Communication and Authority with a Partially Informed Expert*

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Abstract

A *partially* informed expert, **A**, strategically transmits information to a principal, **P**. The residual uncertainty faced by the expert effectively causes the bias between **P** and **A** to be random, with two consequences. First, by misreporting **A** is likely to induce a decision choice by **P**, after the resolution of the residual uncertainty, that is either close to **A**'s ideal position or too far from it, whereas truthful reporting keeps such variations more balanced. A convex loss function of **A** thus favors truthful reporting. Second, by retaining authority of decision making and communicating with **A**, **P** avoids exposure to risks due to **A**'s biased decisions. Better information transmission and the associated insurance benefit thus often imply **P** preferring control over delegation, despite **A** having superior information.

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1 Introduction

In many economic situations where an action affects the welfare of two parties, the formal decision rights belong to one, say a receiver (Principal or **P**) whereas relevant information lies with another, a sender (Agent or **A**). In many such settings, it is not possible to write binding contracts and construct payment schemes that are contingent on chosen actions. One possible avenue for **P** is to elicit information from **A** and then take a decision. However, as the seminal contribution Crawford and Sobel (1982) shows, the ability of **P** to elicit information from **A** to make an informed choice is limited as the latter strategically controls the information she transmits. Aghion and Tirole (1997) and Dessein (2002) explore another avenue – the value of **P** delegating decision rights to **A**. A significant literature has since developed examining the merits of delegating authority. An assumption implicit across this entire literature (which we shall discuss presently) is that the sender is an expert at *everything* that is payoff relevant.[1] We depart from this approach by assuming that the sender is only partially informed. It turns out that not only there is now an increased cost of delegation but also the partial knowledge of the sender improves the quality of information transmission. We will argue therefore that the decision authority should typically remain with the principal.

A rough intuition for the costs and benefits of delegation and authority is as follows. We are considering a scenario where the information relevant for **P**'s optimal decision is partly with **A** but the rest of it, that we call *residual uncertainty*, will become known at a later stage when the decision is to be taken. The expert must be consulted early on in a deliberation process. Giving the expert a control over decisions (rather than just solicit her recommendation), and such a choice must be made in advance, would mean the principal exposes himself to risks of residual uncertainty. By retaining control **P** avoids this risk but exposes himself to potential loss of information due to strategic reporting by **A**. Surprisingly, the loss of information could also be mitigated for the very fact that **P**'s decision will be random due to the residual uncertainty. Basically when contemplating misreporting her information, **A** weighs the possibility of triggering a decision close to her ideal position, if **P**'s bias happens to be congenial, against the possibility that the decision could go very wrong with **P**'s bias turning out in a different direction. The usual concavity of the payoff function now creates an incentive for **A** to make her information more precise to reduce the likelihood of these extreme decisions. As we shall see, the tradeoffs could be such that even truthful revelation becomes a possibility.

In Section 3, we formally present a game of *strategic information transmission under uncertainty* (hereafter, the SITU game). A *random vector* summarizing the state of the world describes the payoff relevant uncertainty; the expert is privately informed of only

[1]Wolinsky (2002) is an exception.

one dimension of the realization of the random vector. We fully characterize all the equilibria of the SITU game with fairly weak assumptions on the probability distributions of the random vector. It turns out that the impact of the extraneous (or residual) uncertainty on the quality of equilibrium information transmission can be captured by a single parameter that we call the *effective bias*. Proposition 2 establishes that a fully revealing equilibrium exists if and only if the effective bias is zero. We also fully characterize all the equilibrium outcomes when the effective bias is non-zero (Proposition 3). In fact, we show that there is an isomorphism between equilibria of a SITU game and a version of Crawford and Sobel's information transmission game (hereafter, the CS-game). It may be recalled that in the CS-game, every equilibrium partitions the agent's type space into finitely many intervals and along the equilibrium path, **P** only finds out in which of these intervals the true type lies. Put differently, Proposition 3 shows that the equilibrium behavior of an expert that only knows some of all relevant information is akin to an expert in an appropriate game in which she knows *all* the information.

The above link between a SITU game and a CS-game is significant. For, we may now borrow a number of comparative statics results from Crawford and Sobel (1982) and study the costs and benefits of delegation in a SITU game, as Dessein (2002) did for the CS-game, with relative ease. Section 4 contains this entire discussion. Through Proposition 5 - Proposition 7, we show how the costs and benefits of delegation vary in relation to the mean and variance of the residual uncertainty and that of type uncertainty. In fact, Proposition 7 is a fairly general result which shows that for arbitrarily small variance in the residual uncertainty, **P** strictly prefers to retain control in all sufficiently informative equilibria. This is in contrast with Dessein (2002)'s conclusion (effectively the case of zero variance of residual uncertainty) that **P** should prefer delegation in all such equilibria.[2]

The rest of the paper is organized as follows. After a brief discussion of the literature, in Section 2, we summarize the main results of the well-known CS-game, a somewhat different presentation relative to the original version in Crawford and Sobel (1982), to aid our analysis in the rest of the paper. Recall from Crawford and Sobel (1982), that the magnitude of a parameter b determines the proximity of players' preferences and bounds the amount of information that is conveyed. A key question, unanswered in Crawford and Sobel (1982) or elsewhere, that we need to resolve for our later discussion in Section 4 is whether **P**'s payoff in a CS-game converges to the fully revealing payoff as b converges to zero (and preferences of **P** and **A** coincide). While it is tempting to conjecture that it does, a formal proof of this fact is actually quite non-trivial. Proposition 1 answers this in the affirmative when a player's payoff in any state depends only on the distance from an optimal action. This result should be of independent interest.[3]

[2] We are not negating Dessein (2002)'s result though; see Remark 3 following Proposition 7.

[3] The question of convergence to the fully revealing equilibrium payoff as the preferences of the two players tend to coincide has also been addressed by Spector (2000). Unfortunately, preferences in his

Literature Review

Strategic Information Transmission

The literature has seen several extensions of the CS-game. By introducing additional elements to the basic CS-model such as multi-dimensional type uncertainty, partial verifiability of actions, multiple experts reporting on the state etc., the broad objectives have been how to improve the extent of information transmission in equilibrium, including even full revelation of information (Gilligan and Krehbiel (1989), Seidmann and Winter (1997), Krishna and Morgan (2001), Krishna and Morgan (2004), Battaglini (2002), Levy and Razin (2007), Ambrus and Takahashi (2008), Li and Madarsz (2008) (where the decision maker is uninformed), Chakraborty and Harbaugh (2010) etc.). Kartik et al. (2007) (also see Kartik (2009)) modify the CS-game so that messages inherently affect the payoffs, but now it is no longer a pure communication game. They show that if the state space is unbounded, there is a fully revealing signalling equilibrium.

A common feature of all such extensions is that the expert or experts are fully aware of the state of the world (but the multiple experts may differ in their preferences). Our work differs from all these papers in that an expert knows only part of the payoff relevant information. This feature allows for possibly full revelation in a *pure communication game* with a compact state space and a *single* expert.

A notable exception to the aforementioned literature is Wolinsky (2002) where the payoff relevant information is fragmented among several experts but they collectively hold all the payoff relevant information. Thus, each individual expert is only partially informed of the payoff relevant state, just as in this paper. At the communication stage, each expert faces a residual uncertainty whose nature is determined by the strategic behavior of the remaining agents.

In contrast to Wolinsky (2002) (except for a brief discussion in Section 5), we consider the case of a single expert for at least two reasons. First, in many organizational settings it is common to have a single expert, typically held on a retainer, and additional opinions are sought only on an *ad hoc* basis. Thus, we believe that the case of a single expert considered for most of this paper is interesting in its own right. Second, retaining the focus on a single expert allows us to lay bare how this distribution impacts the equilibrium outcome. Indeed, we pose very mild restrictions on the distribution of uncertainty. This allows us to illustrate, for instance, how our analysis on full revelation can be carried through for multiple experts in a relatively straightforward manner. (See Section 5.)

Finally, Harris and Raviv (2005) consider a case where **P** can acquire private infor-

setting differ from those typically used in applications of the CS-game and cannot be used here. The reader will notice that the somewhat involved proof does seem not readily generalizable to more general payoff functions and a proof for the most general payoff functions considered by Crawford and Sobel (1982) remains open.

mation. These two-sided private information games can be thought of as particular cases of a SITU game (see Remark 2 in Section 3.1). Their results can be readily understood using the notion of an effective bias that is developed here. McGee (2009) also considers the case where **P** and **A** hold different pieces of information. His is a different framework where **P**'s information affects the support of **A**'s type which makes a direct comparison with our analysis difficult. Moreover, in his framework, the ex-post optimal actions of **P** and **A** differ by a (non-zero) constant, which makes it impossible for the existence of a fully revealing equilibrium.

On Delegation vs. Control

There is a strand of the literature that discusses the merits of delegation in two-agent settings where the decision variable is contractible. These include Holmstrom (1984), Armstrong (1994), Melumad and Shibano (1991), Alonso and Matouschek (2007), Alonso and Matouschek (2008), Armstrong and Vickers (2010), among others. Unlike in this literature, in the present model, the decision maker cannot commit ex-ante to any of the actions.

The key papers concerning delegation and communication when the decision variable is not contractible include Aghion and Tirole (1997), Ottaviani (2000), Dessein (2002), and Krahmer (2006). In all of these models the agent with the superior information is fully informed but may have possibly multi-dimensional information. In our model here, the type uncertainty of the expert is still one-dimensional, just as in Dessein (2002). By varying the size (i.e., the variance) of the residual uncertainty faced by our partially informed agent, we are able to offer a wider perspective to the results in Dessein (2002) in particular. We show how even an arbitrarily small amount of residual uncertainty is often enough to ensure the superiority of control over delegation. An elaborate discussion of these issues is deferred to Section 4.

2 Crawford-Sobel Game of Strategic Information Transmission

In the Crawford-Sobel game of strategic information transmission, hereafter the CS-game, there are two players, **P** and **A**. The authority for choosing an action $\xi \in \mathbb{R}$ rests with **P**, although the choice affects the payoffs of both players. Their payoffs also depend on the realization of a real-valued random variable, denoted by θ, that is distributed with a density f that is continuous and positive on a (non-degenerate) interval support $\Theta = [\theta_\ell, \theta_h]$. Let $U_p(\xi, \theta)$ and $U_a(\xi, \theta)$ denote the respective payoffs of the two agents when ξ is chosen in state θ. Also define

$$\begin{aligned} x_p(\theta) &:= \operatorname{argmax}_\xi \ U_p(\xi, \theta) \\ x_a(\theta) &:= \operatorname{argmax}_\xi \ U_a(\xi, \theta) \end{aligned}$$

to be the ex-post optimal actions of the two players. Assume that payoffs are strictly concave in actions so that the above are well-defined.

