The battle of regimes

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April 2011

Abstract

In spite of long years of high regime stability with a weak opposition and a dominant non-contested ruler, several authoritarian regimes are now dissolving. We show that in contests with incumbent-challenger turnover i) inequality of power may magnify conflicts, ii) more severe conflicts can go together with lower turnover of incumbents, and iii) power can be self-defeating as cost advantages can reduce payoffs. These three propositions of our paper are contrary to the implications of static conflict models. They follow from incorporating positional dynamics into the standard static approach. Such positional dynamics are relevant for competition in battlefields, politics, and market places.

JEL Codes: C73, D72, D74

Keywords: Contests, political stability, war, incumbency advantage

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1 Introduction

Regime stability is associated with a weak opposition and a dominant non-contested ruler. Yet, after serious uprisings several stable authoritarian regimes are now dissolving. In Tunisia what has become known as the Jasmine Revolution has swept president Zine el-Abidine Ben Ali out of office, and finally out of the country. In Egypt huge mass demonstrations have knocked down president Hosni Mubarak. In Libya a similar uprising with more deadly clashes, can put an end to the regime of the Muammar Abu Minyar al-Gaddafi. Also in other Arab countries a seemingly weak opposition has mobilized heavily to overthrow authoritarian rulers of stable regimes.

This paper presents a theory of how fighting can be stimulated, and not mitigated, by the unequal power that characterizes countries with stable regimes. The logic of the battle of regimes, say between an authoritarian ruler and a democratic opposition, is simple. Regime stability is associated with an incumbency advantage that raises the stakes for both the ruler and the opposition. More precisely, once established each regime is more stable the stronger its incumbency edge. Mobilizations are therefore decisive not only for whether the ruler is replaced for a while or not, but also for the relative strengths in future power struggles between the groups that favor each of the two regimes.

If the democratic opposition wins, it can implement its favored policy, and in addition remain in power for long as the nondemocratic groups have a low chance of a reintroducing the authoritarian regime. If the ruler wins, however, it is his group that can (continue to) implement its favored policy and in addition remain in power for a long period as the opposition has a low chance of overthrowing the ruler in every future period. Accordingly, the gains of winning and losses of losing are higher the more stable the two regimes are.

Finally, as the gains of winning are magnified by the low chances of winning, a stronger incumbent raises the stakes for both the ruler and the opposition, even when the increase in power is asymmetrically reserved for the present ruler. The resulting high stakes motivate serious mobilization and fighting.

In other words, since the gain of winning is the value of not losing, the circumstances that lead to a long tenure of the ruler also give special reasons to fight hard to overthrow him—once the opportunity is there. The fighting can be motivated by a strong wish to prevent a continuation of bad governance. This is why the opposition in many countries now feel that so much is at stake.

To study the mechanisms of how power asymmetries affect fighting more generally we incorporate incumbency advantages and other heterogeneities into an otherwise simple theoretical conflict model with repeated battles and positional dynamics. The dynamic extensions we incorporate turn around the results from static conflict models and thus shed new light on real internal conflicts of governance regimes. We explore the logic of contestable power where the fighting is both over political rents, positions and the fighting-edge in the future battles. In all cases the ruler of the regime is more powerful than the opposition, but if the regime opposition today becomes the new ruler tomorrow, it may acquire a quite different fighting advantage than the present ruler enjoys. Power is asymmetric across regimes, both between incumbent and opposition and between incumbents.

While the immediate rent of a contestant is the difference in utilities between his own favored policy and that of his opponents, the advantages in future battles are captured by differences in the unit cost of influence. The effective prize of winning a battle contains both an immediate rent and the value of the incumbency cost advantage and will generally be different for the two groups.

The four papers closest to ours are the contributions by Joan Esteban and Debraj Ray (1999), by William Rogerson (1982), by Stergios Skaperdas and Constantinos Syropoulos (1996), and by Mattias Polborn (1996). Our main addition to this literature lays in combining positional dynamics with power asymmetries between regimes. While Esteban and Ray construct an elegant and general model of multi-group conflicts with heterogenous prizes without dynamics, Skaperdas and Syropoulos consider the problem of achieving cooperation when an early victory to one group improves the group's position in subsequent periods. A status quo bias is also an important part of the innovative paper by Polborn who analyze the strategic timing of attacks. As result of the status quo bias the return to incumbency is high and consequently the incentive for the challenger to try to replace him. Finally, the somewhat overlooked contribution by Rogerson (1982) focuses on insiders and outsiders in a symmetric lobbying game over homogenous prizes where a winning outsider becomes the new insider. We go beyond Rogerson, however, by focusing asymmetric prizes and costs, an asymmetric equilibrium, and by incorporating a more general contest success function.[1]

There is also an article, by Johannes Hörner (2004), that like us, adds a richer structure to the Rogerson setup, but in another dimension. Hörner considers a continuing R&D competition where two firms compete to stay ahead and where the one ahead has lower cost of R&D effort and higher income. In his model incumbency is not a binary feature as the incumbent may stay one, two, or more steps ahead.

