Menu Contracts in Teams

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ABSTRACT

The existing literature on moral hazard in teams consider two types of team production technology: (i) deterministic technology; (ii) non-deterministic technology where the random factor is unobservable *expost* and cannot be contracted upon. However, we often observe non-deterministic team production settings where the random element can be observed *expost* after the resolution of the uncertainty. In such situations it is natural to allow the sharing rule agreed *exante* by the team to also depend on the observed realisation of the random element. Thus, we examine a moral hazard in team problem in which the share of each team member is a function of the observed realisations of both the final output and the random element. In the case of fairly general utility functions and production technology, we provide a necessary and sufficient condition for implementing an outcome. When the utility functions are quasi-linear and the production function has a specific property, we also show that the efficient outcome cannot be implemented. Once we restrict attention to the class of sharing rules where the first-order approach is applicable, even with the more general utility functions and production technology, we are able to show that it is sufficient to consider only the class of sharing rules that are linear in the final output. As a consequence of this result, we also show that efficient outcomes are not implementable in the more general case if we only consider sharing rules where the first-order approach is applicable.

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1. INTRODUCTION

The phenomenon of moral hazard in teams can be observed within a variety of situations: partnerships, labour-managed firms, share cropping arrangements, non-point source pollution, etc. In all of these situations an aggregate measure, which can be output, profits or ambient pollution is the only observable and verifiable indicator of inputs or emission levels. Moral hazard in teams arises because while the total welfare of the team would be higher if all team members exerted high levels of effort, there is an incentive for each member to exert less effort because such effort is costly. Since individual effort levels cannot be observed (at reasonable cost) but only the final output, identification and subsequent punishment of the shirkers is not possible. The scenario just described can apply equally well to the non-point source pollution context. Individual emissions are unobservable but aggregate or ambient pollution level can be observed through monitoring of the receptor. Polluters are treated as a group which must share among themselves the cost of environmental damage due to the ambient level of pollution. Introducing uncertainty in the relationship between inputs/emissions and output/pollution further compounds the problem of moral hazard as individuals or firms can hide behind the veil of uncertainty concerning who was at fault (Holmstrom (1982)).

The literature on moral hazard in teams focuses on ways to design appropriate incentive mechanisms to mitigate the incentive to free-ride and so achieve an efficient outcome (e.g. Alchian and Demsetz (1972), Eswaran and Kotwal (1984), Holmstrom (1982), Legros and Matsushima (1991), Legros and Matthews (1993), Radner and Williams (1992), Rasmusen (1987)). The literature on non-point source pollution has focused on how to apply the theory of moral hazard in teams to the environmental context (e.g. Herriges *et al.* (1994), Segerson (1988), Strand (1999)).[2] Where perfect monitoring is possible, the efficient outcome is achievable by employing a forcing contract which penalises cheating (Holmstrom (1979)). More realistically, perfect monitoring is usually impossible or prohibitively costly and instead, imperfect indicators of individual effort or emissions such as final output or ambient pollution are used as a basis for contracting. The contract may be improved upon if other factors which yield information about actions can be included within the information base (e.g. Nandeibam (2003) widened the information base of the sharing rule to include intermediate as well as final outputs.).

Uncertainty enters into the analysis through the relationship between inputs and output. In the traditional setting, it is implicitly assumed that it is not possible to disentangle the random component affecting how actions are converted into output from the actions themselves even after the uncertainty is resolved, so a low output cannot be taken to imply inadequate actions. An example of this is Eswaran and Kotwal (1985) who suggest weather as a possible stochastic variable which can alter the relationship between inputs and agricultural output. In that case,

[2]Both Segerson's and Strand's contributions allow for budget-breaking, thereby enabling the design of incentive mechanisms to induce polluters to act efficiently.

a bad harvest could be due to adverse weather conditions and not lack of effort. Their analysis does not allow for the fact that adverse weather conditions are observable *expost*, information which could be taken into account when designing contracts. Thus, we maintain that in certain situations it may be possible to disentangle the realisation of the random element from the actions *expost*, thereby allowing it to be observed. In the non-point source pollution context, ambient pollution is a function of individual emission levels and stochastic factors such as rainfall. Prior to emitting pollutants future rainfall in unknown, but it becomes known at the time of monitoring the ambient level of pollution, because rainfall events contributing to the ambient level of pollution have already occurred. We can think of other contexts which share a similar characteristic as that just described. Consider a team's decision on the levels of inputs other than individual actions to be employed to generate maximum profits. The optimal mix of inputs will depend on the expected prices of inputs and outputs in the market. Once the final output is generated and sold, the profits as well as the realised input and output prices are observable. Eswaran and Kotwal (1985) give several examples of this within the agrarian context, where the choice of crops is dependent on a range of factors such as expected prices, water availability, etc. By acknowledging that there is a wide range of situations in which the random element is observable *expost*, we depart from the standard analysis in which the sharing rule is based solely on the final joint output. This allows us to provide a more general analysis.

The model we consider closely resembles the non-deterministic team production model in Holmstrom (1982). The moral hazard problem is caused by joint participation of the team members in the production process and a random element. However, unlike Holmstrom (1982), we assume that after the resolution of the uncertainty the random element can be observed *expost*, and hence, each member's share of the final output is a function of the observed final output itself and the random element. An outcome of our model is represented by the actions chosen by the team members and a distribution of the final output corresponding to each realisation of the random element. The actual production is a noncooperative game played prior to resolution of the uncertainty among the team members conditional on the sharing rule they adopt. This means that, whatever criteria we use to select an outcome, at the very least, it must belong to the class of outcomes that are implementable noncooperatively. An outcome is implementable noncooperatively if it can be realised in pure strategy Nash equilibrium of the game played in production conditional on some sharing rule.