The game unfolds with **A** privately observing θ and then recommending an action to **P**. **P** observes **A**'s recommendation and chooses an action. A strategy of **A** is therefore a mapping from Θ to $\mathbb{R}$ and a strategy of **P** is a mapping from $\mathbb{R}$ to $\mathbb{R}$. Observe that, the composition of **A**'s strategy with that of **P** in a given strategy profile yields an *outcome function* $x : \Theta \longrightarrow \mathbb{R}$ where $x(\theta)$ is the action that will be chosen in state θ if those strategies are played out.

Remark 1 Every game that we encounter in this paper is a multi-stage game of incomplete information with a continuum of types. Throughout, by an equilibrium, we mean *Perfect Bayesian Nash Equilibrium* (see Fudenberg and Levine (1990)). Our results remain unaffected if one takes the original approach of CS by looking at distributional strategies.

An *equilibrium outcome function* EOF of the CS-game is an outcome function $x : \Theta \longrightarrow \mathbb{R}$ that is obtained from an equilibrium strategy profile of this game. An equilibrium is *fully revealing* if the state θ becomes common-knowledge prior to **P**'s choice. Of course, in this case **P** would take his ex-post optimal action. Therefore x_p is necessarily the EOF in a fully revealing equilibrium. It is therefore clear that unless $x_a(\theta) = x_p(\theta)$ for (almost) all θ, i.e. preferences are essentially identical, a fully revealing equilibrium is impossible. Crawford and Sobel's main result involves a complete characterization of all equilibria when the two players' preferences differ. The main characterization theorem can be restated, in terms of outcome functions, as follows.

First define for any $a < a'$,

$$x(a, a') = \text{argmax}_\xi \int_a^{a'} U_p(\xi, \theta) f(\theta) \mathrm{d}\theta, \tag{1}$$

the optimal action of **P** in the event he knows that θ lies in the interval $[a, a']$. Also let $\mathbf{a} = (a_0, a_1, \ldots, a_n)$ denote a typical partition of Θ into n sub-intervals where $\theta_\ell = a_0 < a_1 < \cdots < a_n = \theta_h$ are the boundary points of the sub-intervals.

Theorem CS. **(Crawford-Sobel)** *Suppose $x_a(\theta) \neq x_p(\theta)$ for all θ.*

1. *If $x : \Theta \longrightarrow \mathbb{R}$ is an EOF of the CS game, then there exists a partition $\mathbf{a} = (a_0, a_1, \ldots, a_N)$ of Θ such that:*

$$x(\theta) = x(a_{i-1}, a_i) \quad \forall \theta \in (a_{i-1}, a_i)\,, \ \forall i = 1, \ldots, N \tag{2}$$

$$U_a(x(a_{i-1}, a_i), a_i) = U_a(x(a_i, a_{i+1}), a_i) \quad \forall i = 1, \ldots, N-1. \tag{3}$$

2. *There exists a finite integer N^* such that an EOF of the form described in part (**1**) exists if and only if $N \leq N^*$.*

In words, any equilibrium of the CS-game involves partitioning the state space Θ into some N sub-intervals. In what follows, we will refer to such an equilibrium as the N-*equilibrium* and the corresponding partition as an *equilibrium partition.* In an N-equilibrium, whenever **A** observes that θ is in the ith interval (a_{i-1}, a_i), she recommends $x_i = x(a_{i-1}, a_i)$, and if $\theta = a_i$ then she is indifferent between recommending $x_i = x(a_{i-1}, a_i)$ and recommending $x_{i+1} = x(a_i, a_{i+1})$. For **P**, upon rationally updating his prior, his payoff from choosing an action x following the report x_i is proportional to the maximand in the RHS of (1). Hence, choosing x_i is optimal. Part (2) of the Theorem captures the essence of the bounds on information transmission posed due to strategic considerations. The divergence in the optimal choice of **P** and **A** causes the latter to withhold information about the true state. The maximal number (N^*) of elements in an equilibrium partition depends on the preferences of the two players as well as the distribution F.

2.1 CS-game with a Fixed Bias

In virtually all applications of the CS-game in economics and political science, it is assumed that $U_p(\xi, \theta) = -\ell_p(|\,\xi - \theta\,|)$ and $U_a(\xi, \theta) = -\ell_a(|\,\xi - \theta - b\,|)$, for a given parameter $|b| \neq 0$, where ℓ_p and ℓ_a are increasing, twice differentiable convex functions. The parameter b is referred to as the agent's "*bias*" since it shifts the ex-post optimal response of **P**, namely $x_p(\theta) = \theta$, to $x_a(\theta) = \theta + b$. We will refer to the strategic information transmission game with the above specifications as a CS-game with a fixed bias.

Since $\ell_a(|x|)$ is necessarily symmetric about zero (say, if the loss function is quadratic), we have the following easy Corollary of Theorem CS for a CS-game with a fixed bias:

Corollary 1 *Consider the family of CS-games with a fixed bias in which ℓ_p is fixed. In any such game, $\mathbf{a}_N^b = (a_0^b, a_1^b, \ldots, a_N^b)$ is an equilibrium partition when the bias is b if and only if*

$$a_i^b \;=\; \frac{x(a_{i-1}^b, a_i^b) + x(a_i^b, a_{i+1}^b)}{2} + b \qquad \forall i = 1, \ldots, N-1, \tag{4}$$

*where $x(a, a')$ is as defined by (1) with $U_p(\xi, \theta) = -\ell_p(|\xi - \theta|)$. **P**'s payoff in this equilibrium is*

$$\pi_N(b) \;=\; -\sum_{i=0}^{n} \int_{a_{i-1}^b}^{a_i^b} \ell_p(|x(a_{i-1}^b, a_i^b) - \theta|)\, \mathrm{d}F(\theta). \tag{5}$$

Fix the loss functions in a CS-game of constant bias. The maximal number of possible elements in an equilibrium partition of Θ, namely N^* in Part (2), Theorem CS, is a function of $|b|$. Given a bias b, let $N_b^* \equiv N^*$. CS establish a number of comparative statics of both the players' payoffs and N_b^* with respect to b. Some of these are reproduced in the Appendix. In particular, they only show that N_b^* is *non-decreasing* in $|b|$. However, for our later discussion on the merits of delegation, it becomes necessary to establish two facts for a general F and ℓ_p: First, that $\lim_{|b|\to 0} N_b^* = \infty$ and second, the *size* of each interval of an N-equilibrium partition converges to zero for all N sufficiently large. *Together*, these facts will imply that when $|b|$ is small enough, loss from strategic information transmission can be made close to zero in all sufficiently informative equilibria and consequently the payoff of $\mathbf{P}$ is arbitrarily close to his payoff under full revelation. Neither of the above facts follow directly from CS. We therefore prove the following Proposition that is crucial for proving Proposition 7, a general result on merits of delegation.

Recall that the *mesh* of a partition $\mathbf{a} = (a_0, a_1, \dots, a_N)$ is the length of its longest sub-interval, denoted by $\| \mathbf{a} \|$.

Proposition 1 *Consider the family of CS-game(s) with a fixed bias, indexed by the bias b, in which ℓ_p is the loss function of $\boldsymbol{P}$. Consider an infinite sequence (b_k) such that* $\lim_{k\to\infty} b_k = 0$. *Then the following are true:*

1. $\lim_{k\to\infty} N_{b_k}^* = \infty$.

2. *Consider any infinite sequence of integers (N_k) such that $N_k \le N_{b_k}^*$ for all k and $N_k \to \infty$. Then, for the corresponding equilibrium partitions $\mathbf{a}^k = (a_0^k, a_1^k, \dots, a_{N_k}^k)$,*

$$\lim_{k\to\infty} \| \mathbf{a}_k \| = 0,$$

$$\textit{and} \qquad \lim_{k\to\infty} \pi_{N_k}(b_k) = \ell_p(0).$$

Proof. See Appendix.

Since $\ell_p(0)$ is the payoff of $\mathbf{P}$ in a fully revealing equilibrium, in game-theoretic terms, Proposition 1 shows the *lower* hemi-continuity at zero of $\mathbf{P}$'s payoff in sufficiently informative equilibria of CS-games parametrized by b. The proof of Proposition 1 requires novel argument that goes beyond Crawford-Sobel analysis and is somewhat involved. A sketch of the argument that offers a quick insight is as follows.

In an N-equilibrium of the CS-game, before taking an action, $\mathbf{P}$ is only informed of the element of *equilibrium* partition $\mathbf{a} = (a_0, a_1, \dots, a_N)$ that the realized θ lies in. This gives him a payoff of $\pi_N(b)$. Instead, suppose $\mathbf{P}$ could *choose* an *arbitrary* partition of Θ (into N sub-intervals) and then he takes an action based on the knowledge of the interval that θ lies in. His most preferred partition, say $\mathbf{a}_N^* = (a_0^*, a_1^*, \ldots, a_N^*)$, typically differs from $\mathbf{a}_N$ and leads to a payoff of say, π_N^*. What we show is that the difference between $\mathbf{a}_N^*$ and $\mathbf{a}_N$ is small when $b \approx 0$ and consequently, $\pi_N(b) \approx \pi_N^*$. Separate arguments are then used to show that $\lim_{N\to\infty} \parallel \mathbf{a}_N^* \parallel = 0$ (and hence $\lim_{N\to\infty} \pi_N^* = 0$) and that $N_b^* \to \infty$ as $b \to 0$ to complete the proof.

It is useful to compare Proposition 1 with the main result of Spector (2000). First, Spector considers payoffs in which the utility of an action ξ is $U(\xi)$ for **P** and $U(\xi)+bV(\xi)$ for **A**, where U and V are sufficiently well behaved functions. Ignoring the different specification of utilities, Proposition 1 is different from his main result in the following way. Spector's result is that for any (b_k) such that $\lim_{k\to\infty} b_k = 0$, there *exists* a sequence of N_k-equilibria along which the size of the largest interval in the corresponding equilibrium partition converges to zero. Clearly $\lim_{k\to\infty} N_k = \infty$. Proposition 1 on the other hand claims that the interval shrinks along *every* sequence of N_k-equilibria such that $\lim_{k\to\infty} N_k = \infty$. At a later stage when we discuss the merits of retaining authority over delegation, there is an equilibrium selection issue. The stronger claim of Proposition 1 goes some way in mitigating those considerations.