Our discussion highlights the many contrasts between a dynamic and static approaches to regime conflicts. Leveling of the battlefield, for instance, does not maximize conflict efforts in our dynamic setting. It does in a static one, since an edge to any one player leaves the prize unchanged, but increases his probability of winning and thus induces the opponent to fight less. In our set-up, in contrast, an edge to the incumbent implies a higher prize of winning as the fighting rents go up. In fact both contestants face higher fighting rents since the gain of winning is the value of not losing. This may help explain why the opposition in many countries feel that so much is at stake when the incumbent gets stronger. As the stakes go up for both as the incumbent becomes stronger, both increase their fighting intensity. Thus while unequal influence dampens the fighting in the standard approach, it generates more fighting in our approach. Likewise, the level of resources wasted in the fighting relative to the total prizes at stake is highest when the contestants are equally strong, in static models, but not in a more dynamic setting. For a survey of models of static rent-seeking contests, see the article by Shmuel Nitzan (1994), and the monograph by Kai Konrad (2009).

[1]More generally we are also inspired by Daron Acemoglu and Jim Robinson's work (2001 and 2006) on political transition and elites, and we build on an expanding basic literature where conflicts are seen as rent seeking contests, going back to Trygve Haavelmo (1954), Gordon Tullock (1980), Jack Hirschleifer (1991), Herschel Grossman (1994), Stergios Skaperdas (1992), Kai Konrad and Stergios Skaperdas (1998), Derek Clark and Christian Riis (1998), and others.

2 Contests with incumbency advantages

There are two contestants (groups), a and b, and two regimes. In the first regime a is the incumbent and b is the challenger. In the other regime b is the incumbent and a the challenger. The first regime can be thought of as an authoritarian regime and the other as a democratic. We are basically interested in the battle between the supporters of the two regimes, abstracting from any turnover of leaders within regimes.

Like Esteban and Ray (1999) we focus on a situation where there is no collective decision rule once the conflict has started, and where groups with opposed interests are willing to spend resources to increase the chances of having it their way. Thus contestants a and b take part in a dynamic game where they in each period t may have a battle with each other. The possibility of a lasting conflict means that in every period with conflict it is a positive probability that the struggle continues for yet another period.

2.1 Basics

The timing of events in each period t is as follows:

1. a and b meet in a simultaneous move contest over who is going to become the ruler.

2. The winner of the contest becomes the ruler and implements his preferred policy.

3. The game moves into period $t+1$ (back to stage 1) with the ruler enjoying a cost advantage.

Winning probabilities

The probability of winning depends positively on own effort and negatively on the opponents effort. We assume that winning the contest requires the relative force, S, to be larger than an threshold. Analogously to probabilistic voting models we assume the threshold to be uncertain. The probability of winning is denoted $\Psi(S)$.

The relative force S_j of the incumbent is given by

$$S_j = \frac{\text{force of } j}{\text{total force of } a \text{ and } b} \tag{1}$$

Throughout we use the convention that capital letters reflect incumbency position and lower case letters reflect a challenger position, such that

$$S_a + s_b = 1 \text{ and } s_a + S_b = 1 \tag{2}$$

The long-lasting consequence of present struggles is the possibility that one of the contestants win a fighting advantage in the coming struggles. These fighting advantages are represented by more influence for a given cost, or lower costs per unit of influence more generally. Parametrically we capture the incumbency advantage by letting the relative force be a function of effort in the following way

$$S_{a,t} = \frac{Y_{a,t}/C_a}{Y_{a,t}/C_a + y_{b,t}/c_b} \tag{3}$$

$$S_{b,t} = \frac{Y_{b,t}/C_b}{Y_{b,t}/C_b + y_{a,t}/c_a} \tag{4}$$

Here Y_j and y_i are the efforts of the incumbent and challenger respectively, while C_j and c_i are the unit costs of force. We assume that $C_a \leq c_b$ and $C_b \leq c_b$ where incumbency advantage for group j implies that $C_j < c_j$. Note that we have no a priori assumptions regarding the cost for contestant a relative to that of contestant b.

The contest success function is a generalization of the Tullock contest success function and the probability of winning is homogeneous of degree zero in force. We have three additional requirement for the relationship $\Psi(.)$

- Anonymity (i.e. symmetry): For all S we require that

  $\Psi(S) = 1 - \Psi(1 - S)$, and consequently that

  $\Psi'(S) = \Psi'(1 - S)$ and

  $\Psi''(S) = -\Psi''(1 - S)$.

- Force pays: $\Psi'(S) > 0$.

- No force implies a sure loss: $\Psi(0) = 0$.

Clearly, the much used Tullock contest success function, $\Psi(S) = S$, is a special case of our set-up.[2]

Pay-offs

The object of the fight is to become the ruler and to implement the preferred policy. The antagonism between the two contestants is captured by this immediate consequences of being in power. When a is in power he implements his optimal incumbency behavior. This choice is valued as U_a for the incumbent a and as u_b for the challenger b.

Sticking to the convention of upper case and lower case letters, the value of U_a reflects a's evaluation of a's own choice as incumbent while u_a reflects a's evaluation of b's choice as incumbent. The difference between the two, D_a, is a's immediate gain of assuming power - his ruler rent. Similarly D_b is b's ruler rent.[3]

$$D_a = [U_a - u_a] \tag{5}$$

$$D_b = [U_b - u_b] \tag{6}$$

In addition to getting the immediate benefits of being the ruler, the winning contestant also starts out as the incumbent in the next period. As incumbent a contestant enjoys an *incumbency advantage* in the sense that the cost of fighting is lower as incumbent as when starting out as challenger. This represent a benefit in the next period's contest.

