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## Holdup: Investment Dynamics, Bargaining and Gradualism

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## Holdup: Motivating example

- What is holdup?
  - *Motivating example.* One buyer, one seller.
  - The seller may, or may not make a *sunk* investment, costing  $C$ . Value from sale is  $\phi_I$  in case of investment, and  $\phi_N$  otherwise.
  - Investing is efficient:

$$\phi_I - C > \phi_N. \quad (1)$$

- Suppose the bargaining power is symmetric. Given that investment is sunk, they will bargain over the gross returns.
- *Holdup!* No investment provided

$$\frac{\phi_I}{2} - C < \frac{\phi_N}{2}. \quad (2)$$

## Holdup

- *Holdup: A classical problem*  
An investor has to make *sunk* investments whose returns are vulnerable to being expropriated.  
⇒ under-investment.
- Intrinsic to many situations:
  - *Bilateral exchanges*: Investment in *specific assets* whose benefits are later shared through negotiations:
    - Firms and workers,
    - Manufacturers and suppliers,
    - *Political lobbying* - campaign contributions as sunk investments.
  - *Team production*.

## Holdup: Resolutions

- The literature takes the holdup problem very seriously, suggesting various safeguards against it:
  - *Vertical integration* - aligns the interests of the investor and the expropriator, e.g. Klein, Crawford, Alchian, JLE, 1978, Williamson (1979).
  - *Hierarchical authority* - puts the investor in control, so that she cannot be expropriated, Aghion and Tirole, JPE, 1997.
  - *Reputation and relational contracts* - repeated interactions, Baker, Gibbons, Murphy, QJE, (2002).
- In this paper we however focus on a relatively recent branch of the literature on holdup, that relies on the idea of *gradualism*.

## Resolutions continued: Gradualism

- *Gradualism*: The efficient investment is broken up into several installments, with each round of investment being followed by reimbursements.
  - Marx and Matthews, RES (2000), Lockwood and Thomas, RES (2002), Pitchford and Snyder, JET (2004).
  - In the motivating example, break  $C$  into three equal parts. Once  $C/3$  is invested, the seller is reimbursed, and so on. Thus at the last step the seller invests provided:

$$\phi_I/2 - C/3 > \phi_N/2. \quad (3)$$

## Resolutions continued: Gradualism

- Observed in practice:
  - *Staged procurement contracts*: allows a party to end the process conditional on past experience.
    - Used for billions of dollars of procurement in the US, from construction of passenger railroads in Atlanta, to affordable housing in Baltimore.
    - Job order contracting (JOC)/ Delivery order contracting (DOC)/ Simplified acquisition of base engineering requirements (SABER).

## Resolutions continued: Gradualism

- The idea of gradualism goes back to Schelling (1960, Strategy of Conflict).
- Has been used in other contexts:
  - Gradual contributions to a *public good*, Admati and Perry, RES, 1991.
  - Gradual *concessions in bargaining*, e.g. Compte, Jehiel, RES, 2004.
  - *Micro-finance*: (Gradual) Repayment schemes involve weekly repayments of small amounts, e.g. Chowdhury, Roy Chowdhury and Sengupta (2011), Fisher and Ghatak (2011).

## Holdup in the presence of Investment Dynamics

- *Issue*: Is holdup necessarily very serious, especially in the presence of dynamic interactions?
- Che-Sakovicz (Econometrica, 2004): No!
- Consider the earlier motivating example. Embed it in a dynamic framework with the following three features:
  - ① *Investment dynamics*, i.e. the possibility of future investments.
  - ② *Bargaining* over the existing pie.
  - ③ *Individual rationality*:

$$\frac{\phi I}{2} - C > 0. \quad (4)$$

- There exists an equilibrium where investment takes place, even though a purely static logic suggests that it should not.



