Education Financing Policy: Income Contingent Loans and Educational Poverty Traps

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Abstract

This paper examines the role of income contingent loan policies in ensuring intergenrational mobility and propelling poor households out of persistent educational poverty traps and concommitant child labour traps. In developing economies, credit market imperfection frequently interacts with other rigidities to create long run 'traps' such that poor households stay poor generation after generation. In most of these cases, mass poverty, low average level of education and child labour go hand in hand. Yet the usual policy prescription of subsidized education is often not viable because of the associated large fiscal burden. In this paper we analyse the dynamic consequences of pursuing an alternative policy - the income contingent loan policy - which is not only self-financing but may also be more effective than many other standard policies in creating dynamic incentives for acquring education and thereby eliminating the long run 'traps'. However, the efficacy of such a policy depends on the productivity of basic education system. Thus effective elimination of long run poverty traps requires a holistic approach towards education, with emphasis on both primary and higher education.

KEYWORDS: Poverty Traps, Education Financing, Income Contingent Loans
JEL CLASSIFICATION: I22, I25, I28, H52

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1 Introduction

" Education is central to development. It empowers people and strengthens nations. It is a powerful "equalizer", opening doors to all to lift themselves out of poverty." ...Thus reads the opening passage of an overview of 'education' in the official website of the World Bank.[1] Indeed, education has long been perceived as the principal means (if not the 'miracle cure') for reducing poverty, inequality, inaptitude, ignorance, crime and such other malaises that are typically associated with underdevelopment. Not surprisingly, 'universal education' features prominently in the eight point development agenda (Millennium Development Goals) of the United Nations, adopted in the year 2000. The recent works on new growth theory have further underscored the importance of education by positing a positive link from education to skill formation to growth, thereby combining the normative 'equity' issue with the positive question of macroeconomic 'efficiency'. Consequently, all countries - developed and developing alike - have strived to put in place an active education policy with the objective of reaching education to the masses. This is reflected in the rise in overall public investment in education which has increased manifold since the second half of the 20th century.

Yet, 'education to the masses' is not an easy plan to implement, especially for a poor, less-developed economy. On the one hand, pervasive inequality along with imperfect (or missing) credit market imply that privately funded education remains the prerogative of the rich. On the other hand, the oft-prescribed public education or education-subsidy policy is also not sustainable in the long run because of the large fiscal burden it imposes on the already precarious macroeconomic balance sheets of these poor countries. Thus the real challenge lies in designing an appropriate government policy which is financially viable and at the same time effective in relieving the majority of the population who remain trapped in a low education- low skill- low income equilibrium generation after generation. In this paper we focus on an alternative policy - the income contingent loan scheme- which serves these dual purposes. We analyse the long run dynamic implications of such a policy from the perspective of a poor economy.

The paper starts on the basic premise that in the absence of active government intervention, poor economies are characterized by existence of educational poverty traps which may perpetuate over a long period of time. There now exists a substantive body of theoretical works, [2] which highlight how credit market imperfection interacts with other rigidities in a

[1] http://web.worldbank.org/WBSITE/EXTERNAL/TOPICS/EXTEDUCATION/0,,contentMDK:20591648~menuPK:14

[2] See for example, Banerjee and Newman (1993); Galor and Zeira (1993); Freeman (1996); Aghion and

poor economy to generate a vicious cycle of poverty: low income implies less investment in education and therefore low skills, which in turn restricts mobility and results in low income not only for the current generation, but for subsequent generations as well. The problem may get aggravated in the presence of uncertainties which make the skill formation process risky. One of the principal messages of all these theoretical constructs is that poverty can be chronic and therefore any policy to counter it must take all the dynamic consequences into account. Nonetheless, the existing analytical frameworks used for designing anti-poverty and concomitant education-financing policies are mostly static in nature.[3] As Mookherjee (2006) points out, "adding a dynamic perspective is likely to yield quite different implications for policy". Our paper is an attempt in this direction.

Another manifestation of chronic poverty is the long run persistence of child labour, or child labour traps. Poverty compels households to send their children to work, which hampers their education and skill formation, thereby restricting their future earning capabilities. This in turn implies that their subsequent generations are forced to work as child labour as well. Thus the vicious cycle continues yet again.[4] We recognize that low education and child labour are interrelated problems with similar long run consequences. Therefore in designing policies to combat child labour, once again it is imperative to take into account the dynamic implications of such policies, a fact which is often ignored in the existing discourses on child labour policies.[5]

In this paper we propose a government sponsored income contingent education loan scheme as an effective long run policy to combat both educational poverty traps as well as child labour traps. Income contingent loans (henceforth, ICLs) are self-financing loan contracts whereby the borrower is given loans to finance education with uncertain returns. The loaned amount is paid directly to the educational institutions. Agents who take this loan are required to pay back with interest if and only if they are successful in getting the degree

Bolton (1997); Maoz and Moav (1999); Mookherjee and Ray (2002, 2003).

[3]Dynamic analytical structures for anti-poverty measures are generally rare. A notable exception is Mookherjee and Ray (2008). We shall come back to a discussion of this paper in the context of our work later.

[4]Glomm (1997); Basu and Van (1999); Ranjan (1999,2001); Baland and Robinson (2000); Hazan and Berdugo (2002) are some examples of theoretical models that show existence of child labour traps in short run as well as long run.

[5]This point has been stressed by Udry (2006), who argues that the most effective policy in combating child labour is to encourage school attendence, which not only reduces child labour in the short run, but also has a long run impact by unleashing the dynamic forces that propel the household towards improved economic conditions over time.

and/or find fruitful employment. The repayment is collected in the form of taxes which are deducted directly from the pay cheque. The scheme is budget-balancing in a dynamic sense: the successful borrowers are charged an interest rate high enough for the government to be able to recover the entire loan amount.

The concept of ICL is not new. Friedman (1945) introduced the notion in the context of a private loan market. But the idea did not take off for the obvious moral hazard problems associated with private financing of human capital. Subsequently, the programme has been implemented successfully with active government support in Australia (HECS), New Zealand (ICL), South Africa (NSFAS); and has been introduced more recently in the United Kingdom (Graduate Tax) under the Higher Education Act, 2004. ICLs have been advocated in the context of higher education in the recent education financing literature[6] as a preferred alternative to education subsidies (e.g., scholarships) or ordinary student loans (e.g., mortgage loans), on the grounds that it involves less budgetary pressures (as compared to education subsidies) and is welfare-improving (as compared to ordinary loans) when agents are risk-averse. Again, most of these papers analyse the effect of ICLs in a static context.[7] We bring in here a dynamic perspective. We argue that in addition to its budgetary and welfare implications, a suitably designed ICL can be used as a feasible and effective anti-poverty strategy precisely because it creates dynamic incentives for poorer households to invest in education and thereby escape long run poverty traps.

There are several advantages of an ICL contract - so designed, which make it particularly attractive for resource-strapped developing economies in comparison to other anti-poverty measures. First, it is self-financing and therefore sustainable in the long run. Second, it is politically much easier to implement than a direct redistributive mechanism.[8] Third, it is more equitable than general educational subsidies as it is availed only by those who need it most. Fourth, since the government can obtain credit at more favourable terms than individuals, these credit market benefits can be passed on to the ICL-takers in the form of an expected interest rate that is lower than the market rate. Finally, it provides a cushion in case of failure so that poor households do not get further impoverished in the event of

[6] See, for example, García-Peñalosa and Wälde (2000); Migali (2006); Cigno and Luporini (2009); Eckwert and Zilcha (2011).

[7] One exception is Eckwert and Zilcha (2011). We shall come back to this in a while.

[8] Notice that our suggested ICL scheme entails an ex post redistribution of income from successful agents to agents who have failed. However, since ex ante all agents face the same probability of success (or failure), it is easy to find political support from all potential beneficiaries of the scheme. Moreover, it is only a subset of the potential beneficiaries who will eventually have to pay, thereby leaving the rest of the population unaffected.

failure. This implies that poorer households are more likely to opt for such a scheme. At the same time, it also ensures that the future generations of these poor households never fall back in abject poverty again, even after multiple adverse shocks in the long run. It is this last feature that plays a crucial role in rendering ICL a successful tool in fighting persistent poverty.

As we have mentioned before, dynamic analytical structures to evaluate various anti-poverty measures are rather scarce. In this context, there are two papers in the literature that are closely related to our work. These are Eckwert and Zilcha (2011) and Mookherjee and Ray (2008). Eckwert and Zilcha appraise the dynamic consequences of various education financing schemes from the point of view of growth. Theirs is a model of convex technologies, whereby the marginal returns to human capital investment (education) are symmetric across rich and poor agents.[9] Thus if ability is evenly distributed across poor and rich households, then the poor households do not face any extra disadvantage. A poor but talented child has as much incentive to educate herself as her rich counterpart. We, however, know that existence of non-convexities (technological or preference-related) along with imperfections in credit market distort the incentive structure faced by the poor vis-a-vis the rich, resulting in perpetuation of poverty. Hence it is important to juxtapose the issue of education financing to the broader question of long run persistence of poverty, or poverty 'traps'. Mookherjee and Ray (2008) do precisely that. They compare between two types of direct cash transfer schemes - unconditional and conditional (subject to sending children to school, health check ups etc.) in the context of a poverty-persistence model along the lines of Mookherjee & Ray (2003). But the anti-poverty measures analysed by Mookherjee and Ray do not work through education financing. Our model is close in spirit to Mookherjee and Ray (2008), but we look at anti-poverty measures that work through provision of education.

The basic results of our paper are as follows. We first show that presence of uncertainty in the process of skill formation may exacerbate the long run persistence of poverty in the sense that it is not just the poor households who get entrapped; in the long run every household eventually ends up in a low education-low skill-low income equilibrium, dragging the entire nation down to a state of abysmal poverty. In such a scenario, the ICL scheme prescribed above can work wonders. By providing a support base for those who go for skill formation but fail, it ensures that the dynamic incentives for skill formation remain intact even for those who fail in the first few attempts (or in the first few generations). At the same time, by relaxing the life-time budget constraints of the poor households, it incentivizes more people

[9]Eckwert and Zilcha differentiates between agents on the basis of innate ability, not wealth.

to go for skill formation. While the second feature is present in other forms of government sponsored education financing programmes (e.g., mortgage loan), it is really the first feature that prevents the economy to stagger back to overall poverty in the long run.

But ICL is not a panacea for all ailments. To the extend that skill formation requires some amount of basic education, the effectiveness of ICL as an anti-poverty measure depends crucially on the efficacy of the basic (primary) education system. The more productive is basic education, the better is the scope and reach of ICLs in pulling households (and economies) out of long run poverty traps. Thus our paper calls for a holistic approach towards education as a solution to persistent poverty.

The structure of the paper is as follows. In section 2 we set up the basic framework. Section 3 analyses the long run wealth dynamics in the absence of ICLs. In section 4 we introduce a suitably designed ICL and show how household choices differ in the presence of an ICL and also examine impact of ICL on the long run wealth dynamics. Section 5 concludes the paper.

2 The Model

We consider a small open economy in a one-good world inhabited by altruistic agents (households) who live for two periods in overlapping generations. There is a large population (a continuum) of agents with identical preferences. The production side of the economy consists of two sectors: while the non-technical sector can accommodate an array of "blue-collar" workers differentiated by their levels of basic education or years of schooling, the technical sector employs only technically skilled "white-collar" workers. Acquiring the technical skill requires a lump-sum investment in specialized human capital on top of basic (high school) education. But this investment is risky: an agent may fail to acquire the technical skill even after making the lump-sum investment, in which case the fall-back option is the highest level of blue-collar job. In her youth, an agent has to decide her level of basic education and whether or not to go for the (uncertain) investment in specialized human capital formation. Depending on the schooling decision and the realization of human capital investment, the agent works as either a blue-collar or a white-collar worker in her old age, consumes part of her income and leaves the remainder as bequest.

2.1 Production Technology

The single good in this economy can be produced in either of the two sectors. Production in the non-technical sector is done using only labour and is described by

$$Y_t^n = \widehat{L}_t w^n$$

where Y_t^n is the output in this sector at time t. $\widehat{L}_t$ is the efficiency units of labour and w^n is the blue-collar wage rate per efficiency unit of labour. $\widehat{L}_t$ is defined as

$$\widehat{L}_t = (1 + \gamma e_t)$$

where $e_t \in [0, 1]$ is the level of basic education attained by the worker and $\gamma > 0$ is the productivity of basic education (or the schooling system). Thus, labour efficiency increases with the level of basic education. For simplicity we identify the level of basic education, e_t, with the time spent in school. γe_t is therefore the net acquired productivity of the agent. We assume that every agent is born with some innate productivity that is normalized to 1. Even if the agent does not go to school at all, she can work as a manual labourer in both periods of life. The total productivity of any agent is therefore $(1 + \gamma e_t)$.

The technical or specialized sector, employs technically skilled workers. These are the white-collar workers and have the highest level of basic education ($e_t = 1$) and some special skill above that. So they may be management graduates who work as managers or personnel trained in special skills. Production in the technical sector is described as

$$Y_t^s = F(K_t, H_t)$$

where Y_t^s, K_t and H_t are output, capital input and skilled labour input respectively. $F(\cdot)$ is a concave production function with constant returns to scale.

2.2 Preference and Occupations

The population at time t is described by a distribution function $G_t(x)$, which gives the measure of the population with wealth less than x. In the first period of life, agents receive their initial wealth in the form of a bequest from their parents. They also have an endowment of one unit of labor in each period. Each agent has one parent and gives birth to one child in period 2; hence there is no population growth.[10]

[10] Since there is no population growth and e is chosen only once by an agent, for the sake of simplicity we drop the time subscript for further analysis.

The agents have identical preferences and differ only in their inherited wealth. The individual utility function is given by

$$U = c_1^{\alpha} + \beta c_2^{1-\delta} b^{\delta}$$

where c_1, c_2 and b are period 1 consumption, period 2 consumption and bequest. The sub-utility in each period takes the standard Cobb-Douglas form with $\alpha \in (0,1)$ and $\delta \in (0,1)$. Period 2 sub-utility reflects agent's altruism in the standard 'warm-glow' fashion (Banerjee and Newman, 1993; Galor and Zeira, 1993). The Cobb-Douglas form of period 2 sub-utility also captures that the agent is risk-neutral in period 2 choices: indirect utility is linear in the realized income in period 2. $\beta \in (0,1)$ is the discount factor.