The value of winning is made up of the immediate gain as ruler and the valuation of starting out as incumbent in the next period.

In regime a the present value of the payoffs, $V_{a,t}$ for incumbent a and $v_{b,t}$ for challenger

[2] See Skaperdas (1996) for a structured discussion of success functions in the n-player case. Our Ψ function satisfies his axiom 1, 2, 3, and 6. In addition to $\Psi(S) = S$ the power function

$$\Psi(S) = \frac{S^\gamma}{(1-S)^\gamma + S^\gamma}$$

satisfies our requirements.

[3] In a multi-group context, Esteban and Ray (1999) use utility differences like these as an indication of inter-group distance. The value of $[U_a - u_a]$ measures the distance from group a to group b, the value of $[U_b - u_b]$ the distance from b to a. The larger these differences, the more antagonism there is between groups and the more polarized are the preferences.

b, can be written as

$$V_{a,t} = \Psi\left(S_{a,t}\right) F_{a,t} + \delta v_{a,t+1} - Y_{a,t} \tag{7}$$

$$v_{b,t} = \left[1 - \Psi\left(S_{a,t}\right)\right] F_{b,t} + \delta v_{b,t+1} - y_{b,t} \tag{8}$$

where

$$F_{a,t} = D_a + \delta\left(V_{a,t+1} - v_{a,t+1}\right) \tag{9}$$

$$F_{b,t} = D_b + \delta\left(V_{b,t+1} - v_{b,t+1}\right) \tag{10}$$

The two last terms of (7) are the fighting effort Y_a and the discounted challenger payoff $\delta v_{a,t+1}$. The first part is the expected prize. The probability of winning is $\Psi(S_{a,t})$. If a wins, he obtains the prize $F_{a,t}$, which is the utility difference D_a plus the excess present value from starting out as incumbent in $t+1$, $\delta\left(V_{a,t+1} - v_{a,t+1}\right)$.

Symmetrically, in regime b the present value of the payoffs, $V_{b,t}$ for incumbent b and $v_{a,t}$ for challenger a, can be written as

$$V_{b,t} = \Psi\left(S_{b,t}\right) F_{b,t} + \delta v_{b,t+1} - Y_{b,t} \tag{11}$$

$$v_{a,t} = \left[1 - \Psi\left(S_{b,t}\right)\right] F_{a,t} + \delta v_{a,t+1} - y_{a,t} \tag{12}$$

Excess return ratios

In order to solve the model we first define *excess return* ratios. These are simply the expected returns relative to the size of the prize. We denote them as H_a and h_a for a as incumbent and challenger respectively and H_b and h_b for b and incumbent and challenger respectively. Hence,

$$H_{a,t+1} \equiv (V_{a,t+1} - \delta v_{a,t+2})/F_{a,t+1} = (\Psi(S_{a,t})F_{a,t+1} - Y_{a,t})/F_{a,t+1} \tag{13}$$

$$h_{a,t+1} \equiv (v_{a,t+1} - \delta v_{a,t+2})/F_{a,t+1} = ((1 - \Psi(S_{b,t}))F_{a,t+1} - Y_{a,t})/F_{a,t+1} \tag{14}$$

$$H_{b,t+1} \equiv (V_{b,t+1} - \delta v_{a,t+2})/F_{b,t+1} = (\Psi(S_{b,t})F_{b,t+1} - Y_{b,t})/F_{b,t+1} \tag{15}$$

$$h_{b,t+1} \equiv (v_{a,t+1} - \delta v_{a,t+2})/F_{b,t+1} = ((1 - \Psi(S_{a,t}))F_{b,t+1} - Y_{b,t})/F_{b,t+1} \tag{16}$$

When subtracting (12) from (7) using (13) and (14) and subtracting (8) from (11) using (15) and (16), we get the prizes as

$$F_{a,t} = D_a + \delta [H_{a,t+1} - h_{a,t+1}] F_{a,t+1} \tag{17}$$

$$F_{b,t} = D_b + \delta [H_{b,t+1} - h_{b,t+1}] F_{b,t+1} \tag{18}$$

Consider contestant a. If he wins, he starts the next contest as the incumbent and anticipates a net return equal to $H_{a,t+1}F_{a,t+1}$. If he loses he starts the next contest as the challenger with an anticipated return equal to $h_{a,t+1}F_{a,t+1}$. The difference $[H_{a,t+1} - h_{a,t+1}] F_{a,t+1}$ is the valuation of the cost advantage. It equals the net gain from starting out as an incumbent rather than as a challenger. When adding the immediate rent D_a, we find the prizes from winning.

In the remainder of the paper we focus on the game's stationary equilibrium. If there is more than one candidate we focus on the one which is in the continuation of a symmetric equilibrium.[4]

Suppressing the time index in (17) and (18) and solving yields the following expressions for the prizes in stationary equilibrium:

$$F_a = \frac{D_a}{1 - \delta [H_a - h_a]} \geq D_a \tag{19}$$

$$F_b = \frac{D_b}{1 - \delta [H_b - h_b]} \geq D_b \tag{20}$$

[4]By "continuation of symmetric equilibrium" we mean that the equilibrium is found by the continuous 2×1 valued function who maps parameters to equilibrium pairs (F_a, F_b) and who maps each and every symmetric parameter configuration into a symmetric equilibrium.

