}{2}]$, and (ii) $\bar{\theta} \in (\hat{\theta}(x), \frac{1}{2})$ for all $x \in (0, x_1)$ and $\bar{\theta} \leq \hat{\theta}(x)$ for all $x \in [x_1, \bar{x})$.

(**Figure 3 to be inserted here**)

The property of $\pi^*_{12}(\theta, 1)$ is qualitatively different between the case of linear homogeneous demand (that is, the case of $b = 1$) and the case of $b = 0.6$. When $b = 1$, $\pi^*_{12}(\theta, 1)$ is monotone decreasing in θ for all $\theta \in [0, \frac{1}{2}]$ (Lemma 1 in the previous section). In contrast, when $b = 0.6$, $\pi^*_{12}(\theta, 1)$ is non-monotone function of θ as depicted in Figure 3.

Suppose that firms 1 and 2 choose θ at a level that induces firm 1 to transfer its knowledge to firm 2 at Stage 2. Under the original model, we found that their optimal choice is $\theta = \hat{\theta}(x, n)$, which is the minimum PEO that induces knowledge transfer. Lemma 1' tells us, however, that this is not true when $b = 0.6$ and $x \in (0, x_1)$. In this case, by choosing $\theta = \bar{\theta}$ $(> \hat{\theta}(x))$, firms 1 and 2 induce firm 1's knowledge transfer to firm 2 at Stage 2, where $\theta = \bar{\theta}$ is the level of PEO that maximizes their joint profit conditional upon knowledge transfer (see Figure 3). In contrast, if $x \in [x_1, \bar{x})$, firms 1 and 2 choose $\theta = \hat{\theta}(x)$ as in the original model. This leads us to Proposition 6 below.

Proposition 6 [Equilibrium characterization]: For any given $x \in (0, \bar{x})$, there exists a unique value $\theta^*(x)$ such that actions taken by firms 1 and 2 in the unique equilibrium of the game are described as follows:
(i) Suppose $0 < x < x_1$. Then firms 1 and 2 choose $\theta = \theta^*(x) \equiv \bar{\theta} \approx 0.384$ at Stage 1 and firm 1 transfers its knowledge to firm 2 at Stage 2.
(ii) Suppose $x_1 \leq x < \bar{x}$. Then firms 1 and 2 choose $\theta = \theta^*(x) \equiv \hat{\theta}(x)$ at Stage 1 and firm 1 transfers its knowledge to firm 2 at Stage 2, where $\theta^*(x)$ is strictly positive and strictly increasing in x for all $x \in (x_1, \bar{x})$.

The key difference between $b = 0.6$ case and the original model is captured by (i). That is, when x is small enough satisfying $x \in (0, x_1)$, firms 1 and 2 choose $\theta = \bar{\theta}$, which is strictly greater than $\hat{\theta}(x)$, the minimum PEO that induces knowledge transfer. In contrast, under the original model, firms 1 and 2 choose $\theta = \hat{\theta}(x)$ whenever knowledge is transferred in the equilibrium. This difference is the driving force of the possibility that partial permission of PEO be the optimal antitrust policy in the case of differentiated oligopoly.

Regarding impacts of PEO on consumer surplus and total surplus, we find that the endogenously determined level of PEO $\theta^*(x)$ increases consumer surplus and/or total surplus when x is relatively small (see Appendix for details).

4.2 Partial permission of PEO: The case of $b = 0.6$

Let us now suppose that at Stage 0, the antitrust authority can announce a maximum permissible level of PEO, denoted $\tilde{\theta} \in [0, 1]$, with the objective of maximizing the equilibrium total surplus, and investigate the authority's optimal action. Note that the qualitative nature of our results remain unchanged under an alternative assumption that the authority's objective is to maximize the equilibrium consumer surplus.

We first establish the following lemma, where $ts^*(\theta, k)$ denotes total surplus in the equilibrium of the Stage 3 subgame represented by (θ, k).

Lemma 3: For any given $k = \{0, 1\}$, $ts^*(\theta, k)$ is strictly decreasing in θ for all $\theta \in [0, \frac{1}{2}]$.

Recall that the minimum PEO that induces knowledge transfer is $\hat{\theta}(x)$. Lemma 3 then implies that the maximum possible total surplus which the antitrust authority could achieve in the equilibrium is $ts^*(0, 0)$ or $ts^*(\hat{\theta}(x), 1)$. We find that the equilibrium total surplus is $ts^*(0, 0)$ if $0 < x < x_2 \cong 0.0118(a - c)$ and $ts^*\hat{\theta}(x), 1)$ otherwise.

Proposition 7-1 (b=0.6) [Antitrust implications]: There exists a value $x_2 \approx 0.0118(a-c)$ such that actions taken by the antitrust authority and firms 1 and 2 in the unique equilibrium are described as follows:
(i) Suppose $0 < x < x_2$. Then the antitrust authority announces $\tilde{\theta} = 0$ at Stage 0, firms 1 and 2 choose $\theta = 0$ at Stage 1, and firm 1 does not transfer its knowledge to firm 2 at Stage 2.
(ii) Suppose $x_2 \leq x < x_1$. Then the antitrust authority announces $\tilde{\theta} = \hat{\theta}(x)$ $(\in (0, \bar{\theta}))$ at Stage 0, firms 1 and 2 choose $\theta = \tilde{\theta}$ at Stage 1, and firm 1 transfers its knowledge to firm 2 at Stage 2.
(iii) Suppose $x_1 \leq x < \bar{x}$. Then the antitrust authority makes no announcement at Stage 0, firms 1 and 2 choose $\theta = \hat{\theta}(x)$ at Stage 1, and firm 1 transfers its knowledge to firm 2 at Stage 2.

Proposition 7-1 tells us that the antitrust authority's optimal policy is to prohibit PEO, or partially permit PEO by imposing a binding constraint on the level of the PEO, or permit any levels of PEO without imposing any restrictions, depending on the value of x. First suppose $x \in (0, x_2)$. The social planner would choose $\theta = 0$, and hence the antitrust authority prohibits PEO by announcing $\tilde{\theta} = 0$ in this case.

Patrial permission of PEO happens in the equilibrium when $x \in [x_2, x_1)$. In this case, firms 1 and 2 would choose $\theta = \bar{\theta} > \hat{\theta}(x)$ if no restriction is imposed as Proposition 6 tells us, while the social planner would induce knowledge transfer by choosing $\theta = \hat{\theta}(x)$. The antitrust authority can achieve the same outcome by announcing a binding restriction of $\tilde{\theta} = \hat{\theta}(x)$ if this restriction induces firms 1 and 2 to choose $\theta = \hat{\theta}(x)$ at Stage 1. We find that $\pi_{12}^*(\hat{\theta}(x), 1) > \pi_{12}^*(\theta, 0)$ holds for all $\theta \in [0, \hat{\theta}(x)]$, which implies that firms 1 and 2 do choose $\theta = \hat{\theta}(x)$ at Stage 1 if the authority announces $\tilde{\theta} = \hat{\theta}(x)$ at Stage 0. This implies (ii). Finally, if $x \in [x_1, \bar{x})$, firms 1 and 2 would choose $\theta = \hat{\theta}(x)$ if no restriction is imposed. Then, since $\theta = \hat{\theta}(x)$ is the social planner's optimal choice, the antitrust authority imposes no restrictions in this case.

4.3 Prohibit merger and encourage PEO: The case of $b = 0.4$

In the remainder of this section, we discuss what happens when $b = 0.4$. The equilibrium characterization result in this case turns out to be very simple. That is, for any given $x \in (0, \bar{x})$, in the unique equilibrium of the game firms 1 and 2 choose $\theta = 1$ (i.e, they choose

to merge) at Stage 1, and firm 1 transfers its knowledge to firm 2 at Stage 2. Merger between firms 1 and 2 eliminates the competition between themselves, but the lack of competition between firms 1 and 2 induces firm 3 to take more aggressive strategy. We find that, when $b = 0.4$, the former effect dominates the latter when the merger is compared to any levels of PEO $\theta \in [0, 1)$. This implies that firms 1 and 2 choose to merge in the equilibrium even though any levels of PEO $\theta \in [0, 1]$ is possible. Regarding impacts of the merger on consumer surplus and total surplus, we find that the merger increases consumer surplus and/or total surplus when x is relatively small (see Appendix for details).