3 Strategic Information Transmission Under Uncertainty

Unlike in CS, we now allow for the possibility that there are other sources of uncertainty in addition to the private information of **A** that affects the players' payoffs. Accordingly, a true state of the world is now a tuple $(\theta, \boldsymbol{s})$ where $\boldsymbol{s}$ is a realization of some (possibly) multivariate random variable. The prior distribution however is such that θ is distributed independently of $\boldsymbol{s}$ with a positive density f (and cdf F) on a non-trivial interval $\Theta = [\theta_\ell, \theta_h]$, while $\boldsymbol{s}$ is drawn from an arbitrary (measurable) set $\mathcal{S} \subseteq \mathbb{R}^k$ according to a probability distribution function G. We will refer to the uncertainty regarding $\mathbf{s}$ as residual uncertainty and θ as type uncertainty.

The vNM utility of **P** and **A** from an action $\xi \in \mathbb{R}$ in state $(\theta, \boldsymbol{s})$ are respectively[4]

$$\begin{aligned} u_p(\xi, \theta, \boldsymbol{s}) &= -\ell_p(|\,\xi - \theta - \boldsymbol{w}_p \cdot \boldsymbol{s}\,|), \\ u_a(\xi, \theta, \boldsymbol{s}) &= -\ell_a(|\,\xi - \theta - \boldsymbol{w}_a \cdot \boldsymbol{s}\,|) \end{aligned}$$

where $\boldsymbol{w}_a, \boldsymbol{w}_p \in \mathbb{R}^k$ are given vectors of coefficients that are common knowledge. Assume throughout that $\ell_p(.)$ and $\ell_a(.)$ are both increasing, twice differentiable convex functions. To keep the analysis interesting, assume that $\boldsymbol{w}_a \neq \boldsymbol{w}_p$, which keeps the preferences of the two players different.[5]

[4] $\mathbf{x} \cdot \mathbf{y}$ denotes the inner product between any two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^k$.

[5] For instance, in the usual CS-game with a fixed bias, $\mathbf{s}, \boldsymbol{w}_p, \boldsymbol{w}_a$ are one-dimensional degenerate at b and $w_p = 0$ and $w_a = 1$.

The *game of strategic information transmission under uncertainty*, hereafter referred to as a SITU game, proceeds as follows. **A** privately observes her "type" $\theta \in \Theta$ and sends **P** a message m, chosen from a given message space $\mathcal{M}$. **P** observes **A**'s report *and* the realization of extraneous uncertainty $\boldsymbol{s}$ and then chooses an action, which ends the game.

Note that in terms of the timing of resolution of uncertainty, the key assumption here is that **A** is unaware of the realization $\boldsymbol{s}$ at the communication stage and that **P** is fully aware of $\boldsymbol{s}$ prior to taking his action. Notice **A** acts only once — at the communication stage. Therefore, whether **A** eventually learns of the realization of $\boldsymbol{s}$ is really a moot point insofar as **A**'s incentives for information transmission are concerned. That is, we can allow $\boldsymbol{s}$ to either be privately observed by **P** or be publicly observable. (In Section 4 it will be necessary to specify if it is in fact publicly observed in order to compare the costs and benefits of delegation.)

3.1 Ex-ante, Ex-post and Effective Bias

Before we formally describe the strategies (and the equilibrium) in a SITU game, we introduce a few key concepts that are necessary for presenting our results. Observe that the ex-post optimal choices of **P** and **A** are respectively

$$\begin{aligned} y_p(\theta, \boldsymbol{s}) &= \theta + \boldsymbol{w}_p \cdot \boldsymbol{s}, \\ y_a(\theta, \boldsymbol{s}) &= \theta + \boldsymbol{w}_a \cdot \boldsymbol{s}. \end{aligned}$$

We will refer to

$$b_{\boldsymbol{s}} := (\boldsymbol{w}_a - \boldsymbol{w}_p) \cdot \boldsymbol{s},$$

the difference between the ex-post optimal action of **A** and that of **P** as the *ex-post bias*. Indeed, it is not hard to see that if G is degenerate at some $\boldsymbol{s}$, then the SITU game is effectively a CS-game with the fixed bias $b_{\mathbf{s}}$. Of course, our concern in this paper is primarily in the case where G is non-degenerate. Let

$$\mu_b = E[b_{\boldsymbol{s}}] \quad \text{and} \quad \sigma_b^2 = E[b_{\boldsymbol{s}} - \mu_b]^2.$$

μ_b is the average of the ex-post bias as $\boldsymbol{s}$ varies and $\sigma_b^2 > 0$ is its variance. We will refer to μ_b as the *ex-ante bias* or simply the mean bias.

With a non-degenerate G, one may be tempted to guess that the ex-ante bias μ_b acts as a proxy for determining the extent of information transmission in the SITU game. It turns out that due to strategic considerations, the relevant differences in players' preferences are instead captured by a statistic that in general differs from ex-ante bias. To define this, we first introduce the function $\varphi : \mathbb{R} \longrightarrow \mathbb{R}$ where

$$\varphi(\xi) \ = \ \int \ell_a(|\xi - b_{\boldsymbol{s}}|) \, \mathrm{d}G(\boldsymbol{s}). \tag{6}$$

Definition 1 (Effective Bias) *The effective bias in a SITU game is*

$$b^* \ := \ \operatorname{argmin}_\xi \varphi(\xi), \tag{7}$$

(*which is unique since* $\ell_a''(\xi) < 0$).

Note that $\varphi(\eta - \theta)$ is the expected loss of type θ if **P** were to believe that she is of type η. Therefore, a type θ would most prefer that **P** believes that she is of type $b^* + \theta$. Hence the term effective bias.

Remark 2 It will become clear in Proposition 2 and Proposition 3 that it is b^* that forms a "sufficient statistic" for determining the extent of information transmission that can occur in an equilibrium of the SITU game. Harris and Raviv (2005) examine a case where **P** can acquire private information prior to communication. Their findings that the nature of this uncertainty does not affect equilibrium behavior can be readily explained by looking at the implied effective bias. For instance, in Harris and Raviv's analysis of delegation within a firm, **P**, the CEO and **A**, the divisional manager, care about two sources of uncertainty. **P** privately observes θ_p and **A** privately observes θ_a and their respective losses from an action ξ are $-(\xi - \theta_a - \theta_p)^2$ and $-(\xi - \theta_a - \theta_p - b)^2$ where b is the usual bias. This specification is readily accommodated in our model by taking $\theta \equiv \theta_a$, $\mathbf{s} = (b, \theta_p)$, $\boldsymbol{w}_a = (1, 1)$ and $\boldsymbol{w}_p = (0, 1)$. With this specification, observe that the ex-post bias (and hence effective bias) is independent of the distribution of θ_p. We shall see (through Proposition 2 and Proposition 3) that only the effective bias determines the extent of information transmission, and hence the distribution of **P**'s private observation θ_p has no impact on equilibrium information transmission.

A point of natural interest is the coincidence of ex-ante bias and the effective bias. Although effective and mean bias can be different, they do coincide under modeling assumptions typically considered in various applications of CS, described by Condition A below:

Condition A. *Either (a) or (b) holds:*

a. ***A****'s loss function is quadratic, i.e.* $\ell_p(\xi) = \xi^2$.

b. The distribution of ex-post bias $b_{\mathbf{s}}$ *is symmetric.*

Lemma 1 *In any SITU game where Condition A holds,*

1. *φ is symmetric around μ_b, i.e. $\varphi(\mu_b + \xi) = \varphi(\mu_b - \xi)$ for all $\xi \in \mathbb{R}$.*

2. *The effective bias is the same as the ex-ante bias, i.e. $b^* = \mu_b$.*

Proof. See Appendix.

3.2 Equilibrium

Let $\mathcal{M}$ be a given message space. A strategy of **A** in the SITU game is any (measurable) function $\sigma_a : \Theta \longrightarrow \mathcal{M}$ and **P**'s strategy is a mapping $\sigma_p : \mathcal{M} \times \mathcal{S} \longrightarrow \mathbb{R}$. Just as we did for the CS-game, for any strategy profile (σ_a, σ_p) we associate an *outcome function*, namely the mapping $Y : \Theta \times \mathcal{S} \longrightarrow \mathbb{R}$ where

$$Y(\theta, \boldsymbol{s}) = \sigma_p(\sigma_a(\theta), \boldsymbol{s}).$$

That is, if the strategy profile (σ_a, σ_p) is played, $Y(\theta, \boldsymbol{s})$ will be the eventual action that will be chosen when the true state is $(\theta, \boldsymbol{s})$. Using this, for an arbitrary strategy profile (σ_a, σ_p), we can write down the expected loss of **A** when she is of type θ and chooses to deviate and mimic the behavior of type θ':

$$L_a(\theta', \theta) \quad = \quad \int \ell_a(|Y(\theta', \boldsymbol{s}) - \theta - \boldsymbol{w}_a \cdot \boldsymbol{s}|) \, \mathrm{d}G(\boldsymbol{s}). \tag{8}$$

Let $R(\sigma_a) \subseteq \mathcal{M}$ denote the range of σ_a.

Definition 2 (Equilibrium) *An equilibrium consists of a strategy profile (σ_a, σ_p) such that*

$$L_a(\theta', \theta) \quad \geq \quad L_a(\theta, \theta) \qquad \forall \theta, \theta' \in \Theta, \tag{9}$$

and for every $m \in R(\sigma_a)$,

$$\sigma_p(\theta, \boldsymbol{s}) \quad \in \operatorname{argmin}_x \int_{\theta' \in \sigma_a^{-1}(m)} \ell_p(|x - \theta' - \boldsymbol{w}_p \cdot \boldsymbol{s}|) f(\theta') \, \mathrm{d}\theta', \tag{10}$$

whenever $\sigma_a^{-1}(m)$ is of non-zero probability.

$Y(\cdot, \cdot)$ is said to be an equilibrium outcome function (EOF) of the SITU game if it is the outcome function of an equilibrium (σ_a, σ_p).

Condition (9) is the usual incentive compatibility requirement on **A**'s behavior. Condition (10) is the requirement that at every θ that is reported along the equilibrium path, **P**'s choice is a best response given his updated Bayesian posterior.