2.1.1 Derivation of equilibrium effort

When a is incumbent, the first order conditions for the choice of efforts follows from (7) and (8) as

$$\frac{\partial V_{a,t}}{\partial Y_{a,t}} = 0 => S_{a,t}\left(1 - S_{a,t}\right) F_{a,t}\Psi'\left(S_{a,t}\right) = Y_{a,t} \tag{21}$$

$$\frac{\partial v_{b,t}}{\partial y_{b,t}} = 0 => S_{a,t}\left(1 - S_{a,t}\right) F_{b,t}\Psi'\left(S_{a,t}\right) = y_{b,t} \tag{22}$$

The second order conditions[5] can be written as

$$-\frac{2}{S_a} < \frac{\Psi''\left(S_a\right)}{\Psi'\left(S_a\right)} < \frac{2}{1 - S_a} \tag{23}$$

In addition, we impose the condition that there exists an equilibrium in pure strategies with positive expected return - i.e. the net rate of returns are positive in equilibrium:

$$H_a \equiv \Psi\left(S_a\right) - Y_a/F_a > 0 \text{ and } h_b \equiv \left(1 - \Psi\left(S_a\right)\right) - y_b/F_b > 0 \tag{24}$$

Using (2) it follows that in a Nash equilibrium where both (21) and (22) are satisfied, relative force is given by

$$S_a = \frac{F_a/C_a}{F_a/C_a + F_b/c_b} \tag{25}$$

and by symmetry when b is incumbent, the relative force is

$$S_b = \frac{F_b/C_b}{F_b/C_b + F_a/c_a} \tag{26}$$

Thus, each contestant's prize relative to the unit cost of force determines the equilibrium force of the two contestants. A contestant with either a high stake or a low cost (or both)

[5] By differentiating (7) twice with respect to Y_a we get

$$\frac{\partial^2 V_a}{\partial S_a^2} = \frac{F_a S_a^2}{Y_a^2}\left(\Psi''\left(S_a\right)\left(1 - S_a\right)^2 - 2\Psi'\left(S_a\right)\left(1 - S_a\right)\right) < 0$$

which implies that

$$\frac{\Psi''\left(S_a\right)}{\Psi'\left(S_a\right)} < \frac{2}{1 - S_a}$$

The second order condition for b is found in an equivalent way by differentiating (8) twice with respect to y_b.

has a high equilibrium relative force.

From (21) and (22) and from the symmetry of Ψ it follows that the net return ratios H_a and h_b both are determined by the following function $h(.)$

$$h(S) = \Psi(S) - S(1-S)\Psi'(S)\ , \ \frac{\partial h(S)}{\partial S} > 0,\ h(0) = 0,\ h(1) = 1 \tag{27}$$

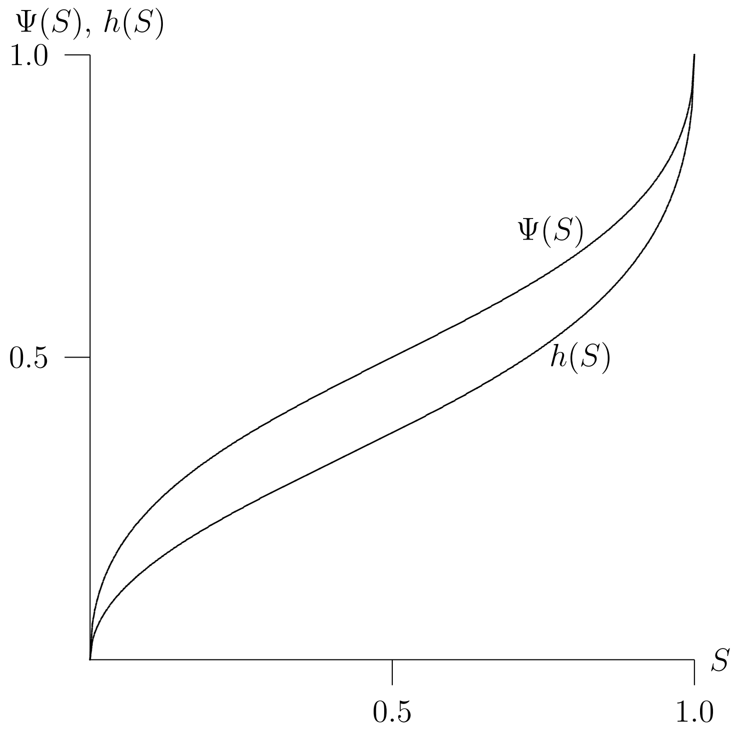
such that

$$H_a = h(S_a) \tag{28}$$

$$h_b = h(s_b) = h(1 - S_a) \tag{29}$$

The function $h(.)$ defines the net return ratio in equilibrium as a function of the equilibrium relative force S. An illustration of the relationship between S, Ψ, and h is given in Figure 1. By definition the h function is below Ψ for all S. The h function

Figure 1: Relationship between S, $\Psi(S)$, and $h(S)$



reaches unity when S is one, since the prize is then won with certainty and for free. The

h function is zero when S is zero since the prize cannot be won and no efforts are wasted.[6]

From (27), (25), (26), (19), and (20) we can note the following, quite natural, homogeneity property.