Now we turn to antitrust implications by introducing the antitrust authority that can impose a maximum permissible level of PEO. As in Proposition 5, we find that for any given $x \in (0, \bar{x})$, there exists the minimum PEO, $\hat{\theta}(x) \in (0, \frac{1}{2})$, that induces firm 1 to transfer its knowledge to firm 2. This leads us to the following result.

Proposition 7-2 (b=0.4) [Antitrust implications]: There exists a value $x_2' \approx 0.00663(a-c)$ such that actions taken by the antitrust authority and firms 1 and 2 in the unique equilibrium are described as follows:
(i) Suppose $0 < x < x_2'$. Then the antitrust authority announces $\tilde{\theta} = 0$ at Stage 0, firms 1 and 2 choose $\theta = 0$ at Stage 1, and firm 1 does not transfer its knowledge to firm 2 at Stage 2.
(ii) Suppose $x_2' \leq x < \bar{x}$. Then the antitrust authority announces $\tilde{\theta} = \hat{\theta}(x)$ at Stage 0, firms 1 and 2 choose $\theta = \hat{\theta}(x)$ at Stage 1, and firm 1 transfers its knowledge to firm 2 at Stage 2.

Proposition 7-2 tells us that the optimal antitrust policy is to "prohibit merger and encourage PEO" unless x is very small. That is, although the merger improves welfare, the antitrust authority can further increase welfare by prohibiting firms 1 and 2 to merge and instead inducing them to choose the minimum PEO for knowledge transfer by announcing $\tilde{\theta} = \hat{\theta}(x)$ as the maximum permissible level of PEO. If options available for the authority were either to prohibit or allow mergers, the authority would allow merger to induce knowledge transfer unless x is very small. Our finding, however, suggests that partial permission of PEO is another option to be considered as an antitrust policy in order to maximize the social efficiency of knowledge transfer.

5 Policy implications

In the United States, cases of PEO in a competitor have gone mostly unchallenged by antitrust agencies (see Gilo (2000) for details). The U.S. antitrust agencies, however, have recently begun to pay increasing attention to the possible antitrust harms of PEO. For example, Deborah Platt Majoras, the then Deputy Assistant Attorney General of the Antitrust Division of the U.S. Department of Justice, mentioned in her speech given in April 2002 that PEO can raise antitrust issues when the two companies or their subsidiaries are competitors. Also, several legal scholars have argued that PEO, even if it is not accompanied by control/influence rights, results in antitrust harms in oligopolistic industries, by reducing quantities and raising prices (Gilo, 2000; O'Brien and Salop, 2000, 2001). Their arguments are consistent with the previous literature on economic theoretical analyses of PEO, in which the level of PEO is exogenously assumed.

Our theoretical analyses presented in previous sections indicate that the link between knowledge transfer and PEO provides new perspectives on antitrust implications of PEO arrangements. In our base model with the linear homogeneous demand, firm 1's PEO in firm 2 reduces not only consumer surplus and total surplus but also the alliance partners' equilibrium joint profit. Hence, in order to induce firm 1 to transfer its knowledge to firm 2, firms 1 and 2 choose the minimum PEO that is necessary for knowledge transfer. Although PEO itself reduces the industry output by weakening competition in the industry, induced knowledge transfer increases the industry output by increasing firm 2's production efficiency. Net welfare consequences of PEO arrangements are therefore not obvious.

We have found that the endogenously determined level of PEO increases total surplus when $n \geq 2$, and, when $n = 1$, it increases total surplus when x is sufficiently large. Also, the endogenously determined level of PEO increases consumer surplus when $n \geq 2$ and x is sufficiently large. As x increases, the equilibrium level of PEO also increases because a higher level of PEO is necessary to compensate for firm 1's loss of competitive advantage through knowledge transfer. Our finding therefore suggests, contrary to the standard intuition, that PEO arrangements benefit consumers and the society when equilibrium levels of PEO are relatively high. We have also found that PEO is more likely to hurt consumers as n decreases, suggesting that the antitrust authority should put more careful eyes on PEO in industries that have a smaller number of competitors.

Consider an antitrust authority whose objective is to maximize total surplus or consumer surplus by imposing restrictions on the level of PEO that firms 1 and 2 can choose. In

our base model, the authority would either impose no restrictions on PEO or completely prohibit PEO, where the authority is more likely to prohibit PEO when the industry is less competitive (in the sense of smaller number of firms) and firm 1's initial competitive advantage (captured by x) is relatively small. An extension of our model to differentiated oligopoly has indicated that partial permission of PEO can be the optimal antitrust policy. Under differentiated oligopoly, firms 1 and 2 may prefer a level of PEO that is higher than the minimum PEO for knowledge transfer. In this case, the authority can increase welfare by imposing the minimum PEO for knowledge transfer as the maximum permissible level of PEO. Firms 1 and 2 may prefer merger under a certain range of parameterizations, where the authority's optimal action is to prohibit merger and encourage PEO arrangements.

6 Conclusion

We have explored oligopoly models in which the link between knowledge transfer and PEO endogenously determines the equilibrium level of PEO. Previous theoretical models of PEO, in which the levels of PEO are exogenously assumed, have shown that PEO arrangements would decrease welfare by reducing the degree of competition in the industry. We have demonstrated that PEO arrangements can increase total surplus and/or consumer surplus because they induce knowledge transfer. Our analysis has indicated that antitrust authorities should either prohibit, partially permit, or permit PEO to maximize total surplus and/or consumer surplus, and identified conditions under which one of these three relevant policy options is optimal.

Appendix (incomplete)

Proofs for Section 3

We first establish the following claim (see Assumption 1 in the text).

Claim 1: Each firm i produces a strictly positive amount in the equilibrium of the Stage 3 subgame for all (θ, k) where $\theta \in [0, \frac{1}{2}] \Leftrightarrow x < \frac{a-c}{2}$.

Proof: Each firm i's profit function given by (1) in the text is strictly concave in q_i when $P(Q) = a - dQ$. Suppose each firm i produces a strictly positive amount in the equilibrium of the Stage 3 subgame for a given (θ, k) where $\theta \in [0, \frac{1}{2}]$. The necessary first order conditions are:

$$\begin{aligned}
\frac{\partial \pi_1(\theta,k,q_1,q_2,q_3)}{\partial q_1} &\equiv [P(Q)-(c-x)]+P'(Q)q_1+\theta P'(Q)q_2=0,\\
\frac{\partial \pi_2(\theta,k,q_1,q_2,q_3)}{\partial q_2} &\equiv (1-\theta)[P(Q)-(c-kx)+P'(Q)q_2]=0,\\
\frac{\partial \pi_3(\theta,k,q_1,q_2,q_3)}{\partial q_3} &\equiv P(Q)-c+P'(Q)q_3=0.
\end{aligned}$$

Substituting $P(Q)=a-dQ$ and $P'(Q)=-d$ and solving these equations imply (2) in the text. For any given (θ,k) where $\theta\in[0,\frac{1}{2}]$, $q_3^*(\theta,k)>0 \Rightarrow q_i^*(\theta,k)>0$ for all $i=1,2,3$. Also, $q_3^*(\theta,k)>0$ for all (θ,k) where $\theta\in[0,\frac{1}{2}] \Rightarrow x<\frac{a-c}{2}$. This proves the necessity part of the claim, and the proof of sufficiency is analogous. *Q.E.D.*

In the paragraph preceding Lemma 1 we claimed that $\pi_1^*(\theta,k)$ is strictly convex in k. Subsequently, using this claim we argued that either firm 1 transfers its entire knowledge or does not transfer any knowledge at all. Below we state and prove this claim.

Claim 2: Firm 1's equilibrium profit in the stage 3 subgame, $\pi_1^*(\theta,k)$, is strictly convex in k and attains its maximum at $k=0$ or $k=1$.