Our first result provides a necessary and sufficient condition for implementing a given outcome under fairly general utility functions and production technology. Roughly speaking, this condition imposes restrictions on the set of unilateral deviations from the given outcome that can be caused by any member of the team and this allows sufficient punishments to be incorporated into the sharing rule to deter such deviations. Using this result, our second result shows that, when the utility functions are quasi-linear and the production technology has a specific property,[3] efficient outcomes cannot be implemented. For our remaining two results, we consider the more general utility functions and production tecnology but concentrate on the case where the first-order approach is applicable. In this case we first show that any implementable outcome can be implemented using a sharing rule which is linear in the final output level. This result can be contrasted with those of Kim and Wang (1998) and Nandeibam (2002). In the former, Kim and Wang show that under uncertainty the linear sharing rule result holds only when the production technology is of a special form and the team members are risk neutral. On the otherhand, Nandeibam shows that the linear sharing rule result is valid with fairly general utilities and production technology when there is certainty. As a consequence of our linear sharing rule result, we are also able to show that efficient outcomes are not implementable in the general case by using sharing rules for which the first-order approach is valid.

In the next section we describe our model and define the notion of implementability of an outcome. In the third section we present a necessary and sufficient condition for implementing a given outcome. In this section we also show that efficient outcomes are not implementable in a special case with quasi-linear utilities and a specific production function. In section 4 we consider the class of sharing rules where the first-order approach is applicable and show that in this case any outcome that is implementable can be implemented using a sharing rule which is linear in output. Using this result, we also show that efficient outcomes cannot be implemented by using sharing rules where the first-order approach is applicable. We conclude in the final section.

2. THE MODEL

In what follows we have two types of cases in mind, although as noted above there are others. In the traditional case we consider a team of workers each exerting effort which when combined produces a final output. We can think of this scenario as a game taking place in two stages. Prior to taking part in the production process, the workers agree, possibly through a process of bargaining, on how they will share the final output *expost*. Conditional on the sharing rule agreed in stage one, the workers choose their efforts non-cooperatively in the second stage. The second case we consider is a team of polluters whose individual emissions are unobservable. These emissions contribute towards an observable ambient level of pollution which produces environmental damages that are costly to society. Here again, invoking the polluter pays principle, we can think of a two stage process where the polluters first agree on how to allocate the total cost of environmental damages among themselves and then choose their respective emission levels non-cooperatively in the second stage.

To prevent confusion, the explanation of the model follows along the lines of the team of workers case. We consider a team comprising on $N \geq 2$ individuals where each individual i's unobserv-

[3]This is the case often considered in the literature, e.g. Bhattacharyya and Lafontaine (1995), Eswaran and Kotwal (1985), Kim and Wang (1998), Romano (1994), etc.

able and unverifiable action is denoted by $a_i \in \Re_+$. The vector of actions $\mathbf{a} = (a_1, ..., a_N) \in \Re_+^N$ of the N individuals together with a random variable $\theta \in \Theta$ determine a joint monetary output according to the production function $f : \Re_+^N \times \Theta \to \Re_+$. We endow the state space Θ with a Borel field $\mathcal{F}$ and represent the distribution of the random variable θ by a probability measure μ on $\mathcal{F}$. Each individual i's utility over money and action pairs is given by $U^i(m_i, a_i)$, where $m_i \in \Re$ is i's income. Throughout this paper we maintain the following assumptions:

A1. f is continuously differentiable in the actions and f_i denotes the derivative of f with respect to the action of individual i; f is strictly increasing in the actions in the interior of $\Re_+^N$; f is concave in the actions.

A2. f is measurable with respect to the random variable.

A3. U^i is continuously differentiable and U_1^i and U_2^i respectively denote the derivative of U^i with respect to income and action; U^i is concave; U^i is strictly increasing in income and strictly decreasing in action; $U_2^i(m, 0) = 0$ and $U_2^i(m, \infty) = -\infty$.

Assumptions **A1** and **A3** are quite standard and need no further explanation. Assumption **A2** is a technical assumption.[4]

The realisation of the random variable takes place after the actions have been chosen. However, at the end of the production process, both the monetary output and the realisation of the random variable are observable.

An outcome comprises of a single action vector and one payment vector for each realisation of the random variable which allocates amongst the team members the final output corresponding to the action vector and this random variable. However, because of the problem of enforcing sharing rules that do not distribute the entire final output within the team (see Eswaran and Kotwal (1984)), we need the payments to add up to the final output in each state of the world. Thus, an outcome is $(\mathbf{a}, \{\mathbf{p}^\theta : \theta \in \Theta\})$, where $\mathbf{p}^\theta = (p_1^\theta, ..., p_N^\theta) \in \Re^N$ for each $\theta \in \Theta$ is such that $\sum_{i=1}^N p_i^\theta = f(\mathbf{a}, \theta)$.[5]

Since actions are unobservable, each individual's share of the final output can only be a function of the observable final output and the observed realisation of the random variable. So a *share function* of individual i is a function $s_i : \Re_+ \times \Theta \to \Re$, where $s_i(q, \theta)$ is the amount individual i receives when q is the final output and θ is the realisation of the random variable. An

[4]The action variable in the case of the non-point source pollution context is the emission level. The monetary transfers refer to the fee that polluters are charged to cover the cost of environmental damages. The utility function for polluter i could be written as $U^i(t_i, e_i)$, where t_i and e_i respectively refer to i's transfer payment and emission level. In the non-point source pollution case, f becomes the cost function of environmental damage caused by pollution. Since the generation of emissions in the production process produces benefits for the individual polluter, we have $U_1^i < 0$ and $U_2^i > 0$.