Homogeneity: What matters for the choice of conflict efforts is the costs of influence relative to the gain. A proportional increase of C_a, c_a, and D_a has no effect for the equilibrium choice of effort for a and hence neither for b. Moreover a $k\%$ increase in costs C_a and c_a has the same effect on equilibrium efforts as a $k\%$ reduction in rents D_a. This implies that, in the following, when we discuss weak versus strong groups we could equally well discuss groups with moderate versus high rents.

2.2 Unequal power

We are now ready to prove the already anticipated result regarding one-sided incumbency advantage.

Proposition 1 *Compared to the case without incumbency advantage an incumbency edge to one contestant raises the prize for both contestants of the struggle. The effects on the prizes are the same irrespective of who gets the incumbency advantage.*

Proof. By differentiating (25) and (26) we get

$$\mathrm{d}S_i = S_i\left(1 - S_i\right)\left(\frac{\mathrm{d}F_i}{F_i} - \frac{\mathrm{d}F_j}{F_j} - \frac{\mathrm{d}C_i}{C_i}\right),\ i \neq j$$

By differentiating (19), and (20), using (28) and (29), we get

$$\frac{\mathrm{d}F_i}{F_i} = \frac{1}{1 - \delta\left[h\left(S_i\right) - h\left(1 - S_j\right)\right]}\delta\left(h'\left(S_i\right)\mathrm{d}S_i + h'\left(1 - S_j\right)\mathrm{d}S_j\right),\ i \neq j$$

[6] In the case where the prizes are fixed and equal to $F_a = F_b = F$, the sum of $h\left(S\right)$ and $h\left(1 - S\right)$ is negatively associated with the social waste of the fight $Y_a + y_b$. We can then define the total waste ratio as total resources spent on the conflict relative to the prize. The waste ratio function $\omega\left(S\right)$ is given by

$$\omega\left(S\right) = 1 - \left(h\left(S\right) + h\left(1 - S\right)\right) = 2S\left(1 - S\right)\Psi'\left(S\right) > 0 \text{ when } S \in \langle 0, 1\rangle$$

Given that Ψ'' is not too large, the waste ratio ω has its maximum at $S = 1/2$.

$\omega\left(S_a\right)$ has a local extrema when $\Psi''/\Psi' = \left(2S_a - 1\right)/\left(S_a - S_a^2\right)$. Given the symmetry of the Ψ-function $S_a = 1/2$ obviously satisfies the first order condition and if Ψ is S-shaped ($\Psi'' < 0$ to the right of $S_a = 1/2$ i.e. a strong decreasing returns to effort) $S_a = 1/2$ is a unique and global maximum. The function $\Psi\left(S\right) = 1/2 - k/4\left(\ln\left(1 - S\right) - \ln\left(S\right)\right)$ (where $\Psi'\left(1/2\right) = k$) happens to have exactly the curvature that makes social waste independent of S, as in this case $\omega\left(S\right) = k/2$ for all S.

Evaluating in a point without incumbency advantage (i.e. where $C_a = c_a$ and $C_b = c_b$ and where, as a result, $S_a + S_b = 1$) yields

$$\frac{\mathrm{d}F_i}{F_i} = -S_a S_b \delta h'(S_i) \left(\frac{\mathrm{d}C_a}{C_a} + \frac{\mathrm{d}C_b}{C_b} \right) \tag{30}$$

$$\mathrm{d}S_i = -S_a^2 S_b^2 \delta (h'(S_i) - h'(1 - S_i)) \left(\frac{\mathrm{d}C_a}{C_a} + \frac{\mathrm{d}C_b}{C_b} \right) - S_a S_b \frac{\mathrm{d}C_i}{C_i} \tag{31}$$

Noting that incumbency advantage to one contestant implies that either $\mathrm{d}C_a/C_a$ or $\mathrm{d}C_b/C_b$ is negative, the proposition follows. ■

An incumbency edge that favors one contestant only, makes the stakes higher for both. The reason is that the prize of winning is high when the payoff of becoming the incumbent is high and/or when the payoff of becoming the challenger is low. The winning contestant obtains the *difference* between these as part of the prize.

The proposition above establishes that the stakes increase for both with an incumbency advantage irrespective of who gets the advantage. Regime stability, measured as the average survival probability of the incumbent, and the extent of effort can go either way. If the payoff structure is not too uneven, however, an incumbency advantage to one contestant may increase efforts by both at the same time as regime stability increases. Hence we also have the following proposition.

Proposition 2 *If both contestants are equally strong, resource use and regime stability go up as one contestant gets an incumbency advantage. If the ruler in regime a is particularly strong ($S_a \approx 1$), resource use goes down for both the ruler and the opposition as the ruler enjoys a stronger incumbency advantage. Resource use goes up for both contestants, however, if challenger b gets the prospect of an incumbency advantage.*

Proof. When both contestants are equally strong it will be the case that $S_a = S_b = 1/2$ and it follows that $\Psi'' = 0$. It then follows from the first order conditions (21) and (22) that

$$\frac{\mathrm{d}Y_i}{Y_i} = \frac{\mathrm{d}y_i}{y_i} = \frac{\mathrm{d}F_i}{F_i}$$

Combined with the results from the previous proposition we know that resource use goes up.