Proof: Using the first-order conditions (stated in the proof of claim 1) we can write $\pi_1^*(\theta,k)$ as

$$\pi_1^*(\theta,k)=d(q_1^*(\theta,k)^2+\theta q_1^*(\theta,k)q_2^*(\theta,k)+\theta q_2^*(\theta,k)^2).$$

Differentiating $\pi_1^*(\theta,k)$ twice and simplifying subsequently we get:

$$\begin{aligned}
\frac{d\pi_1^{*2}(\theta,k)}{dk^2} &= 2d(1-\theta)(\frac{dq_1^*(\theta,k)}{da})^2+2d\theta[(\frac{dq_1^*(\theta,k)}{da})^2+(\frac{dq_2^*(\theta,k)}{da})^2+\frac{dq_1^*(\theta,k)}{da}\frac{dq_2^*(\theta,k)}{da}]\\
&= 2d(1-\theta)(\frac{dq_1^*(\theta,k)}{da})^2+2d\theta[(\frac{dq_1^*(\theta,k)}{da}+\frac{dq_2^*(\theta,k)}{da})^2-\frac{dq_1^*(\theta,k)}{da}\frac{dq_2^*(\theta,k)}{da}]>0
\end{aligned}$$

where the strict inequality follows from noting that $d>0$, $\frac{dq_1^*(\theta,k)}{dk}=\frac{-(1+2\theta)x}{d(4-\theta)}<0$ and $\frac{dq_2^*(\theta,k)}{dk}=\frac{3x}{d(4-\theta)}>0$.

Now we turn to prove the second part of the claim. Since k lies in a compact interval $[0,1]$ and $\pi_1^*(\theta,k)$ is continuous in k it is immediate that $\pi_1^*(\theta,k)$ attains its maximum for some $k=k^*\in[0,1]$. Suppose $k^*\in(0,1)$. Then (a) $\frac{d\pi_1^*(\theta,k)}{dk}=0$ and (b) $\frac{d\pi_1^*(\theta,k)^2}{dk^2}<0$ must hold. However, (b) cannot hold which imply $k^*\notin(0,1)$. *Q.E.D.*

Proof of Proposition 1: Define $\Delta(\theta,n)\equiv\pi_1^*(\theta,1,n)-\pi_1^*(\theta,0,n)$, where we find that $\Delta(\theta,n)=\frac{x}{d(n+3-\theta)^2}[[(n+1)\theta-(2n+3)]x-[\theta^2-(n+5)\theta+2](a-c)]$. Firm 1 transfers

its knowledge to firm 2 in the equilibrium if and only if $\Delta(\theta,n)\geq 0$ holds. We have that $\Delta(0,n)=-\frac{x}{d(n+3)^2}[2(a-c)+(2n+3)x]<0$, and $\frac{d}{d\theta}\Delta(\theta,n)=\frac{x}{d(n+3-\theta)^2}[(a-c-x)(n^2-(n+1)\theta))+(2n^2-3)x+8(a-c)n+11(a-c)]$. We find that $\Delta'(\theta)>0$ holds for all $n\geq 2$, and, when $n=1$, $\Delta'(\theta)=\frac{2x[10a-10c-x-\theta(a-c-x)]}{d(4-\theta)^3}>\frac{2x(10-\theta)(a-c-x)}{d(4-\theta)^3}>0$. Also, $\Delta(\frac{1}{2},n)=\frac{x}{d(2n+5)^2}[(2n+1)(a-c)-(6n+10)x]$, where $\Delta(\frac{1}{2},n)>(=,<)\ 0$ holds if $x<(=,>)\ x_1(n)\equiv\frac{2n+1}{6n+10}(a-c)$, and $x_1(n)<\bar{x}$ holds for all $n\geq 1$. This implies that for any given $x\in(0,x_1(n))$, there exists a unique value $\hat{\theta}(x,n)$ such that $\Delta(\theta,n)=0\Leftrightarrow\theta=\hat{\theta}(x,n)$. This proves (i) and (ii) except for the property that $\hat{\theta}(x,n)$ is strictly increasing in x and strictly decreasing in n for all $x\in(0,x_1(n))$ and $n\geq 1$. Suppose $x\in(0,x_1(n))$. We have that $\Delta(\theta,n)=0\Leftrightarrow x=\frac{\theta^2-(n+5)\theta+2}{(n+1)\theta-(2n+3)}(a-c)\equiv\tilde{x}(\theta,n)$, where $\frac{d}{d\theta}\tilde{x}(\theta,n)=\frac{(n+1)\theta^2-2(2n+3)\theta+2n^2+11n+13}{((n+1)\theta-(2n+3))^2}(a-c)>0$ and $\frac{d}{dn}\tilde{x}(\theta,n)=\frac{(4-\theta)(1-\theta)^2}{[(n+1)\theta-(2n+3)]^2}>0$ for all $\theta\in[0,\frac{1}{2}]$ and $n\geq 1$. This implies the desired property of $\hat{\theta}(x,n)$. *Q.E.D.*

Proof of Lemma 1: First let $k=1$. We find that $\frac{\partial}{\partial\theta}\pi_{12}^*(\theta,1,n)=-\frac{(n-1+\theta)(a-c+(n+1)x)^2}{d(n+3-\theta)^3}\leq 0$ for all $\theta\in[0,\frac{1}{2}]$ where the inequality holds with strict inequality for all $\theta\in(0,\frac{1}{2}]$. Also, for any given $n\geq 1$, $q_i^*(\theta,1,n)$ is constant for all $\theta\in(\frac{1}{2},1]$, and hence $\pi_{12}^*(\theta,1,n)$ is also constant for all $\theta\in(\frac{1}{2},1]$. We find that $\pi_{12}^*(\frac{1}{2},1,n)-\pi_{12}^*(\theta',1,n)=\frac{(2n^2+4n-1)(a-c+(n+1)x)^2}{(2n+5)^2(n+2)^2d}>0$ for any $\theta'\in(\frac{1}{2},1]$. This proves the lemma for $k=1$.

Next let $k=0$. We find that $\frac{\partial}{\partial\theta}\pi_{12}^*(\theta,0,n)=-\frac{(a-c-x)[(n-1+\theta)(a-c)+x(n^2+(3-\theta)n+4-2\theta)]}{d(4-\theta)^3}<0$ for all $\theta\in[0,\frac{1}{2}]$. We also find that $\pi_{12}^*(\theta,0,n)$ is constant for all $\theta\in(\frac{1}{2},1]$, and that for any $\theta'\in(\frac{1}{2},1]$, $\pi_{12}^*(0,0,n)-\pi_{12}^*(\theta',0,n)=\frac{(a-c-x)}{(n+3)^2(n+2)^2d}[(n^2+2n-1)(a-c)-(3n^2+12n+11)x]$. Hence $\pi_{12}^*(0,0,n)-\pi_{12}^*(\theta',0,n)>(=,<)\ 0$ holds if $x<(=,>)\ x_2(n)\equiv\frac{n^2+2n-1}{3n^2+12n+11}(a-c)$, where $x_2(n)<\bar{x}$ holds for all $n\geq 1$. *Q.E.D.*

Proof of Proposition 2: Note that we have $x_1(n)-x_2(n)=\frac{5n^2+20n+21}{2(3n+5)(3n^2+12n+11)}>0$. Proposition 1 and Lemma 1 together imply that at Stage 1, firms 1 and 2 have following three relevant options if $x\in(0,x_1(n)(a-c)]$: (i) Choose $\theta=\hat{\theta}(x,n)$, the minimum PEO that induces knowledge transfer, (ii) choose $\theta=0$, or (iii) choose $\theta=1$. Consider $\pi_{12}^*(\hat{\theta}(x,n),1,n)-\pi_{12}^*(0,0,n)\equiv g1(x,n)$. By definition of $\tilde{x}(\theta,n)$ (see the proof of Lemma 1), we have that $\theta=\hat{\theta}(x,n)\Leftrightarrow x=\tilde{x}(\theta,n)$, where $\frac{d}{d\theta}\tilde{x}(\theta,n)>0$ for all $\theta\in[0,\frac{1}{2}]$. Given this, for algebraic convenience we substitute $x=\tilde{x}(\theta,n)$ in $g1(x,n)$ so that it becomes a function of θ. We find that $g1(\tilde{x}(\theta,n),n)=\frac{1-\theta}{d(n+3)^2(n+3-\theta)^2(2n+3-(n+1)\theta)^2}f1(\theta,n)(a-c)^2$ where $f1(\theta,n)\equiv(n^2+4n+5)\theta^5+(n^4+4n^3-7n^2-52n-64)\theta^4+(-2n^5-19n^4-54n^3+9n^2+266n+310)\theta^3+(n^6+14n^5+73n^4+140n^3-106n^2-736n-716)\theta^2+(10n^4+124n^3+559n^2+1090n+777)\theta-(4n^4+44n^3+180n^2+324n+216)$. We now establish the following claim.

