

# Economic growth, population and pollution interdependence under stochastic shocks: Does convergence speed matter?\*

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## Abstract

We study dynamics of interactions among environmental pollution, population and output growth showing that shocks are persistent in these systems and that they converge slowly to the steady state. It is demonstrated that the extent of persistence in economic growth is determined by the convergence speed of shocks in both environmental pollution and population. A modified Solow-Swan economy is simulated to illustrate that non-mean convergent shocks in these series alter growth trajectories of output in the long run. An empirical analysis is carried out to examine persistence and co-movement pattern of pollution, population and per capita income time series for a sample of highly-polluted OECD and non-OECD countries over the period 1950-2003. The magnitude of shocks found are high enough to oscillate long-run economic growth. Moreover, the population, output and population growth are found to be (fractionally) co-integrated where the co-evolutionary pattern is characterized by slow-adjustment to long-run equilibrium. The implication is that a joint shock control policy in population and environmental pollution would provide stability to long-run economic growth, although the effectiveness of such policy would depend on the magnitude of shocks in environmental and demographic systems.

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# I Introduction

Whether a permanent change in environmental and demographic systems affects the long-run growth rate of an economy is an empirical question that many researchers and policy makers have been interested in over the past decades. Moreover, it is also a distinguishing characteristic between endogenous and exogenous growth models in the sense that the ‘change’ leads to a growth effect in the former class of models but only a level effect in the latter (see, for example, Romer, 1986 and Lucas, 1988). Indeed, from the point of view of a variety of economic and policy reasons<sup>1</sup> it is important to understand how the extent of such ‘change’ in environmental and demographic systems affect the long-term pattern of economic growth. To understand the dynamics of such changes, an important approach is to study the (co-)evolutionary pattern of environmental, demographic and economic growth (in short, EDG) systems from temporal perspective. Apart from its methodological appeal, time series characteristics of a growing system, such as EDG can be modeled in light of modern economic growth theory. But interlinking empirics to theory has not been so straightforward in this regard. In fact, the extant literature till date has produced parallel analysis, i.e., either with a well-founded theory investigating the predictive performance of one of the variables in EDG on others (see for instance, Dasgupta, 1995; Dasgupta and Heal, 1995; Soretz, 2003; Maler and Vincent, 2005, and Henderson and Millimet, 2007, among others) or estimating a reduced form econometric specification of the EDG system (for recent analysis, see for instance, Nguyen van and Azomahou, 2007; Dinda and Coondoo, 2008).

An important aspect which has not been explicitly investigated in both theory and empirics of EDG relationship is the effect of slowly-converging stochastic shocks. The immediate effect of such shocks in any dynamic system can be gauged from the perceptible changes it incurs on the long-run growth pattern.<sup>2</sup> Additionally, a fractional

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<sup>1</sup>The persistent emphasis on the negative consequences of rising volatility in environmental and demographic systems have been duly stressed in various Inter-government Panel for Climate Change (IPCC) reports. For details, please refer to [http : //www.ipcc.ch/publications\\_and\\_data/publications\\_and\\_data\\_reports.shtml](http://www.ipcc.ch/publications_and_data/publications_and_data_reports.shtml)

<sup>2</sup>Emphasizing on the sizable impact stochastic shocks have on long-term economic growth Michelacci and Zaffaroni (2000) investigate the importance of fractional dynamics of shocks in understanding convergence of output.

rate of convergence of shocks (i.e., having hyperbolic decay) in environmental (especially environmental pollution) and demographic (especially population growth) systems may imply a slower adjustment to the long-run equilibrium which also affects the speed of adjustment in their co-evolutionary processes. While majority of research (as will be explained shortly) studying the impact of environmental and demographic stochasticities on economic growth concentrate on either testing for unit root (and in a relatively small number of cases the fractional integration) nature of pollution, population and economic growth in cross country setting, understanding their implications within a conventional growth framework appears less distinct. In this paper, we build a modified Solow-Swan economy and study the implications of slowly-converging population and pollution shocks on long-run economic growth.

Indeed, understanding the implications of convergence pattern of shocks within a defined system is important both for pure academic and policy reasons or both. Although neither of the individual systems in EDG framework can be studied while neglecting others, recent research has invariably focused on ‘economy’ as being the central agent where the effects of volatile demography and environmental growth is perceived. Although the relationship can easily run from either direction<sup>3</sup>, research over the past decades have tended to consider economy as the target variable and demography along with environment as impact variables. Investigating the relationship between population growth and per capita income for instance, Boucekine, de la Croix and Licandro (2002) showed that the relationship is hump-shaped and that increases in longevity can be responsible for a switch from a no-growth regime to a sustained growth regime. Dynamics between environment and demographic components, such as, life-expectancy has also been explored in numerous studies. Mariani et al. (2010) recently use an overlapping generations model and show that environmental conditions affect life-expectancy which gives rise to the possibility of multiple-equilibria so that some countries might be caught in a low-life-expectancy and low-environmental quality trap. In light of the above, it is reasonable

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<sup>3</sup>Ehrlich and Holdren’s (1971)  $I = PAT$  identity is one of the first to explain the logical relationship between environmental impact (I), population (P), affluence (A), and technological efficiency (T). Which one of the three factors act (upon) less on others actually defines the weight of the problem - i.e., whether it is demography-pressure pull environmental problem or excess consumption-push environmental problem.

to assume that demographic and environmental variations taken together can explain a large part of the stochastic behavior of output fluctuations.

In light of the above, a natural question that arises is ‘how do stochastic shocks in environmental and demographic systems affect transitional and evolutionary dynamics of output growth?’ Can the rate of convergence of stochastic shocks in environmental and demographic systems determine convergence pattern of output (both at regional and cross-country level)? What impact does the convergence rate of shocks in these systems have on long-run output growth path of an economy? Answers to these questions are not always straightforward as conventional approach to modeling environmental degradation, economic growth and population dynamics has invariably thrived on stationary relationship among these variables. Wherever the existence of stochastic shocks has been considered, they are often based on the idea that the duration of these shocks are ‘duration-dependent’ and are of ‘one-off’ type such that a stochastic shock either escapes certain period and becomes permanent thereafter. The slow convergence rate of shocks with explicit temporal dynamics was evidently missing from modeling economic growth with environment and demography being the main facilitators. Moreover, the interrelationships among these variables (which we refer to here and in subsequent analysis as EDG system) is exceedingly complex, often involving non-linear interactions. Therefore, a stochastic shock in one of the variables may be argued not to completely wither-away in others.

There are basically two ways economists can address such complexity: either by means of a full-blown theoretical model or through some reduced-form approach. The former approach has been celebrated in works of Dasgupta and Heal (1995) and in recent research e.g., Henderson and Millimet, 2007; Xepapadeas, 2003 and Soretz (2003). The latter option is way too complex, as it is already quite challenging to build a convincing economic growth model that takes population exogenous and does not model the environment. Although theoretical models of economic growth incorporating demography or the environment have been proposed in the literature, there are very few cases in which all the three dimensions have been brought together. One such case, in which a dynamic

general equilibrium model is simulated, is Dalton et al. (2008).<sup>4</sup> The alternative is to use some reduced-form approach based on the statistical analysis of time series (and cross-sectional) data. Here there are a few alternative routes that can be taken, one of them is for instance to estimate population-augmented Environmental Kuznets Curves (e.g. Cole and Neumayer, 2006).

In this paper we follow a third approach, one that is a mix of the two aforementioned strategies. We first take a very simple growth model – the celebrated Solow-Swan model – and modify it to include the dynamics of the labor input that depends upon the growth rate of population. This rate in turn is made dependent upon environmental quality.<sup>5</sup> Finally, the dynamics of environmental quality is allowed to functionally related to population growth. In so doing the link between environmental quality, population, and then economic growth is established. What we explore here thus is the link running from environment and population to output, although the reverse link is possible. However, the analytical results we derive in the paper would not change implications while exploring the reverse causation of feedback effects.

Our idea is to model and examine the interrelationships among variables in the EDG system using (long-memory) time series framework. In this setting, it would be possible to examine if shocks in these systems were converging slowly, that is if they are characterized by long-memory. Additionally, one may model the co-evolutionary paths of the systems in the presence of slow-converging shocks such that the long-run equilibrium of these systems will be governed by the rate of convergence of shocks. Thus persistent shock in one of the systems would easily transcend to other systems making the co-evolutionary pattern of the integrated system unpredictable and volatile. A simulation experiment is carried out using an extended Solow-Swan economy with long-memory population growth.

The rest of the paper is structured as follows. Section II describes the basic framework and argues why environment and demographic systems can be modeled in a long-memory framework. In section III we provide analytical arguments of economic growth

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<sup>4</sup>One should also add, for the sake of completeness, that a fourth factor should not be neglected, and that is technological progress.

<sup>5</sup>One may consider reverse causality, however this will not significantly alter the implications of our main argument.

stochasticity arising from long-memory environmental and demographic systems. Empirical results are described in section IV. Section V presents results of simulation of the extended Solow-Swan economy. Section VI summarizes and discusses the implication of the main findings.