Formally regime stability is given by

$$\frac{1-\Psi\left(S_b\right)}{2-\Psi\left(S_a\right)-\Psi\left(S_b\right)}\Psi\left(S_a\right)+\frac{1-\Psi\left(S_a\right)}{2-\Psi\left(S_a\right)-\Psi\left(S_b\right)}\Psi\left(S_b\right)$$

(where $\frac{1-\Psi(S_b)}{2-\Psi(S_a)-\Psi(S_b)}$is the frequency of a as incumbent and $\frac{1-\Psi(S_a)}{2-\Psi(S_a)-\Psi(S_b)}$is the frequency of b as incumbent). That stability goes up follows directly from (31). As $h'\left(S_i\right)-h'\left(1-S_i\right)=0$ when $S_a=S_b=1/2$ it follows that S_a goes up while S_b is unchanged when C_a goes down.

With uneven positions these results are altered. When $S_a\to 1$ it follows from (30) and (31) that the Fs and the Ss are unaffected by a change in the Cs. It thus follows from (25) and (26) that

$$\frac{\mathrm{d}Y_a}{Y_a}=\frac{\mathrm{d}y_b}{y_b}=\frac{\mathrm{d}C_a}{C_a}\text{ and }\frac{\mathrm{d}Y_b}{Y_b}=\frac{\mathrm{d}y_a}{y_a}=-\frac{\mathrm{d}C_b}{C_b}$$

■

The proposition establishes that more fighting from both contestants can go together with higher regime stability. As both contestants raise their conflict spending equally, the probability of winning for the contestant that obtains the edge increases and the average regime stability goes up. The main result is that an incumbency edge to one group raises the stakes for both groups since it is equally important to obtain the incumbency edge as it is to prevent the opponent from getting it. The prize of winning goes up for both groups. For the incumbency group it goes up as the payoff of winning increases. For the challenger group it goes up as the payoff of losing declines. With higher stakes both fight harder either to win the edge for future battles, or to prevent the opponent from winning it. As a result the amount of resources wasted in the conflict increases. Hence, more unequal strengths may imply that more resources are spoiled in the struggle even though the incumbent who obtains the edge wins the battle more often than the challenger.

The dynamics of conflict thus have clear bearings on the link between regime stability and fighting. While the average turnover of the incumbent must be highest when the probability of winning is fifty-fifty in each round, more unequal power reduces average turnover and can increase fighting.

The result that an incumbency advantage raises the prize for both contestants is not only a local phenomenon valid only for approximately equally strong groups. Let us define an absolute incumbency advantage where the unit cost of force approaches zero. Clearly, when a contestant with an absolute advantage becomes the incumbent, he stays forever. When one contestant gets an absolute incumbency advantage we get the following result:

Proposition 3 *Compared to the case without incumbency advantage, the introduction of an absolute incumbency advantage to one contestant raises the prize for both contestants, but more so for the contestant who gets the advantage. In the limit case where both groups have absolute advantage the valuation of incumbency is the same for both contestants and equal to* $1/(1-\delta) D_i$.

Proof. Consider the case where a gets an absolute incumbency advantage ($C_a \to 0$). It follows from (25) and (26) that $S_a = 1$ and $S_b < 1$. From (27) it in turn follows that

$$h(S_a) - h(1 - S_b) = 1 - h(1 - S_b) > h(S_b) - h(1 - S_a) = h(S_b)$$

When both get absolute incumbency power ($C_a \to 0$ and $C_b \to 0$) It follows from (25), (26) that $S_a = S_b = 1$ and that $\Psi(S_a) = \Psi(S_b) = 1$. From (27) it in turn follows that $h(S_a) - h(1 - S_b) = h(S_b) - h(1 - S_a) = 1 - 0$ ■

An absolute incumbency advantage, say in regime a, means that a victory for a implies that regime a lasts forever. As long as only one contestant has an absolute incumbency advantage and he remains the challenger, fighting is hard as the stakes are high for both.

We have shown that (i) introducing a minor incumbency advantage to one contestant, and (ii) introducing an absolute incumbency advantage to one contestant or both, raises the prize for both contestants in the conflict. The prize for each contestant does however not always increase as the unit cost of influence decreases for one contestant. To see this consider the simple case where $D_a = D_b$, where $c_a = c_b = c$, where $C_a = C_b = C < c$, where $\Psi(S) = S$, and where, from (27), $h(S) = S^2$. Then a further reduction in the cost of influence for a as incumbent (C_a down from C) would lower the prize to b, F_b, if the discount factor is high enough[7]

$$\frac{\partial F_b}{\partial C_a} > 0 \iff \delta > \frac{1 + 2(C/c) + (C/c)^2}{3 - 2(C/c) - (C/c)^2} \tag{32}$$

Therefore, if both contestants initially have a strong incumbency advantage, an even stronger advantage for one group will lower the prize for the other. This result is a combination of two effects. First, when both contestants have strong incumbency advantages, the challenger position is dismal for both. Hence, both v_a and v_b are low and cannot be much affected by a further reduction in the influence costs of the incumbents. Now, if contestant a gets an even stronger incumbency advantage, implying that C_a goes down, contestant a will fight harder as challenger, lowering V_b. If the future matters sufficiently for contestant a (δ is high) the value of V_b will go so much down that F_b also declines. From condition (32) it is clear that $\delta > 1/3$ is an absolute requirement for this to be possible for any positive C.