The timing structure is as follows. In the first period (youth) the agent is born with one unit of time and some wealth x that she inherits from her parents. She can use that time to either work or acquire education. As described earlier, e is the amount of time she spends in school. $e = 0$ implies no schooling and $e = 1$ implies the agent has completed basic schooling. We can interpret this range of school levels according to our convenience. To fix ideas let us say that the range is from no schooling to high school. $(1 - e)$ is the amount of time the agent spends working as a child labour. Additionally she can attain specialized skill. However, in order to attain special (technical) skills it is imperative that she has completed basic schooling ($e = 1$) and makes a lumpy investment $h > 0$. One can interpret this as a management degree, which will enable the agent to work as a manager or an engineering degree.

In period 1, as the agent is still acquiring skills, she gets paid $(1 - e)w^n$ if she works as a child labour. She can consume out of her inheritance and her earnings as a child labour. Note that, in our model, there is *no pecuniary cost* of acquiring basic education. The only cost is the associated opportunity cost, that is, the forgone earnings working as a child labour.

In period 2, the agent works, consumes, gives birth to an offspring, leaves a bequest and then dies. If she works as a blue-collar worker then she gets a wage equal to $(1 + \gamma e)w^n$ depending on her choice of e in period 1. Alternatively, she can work as a "white-collar" worker if she had invested in special skill formation. As noted earlier, investment in specialized human capital is risky. We define an exogenous probability of success and failure equal to p and $1 - p$ respectively. This probability captures risk due to failure while studying or random shocks to employment. So even after completing this technical/high-skilled education one may not get the white-collar job. If the agent is successful she gets employed as a white-collar worker and gets a wage w^s. If she fails however, because she had completed her basic education ($e = 1$), she gets the highest blue-collar wage $(1 + \gamma)w^n$. We assume that the white-collar wage is higher than the highest blue-collar wage:

$$w^s > (1+\gamma)w^n. \qquad \text{(Assumption 1)}$$

Assumption 1 ensures that any agent who invests in special skill formation always prefers a "white-collar" job to a "blue-collar" job.

2.3 Markets

Capital is assumed to be perfectly mobile so that both firms and individuals have free access to international capital markets. The world interest rate is equal to r and assumed to be constant over time. Individuals can lend any amount at this rate. Although we assume that for borrowing individuals can evade debt payments by moving to other places etc., but this activity is costly. This renders the capital market imperfect and the borrowing interest rate i is higher than the lending interest rate r:

$$i > r.^{11} \qquad \text{(Assumption 2)}$$

Firms, however, are unable to evade debt payments, due to reasons such as reputation. Hence, firms can borrow at the rate r. The amount of capital in the skilled labour sector is then adjusted each period so that

$$D_1F(K_t, H_t) = r.$$

Constant returns to scale implies that the capital-labour ratio and hence the wage rate w^s in this sector is constant over time. w^s depends only on r and the technology.

We further assume that both labour markets and the good market are perfectly competitive. We also assume that the gross borrowing rate is higher than the productivity of schooling which is greater than the gross lending rate:

$$(1+i) > \gamma > (1+r). \qquad \text{(Assumption 3)}$$

3 Optimal Choices and Wealth Dynamics

Each agent is faced with the following choices. She has to decide whether or not to make the lumpy investment in skill formation. If not then she has to choose the extent to which she

[11] This assumption is standard in the literature (see, Galor and Zeira (1993), for example).

should study, that is, choose $e \in [0,1]$. This will determine her productivity as a blue-collar worker. The lifetime behaviour and choices of the agents will depend on their inherited wealth, the relative wages, the productivity of the schooling system, the borrowing and lending rates, the relative importance of consumption in both periods and the inter-temporal discount factor.

We approach the problem in two steps because of the indivisibility of the amount of investment in human capital. First we look at the agent's choices when she is not making the lumpy investment. Next we look at her choices when she does make the lumpy investment. Then we will compare the expected indirect utilities to derive the actual optimal lifetime behaviour of the agents. Depending upon their optimal choices we will then analyze the long-run wealth dynamics.

3.1 No Investment in Technical-Skill Formation

The agent's decision problem is as follows:

$$\max_{c_1, c_2, b, 0 \le e \le 1} U(c_1, c_2, b) = c_1^{\alpha} + \beta c_2^{1-\delta} b^{\delta},$$

subject to

$$c_1 \le (1-e)w^n + x + B - S, \tag{1}$$

$$c_2 + b \le (1+\gamma e)w^n + S(1+r) - B(1+i), \tag{2}$$

where $B \ge 0$ is borrowing and $S \ge 0$ is savings. (1) and (2) are period 1 and period 2 budget constraints respectively. In this problem the agent may borrow to finance consumption. In period 1, consumption and savings (if any) has to be funded out of earnings as child labour depending on the choice of e, inherited wealth x and borrowings B. In period 2, the consumption, bequest and loan repayments have to be funded out of earnings as a blue-collar worker getting a wage $(1+\gamma e)w^n$ and interest earnings on savings. The agent chooses c_1, c_2, b and e to maximize her utility subject to her budget constraints (1) and (2).

We make two useful observations here. First, note that the fact that the second period utility takes a Cobb-Douglas form, optimally will imply[12]

$$b = \left(\frac{\delta}{1-\delta}\right) c_2 \tag{3}$$

Second, using (3) we can understand the reduced form budget set for the agent in c_1-c_2 space. This is important because, as we shall see, the choice problem is not a standard

[12] Detailed calculations are shown in Appendix A.

consumer theory utility maximization exercise. Using (3), the period 2 budget constraint can be written as[13]

$$c_2 = (1-\delta)\left[(1+\gamma e)w^n + S(1+r) - B(1+i)\right]. \quad (4)$$

Fix an agent with an inherited wealth x. If she does not borrow and save and chooses $e = 1$, then the consumption point (marked as A in Figure 1) is given by $(c_1, c_2) = (x, (1-\delta)(1+\gamma)w^n)$. This can be easily seen using equations (1) and (4). Additionally, she can save and consume points to the left of A. Therefore, the slope of the budget line to the left of A is $-(1-\delta)(1+r)$. The agent can also consume points to the right of A in two ways. First, she can simply borrow and consume more in period 1. In this case the slope of the budget line will be $-(1-\delta)(1+i)$. However, there is an efficient option available. The agent can choose any value of $e \in [0,1]$. By choosing a smaller value of e, she can allow herself to consume more in period 1 and less in period 2.

For example, for $e = 0.5$, no borrowing and no savings, the consumption point is marked as point B in the figure. Assumption 3 implies that the slope of consumption possibility frontier generated by choosing different values of e is flatter than that generated by borrowing for any point till $e = 0$. Hence, the slope of the budget set will be $-(1-\delta)\gamma$ till $e = 0$, marked as point C. At point like A and B the agent can also save and borrow but because of Assumption 3 the consumption possibility set will be included within the reduced form budget set. Now to consume points beyond point C, there is only one instrument, to borrow. Therefore, beyond C, the slope of the budget set is $-(1-\delta)(1+i)$.

This is also very intuitive. In this model there are three ways to substitute intertemporally between period 1 and period 2: saving, borrowing and acquiring education. Assumption 3 implies that augmenting period 2 income by acquiring education is the most efficient means followed by savings and the least efficient means is to borrow (it reduces period 2 income). This shows up in the reduced form budget set as the budget frontier is piece-wise linear with three pieces. As we shall see further that this fact will also show up in the optimal choices of the agents. Finally, like any standard budget set, higher inherited wealth x shifts the budget set out. We illustrate two more budget sets in the figure for someone with inherited wealth $x' > x$ and $x'' < x$.

[13]Since utility is strictly increasing in c_1, c_2 and b, we can replace the inequalities with equalities in the budget constraints.

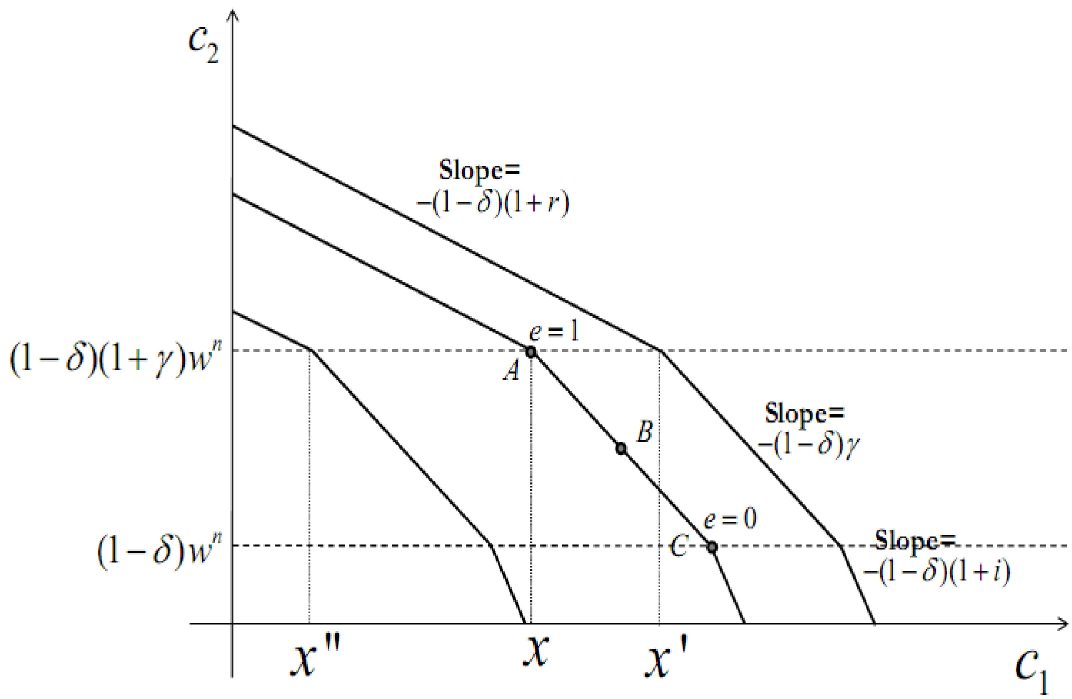

FIGURE 1: NO INVESTMENT BUDGET SET.

The optimal choices of an agent, in this no investment case is described in the following proposition.

Proposition 1 *There exist wealth thresholds* $0 < \underline{x}^{NI} < \widetilde{x}^{NI} < \overline{x}^{NI}$ *such that the optimal choices for* B, e *and* S *are characterized by the following:*

(a) for $x \leq \underline{x}^{NI}$, $B^* = 0, e^* = 0$ *and* $S^* = 0$;

(b) for $x \in \left(\underline{x}^{NI}, \widetilde{x}^{NI}\right)$, $B^* = 0$, $e^* \in (0,1)$ *and* $S^* = 0$, *with* $D_1 e^*(x) > 0$;

(c) for $x \in \left(\widetilde{x}^{NI}, \overline{x}^{NI}\right)$, $B^* = 0, e^* = 1$ *and* $S^* = 0$; *and*

(d) for $x \geq \overline{x}^{NI}$, $B^* = 0, e^* = 1$ *and* $S^* \geq 0$, *with* $D_1 S^*(x) > 0$.[14]

Proof. See Appendix A. ■

We can see from Proposition 1 that the agents augment period 2 income first by acquiring education and then by savings.

[14]We rule out consumption borrowing under this no-investment case by assuming that the earning from child labour, w^n, is high enough: $w^n > \eta_i \equiv \left(\frac{\alpha}{\beta(1+i)\delta^\delta(1-\delta)^{1-\delta}}\right)^{\frac{1}{1-\alpha}}$.

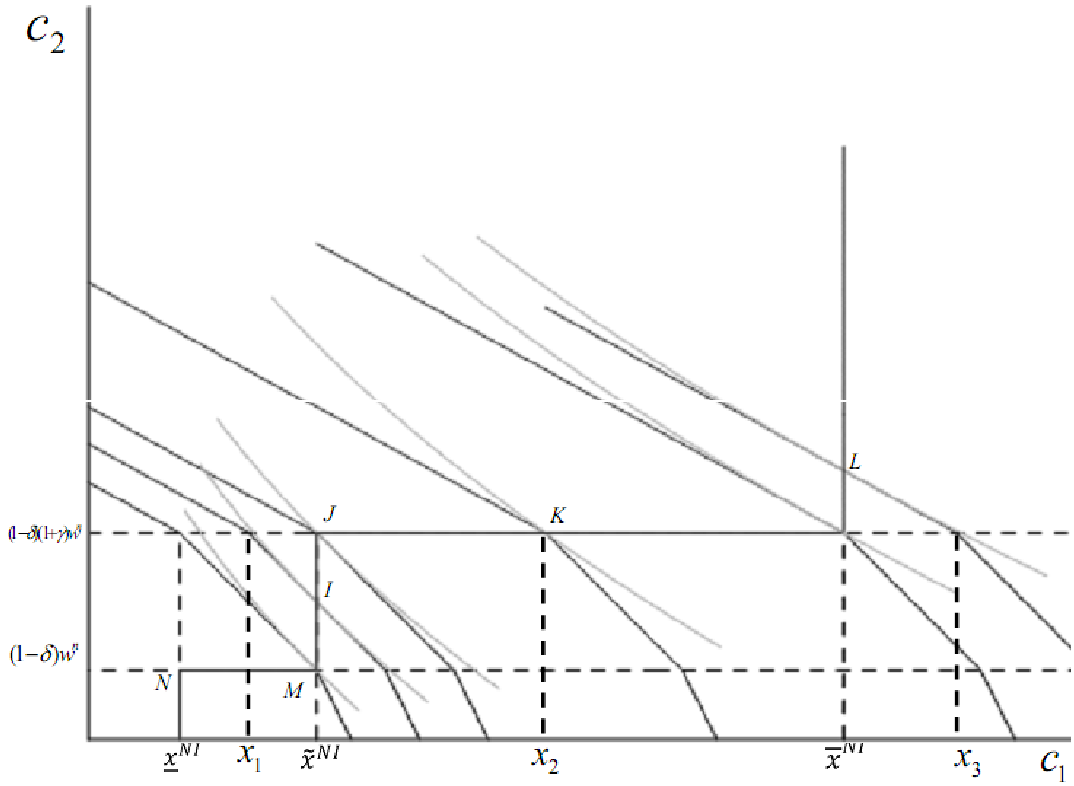

Figure 2: Optimal Choices and Wealth Expansion Path.