Equilibrium beliefs. Strictly speaking, (9) and (10) are only necessary conditions for an equilibrium. In order to complete the description of an equilibrium, one must also specify beliefs and behavior at unreached information sets. Insofar as our concern is only in the characterization of the EOF, this is without loss of generality. For, given a strategy profile (σ_a, σ_p) such that (9) and (10) hold, pick $\hat{\theta}$ arbitrarily and let $\hat{m} = \sigma_a(\hat{\theta})$. For any $m \in \mathcal{M} \setminus R(\sigma_a)$, which represents an unreached node in the candidate equilibrium (σ_a, σ_p), prescribe the beliefs of **P** at m to be the same as those at $\hat{m}$ and redefine $\sigma_p(m, \mathbf{s}) = \sigma_p(\hat{m}, \mathbf{s})$. That is, **P** behaves at any unreached equilibrium message exactly as he does upon hearing $\hat{m}$. Since the original incentive compatibility conditions prevent any type from mimicking the behavior of $\hat{\theta}$, with the above prescribed beliefs, every type of **A** has an incentive to weakly report $\sigma_a(\theta)$ and makes (σ_a, σ_p) a Perfect Bayesian Equilibrium, in the sense of Fudenberg and Levine (1990).

Observe that in any equilibrium **P** chooses an action only *after* observing $\boldsymbol{s}$. Since the ex-post optimal action is additively separable in θ and $\boldsymbol{s}$, it is intuitive that an EOF is similarly separable. The following Lemma makes this precise.

Lemma 2 *For every EOF* Y *of the SITU game, there is a correspondingly unique function* $\psi : \Theta \longrightarrow \mathbb{R}$ *such that* $Y(\theta, \boldsymbol{s}) = \psi(\theta) + \boldsymbol{w}_p \cdot \boldsymbol{s}$.

Proof. See Appendix.

3.3 Fully Revealing Equilibrium

Before presenting the result formally, let us see how full revelation can arise. To illustrate, suppose the extraneous uncertainty is one dimensional so that the state of the world is a pair of real numbers (θ, s), in which case **P**'s ex-post optimal action is $y_p(\theta, s) = \theta + s$. Further assume that the loss of type-θ **A** if **P** chooses an action y in state θ is independent of s (i.e. $w_a = 0$) and is therefore given by the symmetric U-shaped curve shown in the diagram below, which has a minimum at θ. Also, suppose s can take on two values $s_0 > 0$ or $-s_0$ with equal probability.

Consider **A** reporting her information truthfully to **P** as a candidate equilibrium strategy. What are the incentives for **A** of type θ to report her type truthfully? When **P** hears the report θ, he becomes fully informed of the true state prior to making his decision, causing $y_p(\theta, -s_0)$ and $y_p(\theta, s_0)$ to be selected with equal probability from **A**'s viewpoint at the time of report. This leads to an expected loss of L_0. Should type θ deviate and mimic report of type $\theta' > \theta$, the resulting actions are $y_p(\theta', -s_0)$ and $y_p(\theta', s_0)$ leading to an ex-post loss L_ℓ and L_h respectively with equal probability. Observe that the gains from that misrepresentation is $(L_0 - L_\ell)$ when $-s_0$ occurs, but this is not enough to offset the increased cost $(L_h - L_0)$ when s_0 occurs, due to the convexity of the loss function,

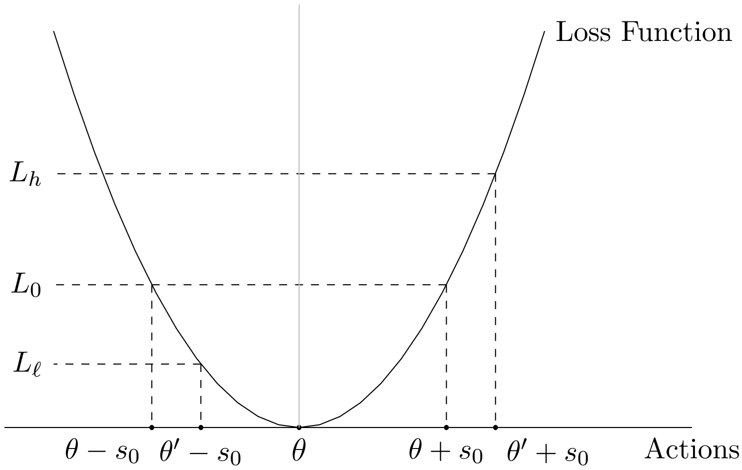

Figure 1: Full revelation in a SITU game

causing a positive net expected loss. Following such logic, it is clear that such defections (and others) are dominated from which we can conclude that perfect revelation can be supported as an equilibrium behavior.[6,7]

More formally, an equilibrium (σ_a, σ_p) is said to be *fully revealing* if $\sigma_a(\theta) = \theta$ for all $\theta \in \Theta$. An easy application of Lemma 2 combined with the definition of equilibrium yields the following necessary and sufficient condition for the existence of a fully revealing equilibrium.

Proposition 2 (Fully revealing equilibria) *A fully revealing equilibrium exists if and only if* $b^* = 0$*.*

Proof. Suppose a fully revealing equilibrium exists. The EOF is then $Y(\theta, \boldsymbol{s}) = \theta + \boldsymbol{w}_p \cdot \boldsymbol{s}$. Substituting in (8) we have

$$\begin{aligned} L_a(\theta', \theta) &= \int \ell_a(|\theta' + \boldsymbol{w}_p \cdot \boldsymbol{s} - \theta - \boldsymbol{w}_a \cdot \boldsymbol{s}|) \, \mathrm{d}G(\boldsymbol{s}) \\ &= \int \ell_a(|\theta' - \theta - (\boldsymbol{w}_a - \boldsymbol{w}_p) \cdot \boldsymbol{s}|) \, \mathrm{d}G(\boldsymbol{s}) \\ &= \varphi(\theta' - \theta), \end{aligned}$$

[6]This feature is somehow reminiscent of how it is impossible to attain ex-post efficiency in a bilateral trade setting and yet it is possible to achieve an efficient dissolution of partnership when the share of a partner in the organization is roughly equal. (Compare Myerson and Satterthwaite (1983) and Cramton et al. (1987).) Intuitively, efficiency obtains in the latter case because, ex-post, a partner can either take on the role of a "seller" or a "buyer" depending on realization of the uncertainty.

[7]In Koessler and Martimort (2010) decision variables are multi-dimensional. The principal, with an upward bias in each dimension relative to the agent's ideal actions, optimally distorts the actions in opposite directions to create countervailing incentives to minimize his loss due to information asymmetry. In this paper, the decision variables are non-contractible and therefore the arguments here are quite different.

and the equilibrium requirement (9) that $L_a(\theta', \theta) \geq L_a(\theta, \theta) = \varphi(0)$ gives $b^* = 0$.

Conversely, assume $b^* = 0$ and suppose that **A** plays $\sigma_a(\theta) = \theta$ for all θ. The equilibrium requirement (10) gives $\sigma_p(\theta, \boldsymbol{s}) = \theta + \boldsymbol{w}_p \cdot \boldsymbol{s}$, which again gives $L_a(\theta', \theta) = \varphi(\theta' - \theta)$. Again, since $b^* = 0$, (9) holds as $L_a(\theta, \theta) = \varphi(0) \leq \varphi(\theta' - \theta) = L_a(\theta', \theta)$ for all θ. ■

Given the intuitive meaning of b^* (following Definition 1), the necessary and sufficient condition for full revelation presented in Proposition 2 is only natural. Full revelation of the expert's private information in a one sender, one receiver cheap-talk communication game for a one-dimensional sender type contrasts with the existing results on information revelation (such as Krishna and Morgan (2004), Battaglini (2002), or Goltsman and Pavlov (2011), among others) that rely either on multi-dimensional type, multiple senders or multiple audiences. Neither do messages affect the payoffs directly, as in Kartik et al. (2007). All that is required in this pure communication game is that the expert's information at the stage of communication (which is also her type) is only a *part* of the array of information that are going to be used in the Principal's decision. The uncertainty in the expert's mind about how her recommendation will play out in the eventual decision, given the importance of other pieces of information, makes the risk-averse expert not to misreport. Moreover, the ex-post preferences of the two agents can be vastly different as it is entirely possible that ex-post bias $b_{\mathbf{s}}$ assumes very large values and yet the effective bias $b^* = 0$.[8]

3.4 Partition Equilibria

When $b^* \neq 0$, full revelation is no longer possible. To provide a complete characterization of all the possible EOF in this case, we first consider a special case of the original CS-game.

Definition 3 (CS*-game) *Given a SITU game, CS*-game is a CS-game of strategic information transmission (i.e. no extraneous uncertainty) with the payoff functions given by*

$$\begin{aligned} U_p(x, \theta) &= -\ell_p(|x - \theta|), \\ U_a(x, \theta) &= -\varphi(x - \theta). \end{aligned}$$

Observe that in the CS*-game, the ex-post optimal actions are $x_p(\theta) = \theta$ and $x_a(\theta) = \theta + b^*$, where b^* is as defined in (7). Occasionally we will refer to the above CS*-game as having an effective bias b^*.

[8] This becomes obvious when we consider the case where Lemma 2 applies so that $b^* = \mu_b$. Clearly, we can have $\mu_b = 0$ and yet $b_{\mathbf{s}}$ take arbitrarily large values.

It is worth emphasizing that a CS* game is not necessarily a CS game with a fixed bias despite the fact that $x_a(\theta)$ is a fixed distance b^* away from $x_p(\theta)$. The distinction is that φ, the "loss function" of **A**, is not necessarily symmetric. If it were the case that φ is symmetric about some point p (i.e. $\varphi(p-\xi) = \varphi(p+\xi)$ for all ξ), then from the convexity of φ and the definition of b^*, we must have $p = b^*$ and the CS* game does become CS game with the fixed bias b^*.

The following proposition offers a complete characterization of all the EOF of the SITU game by showing them to be isomorphic to the equilibria of the CS*-game.

Proposition 3 *Suppose $b^* \neq 0$. $Y : \Theta \times \mathcal{S} \longrightarrow \mathbb{R}$ is an equilibrium outcome function of the SITU game if and only if $\mathbf{a} = (a_0, a_1, \ldots, a_N)$ is an equilibrium partition of the CS* game and*

$$Y(\theta, \boldsymbol{s}) = x(a_{i-1}, a_i) + \boldsymbol{w}_p \cdot \boldsymbol{s} \qquad \text{for all } \theta \in [a_{i-1}, a_i],\ \boldsymbol{s} \in \mathcal{S}$$

and for $i = 1, \ldots, N$.