2.3 Self-defeating power

We have seen that incumbency advantages can explain higher fighting efforts and higher conflict spending for the contestants. It is even possible that situation becomes worse for the contestant who improves his incumbency. A strengthening of the incumbency advantage for a present challenger may represent a serious threat for the present incumbent, and the challenger could as a result be met with a much heavier resistance. The fact that the challenger may actually lose by getting the prospect of incumbency advantage could make it optimal for a challenger to try to commit to abstaining from using some of his incumbency power. More precisely:

Proposition 4 *Power can be self-defeating: When a weak challenger gets the prospect*

[7] This can be shown using (19), (20), (25), (26), (27), (28) and (29), when differentiation with respect to C_a.

of an incumbency advantage, it may induce so much fighting that the expected pay-off to the challenger goes down. Even for a weak and farsighted incumbent a strengthened incumbency advantage may lower his expected pay-off.

Proof. Consider a marginal incumbency advantage for a weak group a ($S_a = s_a$ small). From (13) and (14) in combination with (27), it follows that

$$\begin{aligned} \frac{\mathrm{d}V_a}{F_a} - \delta\frac{\mathrm{d}v_a}{F_a} &= h\left(S_a\right)\frac{\mathrm{d}F_a}{F_a} + h'\left(S_a\right)\mathrm{d}S_a \\ \left(1-\delta\right)\frac{\mathrm{d}v_a}{F_a} &= h\left(S_a\right)\frac{\mathrm{d}F_a}{F_a} - h'\left(S_a\right)\mathrm{d}S_b \end{aligned}$$

It follows by combining with (30) and (31) that

$$\begin{aligned} \frac{\mathrm{d}v_a}{F_a} &= \frac{\delta}{1-\delta}h'\left(S_a\right)S_aS_b\left[\left(h'\left(S_b\right)-h'\left(S_a\right)\right)S_aS_b - h\left(S_a\right)\right]\frac{\mathrm{d}C_a}{C_a} \\ \frac{\mathrm{d}V_a}{F_a} &= \frac{\mathrm{d}v_a}{F_a} - h'\left(S_a\right)S_aS_b\frac{\mathrm{d}C_a}{C_a} \end{aligned}$$

Using (27) it follows that, when S_a is small

$$\begin{aligned} \frac{\mathrm{d}v_a}{F_a} &\approx \frac{2\delta}{1-\delta}h'\left(S_a\right)S_a^2S_b\Psi'\left(0\right)\frac{\mathrm{d}C_a}{C_a} > 0 \\ \frac{\mathrm{d}V_a}{F_a} &\approx \frac{1}{1-\delta}S_aS_bh'\left(S_a\right)\left[-1+\delta\left(2\Psi'\left(0\right)+1\right)\right]\frac{\mathrm{d}C_a}{C_a}\begin{cases} > 0 & \text{when } \delta \text{ close to } 1 \\ < 0 & \text{when } \delta \text{ close to } 0 \end{cases} \end{aligned}$$

■

The result that power can be self-defeating, is in stark contrast to the result in static contests where the return unambiguously goes up when the costs go down. The intuition is simple enough when a is the challenger, he has no direct gain from his incumbency advantage. If the incumbency advantage increases, the probability of becoming the incumbent may go so much down that there is a net loss. This mechanism may even be the dominant one for a as incumbent. The weak incumbent a will know that the larger part of his future periods will be played as challenger. The value V_a will largely be determined by v_a. Hence, if δ is large and S_a is low, an incumbency advantage that lowers v_a may also lower V_a.

The proposition is relevant for weak groups. As noted during solution of the model costs and gains enters symmetrically in the model. Hence, a weak group is weak either as a result of having high costs, C and c, or having little to fight for D. Hence, a group who only modestly prefers own rule over the opposition (a low D), could prefer not having an incumbency advantage. It may in fact have an incentive to, if possible and credible, commit to limit its incumbency advantage.

3 Concluding remarks

The recent strong mobilizations of opposition force in authoritarian countries go against the traditional perception in conflict models of how fighting efforts are mitigated by unequal power and high regime stability. How is the opposition able to mobilize so hard when the chances of winning seem so low at the outset? To understand this paradox of authority one should notice that the mobilization is enforced by a shared conviction that much is at stake. The conflict is a battle of regimes. It can be viewed, perhaps optimistically, as a battle between an authoritarian regime and a democratic regime. Normally the authoritarian regime is characterized by 'indefinite political tenure of the ruler', implying low winning chances of the opposition. Once established, however, the democratic regime might be equally stable, implying low chances of the authoritarian ruler to resume power.

Disparities play an essential role in lasting power struggles. They shape the dynamics of the conflicts. A specific incumbency advantage implies that a victory today may to some degree guarantee the victory also tomorrow, and as the expected outcome today depends on who is the incumbent, the victory of tomorrow actually depends on who was yesterday's winner. The past thus affects present fighting efforts, which again affects the future path of the struggle.