## Investment Dynamics: Che-Sakovicz

- Formally, consider an infinite horizon framework:
  - Time is discrete and goes from  $1, 2, \dots, \infty$ .
  - Let  $\delta$ ,  $0 < \delta < 1$ , denote the common discount factor.
  - At every period there are two stages.
    - *Stage 1*: The seller can invest  $C$ , assuming she has not already done so.
    - *Stage 2*: There is bargaining following a random offers protocol (Binmore, 1987), where each agent is selected as the proposer with equal probability.
- Che-Sakovicz (2004):

Suppose investing is *individually rational*, i.e.  $\frac{\phi_I}{2} - C \geq 0$ .  
Then  $\exists \delta^* < 1$ , such that  $\forall \delta > \delta^*$ , the efficient outcome, i.e. investing, can be implemented as a Markov perfect equilibrium.
- *Implications*: Holdup may be resolved provided the *individual rationality* condition is satisfied.

## Investment Dynamics: Che-Sakovicz

- *Idea*: The possibility of future investment changes the *reference payoff from not investing*, thus making investment more attractive.
  - In a static model, the payoff from not investing is  $\frac{\phi_N}{2}$ .
  - Whereas under a dynamic framework its  $\phi_N - \frac{\delta\phi_I}{2}$ .
    - Consider the following *Markov strategy*: The seller invests immediately, if she has not already done so.
    - The payoff from not investing (in case the seller becomes the proposer) is now  $\phi_N - \frac{\delta\phi_I}{2}$ .
  - This is less than  $\phi_N/2$  for  $\delta$  large.
  - Thus, under the dynamic framework, the strategy of not investing is less attractive. This follows since the buyer asks for a lot, as she anticipates that there will be investment in the next period.

## Investment Dynamics: Che-Sakovicz

- *Remarks:*
  - Che-Sakovicz proves the result for a general production function  $\phi(b, s)$  - the buyer invests  $b$ , and the seller invests  $s$ .
  - The result is *not* a folk theorem, as the game is not a repeated one - ending as soon as an agreement is reached.
  - Does not require investment to be divisible, unlike the *gradualism* literature.

## The Research Questions

- *The Research Questions:* Given Che-Sakovicz:
  - Is there still a role for gradualism?
  - Is individual rationality necessary for reaching efficiency?
    - Important as individual rationality is likely to be violated in many cases, e.g. if the bargaining powers are asymmetric.
- This paper argues that a natural modification of the Che-Sakovicz framework allows one to address both these issues at the same time.

## The Research Questions

- Suppose the agents are allowed to make *pecuniary transfers* to each other:
  - Then, if the seller alone can invest, then 'efficiency' obtains *unconditionally*, i.e. irrespective of whether IR holds or not.
  - Further, if the buyer can also invest, and the investments are *substitutes*, then we get back *gradualism* - but for a reason very different from that in the literature.

## The Framework

- Extend the motivating example by allowing for:
  - (a) continuous investments, and
  - (b) generalized bargaining power.
- The project value is  $\phi(s)$  in case the seller invests an amount  $s$ :
  - $\phi(s)$  is increasing and concave in  $s$ ,  $\phi(0) = 0$ , and satisfies the Inada conditions. Cost of investment  $s$ .
- Let  $s^*$  be the efficient level of investment where

$$s^* = \operatorname{argmax} \phi(s) - s.$$

- Let the seller's bargaining power be denoted by  $1 - \alpha$ .

## The Framework

- Infinitely repeated game,  $\delta$  common discount factor of all agents,  $0 < \delta < 1$ .
- Every period there are three stages:
  - *Stage 1*: The buyer can make a non-negative transfer to the seller.
  - *Stage 2*: The seller decides on how much to invest.
  - *Stage 3*: There is random offers bargaining, with the buyer being the proposer with probability  $\alpha$ .