[5]In the non-point source pollution case an outcome consists of an emissions vector and a charge on each polluter for each realisation of the random variable such that the sum of the charges paid by all polluters must cover the total cost of environmental damages arising from emissions in each state.

individual's share function can be seen as a menu of contracts in which each contract corresponds to a realisation of the random variable and specifies the relationship between the final output and the individual's share at that realisation of the random variable. Because of the reasons mentioned above, we will require the team to balance its budget at each realisation of the random variable. Thus, a *sharing rule* is an N-tuple of share functions $\mathbf{s} = (s_1, ..., s_N)$ such that the following budget balancing condition holds:

$$\sum_{i=1}^{N} s_i(q, \theta) = q \quad \text{for all } (q, \theta) \in \Re_+ \times \Theta.$$

We will consider the class of sharing rules in which each share function is measurable with respect to the product measure formed by the Lebesque measure on $\Re_+$ and the probability measure μ on Θ.

Once the team adopts a sharing rule $\mathbf{s}$, the members of the team play a noncooperative game in choosing their actions in the production process. In this game conditional on $\mathbf{s}$, the payoff of each individual i is given by

$$\int_{\Theta} U^i(s_i(f(\mathbf{a}, \theta), \theta), a_i) d\mu, \tag{1}$$

when the actions in $\mathbf{a}$ are chosen. Clearly, given a sharing rule $\mathbf{s}$, $\mathbf{a} \in \Re_+^N$ is a Nash equilibrium conditional on $\mathbf{s}$ if, for each i and for all $a_i' \in \Re_+$,

$$\int_{\Theta} U^i(s_i(f(\mathbf{a}, \theta), \theta), a_i) d\mu \geq \int_{\Theta} U^i(s_i(f(\mathbf{a}_{-i}, a_i', \theta), \theta), a_i') d\mu$$

We implicitly assume that, before production takes place, the team uses some procedure to select a sharing rule, for example, this could be a bargaining process or a welfare maximization problem for the team. This will ultimately generate an outcome which is realized in a Nash equilibrium of the game conditional on the sharing rule adopted. Thus, the teams problem of deciding a sharing rule can be reduced to an equivalent problem of selecting an outcome which can be implemented non-cooperatively in the following sense:

Implementation: An outcome $(\mathbf{a}, \{\mathbf{p}^\theta : \theta \in \Theta\})$ is implementable if there exists a sharing rule $\mathbf{s}$ such that: (i) $\mathbf{a}$ is a Nash equilibrium conditional on $\mathbf{s}$ and (ii) $p_i^\theta = s_i(f(\mathbf{a}, \theta), \theta)$ for each i and for all $\theta \in \Theta$.

3. IMPLEMENTABLE OUTCOMES

Without loss of generality, let us fix an outcome $(\hat{\mathbf{a}}, \{\hat{\mathbf{p}}^\theta : \theta \in \Theta\})$ to be implemented non-cooperatively. Discouraging each team member from unilaterally deviating from his/her action in the outcome is very important for implementing it. The most problematic deviations are those that can be caused by any team member unilaterally and we need to impose restrictions on these for implementation.

For each i and each $\theta \in \Theta$, let $\underline{q}_i^\theta$ be the lowest output level that could be generated in state θ if individual i deviates unilaterally, i.e. $\underline{q}_i^\theta = f(\hat{\mathbf{a}}_{-i}, 0, \theta)$. Also, for each $\theta \in \Theta$, let

$Q(\theta) = \{q \in \Re_+ : q \geq \underline{q}_i^\theta$ for all $i\}$. So, for each $\theta \in \Theta$, $Q(\theta)$ is the most problematic set of deviations mentioned above.

For each i and each $a_i \in \Re_+$, let

$$W_i(a_i) = \left\{ \{w^\theta : \theta \in \Theta\} : w^\theta \in \Re \text{ for each } \theta \in \Theta;\ \int_\Theta U^i(w^\theta, a_i) d\mu = \int_\Theta U^i(\hat{p}_i^\theta, \hat{a}_i) d\mu \right\}$$

It can be verified using assumption **A1** that, for any i and any $(q, \theta) \in \Re_+ \times \Theta$, there is a unique action to which individual i could deviate unilaterally to generate the final output q in state θ if $q \geq \underline{q}_i^\theta$ and we denote this unique action by $a_i^{q\theta}$, i.e. $a_i^{q\theta} \in \Re_+$ is the unique action such that $f(\hat{\mathbf{a}}_{-i}, a_i^{q\theta}, \theta) = q$.

It is clear from the definition of $W_i(a_i)$ that, if we could make one of the menu of payments in $W_i(a_i)$ act as an upper bound (state by state) on individual i's shares when he/she deviates unilaterally to a_i, then individual i could be discouraged from deviating to a_i. However, we have to be able to do this simultaneously for the entire team at every possible deviation in $Q(\theta)$ for all $\theta \in \Theta$, whilst keeping in mind that the budget-balancing conditions must be maintained. This leads us to the following condition for implementability of the given outcome $(\hat{\mathbf{a}}, \{\hat{\mathbf{p}}^\theta : \theta \in \Theta\})$:

(I) For each i and each $a_i \in \Re_+$, there exists $\{w_i^\theta(a_i) : \theta \in \Theta\} \in W_i(a_i)$ such that, for every $\theta \in \Theta$: $\sum_{i=1}^N w_i^\theta(a_i^{q\theta}) \geq q$ for all $q \in Q(\theta)$.

Our first result shows that condition **(I)** is necessary and sufficient for implementing the given outcome $(\hat{\mathbf{a}}, \{\hat{\mathbf{p}}^\theta : \theta \in \Theta\})$. Before we state and prove this characterisation of the set of implementable outcomes, we will first construct a sharing rule which will be used in establishing the result.