## II Basic framework

The objective of this section is to provide a basic setting under which non-stationary relationship among environment, economic growth and demography can be studied. The framework we use for illustration relies on the fact that population growth dynamically depends on the growth of environmental quality as environmental degradation inflicts adverse effect on fertility. In numerous research published in medical science, it has been found that degrading environmental quality (especially the presence of common chemicals in the home and workplace) affects human infertility to a great extent.<sup>6</sup>

On the other hand, economic growth depends on population growth with a lag due to the inevitability that the economy takes time to respond to a shock in  $n_t$  necessitating thus the EDG to thrive on the natural feedback effects. Easterlin (1966) provided the cornerstone of the widely discussed economic-demographic interactions with feedback effects. To motivate analysis, we specify a model which accounts for the negative effects of accumulating pollution on the supply side through the reduction of labour productivity and population growth by environmental pollution. Let us consider an aggregate Cobb-Douglas production function

$$Y(t) = K(t)^\alpha [A(t)N(t)]^{1-\alpha}, 1 < \alpha < 0 \quad (1)$$

with the standard notations:  $Y(t)$  the output at time  $t$ ,  $K(t)$  the level of physical capital,  $N(t)$  the labour input,  $A(t)$  the labour augmenting technical progress. Then,  $A(t)N(t)$

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<sup>6</sup>Dr. Hein Strokum, Institute of Sterility Treatment, Vienna, Austria in an interesting research published in the American Journal of Industrial Medicine (Vol. 24:587-592, 1983) found that men experiencing infertility were found to be employed in agricultural/pesticide related jobs 10 times more often than a study group of men not experiencing infertility. It was also found that mothers who lived near crops where certain pesticides were sprayed faced a 40 to 120 percent increase in risk of miscarriage due to birth defects.

is the effective labour. We denote  $g(E)$  the growth rate of labour augmenting technical progress which depends on environmental quality  $E$ , with  $g'(E) < 0$ . This translates the negative effects of accumulated pollution on labour productivity. The labour input is governed by the growth rate of population,  $n(E)$  so that

$$N(t) = (1 + n(E))N(t - 1) \quad (2)$$

where  $n(E)$  represents the growth rate of population that also depends on environmental quality. We assume that  $n'(E) < 0$ . Both  $g(E)$  and  $n(E)$  imply that we can endogenize technical progress and population growth in terms of environmental quality.

Since we focus on population and environmental quality (where population exerts some pressure on environment via consumption and production), the latter can be viewed as a physical good. Following Aghion and Howitt (1998, Chap.5), we assume that there is an upper limit to environmental quality, denoted by  $E_{\max}$ . We measure  $E(t)$  as the difference between the actual quality and this upper limit. Thus, environmental quality will always be negative. The equation of motion of environmental quality is given by

$$\dot{E}(t) = -\theta E(t) - \psi n(E) \quad (3)$$

where  $\theta E(t) > 0$  in (3) indicates the maximum potential rate of recovery of the environment, and  $\psi > 0$  measures the environmental damage following from demographic pressures. Furthermore, from sustainable economic perspective, we assume that environmental quality also has a lower limit,  $E_{\min}$  referred to as catastrophic. This, implies that the optimal growth path, if it exists, will be constrained as

$$E_{\min} \leq E \leq 0. \quad (4)$$

The function  $A(t)$  describing the level of labour augmenting technical progress specified as

$$A(t) = \Omega(t)E(t)^{-\beta}, \beta > 0. \quad (5)$$

This formulation has two parts. The first part like Solow (1956) and Swan (1956) represents some exogenous portion of technical progress:  $\Omega(t) = \Omega(0)e^{\mu t}$  where  $\mu$  is its constant rate of growth. The second part implies that as  $t \rightarrow 1$ , and  $E(t) \rightarrow \infty$  negatively (recall that  $E(t)$  is always negative),  $A(t) \rightarrow 0$ . This means that more environmental pollution will lead labour productivity to decline increasingly more slowly and thus approaches zero only asymptotically. It can be easily seen from relations (3) and (5) that

$$g(E) = \frac{\dot{A}(t)}{A(t)} = \mu - \beta \frac{\dot{E}}{E} = \mu + \beta\theta + \frac{\psi n(E)}{E} \quad (6)$$

from which one observes that  $g'(E) < 0$  if  $n(E) = n$ . Since  $n(E)$  is not constant and incorporates some stochastic feature, it will be useful to study its properties using a flexible framework where stationarity of shocks is a limiting case of the broader problem.<sup>7</sup> This can be done by describing relation (3) in a fractional integration framework such that  $\dot{E}_t \equiv E_t - E_{t-1}$  is re-written as  $(1 - L)^d E_t$  where  $d$  is the integration parameter and  $L$  is backward shift operator with  $LE_t = E_{t-1}$ . When  $d = 1$ ,  $(1 - L)^d E_t$  is equivalent to  $\dot{E}_t \equiv E_t - E_{t-1}$ . However, allowing  $d$  to assume a fractional value allows us to study various convergence patterns of shocks which not only determines its growth dynamics but also the evolutionary pattern of interacting systems, such as demography and economic growth. Describing equation (3) by

$$(1 - L)^d E_t = \epsilon_t \quad (7)$$

where  $\epsilon_t \sim (0, \sigma^2)$  is a gaussian fractional noise, we may thus study shock convergence pattern of this system by looking at the decay of the autocovariance function along with the binomial expansion of  $(1 - L)^d$ :

$$(1 - L)^d = \sum_0^{\infty} h_j L^j = 1 - dL + \frac{d(d-1)}{2!} L^2 - \frac{d(d-1)(d-2)}{3!} L^3 - \dots \quad (8)$$

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<sup>7</sup>The reason why demographic and environmental systems are characterized by long-memory can be traced to the fact that an growing system, small or big is often subject to continuous perturbations, the accumulations of which in the long-run may result in a behavior of shocks which may not converge in the long-run. Moreover, presence of such shocks in an interdependent system like EDG affect the whole system's evolutionary process.



$h_0 \equiv 1$ ,  $L^j$  is backward operator  $j$  times, and  $h_j \equiv (1/j!)(d + j - 1)(d + j - 2)(d + j - 3) \cdots (d + 1)(d)$ . It may be noted from the above that the coefficient of lagged  $E_t$  provides the rate of declining weights. However, based on the non-integer values and sign of  $d$ , the following memory properties are observed.

With  $d = 0$  in the above, the process exhibits ‘short memory’ as the autocorrelations in this case is summable and decay fairly rapidly so that a shock has only a temporary effect completely disappearing in the long run. Long memory and persistence is observed for  $d > 0$ . In this case, the shock affects the historical trajectory of the series. However, greater is the magnitude of  $d$ , stronger is the memory and shock persistence. For  $d \in (0, 0.5)$ , the series is covariance stationary and the autocorrelations take much longer time to die out. When  $d \in [0.5, 1)$ , the series is a mean reverting long-memory and non-stationary process. This implies even though remote shocks affect the present value of the series, this will tend to the value of its mean in the long run. For  $-1/2 < d < 0$  the process is known to be fractionally over-differenced. In this case, there is still short memory with summable autocovariances, but the autocovariance sequence sums to 0 over  $(-\infty, +\infty)$ . For  $d < -1/2$  the series is covariance stationary but not invertible. And finally, when  $d \geq 1$  the series is nonstationary and exhibits ‘perfect memory’ or ‘infinite memory’. There is no unconditional mean defined for the series in this case. The process defined by this value of  $d$  is non-stationary and non-mean reverting. In this case, the mean of the series has no measured impact on the future values of the process.

### III Stochasticity in economic growth

In this section we investigate if the presence of long-memory in environment and demographic variables imply a long-memory in economic growth. In other words, is it right to argue that (possible) slow convergence of shocks in international output in recent studies (e.g., Michelacci and Zaffaroni, 2000) can be attributed to the slow-convergent shocks in environmental and demographic variables? To motivate such setting, recall that the relation between environment and demography is expressed in a way that allows population growth  $n$  to be a function of environmental quality  $E$ , which is both empirically

well-supported and theoretically well-explored. Moreover, using conventional framework of economic growth and demography literature per capita output growth,  $Y$  is assumed to be functionally dependent on population growth  $n$ . A simple model of economic growth viz., Solow-Swan economy<sup>8</sup> is considered next to demonstrate how a long-memory in population growth can affect growth behavior of the economy with environmental stochasticity implicitly governing the dynamics of the demographic system.<sup>9</sup>

To begin with, recall the environmental growth equation:

$$(1 - L)^d E_t = -\theta E_t - \psi n_t \quad (9)$$

then

$$E_t = -\psi n_t [(1 - L)^d + \theta]^{-1} \quad (10)$$

Observe that (3) is conceptualized to be a function of the level of environmental quality and the evolution of population, viz., the population growth in the model. Here, we assume that demographic pressure impacts upon environmental quality and this consequently affects population growth via mortality and fertility changes. Thus, the equation accommodates the feedback effect from demographic process to environmental quality. Moreover,  $E_t$  can be described by

$$(1 - L)^d \Phi(L) E_t = \Theta(L) \epsilon_t. \quad (11)$$

$L$  is the lag operator as defined before and  $\Phi(L) = (1 + \phi_1 L + \dots + \phi_p L^p)$  and  $\Theta(L) = (1 - \vartheta_1 L - \dots - \vartheta_q L^q)$  are *AR* and *MA* polynomials respectively. Following the expressions in (9) and (10), we re-write the population growth equation as a function of stochastic

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<sup>8</sup>We emphasize that the results from this exercise need to be studied with caution due to the simplicity of model assumptions of Solow-Swan economy especially because Solow-Swan model has no endogenous (optimal) capital accumulation, and is based on exogenous saving rate and have no mechanism to describe evolution of technology. Nevertheless, our approach provides insights into the behavior of the economic growth system while demographic system as endogenous with slow-converging shocks. The results presented here are of statistical in nature. A more formal growth-theoretic implication would have an interesting exercise, however, this is beyond the scope of this exercise.