*Moreover, the ex-ante equilibrium payoff of **P** is given by*

$$-\sum_{i=1}^{N} \int_{a_{i-1}}^{a_i} \ell_p(|x(a_{i-1}, a_i) - \theta|) f(\theta)\, \mathrm{d}\theta. \tag{11}$$

Proof. See Appendix.

Just as in the original CS game of strategic information transmission, with a residual uncertainty, in an equilibrium of the SITU game the agent partitions her type space Θ into finitely many sub-intervals $\mathbf{a} = (a_1, \ldots, a_N)$ and reports the interval she has observed. **P** then chooses the action $x(a_{i-1}, a_i)$ with the correction $\boldsymbol{w}_p \cdot \mathbf{s}$ for the residual uncertainty.

The extent of strategic information transmission clearly depends on the distribution of the residual uncertainty. However, since the equilibrium action of **P** corrects for the residual uncertainty, the equilibrium payoff of **P** in a SITU game is independent of the residual uncertainty and only suffers the loss due to strategic transmission of information. In fact, the payoff of **P** in an N-equilibrium of the SITU game is the same as his payoff in the CS* game, given in (11).

The relation between CS game and SITU game is further simplified under Condition A, as summarized in the following proposition.

Proposition 4 *Consider a SITU game with a given ℓ_p and such that Condition A holds. $\mathbf{a}_N$ is an equilibrium partition of the SITU game if and only if it is an equilibrium partition of the corresponding CS game with bias μ_b. In particular, **P**'s payoff in the SITU game is $\pi_N(\mu_b)$, as defined in (5).*

Proof. Lemma 1 applies. The effective bias is therefore μ_b and φ is symmetric about μ_b. We can now apply Corollary 1 to conclude that the equilibrium partition is the same as the one in a CS-game with the fixed bias μ_b, thus completing the proof. ■

4 Delegation vs. Authority

The complete characterization of the SITU game given in Proposition 2 and Proposition 3 enables us to evaluate the incentives of **P** to delegate authority to **A** instead of eliciting **A**'s information through communication, in the spirit of Dessein (2002).

If the hypothesis of Proposition 2 holds, there is after all a fully revealing equilibrium. Accordingly, we may conclude that:

- It can be optimal for **P**, even in an ex-post sense, to retain control, despite the fact that **A** is better informed than **P**.
- If **P** could *choose* when to seek advice from **A**, it is optimal for **P** to do so *before* the public signal is realized.

In fact, we may even assume that $\boldsymbol{s}$ is privately observed by **P**.

However, when Proposition 2 does not hold, to evaluate the costs of delegation we need to specify in greater detail **A**'s information regarding the residual uncertainty at the time of making her choice, should **P** delegate authority. For most of the analysis, we shall examine the case where, if delegated the authority, **A** can choose an action after observing the realization of the residual uncertainty, just as **P** could in the SITU game. Toward the end we consider some variations in the timing of resolution of uncertainty and the delegation decision.

With the above assumption, it is clear that if **P** were to delegate authority, **A** takes the action $x_a(\theta, \mathbf{s}) = \theta + \boldsymbol{w}_a \cdot \mathbf{s}$ in state $(\theta, \mathbf{s})$. Hence, **P**'s payoff from delegation is $-E[\ell_p(|b_{\mathbf{s}}|)]$. Let $K := \min_{\xi \in \Theta} \ell_p''(\xi)/2$.[9] Using Taylor's expansion of ℓ_p about μ_b with the Lagrange form for the remainder term, shows that **P**'s payoff from delegation is bounded above by $-\bar{\pi}_D(\mu_b, \sigma_b^2)$, where

$$\bar{\pi}_D(\mu_b, \sigma_b^2) \quad = \quad \ell_p(|\mu_b|) + K\sigma_b^2. \tag{12}$$

If **P** keeps authority, since he takes an action after observing $\boldsymbol{s}$, he is completely protected from the variability in the residual uncertainty. On the other hand, once **P** delegates, he has no means of insuring against the variability of **A**'s choice as it varies with the

[9] K is well-defined since ℓ_p was assumed to be twice continuously differentiable. Further, since we assumed that ℓ_p is strictly convex, $K > 0$.

residual uncertainty, as is evident from the above bound on **P**'s payoff from delegation. By keeping authority however, **P** has to endure the loss of information regarding θ. The extent of this loss of course depends on a number of factors, including the selection of an equilibrium.

If, for example, the conditions of Proposition 2 were to apply, in the fully revealing equilibrium **P** would choose $x_p(\theta, \mathbf{s}) = \theta + \boldsymbol{w}_p \cdot \mathbf{s}$. There is of course no loss of information and **P** is clearly strictly better off from retaining authority. Under Condition A, we have the following simple sufficient condition, directly in terms of the residual uncertainty, for superiority of authority. It is a direct corollary of Proposition 2 and Lemma 1.

Corollary 2 *Suppose Condition A holds.* $\mu_b = 0$ *is* (*a necessary and sufficient condition for the existence of a fully revealing equilibrium and hence*) *a sufficient condition for authority to dominate delegation.*

The existence of a fully revealing equilibrium presents us with an easy case for arguing the superiority of retaining authority over delegation from **P**'s point of view. More generally, in determining the actual tradeoff between delegation and authority, we are presented with an equilibrium selection problem.[10] In the remainder of this section, we shall focus on the case where a fully revealing equilibrium does *not* exist. In order to keep the discussion tractable, we shall impose Condition A. We do emphasize, however, that results to follow do not require its full force.[11]

4.1 Large Variance of Ex-Post Bias and Delegation

Condition A, via Proposition 4, tells us that **P**'s payoff in an N-equilibrium of the SITU game is $\pi_N(\mu_b)$, the payoff that **P** would have received in an N-equilibrium of a CS-game with a fixed bias μ_b. Notably, this payoff does not depend on σ_b^2. It is then immediate that, for any SITU game of a given ex-ante bias μ_b, **P** strictly prefers retaining authority provided the variability in the residual uncertainty, σ_b^2 is sufficiently large. Further, the payoff **P** would have received from delegation in the CS-game is $-\ell_p(|\mu_b|)$, which is more than the upper bound $-\bar{\pi}_D(\mu_b, \sigma_b^2)$ on the analogous payoff in the SITU game. Therefore, all the conclusions of Dessein (2002) that establish the superiority of authority over delegation are also now directly applicable. This includes the following analogue of his Proposition 4.

[10]Note that a fully revealing equilibrium is always accompanied by many non-revealing equilibria. Therefore, the problem of equilibrium selection remains even in this case.

[11]In fact, the main property that is being used here is that function φ defined in (6) is symmetric. Condition A is a sufficient condition for this to be the case. Whenever symmetry of φ holds (even if Condition A does not hold), one may repeat all the results to follow, ad verbatim, by replacing μ_b with the effective bias b^*.

Proposition 5 *Assume that F is symmetric, $|\mu_b| < (\theta_h - \theta_\ell)/4$ and Condition A holds. Then, in the SITU game, any informative equilibrium dominates delegation if F is such that*

$$\int_{\theta \in \Theta} \ell_p(|\theta|)\, \mathrm{d}F(\theta) \ \leq \ \bar{\pi}_D(\mu_b, \sigma^2). \tag{13}$$

Proof. The proof is omitted as it follows directly from Dessein (2002).

4.2 Small Variance of Ex-Post Bias and Delegation

The two salient features in the above discussion were that the variance of ex-post bias was allowed to be sufficiently large *and* we were agnostic about which equilibrium is played. However, it turns out that the information that is conveyed through N-equilibria for higher values of N, *rapidly* reduces the minimal size of σ_b^2 that is required for delegation to be a poor strategy. To appreciate this, consider for the moment (we will consider more general preferences later) the SITU game where **P** has a quadratic loss function. By borrowing some of the calculations regarding the linear quadratic CS-game of a fixed bias in Section 4 of Crawford and Sobel (1982), we have the following proposition.

Proposition 6 *Consider a SITU game in which $\ell_p(\xi) = -\xi^2$, Condition A holds and θ is uniformly distributed over $[0, 1]$.*

1. ***P** strictly prefers to retain authority in an N-equilibrium game if and only if*

$$\sigma_b^2 \ > \ \frac{1}{12N^2} + \mu_b^2(\frac{(N^2-4)}{3}). \tag{14}$$

2. ***P** strictly prefers to retain authority in an N-equilibrium whenever*

$$\sigma_b^2 \ > \ \frac{(2N^2+2N-3)}{N^2(N+1)^2}\ \sigma_\theta^2. \tag{15}$$

Proof. See Appendix.

Part 2, Proposition 6 offers a sufficient condition for dominance of authority by relating the variance in type uncertainty with the variance in the residual uncertainty. The actual magnitudes are interesting. For instance, if $N = 2$ then the ex-post bias need only be 25% more variable than type-uncertainty; with $N = 4$, ex-post bias needs to be only 10% more variable. In fact, the rate at which the ratio $\sigma_b^2/\sigma_\theta^2$ must fall is of the order of $O(N^{-2})$. In other words, as the communication equilibrium becomes more informative, the incentive to retain authority becomes more attractive at a fairly rapid rate. Of course, one needs to bear in mind that in order to ensure the existence of an N-equilibrium, μ_b must be sufficiently small.