Suppose condition **(I)** is satisfied. Given $(q, \theta) \in \Re_+ \times \Theta$ with $q \in Q(\theta)$, let $w_i(q, \theta) \in \Re$ for each i be such that:

(i) $w_i(q, \theta) \leq w_i^\theta(a_i^{q\theta})$;

(ii) $\sum_{i=1}^N w_i(q, \theta) = q$.

Also, for each $(q, \theta) \in \Re_+ \times \Theta$, let $\eta^{q\theta} = \{i : q \geq \underline{q}_i^\theta\}$. So $|\eta^{q\theta}| = N$ (where $|\eta^{q\theta}|$ is the number of individuals in $\eta^{q\theta}$) if and only if $q \in Q(\theta)$.

We will now define $\hat{\mathbf{s}} = (\hat{s}_1, ..., \hat{s}_N)$ to be the sharing rule which satisfies the following for each i and each $(q, \theta) \in \Re_+ \times \Theta$:

(A) if $q = f(\hat{\mathbf{a}}, \theta)$, then $\hat{s}_i(q, \theta) = \hat{p}_i^\theta$;

(B) if $q \in Q(\theta)$ but $q \neq f(\hat{\mathbf{a}}, \theta)$, then $\hat{s}_i(q, \theta) = w_i(q, \theta)$;

(C) if $q \notin Q(\theta)$ but $i \in \eta^{q\theta}$, then $\hat{s}_i(q, \theta) = w_i^\theta(a_i^{q\theta})$;

(D) if $i \notin \eta^{q\theta}$, then $\hat{s}_i(q, \theta) = [q - \sum_{j \in \eta^{q\theta}} w_j^\theta(a_j^{q\theta})]/[N - |\eta^{q\theta}|]$.

It can be checked that $\hat{\mathbf{s}}$ as defined by (A)-(D) is indeed a sharing rule.

Proposition 1: *The outcome* $(\hat{\mathbf{a}}, \{\hat{\mathbf{p}}^\theta : \theta \in \Theta\})$ *can be implemented if and only if it satisfies condition* **(I)**.

Proof: Suppose $(\hat{\mathbf{a}}, \{\hat{\mathbf{p}}^\theta : \theta \in \Theta\})$ can be implemented. Let $\check{\mathbf{s}}$ be a sharing rule such that $\hat{\mathbf{a}}$ is a Nash equilibrium conditional on $\check{\mathbf{s}}$ and $\check{s}_i(f(\hat{\mathbf{a}}, \theta), \theta) = \hat{p}_i^\theta$ for all i and for all $\theta \in \Theta$. Consider $(\check{q}, \check{\theta})$ such that $\check{q} \in Q(\check{\theta})$. For each i, let $\check{a}_i \in \Re_+$ be such that $f(\hat{\mathbf{a}}_{-i}, \check{a}_i, \check{\theta}) = \check{q}$. We need to find $\{w_i^\theta(\check{a}_i) : \theta \in \Theta\} \in W_i(\check{a}_i)$ for each i such that $\sum_{i=1}^N w_i^{\check{\theta}}(\check{a}_i) \geq \check{q}$. Clearly, $\sum_{i=1}^N \check{s}_i(\check{q}, \check{\theta}) = \check{q}$ and for each i,

$$\int_\Theta U^i(\check{s}_i(f(\hat{\mathbf{a}}_{-i}, \check{a}_i, \theta), \theta), \check{a}_i) d\mu \leq \int_\Theta U^i(\check{s}_i(f(\hat{\mathbf{a}}, \theta), \theta), \hat{a}_i) d\mu$$

So there exists $\epsilon_i \geq 0$ for each i such that

$$\int_\Theta U^i(\check{s}_i(f(\hat{\mathbf{a}}_{-i}, \check{a}_i, \theta), \theta) + \epsilon_i, \check{a}_i) d\mu = \int_\Theta U^i(\hat{p}_i^\theta, \hat{a}_i) d\mu$$

For each i, let $\check{w}_i^\theta = \check{s}_i(f(\hat{\mathbf{a}}_{-i}, \check{a}_i, \theta), \theta) + \epsilon_i$ for all $\theta \in \Theta$. Then $\{\check{w}_i^\theta : \theta \in \Theta\} \in W_i(\check{a}_i)$ for each i and

$$\sum_{i=1}^N \check{w}_i^{\check{\theta}} = \sum_{i=1}^N [\check{s}_i(\check{q}_i, \check{\theta}) + \epsilon_i] \geq \check{q}.$$

We will next prove the converse using the sharing rule $\hat{\mathbf{s}}$ constructed above. Consider any i and any $a_i \in \Re_+$. Because of (A) in the definition of $\hat{\mathbf{s}}$, it is sufficient to show that

$$\int_\Theta U^i(\hat{s}_i(f(\hat{\mathbf{a}}_{-i}, a_i, \theta), \theta), a_i) d\mu \leq \int_\Theta U^i(\hat{s}_i(f(\hat{\mathbf{a}}, \theta), \theta), \hat{a}_i) d\mu$$

From (B) and (C) in the definition of $\hat{\mathbf{s}}$, it can be verified that $\hat{s}_i(f(\hat{\mathbf{a}}_{-i}, a_i, \theta), \theta) \leq w_i^\theta(a_i)$ for every $\theta \in \Theta$. We also know that $\{w_i^\theta(a_i) : \theta \in \Theta\} \in W_i(a_i)$. Thus, the monotonicity condition in assumption **A3** implies

$$\int_\Theta U^i(\hat{s}_i(f(\hat{\mathbf{a}}_{-i}, a_i, \theta), \theta), a_i) d\mu \leq \int_\Theta U^i(\hat{s}_i(f(\hat{\mathbf{a}}, \theta), \theta), \hat{a}_i) d\mu. \quad \|$$

In our model an efficient outcome is simply an outcome which cannot be dominated in the Pareto sense *exante*.