<sup>9</sup>Mishra et al. (2010) studies consequences of long-memory population shocks on economic growth.

environment:

$$n_t = -E_t \left( (1 - L)^d + \theta \right) \psi^{-1} \quad (12)$$

$$\equiv - \left( E_t \psi \left[ \sum_{j=0}^{\infty} \frac{\Gamma(j-d)}{\Gamma(j+1)} \Gamma(-d) L^j + \theta \right]^{-1} \right) \quad (13)$$

Now since  $Y_t$  is described by (1), then inducting the growth of population equation with long memory in (12) in the production function it appears that

$$Y_t = K_t^\alpha \left[ A_t \left( 1 - E_t \left[ \psi \sum_{j=0}^{\infty} \frac{\Gamma(j-d)}{\Gamma(j+1)\Gamma(-d)} L^j + \theta \right]^{-1} N_{t-1} \right) \right]^{1-\alpha} \quad (14)$$

We will use this modified production function along with savings and consumption function with an assumed depreciation rate of capital stock and proportion of labor and capital for simulation. The investment,  $I_t$  and capital stock equations are described as

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (15)$$

where capital stock is assumed to decline at a constant rate of  $\delta(0 < \delta < 1)$  per period. Given that  $s$  is the fraction of  $Y$  to be invested, then

$$I_t = sY_t \quad (16)$$

and consumption is defined according to

$$C_t = (1 - s)Y_t \quad (17)$$

The system of equations (14-17) describing modified Solow-Swan economy will be utilized for simulation exercise in section 6.

Having described the output growth equation with stochastic long memory in population and environment, it is necessary to outline in brief the main results of variation of  $d$  in this system. To elucidate, if we assume that environmental quality is governed by

the mean of the process, i.e.,

$$E_t = \bar{E} + \epsilon_t \quad (18)$$

then the spectral density of  $E_t$  is written as

$$f_E(\lambda) = \frac{\sigma_\epsilon^2}{2\pi} \quad (19)$$

However, if a stochastic shock persists in  $E_t$  or in its growth, i.e., if  $(1 - L)^d E_t = \epsilon_t$ , then the spectrum is governed by the stochastic memory in the environmental equation:

$$f_E(\lambda) = |1 - e^{i\lambda}|^{-2d} f_\epsilon(\lambda) = \left| 2 \sin \frac{\lambda}{2} \right|^{-2d} f_\epsilon(\lambda) \quad (20)$$

where  $f_\epsilon(\lambda)$  denotes the spectral density of the error term. The following results are obtained which explain the dynamics of interrelationships among environment, economic and population growth in the presence of stochastic shocks.

**Proposition 1:** *Under the assumption of the environmental and economic growth system described in Eqs. 1-3, the long memory in output growth,  $y_t$ , can be described as a function of long memory in the growth of environmental quality and population growth.*

**Proof:**

Empirical verification has already proven that demography-push led environmental problems have inflicted substantial fluctuations to economic growth. Since long memory in output growth can be written as

$$(1 - L)^d y_t = u_t \quad (21)$$

with the usual restrictions of  $d$  on the real line. If  $y_t$  is described by  $y_t = \bar{y} + u_t$ , i.e., the process is independently distributed around the mean, then the spectral density of  $y_t$  is  $f_y(\lambda) = \frac{\sigma_u^2}{2\pi}$ . If shocks persist in  $y_t$  and is characterized in long memory setting,

then  $y_t$  follows  $(1 - L)^d y_t = u_t$  with the spectral density  $f_y(\lambda) = |1 - e^{i\lambda}|^{-2d} f_u(\lambda) = |2\sin(\frac{\lambda}{2})|^{-2d} f_u(\lambda)$ , where  $f_u(\lambda)$  is the spectral density of the error term. A natural way to present whether, say  $y_t$  is a short-memory or a long memory process, is to know the shape of the spectral density of  $y_t$  and decompose spectral frequencies according to different components, in our case environment and population. Since we can define the persistent properties of  $E_t$  (growth of environment) and  $n_t$  (growth of population) in terms of long-memory process, then the possible source of long-memory in  $y_t$  can be expressed as a product of the stochastic long-memory components from  $E_t$  and  $n_t$ . Therefore, the long-memory in  $y_t$ , can be expressed as a product of  $f_n(\lambda)$  and  $f_E(\lambda)$ , such that  $f_y(\lambda) = [(|1 - e^{i\lambda}|^{-2d} f_u(\lambda))(|1 - e^{i\lambda}|^{-2d} f_\epsilon(\lambda))]$ , where  $u_t$  and  $\epsilon_t$  are the *iid* error processes of population growth and environment. Thus the likelihood of a possible stochastic shock in the output growth equation can be expressed as the joint likelihood of the stochastic shocks from demographic and environmental system.

One can also express the variance in output growth as the sum of the variance of environment and demographic system and the covariance between them such that

$$\text{Var}[y_t] = \text{Var}[N_t] + \text{Var}[E_t] + \text{Cov}[\mathbf{E}_t, \mathbf{N}_t] \quad (22)$$

As  $t \rightarrow \infty$ , the contribution of  $E_t$  and  $N_t$  to the variance of  $y_t$  increases and under deterministic assumption there would be steady state equilibrium. However, under alternative assumption of stochasticity, both  $E_t$  and  $N_t$  tend to experience heavy spurts and the spectral variance of their respective shocks contribute to the total variance of output.

**Proposition 2:**

*If  $d$  is mean converging/covariance stationary, then it is possible to achieve  $g'(E) < 0$  that is, change in output growth as a function of environmental growth is negative.*

**Proof:**

Recall the relation where  $g(E) = \frac{A(t)}{A(t)} = \mu - \beta \frac{\dot{E}}{E} = \mu + \beta\theta + \frac{\psi n(E)}{E}$ ,  $g'(E) < 0$  if  $n(E) = n$ . Now since,  $n(E)$  is stochastic and is given by Eq. 12, where  $(1 - L)^d$  is given by the power

series expansion:  $(1-L)^d = \sum_0^\infty h_j L^j = 1 - dL + \frac{d(d-1)}{2!} L^2 - \frac{d(d-1)(d-2)}{3!} L^3 - \dots$ , then  $g'(E) < 0$  occurs when  $d$  has convergent properties in the sense that  $d \in (0, 0.5)$ . Because with  $d \leq 1/2$ , the population growth series is covariance stationary with summable autocorrelation function and which is positive, then (local) stability is achieved. Moreover, under the assumption that  $n(E) = n$ , that is under deterministic setting population growth can at least be zero and will be positive normally. This implies that the change in environmental quality ( $g'(E) < 0$ ) will be negative even under stochastic formulation where the sum of stochastic shocks are added to a deterministic constant. The intuition is as long as stochastic shocks accumulate and has convergent properties such that  $0 < d \leq 1/2$ , the numerator still can be positive and therefore  $g'(E) < 0$ .

**Proposition 3:** *Under the assumption that  $n(E)$  is stochastic and is characterized by stochastic memories,  $n'(E) < 0$  only if  $0 < d < 1/2$ .*

**Proof:**

This is a corollary of the main result. The explanation follows from result 2. The idea is that stochastic population growth does not necessarily pertain to constant and positive numerator as in Eq. 6. The conditions under which  $n'(E) < 0$  can be given by the convergence properties of the  $n(E)$  function under stochastic memory properties.  $n'(E) < 0$  implies that a unit change in  $E$  will induce a negative response from population growth. The converse is also true. To prove the proposition assume that  $d$  lies in the region  $(1/2, 1)$  and second  $d \geq 1$ , then the autocorrelation function is not summable and the series exhibit non-stationary long-memory. There is no convergence of shocks to the mean value, thus the growth of the population series has no constant value in the numerator. Only when  $0 < d < 1/2$ ,  $(1-L)^d = \sum_0^\infty h_j L^j = 1 - dL + \frac{d(d-1)}{2!} L^2 - \frac{d(d-1)(d-2)}{3!} L^3 - \dots$  has finite sum and has a positive constant on the numerator thus giving rise to  $n'(E) < 0$ . Local stability and long-run convergence (to steady state) occurs only when the stochastic  $E_t$  and  $n_t$  have  $(1-L)^d$  with  $d \in (0, 0.5)$ . Chaotic EDG system occurs when  $d \geq 1$  due to the high sensitivity of the system to their initial values and high non-linearity due to

propagation of shocks.