Figure 2 depicts the wealth expansion path. Consider an agent who inherits a wealth $x_1 \in \left(\underline{x}^{NI}, \widetilde{x}^{NI}\right)$. The optimal choice of such an agent (point I) lies on the piece of the budget set which has a slope $-(1-\delta)\gamma$. Next consider an agent who has a higher inherited wealth $\widetilde{x}^{NI}$. As depicted, her optimal choice (point J) is where $e = 1$. Now for any agent who has an inheritance, say $x_2 \in \left(\widetilde{x}^{NI}, \overline{x}^{NI}\right)$, the optimal choice (marked as point K) is again $e = 1$. However, an agent with an inheritance $x_3 > \overline{x}^{NI}$ additionally chooses to save. Hence, her optimal point (point L) is on that piece of the budget which has a slope $-(1-\delta)(1+r)$. Finally someone with a very low inheritance, say $\underline{x}^{NI}$ chooses a point M on the budget line where $e = 0$. $\underline{x}^{NI}$NMIJKL is therefore the wealth expansion path.

From figure 2 we can also see the influence of the quasi-linear nature of the reduced form utility function.[15] For different ranges of inherited wealth, there is an optimal period 1 consumption which is achieved first. Then, as wealth increases, period 1 consumption remains fixed and the remaining wealth is used to augment period 2 consumption. This gives rise to a wealth expansion path which looks like a step function.

[15] Using $b = \left(\frac{\delta}{1-\delta}\right) c_2$,the utility function becomes $c_1^{\alpha} + \beta\delta^{\delta}(1-\delta)^{(1-\delta)} c_2$.

3.2 Investment in Technical-Skill formation

In this case we analyse household behaviour when agents choose to invest in skill formation. To begin with there is uncertainty in second period now. As the agent invests in skill formation, her second period wage realization depends on whether she is successful or she fails. Formally, there are two states of nature - success and failure. We denote them by s and f respectively. Accordingly, c_{2s} and b_s are the agent's period 2 consumption and bequest if she is successful and c_{2f} and b_f are the consumption and bequest if she fails. Our agent is a von-Neumann Morgenstern expected utility maximizer. Her expected utility function is thus

$$\begin{aligned} E\left[U(c_1, c_{2s}, c_{2f}, b_s, b_f)\right] &= p\left[c_1^{\alpha} + \beta c_{2s}^{1-\delta} b_s^{\delta}\right] + (1-p)\left[c_1^{\alpha} + \beta c_{2f}^{1-\delta} b_f^{\delta}\right] \\ &= c_1^{\alpha} + \beta\left[p c_{2s}^{1-\delta} b_s^{\delta} + (1-p) c_{2f}^{1-\delta} b_f^{\delta}\right]. \end{aligned}$$

The decision problem for the agent can be written as:

$$\max_{c_1, c_{2s}, c_{2f}, b_s, b_f} E\left[U(.)\right] = c_1^{\alpha} + \beta\left[p c_{2s}^{1-\delta} b_s^{\delta} + (1-p) c_{2f}^{1-\delta} b_f^{\delta}\right],$$

such that,

$$c_1 \leq x - h + B - S, \tag{5}$$

$$c_{2s} + b_s \leq w^s - B(1+i) + S(1+r), \tag{6}$$

$$c_{2f} + b_f \leq (1+\gamma) w^n - B(1+i) + S(1+r). \tag{7}$$

Analogous to the no-investment case, equations (5), (6) and (7) are the period 1 budget constraint and the period 2 state wise budget constraint. In period 1, consumption, savings and the investment h has to be financed out of the inherited wealth x and by borrowing from the credit market B. In period 2, in each state, the consumption, bequest and the interest payment on savings have to be financed out of second period earnings which comprise of interest accrued on savings and the realized wage income. When the agent is successful in acquiring the specialized skill, she earns the white-collar wage w^s.In case she fails to acquire this skill, she earns the highest blue-collar wage $(1+\gamma)w^n$. Note that because it is necessary for the agent to choose $e = 1$ to be able to make investment in skill formation, there is no wage income in period 1. For future reference let us also note down the period 2 budget constraint in expected terms:

$$E(c_2) + E(b) \leq E(w) - B(1+i) + S(1+r) \tag{8}$$

where $E(w) = pw^s + (1-p)(1+\gamma)w^n$.

Again we use the fact that the utility function in each state in the second period takes the Cobb-Douglas form to conclude that:

$$b_i = \left(\frac{\delta}{1-\delta}\right) c_{2i}, \qquad i = s, f.$$

If we take expectations, we can write the above equation as

$$E[b] = \left(\frac{\delta}{1-\delta}\right) E[c_2]. \tag{9}$$

As before, we use this relation to understand the reduced form budget set for the investment case.

We now plot the expected consumption in period 2 on the y-axis. Again consider the same agent with inherited wealth x. If she does not save and borrow, her consumption in period 1 is $x - h$, as can be seen from the period 1 budget constraint (5).

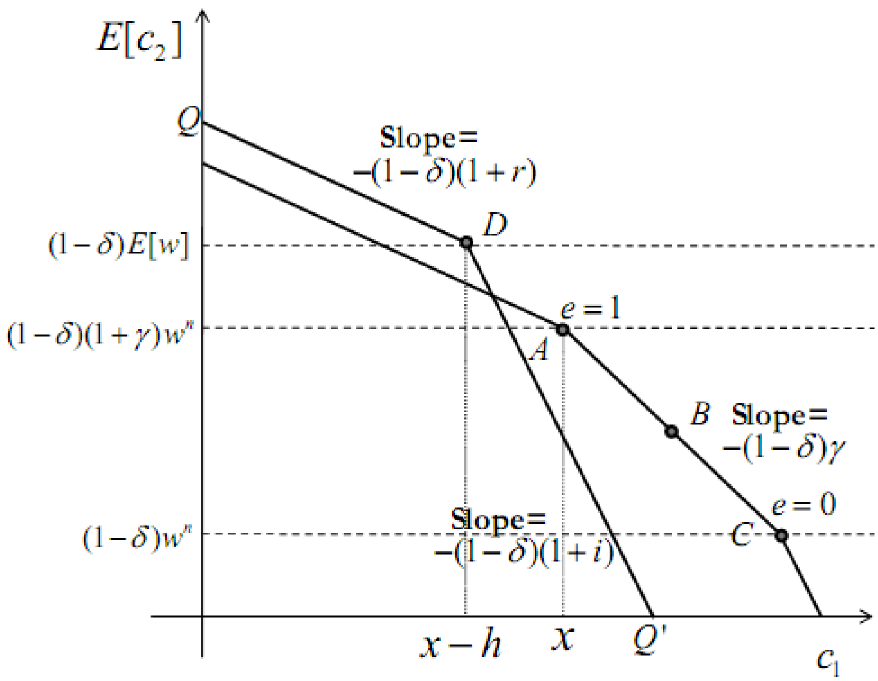

Figure 3: Investment Budget Set.

In period 2, the agent can consume $(1-\delta)E(w)$, in expected terms. This point is marked as D in figure 2. Unlike the no-investment case, here inter-temporal transfer of income can take place only by savings or borrowings. (Since e is fixed to be 1, that instrument is not available.) Thus, the slope of the budget set to the right of D, where the agent borrows is $-(1-\delta)(1+i)$. Similarly, the slope of the budget set to the left of D, where the agent is saving is $-(1-\delta)(1+r)$. QDQ' gives the budget line for our agent when she chooses to invest in technical-skill formation.

The optimal choice of the agent under the investment case is summarized in the following proposition.

Proposition 2 *There exist wealth thresholds* $0 < \underline{x}^I < \overline{x}^I$ *such that the optimal choices for* B *and* S *are characterized by the following:*

(a) for $x < \underline{x}^I$, $B^* > 0$ *and* $S^* = 0$,*with* $D_1 B^*(x) < 0$;

(b) for $x \in \left[\underline{x}^I, \overline{x}^I\right]$, $B^* = 0$ *and* $S^* = 0$; *and*

(c) for $x > \overline{x}^I$, $B^* = 0$ *and* $S^* > 0$, *with* $D_1 S^*(x) > 0$.

Proof. See Appendix B. ■

Therefore, we can see that the poorer individuals borrow to finance the investment in skill-formation (and consumption) while the richer individuals save, on top of the investment in human capital.

3.3 Optimal Choice between No Investment and Investment

Once we have solved for the optimal choices of the agent when she does not invest in technical-skill formation (Section 3.1) and when she does (Section 3.2), we can now analyse these two cases together and identify the actual optimal lifetime behaviour of the agent given her level of inheritance. The trade-off that the agent faces while deciding whether to invest or not is that she must sacrifice some current income for a higher income in the future. Sacrificing current income makes fulfilling current consumption needs harder for the agent. This is in conflict with the consumption smoothing motive and the preference for current consumption manifested in the quasi-linearity of the utility function. Hence, the agents with low inheritance find it difficult to invest in skill formation owing to a pressing need for current consumption coupled with the uncertainty in the future. However, as inheritance increases, there is a greater tendency towards investment.

We assume that the 'very rich' (those with inheritance $x \geq \bar{x}^I$) always find it optimal to invest in skill formation. In particular:

$$E(w) - h(1+r) > w^n(1+\gamma) \qquad \text{(Assumption 4)}$$

For agents with $x < \bar{x}^I$, we denote the inheritance level which makes the agent indifferent between investing and not investing as f and define it such that all those with $x < f$ prefer not to undertake the lumpy investment whereas those above this threshold derive more expected utility from the risky investment. The exact position of f depends on the actual values of the parameters in the model (w^s, γ, and p). For the purpose of exposition, we assume and illustrate one particular position as shown in Figure 4.

In the figure, V^{NI} is the expected indirect utility plot for the agent given that she is not investing in skill formation and V^I represents the same when she is investing[16]. The former is made of up the different curves and lines which are active in the respective ranges. Therefore, f is that level of inheritance x, such that $V^I(f) = V^{NI}(f)$. We assume conditions that ensure that the V^I line is below the V^{NI} curve at $x = 0$ and $x = \tilde{x}^{NI}$, implying that $V^I(x) < V^{NI}(x), \forall x < \tilde{x}^{NI}$. Next we assume that the converse happens at $x = \bar{x}^{NI}$ such that $V^I(x) > V^{NI}(x), \forall x > \bar{x}^{NI}$. In particular:

$$(w^n)^\alpha + \mu w^n > \eta_i^\alpha + \mu \left[E(w) - h(1+i) - (1+i)\eta_i\right]^{17} \qquad \text{(Assumption 5)}$$

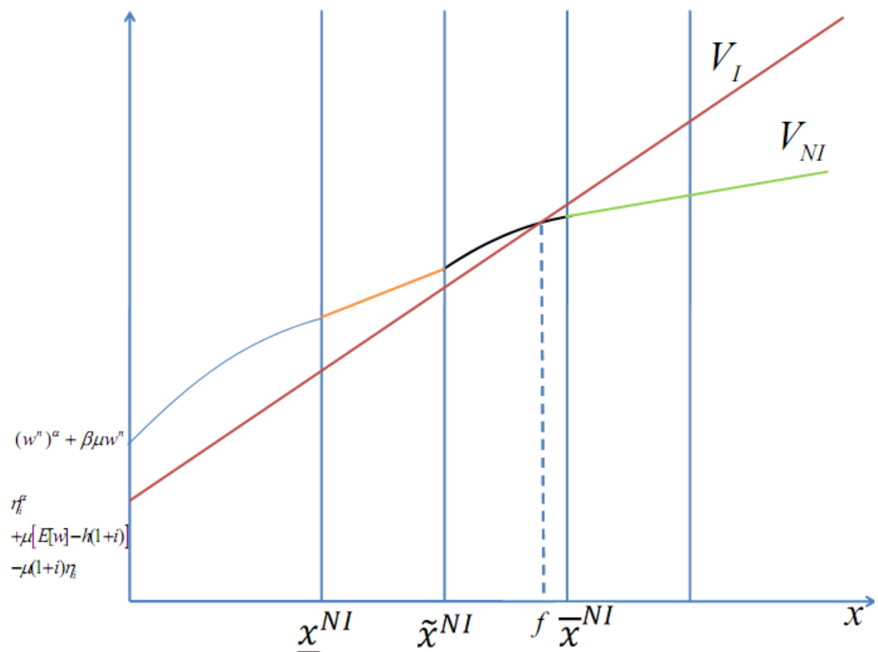

FIGURE 4: EXPECTED INDIRECT UTILITIES AND f.

$$\eta_\gamma^\alpha + \mu[(1+\gamma)w^n] > \eta_i^\alpha + \mu[E(w) - h(1+i)] + \mu(1+i)(\eta_\gamma - \eta_i)^{18} \qquad \text{(Assumption 6)}$$

$$\eta_i^\alpha + \mu[E(w) - h(1+i)] + \mu(1+i)(\eta_r - \eta_i) > \eta_r^\alpha + \mu[(1+\gamma)w^n] \qquad \text{(Assumption 7)}$$

[16] The exact expressions can be found in appendices A and B.

[17] $\beta(1-\delta)\left(\frac{\delta}{1-\delta}\right)^\delta = \mu$ and $\eta_i = \left(\frac{\alpha}{\beta(1+i)\delta^\delta(1-\delta)^{1-\delta}}\right)^{\frac{1}{1-\alpha}}$

[18] $\eta_r = \left(\frac{\alpha}{\beta(1+r)\delta^\delta(1-\delta)^{1-\delta}}\right)^{\frac{1}{1-\alpha}}$,

$\eta_\gamma = \left(\frac{\alpha}{\beta\gamma\delta^\delta(1-\delta)^{1-\delta}}\right)^{\frac{1}{1-\alpha}}$

Then the Intermediate Value Theorem[19] implies that $f \in \left(\tilde{x}^{NI}, \overline{x}^{NI}\right)$. Note that by construction it is the case that $V^I(x) < V^{NI}(x), \forall x < f$ and $V^I(x) > V^{NI}(x), \forall x > \tilde{x}^{NI}$. Since the expected indirect expected utility from no investment is higher than investment when $x < f$ and the converse is true for $x > f$, all agents with inheritance above f always invest in technical skill formation and those with inheritance below f choose otherwise. This discussion is summarized in the proposition below.

Proposition 3 *Define f as $V^I(f) = V^{NI}(f)$. Under assumptions 5, 6 and 7 $f \in \left(\tilde{x}^{NI}, \overline{x}^{NI}\right)$. Further, $V^I(x) < V^{NI}(x), \forall x < f$ and $V^I(x) > V^{NI}(x), \forall x > f$.*

The initial wealth distribution will now determine the long-run wealth dynamics and the long-run equilibrium occupational structure of the economy. The agents who inherit an amount less than f do not invest in skill formation and hence can at most earn the highest blue-collar wage. However, those who are richer in terms of inheritance ($x > f$) take the opportunity to develop the specialized skills required to be eligible for the highest wage level. We discuss the dynamics in detail in the next section.