Part 1, Proposition 6 offers a comparison that is based only on parameters of the residual uncertainty (μ_b and σ_b^2). As such, it allows us to place the conclusions of Dessein (2002) in a wider context. For, $\sigma_b^2 = 0$ and a bias $\mu_b > 0$ is precisely the case considered by Dessein (2002) and hence his is conclusion that authority must dominate delegation. In fact, his conclusion continues to be valid even in the presence of some residual uncertainty, i.e., $\sigma_b^2 > 0$. For example, the curve C in Figure 2 describes the pairs (μ_b, σ_b^2) such that (14) holds as an equality under the further assumption that the players coordinate on the most informative equilibrium N_{μ_b}. Whenever (μ_b, σ_b^2) lies below

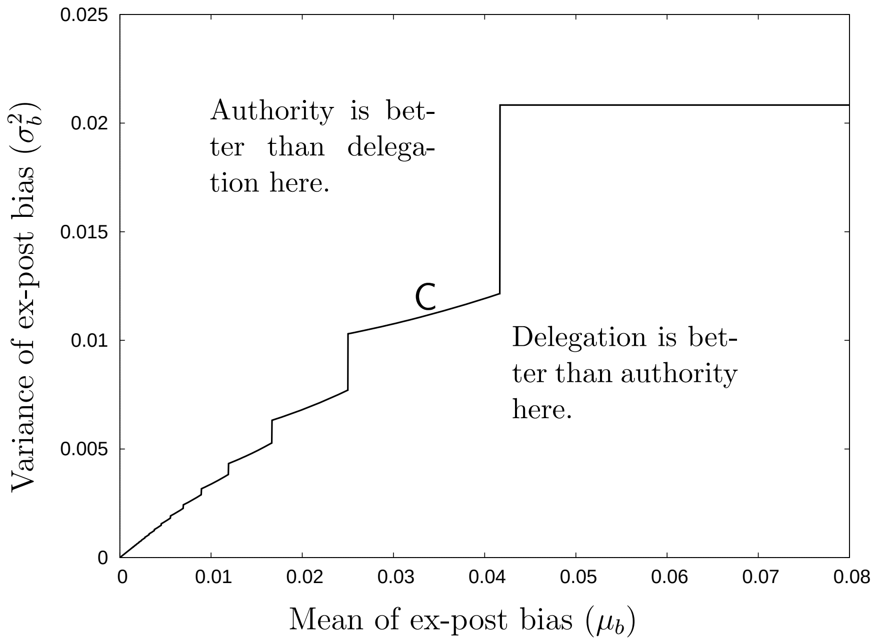

Figure 2: Delegation vs. Authority

C, Dessein's result continues to hold *despite residual uncertainty*. Observe however that for *any* positive σ_b^2, there is a threshold mean bias μ_b below which (i.e., for (μ_b, σ_b^2) above C), the *opposite* of Dessein's conclusion holds. This observation is quite general. As Proposition 7 below shows, even with an arbitrarily small threshold on the variance of the bias, if the mean bias is small enough to allow a sufficiently informative equilibrium in the communication game, **P** would strictly prefer authority to delegation in all such equilibria.

Proposition 7 *Choose any $\epsilon > 0$. Consider the family of SITU games in which **P**'s loss function ℓ_p is fixed and Condition A holds. If $\sigma_b^2 > \epsilon$, then **P** strictly prefers to retain authority in all sufficiently informative equilibria.*

Proof. We will show that there exists an integer N_ϵ such that **P** strictly prefers to retain authority in any N-equilibrium where $N \geq N_\epsilon$. Under the given hypothesis, applying (12), the payoff from delegation is at most $-K\epsilon$. Let $b_\epsilon = \sup\left\{|b| : \pi_{N_b^*}(b) \geq -K\epsilon\right\}$. Note that b_ϵ is well-defined, due to Proposition 1. Define $N_\epsilon = N_{b_\epsilon}^* + 1$. For the existence of an N-equilibrium in a CS-game of fixed bias b, where $N \geq N_\epsilon$, we must have[12] $|b| < |b_\epsilon|$ and by construction then $\pi_N(b) > -K\epsilon$. The proof is complete on noting that, by Proposition 4, the payoff from retaining authority in an N-equilibrium of the SITU game is $\pi_N(\mu_b)$. ■

Remark 3 The above result should be interpreted carefully. It points out the "insurance value" of retaining authority which is absent when there is no residual uncertainty. That is, our model introduces an additional benefit of centralization that was absent in Dessein (2002). This, however, should not be seen as lessening one of Dessein's key observation which is that as the preferences of **P** and **A** become more aligned (i.e., bias disappears in the limit), the delegation turns out to be optimal. Recast in our setting, Dessein's result still holds for the following reason. While our comparative statics are performed in terms of the expected bias $\mu_b = E[b_\mathbf{s}]$ and the variance $\sigma_b^2 = E\{[b_\mathbf{s} - \mu_b]^2\}$, the latter being the source of the insurance motive, the two are related. In particular, since the true ex-post bias is $b = (\boldsymbol{w}_a - \boldsymbol{w}_p) \cdot \boldsymbol{s}$, therefore the ex-ante bias $\mu_b = (\boldsymbol{w}_a - \boldsymbol{w}_p) \cdot E[\mathbf{s}]$ and $\sigma_b^2 = (\boldsymbol{w}_a - \boldsymbol{w}_p)^T \cdot \Sigma_\mathbf{s} \cdot (\boldsymbol{w}_a - \boldsymbol{w}_p)$, where $\Sigma_\mathbf{s}$ is the variance-covariance matrix of $\boldsymbol{s}$. Now, talking about incentive alignment *à la* Dessein (2002) amounts to taking $||\boldsymbol{w}_a - \boldsymbol{w}_p|| \to 0$ which has nothing to do with the shape of the underlying uncertainty itself. But as $||\boldsymbol{w}_a - \boldsymbol{w}_p|| \to 0$, both μ_b and σ_b^2 become small. Further, σ_b^2 converges to zero faster than μ_b and the insurance motive disappears in alignment as well, confirming Dessein's observation. This, however, does not negate the basic point of our exercise which is that, if we increase the diagonal elements of $\Sigma_\mathbf{s}$ (keeping off-diagonal terms zeros, say), the cost of delegation increases because **A** would respond in a manner disliked by **P**.

4.3 Timing of Communication and Uncertainty

So far in our discussion of control vs. delegation, we have assumed that **P** elicits information from **A** *before* $\boldsymbol{s}$ is publicly observed. A natural question is whether **P**, prior to the resolution of any uncertainty, has an incentive to commit to postpone the decision to either elicit information about θ or to delegate until *after* $\boldsymbol{s}$ is publicly observed. Proposition 7 offers a ready answer: if the mean bias is close to zero, **P** has no such incentives. For, if communication occurs after $\boldsymbol{s}$ is publicly observed, then it is as if **P** and **A** play the CS-game with a constant bias. On the other hand, if the residual uncertainty is such that $|b_\mathbf{s}| \approx 0$ for all $\boldsymbol{s}$, we know from Dessein (2002) that **P** is better off by delegating authority giving him a loss of $\ell_p(|b_\mathbf{s}|)$. Therefore, the expected loss from delaying either

[12]See Part CS2, Lemma A in the Appendix.

communication or delegation until after the realization of $\boldsymbol{s}$ leads to an expected loss for $\mathbf{P}$ that is bounded away from zero provided $\sigma_b^2 > 0$ and the support of $b_{\mathbf{s}}$ contains a set of positive measure close to 0. However, if the mean bias is sufficiently close to zero, then eliciting information on θ prior to public revelation of $\boldsymbol{s}$ allows the information loss to be arbitrarily close to zero (see Proposition 1, Corollary 2 and Proposition 7). In this case authority dominates delegation.

5 Concluding Remarks

In this section, we shall discuss a few future research possibilities.

Interim Delegation

We considered the question of delegation vs. control in a hierarchy with a sequential resolution of multiple sources of uncertainty. An introduction of an additional uncertainty arising in the post-delegation stage exposes the principal to new risks as the agent's ex-post optimal decisions might significantly differ from that of the principal. Even for small ex-ante differences between agent's preferences and that of the principal (as measured by the average bias of the agent), the principal would like to retain authority and tolerate loss of information due to agent's strategic communication, thus uncovering a completely different aspect of an important observation earlier in the literature (Dessein (2002)) that the authority should be delegated to the agent. Given that the extraneous uncertainty considered in this paper is a very reasonable description, the lesson from our analysis is equally important.

A scenario that we have not covered is the case where $\mathbf{P}$ observes $\boldsymbol{s}$ privately and then decides whether to delegate or elicit information. We may call this *interim delegation*. In principle, the entire analysis of Section 3 can be brought to bear on this case. For, now $\mathbf{P}$'s decision to retain authority provides information to $\mathbf{A}$ about the former's observation of $\boldsymbol{s}$. Given $\mathbf{P}$'s equilibrium behavior, at any part of the game tree where he retains authority, there is a SITU game with $\mathbf{A}$'s posterior on $\boldsymbol{s}$ determining the nature of uncertainty. Our earlier analysis in this paper helps to characterize equilibrium behavior on that sub-tree. One may then work recursively to solve the entire game. We expect to pursue this in future work.

Multiple Experts

Throughout we have considered only a single agent. Our analysis can be extended in certain ways to discuss communication by multiple experts. Indeed, suppose that there are K agents and agent i privately observes a signal s_i. Let $\mathbf{w}_p = (1, \ldots, 1) \in \mathbb{R}^K$ and for each $i = 1, \ldots, K$, let $\mathbf{W}$ be a $K \times K$ matrix with '1' as every diagonal entry. Let $\mathbf{w}_i$ denote the ith row of this matrix. Let G denote the joint probability distribution $\boldsymbol{s}$ on a compact support $\mathcal{S} \subset \mathbb{R}^K$. Let $\ell_p(|\xi - \mathbf{w}_p \cdot \boldsymbol{s}|)$ and $\ell_i(|\xi - \mathbf{w}_i \cdot \boldsymbol{s}|)$ denote the payoffs of $\mathbf{P}$ and agent i when the true state is $\boldsymbol{s} = (s_1, \ldots, s_K)$. Assume that agents submit reports simultaneously.

The above informational setting is precisely the one found in Wolinsky (2002). However, the specification of preferences is different. In Wolinsky (2002) *all* the experts are biased in the same direction whereas here, two experts may be biased in different directions relative to $\mathbf{P}$ depending on the realization of the uncertainty. In terms of the SITU game, in the current setting, given the behavior of remaining agents, it is as if agent i knows her type s_i (which is θ in our earlier notation) but has a residual uncertainty about $\boldsymbol{s}_{-i}$. We can therefore proceed just as we did in Section 3 and define the ex-post bias of i and the φ_i function:

$$b^i_{\boldsymbol{s}} = (\mathbf{w}_i - \mathbf{w}_p) \cdot \boldsymbol{s} \tag{16}$$

$$\varphi_i(\xi) = \int_{\boldsymbol{s} \in \mathcal{S}} \ell_i(|\xi - b_{\boldsymbol{s}_{-i}}|) \mathrm{d}G(\boldsymbol{s}). \tag{17}$$

Finally, define the effective bias of agent i as

$$b^*_i = \arg\min_{\xi} \varphi_i(\xi). \tag{18}$$

Proceeding exactly as in Section 3.2, we may now readily conclude that

Proposition 8 *A necessary and sufficient condition for full revelation in the above game is that $b^*_i = 0$ for all $i = 1, \ldots, K$.*

One may also, possibly with further restrictions, extend the analysis of Section 3.2.