## IV Empirical analysis

### IV.1 Data characteristics

We discuss in this section the persistence behavior of aggregate population, pollution and economic growth for a sample of OECD and non-OECD countries chosen according to the highest volume of CO<sub>2</sub> emissions. Pollution data is presented by national CO<sub>2</sub> emission per capita, measured in metric tons. The data have been collected from *Carbon Dioxide Information Analysis Center* (CDIAC) of the *Oak Ridge National Laboratory* (see Marland et al., 2004). The aggregate population data, measured in thousand numbers has been collected from Maddison (2004). Finally, the real GDP per capita series, measured in thousand dollars in 2001 international prices, were extracted from the *Penn World Table 6.1* (Summer and Heston, 2006). All data are annual time series. For OECD countries CO<sub>2</sub> emissions and aggregate population cover the period 1870-2003. The same variables for non-OECD countries are available for the period 1900-2003. Finally, per capita GDP for both OECD and non-OECD countries covers the period 1950-2003 as the historical data before 1950 are not available for the chosen non-OECD countries.

Figures 1-4 present plots of population, CO<sub>2</sub> and per capita real GDP for the chosen OECD and non-OECD countries after logarithmic transformations. The real GDP plots for OECD (Figure 3) and Non-OECD (Figure 4) countries show steady trends. Fluctuations occur at different periods but OECD countries on the whole display similar growth trend, where Non-OECD countries evince differential trends. The striking common feature in figures 1-4 is that CO<sub>2</sub> emissions display significant fluctuations for both OECD and non-OECD countries whereas trend in population growth display steady pattern reflecting contrasting features of the growth processes of population and pollution. Long-range temporal dependence pattern of population and CO<sub>2</sub> emissions can be discerned by looking at their autocorrelations functions. For both sets of countries, the autocorrelation lag has been set to 24 indicating 24 years given our annual time series

data for these variables. From figures 5-8, one can easily observed that autocorrelations among observations are still strong and positive even at higher lag. The rate of decline of autocorrelated lags are slow-paced indicating possible presence of long-memory in these variables.

To explore equilibrium relationship among real GDP per capita, population and pollution we model them in fractional co-integration framework. All data used for this purpose range from 1950-2003. It is important to note that growth in the variables are calculated by taking logarithmic differences between period  $t$  and  $t - 1$ . In case of pollution growth, it may be described by  $\Delta \ln(E_t) = (1 - L) \ln(E_t)$  where  $\Delta = (1 - L)$  and  $LE_t = E_{t-1}$ . Similarly, fractional difference of  $\ln(E_t) = \Delta^d \ln(E_t) = (1 - L)^d \ln(E_t)$ , where  $d$  is estimated using either parametric or semi-parametric methods.

## **IV.2 Fractional (co-)integration in GDP per capita, pollution and population**

### **IV.2.1 Testing long-memory**

To further investigate, our next step is to test for fractional integration for population and CO<sub>2</sub> emissions after log-transformation. Accordingly, we have estimated fractional integration parameter  $d$  for the process  $(1 - L)^d X_{it} = u_{it}$  where  $i$  represents population and CO<sub>2</sub> emissions. There exist many approaches for estimating  $d$ . Some of them are parametric, in which the model is specified up to a finite number of parameters. However, parametric approaches suffer from the drawback that unless correct model is specified, the estimates are liable to be inconsistent. In fact, misspecification of short-run components may invalidate the estimation of the long-run parameter. Thus, there may be some advantages on estimating  $d$  with semiparametric techniques. In this paper, we have employed Robinson's (1995) gaussian semi-parametric procedure. This is primarily a Whittle estimate (1951) in the parametric domain, considering a band of frequencies that



degenerates to zero. The estimate is defined by

$$\hat{d} = \arg \min_d \left( \log \overline{V(d)} - 2d \frac{1}{m} \sum_{j=1}^m \log \omega_j \right) \quad (23)$$

where  $\overline{V(d)} = \frac{1}{m} \sum_{j=1}^m I(\omega_j) \omega_j^{2d}$ ,  $\omega_j = \frac{2\pi j}{T}$ ,  $\frac{m}{T} \rightarrow 0$ .  $I(\omega_j)$  is the periodogram of the raw time series,  $m$  is bandwidth parameter and  $d \in (-0.5, 0.5)$ . Under finiteness of the fourth moment and other mild conditions, Robinson (1995) proved that  $\sqrt{m}(\hat{d} - d_a) \rightarrow N(0, 1/4)$  as  $T \rightarrow \infty$ , where  $d_a$  is the true value of  $d$  and with the additional requirement that  $m \rightarrow \infty$  slower than  $T$ . Robinson (1995) showed that  $m$  must be smaller than  $T/2$  to avoid aliasing effects. One may also use other semi-parametric approaches such as log periodogram regression of Geweke and Porter-Hudak (1983) or modified log-periodogram regression of Kim and Phillips (2000). In Robinson's (1995) approach we do not have to assume Gaussianity to obtain an asymptotic normal distribution. The Whittle method is argued to be more efficient than log-periodogram regression approaches.

A choice must be made of the number of harmonic ordinates to be included in the spectral regression. One of the innovations of Robinson's estimator is that it is not restricted to using a small fraction of the ordinates of the empirical periodogram of the series. The estimator also allows for the removal of one or more initial ordinates and for the averaging of the periodogram over adjacent frequencies. Since some researchers have found that exclusion of initial ordinates from the log-periodogram regression improves the properties of tests, in this paper we have excluded two initial ordinates. A choice of bandwidth, however, needs to be made, for which we have performed a Monte-Carlo simulation using Davidson's (2010) time series modelling software and found that bandwidth = 0.90 possess lowest bias.<sup>10</sup>

In tables 1 and 2 we present estimates of  $d$  for logarithm of total population and CO<sub>2</sub> emissions in case of both OECD and non-OECD countries. The estimates are reported for various bandwidths ( $\tau = 0.70 - 0.90$ ). The null hypothesis we test is  $d = 0$ , that is short memory against an alternative hypothesis of long-memory. As is evident, all estimates of  $d$  for both time series and for both group of countries indicate  $1 > d > 0.5$ .

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<sup>10</sup>We have not presented the detailed results here. However, these can be obtained from the authors.

Indeed, our choice of  $\tau = 0.90$ , which is the default value of Robinson (1995) provides us with the same conclusion. The implication is that the variables are characterized by long-memory processes and therefore a shock in the series in the distant past will still have a long-lasting influence on the present. The  $d$  estimates are however smaller than 1 indicating the long-memory persistence with the possibility of convergent shocks in the long-run. No distinguishable results can be observed across OECD and non-OECD countries's magnitude of persistence for both population and CO<sub>2</sub> emissions series as  $d$  estimates are all greater than 0.5 but less than 1. The autocorrelation functions for the values of  $d$  between 0.5 and 1 have same economic and statistical implications.

Table 1: Robinson's (1995) semi-parametric estimation of  $d$  for CO<sub>2</sub> emissions  
(Note:  $H_0: d = 0$ . Standard errors are in parentheses)

$T^\tau$	$\tau = 0.70$	$\tau = 0.75$	$\tau = 0.80$	$\tau = 0.85$	$\tau = 0.90$
Periodogram					
Ordinates	25	33	41	51	65
<b>OECD</b>					
USA	0.820 (0.051)	0.767 (0.041)	0.687 (0.041)	0.652 (0.034)	0.569 (0.034)
Japan	0.921 (0.060)	0.971 (0.050)	0.872 (0.041)	0.840 (0.031)	0.767 (0.029)
UK	0.838 (0.151)	0.794 (0.134)	0.775 (0.125)	0.634 (0.104)	0.532 (0.087)
France	0.708 (0.154)	0.708 (0.117)	0.760 (0.093)	0.749 (0.072)	0.714 (0.062)
Canada	0.999 (0.042)	0.929 (0.038)	0.863 (0.034)	0.818 (0.029)	0.745 (0.026)
<b>Non-OECD</b>					
China	0.706 (0.134)	0.519 (0.154)	0.529 (0.131)	0.514 (0.137)	0.583 (0.126)
India	0.957 (0.018)	0.937 (0.020)	0.912 (0.019)	0.879 (0.018)	0.808 (0.022)
Brazil	1.004 (0.048)	0.972 (0.040)	0.954 (0.034)	0.906 (0.030)	0.829 (0.030)
South Africa	0.908 (0.024)	0.892 (0.023)	0.869 (0.020)	0.827 (0.020)	0.755 (0.023)
Mexico	1.077 (0.153)	0.987 (0.117)	0.930 (0.098)	0.871 (0.081)	0.789 (0.068)