3.4 Inter-generational Wealth Dynamics

The agents in the current period leave a certain bequest for their progeny, which will determine her level of skill attainment in the next period. This highlights the linkage between the wealth distribution today and the distribution of wealth and skills in the future. Given, the position of f (as defined above), we have the following bequest equations for the respective thresholds of inheritance in the current period.

Agents with inherited wealth less than f choose not to invest in specialized-skill formation and hence there is no uncertainty in their bequest. This is described below

$$x_{t+1} = \begin{cases} \delta w^n, & \text{if } x_t \leq \underline{x}^{NI}, \\ \delta \left[(1+\gamma) w^n - \gamma \eta_\gamma + \gamma x_t \right], & \text{if } \underline{x}^{NI} < x_t \leq \tilde{x}^{NI}, \\ \delta (1+\gamma) w^n, & \text{if } \tilde{x}^{NI} < x_t \leq f. \end{cases}$$

Agents with inherited wealth greater than f, choose to make the risky investment in specialized skill formation. Thus, their bequest lines exhibit uncertainty.

[19] Both the functions in consideration are continuous in the relevant range and the set is compact.

(a) if $f < x_t \leq \underline{x}^I$

$$x_{t+1} = \begin{cases} \delta\left[w^s + (x_t - h)(1+i) - \eta_i(1+i)\right], & \text{with prob } p, \\ \delta\left[(1+\gamma)\, w^n + (x_t - h)(1+i) - \eta_i(1+i)\right], & \text{with prob } 1-p. \end{cases}$$

(b) if $\underline{x}^I < x_t \leq \bar{x}^I$

$$x_{t+1} = \begin{cases} \delta w^s, & \text{with prob } p, \\ \delta(1+\gamma)w^n, & \text{with prob } 1-p. \end{cases}$$

(c) if $x_t > \bar{x}^I$

$$x_{t+1} = \begin{cases} \delta\left[w^s + (x_t - h)(1+r) - \eta_r(1+r)\right], & \text{with prob } p, \\ \delta\left[(1+\gamma)\, w^n + (x_t - h)(1+r) - \eta_r(1+r)\right], & \text{with prob } 1-p. \end{cases}$$

Notice that optimal bequests are a fixed proportion of the second period income or the total lifetime income. This is essentially because of the Cobb-Douglas utility function that we assume. The bequest dynamics is shown in figure 5 and 6, where the solid lines represent the actual bequest lines and the dotted lines correspond to the expected bequests.

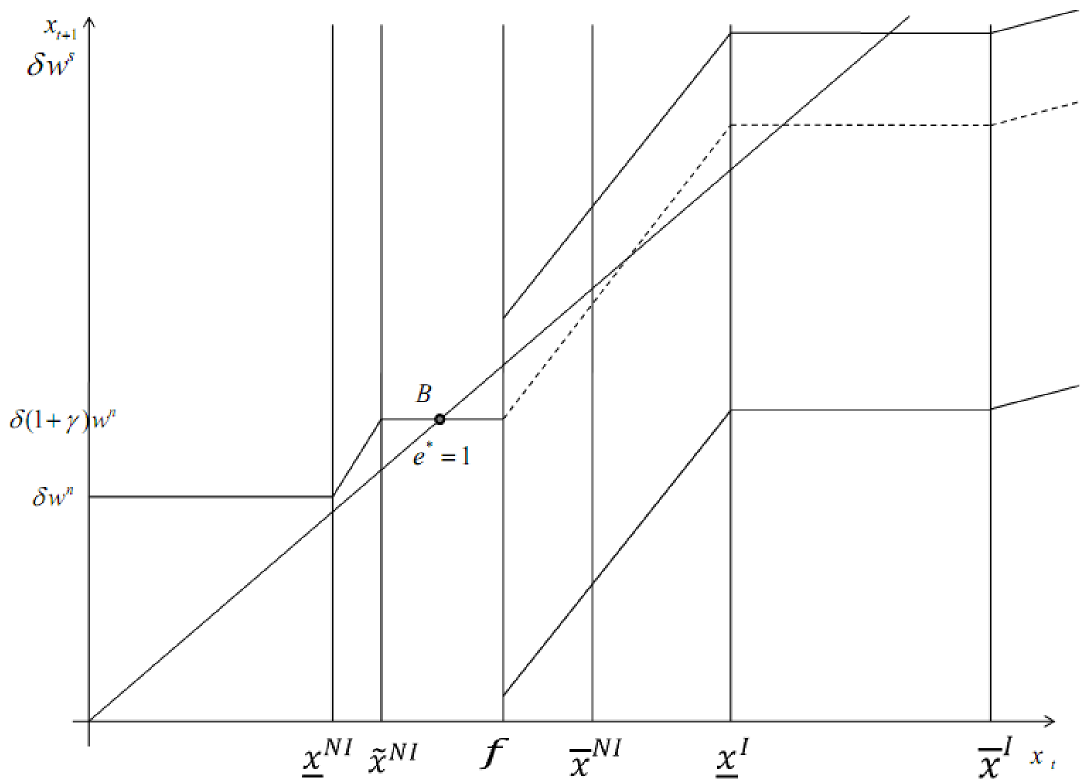

FIGURE 5: BEQUEST DYNAMICS WITH HIGH γ.

Figure 5 has been drawn for a high value of γ. Notice that the intersection with the 45 degree line for the agents with $x \leq f$ is in the range $(\tilde{x}^{NI}, \bar{x}^{NI})$, where $e^* = 1$. For the agents who invest in skill formation, the bequest line in the bad state has no intersection with the 45 degree line. So, consider an agent above f. If she gets a good shock, her child will receive a high bequest and invest in skill formation again. However, if she receives bad shock, then the next generation will receive a low bequest. In this way, if a dynasty continues to get a series of bad shocks, then it will fall below f and will be trapped there forever. Hence, the only long-run steady state in this case is point B which we call a 'blue-collar employment' trap.

Figure 6 has a low value of γ as compared to that for figure 5. Here, the intersection with the 45 degree line of those below f happens in the range $(0, \underline{x}^{NI})$ where $e^* = 0$, and the bad state bequest line for those who invest in skill formation does not have any intersection with the 45 degree line. Hence, arguing as before we can see that the only long-run equilibrium in this economy is $e^* = 0$ at point C, which we refer to as a 'child labour trap'.

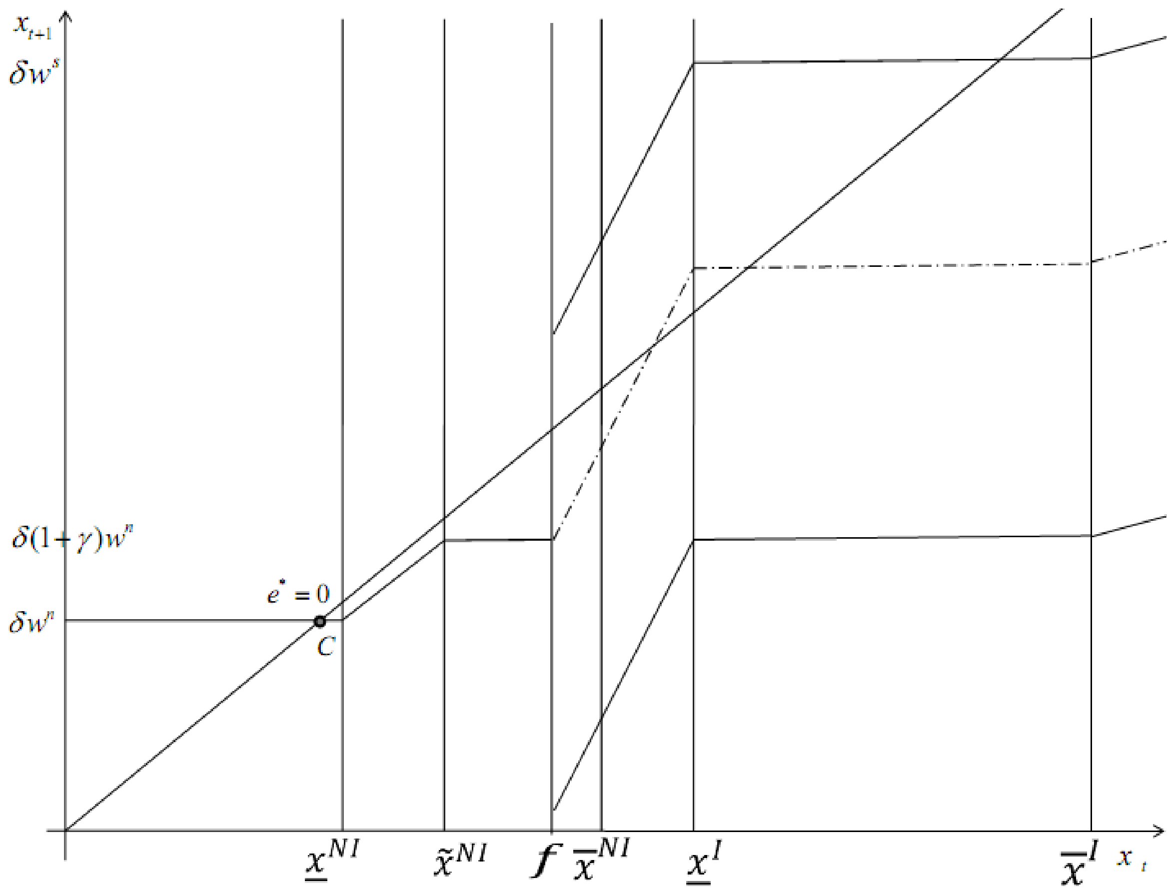

FIGURE 6: BEQUEST DYNAMICS WITH LOW γ.

Thus, evidently γ plays an important role in this model. γ represents the productivity of the primary education system in the economy. So, the previous two figures illustrate that if the productivity of the schooling system is not very high then the entire economy may be stuck in a child labour trap in the presence of imperfect credit markets. If the schooling system is efficient, however, then the economy is more likely to have a blue-collar trap. For the mid-range values of γ, both traps might coexist This role of γ will become clearer in the next section.

The above discussion is summarized in the proposition below.

Proposition 4

(a) *If the basic schooling system is very productive (sufficiently high γ), the wealth distribution of the economy in the long-run converges to the wealth level corresponding to point B in figure 5. In the long-run the economy is in a "blue-collar" employment trap, where members of all dynasties complete basic schooling ($e = 1$) and work as "blue-collar" workers in the second period of their lives.*

(b) *If the basic schooling system is unproductive (sufficiently low γ), the wealth distribution of the economy in the long-run converges to the wealth level corresponding to point C in figure 6. In the long-run the economy gets stuck in a "child-labour" trap, where members of all dynasties never go to school ($e = 0$), work as child labour in first period and as unskilled workers in the second period of their lives.*

4 Income Contingent Loans

As the previous discussion has shown, imperfections in credit markets may lead to educational poverty traps. Depending on history (initial wealth), dynasties get trapped in a child-labour trap or a blue-collar employment trap. The risk associated with the investment in technical skills, need for present consumption and an inefficient schooling system forces dynasties to get locked in these "bad" equilibria. We shall now introduce a very stylized version of ICLs and study its consequences on the long-run wealth dynamics.

For the purpose of exposition in our model we assume that the ICL scheme is implemented by the government. The fact that the loan giving agency needs to have full information about the earnings of an individual, an efficient mechanism of income linked repayment collection and that the agency should itself be credible enough to sustain the scheme makes government the best suited agency. This is not a very esoteric assumption.[20] Also, if this scheme is run

[20] See García-Peñalosa and Wälde (2000) and Eckwert and Zilcha (2011), for example.

by a private agency then there may be a conflict of interest and other moral hazard and adverse selection issues. (For a detailed discussion see Nerlov (1975)).

In this stylized economy, the government provides an education loan to anyone who wishes to make an investment in technical skill formation. There is no need for collateral. The loan is transferred directly to the educational institution and therefore there is no possibility to run away with the loan amount. Anyone who has taken this loan has to pay an additional tax at the rate τ if she is successful. In case of failure however, she is exempted from making any repayment. Since the government is a credible agency, it can borrow from the international markets at the rate r.[21] Suppose λ proportion of people take this loan so that the total government expenditure amounts to $h(1+r)\lambda$. For a large enough population the proportion of people who will be successful will be λp and hence the total tax collection from them is $\lambda p \tau w^s$. The government sets the tax rate to balance budget in expected terms. Therefore we have

$$\lambda p \tau w^s = h(1+r)\lambda.$$

or, equivalently,

$$\tau w^s = \frac{h(1+r)}{p}. \tag{10}$$

Note two important features of such a scheme:

1. There is an insurance component for those who fail.
2. The effective interest rate that agents pay is $(1+r)$ which is lower than the credit market interest rate $(1+i)$.[22]

Further note that in our model the agent is risk-neutral in period two.[23] This shuts down the first channel. Hence, in an environment where the agents are risk-averse, our results will only get strengthened.

4.1 Agent's Decision Problem under ICL scheme

As we had done earlier, we will now compute the optimal choices when the agent invests under ICL scheme. Then we will compare it with the no investment case (Section 3.1) to

[21] This is again a simplifying assumption. Most of our results will hold as long as the government can borrow at some rate below i.

[22] Suppose there is an implicit interest rate $\widehat{i}$. Then $p\tau w^s = h(1+\widehat{i})$. Using this and (10) it is easy to see that $\widehat{i} = r$.

[23] This follows from the expected utility function and (9)

compute the optimal lifetime behaviour of the agents and the long-run equilibria. Finally, we compare the long-run dynamics thus generated with that of the imperfect credit market case. We can write the agent's decision problem as follows:

$$\max_{c_1, c_{2s}, c_{2f}, b_s, b_f} E\left[U(.)\right] = c_1^{\alpha} + \beta\left[pc_{2s}^{1-\delta}b_s^{\delta} + (1-p)c_{2f}^{1-\delta}b_f^{\delta}\right].$$

such that,

$$c_1 \leq x + B_C - S, \tag{11}$$

$$c_{2s} + b_s \leq \left(w^s - \frac{h(1+r)}{p}\right) - B_C(1+i) + S(1+r), \tag{12}$$

$$c_{2f} + b_f \leq (1+\gamma)w^n - B_C(1+i) + S(1+r). \tag{13}$$

where $B_C \geq 0$, stands for consumption borrowing. We can write the period 2 budget constraint in expected terms as

$$E(c_2) + E(b) \leq E(w_{ICL}) - B_C(1+i) + S(1+r). \tag{14}$$

where $E(w_{ICL}) = p\left(w^s - \frac{h(1+r)}{p}\right) + (1-p)(1+\gamma)w^n$ is the expected earning under ICL scheme. Note that the expected wage has changed only by the fact that in the good state the additional negative term is the tax payment because one had taken ICL.