## The Framework: Preliminary Results

- Let

$$s_\delta = \operatorname{argmax} (1 - \alpha)\phi(s) - (1 - \alpha\delta)s.$$

- $s_\delta$  maximizes the seller's payoff when she (a) has a starting investment of  $s$ , (b) is planning to increase her immediate investment to  $s'$ , and (c) follows the Markov strategy that in the next period will increase her investment to  $s_\delta$ .
- $s_\delta$  is increasing in the discount factor  $\delta$ , and  $s_\delta|_{\delta \rightarrow 1} = s^*$ .
- *Importance*: if we can show that there is an equilibrium where  $s_\delta$  can be sustained in at most a 'few' steps, then we are done.



## The Framework: Preliminary Results

### Proposition (Che-Sakovicz: Continuous seller investment)

Let  $IR$  be satisfied, i.e.  $(1 - \alpha)\phi(s^*) - s^* > 0$ . Then, for  $\delta$  large, we can sustain an investment of  $s_\delta$  in the first period.

- *Markov perfect investment strategies:* At any period  $t$  with initial investment of  $s$ , the seller's strategy is to invest till  $s_\delta$  in case  $s < s_\delta$ , otherwise no further investment.
- Why  $s_\delta$ ? It maximizes the seller's expected payoff if she is planning to increase her investment level from  $s$  to  $s'$ ,  $s' \leq s_\delta$ .

## The Framework: Preliminary Results

- Checking for optimality:

- Say starting level  $s$ ,  $s < s_\delta$ , deciding what level of  $s'$  to invest at. Then expected payoff

$$\begin{aligned}(1 - \alpha)[\phi(s') - \delta\alpha\phi(s_\delta)] + \alpha\delta[(1 - \alpha)\phi(s_\delta) - (s_\delta - s')] - (s' - s) \\ = (1 - \alpha)\phi(s') - s'(1 - \alpha\delta) - \alpha\delta s_\delta + s.\end{aligned}$$

- Say,  $s > s_\delta$ . The payoff of the seller from increasing the investment to  $s' > s$  is

$$(1 - \alpha)\phi(s') - (s' - s),$$

which is maximized at  $s_0$ . The result follows as  $s_0 < s_\delta \leq s'$ .

## Asymptotic Efficiency when Individual Rationality Fails

- Let *Individual Rationality fail*:  $(1 - \alpha)\phi(s^*) - s^* < 0$ .
- Let  $\bar{s}$  solve:

$$(1 - \alpha)\phi(s_\delta) - \bar{s} = 0,$$

so that in case the seller has to invest only  $\bar{s}$  (or less), to reach  $s_\delta$ , then it is individually rational for the seller to do so.

## Asymptotic Efficiency when IR Fails

### Proposition (1)

*For  $\delta$  sufficiently large, an asymptotically efficient equilibrium exists, where an aggregate investment of  $s_\delta$  can be attained in at most two periods. Along the equilibrium path:*

- At  $t = 1$ , Stage 2: the seller invests  $s_1 = s_\delta - \bar{s}$ .
- At  $t = 2$ :
  - Stage 1: the buyer transfers  $\frac{s_1}{\delta}$  to the seller.
  - Stage 2: the seller invests  $\bar{s}$ .
  - Stage 3: the agents bargain using the random offers protocol.

## Sketch of Proof

- *Sketch of proof:*
  - *Period 1:*
    - *Stage 2:* If the seller invests less than  $s_\delta - \bar{s}$ , then there is no reimbursement, and the seller makes up the investment in the next period.
    - *Stage 3:* For  $s_1 = s_\delta - \bar{s}$ , can be shown that the selected proposer makes an *unacceptable offer* to the responder.
  - *Period 2, Stage 1:* If the seller invests to  $s_\delta - \bar{s}$ , but the buyer does not reimburse, then no further investments in this period, and the buyer reimburses in the next period.

## Implications

- *Implications:*
  - Efficiency is 'asymptotic' as
    - (i)  $s_\delta < s^*$ , and
    - (ii) agreement is reached at the *second* period.
  - For  $\delta$  large, however, both these inefficiencies are small.
  - Significantly extends the Che-Sakovicz analysis, showing that the holdout problem may be resolved even when there are individual rationality issues.
  - The reason for 'delay' is very different from that under gradualism - it is to ensure that individual rationality is not an issue.