Claim 3: There exists a unique value $x_3(n) \in (0, x_1(n))$ such that $g1(x,n) > 0$ if $x \in (x_3(n), x_1(n)]$, $g1(x,n) = 0$ if $x = x_3(n)$, and $g1(x,n) < 0$ if $x \in (0, x_3(n))$.
Proof: We have that $f1(0,n) < 0$ and $f1(\frac{1}{2},n) > 0$. Also, $\frac{d}{d\theta}f1(\theta,n) = 2\theta n^6 + (-6\theta^2 + 28\theta)n^5 + (4\theta^3 - 57\theta^2 + 146\theta + 10)n^4 + (16\theta^3 - 162\theta^2 + 280\theta + 124)n^3 + (5\theta^4 - 28\theta^3 + 27\theta^2 - 212\theta + 559)n^2 + (20\theta^4 - 208\theta^3 + 798\theta^2 - 1472\theta + 1090)n + 25\theta^4 - 256\theta^3 + 930\theta^2 - 1432\theta + 777 > 0$ for all $\theta \in [0, \frac{1}{2}]$ and $n \geq 1$. Hence there exists a unique value $\theta' \in (0, \frac{1}{2})$ such that $f1(\theta,n) > (=, <)\ 0$ if $\theta > (=, <)\ \theta'$. We find that $\tilde{x}(0,n) < 0 < \tilde{x}(\frac{1}{2},n) = x_1(n)$ for all $n \geq 1$, which implies that $\hat{\theta}(0,n) > 0$. we then have that $g1(0,n) = \pi_{12}^*(\hat{\theta}(0,n),1,n) - \pi_{12}^*(0,1,n) < 0$, which implies the result. *Q.E.D.*

Finally, noting that $\pi_{12}^*(1,0,n) = \pi_1^*(0,0,n-1)$, we find that $\pi_{12}^*(\hat{\theta}(\tilde{x}(\theta,n),n),1,n) - \pi_{12}^*(1,0,n) = \frac{[(n+1)\theta^2 - (n+1)(n+4)\theta - 1]^2(\theta + n^2 + 2n - 1)(1-\theta)}{d(n+2)^2[(n+1)\theta - (2n+3)]^2[\theta - (n+3)]^2}(a-c)^2 > 0$. This and Claim 3 along with Lemma 1 imply Proposition 2 except for the property of $x_3(n)$. Then Claim 4 below completes the proof.

Claim 4: $x_3(n)$ is strictly increasing in n for all $n \geq 1$.
Proof: First note that $g1(0,n) < 0$ for all $n \geq 1 \Rightarrow f1(\hat{\theta}(0,n),n) < 0$ for all $n \geq 1$. We find by using mathematical software Maple that $f1(\hat{\theta}(0,n),n+1) - f1(\hat{\theta}(0,n),n) = -\frac{1}{2}n^6 - \frac{13}{2}n^5 - \frac{59}{2}n^4 - 50n^3 + \frac{63}{2}n^2 + 217n + 205 + (\frac{1}{2}n^5 + 4n^4 + \frac{23}{2}n^3 + \frac{13}{2}n^2 - 20n - 19)(n^2 + 10n + 17)^{\frac{1}{2}} > 0$ for all $n \geq 1$. Furthermore, we have that $\frac{d^2}{dnd\theta}f1(\theta,n) = 12\theta n^5 + 5(-6\theta^2 + 28\theta)n^4 + 4(4\theta^3 - 57\theta^2 + 146\theta + 10)n^3 + 3(16\theta^3 - 162\theta^2 + 280\theta + 124)n^2 + 2(5\theta^4 - 28\theta^3 + 27\theta^2 - 212\theta + 559)n + 20\theta^4 - 208\theta^3 + 798\theta^2 - 1472\theta + 1090 > 0$ for all $\theta \in [0, \frac{1}{2}]$ and $n \geq 1$. This implies the result. *Q.E.D.*

Proof of Proposition 3: Consumer surplus can be expressed as $CS(q_1,q_2,q_3) \equiv \frac{1}{2}d(q_1 + q_2 + q_3)^2$. Consumer surplus in the equilibrium of the Stage 3 subgame represented by (θ,k) is given by $CS(q_1^*(\theta,k), q_2^*(\theta,k), q_3^*(\theta,k)) \equiv CS^*(\theta,k)$. Consider $CS^*(\hat{\theta}(x),1) - CS^*(0,0) \equiv g2(x)$. As in the proof of Proposition 1, we substitute $x = \tilde{x}(\theta)$ in $g2(x)$. We find that $g2(\tilde{x}(\theta)) = \frac{1-\theta}{32d(5-2\theta)^2(4-\theta)^2}f2(\theta)(a-c)^2$ where $f2(\theta) \equiv (3\theta^2 - 17\theta + 8)(5\theta^3 - 28\theta^2 - \theta + 96)$, and that $f2(\theta) < 0$ for all $\theta \in [0, \frac{1}{2}]$. Also, $CS^*(1,0) - CS^*(0,0) = -\frac{(a-c-x)[17(a-c)+7x]}{288d} < 0$ for all $x \in (0, \bar{x})$. This implies the result. *Q.E.D.*

Proof of Proposition 3: Total surplus in the equilibrium of the Stage 3 subgame represented by (θ,k) is $CS^*(\theta,k) + \pi_1^*(\theta,k) + \pi_2^*(\theta,k) + \pi_3^*(\theta,k) \equiv TS^*(\theta,k)$. Consider $TS^*(\hat{\theta}(x),1) - TS^*(0,0) \equiv g3(x)$. We find that $g3(\tilde{x}(\theta)) = \frac{(1-\theta)(5-\theta)}{32d(5-2\theta)^2(4-\theta)^2}f3(\theta)(a-c)^2$ where $f3(\theta) \equiv 25\theta^4 - 242\theta^3 + 477\theta^2 + 536\theta - 256$, and that $f3(\theta) > (=, <)\ 0$ holds if $\theta < (=, >)\ 0.375$ where

$xx(0.375, a, c) = 0.0260(a - c)$. Also, $TS^*(1,0) - TS^*(0,0) = -\frac{(a-c-x)[7(a-c)-31x]}{288d} > 0$ $(=, <)$ 0 holds if $x > (=, <)$ 0.0226. This implies the result. *Q.E.D.*

Proofs for Section 4

In the paragraph preceding subsection 4.2.1 we claimed that as in the homogeneous products case, either firm 1 transfers its entire knowledge or it does not transfer any knowledge at all in the equilibrium of the stage 2 subgame. To prove the claim it suffices to show that Firm 2's equilibrium profit in the stage 3 subgame is strictly convex in k.

Claim 3: $\pi_1^*(\theta, k)$, given by (7), is strictly convex in k.
Proof: Using (5), (6) and the relevant first-order conditions we can write $\pi_1^*(\theta, k)$ in (7) as

$$\pi_1^*(\theta, k) = (1-\theta)q_1^*(\theta, k)^2 + \theta(q_1^*(\theta, k)^2 + bq_1^*(\theta, k)q_2^*(\theta, k) + q_2^*(\theta, k)^2).$$

Differentiating $\pi_1^*(\theta, k)$ twice and simplifying subsequently we get:

$$\frac{d\pi_1^{*2}(\theta, k)}{dk^2} = 2[(1-\theta)(\frac{dq_1^*(\theta, k)}{dk})^2 + \theta(\frac{dq_1^*(\theta, k)}{dk} + \frac{dq_2^*(\theta, k)}{dk})^2 - \theta(2-b)\frac{dq_1^*(\theta, k)}{dk}\frac{dq_2^*(\theta, k)}{dk}] > 0$$

where the strict inequality follows from noting that $2 - b > 0$, $\frac{dq_1^*(\theta,k)}{dk} = \frac{-(b+\frac{2b\theta}{2-b})x}{4+2b-(2+\theta)b^2} < 0$ and $\frac{dq_2^*(\theta,k)}{dk} = \frac{(2+b)x}{4+2b-(2+\theta)b^2} > 0$. *Q.E.D.*