Table 2: Robinson's (1995) semi-parametric estimation of  $d$  for total population  
(Note:  $H_0: d = 0$ . Standard errors are in parentheses)

$T^\tau$	$\tau = 0.70$	$\tau = 0.75$	$\tau = 0.80$	$\tau = 0.85$	$\tau = 0.90$
Periodogram					
Ordinates	25	33	41	51	65
<b>OECD</b>					
USA	0.958 (0.006)	0.941 (0.006)	0.909 (0.008)	0.860 (0.011)	0.774 (0.018)
Japan	0.965 (0.009)	0.946 (0.008)	0.913 (0.009)	0.862 (0.012)	0.775 (0.018)
UK	0.960 (0.022)	0.943 (0.019)	0.912 (0.017)	0.859 (0.017)	0.773 (0.021)
France	0.967 (0.046)	0.964 (0.034)	0.940 (0.026)	0.896 (0.021)	0.809 (0.023)
Canada	0.945 (0.010)	0.933 (0.008)	0.905 (0.008)	0.857 (0.011)	0.773 (0.017)
<b>Non-OECD</b>					
China	0.999 (0.014)	0.971 (0.013)	0.941 (0.013)	0.896 (0.016)	0.814 (0.022)
India	0.987 (0.027)	0.969 (0.023)	0.934 (0.022)	0.893 (0.022)	0.810 (0.026)
Brazil	0.985 (0.008)	0.961 (0.009)	0.932 (0.011)	0.889 (0.014)	0.808 (0.021)
South Africa	0.589 (0.296)	0.969 (0.379)	0.751 (0.304)	0.731 (0.235)	0.593 (0.203)
Mexico	0.997 (0.015)	0.971 (0.013)	0.940 (0.014)	0.895 (0.016)	0.812 (0.022)

### IV.3 (Fractional-)cointegration analysis

In order to test if environmental pollution, population growth and per capita income growth are fractionally co-integrated, we adopt the two-step strategy as in Caporale and Gil-Alana (2004, 2005) and discussed succinctly in Gil-Alana and Hualde (2009). The strategy is to employ Robinson (1995) test for fractional integration of a time series in various stages. Accordingly, in the first step, we test for the order of integration of each series, and if they are found to be of the same order, we test, in the second step, the order of integration of the estimated residuals of the cointegration relationship. Let us call  $\epsilon_t$ , the estimated equilibrium errors among three series, real GDP per capita, aggregate population and CO<sub>2</sub> emissions for each country:

$$\epsilon_t = \ln(Y_t) - \hat{\alpha}_1 \ln P_t - \hat{\alpha}_2 \ln E_t \quad (24)$$

where  $Y_t$ ,  $P_t$  and  $E_t$  are real GDP per capita, population and CO<sub>2</sub> emissions respectively and  $\hat{\alpha}$  are the OLS estimator of the cointegrating parameter. Let us consider the model:

$$(1 - L)^{d+\theta} = u_t \tag{25}$$

where  $u_t$  is a  $I(0)$  process ; we applied the Robinson (1995)'s testing procedure in order to test the null hypothesis  $H_0 : \theta = 0$  against the alternative  $H_1 : \theta < 0$ . If the null hypothesis is rejected, it implies that the equilibrium error exhibits a smaller degree of integration than the original series:  $Y_t$ ,  $P_t$  and  $E_t$  are thus fractionally cointegrated. On the opposite, if the null hypothesis is not rejected, the series are not cointegrated because the order of integration of  $\theta$  is the same as the order of the original series. As a first step to testing this hypothesis, we have saved residuals from regression of real GDP per capita on total population and CO<sub>2</sub> emissions for each country.<sup>11</sup> Due to the unavailability of real GDP data before 1950 for some countries and for the sake of comparison, the regression has been run for the truncated sample over the period 1950-2003. In the next step, the equilibrium errors  $\hat{\epsilon}_i$  where  $i$  is indexed for each country, are tested for short or long-memory using Robinson's (1995) semi-parametric log periodogram regression. Table 3 presents results of the  $d$  estimates of equilibrium errors for each country. It is observed that at  $\tau = 0.9$ , the default value as in Robinson (1995), USA, Japan and UK have  $d < 0.5$  implying that shocks in the equilibrating mechanism will converge and that there is a stable co-movement among GDP per capita, population and CO<sub>2</sub> emissions in these countries. For others, we find that  $d$  values range from 0.572 - 0.959, that is  $1 > d > 0.5$ . The co-movement of GDP, population and CO<sub>2</sub> emissions in these countries contain non-stationary long-memory with a possibility of mean convergence in the long-run. Among countries with values of  $d$  in the range 0.5-0.9, China has highest  $d$  (0.959) for equilibrium errors, while South Africa has the lowest  $d$  value (0.572). All  $d$  values are statistically significant at 5 percent significance level.

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<sup>11</sup>The detailed results have not been reported here but are available with the authors.

Table 3: Robinson's (1995) semi-parametric estimation of  $d$  for estimated equilibrium errors

(Note:  $H_0: d = 0$ . Standard errors are in parentheses)

$T^\tau$	$\tau = 0.70$	$\tau = 0.75$	$\tau = 0.80$	$\tau = 0.85$	$\tau = 0.90$
Periodogram					
Ordinates	25	33	41	51	65
<b>OECD</b>					
USA	0.217 (0.085)	0.233 (0.069)	0.287 (0.081)	0.205 (0.070)	0.183 (0.069)
Japan	0.443 (0.314)	0.449 (0.252)	0.635 (0.175)	0.452 (0.166)	0.426 (0.115)
UK	1.191 (0.331)	0.920 (0.334)	0.752 (0.249)	0.648 (0.209)	0.448 (0.162)
France	1.023 (0.464)	1.051 (0.373)	1.190 (0.255)	1.112 (0.223)	0.896 (0.205)
Canada	1.074 (0.330)	1.046 (0.265)	1.066 (0.214)	1.004 (0.171)	0.711 (0.169)
<b>Non-OECD</b>					
India	1.065 (0.238)	0.919 (0.220)	0.5915 (0.222)	0.847 (0.199)	0.665 (0.153)
China	0.394 (0.121)	0.483 (0.128)	0.993 (0.233)	1.467 (0.307)	0.959 (0.284)
Brazil	1.039 (0.262)	0.978 (0.214)	0.943 (0.218)	0.683 (0.217)	0.680 (0.152)
South Africa	0.894 (0.486)	0.775 (0.400)	0.725 (0.259)	0.761 (0.208)	0.572 (0.172)
Mexico	1.488 (0.435)	1.104 (0.452)	1.006 (0.288)	0.813 (0.247)	0.687 (0.235)

## V Simulation experiments

The simulation experiment carried out in this section aims to lend additional support for the long-memory dynamics of Economy-Demography-Environment framework. The simulation economy closely follows the modified Solow-Swan growth model by embedding stochastic long-memory characters. Our idea is to show that as stochastic shocks move from convergent to high degree of non-convergence, i.e., as  $d$  moves from 0 to 1, the response of the economy and the environment over time becomes fairly stochastic in nature. Different regimes of change for economy and environment arise due to shifting patterns of stochastic memory from convergence to high degree of non-convergence. Our standard Solow-Swan economy has global assumption about labor and capital usage in production (that is, 2/3rd and 1/3rd) in line with many empirical research. The depreciation rate has been kept constant following the tradition of constant scrapping rule. Time dimension is set to 50 years so that the effect of long memory can be gauged over five decades.

From Figures 9-17, we observe interesting features of the response of environment to long-memory shocks. Essentially, three distinct patterns can be noted. For instance, we observe that depending on the value of  $d$ , there is a change of regime starting from approximately year 15-16 (cf. figures 9 and 12). So, there are basically two regimes for the first type. Figures 10 and 11, 13 and 14 depict different pattern (one regime with no change). The third pattern is observed in Figure 14 where now, two dates matter, year 8 and year 12. There are a total of three phases: before year 8, between year 8 and year 12, and after year 12. Moreover, for this third pattern, in each phase, the place of curves varies depending on  $d$ . Thus, before year 8, the curve for  $d = 1$  is above all curves. Between year 8 and year 12, the curve for  $d = 1$  is below all curves, and after year 12, we still obtain the same curve ranking as in phase 1 (before year 8). So, here, we have in total 3 regimes. So, generally, we can say that there exists a chaotical dynamic structure depending on the sensitivity of the initial values.