## Gradualism: Buyer Investments

- Analysis so far addresses the first question, as to whether 'efficiency' can be sustained without IR.
- Next turn to the second question, i.e. whether one can sustain gradualism. Allow the buyer to invest also.
- We show that in this case:
  - The result critically depends on whether these investments are *substitutes*, or *complements*, and
  - Interestingly, there is a role for gradualism if the investments are substitutes, but not otherwise.

## Substitutes

- Consider the case where the buyer also can invest an amount  $s$ , at a cost of  $\beta s$ , where  $\beta > 1$ .
  - Since  $\beta > 1$ , efficiency demands that the seller alone invests.
  - Modify the earlier game so that at stage 2 of every period, the buyer and the seller simultaneously decide on how much to invest.
- Let IR fail:  $(1 - \alpha)\phi(s^*) - s^* < 0$ .
- *The issue*: If the seller invests too much, then the buyer may have an incentive to complete the project using her 'inefficient' technology and then bargain, rather than paying the buyer the amount due from earlier investments.



## Substitutes

### Proposition (2)

*Suppose the buyer is not very efficient, i.e.  $\beta\bar{s} > \frac{s_1 - \bar{s}}{\delta}$ . Then the asymptotically efficient equilibrium described in Proposition 1 earlier can be implemented for  $\delta$  large.*

- Consider the strategies described in Proposition 1. The only possible deviation will be in period 2. Suppose the buyer refuses to pay the seller for the previous investment and instead does the investment herself (once she does it then there will be bargaining over  $\phi(s_\delta)$ ). The buyer will not deviate iff

$$\beta\bar{s} > \frac{s_1}{\delta}.$$

## Buyer investment is a substitute

- Hence we restrict attention to the interesting case where

$$\beta\bar{s} < \frac{s_\delta - \bar{s}}{\delta}.$$

### Proposition (3)

Let  $\beta\bar{s} < \frac{s_\delta - \bar{s}}{\delta}$ . Then, for  $\delta$  sufficiently large, an investment of  $s_\delta$  can be implemented using a gradual investment scheme (with the seller alone making the investment).

- *Implication:*
  - *Role for Gradualism:* Resolving the holdout problem may require gradualism in case (a) the seller faces an individual rationality constraint, and (b) the investment are substitutes.

## Gradualism: Proof

- *Proof by Construction.* We define an  $n$ -period *Investment with Monetray Transfer* scheme involving *gradualism*:

$$I_n = \langle (p_1, s_1), \dots, (p_n, s_n) \rangle,$$

where, for every time period  $i$ ,  $s_i$  denotes the investment made by the seller, and  $p_i$  denotes the pecuniary transfer made by the buyer to the seller.

- The scheme is constructed as follows:
  - No payment at  $t = 1$ , and in the last period the quantum of investment is  $\bar{s}$ . Thus  $p_1 = 0$  and  $s_n = \bar{s}$ .
  - At every period the seller is recompensed for the investment she did in the last period, so that  $p_i = \frac{s_{i-1}}{\delta}$ , where  $i \geq 2$ .
  - At  $t = n$ , the *aggregate* investment reaches  $s_\delta$ , whence the buyer and the seller reach an immediate agreement.

## Gradualism: Proof

- It remains to construct the investment sequence till  $s_{n-1}$ .
- The idea is to construct it in such a way such that at every  $i$ , the buyer is indifferent between making the promised payment, and doing the investment herself and completing the project:

- $s_{n-1}$  solves:

$$\frac{s_{n-1}}{\delta} = \beta \bar{s}.$$

- $s_{n-2}$  solves:

$$-\frac{s_{n-2}}{\delta} - s_{n-1} + \alpha \delta \phi(s_\delta) = \alpha \phi(s_\delta) - \beta(s_{n-1} + \bar{s}).$$

- We proceed inductively, with  $s_1$  being just enough such that  $\sum_1^{n-1} s_i = s_\delta - \bar{s}$ .