Proof of Lemma 1' (b=0.6): Define $\Delta(\theta) \equiv \pi_1^*(\theta, 1) - \pi_1^*(\theta, 0)$, where we find that $\Delta(\theta) = \frac{5}{49}\frac{(-1800\theta^2+21728\theta-16905)x-7(549\theta^2-6370\theta+1470)(a-c)}{(112-9\theta)^2}$. Firm 1 transfers its knowledge to firm 2 in the equilibrium if and only if $\Delta(\theta) \geq 0$ holds. We have that $\Delta(0) = -\frac{75x[14(a-c)+23x]}{12544} < 0$, and $\Delta'(\theta) = \frac{(152089-14832\theta)x+7(49070-4689\theta)(a-c)}{7(112-9\theta)^3} > 0$. Also, $\Delta(\frac{1}{2}) = \frac{x[44177(a-c)-25964x]}{453005} > \frac{5951(a-c)^2}{166410} > 0$ where the first inequality is implied by Assumption 1'. This implies that for any given $x \in (0, 1.17(a-c))$, there exists a unique value $\hat{\theta}(x)$ such that $\Delta(\theta) = 0 \Leftrightarrow \theta = \hat{\theta}(x)$. Also, we have that $\Delta(\theta) = 0 \Leftrightarrow x = \frac{7(549\theta^2-6370\theta+1470)}{-1800\theta^2+21728\theta-16905}(a-c) \equiv \tilde{x}(\theta)$, where $\tilde{x}'(\theta) = \frac{4998(648\theta^2-18585\theta+106085)}{(1800\theta^2-21728\theta+16905)^2}(a-c) > 0$ for all $\theta \in [0, \frac{1}{2}]$. Furthermore, we find that $\tilde{x}(\theta) = 0 \Leftrightarrow \theta \approx 0.236$ and $\tilde{x}(\theta) = 1.17(a-c) \Leftrightarrow \theta \approx 0.446$. This implies the result. *Q.E.D.*

Proof of Lemma 2' (b=0.6): We have that $\frac{\partial}{\partial\theta}\pi_{12}^*(\theta, 1) = \frac{45[7(a-c)+10x]^3}{7(112-9\theta)^3}(28 - 73\theta) > 0$ $(=, <)$ 0 if $\theta < (=, >)$ $\bar{\theta} \equiv \frac{28}{73} \approx 0.384$. Recall $\theta = \hat{\theta}(x) \Leftrightarrow x = \tilde{x}(\theta)$, and we find $\tilde{x}(\bar{\theta}) = \frac{679462}{960937}(a-c) \approx 0.707(a-c)$. This implies the result. *Q.E.D.*

Proof of Proposition 1' (b=0.6): We have that $\frac{\partial}{\partial\theta}\pi_{12}^*(\theta, 0) = -\frac{45(7(a-c)-3x)}{(112-9\theta)^3}[(73(a-c) - 39x)\theta - (28(a-c) - 108x)]$, where $112 - 9\theta > 0$ holds for all $\theta \in [0, \frac{1}{2}]$, and $7(a-c) - 3x > 0$,

$73(a-c)-39x>0$, and $73(a-c)-39x>28(a-c)-108x$ hold for all $x\in(0,1.17(a-c))$. Noting that $28(a-c)-108x>(=,<)\ 0$ if $x<(=,>)\ 0.259(a-c)$, define $\breve{\theta}(x)$ by $\breve{\theta}(x)=\frac{28(a-c)-108x}{73(a-c)-39x}$ if $x\in(0,0.259(a-c))$, and 0 if $x\in[0.259(a-c),1.17(a-c))$. Then, $\theta=\breve{\theta}(x)$ maximizes the value of $\pi_{12}^*(\theta,0)$ when $\theta\in[0,\frac{1}{2}]$. Also, given Lemmas 1' and 2', we define $\theta^*(x)$ by $\theta^*(x)=\bar{\theta}$ if $x\in(0,0.707(a-c))$ and $\theta^*(x)=\hat{\theta}(x)$ if $x\in[0.707(a-c),1.17(a-c))$. It can be shown by using mathematical software Maple that $\pi_{12}^*(\theta^*(x),1)>\pi_{12}^*(\theta,k)$ for any $\theta\in(\frac{1}{2},1]$ and $k=\{0,1\}$ (computational details are available upon request). Then, Lemma 1', Lemma 2', and Claim 2 (see below) together imply the result.

Claim 2: $\pi_{12}^*(\theta^*(x),1)-\pi_{12}^*(\breve{\theta}(x),0)>0$ for all $x\in(0,1.17(a-c))$.
Proof: We first find that,
(i) if $x\in(0,0.259(a-c))$, $\pi_{12}^*(\theta^*(x),1)-\pi_{12}^*(\breve{\theta}(x),0)=(5/110936)[6482(a-c)+619x]x>0$, and
(ii) if $x\in[0.259(a-c),0.707(a-c))$, $\pi_{12}^*(\theta^*(x),1)-\pi_{12}^*(\breve{\theta}(x),0)=(45/36224)(a-c)^2+(35825/126784)(a-c)x+(82325/1774976)x^2>0$.

Recall from the proof of Lemma 1' that $\theta=\hat{\theta}(x)\Leftrightarrow x=\tilde{x}(\theta)$, where $\tilde{x}(\theta)$ is strictly increasing in θ for all $\theta\in[0,\frac{1}{2}]$. Define $\pi_{12}^{**}(y,1)$ and $\pi_{12}^{**}(y,0)$ by $\pi_{12}^*(\theta^*(\tilde{x}(y)),1)\equiv\pi_{12}^{**}(y,1)$ and $\pi_{12}^*(\breve{\theta}(\tilde{x}(y)),0)\equiv\pi_{12}^{**}(y,0)$. Also, define y_1 and y_{max} by $\tilde{x}(y_1)=0.707(a-c)$ and $\tilde{x}(y_{max})=1.17(a-c)$, where we find that $y_1\approx0.384$ and $y_{max}\approx0.446$. We then find that,
(iii) if $y\in[y_1,y_{max})\Leftrightarrow x\in[0.707(a-c),1.17(a-c))$, $\pi_{12}^{**}(y,1)-\pi_{12}^{**}(y,0)=-(15/128)[(13181827815y^6-332117373420y^5+2589364899282y^4-7940837629788y^3+25636640848735y^2-25642413008800y+4698430464000)/((1800y^2-21728y+16905)^2(112-9y)^2)](a-c)^2$, where we find by using mathematical software Maple that $\pi_{12}^{**}(y,1)-\pi_{12}^{**}(y,0)>0$ holds for all $y\in[y_1,y_{max})$. (i), (ii) and (iii) together imply Claim 2. *Q.E.D.*

Proof of Propositions 2' and 3' (b=0.6): Given (θ,k,q_1,q_2,q_3), total surplus is $ts(\theta,k,q_1,q_2,q_3)\equiv a(q_1+q_2+q_3)-\frac{1}{2}(q_1^2+q_2^2+q_3^2)-0.6(q_1q_2+q_2q_3+q_3q_1)-(c-x)q_1-(c-kx)q_2-cq_3$. Let $ts^*(\theta,k)$ and $cs^*(\theta,k)$ respectively denote total surplus and consumer surplus in the equilibrium of the Stage 3 subgame represented by (θ,k). We have that $ts^*(\theta,k)=ts(\theta,k,q_1^*(\theta,k),q_2^*(\theta,k),q_3^*(\theta,k))$ and $cs^*(\theta,k)=ts^*(\theta,k)-\Sigma_{i=1}^3\pi_i^*(\theta,k)$. Proposition 1' then implies $TS(\theta^*(x))=ts^*(\bar{\theta},1)$ and $CS(\theta^*(x))=cs^*(\bar{\theta},1)$ for all $x\in(0,0.707(a-c))$, and $TS(\theta^*(x))=ts^*(\hat{\theta}(x),1)$ and $CS(\theta^*(x))=cs^*(\hat{\theta}(x),1)$ for all $x\in[0.707(a-c),1.17(a-c))$. Note also that $CS(0)=cs^*(0,0)$ and $TS(0)=ts^*(0,0)$.

First suppose $x\in(0,0.707(a-c))$. We find by using mathematical software Maple that $CS(\theta^*(x))-CS(0)=-(5/2009272832)[21(a-c)-253x][294133(a-c)+136907x]<$

$(=,>)$ 0 if $x < (=,>)$ $0.083(a-c)$, and $TS(\theta^*(x)) - TS(0) = (320595/41005568)(a-c)^2 - (54640665/143519488)(a-c)x + (359622405/2009272832)x^2 < (=,>)$ 0 if $x < (=,>)$ $0.0203(a-c)$.