From the above, it appears that response of output and environment to long memory shock varies with the level of  $d$  values. The output growth equation which embeds stochastic features of environment and population growth responds to the long-memory shock in expected pattern: that stochastic shocks to environment and demography would result in stochasticity in output growth in the long-run. From environmental perspective we observed that the stochasticity of pollution growth, broadly the environmental system grows over time. That is what we observe for the output figures. Regimes changes are also presented in Figures 15-17 where distinction is made between one regime with no regime change. To summarize, the simulation experiment carried out for a modified Solow-Swan economy shows that economy-demography and environmental interactions are exceedingly complex and that the persistence of stochastic shocks in one system easily filters into the others, accelerating over time and pressing the system to behave in a stochastic pattern in the long-run. Different regime changes observed in our experiment due to variations in the magnitudes of the long-memory parameter indicate how initial distribution can change the growth profile while stochastic shocks move from convergence to non-convergence. Different economic and thus environmental systems would thus be observed depending on how fast  $d$  moves from 0 to 1 and how initial values change.

## VI Conclusion

We modeled the dynamics of interactions between economy-demography-environmental systems under slowly-convergent shocks. It was demonstrated that presence of long memory in environment and population gave rise to long-memory in economic growth. Time series characteristics of population, economic growth and pollution series were exploited emphasizing on the fact that the evolutionary paths of the series are bound to contain perturbations, the effect of which may extend beyond mean reversion. The source of stochasticity in economic growth in this paper, was therefore illustrated to result from persistence of shocks in population and environmental systems. It was shown that the possible mean reversion of persistent shocks in economic growth will depend upon the shock convergence pattern in both population and environmental systems. Empirical illustration for a set of highly polluting OECD and non-OECD countries evinced that pollution and population series displayed high degree of persistence, often reflecting non-mean convergence.

The co-evolutionary path of the three environment, demographic and economic growth system was modeled using fractional co-integration framework. It was found that the equilibrium error from the regression of economic growth on environment and population displayed stochastic long-memory, implying that unless volatility and stochasticity in population and environmental systems are checked, economic growth system will tend to be more chaotic across globe. Simulation experiment for a modified Solow-Swan economy provided additional support to our proposition that rate of convergence of demographic and environmental shocks would determine the rate of convergence of output shocks. Regime changes in economy and environment were observed to be consequences of the presence of persistence in environment and demographic system.

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# Appendix: Figures

Figure 1: Logarithmic plots of CO<sub>2</sub> emissions and population: OECD countries

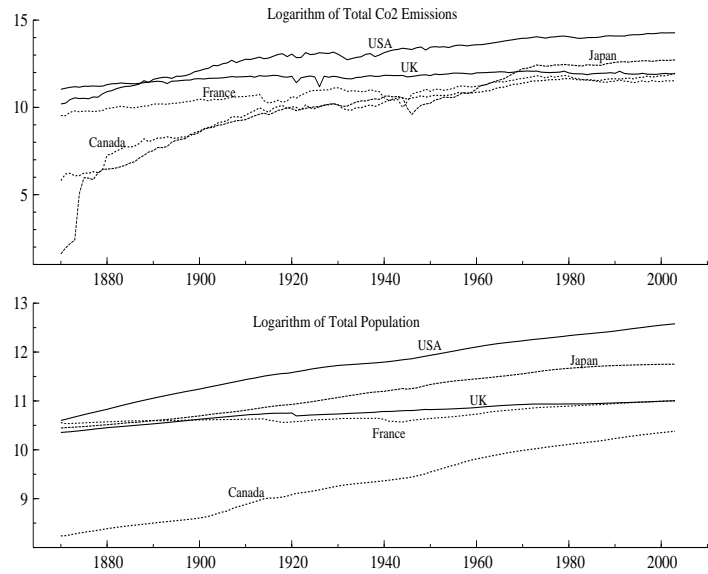


Figure 2: Logarithmic plots of CO<sub>2</sub> emissions and population: Non-OECD countries