Here everything is analogous to the investment case. Although now education finance takes place through ICLs, borrowing for consumption still takes place at the rate $(1+i)$. The budget constraints are defined analogously to the investment case with minor changes. First, in the period 1 budget constraint (11) there is no $-h$ term because education finance now takes place through ICL which is transferred directly to the educational institution. Second, the good state period 2 budget (12) has a $-\frac{h(1+r)}{p}$ term which is the repayment of the ICL. In the bad state there is no repayment. (14) is still the expected period 2 budget constraint, with the new definition of $E(w_{ICL})$.

As before, we have:

$$E[b] = \left(\frac{\delta}{1-\delta}\right)E[c_2] \tag{15}$$

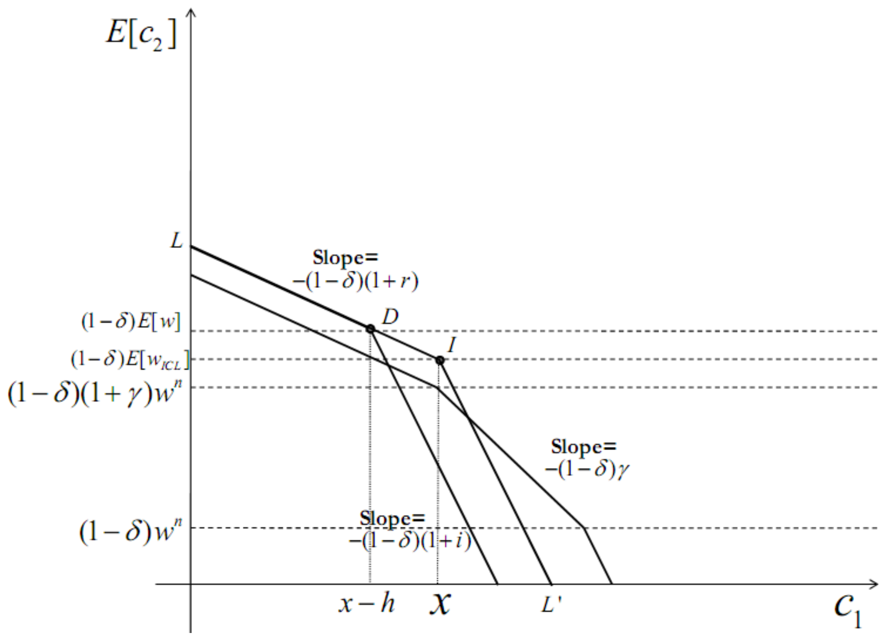

FIGURE 7: ICL BUDGET SET.

The ICL budget set for our agent who had an inherited wealth x is shown in figure 7. If she does not borrow for consumption and does not save then her period 1 consumption is x and in period 2 she consumes her expected wage, as can be seen from (11) and (14). This point is marked as I in the figure. Given that she can borrow and save, the budget set is analogous to the investment budget set. The fact that this agent can borrow the h amount at the expected rate $(1+r)$ is very clear from the figure. The slope of the budget between D and I is exactly $-(1-\delta)(1+r)$ and the horizontal distance between these two points is exactly h. Since the expected wage in this case is lower than the investment case in section 3.2 we label that specifically. Thus, LIL' is the ICL budget for the agent who has inherited wealth equal to x.

The optimal choice of the agent is summarized in the following proposition.

Proposition 5 *There exist wealth thresholds* $0 < \underline{x}^{ICL} < \overline{x}^{ICL}$ *such that the optimal choices for* B_C^* *and* S *are characterized by the following:*

(a) for $x < \underline{x}^{ICL}$, $B_C^* > 0$ *and* $S^* = 0$,*with* $D_1 B_C^*(x) < 0$;

(b) for $x \in [\underline{x}^{ICL}, \overline{x}^{ICL}]$, $B_C^* = 0$ *and* $S^* = 0$; *and*

(c) for $x > \overline{x}^{ICL}$, $B_C^* = 0$ *and* $S^* > 0$, *with* $D_1 S^*(x) > 0$.

Proof. See Appendix C. ∎

4.2 Optimal Choice between No Investment and Investment in Technical Skill-Formation with ICLs

To derive the optimal choice between not investing in technical skill formation and investing using ICLs, we compare the expected indirect utilities obtained from the optimization problems in sections 3.1 and 4.1. Under Assumption 4, the 'rich' ($x > \bar{x}^{NI} = \bar{x}^{ICL}$) always prefer to invest in skill formation using ICLs than not investing. For the poorer agents, the same trade-off between lower current income and higher future income holds. Hence, there will be a threshold level of inheritance beyond which agents will prefer ICLs to not investing in skill formation. We call this threshold, f_{ICL}.

Notice that f_{ICL} will always be lower than f. This is because f is the inheritance level that makes one indifferent between borrowing and not borrowing at the interest rate $(1+i)$, while f_{ICL} is the threshold inheritance that does the same for a lower (effective) interest rate $(1+r)$. Hence, $f_{ICL} < f$.

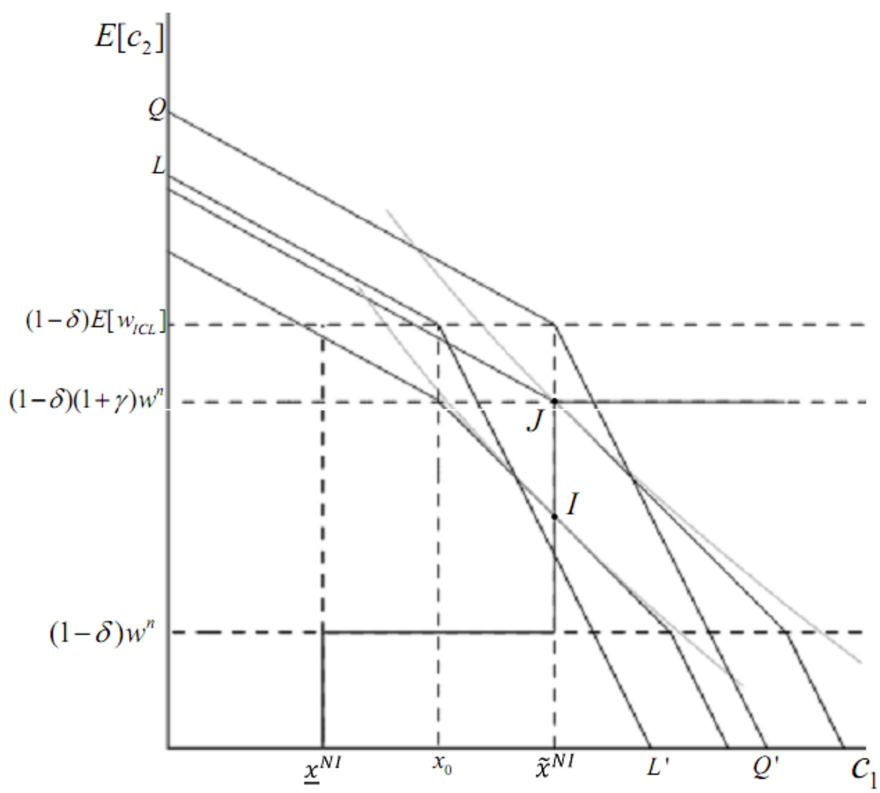

Figure 8: Location of f_{ICL}.

Also, $f_{ICL} \leq \tilde{x}^{NI}$ always. The argument here is illustrated in the figure 8. As can be seen, for an agent having inherited wealth $\tilde{x}^{NI}$, the optimal choice when she is not investing in skill formation (J) lies inside the budget set for the case she invests using ICLs (QQ'). Thus, using the Weak Axiom of Revealed Preference we can infer that the agent will always prefer to invest using ICLs at $\tilde{x}^{NI}$ and the same argument continues for agents with inherited wealth beyond $\tilde{x}^{NI}$ as well. However, the same cannot be said for all agents having inherited wealth below $\tilde{x}^{NI}$ as the best point when not investing could be inside or outside the ICL budget. An example of such a case is shown for someone who has inherited a wealth of x_0. Her optimal choice when she is not investing in skill formation is I and that is outside her ICL budget, LL'. Hence, we are not sure how much below $\tilde{x}^{NI}$, f_{ICL} will be.

In what follows we take different parametric assumptions to pin down the location of f_{ICL} and show an exhaustive class of results.

4.3 Long-Run Wealth Dynamics under ICLs

The bequest lines for any agent who takes ICL are as follows:

(a) $x < \underline{x}^{icl}$

$$x_{t+1} = \begin{cases} \delta \left[w^s - \frac{h(1+r)}{p} + x(1+i) - \eta_i(1+i) \right] & \text{with prob } p \\ \delta \left[(1+\gamma)w^n + x(1+i) - \eta_i(1+i) \right] & \text{with prob } 1-p \end{cases}$$

(b) $x \in [\underline{x}^{icl}, \bar{x}^{icl}]$

$$x_{t+1} = \begin{cases} \delta \left[w^s - \frac{h(1+r)}{p} \right] & \text{with prob } p \\ \delta \left[(1+\gamma)w^n \right] & \text{with prob } 1-p \end{cases}$$

(c) $x > \bar{x}^{icl}$

$$x_{t+1} = \begin{cases} \delta \left[w^s - \frac{h(1+r)}{p} + x(1+r) - \eta_r(1+r) \right] & \text{with prob } p \\ \delta \left[(1+\gamma)w^n + x(1+r) - \eta_r(1+r) \right] & \text{with prob } 1-p \end{cases}$$

The final bequest lines will depend on the position of f_{ICL}. Again, the bequest lines after f_{ICL} have two components because of the uncertainty. We now analyze the dynamics with ICLs and compare them with that of the non-ICL case. A lot of cases are possible depending on the value of γ and the exact position of f_{ICL}. All the cases can be summarized into two classes: (1) $f_{ICL} < \underline{x}^{NI}$ and (2) $f_{ICL} > \underline{x}^{NI}$. We illustrate each case with two sub-cases,low or high γ.

Figure 9.1 depicts the Class I ($f_{ICL} < \underline{x}^{NI}$) bequest dynamics with a high value of $\gamma(> 1)$. The dotted lines represent the bequest lines for the non-ICL case (as shown in Figure 5) while the solid lines are drawn for the optimal bequests when agents invest in skill formation using

ICLs. Recall that as in Figure 5, the only long-run steady state for the non-ICL (dotted) bequests is at point B which is a 'blue-collar employment trap'. Let us look at the bequest lines for the ICLs now. Before f_{ICL}, the lines are identical to those for the non-ICL case. At f_{ICL}, the bequests jump to a flat stretch, the height of which depends on the realized state of nature, and they start rising after $\bar{x}^{ICL}$.

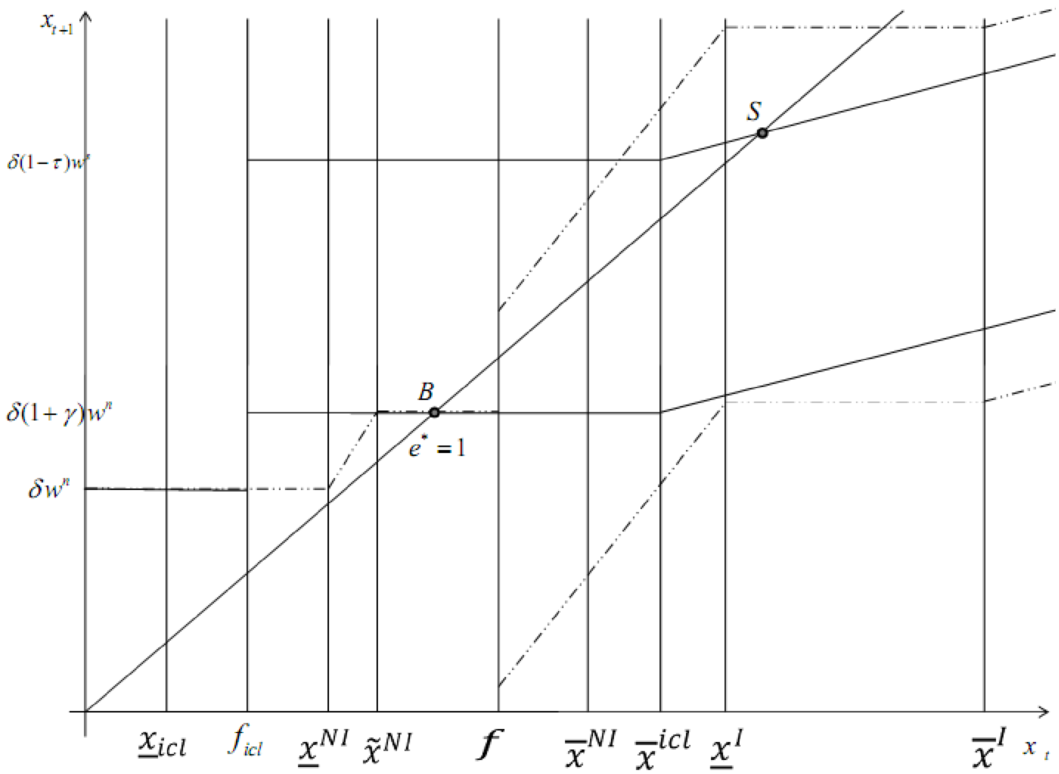

FIGURE 9.1: CLASS I BEQUEST DYNAMICS WITH ICL AND HIGH γ.

Before f_{ICL}, there is no intersection of the bequest line with the 45 degree line. After f_{ICL}, however, there two intersections, one in the good state (point S) and another in the bad state (point B). Now, consider an agent who inherits a wealth greater than f_{ICL}. This agent invests in skill formation. If she gets a good shock, she leaves a bequest corresponding to the higher bequest line. Subsequent consecutive good shocks will make her dynasty move towards S. A series of consecutive bad shocks, on the other hand, take the dynasty to the point B. Since, B is a stable point, any number of bad shocks can never push the dynasty below B. Now consider an agent with inherited wealth below f_{ICL} to begin with. This agent does not invest in skill formation, but leaves a bequest higher than she receives (corresponding to the flat stretch before f_{ICL}). This pattern continues till her dynasty crosses the threshold f_{ICL} and starts investing in skill formation, after which the dynamics are the same as for an individual who was rich enough to invest given her bequest.