## Gradualism: Properties

- The investments decrease from the second step onwards - similar to Pitchford Snyder (2002). For  $\delta$  close to 1:
  - $s_{n-1} = \beta\bar{s}$ ,
  - $s_{n-2} = \beta^2\bar{s}$ , etc.
- However, in this paper, the reason for such a structure is that it prevents the buyer from investing herself (which is inefficient), whereas in Pitchford-Snyder (2002) this is to ensure that the seller herself has incentive to invest.
- Consequently, a finite scheme exists.
  - Pitchford Snyder (2002) - “to avoid unravelling there can be no known finite end to the number of installments,” not true in our context - as we can invoke Che-Sakovicz to implement  $\bar{s}$  in the last period.

## Gradualism: Properties

- While this scheme generates gradualism, in the sense of sequential investment and payments, it is possible that the lag between subsequent periods is very small, so that everything happens very quickly.
  - Thus this theory generates 'delay' provided we make the additional assumption that every single transaction takes some time.
  - True for Pitchford Snyder (2002) also.

## Gradualism: Conjectures

- Let  $\hat{n}$  denote the least number of periods in which  $s_\delta$  can be implemented. Given Proposition 3,  $\hat{n}$  is well defined.
- Then we have the following *conjectures*:
  - For any  $n > \hat{n}$  we can construct a gradual scheme involving  $n$  periods.
  - The scheme described in Proposition 3 involves exactly  $\hat{n}$ .
  - $\hat{n}(\beta)$  is decreasing in  $\beta$ , and  $\lim_{\beta \rightarrow 1} \lim_{\delta \rightarrow 1} \hat{n}(\beta) = \frac{s^*}{s}$ .

## Complementary Investments

- Suppose the buyer can make investments that are 'complementary' to that made by the seller.
- Denoting the buyer's investment by  $b$ , let the production function be  $b\phi(s)$ , where  $b \in [0, 1]$  and the cost of investing  $b$  is  $b$ .
- Let the efficient outcome involve  $s = s^*$  and  $b = 1$ .
- The game is as earlier.
- Let

$$(1 - \alpha)\phi(s^*) - s^* < 0.$$



## Complementarity in investments

- Let  $s_\delta$  and  $\bar{s}$  be defined as earlier.

### Proposition (4)

Let  $\delta$  be large. An outcome where the aggregate investment reaches  $s_\delta$  at  $t = 2$  can be sustained. Along the equilibrium path:

- At  $t = 1$ , Stage 2: the seller invests  $s_1 = s_\delta - \bar{s}$ .
- At  $t = 2$ :
  - Stage 1: the buyer transfers  $\frac{s_1}{\delta}$  to the seller.
  - Stage 2: the seller invests  $\bar{s}$  and the buyer invests  $b = 1$  (sustain using Che-Sakovicz).

## Complementarity in investments

- Intuition:
  - Given that the investments are complementary, as  $b$  increases, the seller's incentive to invest increases.
  - Given efficiency, and that the seller's IR fails, it follows that the buyer's IR condition is satisfied for  $\delta$  large, so that  $\alpha\phi(s_\delta) - 1 > 0$ , so that the buyer's payoff is increasing in  $b$ .

## Summary

- This paper examines the holdup problem in a *dynamic* framework, that allows one to study the *interaction* between investment dynamics and gradualism.
- We find that:
  - In case only one of the agents can invest, *holdup is not too serious!*
    - 'asymptotic efficiency' obtains *unconditionally*, i.e. irrespective of whether individual rationality holds, or not.
    - carries forward the theme in Che-Sakovicz, arguing that in many situations vertical integration etc. may not be required.
  - In case both the agents can invest, and the investments are substitutes, then 'efficiency' may require gradualism.
    - The role of gradualism is to prevent over-investment. This is in contrast to the literature where gradualism prevents under-investment.

Thanks!

Thanks!