Thus the struggle between groups over the control of a country plays out differently in countries where the control entails access to a strong state apparatus compared to countries where the state apparatus is weak. In divided societies, a strong state may fuel conflicts rather that mitigate them. Control of the state apparatus makes the incumbent stronger, but a stronger incumbent makes the control of the state apparatus more valuable. As a result the struggle for state control is intense and the amount of resources wasted is high. Therefore, regimes may be long lasting without deterring fighting by opposing groups. The attractiveness of taking over a strong incumbency position may dominate the low odds of a short run success. The end of the cold war represented a dramatic shift in many internal conflicts of the world. Many groups that had previously been supported by the East and the West respectively, were now left to cater for themselves. As a result, the incumbency advantage associated with international recognition and access to government resources became relatively more important. Our analysis may explain why many civil wars continued with high intensity even after 1990. The reason could simply be that victory became more important when a higher incumbency advantage became part of the prize of winning.

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A Appendix Non-stationary solutions

In our analysis we have assumed that we are in a stationary Markov equilibrium where (19) and (20) holds. In this section we will prove that there always exists at least one stationary Markov equilibrium. We will also discuss the possibility of nonstationary equilibria and the possibility of more than one stationary equilibrium.

A Markov equilibrium (not necessarily stationary) of the game defines time paths of F_a and F_b such that (17) and (18) holds hence

$$\begin{aligned} F_{a,t-1} &= F_a\left(F_{a,t}, F_{b,t}\right) \equiv D_a + \delta\left(h\left(S_{a,t}\right) - h\left(s_{a,t}\right)\right) F_{a,t} \\ F_{b,t-1} &= F_b\left(F_{a,t}, F_{b,t}\right) \equiv D_b + \delta\left(h\left(S_{b,t}\right) - h\left(s_{b,t}\right)\right) F_{b,t} \end{aligned} \tag{33}$$

where

$$S_{a,t} = \frac{F_{a,t}c_b}{F_{a,t}c_b + F_{b,t}C_a} \text{ and } S_{b,t} = \frac{F_{b,t}c_a}{F_{b,t}c_a + F_{a,t}C_b} \tag{34}$$

A fixpoint of the system (33) is defined as $\left(F_a^{**}, F_b^{**}\right)$ such that

$$F_a^{**} = F_a\left(F_a^{**}, F_b^{**}\right) \text{ and } F_b^{**} = F_b\left(F_a^{**}, F_b^{**}\right) \tag{35}$$

In order to find the fix points we first find the fixpoint, F_a^*, for F_a given F_b and vice versa such that

$$F_a^* = F_a\left(F_a^*, F_{b,t}\right) \tag{36}$$
$$F_b^* = F_b\left(F_{a,t}, F_b^*\right)$$

From (27) it follows that

$$0 \leq \left(h\left(S_i\right) - h\left(s_i\right)\right) \leq 1$$

therefore a fixpoint for F_i has to be larger than D_i and less than $D_i/\left(1-\delta\right)$. From (27), (23), (24) and (34) it follows that

$$\frac{\partial F_{i,t-1}}{\partial F_{i,t}} = \delta\left(\left(h_i + \frac{\partial h\left(S_{i,t}\right)}{\partial S_{i,t}}\frac{\partial S_{i,t}}{\partial F_{i,t}}F_{i,t}\right) - \left(h_i + \frac{\partial h\left(s_{i,t}\right)}{\partial s_{i,t}}\frac{\partial s_{i,t}}{\partial F_{i,t}}F_{i,t}\right)\right) < 1$$

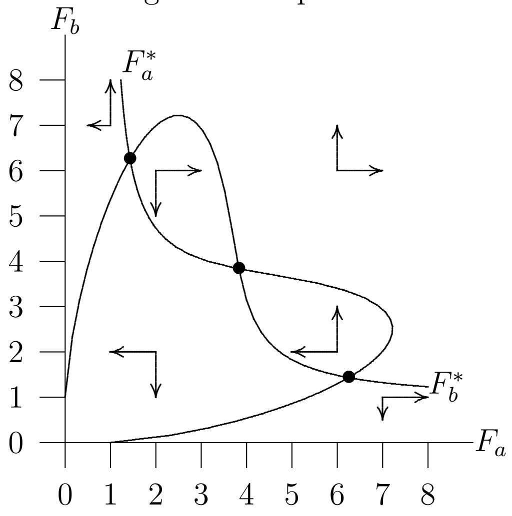
It therefore follows that (36) has unique solutions.

$$F_a^* \in R_a \equiv \left[D_a, \frac{D_a}{(1-\delta)}\right]$$
$$F_b^* \in R_b \equiv \left[D_b, \frac{D_b}{(1-\delta)}\right]$$

The combined problem (35) therefore always has at least one solution. Moreover, all solutions are found within the rectangle R_a, R_b. A simple case is the one where δ is low or $c_i \approx C_i$ for both contestants. Then F_i^* is close to D_i, and F_a^* and F_b^* will only cross once.

If there are strong incumbency advantages combined with a high discount factor δ, however, the fixpoint curves may get sufficient curvature to generate multiple stationary equilibria. One illustration of this possibility is provided in Figure (2). Here, the parameter configurations are symmetric ($C_a = C_b \ll 1$ and $D_a = D_b = 1$). We see that one of the equilibria is symmetric, reflecting a situation where each contestant inherits the other's behavior when they change status. The upper left equilibrium is a case where group a (caused by a low F_a) takes on a more passive role as incumbent causing a low F_a. Group b however (caused by the high F_b) takes on a more aggressive role as incumbent causing

Figure 2: Fixpoints



a high F_b. A similar skewed equilibrium exists down and to the right. These three points all satisfy the conditions for a stationary equilibrium.