Efficiency: An outcome $(\mathbf{a}, \{\mathbf{p}^\theta : \theta \in \Theta\})$ is an efficient outcome if there does not exists another outcome $(\mathbf{a}', \{\mathbf{p}'^\theta : \theta \in \Theta\})$ such that

$$\int_\Theta U^i(p_i'^\theta, a_i') d\mu \geq \int_\Theta U^i(p_i^\theta, a_i) d\mu \quad \text{for all } i$$

with strict inequality holding for some i.

An obvious consequence of Proposition 1 is that, an efficient outcome can be implemented if and only if it satisfies condition **(I)**. So Proposition 1 can help us understand whether the set of efficient outcomes that are implementable is empty or not, and how does it look like when nonempty.

A special case:

We will now explore the issue of implementability of efficient outcomes in slightly more detail in a special case. As mentioned earlier, this special case has been considered by some authors in the existing literature, although, unlike the current paper, they assume that the random variable is not observable *expost* and cannot be contracted upon.

In addition to assumption **A3**, we will also assume that the utility functions are quasi-linear, i.e. for each i, $U^i(m_i, a_i) = m_i - c_i(a_i)$, where c_i is the cost or disutility function of individual i's action. Consider the following expected welfare maximization problem:

$$\max_{\mathbf{a}\in\Re^N_+} \left[\int_\Theta f(\mathbf{a}, \theta) d\mu - \sum_{i=1}^{N} c_i(a_i) \right]$$

For expositional convenience, we will assume that the above problem has an interior solution $\mathbf{a}^*$ and is unique. We will also let $x^* = \int_\Theta f(\mathbf{a}^*, \theta) d\mu$.

The second additional feature of our special case is that, apart from assumptions **A1** and **A2**, the production function f satisfies the following condition at $\mathbf{a}^*$:

(E) For every i, j and every $a_i, a_j \in \Re_+$, if $f(\mathbf{a}^*_{-i}, a_i, \theta') = f(\mathbf{a}^*_{-j}, a_j, \theta')$ for some $\theta' \in \Theta$, then $f(\mathbf{a}^*_{-i}, a_i, \theta) = f(\mathbf{a}^*_{-j}, a_j, \theta)$ for all $\theta \in \Theta$.

Roughly speaking, condition **(E)** would be satisfied if the final output is a function of the random variable and a composite action determined by the actions all team members, i.e. if f is of the form $f(\mathbf{a}, \theta) = F(A(\mathbf{a}), \theta)$ for each $(\mathbf{a}, \theta) \in \Re^N_+ \times \Theta$, where the function A determines a composite action which together with the random variable generates the final output according to the function F.

Let $\underline{x} = \max\{\int_\Theta f(\mathbf{a}^*_{-i}, 0, \theta) d\mu : 1 \leq i \leq N\}$. Then for any $x \geq \underline{x}$, let a^x_i be the unique action of individual i such that $\int_\Theta f(\mathbf{a}^*_{-i}, a^x_i, \theta) d\mu = x$, where the uniqueness follows from the monotonicity condition in assumption **A1**. It can be verified by using condition **(E)** that, for any two individuals i and j and for any $x \geq \underline{x}$, $f(\mathbf{a}^*_{-i}, a^x_i, \theta) = f(\mathbf{a}^*_{-j}, a^x_j, \theta)$ for all $\theta \in \Theta$, and we let $q^{x\theta} = f(\mathbf{a}^*_{-i}, a^x_i, \theta)$.

For each i, let $v_i : [\underline{x}, \infty) \to \Re$ be the function defined by $v_i(x) = c_i(a^x_i)$ for all $x \geq \underline{x}$. Given our assumptions, it can be verified that v_i is continuously differentiable and $v_i(x^*) = c_i(a^*_i)$ for each i. Since $\mathbf{a}^*$ maximizes expected welfare, we know that $x - c_i(a^x_i) - \sum_{j\neq i} c_j(a^*_j) \leq x^* - \sum_{j=1}^N c_j(a^*_j)$ for each i and for all $x \geq \underline{x}$. So for each i, $x - v_i(x) \leq x^* - v_i(x^*)$ for all $x \geq \underline{x}$. Thus, for each i, x^* maximizes $x - v_i(x)$, and hence, $v'_i(x^*) = 1$.

Because of the quasi-linear utilities, it is clear that an efficient outcome is implementable if and only if it has the actions in $\mathbf{a}^*$. Consider an arbitrary efficient outcome $(\mathbf{a}^*, \{\mathbf{p}^{*\theta} : \theta \in \Theta\})$

and suppose it is implementable. Using Proposition 1, for each i and each $x \geq \underline{x}$, let $\{w_i^\theta(a_i^x) : \theta \in \Theta\} \in W_i(a_i^x)$ be such that

$$\sum_{i=1}^N w_i^\theta(a_i^x) \geq q^{x\theta} \quad \text{for all } \theta \in \Theta.$$

For each i and each $x \geq \underline{x}$, since $\int_\Theta w_i^\theta(a_i^x)d\mu - c_i(a_i^x) = \int_\Theta p_i^{*\theta}d\mu - c_i(a_i^*)$, we also have

$$\int_\Theta p_i^{*\theta}d\mu - c_i(a_i^*) + v_i(x) = \int_\Theta w_i^\theta(a_i^x)d\mu.$$