Appendix

Many of the proofs in this Appendix rely on some of the comparative statics results for the CS-games with a fixed bias. These results, available from Crawford and Sobel (1982), are collected as Lemma A below and stated without proofs.

Lemma A *Consider the family of CS-game(s) with a fixed bias, indexed by the bias b, in which ℓ_p is the loss function of $\boldsymbol{P}$. Then,*

CS*1.* $\pi_{N-1}(b) < \pi_N(b)$ *whenever* $N \leq N^*_b$. (Theorem 3, CS)

CS*2.* $N^*_b \leq N^*_{b'}$ *whenever* $|b'| < |b|$. (Lemma 6, CS)

CS*3.* $\pi_N(b) < \pi_N(b')$ *whenever* $|b'| < |b|$. (Theorem 4, CS)

In order to prove Proposition 1, we will first establish two auxiliary results, Lemma 3 and Lemma 4. Lemma 3 considers what payoffs **P** can achieve in a CS-game with constant bias if his information about θ is not constrained by *strategic* considerations but instead he could choose *any* partition of Θ into N sub-intervals. Lemma 4 then relates these payoffs in an N-equilibrium when the bias is small.

Let Δ_N represent all possible ways of dividing Θ into at most N sub-intervals. That is, setting $a_0 := \theta_\ell$,

$$\Delta_N \;=\; \left\{(a_1,\ldots,a_N)\in\Theta^N \;:\; a_{i-1}\le a_i, \quad i=1,\ldots,N \quad \text{and} \quad a_N=\theta_h\right\}.$$

Given $\mathbf{a}_N\in\Delta_N$ and any vector $\mathbf{x}=(\xi_1,\ldots,\xi_N)\in\mathbb{R}^N$ let

$$\Pi_N(\mathbf{x},\mathbf{a}_N) \;=\; -\sum_{k=1}^{N}\int_{a_{k-1}}^{a_k}\ell_p(|\xi_k-\theta|)f(\theta)\,\mathrm{d}\theta. \tag{19}$$

$\Pi_N(\mathbf{x},\mathbf{a}_N)$ is the ex-ante payoff of **P** *if* his information upon realization of θ is given by $\mathbf{a}_N$ and he chooses according to $\mathbf{x}$. Since ℓ_p is continuous and convex, and Δ_N is compact, each of the following is well-defined:

$$h_N(\mathbf{a}_N) \;:=\; \arg\max_{\mathbf{x}}\Pi_N(\mathbf{x},\mathbf{a}_N), \qquad a_N^* \;:=\; \arg\max_{\mathbf{a}_N\in\Delta_N}\Pi_N(h(\mathbf{a}_N),\mathbf{a}_N). \tag{20}$$

Set $\Pi_N(h(\mathbf{a}_N^*),\mathbf{a}_N^*) \;=\; \Pi_N^*$. In words, if **P** were given the option of dividing Θ into at most N sub-intervals with the knowledge that it would be revealed later which of these sub-intervals that θ lies in, he would cut Θ according to $\mathbf{a}_N^*$.

Lemma 3 $\lim\limits_{N\to\infty}\Pi_N^*=-\ell_p(0)$.

Proof. Let $\Pi_N(\mathbf{a}_N)=\Pi_N(h_N(\mathbf{a}_N),\mathbf{a}_n)$. Since, ℓ_p is uniformly continuous on Θ, for any $\epsilon>0$, there exists a $\delta>0$ such that $\ell_p(|\xi|)<\ell_p(0)+\epsilon$ whenever $|\xi|<\delta$. Consequently, for any partition $\mathbf{a}_N$ with a norm $\parallel\mathbf{a}_N\parallel<\delta$, we must have $\max_\xi\int_{a_{i-1}}^{a_i}\ell_p(|\xi-\theta|)f(\theta)\mathrm{d}\theta < (F(a_i)-F(a_{i-1}))(\ell_p(0)+\epsilon)$ and hence $\Pi_N(\mathbf{a}_N)>-\ell_p(0)-\epsilon$. Further, since $\ell_p(|\xi|)\ge\ell_p(0)$ for all ξ, we also have $\Pi_N(\mathbf{a}_N)\le-\ell_p(0)$. In other words, for any sequence of partitions $(\mathbf{a}_N)$, $\parallel\mathbf{a}_N\parallel\to 0$ implies $\Pi_N(\mathbf{a}_N)\to 0$. Therefore, the proof is complete on showing that $\lim_{N\to\infty}\parallel\mathbf{a}_N^*\parallel=0$.

Given any $\alpha<\xi<\beta$, define

$$L(\xi,\alpha,\beta) \;=\; \min_{\xi_1}\int_\alpha^\xi\ell_p(|\xi_1-\theta|)f(\theta)\,\mathrm{d}\theta+\min_{\xi_2}\int_\xi^\beta\ell_p(|\xi_2-\theta|)f(\theta)\,\mathrm{d}\theta. \tag{21}$$

Observe that L is continuous and for a fixed ξ, it is strictly decreasing in α and strictly increasing in β. Therefore, given any $\varepsilon > 0$, define

$$\delta(\varepsilon,\xi) \;=\; \max_{\theta_\ell \le \alpha \le \xi-\varepsilon, \xi+\varepsilon \le \beta \le \theta_h} \big(L(\xi,\alpha,\beta) - L(\alpha,\alpha,\beta)\big).$$

Observe that $L(\xi,\alpha,\beta) < L(\alpha,\alpha,\beta)$ for any such α,ξ,β and hence $\delta(\varepsilon,\xi) < 0$.

Now suppose, by way of contradiction, that $\lim_{N\to\infty} \parallel \mathbf{a}_N^* \parallel > 0$. It follows that there must exist a ξ and $\varepsilon > 0$ such that the interval $[\xi-\varepsilon, \xi+\varepsilon]$ is contained in some sub-interval of $\mathbf{a}_N^*$, say $[a_{k-1}^*, a_k^*]$. Now cut this interval in two at ξ and let $\hat{\mathbf{a}}_N^*$ denote the obvious partition of $N+1$ sub-intervals obtained from $\mathbf{a}_N^*$. Then,

$$\begin{aligned}
\Pi_N^* = \Pi_N(\mathbf{a}_N^*) \;&=\; -\sum_{i=1}^{k-1} L(a_{i-1}^*, a_{i-1}^*, a_i^*) - \sum_{i=k+1}^{N} L(a_{i-1}^*, a_{i-1}^*, a_i^*) \\
&\qquad -L(a_{k-1}^*, a_{k-1}^*, a_k^*) \\
&\le\; -\sum_{i=1}^{k-1} L(a_{i-1}^*, a_{i-1}^*, a_i^*) - \sum_{i=k+1}^{N} L(a_{i-1}^*, a_{i-1}^*, a_i^*) \\
&\qquad -L(\xi, a_{k-1}^*, a_k^*) + \delta(\varepsilon,\xi) \\
&=\; \Pi_{N+1}(\hat{\mathbf{a}}_{N+1}^*) + \delta(\varepsilon,\xi) \\
&\le\; \Pi_{N+1}^* + \delta(\varepsilon,\xi).
\end{aligned}$$

Not that (Π_N^*) is a non-decreasing sequence bounded above by 0 and hence its limit exists. Taking the limit as $N \to \infty$ on both sides of the above inequality, we readily arrive at the contradiction that $\delta(\varepsilon,\xi) < 0$. ■

Lemma 4 *Consider the family of CS-game(s) with a fixed bias, indexed by the bias b, in which ℓ_p is the loss function of $\boldsymbol{P}$. Then,*

$$\lim_{b\to 0} \pi_N(b) \;=\; \Pi_N^*. \tag{22}$$

Furthermore, given any integer N, for all b sufficiently close to zero, an N-equilibrium of the corresponding CS game with the bias b exists.

Proof. The N-equilibrium of the CS-game with a fixed bias 0 (see Corollary 1), if it exists, is fully characterized by the partition $\mathbf{a}^0 = (a_1^0, \ldots, a_N^0)$ given as the solution to the following system of equations for all $i = 1, \ldots, N-1$:

$$\xi_i^0 \;=\; \arg\min_{\xi} \int_{a_{i-1}^0}^{a_i^0} \ell_p(|\xi-\theta|) f(\theta)\,\mathrm{d}\theta, \tag{23}$$

$$a_i^0 \;=\; \frac{\xi_i^0 + \xi_{i+1}^0}{2}. \tag{24}$$

Now let $h_N(\mathbf{a}_N^*) = (\xi_1^*, \ldots, \xi_N^*)$. Given the definition of $\mathbf{a}_N^*$ (in (20)), upon applying the envelope theorem, we have

$$\xi_i^* = \arg\min_{\xi} \int_{a_{i-1}^*}^{a_i^*} \ell_p(|\xi - \theta|) f(\theta)\, \mathrm{d}\theta, \tag{25}$$

$$a_i^* = \frac{\xi_i^* + \xi_{i+1}^*}{2}, \tag{26}$$

where (26) is merely the first-order condition for maximizing $\Pi_N(h(\mathbf{a}_N), \mathbf{a}_N)$ with respect to $\mathbf{a}_N$. In addition, following Theorem 2 of CS, for any N, there is at most one solution satisfying (23) and (24). This implies $\mathbf{a}_N^* = \mathbf{a}_N^0$ and hence $\pi_N(0) = \Pi_N^*$. Hence an N-equilibrium exists when the bias is zero. Comparing $\mathbf{a}_N^b$, the equilibrium partition described in Corollary 1, with (23) and (24), it is clear that $\mathbf{a}_N^b \approx \mathbf{a}_N^0$ when $b \approx 0$. The Lemma now follows from these observations and CS2, Lemma A. ■

Proof of Proposition 1. Part 1 of the Proposition follows from Lemma 4. To prove Part 2, let (N_k) and (b_k) be as given in the Proposition. For any integer m, define

$$n_m = \inf_{k \geq m} N_k. \tag{27}$$

Since $n_m \leq N_k \leq N_{b_k}^*$ for all $k \geq m$, we know that for all such k, an n_m-equilibrium exists when the bias is b_k with the property that

$$\pi_{N_k}(b_k) \geq \pi_{n_m}(b_k) \quad \text{(by CS1, Lemma A)}, \tag{28}$$

$$\text{and therefore} \qquad \liminf_{k \to \infty} \pi_{N_k}(b_k) \geq \Pi_{n_m}^* \quad \text{(by Lemma 4)}. \tag{29}$$

Now take the limit as $m \to \infty$ in the above inequality to deduce from Lemma 3 that the above RHS converges to $-\ell_p(0)$, since $\pi_{N_k}(b_k) \leq -\ell_p(0)$. ■

Proof of Lemma 1. First suppose Condition A holds because **A** has quadratic preferences. Then, $\varphi(x) = (x - \mu_b)^2 + \sigma_b^2$. Clearly, φ achieves a minimum at μ_b and is symmetric about it.