Hence, in the long-run, the wealth distribution of the entire economy will lie between the points B and S. There will be a lot of mobility within this range depending on whether the agent gets a good shock or bad shock, but no dynasty will ever fall below point B. This is a clear improvement from the non-ICL case in two respects:

1. The economy is not stuck at point B. In fact, in the long run, it has an invariant wealth distribution with wealth levels higher than that at point B.

2. Since, the entire range of the invariant distribution lies above f_{ICL}, all individuals in the economy continually invest in skill formation. In the non-ICL case, however, the wealth corresponding to point B was below f, due to which in the long run, nobody invested in skill formation. ICLs, therefore, are successful in eliminating such stagnation.

The economy, in the long-run, is thus richer in terms of both education and wealth with ICLs.

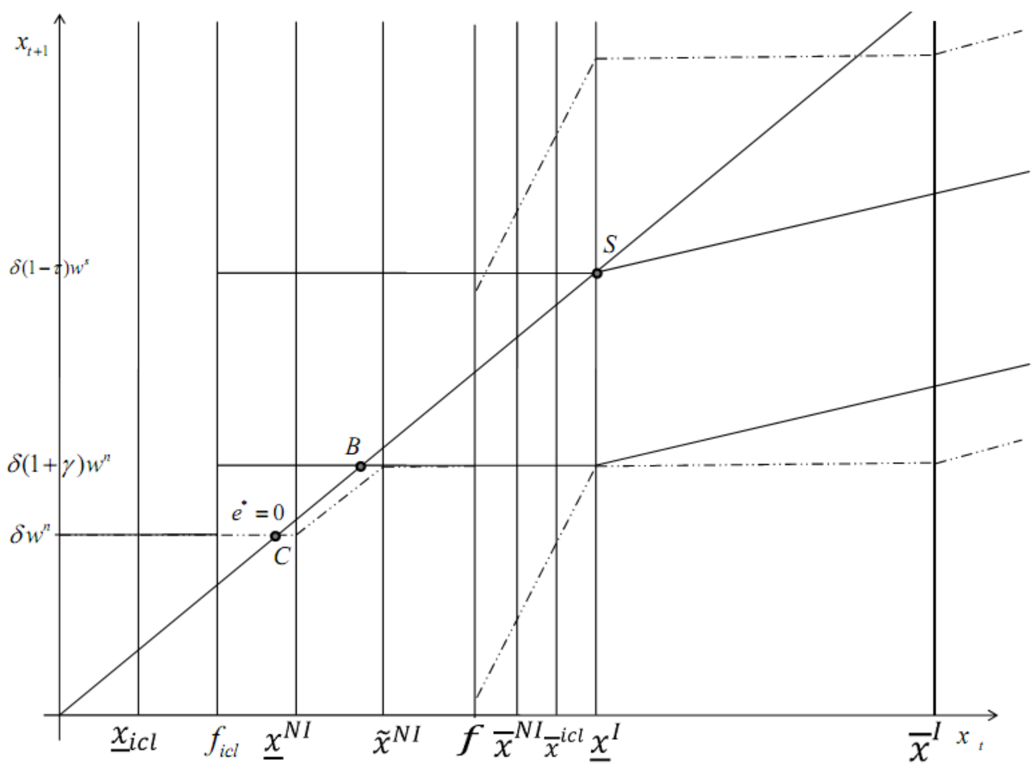

FIGURE 9.2: CLASS I BEQUEST DYNAMICS WITH ICL AND LOW γ.

Figure 9.2 has the same relative positions of f_{ICL} and $\underline{x}^{NI}$ but differs in the value of γ which is lower ($\gamma < \frac{1}{\delta}$) here. The dotted lines now correspond to those in Figure 6, with the unique long-run equilibrium being a 'child-labour trap' at point C. The solid bequest lines for the ICL case are exactly analogous to Figure 9.1 with the two intersections with the 45 degree line being at B and S and the long-run invariant wealth distribution lying between these points.

The difference here is in the effects of ICL: The wealth levels in long-run invariant distribution lie strictly above the one corresponding to point C. Also, the education level of all individuals is higher than at C, where actually agents are completely illiterate. With ICLs, however, they not only complete primary education but in fact go all the way to invest in specialized skill formation as well. This illustrates a 'light-at-the-end-of-the-tunnel' effect of ICLs. That is, very poor individuals might decide not to send their children to school because primary education does not lead to enough increase in the wage income of the household, while the opportunity cost in terms of forgone (current) child labour income is very high. Hence, the economy ends up in a child labour trap. The presence of ICLs implies that these families have the opportunity to educate their children even after the basic schooling with the help of ICLs. This encourages them to send their children to school and pulls the economy out of the child labour trap in the long run. Hence, ICLs are able to break the 'low-income-low-education' vicious cycle.

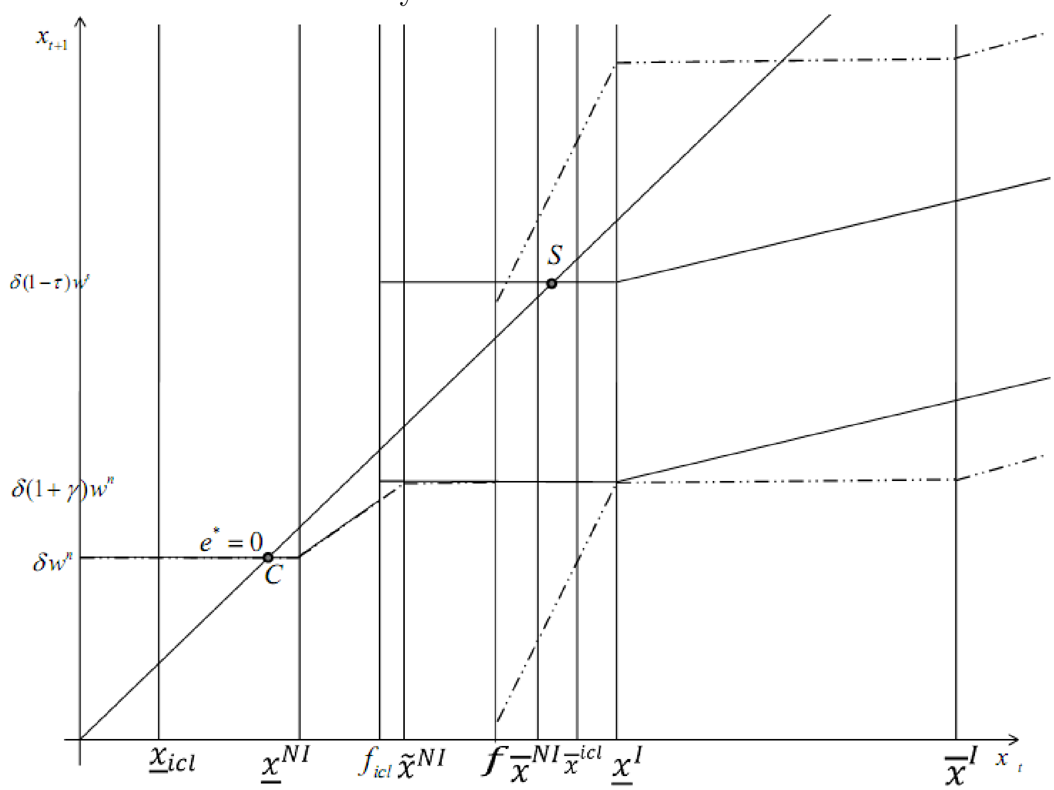

FIGURE 10.1: CLASS II BEQUEST DYNAMICS WITH ICL - CASE 1.

We now come to the second class of dynamics where $f_{ICL} > \underline{x}^{NI}$. When the value of γ is high, the dynamics is qualitatively the same as in previous class. Hence, we only discuss the case when γ has a low value ($< \frac{1}{\delta}$) in detail here.

The dotted lines in Figure 10.1 are same as the solid lines in Figure 6 with the unique long-run steady state at point C. Notice, however, that the ICL bequest lines (solid) are different than the ones in the previous class. Since, f_{ICL} lies above $\underline{x}^{NI}$, the jump to the flat stretch happens much later as compared to Figure 9.1 and 9.2. Hence, now it is possible that the bequest line with ICLs in the bad state (lower solid line) does not have any intersection with the 45 degree line as opposed to the previous class. And this is precisely the case illustrated in Figure 10.1.

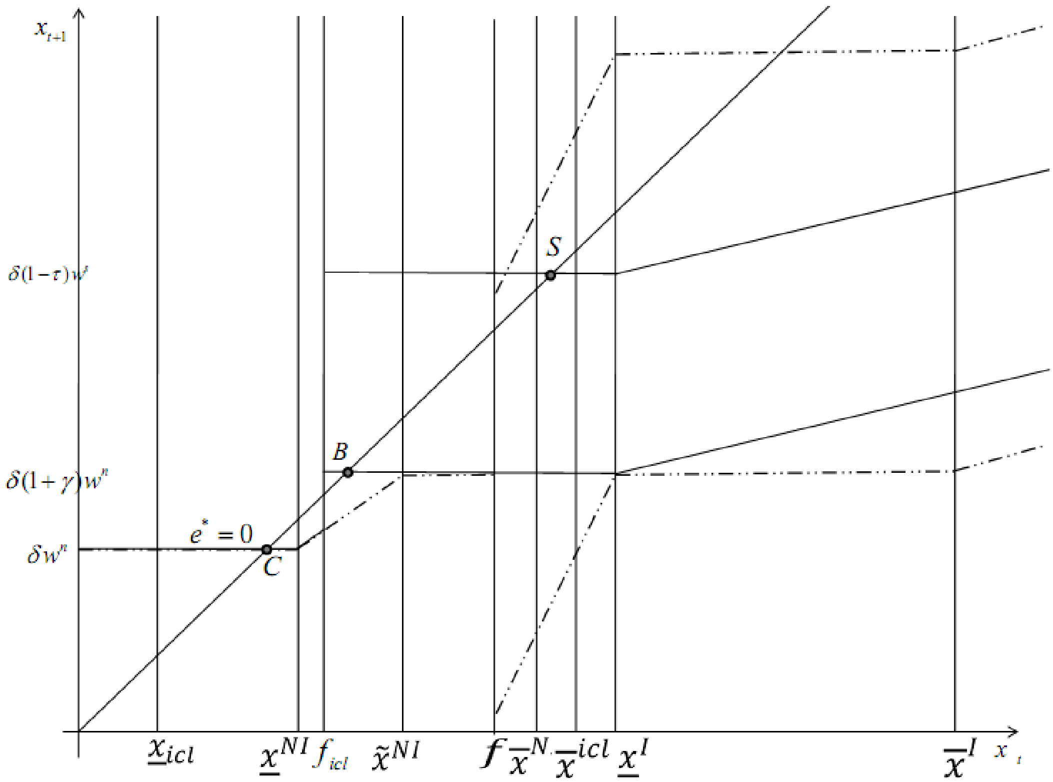

FIGURE 10.2: CLASS II BEQUEST DYNAMICS WITH ICL - CASE 2.

Till f_{ICL}, the solid and dotted lines coincide and then the solid ones jump up. The intersection of the ICL bequest lines with the 45 degree line happens at points C and S in Figure 10.1. Since C is a stable point, all those below f_{ICL} will move towards this C. Consider an agent who inherits an amount greater than f_{ICL}. If she invests in skill formation and gets a good shock, she moves towards S. But one bad shock will bring her to the lower solid line. A sufficiently high number of consecutive bad shocks after that will make her dynasty fall below f_{ICL} and then her dynasty will stop investing in skill formation and move towards C where it will be trapped forever. If once a dynasty falls below f_{ICL}, there is no way of moving up again. Hence, in the long-run, the entire economy will be stagnant at the point C where all agents are illiterate and nobody ever invests in skill formation. Thus, evidently, ICLs are a complete failure in this case.

This suggests that ICLs are not a magic wand. The primary education needs to be productive enough (γ high) for the ICLs to be helpful in pulling the economy out of stagnation. Hence, any anti-poverty policy which has a focus on education must consider primary and higher education as complements to each other. Without either one of these, the policy could be completely self-defeating.

In this class, the pulling-out effect of ICLs can be sustained if the position of f_{ICL} is such that even with a low value of γ, the bad state bequest line has an intersection with the 45 degree line. Such a case is illustrated in Figure 10.2 where the long-run invariant wealth distribution of the dynasties that initially qualify for ICL ($x_0 > f_{ICL}$) is again between wealth levels corresponding to the points B and S.

The results can be summarized as follows:

Proposition 6

(a) *When* $f_{ICL} < \underline{x}^{NI}$, *irrespective of the value of* γ, *the long-run invariant wealth distribution is between wealth levels corresponding to points B and S in figures 9.1 and 9.2. Thus, in the long-run dynasties never get locked in a "child-labour" trap and further, they keep investing in special skill formation.*

(b) *When* $f_{ICL} > \underline{x}^{NI}$, *and* γ *is sufficiently low, the long-run wealth distribution converges to the wealth level corresponding to point C in figure 10.1 and the economy is stuck in the "child-labour" trap.*
However, when $f_{ICL} > \underline{x}^{NI}$, *but* γ *is only moderately low (figure 10.2), the long-run wealth distribution is characterized by the following: while the dynasties with initial wealth* $x_0 < f_{ICL}$ *get stuck at the child labour trap C, the long-run wealth distribution of dynasties with* $x_0 \geq f_{ICL}$ *is invariant between wealth levels corresponding to points B and S.*

5 Conclusion

In this paper we have analysed the efficacy of ICL as an anti-poverty instrument. To our mind, lack of education lies at the root of many (if not all) development-related problems including persistence of poverty and child labour. Thus, educating the masses constitutes an important poverty alleviation strategy. However, successful eradication of poverty in the long run requires a careful designing of education policy which not only maintains the right incentive structure for the poor to go for skill formation, but, at the same time, creates the right foundation in terms of basic education so that people can fruitfully utilize this option. We have shown here that a suitably designed ICL along with an effective primary education policy can go a long way in achieving this goal.

In this context one might like to see how ICL compares with other types of education and/or poverty alleviation strategies. Education policies are usually politically easier to implement than a direct redistributive tax-transfer scheme. Among education policies, we have already noted the advantages of ICLs vis-a-vis education subsidies or mortgage loans. One such advantage stems from the fact the ICL provides insurance to the potential borrowers in the event of failure. The implicit insurance component may impact on optimal choices of household through two distinct channels: (a) existence of insurance encourages more people to invest in high skills (which we shall call the 'incentive effect'); and (b) insurance prevents the poor borrowers from falling back into the poverty trap (which we shall call the 'cushioning effect'). It has been shown in the literature that when agents are risk averse, existence of such an implicit insurance factor makes ICL welfare-improving over ordinary mortgage loans. However, we should emphasize here that in our model the efficacy of ICL does not work through risk aversion. In fact we have assumed that agents are risk neutral. Allowing for risk averse agents will make the case for ICL even stronger.