Then by summing over all i we get

$$\sum_{i=1}^N \left[\int_\Theta p_i^{*\theta}d\mu - c_i(a_i^*) + v_i(x)\right] = \sum_{i=1}^N \int_\Theta w_i^\theta(a_i^x)d\mu \geq \int_\Theta q^{x\theta}d\mu = x.$$

Hence, for each $x \geq \underline{x}$, we have

$$x - \sum_{i=1}^N v_i(x) - x^* + \sum_{i=1}^N c_i(a_i^*) \leq 0.$$

Let $h(x) = x - \sum_{i=1}^N v_i(x) - x^* + \sum_{i=1}^N c_i(a_i^*)$ for all $x \geq \underline{x}$. So $h(x) \leq 0$ for all $x \geq \underline{x}$ and $h(x^*) = 0$. Because of our assumptions, h is continuously differentiable and $h'(x^*) = 1 - \sum_{i=1}^N v_i'(x^*) = 1 - N < 0$. This implies the existence of $x \in [\underline{x}, x^*)$ such that $h(x) > 0$, which is inconsistent with $h(x) \leq 0$ for all $x \geq \underline{x}$. Therefore, we have established the following result regarding the efficiency property of our special case.

Proposition 2: *Efficient outcomes are not implementable in the special case.*

4. LINEAR SHARING RULES

In this section we will drop the quasi-linearity of the utility functions and condition **(E)** and revert back to the more general framework. However, we will only concentrate on the class of outcomes with positive actions that are implementable by using sharing rules for which the first-order approach is applicable.

In the literature, the random variable θ is often suppressed by viewing the output q as a random variable with a distribution function $G(q|\mathbf{a})$ parametrised by the actions (e.g. see Holmstrom (1982), Kim and Wang (1998)) and conditions are often imposed on G to generate contracts that are nondecreasing in output. We do not want to suppress the random variable θ in our context as it has an essential role to play in the analysis, but we could explore conditions on the production function f and the distribution function μ to avoid a negative relationship between payments and output. However, instead of this approarch, we will rely on the fairly usual casual empiricism that individual payments are often related non-negatively to the final output level. Thus, throughout this section we will also consider only the class of outcomes that are implementable by using sharing rules in which the share functions are nondecreasing in the final output.

Consider an implementable outcome $(\bar{\mathbf{a}}, \{\bar{\mathbf{p}}^\theta : \theta \in \Theta\})$ and let the sharing rule $\bar{\mathbf{s}}$ implement this outcome. So we have

$$\bar{s}_i(f(\bar{\mathbf{a}},\theta),\theta) = \bar{p}_i^\theta \quad \text{for each } i \text{ and for all } \theta \in \Theta. \tag{2}$$

From the first-order conditions of Nash equilibrium, we also have the following for each i:

$$\int_\Theta \left[U_1^i(\bar{s}_i(f(\bar{\mathbf{a}},\theta),\theta),\bar{a}_i)\bar{s}_i'(f(\bar{\mathbf{a}},\theta),\theta)f_i(\bar{\mathbf{a}},\theta) + U_2^i(\bar{s}_i(f(\bar{\mathbf{a}},\theta),\theta),\bar{a}_i)\right] d\mu = 0 \tag{3}$$

where $\bar{s}_i'$ denotes the derivative of $\bar{s}_i$ with respect to the output.

For each i and each $\theta \in \Theta$, define $\tilde{\gamma}_i^\theta$ and $\tilde{\alpha}_i^\theta$ as follows:

$$\tilde{\gamma}_i^\theta = \bar{s}_i'(f(\bar{\mathbf{a}},\theta),\theta) \tag{4}$$

$$\tilde{\alpha}_i^\theta = \bar{s}_i(f(\bar{\mathbf{a}},\theta),\theta) - \tilde{\gamma}_i^\theta f(\bar{\mathbf{a}},\theta) \tag{5}$$

Since $\bar{s}_i$ is nondecreasing in output, $\tilde{\gamma}_i^\theta \geq 0$ for all i and for all $\theta \in \Theta$. The budget balancing condition gives

$$\sum_{i=1}^N \bar{s}_i(f(\bar{\mathbf{a}},\theta),\theta) = f(\bar{\mathbf{a}},\theta) \quad \text{for each } \theta \in \Theta.$$

By differentiating both sides with respect to the action of any individual, it can be verified that $\sum_{i=1}^N \bar{s}_i'(f(\bar{\mathbf{a}},\theta),\theta) = 1$ for each $\theta \in \Theta$. So equation (4) implies

$$\sum_{i=1}^N \tilde{\gamma}_i^\theta = 1 \quad \text{for each } \theta \in \Theta. \tag{6}$$

Also, it can be checked by using equations (5) and (6) and the budget balancing condition that

$$\sum_{i=1}^N \tilde{\alpha}_i^\theta = 0 \quad \text{for each } \theta \in \Theta. \tag{7}$$

Using the above derivations, we will now construct a sharing rule which is linear in the output q for each realisation of the random variable θ and show that this sharing rule also implements the given outcome $(\bar{\mathbf{a}}, \{\bar{\mathbf{p}}^\theta : \theta \in \Theta\})$. Let $\tilde{\mathbf{s}} = (\tilde{s}_1, ..., \tilde{s}_N)$ be the sharing rule such that, for each i,

$$\tilde{s}_i(q,\theta) = \tilde{\alpha}_i^\theta + \tilde{\gamma}_i^\theta q \quad \text{for all } (q,\theta) \in \Re_+ \times \Theta. \tag{8}$$

It can be verified by using equations (6) and (7) that the above definition does satisfy the budget balancing condition.

Proposition 3: *The outcome* $(\bar{\mathbf{a}}, \{\bar{\mathbf{p}}^\theta : \theta \in \Theta\})$ *can be implemented by using the sharing rule* $\tilde{\mathbf{s}}$.