Now consider the case where Condition A holds because G is a symmetric probability distribution. Then, the density satisfies $g(\mu_b + b) = g(\mu_b - b)$. Let $\mu_b - B$ and $\mu_b + B$ denote the end points of the support of G. Then write

$$\begin{aligned} \varphi(\mu_b + x) &= \int_{\mu_b - B}^{\mu_b + B} \ell_a(|\mu_b + x - b|) g(b)\, \mathrm{d}b \\ &= \int_0^B (\ell_a(|x - t|) + \ell_a(|x + t|)) g(\mu_b + t)\, \mathrm{d}t \\ \text{and} \quad \varphi(\mu_b - x) &= \int_0^B (\ell_a(|-x - t|) + \ell_a(|-x + t|)) g(\mu_b + t)\, \mathrm{d}t. \end{aligned}$$

Due to symmetry of ℓ_a around zero, $\ell_a(|-x-t|) = \ell_a(|x+t|)$ and $\ell_a(|-x+t|) = \ell_a(|x-t|)$. Hence $\varphi(\mu_b - x) = \varphi(\mu_b + x)$, i.e. φ is symmetric about μ_b. That $b^* = \mu_b$ now follows from the fact that φ is symmetric about μ_b and convex (because $\ell_a(.)$ is convex). ■

Proof of Lemma 2. Choose any equilibrium strategy profile (σ_a, σ_p). At θ, **P** hears the report $m = \sigma_a(\theta)$. The support of his posterior is $\sigma_a^{-1}(m)$. His expected loss from selecting an action ξ' after observing $\boldsymbol{s}$ is proportional to

$$\int_{\theta\in\sigma_a^{-1}(\xi)} \ell_p(|\xi' - \theta - \boldsymbol{w}_p \cdot \boldsymbol{s}|) f(\theta)\, \mathrm{d}\theta.$$

The best-response property requires choosing an action that minimizes the above expression. Now define ψ as follows:

$$\psi(\theta) \;:=\; \arg\min_{\xi'} \int_{\theta\in\sigma_a^{-1}(\xi)} \ell_p(|\xi' - \theta|) f(\theta)\, \mathrm{d}\theta. \tag{30}$$

Comparing the minimand expression in (30) with the payoff of **P** given above, we have $\sigma_p(m, \boldsymbol{s}) = \psi(\theta) + \boldsymbol{w}_p \cdot \boldsymbol{s}$. ■

The proof Proposition 3 and Lemma 5 below require the following preliminaries. Fixing **P**'s equilibrium behavior, the payoff of **A** of type θ from mimicking to be type θ' and sending a signal $\xi' = \psi(\theta')$ is $-\varphi(\xi' - \theta)$.

For any $\xi_1 = \psi(\theta_1) < \psi(\theta_2) = \xi_2$, write

$$D(\theta, \xi_1, \xi_2) \;=\; \frac{\varphi(\xi_2 - \theta) - \varphi(\xi_1 - \theta)}{\xi_2 - \xi_1}. \tag{31}$$

Therefore, a type θ would prefer to mimic being type θ_2 instead of type θ_1 provided $(\xi_2 - \xi_1)D(\theta, \xi_1, \xi_2) \leq 0$ and conversely otherwise. $D(\theta, \xi, \xi')$ is the slope of φ between the points $\xi - \theta$ and $\xi' - \theta$. Since φ is strictly convex, this slope must be *decreasing* in θ.

Lemma 5 *Consider an equilibrium where $\psi(\theta_1) = \xi_1$ and $\psi(\theta_2) = \xi_2$ are such that $\xi_1 \neq \xi_2$. Then $|\xi_1 - \xi_2| \geq |b^*|$.*

Proof. Assume, with no loss in generality, that $\xi_1 < \xi_2$. Given **P**'s behavior described by Lemma 2, the payoff of a type θ from reporting ξ_i is $-\varphi(\xi_i - \theta)$. By incentive compatibility of equilibrium behavior of θ_1, we must have $D(\theta_1, \xi_1, \xi_2)(\xi_2 - \xi_1) \geq 0$, and similarly incentive compatibility of θ_2 requires $(\xi_2 - \xi_1)D(\theta_2, \xi_1, \xi_2) \leq 0$. By continuity of $D(\cdot, \xi_1, \xi_2)$, there must exist some $\theta^* \in [\theta_1, \theta_2]$ such that $D(\theta^*, \xi_1, \xi_2) = 0$. By monotonicity, those types to the right of θ^* would strictly prefer to report ξ_2 and those to its left strictly prefer to report ξ_1. Therefore, when **P** hears ξ_1 or ξ_2, he knows the true type is respectively bounded above or below by θ^*. Looking at the definition of ψ in (30), we can readily conclude that

$$\xi_1 \ \leq \ \theta^* \ \leq \xi_2. \tag{32}$$

Furthermore, since φ is convex with a minimum at b^*, for $D(\theta^*, \xi_1, \xi_2)$ to be zero, we must have

$$\xi_1 - \theta^* \ < \ b^* \ < \xi_2 - \theta^*. \tag{33}$$

Combining (32) and (33), we obtain

$$\begin{aligned} \xi_1 < \theta^* < \theta^* + b^* < \xi_2 \qquad & \text{for } \ b^* > 0, \\ \xi_1 < \theta^* + b^* < \theta^* < \xi_2 \qquad & \text{for } \ b^* < 0, \end{aligned}$$

implying $\xi_2 - \xi_1 \ \geq \mid b^* \mid$. Since $b^* \neq 0$ and Θ is a compact set, it follows that there can only be finitely many outcomes in any equilibrium. ■

Proof of Proposition 3. Pick an EOF Y. It follows from Lemma 5 and Lemma 2 that in any EOF, there are finitely many values, say $\xi_1 < \xi_2 < \cdots < \xi_N$, such that $Y(\theta, \boldsymbol{s}) = \xi_i + \boldsymbol{w}_p \cdot \boldsymbol{s}$ for $1 \leq i \leq N$. Each ξ_i is a possible report that will be sent by some θ in the equilibrium. We now determine which types send a given report ξ_i. To this end, with no loss in generality assume that $N \geq 2$ and first define a_i by the equation

$$\varphi(\xi_i - a_i) \quad = \quad \varphi(\xi_{i+1} - a_i) \qquad \text{for } i = 1, \ldots, N-1. \tag{34}$$

Next, recall that the loss of a type θ from reporting ξ_i, given the behavior of $\mathbf{P}$, is $\varphi(\xi_i - \theta)$ and hence $(\xi_{i+1} - \xi_i) D(\theta, \xi_i, \xi_{i+1})$ is the difference in type θ's payoff from reporting ξ_{i+1} instead of ξ_i. Since $D(\cdot, \xi_i, \xi_{i+1})$ is decreasing, all the types in $[\theta_\ell, a_i)$ would strictly prefer reporting ξ_i instead of ξ_{i+1} whereas the opposite is true for types in $(a_i, \theta_h]$ and type a_i, by its definition above, is indifferent between either report. Consequently (a_{i-1}, a_i) is the set of types that strictly prefer to report ξ_i to any of the other reports. Type a_i is indifferent between reporting ξ_i and ξ_{i+1}, and strictly prefers those over any other report. Therefore, (30) reduces to

$$\xi_i = \arg\min_\xi \int_{a_{i-1}}^{a_i} \ell_p(\xi - \theta) f(\theta) \, \mathrm{d}\theta \quad \forall \, i = 1, \ldots, N. \tag{35}$$

The proof of the Proposition is now complete upon recalling that (34) and (35) are precisely the conditions that describe an N-equilibrium in the CS-game with the stated payoff functions. ■

Proof of Proposition 6. By virtue of Lemma 1, Corollary 1 and Proposition 3, the payoff from retaining authority and playing the SITU game is the same as $\mathbf{P}$'s payoff in the CS-game with quadratic loss functions. In an N-equilibrium of the latter, the expected loss of **P** is simply the residual variance of θ that **P** expects after hearing **A**'s report. That is, if $\xi_N^* : \Theta \longrightarrow \mathbb{R}$ denotes the equilibrium outcome function of the CS-game, then **P**'s loss from retaining authority, as shown in Section 4 of CS with the bias $b = \mu_b$, is

$$E\left[\ell_p(\xi_N(\theta))\right] = \frac{1}{12N^2} + \mu_b^2 \frac{(N^2-1)}{3}.$$

On the other hand, if **P** delegates, **A** chooses the action $\theta + \boldsymbol{w}_a \cdot \boldsymbol{s}$ in state $(\theta, \boldsymbol{s})$ leaving **P** with an ex-post payoff of $\ell_p(b_{\boldsymbol{s}})$. Given that ℓ_p is assumed to be quadratic, the loss from delegation is $\mu_b^2 + \sigma_b^2$. Retaining authority is therefore a superior choice whenever

$$\begin{aligned} \mu_b^2 + \sigma_b^2 &> \frac{1}{12N^2} + \mu_b^2 \frac{(N^2-1)}{3} \\ \Leftrightarrow \qquad \sigma_b^2 &> \frac{1}{12N^2} + \mu_b^2 (\frac{(N^2-4)}{3}). \end{aligned} \tag{36}$$

(36) establishes Part 2. For Part 1, recall that CS show that the existence of an equilibrium which divides Θ into N sub-intervals requires that

$$N \leq -\frac{1}{2} + \frac{1}{2}(1 + \frac{2}{\mu_b})^{1/2} \quad \text{or equivalently,} \quad \mu_b \leq \frac{2}{(2N+1)^2 - 1}.$$

Therefore

$$\begin{aligned} \sigma_b^2 &> \frac{1}{12N^2} + (\frac{2}{(2N+1)^2-1})^2 \frac{(N^2-4)}{3} \\ &= \frac{(2N^2+2N-3)}{N^2(N+1)^2} \sigma_\theta^2, \end{aligned}$$

which completes Part 1. ■

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