Finally, in our model we have implicitly assumed that the government can avail credit at a cheaper rate than individual households because as an institution it cannot run away with the money, while individual borrowers can. In fact we have assumed that there is absolutely no moral hazard problem associated with government borrowing, such that it can borrow exactly at the lenders' rate. This begs the following question: if indeed the government can borrow at a cheaper rate and can enforce repayment through taxation (as it does in case of ICL), then why does not it simply offer an ordinary mortgage loan at lower interest rate? In other words, is there any extra advantage of ICL vis-a-vis a government-backed mortgage loan, which basically replicates the perfect credit market? Our answer is in the affirmative. ICLs performs better than the perfect credit market even when agents are risk neutral, precisely because of the existence of the 'cushioning effect'. Moral hazard problems for government loans and/or leakage in the tax collection mechanism would somewhat undermine the cushioning effect of ICL, but to what extent is a moot point. It remains a part of the future research agenda.

6 Appendix

6.1 Appendix A: Agent's Optimization Problem - No Investment in Technical-Skill Formation.

$$\max_{c_1,c_2,b,e} U(c_1,c_2,b,e) = c_1^{\alpha} + \beta c_2^{1-\delta} b^{\delta}$$

such that:

$$c_1 = (1-e)w^n + x + B - S^{24}$$

$$c_2 + b = (1+\gamma e)w^n + S(1+r) - B(1+i)$$

$$B \geq 0$$

$$S \geq 0$$

$$e \geq 0$$

$$(1-e) \geq 0$$

6.1.1 Solution

The Lagrangian for this problem is defined as follows:

$$\mathcal{L} = [(1-e)w^n + x + B - S]^{\alpha} + \beta\left[(1+\gamma e)w^n + S(1+r) - B(1+i) - b\right]^{1-\delta} b^{\delta} \\ + \lambda_1 B + \lambda_2 S + \lambda_3 e + \lambda_4 (1-e)$$

First Order Conditions:

$$\frac{\partial \mathcal{L}}{\partial e} = -\alpha w^n c_1^{\alpha-1} + \beta\gamma w^n (1-\delta)\, c_2^{-\delta} b^{\delta} + \lambda_3 - \lambda_4 = 0$$

$$\frac{\partial \mathcal{L}}{\partial B} = \alpha c_1^{\alpha-1} - \beta(1+i)(1-\delta)\, c_2^{-\delta} b^{\delta} + \lambda_1 = 0$$

$$\frac{\partial \mathcal{L}}{\partial S} = -\alpha c_1^{\alpha-1} + \beta(1+r)(1-\delta)\, c_2^{-\delta} b^{\delta} + \lambda_2 = 0$$

$$\frac{\partial \mathcal{L}}{\partial b} = -\beta(1-\delta)\, c_2^{-\delta} b^{\delta} + \beta\delta c_2^{1-\delta} b^{\delta-1} = 0$$

The last equation implies that:

$$\boxed{b = \left(\tfrac{\delta}{1-\delta}\right) c_2}$$

[24] Because the utility function is increasing in c_1, c_2 and b, we can replace the inequality in the budget constraints by an equality sign.

Using $b = \left(\frac{\delta}{1-\delta}\right) c_2$, we can rewrite the FOCs in a reduced form as:

$$-\alpha w^n c_1^{\alpha-1} + \beta\gamma w^n (1-\delta)\left(\frac{\delta}{1-\delta}\right)^{\delta} + \lambda_3 - \lambda_4 = 0$$

$$\alpha c_1^{\alpha-1} - \beta(1+i)(1-\delta)\left(\frac{\delta}{1-\delta}\right)^{\delta} + \lambda_1 = 0$$

$$-\alpha c_1^{\alpha-1} + \beta(1+r)(1-\delta)\left(\frac{\delta}{1-\delta}\right)^{\delta} + \lambda_2 = 0$$

To simplify notations, we depict some recurring expressions in a concise form as follows:

$$\mu \equiv \beta\left(\frac{\delta}{1-\delta}\right)^{\delta}(1-\delta)$$

$$\eta_i \equiv \left[\frac{\alpha}{\beta\delta^{\delta}(1-\delta)^{(1-\delta)}(1+i)}\right]^{\frac{1}{1-\alpha}}$$

$$\eta_\gamma \equiv \left[\frac{\alpha}{\beta\delta^{\delta}(1-\delta)^{(1-\delta)}(\gamma)}\right]^{\frac{1}{1-\alpha}}$$

$$\eta_r \equiv \left[\frac{\alpha}{\beta\delta^{\delta}(1-\delta)^{(1-\delta)}(1+r)}\right]^{\frac{1}{1-\alpha}}$$

Note that: $\eta_r > \eta_\gamma > \eta_i$

There are several cases in the solution depending on which constraints are slack and which ones are active. We show the relevant cases first and then list the all the ruled out cases along with the respective arguments.

Case 1: $e = 0, 1-e > 0, B = 0$ **and** $S = 0$ Due to complementary slackness, the respective multipliers are: $\lambda_1, \lambda_2, \lambda_3 \geq 0; \lambda_4 = 0$. Substituting these in the reduced-form FOCs, we get:

$$c_1 \leq \eta_\gamma, \quad c_1 \geq \eta_i \quad \text{and} \quad c_1 \leq \eta_r$$

which implies that:

$$\eta_i \leq c_1 \leq \eta_\gamma$$

Now, from the budget constraints we have,

$$c_1^* = w^n + x \tag{A1}$$

$$c_2^* = (1-\delta)\, w^n \tag{A2}$$

It follows that this case arises when

$$\eta_i - w^n \leq x \leq \eta_\gamma - w^n$$

We assume that $w^n > \eta_i$ and denote $\underline{x}^{NI} \equiv \eta_\gamma - w^n$. Hence, for agents who inherit an amount less that $\underline{x}^{NI}$, optimal consumption is given by equation (A1) and (A2), and the indirect utility and bequest equation are as follows:

$$V_1^{NI} = c_1^\alpha + \beta c_2^{1-\delta} b^\delta = (w^n + x)^\alpha + \mu w^n$$

$$b^* = \delta w^n$$

Case 2: $e \in (0,1), 1-e > 0, B = 0$ **and** $S = 0$ Multipliers are therefore: $\lambda_1, \lambda_2 \geq 0; \lambda_3, \lambda_4 = 0$. Again, from the first order conditions we have:

$$c_1^* = \eta_\gamma \quad \text{and} \quad \eta_i \leq c_1^* \leq \eta_r \tag{A3}$$

and from the budget equations we have,

$$c_1 = (1-e)w^n + x$$

$$c_2 = (1-\delta)(1+\gamma e)w^n$$

Eliminating e from the above two equations, we can get the expression for the optimal period 2 consumption:

$$c_2^* = (1-\delta)\left[(1+\gamma)w^n + \gamma x - \gamma\eta_\gamma\right] \tag{A4}$$

Also, the optimal value of e is given by:

$$e^*(x) = \left(\frac{1}{w_n}\right)\left[w^n + x - \eta_\gamma\right]$$

Using this equation, we can see that

$$e^*(\cdot) = 0 \quad \Rightarrow \quad x = \eta_\gamma - w^n \equiv \underline{x}^{NI}$$

and

$$e^*(\cdot) = 1 \quad \Rightarrow \quad x = \eta_\gamma \equiv \tilde{x}^{NI}$$

Hence, for the agents with inheritance in the range $(\underline{x}^{NI}, \tilde{x}^{NI})$, the optimal consumption in either period are given by (A3) and (A4), while the indirect utility and bequest are given by the following:

$$V_2^{NI} = (\eta_\gamma)^\alpha + \mu\left[(1+\gamma)w^n + \gamma x - \gamma\eta_\gamma\right]$$

$$b^* = \delta\left[(1+\gamma)w^n + \gamma x - \gamma\eta_\gamma\right]$$

Case 3: $e = 1, 1-e = 0, B = 0$ **and** $S = 0$ By complementary slackness, $\lambda_1, \lambda_2, \lambda_4 \geq 0; \lambda_3 = 0$. Now, the FOCs give us the following inequality:

$$\eta_\gamma \leq c_1^* \leq \eta_r$$

From the constraints we have:

$$c_1^* = x$$

$$c_2^* = (1-\delta)\left[(1+\gamma)w^n\right]$$

Hence, this case holds when

$$\eta_\gamma \leq x \leq \eta_r \quad \Rightarrow \quad \tilde{x}^{NI} \leq x \leq \bar{x}^{NI}$$

Indirect Utility and bequest function is given by:

$$V_3^{NI} = x^\alpha + \mu\left[(1+\gamma)w^n\right]$$

$$b^* = \delta(1+\gamma)w^n$$

Case 4: $e = 1, 1-e = 0, B = 0$ **and** $S > 0$ Again, the multipliers are: $\lambda_1, \lambda_4 \geq 0, \lambda_2 = \lambda_3 = 0$. Hence, it must be that:

$$c_1^* = \eta_r \quad \text{and} \quad \eta_i \leq \eta_\gamma \leq c_1^*$$

Solving for the equilibrium value of savings:

$$S^*(x) = x - c_1^* = x - \eta_r$$

Hence, for $S^*(x) \geq 0$, we require that $x \geq \eta_r \equiv \bar{x}^{NI}$. The optimal choice are given by the following:

$$c_2^* = (1-\delta)\left[(1+\gamma)w^n + (1+r)x - (1+r)\eta_r\right]$$

$$V_4^{NI} = (\eta_r)^\alpha + \mu\left[(1+\gamma)w^n + (1+r)x - (1+r)\eta_r\right]$$

$$b^* = \delta\left[(1+\gamma)w^n + (1+r)x - (1+r)\eta_r\right]$$

Case 5: $e = 0, 1-e > 0, B = 0$ **and** $S > 0$ Then, the multipliers will be: $\lambda_2, \lambda_4 = 0$ and $\lambda_1, \lambda_3 \geq 0$. Then, the FOCs and constraints together require that:

$$c_1^* = \eta_r \quad \text{and} \quad c_1^* \leq \eta_\gamma$$

But that is a contradiction because $(1+r) < \gamma$ implies that $\eta_r > \eta_\gamma$. Hence, this case is not possible.

Case 6:$e = 0, 1-e > 0, S = 0$ **and** $B > 0$ Multipliers must be: $\lambda_1, \lambda_4 = 0$ and $\lambda_2, \lambda_3 \geq 0$. FOCs and constraints then require that:

$$c_1^* = \eta_i \quad \text{and} \quad c_1^* \leq \eta_\gamma \leq \eta_r$$

The above conditions are consistent but we rule this case out by assuming that $w^n > \eta_i$, which implies that the range in which this case holds we will have $x < 0$.

Case 7: $e = 0, 1-e > 0, S > 0$ **and** $B > 0$ This case requires that $\eta_i = \eta_r$, which is a contradiction because $i > r$ by assumption. Hence, this case is ruled out.

Case 8: $e \in (0,1), 1-e > 0, B = 0$ **and** $S > 0$ Now, we must have: $\lambda_2, \lambda_4 = 0$ and $\lambda_1, \lambda_3 \geq 0$. The FOCs and constraints require:

$$c_1^* = \eta_\gamma = \eta_r$$

Again, this is a contradiction because $\gamma > (1+r)$.

Case 9: $e \in (0,1), 1-e > 0, S = 0$ **and** $B > 0$ Multipliers must be: $\lambda_1, \lambda_4 = 0$ and $\lambda_2, \lambda_3 \geq 0$. This implies that $\eta_\gamma = \eta_i$, which is not possible given $(1+i) > \gamma$. Hence, this case gets ruled out.

Case 10: $e = 1, 1-e = 0, B > 0$ **and** $S = 0$ Now, the multipliers are: $\lambda_1, \lambda_3 = 0$ and $\lambda_2, \lambda_4 \geq 0$. These values of the multipliers and the FOCs together imply that for this case to hold, we must have $c_1^* = \eta_i$ and $c_1^* \geq \eta_\gamma$, both of which cannot happen together. Hence, this case is ruled out.

Note that all the cases that have both $B > 0$ and $S > 0$ are invalid because these are two separate instruments to augment income and have different interest rates. Hence both cannot be used together.

6.1.2 Summary

- $x \leq \underline{\mathrm{x}}^{NI} : B = 0, e = 0, S = 0$

$$c_1^* = w^n + x$$

$$c_2^* = (1-\delta)\, w^n$$

$$V_1^{NI} = c_1^\alpha + \beta c_2^{1-\delta} b^\delta = (w^n + x)^\alpha + \mu w^n$$

$$b^* = \delta w^n$$

- $x \in (\underline{\mathrm{x}}^{NI}, \tilde{x}^{NI}] : B = 0, 0 < e < 1, S = 0$

$$c_1^* = \eta_\gamma$$

$$c_2^* = (1 - \delta) \left[(1 + \gamma)w^n + \gamma x - \gamma\eta_\gamma\right]$$

$$V_2^{NI} = (\eta_\gamma)^\alpha + \mu \left[(1 + \gamma)w^n + \gamma x - \gamma\eta_\gamma\right]$$

$$b^* = \delta \left[(1 + \gamma)w^n + \gamma x - \gamma\eta_\gamma\right]$$

- $x \in (\tilde{x}^{NI}, \bar{x}^{NI}] : B = 0, e = 1, S = 0$

$$c_1^* = x$$

$$c_2^* = (1 - \delta) [(1 + \gamma)w^n]$$

$$V_3^{NI} = x^\alpha + \mu [(1 + \gamma)w^n]$$

$$b^* = \delta(1 + \gamma)w^n$$

- $x > \bar{x}^{NI} : B = 0, e = 1, S > 0$

$$c_1^* = \eta_r$$

$$c_2^* = (1 - \delta) [(1 + \gamma)w^n + (1 + r)x - (1 + r)\eta_r]$$

$$V_4^{NI} = (\eta_r)^\alpha + \mu [(1 + \gamma)w^n + (1 + r)x - (1 + r)\eta_r]$$

$$b^* = \delta [(1 + \gamma)w^n + (1 + r)x - (1 + r)\eta_r]$$

where:

- $\underline{x}^{NI} = \eta_\gamma - w^n$
- $\tilde{x}^{NI} = \eta_\gamma$
- $\bar{x}^{NI} = \eta_r$