Proof: Equations (2), (5) and (8) imply

$$\tilde{s}_i(f(\bar{\mathbf{a}},\theta),\theta) = \bar{s}_i(f(\bar{\mathbf{a}},\theta),\theta) = \bar{p}_i^\theta \quad \text{for each } i \text{ and for all } \theta \in \Theta. \tag{9}$$

Furthermore, from equations (3), (4), (8) and (9), we can derive the following first-order conditions for $\bar{\mathbf{a}}$ to be a Nash equilibrium when $\tilde{\mathbf{s}}$ is used:

$$\int_\Theta \Big[U_1^i(\tilde{s}_i(f(\bar{\mathbf{a}},\theta),\theta),\bar{a}_i)\tilde{s}_i'(f(\bar{\mathbf{a}},\theta),\theta)f_i(\bar{\mathbf{a}},\theta) + U_2^i(\tilde{s}_i(f(\bar{\mathbf{a}},\theta),\theta),\bar{a}_i)\Big]\,d\mu = 0 \quad \text{for all } i.$$

To complete the proof, it is then sufficient to show that, for each i, $U^i(\tilde{s}_i(f(\mathbf{a},\theta),\theta),a_i)$ is concave in $\mathbf{a}$ for every $\theta \in \Theta$.

Let $\mathbf{a}',\mathbf{a}'' \in \Re_+^N$ and $\lambda \in (0,1)$. Also, let $\mathbf{a}^\lambda = \lambda\mathbf{a}' + (1-\lambda)\mathbf{a}''$. For each i, the concavity of U^i implies

$$\begin{aligned} U^i(\lambda\tilde{s}_i(f(\mathbf{a}',\theta),\theta) + (1-\lambda)\tilde{s}_i(f(\mathbf{a}'',\theta),\theta),a_i^\lambda) \quad &\geq \\ \lambda U^i(\tilde{s}_i(f(\mathbf{a}',\theta),\theta),a_i') + (1-\lambda)U^i(\tilde{s}_i(f(\mathbf{a}'',\theta),\theta),a_i'') \quad &\text{for all } \theta \in \Theta. \end{aligned} \tag{10}$$

Using (8), $\lambda\tilde{s}_i(f(\mathbf{a}',\theta),\theta) + (1-\lambda)\tilde{s}_i(f(\mathbf{a}'',\theta),\theta) = \tilde{\alpha}_i^\theta + \tilde{\gamma}_i^\theta[\lambda f(\mathbf{a}',\theta) + (1-\lambda)f(\mathbf{a}'',\theta)]$ for each i and each $\theta \in \Theta$. For each i and each $\theta \in \Theta$, the concavity of f and $\tilde{\gamma}_i^\theta \geq 0$ also imply $\tilde{\gamma}_i^\theta f(\mathbf{a}^\lambda,\theta) \geq \tilde{\gamma}_i^\theta[\lambda f(\mathbf{a}',\theta) + (1-\lambda)f(\mathbf{a}'',\theta)]$. So, for each i and each $\theta \in \Theta$, we have

$$\tilde{\alpha}_i^\theta + \tilde{\gamma}_i^\theta f(\mathbf{a}^\lambda,\theta) \geq \lambda\tilde{s}_i(f(\mathbf{a}',\theta),\theta) + (1-\lambda)\tilde{s}_i(f(\mathbf{a}'',\theta),\theta) \tag{11}$$

Hence, (8), (10), (11) and the monotonicity of the utilities in income imply $U^i(\tilde{s}_i(f(\mathbf{a}^\lambda,\theta),\theta),a_i^\lambda) \geq \lambda U^i(\tilde{s}_i(f(\mathbf{a}',\theta),\theta),a_i') + (1-\lambda)U^i(\tilde{s}_i(f(\mathbf{a}'',\theta),\theta),a_i'')$ for each i and each $\theta \in \Theta$. Thus, for each i, $U^i(\tilde{s}_i(f(\mathbf{a},\theta),\theta),a_i)$ is concave in $\mathbf{a}$ for every $\theta \in \Theta$. $\quad \|$

Needless to say, Proposition 3 shows that any outcome which belongs to the class of implementable outcomes considered in this section can be implemented by a sharing rule that is linear in the output for each realisation of the random variable. We will now look at the efficiency issue for this class of implementable outcomes.

Consider an efficient outcome $(\mathbf{a}^E,\{\mathbf{p}^{E\theta} : \theta \in \Theta\})$. It can be verified that, for each i, a_i^E must be a solution of the following problem:

$$\max_{a_i \in \Re_+} \int_\Theta U^i(f(\mathbf{a}_{-i}^E,a_i,\theta) - \sum_{j\neq i} p_j^{E\theta},a_i)d\mu$$

For each i, by substituting $p_i^{E\theta} = f(\mathbf{a}^E,\theta) - \sum_{j\neq i} p_j^{E\theta}$ for all $\theta \in \Theta$ into the first-order condition of the above problem, we get

$$\int_\Theta \Big[U_1^i(p_i^{E\theta},a_i^E)f_i(\mathbf{a}^E,\theta) + U_2^i(p_i^{E\theta},a_i^E)\Big]\,d\mu = 0. \tag{12}$$

Suppose $(\mathbf{a}^E,\{\mathbf{p}^{E\theta} : \theta \in \Theta\})$ can be implemented. According to Proposition 3, it must be implementable by a linear sharing rule $\mathbf{s}^E$ such that, for each i, $s_i^E(q,\theta) = \alpha_i^{E\theta} + \gamma_i^{E\theta}q$ for all $(q,\theta) \in \Re_+ \times \Theta$, with $\sum_{j=1}^N \gamma_j^{E\theta} = 1$ and $\sum_{j=1}^N \alpha_j^{E\theta} = 0$. Then from the first-order condition for Nash equilibrium, we can derive the following for each i:

$$\int_\Theta \Big[U_1^i(p_i^{E\theta},a_i^E)\gamma_i^{E\theta}f_i(\mathbf{a}^E,\theta) + U_2^i(p_i^{E\theta},a_i^E)\Big]\,d\mu = 0. \tag{13}$$

However, it can be seen clearly that equations (12) and (13) cannot hold simultaneously for some i. Therefore, we have established the following efficiency result for the case considered in this section.