Next suppose $x \in [0.707(a-c), 1.17(a-c))$. As in the proof of Proposition 1', we substitute $x = \tilde{x}(y)$ in $TS(\theta^*(x))$ and $CS(\theta^*(x))$, and let $TS(\theta^*(\tilde{x}(y))) \equiv TS^*(y)$ and $CS(\theta^*(\tilde{x}(y))) \equiv CS^*(y)$. Recall that $x \in [0.707(a-c), 1.17(a-c)) \Leftrightarrow y \in [y_1, y_{max})$. We find by using mathematical software Maple that $CS^*(y) - CS^*(0) = (15/512)[(16011y^3 - 244734y^2 + 720055y - 164640)(1978623y^3 - 22827910y^2 - 3322053y + 67612160)/((1800y^2 - 21782y + 16905)^2(112-9y)^2)](a-c)^2 > 0$, and $TS^*(y) - TS^*(0) = (-15/512)[(3422829717y^6 + 149538030012y^5 - 4750899194874y^4 + 28881577705036y^3 + 16957636927245y^2 - 9461652065120y + 21251362406400)/\ ((1800y^2 - 21782y + 16905)^2(112-9y)^2](a-c)^2 > 0$ for all $y \in [y_1, y_{max})$. This implies the result. *Q.E.D.*

Proof of Lemma 3: Let $z \equiv \frac{x}{a-c}$. We find that
(i) $\frac{\partial}{\partial \theta} ts^*(\theta, 1) = -(15/7)(a-c)^2(7+10z)[(4130+645\theta)z + 1715 + 546\theta]/(112-9\theta)^3 < 0$ for all $\theta \in [0, \frac{1}{2}]$, and
(ii) $\frac{\partial}{\partial \theta} ts^*(\theta, 0) = -15(a-c)^2(7-3z)[(935-117\theta)z + 245 + 78\theta]/(112-9\theta)^3 < 0$ for all $\theta \in [0, \frac{1}{2}]$. *Q.E.D.*

Proof of Proposition 4 (b=0.6): Let $ts^*(\hat{\theta}(x), 1) - ts^*(0,0) \equiv \psi(x)$. As in the proof of Proposition 1' (b=0.6), we substitute $x = \tilde{x}(y)$ in $\psi(x)$ and find $\psi(\tilde{x}(y)) = -(15/512)$ $[(3422829717y^6 + 149538030012y^5 - 4750899194874y^4 + 28881577705036y^3 + 16957636927245y^2 - 94616152065120y + 21251362406400)/((1800y^2 - 21728y + 16905)^2(112-9y)^2)](a-c)^2$. We find by using mathematical software Maple that there exists a unique threshold value $y_{t0} \approx 0.239$ in the relevant range of y such that $\psi(\tilde{x}(y)) < (=,>)$ 0 if $y < (=,>)$ y_{t0}, where $\tilde{x}(y_{t0}) \approx 0.0118(a-c)$. Through a procedure analogous to the proof of Claim 2 above, we find that $\pi_{12}^*(\hat{\theta}(x), 1) - \pi_{12}^*(\breve{\theta}(x), 0) > 0$ holds for all $x \in [0.0118(a-c), 1.17(a-c))$. This implies that, if the antitrust authority announces $\bar{\theta}(x) = \hat{\theta}(x)$ at Stage 0, firms 1 and 2 choose $\theta = \hat{\theta}(x)$ at Stage 1 in the subsequent equilibrium for all $x \in [0.0118(a-c), 1.17(a-c))$. Then Lemma 3 and Proposition 1' (b=0.6) together imply (ii) and (iii) of the proposition. Now suppose $x \in (0, 0.0118(a-c))$ so that $ts^*(0,0) > ts^*(\hat{\theta}(x), 1)$. From the proof of Proposition 1' (b=0.6), we have $\frac{\partial}{\partial \theta} \pi_{12}^*(0,0) > 0$ for all $x \in (0, 0.0118(a-c))$. This implies that, for all $x \in (0, 0.0118(a-c))$, the antitrust authority must announce $\bar{\theta}(x) = 0$ in order to induce firms 1 and 2 to choose $\theta = 0$ at Stage 1 in the subsequent equilibrium. This implies (i) of the proposition. *Q.E.D.*

Proof of Proposition 1' (b=0.4): Let $\theta \in [0, \frac{1}{2}]$ be given and consider firm 1's decision at Stage 2 in the equilibrium of the Stage 2 subgame. Similar to Lemma 1' (b=0.6), we find that there exists a unique value $\hat{\theta}(x) \in (0, \frac{1}{2})$ such that firm 1 transfers its knowledge to firm 2 if and only if $\theta \in [\hat{\theta}(x), \frac{1}{2}]$. We find that $\theta = \hat{\theta}(x) \Leftrightarrow x = \frac{8(19\theta^2-480\theta+80)}{-75\theta^2+1912\theta-880}(a - c) \equiv \tilde{x}(\theta)$, where $\tilde{x}(\theta)$ is strictly increasing in θ for all relevant range of θ. We have that $\frac{\partial}{\partial\theta}\pi_{12}^*(\theta, 1) = \frac{5[4(a-c)+5x]^2}{4(28-\theta)^3}(6 - 11\theta) > 0$ for all $\theta \in [0, \frac{1}{2}]$, and hence $\pi_{12}^*(\frac{1}{2}, 1) > \pi_{12}^*(\theta, 1)$ holds for all $\theta \in [\hat{\theta}(x), \frac{1}{2})$. It can be shown by using mathematical software Maple that $\pi_{12}^*(1, 1) > \pi_{12}^*(\theta, 1)$ for all $\theta \in [\frac{1}{2}, 1)$ and $\pi_{12}^*(1, 1) > \pi_{12}^*(\theta, 0)$ for all $\theta \in [0, 1]$ (computational details are available upon request). This implies the result. *Q.E.D.*

Proof of Propositions 2' and 3' (b=0.4): We find by using mathematical software Maple that $CS(1)-CS(0) = -(11155/213444)(a-c)^2+(24005/213444)(a-c)x+(170915/3415104)x^2 < (=, >)\ 0$ if $x < (=, >)\ 0.395(a - c)$, and $TS(1) - TS(0) = -(7325/213444)(a - c)^2 + (82255/213444)(a-c)x+(586765/3415104)x^2 < (=, >)\ 0$ if $x < (=, >)\ 0.0858(a-c)$. *Q.E.D.*

Proof of Propositions 4 (b=0.4): We first establish the following claim, which says that Lemma 3 holds in the case of $b = 0.4$ as well.

Claim 3: For any given $k = \{0, 1\}$, $ts^*(\theta, k)$ is strictly decreasing in θ for all $\theta \in [0, \frac{1}{2}]$.
Proof: Let $z \equiv \frac{x}{a-c}$. Note that $x \in (0, 2(a - c)) \Leftrightarrow z \in (0, 2)$. We find that
(i) $\frac{\partial}{\partial\theta}ts^*(\theta, 1) = -(5/16)(a - c)^2(4 + 5z)[(540 + 85\theta)z + 320 + 72\theta]/(28 - \theta)^3 < 0$ for all $\theta \in [0, \frac{1}{2}]$, and (ii) $\frac{\partial}{\partial\theta}ts^*(\theta, 0) = -(5/2)(a-c)^2(4-z)[(95-6\theta)z+40+9\theta]/(28-\theta)^3 < 0$ for all $\theta \in [0, \frac{1}{2}]$. *Q.E.D.*