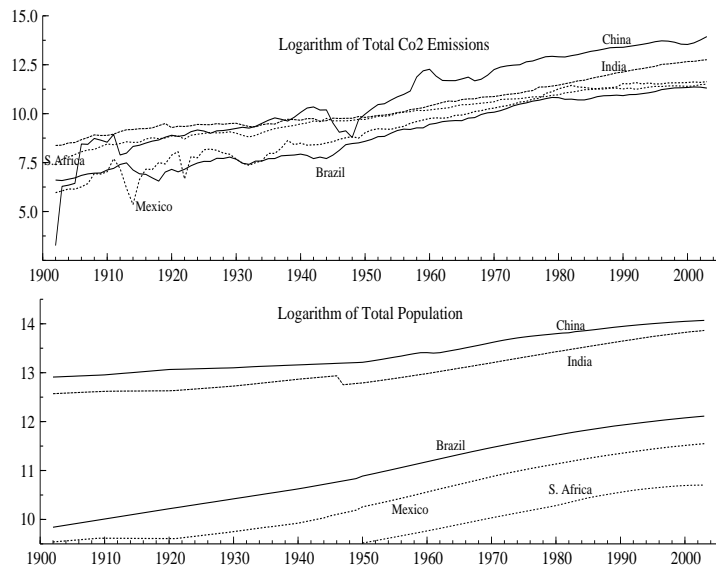


Figure 3: Logarithmic plots of real GDP: OECD countries

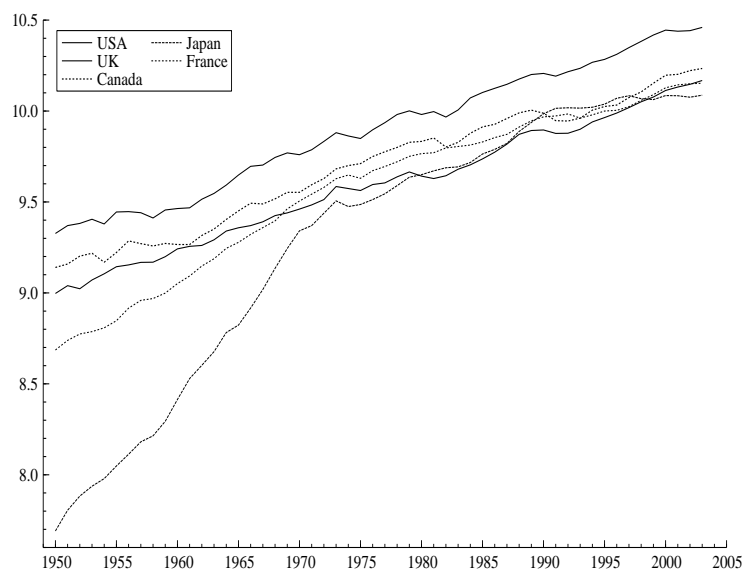


Figure 4: Logarithmic plots of real GDP: Non-OECD countries

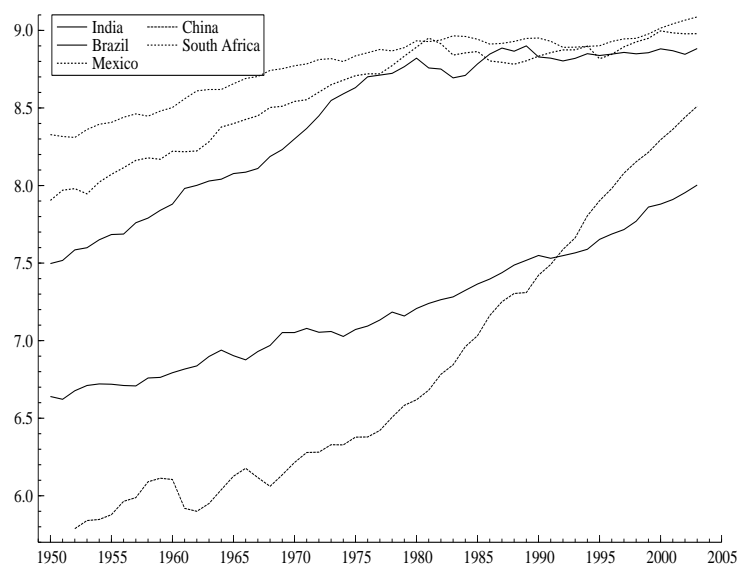


Figure 5: Autocorrelation functions for  $CO_2$  emissions: non-OECD countries

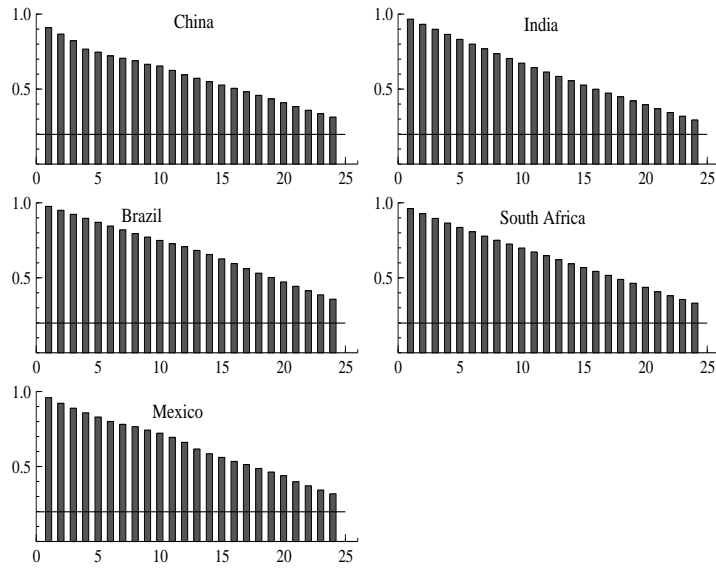


Figure 6: Autocorrelation functions for  $CO_2$  emissions: OECD countries

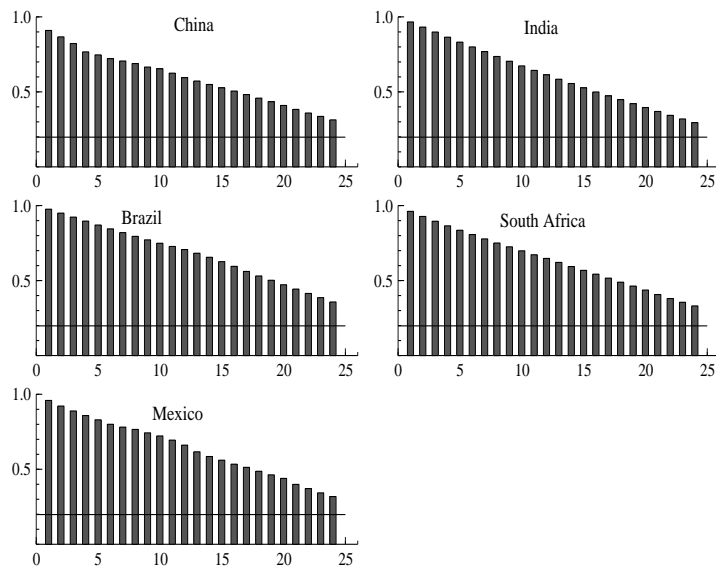


Figure 7: Autocorrelation functions for total population: non-OECD countries

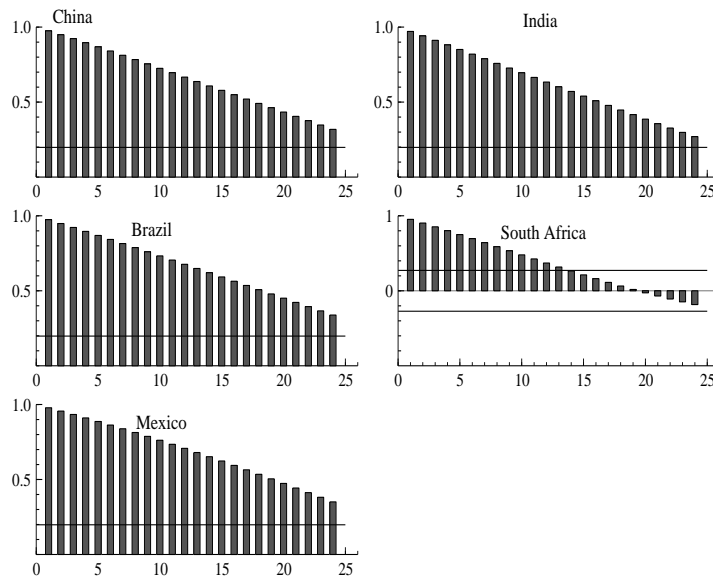
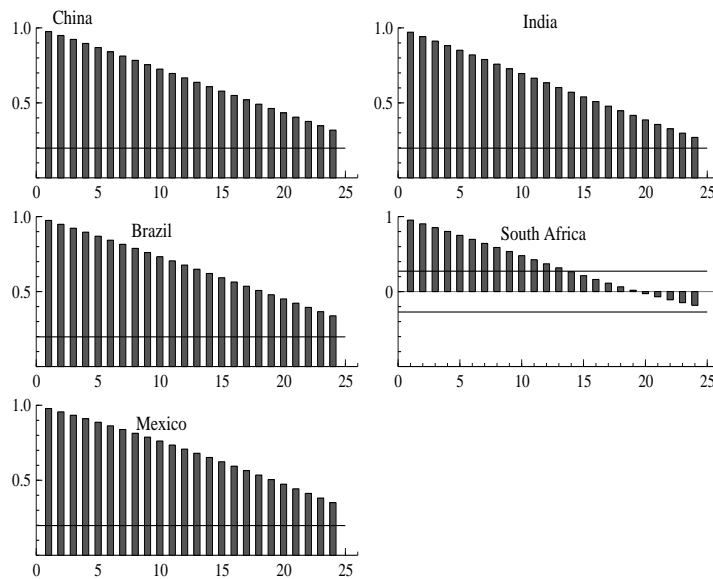


Figure 8: Autocorrelation functions for total population: OECD countries



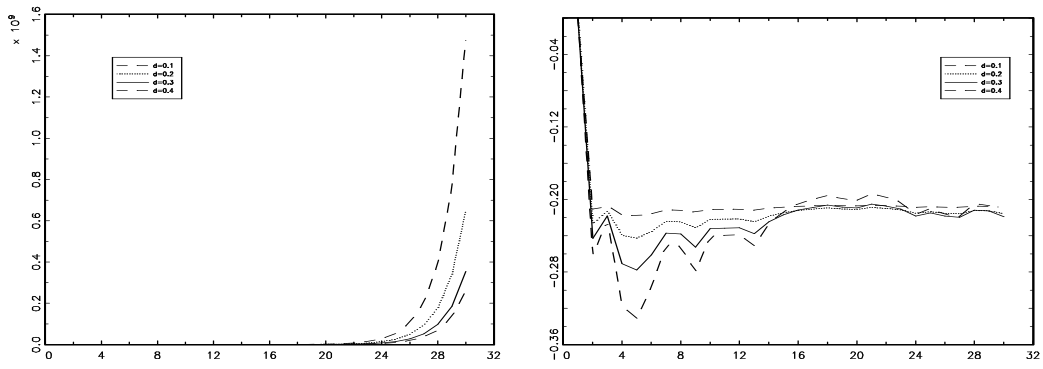


Figure 9: Simulation of the dynamics of the environment demographic growth system. [Left]: Response of environment to long-memory shock. [Right]: Response of output to long-memory shock.

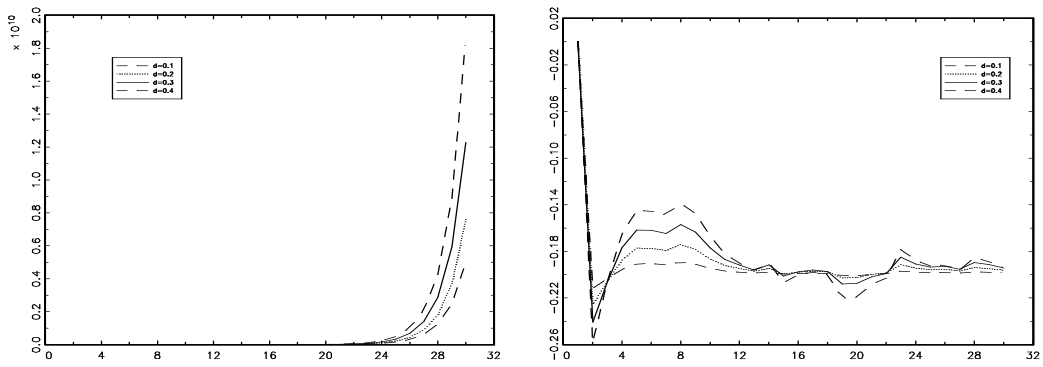


Figure 10: Simulation of the dynamics of the environment demographic growth system. [Left]: Response of environment to long-memory shock. [Right]: Response of output to long-memory shock.

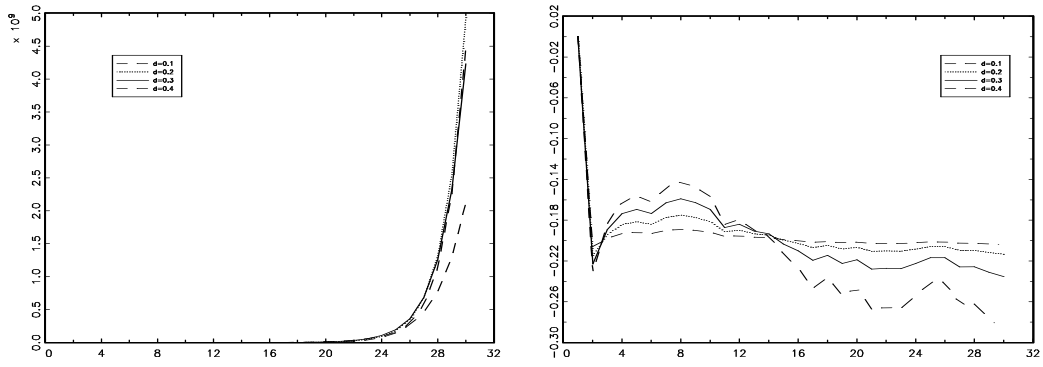


Figure 11: Simulation of the dynamics of the environment demographic growth system. [Left]: Response of environment to long-memory shock. [Right]: Response of output to long-memory shock.