6.2 Appendix B:Agent's Optimization Problem - Investment in Technical-Skill Formation without ICLs.

$$\max_{c_1,c_{2s},c_{2f},b_s,b_f} E\left[U(.)\right] = c_1^{\alpha} + \beta\left[pc_{2s}^{1-\delta}b_s^{\delta} + (1-p)c_{2f}^{1-\delta}b_f^{\delta}\right]$$

such that,

$$c_1 = x - h + B - S$$

$$c_{2s} + b_s = w^s - B(1+i) + S(1+r)$$

$$c_{2f} + b_f = (1+\gamma)w^n - B(1+i) + S(1+r)$$

In expected terms,

$$E(c_2) + E(b) = E(w) - B(1+i) + S(1+r)$$

$$S \geq 0$$

$$B \geq 0$$

6.2.1 Solution

The Lagrangian for this problem is as follows:

$$\mathcal{L} = \left[x - h + B - S\right]^{\alpha} + \beta p {b_s}^{\delta}\left[w^s - B(1+i) + S(1+r) - b_s\right]^{1-\delta}$$
$$+ \beta(1-p){b_f}^{\delta}\left[(1+\gamma)w^n - B(1+i) + S(1+r) - b_f\right]^{1-\delta} + \lambda_1 S + \lambda_2 B$$

The First Order Conditions then are:

$$-\alpha {c_1}^{\alpha-1} + \beta\left(1-\delta\right)\left(1+r\right)\left\{E\left[c_2^{-\delta}b^{\delta}\right]\right\} + \lambda_1 = 0$$

$$\alpha {c_1}^{\alpha-1} - \beta\left(1-\delta\right)\left(1+i\right)\left\{E\left[c_2^{-\delta}b^{\delta}\right]\right\} + \lambda_2 = 0$$

$$b_f = \left(\frac{\delta}{1-\delta}\right)c_{2f}.$$

$$b_s = \left(\frac{\delta}{1-\delta}\right)c_{2s}.$$

Thus, using $b = \left(\frac{\delta}{1-\delta}\right)c_2$, the FOCs can be written in a reduced form as:

$$-\alpha {c_1}^{\alpha-1} + \beta\left(1-\delta\right)\left(1+r\right)\left(\frac{\delta}{1-\delta}\right)^{\delta} + \lambda_1 = 0.$$

and

$$\alpha {c_1}^{\alpha-1} - \beta\left(1-\delta\right)\left(1+i\right)\left(\frac{\delta}{1-\delta}\right)^{\delta} + \lambda_2 = 0.$$

Case 1: $S = 0$ **and** $B > 0$ By complementary slackness, the multipliers here will be: $\lambda_1 \geq 0$ and $\lambda_2 = 0$. Then, the FOCs imply that:

$$c_1^* = \eta_i$$

Now, from the budget constraints, we get that:

$$c_1 = x - h + B$$

$$E(c_2) = (1 - \delta)\left[E(w) - B(1 + i)\right]$$

Eliminating B, we can solve for the optimal period 2 consumption:

$$E(c_2^*) = (1 - \delta)\left[E(w) + (x - h)(1 + i) - \eta_i(1 + i)\right]$$

Also, we get the equilibrium value of borrowing as follows:

$$B^*(x) = \eta_i - x + h$$

We define $\underline{x}^I$ as the threshold value of inherited wealth below which agents will borrow to meet their optimal choices. Hence, it must be that $B^*(\underline{x}^I) = 0$, which gives us:

$$\underline{x}^I \equiv \eta_i + h$$

The expected indirect utility and the expected bequest line are as follows:

$$V_1^I = (\eta_i)^\alpha + \mu\left[E(w) + (x - h)(1 + i) - \eta_i(1 + i)\right]$$

$$E(b^*) = \delta\left[E(w) + (x - h)(1 + i) - \eta_i(1 + i)\right]$$

Case 2: $S = 0$ **and** $B = 0$ Here, both multipliers, λ_1 and $\lambda_2 \geq 0$. Hence, both the FOCs together imply the following inequality:

$$\eta_i \leq c_1^* \leq \eta_r$$

And from the budget constraints we get that:

$$c_1^* = x - h$$

$$E(c_2^*) = (1 - \delta)E(w)$$

Therefore, we get:

$$\eta_i + h \leq x \leq \eta_r + h$$

We denote the latter threshold as $\bar{x}^I$. So, when the inheritance of the agent is in range $[\underline{x}^I, \bar{x}^I]$, the expected indirect utility and bequest are as follows:

$$V_2^I = (x - h)^\alpha + \mu E(w)$$

$$E(b^*) = \delta E(w)$$

Case 3: $B = 0$ **and** $S > 0$ The multipliers will now be: $\lambda_1 = 0$ and $\lambda_2 \geq 0$. Hence, from the FOCs, we have:

$$c_1^* = \eta_r$$

Using $B = 0$, the budget constraints can be reduced to:

$$c_1 = x - h - S$$

$$E(c_2) = (1 - \delta)[E(w) + S(1 + r)]$$

Again, eliminating S, we get:

$$E(c_2^*) = (1 - \delta)\left[E(w) + (x - h)(1 + r) - \eta_r(1 + r)\right]$$

Also, the equilibrium savings function is:

$$S^*(x) = x - h - \eta_r$$

Hence, the threshold of inherited wealth from where the agents start investing using their own means and also saving is $\bar{x}^I = h + \eta_r$ such that $S^*(\bar{x}^I) = 0$. Thus, the expected indirect utility and expected bequests for these agents is given by:

$$V_3^I = (\eta_r)^\alpha + \mu\left[E(w) + (x - h)(1 + r) - \eta_r(1 + r)\right]$$

$$E(b^*) = \delta\left[E(w) + (x - h)(1 + r) - \eta_r(1 + r)\right]$$

Case 4: $B > 0$ **and** $S > 0$ This case is ruled out because $(1 + i) > (1 + r)$ and so both these instruments can never be used together optimally.

6.2.2 Summary

- $x < \underline{\mathrm{x}}^I : B > 0, S = 0$

$$c_1^* = \eta_i$$

$$E(c_2^*) = (1 - \delta)\left[E(w) + (x - h)(1 + i) - \eta_i(1 + i)\right]$$

$$V_1^I = (\eta_i)^\alpha + \mu\left[E(w) + (x - h)(1 + i) - \eta_i(1 + i)\right]$$

$$E(b^*) = \delta\left[E(w) + (x - h)(1 + i) - \eta_i(1 + i)\right]$$

- $x \in \left[\underline{\mathrm{x}}^I, \bar{x}^I\right] : B = 0, S = 0$

$$c_1^* = x - h$$

$$E(c_2^*) = (1-\delta)E(w)$$

$$V_2^I = (x-h)^\alpha + \mu E(w)$$

$$b^* = \delta E(w)$$

- $x > \bar{x}^I : B = 0, S > 0$

$$c_1^* = \eta_r$$

$$E(c_2^*) = (1-\delta)\left[E(w) + (x-h)(1+r) - \eta_r(1+r)\right]$$

$$V_3^I = (\eta_r)^\alpha + \mu\left[E(w) + (x-h)(1+r) - \eta_r(1+r)\right]$$

$$E(b^*) = \delta\left[E(w) + (x-h)(1+r) - \eta_r(1+r)\right]$$

where:

- $\underline{x}^I = h + \eta_i$
- $\bar{x}^I = h + \eta_r$

6.3 Appendix C:Agent's Optimization Problem - Investment in Technical-Skill Formation with ICLs

$$\max_{c_1, c_{2s}, c_{2f}, b_s, b_f} E\left[U(.)\right] = c_1^\alpha + \beta\left[p c_{2s}^{1-\delta} b_s^\delta + (1-p) c_{2f}^{1-\delta} b_f^\delta\right]$$

such that,

$$c_1 = x + B_C - S$$

$$c_{2s} + b_s = \left(w^s - \frac{h(1+r)}{p}\right) - B_C(1+i) + S(1+r)$$

$$c_{2f} + b_f = (1+\gamma)w^n - B_C(1+i) + S(1+r)$$

In expected terms,

$$E(c_2) + E(b) = E(w) - B_C(1+i) + S(1+r)$$

and the non-negativity constraints

$$S \geq 0$$

$$B_C \geq 0 \tag{16}$$

6.3.1 Solution

The Lagrangian is given by:

$$\mathcal{L} = [x + B_C - S]^{\alpha} + \beta p {b_s}^{\delta} \left[\left(w^s - \frac{h(1+r)}{p} \right) - B_C(1+i) + S(1+r) - b_s \right]^{1-\delta} + \beta(1-p){b_f}^{\delta} \left[(1+\gamma)w^n - B_C(1+i) + S(1+r) - b_f \right]^{1-\delta} + \lambda_1 S + \lambda_2 B_C$$

B_C here denotes borrowing for consumption. The First Order Conditions are as follows:

$$(-\alpha){c_1}^{\alpha-1} + \beta\left(1-\delta\right)\left(1+r\right)\left\{E\left[c_2^{-\delta}b^{\delta}\right]\right\} + \lambda_1 = 0$$

$$\alpha{c_1}^{\alpha-1} - \beta\left(1-\delta\right)\left(1+i\right)\left\{E\left[c_2^{-\delta}b^{\delta}\right]\right\} + \lambda_2 = 0$$

$$b_s = \left(\frac{\delta}{1-\delta}\right) c_{2s}.$$

$$b_f = \left(\frac{\delta}{1-\delta}\right) c_{2f}.$$

Thus, we have:

$$b = \left(\frac{\delta}{1-\delta}\right) c_2$$

and using this we can rewrite reduced forms of the FOCs as:

$$-\alpha{c_1}^{\alpha-1} + \beta\left(1-\delta\right)\left(1+r\right)\left(\frac{\delta}{1-\delta}\right)^{\delta} + \lambda_1 = 0.$$

and

$$\alpha{c_1}^{\alpha-1} - \beta\left(1-\delta\right)\left(1+i\right)\left(\frac{\delta}{1-\delta}\right)^{\delta} + \lambda_2 = 0.$$

Case 1: $S = 0$ and $B_C > 0$ Multipliers here will have values such that: $\lambda_1 \geq 0$ and $\lambda_2 = 0$. Then, the FOCs imply that:

$$c_1^* = \eta_i$$

Substituting $S = 0$ in the budget constraints, we get:

$$c_1 = x + B_C$$

$$E(c_2) = (1-\delta)\left[E(w) - B_C(1+i)\right]$$

Eliminating B_C:

$$E(c_2^*) = (1-\delta)\left[E(w) + x(1+i) - \eta_i(1+i)\right]$$

Also,

$$B_C^*(x) = \eta_i - x$$

We define $\underline{x}^{ICL}$ as the inherited wealth level below which agents invest using ICLs but borrow to augment period 1 consumption. Hence, $\underline{x}^{ICL}$ must be such that

$$B_C^*(\underline{x}^{ICL}) = 0 \quad \Rightarrow \quad \underline{x}^{ICL} \equiv \eta_i$$

The expected indirect utility and expected bequest below this threshold can be written as:

$$V_1^{ICL} = (\eta_i)^\alpha + \mu\left[E(w) + x(1+i) - \eta_i(1+i)\right]$$

$$E(b^*) = \delta\left[E(w) + x(1+i) - \eta_i(1+i)\right]$$

Case 2: $S = 0$ **and** $B_C = 0$ Both multipliers here are greater than or equal to zero. Hence, the FOCs imply that:

$$\eta_i \leq c_1^* \leq \eta_r$$

And from the budget constraints:

$$c_1^* = x$$

$$E(c_2^*) = (1-\delta)E(w)$$

Therefore, using the equations above, we get:

$$\underline{x}^{ICL} \equiv \eta_i \leq x \leq \eta_r \equiv \bar{x}^{ICL}$$

So in the range $[\underline{x}^{ICL}, \bar{x}^{ICL}]$, the expected indirect utility and bequest are as follows:

$$V_2^{ICL} = (x)^\alpha + \mu E(w)$$

$$E(b^*) = \delta E(w) \tag{17}$$

Case 3: $S > 0$ **and** $B_C = 0$ Complementary slackness implies: $\lambda_1 = 0$ and $\lambda_2 \geq 0$. From the FOCs, then, we get:

$$c_1^* = \eta_r$$

Using $B_C = 0$, the budget constraints can be reduced to:

$$c_1 = x - S$$

$$E(c_2) = (1-\delta)[E(w) + S(1+r)]$$

Again, eliminating S, we get:

$$E(c_2^*) = (1-\delta)\left[E(w) + x(1+r) - \eta_r(1+r)\right]$$

Also, the equilibrium savings function is:

$$S^*(x) = x - \eta_r$$

Hence, the threshold of inherited wealth from where the agents start is $\bar{x}^{ICL} = \eta_r$ such that $S^*(\bar{x}^{ICL}) = 0$. Thus, the expected indirect utility and expected bequests for these agents is given by:

$$V_3^{ICL} = (\eta_r)^\alpha + \mu\left[E(w) + x(1+r) - \eta_r(1+r)\right]$$
$$E(b^*) = \delta\left[E(w) + x(1+r) - \eta_r(1+r)\right]$$

Case 4: $B_C > 0$ and $S > 0$ Again, this case is ruled out because $(1+i) > (1+r)$.

6.3.2 Summary

- $x < \underline{\mathrm{x}}^{ICL} : B_C > 0, S = 0$

$$c_1^* = \eta_i$$
$$E(c_2^*) = (1-\delta)\left[E(w) + x(1+i) - \eta_i(1+i)\right]$$
$$V_1^{ICL} = (\eta_i)^\alpha + \mu\left[E(w) + x(1+i) - \eta_i(1+i)\right]$$
$$E(b^*) = \delta\left[E(w) + x(1+i) - \eta_i(1+i)\right]$$

- $x \in \left[\underline{\mathrm{x}}^{ICL}, \bar{x}^{ICL}\right] : B_C = 0, S = 0$

$$c_1^* = x$$
$$E(c_2^*) = (1-\delta)E(w)$$
$$V_2^{ICL} = (x)^\alpha + \mu E(w)$$
$$E(b^*) = \delta E(w)$$

- $x > \bar{x}^{ICL} : B_C = 0, S > 0$

$$c_1^* = \eta_r$$
$$E(c_2^*) = (1-\delta)\left[E(w) + x(1+r) - \eta_r(1+r)\right]$$
$$V_3^{ICL} = (\eta_r)^\alpha + \mu\left[E(w) + x(1+r) - \eta_r(1+r)\right]$$
$$E(b^*) = \delta\left[E(w) + x(1+r) - \eta_r(1+r)\right]$$

where:

- $\underline{x}^{ICL} = \eta_i$
- $\bar{x}^{ICL} = \eta_r$

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