It can be shown by using mathematical software Maple that $ts^*(\hat{\theta}(x), 1) > ts^*(\theta, k)$ for all $\theta \in (\frac{1}{2}, 1]$ and $k = \{0, 1\}$. We substitute $x = \tilde{x}(y)$ in $ts^*(\hat{\theta}(x), 1)$ and $ts^*(0, 0)$ to find that $ts^*(\hat{\theta}(\tilde{x}(y)), 1)-ts^*(0, 0) = (-5/196)[(545087y^6-15553288y^5-250838624y^4+7407812096y^3+2972174080y^2 - 32346388480y + 5338726400)/((75y^2 - 1912y + 880)^2(y - 28)^2](a - c)^2 < (=, >)\ 0$ if $y < (=, >)\ 0.169$, where $\tilde{x}(0.169) \approx 0.00663(a-c)$. As in the proof of Proposition 4 (b=0.6), we find that $\pi_{12}^*(\hat{\theta}(x), 1) - \pi_{12}^*(\theta, 0) > 0$ holds for all $\theta \in [0, 1]$ for any given $x \in [0.00663(a - c), 2(a - c))$. Hence, if the antitrust authority announces $\bar{\theta}(x) = \hat{\theta}(x)$ at Stage 0, firms 1 and 2 choose $\theta = \hat{\theta}(x)$ at Stage 1 in the subsequent equilibrium for all $x \in [0.00663(a - c), 2(a - c))$. Claim 3 then implies (ii) of the proposition. We also find that $\frac{\partial}{\partial\theta}\pi_{12}^*(0, 0) > 0$ holds for all $x \in (0, 0.00663(a - c))$, which implies that, for all $x \in (0, 0.00663(a - c))$, the antitrust authority must announce $\bar{\theta}(x) = 0$ in order to induce firms 1 and 2 to choose $\theta = 0$ at Stage 1 in the subsequent equilibrium. Claim 3 then implies

(i) of the proposition. *Q.E.D.*

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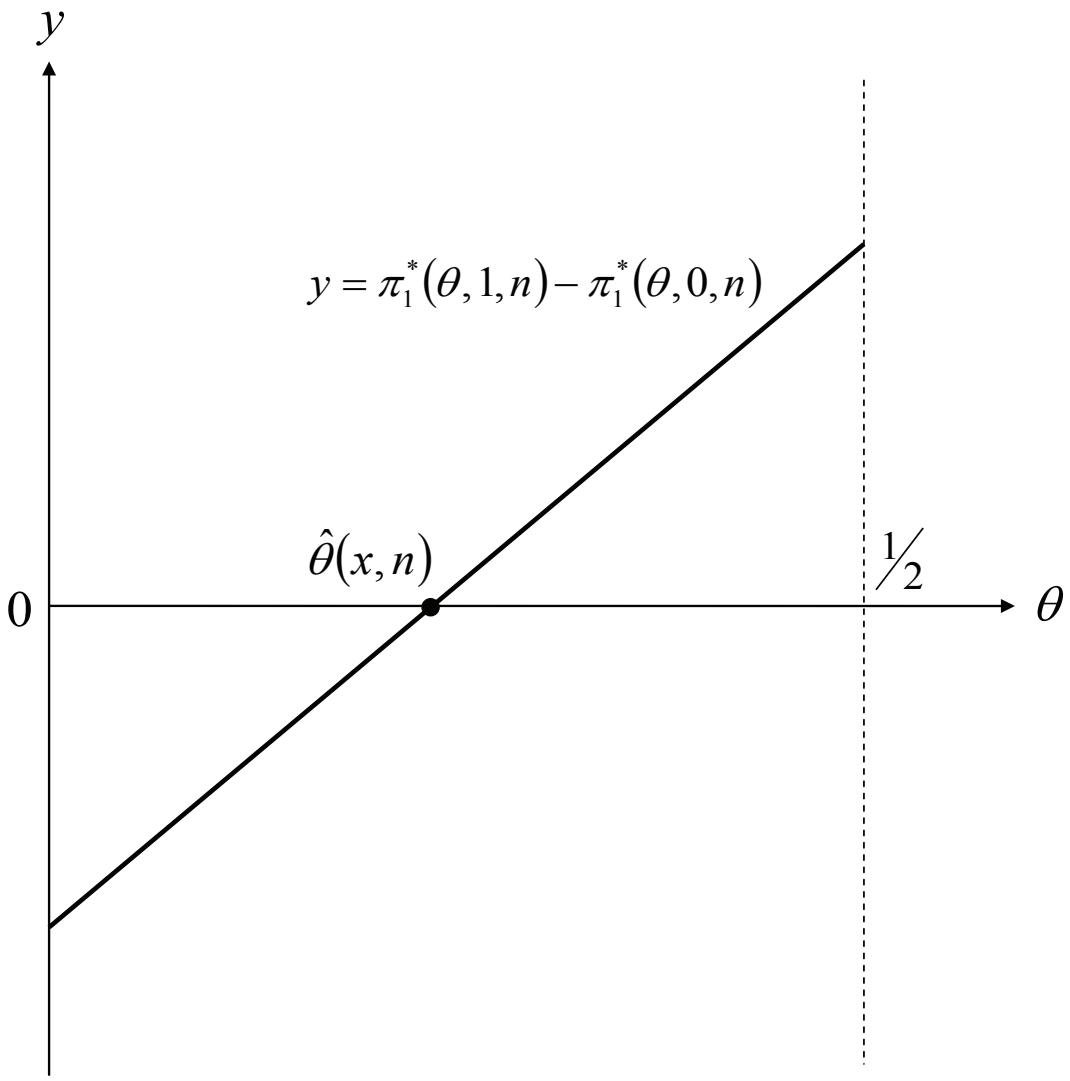