Proposition 4: *The efficient outcome* $(\mathbf{a}^E, \{\mathbf{p}^{E\theta} : \theta \in \Theta\})$ *cannot be implemented.*

Thus, according to Proposition 4, the class of efficient outcomes does not contain any outcome that can be implemented by using a sharing rule for which the first-order approach is applicable and each individual share function is nondecreasing in the output.

A straightforward corollary of Proposition 3 is the following characterisation of the class of implementable outcomes for the case considered in this section: An outcome $(\mathbf{a}, \{\mathbf{p}^\theta : \theta \in \Theta\})$ is implementable if and only if there exist $\gamma_i^\theta \geq 0$ for each i and each $\theta \in \Theta$ such that

(i) $\sum_{i=1}^N \gamma_i^\theta = 1$ for all $\theta \in \Theta$;

(ii) $\int_\Theta [U_1^i(p_i^\theta, a_i)\gamma_i^\theta f_i(\mathbf{a}, \theta) + U_2^i(p_i^\theta, a_i)] d\mu = 0$ for each i.

Suppose an outcome $(\mathbf{a}, \{\mathbf{p}^\theta : \theta \in \Theta\})$ is such that, for every $\theta \in \Theta$,

$$\sum_{i=1}^N \left[\frac{U_2^i(p_i^\theta, a_i)}{U_1^i(p_i^\theta, a_i) f_i(\mathbf{a}, \theta)} \right] = -1.$$

Then by letting $\gamma_i^\theta = -U_2^i(p_i^\theta, a_i)/U_1^i(p_i^\theta, a_i) f_i(\mathbf{a}, \theta)$ for each i and each $\theta \in \Theta$, it is possible to obtain $\int_\Theta [U_1^i(p_i^\theta, a_i)\gamma_i^\theta f_i(\mathbf{a}, \theta) + U_2^i(p_i^\theta, a_i)] d\mu = 0$ for each i. Thus, another corollary we can derive is that: A sufficient condition for an outcome $(\mathbf{a}, \{\mathbf{p}^\theta : \theta \in \Theta\})$ to be implementable is

$$\sum_{i=1}^N \left[\frac{U_2^i(p_i^\theta, a_i)}{U_1^i(p_i^\theta, a_i) f_i(\mathbf{a}, \theta)} \right] = -1 \quad \text{for all } \theta \in \Theta.$$

5. SUMMARY AND CONCLUSION

In a moral hazard in team setting, apart from the final output, any additional information that allows us to better infer the actions may bring us closer to the first-best outcome. We argue that there are many situations where, although uncertainty enters into the relationship between the actions and the final output *exante*, this uncertainty is resolved and more critically, observable *expost*. For example, we can think of the case of non-point source pollution problem where a random element like rainfall combines with individual emissions to produce an ambient level of pollution. Prior to pollution discharge, future rainfall is not known with certainty. However, at the time when the ambient level of pollution is observed, the rainfall that preceded this point in time is also known and observable. There are other situations, some of which we have already mentioned, that are similar to the non-point source pollution problem. Thus, the potential to observe the random factor *expost* after the resolution of the uncertainty seems quite prevalent in problems characterised by moral hazard in teams. This observation suggests that it makes sense to include this knowledge in the construction of a sharing rule aimed at reducing free-riding incentives and may improve on current schemes suggested in the literature.[6] Accordingly, we considered a set up in which the team's sharing rule took the form of a menu contract by incorporating the random factor as an additional variable that could be contracted upon.

In the case of our framework with fairly general utility functions and production technology, we first provided a necessary and sufficient condition for implementing an outcome. Intuitively, this condition imposed restrictions on the set of unilateral deviations that could be caused by anyone and this allowed sufficient punishments to be incorporated into the sharing rule to deter such deviations. Using this characterisation, we showed that efficient outcomes could not be implemented in a special case with quasi-linear utilities and a specific production technology. With the more general utility functions and production technology, when we looked at the case in which the first-order approach could be used and the share functions were nondecreasing in output, we found that it was possible to restrict the search for suitable sharing rules to simple rules that are linear in output. This may also provide some explanation as to why, although contrary to predictions by theory, simple linear sharing rules are prevalent.[7] Using the linear sharing rule result, we showed that efficient outcomes are not implementable in the restricted case considered in section 4. In addition to the characterisation in section 3 for the general case, we also provided a characterisation of implementable outcomes for the restricted case in section 4 by using the linear sharing rule result. This naturally raises the interesting issue of the relationship between the two classes of implementable outcomes we looked at in sections 3 and 4 respectively. At the moment, the nature of this relationship remains an open question and is a possible future line of research.

[6]By not accounting for the random factor, polluters may find that they have to pay a tax because the ambient level of pollution exceeded a specified target due to the random factor and not because they over emitted pollution (Herriges *et al.* (1994), Horan *et al.* (1998), Segerson (1988), Xepapadeas (1995)).

[7]Romano (1994) and Bhattacharyya and Lafontaine (1995) provide another explanation of why linear sharing rules tend to be the norm in practice. They find that, in the presence of double moral hazard and risk neutrality of the agent, a simple linear sharing rule implements the desired outcome.

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