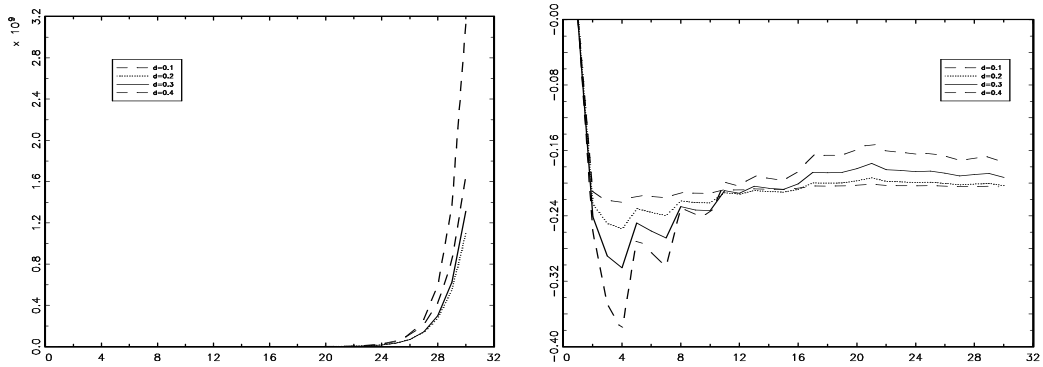


Figure 12: Simulation of the dynamics of the environment demographic growth system. [Left]: Response of environment to long-memory shock. [Right]: Response of output to long-memory shock.



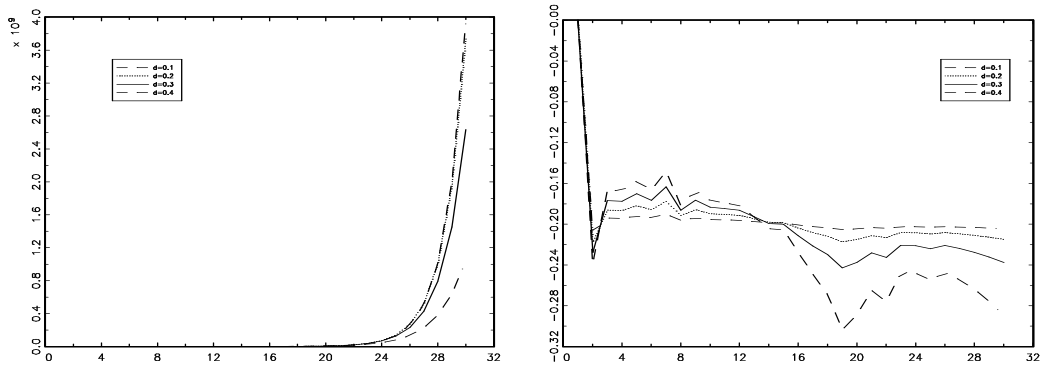


Figure 13: Simulation of the dynamics of the environment demographic growth system. [Left]: Response of environment to long-memory shock. [Right]: Response of output to long-memory shock.

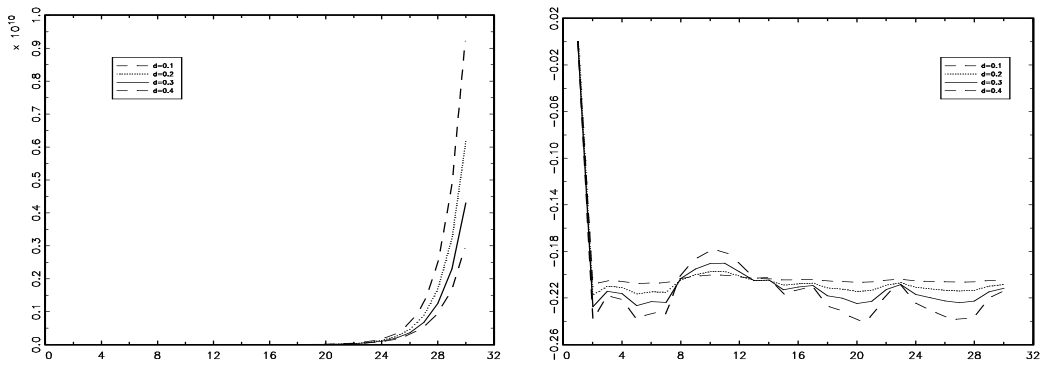


Figure 14: Simulation of the dynamics of the environment demographic growth system. [Left]: Response of environment to long-memory shock. [Right]: Response of output to long-memory shock.

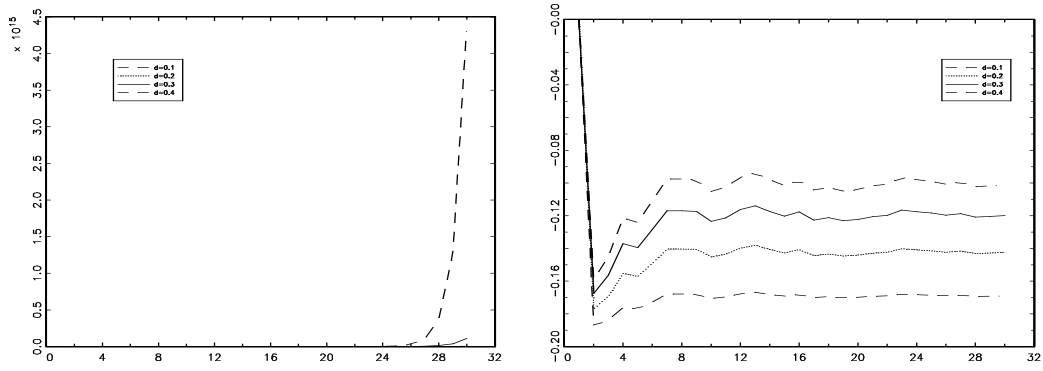


Figure 15: Simulation of the dynamics of the environment demographic growth system: two regimes. [Left]: Response of environment to long-memory shock. [Right]: Response of output to long-memory shock.

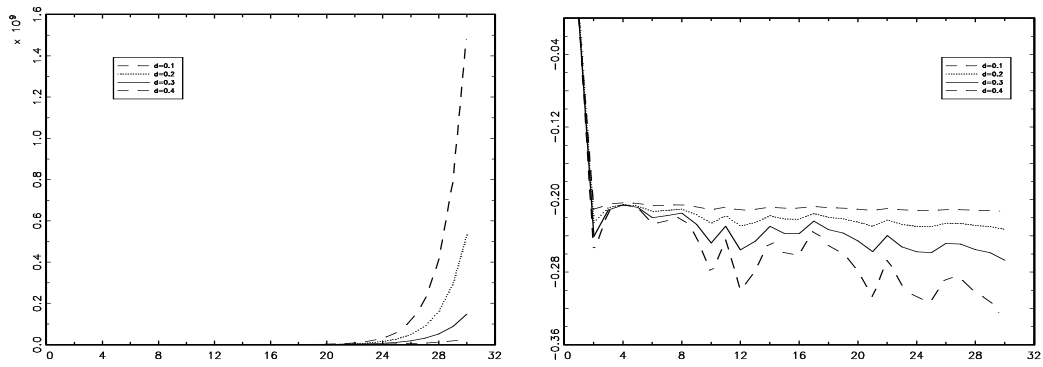


Figure 16: Simulation of the dynamics of the environment demographic growth system: three regimes. [Left]: Response of environment to long-memory shock. [Right]: Response of output to long-memory shock.

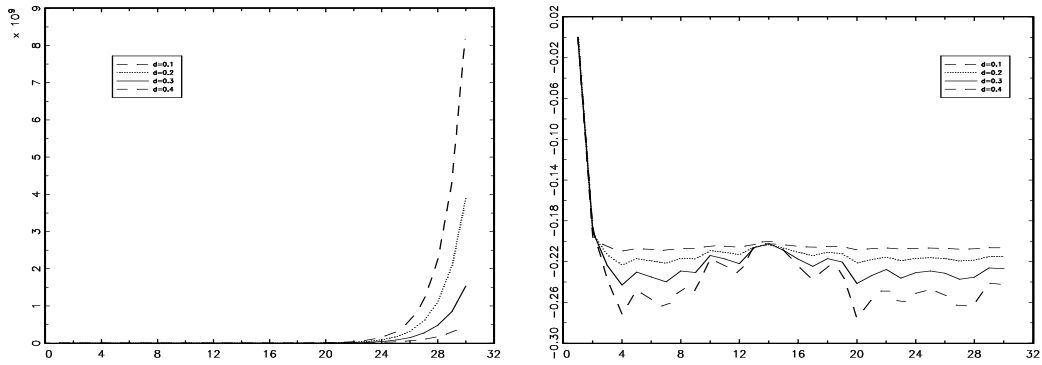


Figure 17: Simulation of the dynamics of the environment demographic growth system: one regime with no regime change. [Left]: Response of environment to long-memory shock. [Right]: Response of output to long-memory shock.