Figure 1

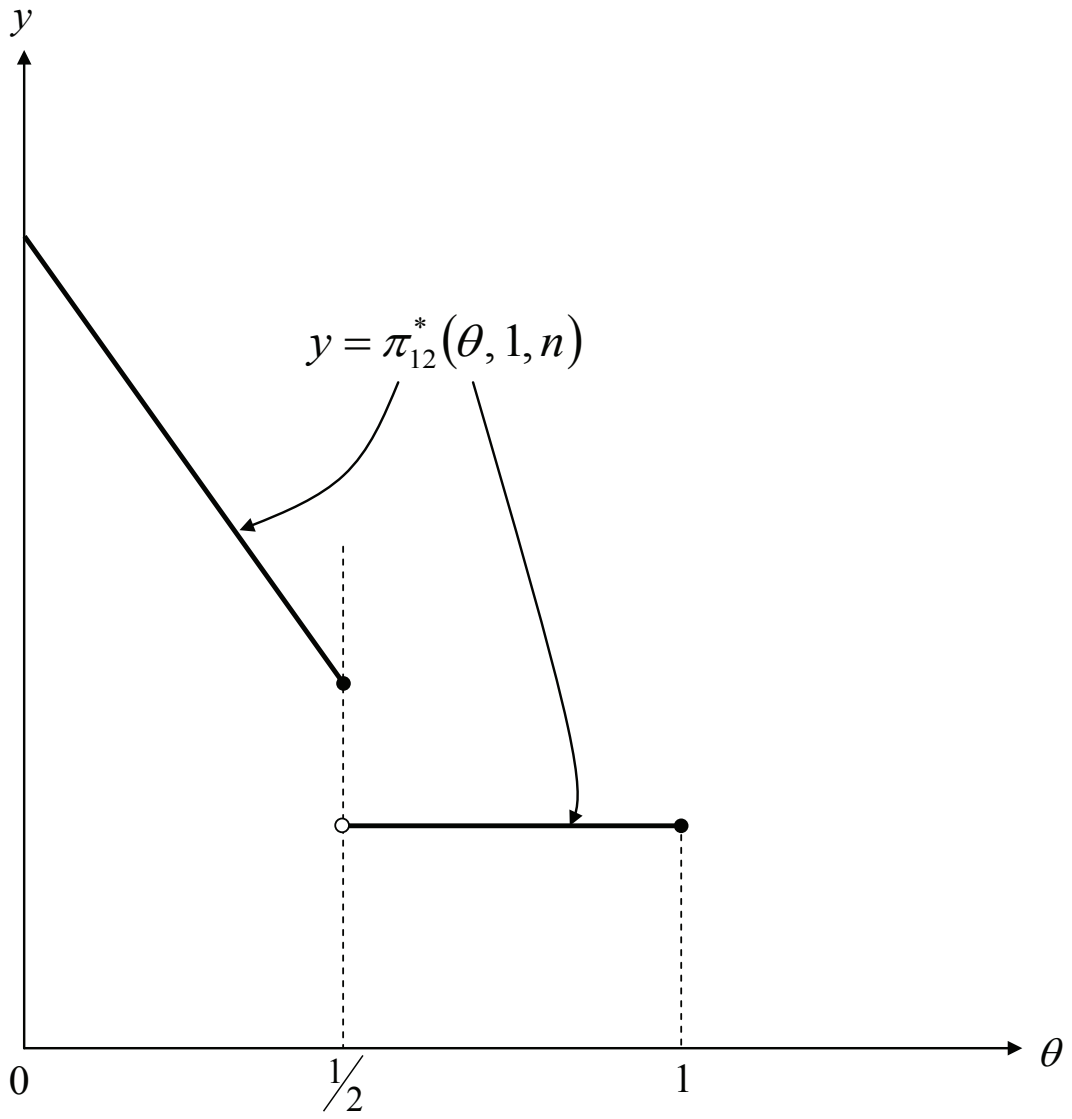

Figure 2-1

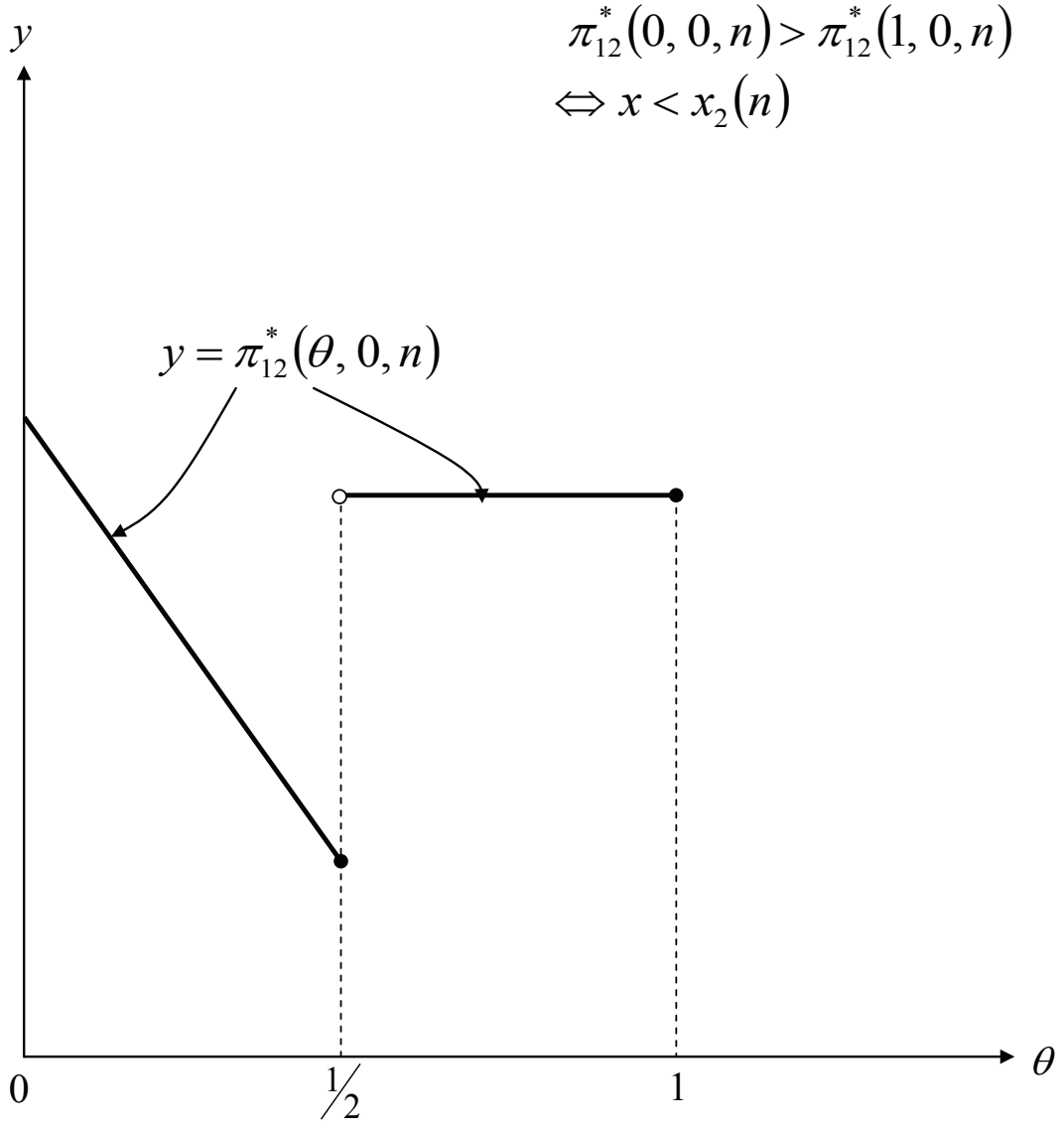
$\pi_{12}^*(0, 0, n) > \pi_{12}^*(1, 0, n)$
$\Leftrightarrow x < x_2(n)$
y
$y = \pi_{12}^*(\theta, 0, n)$
0
$\frac{1}{2}$
1
θ

Figure 2-2

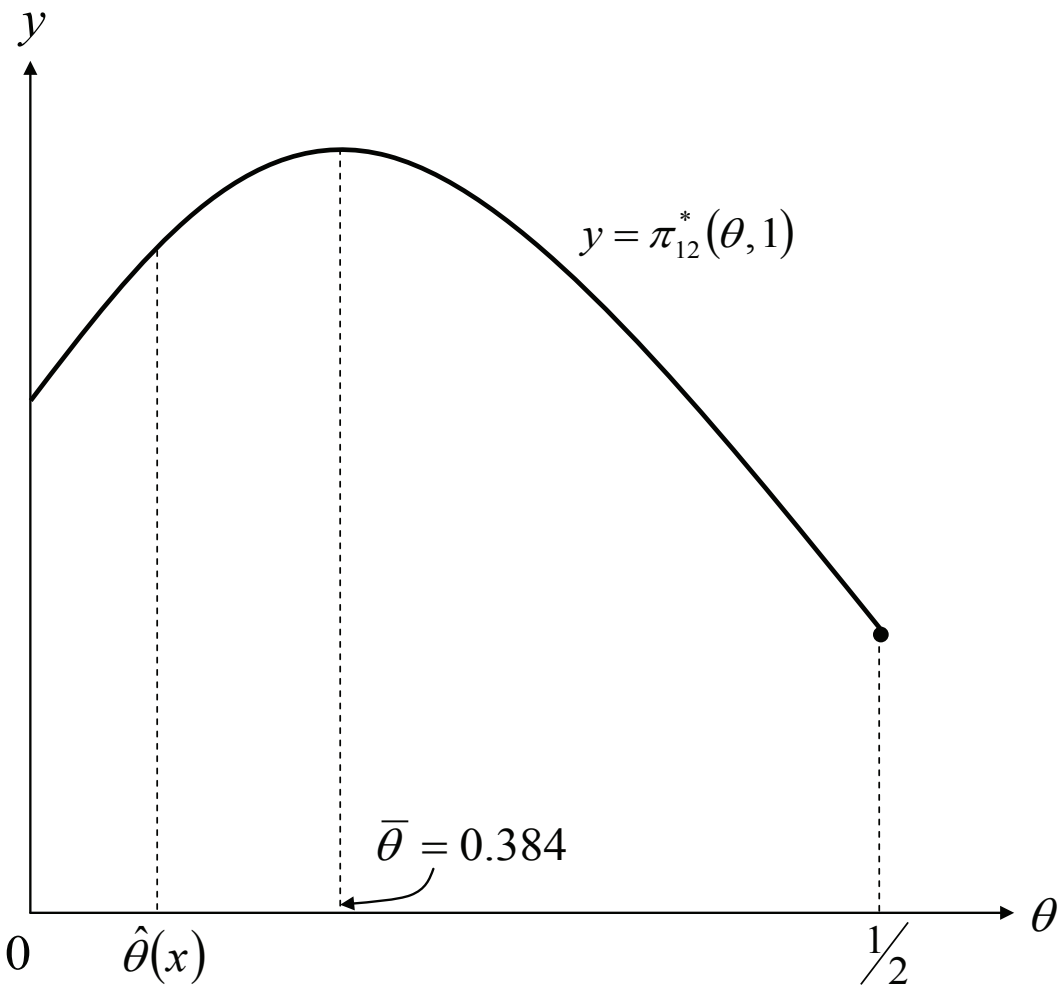

Figure 3