Mass Consumption and Bounded Learning by Doing:

Some Demand-side Implications of Income Distribution for Growth

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Abstract

The stylized facts relevant to the analysis of economic growth traditionally focus on the supply-side of the economy. Little reference is made to mass consumption which has accompanied industrial revolutions, and which today characterizes economic development in large emerging countries. This paper provides an endogenous growth model with bounded learning by doing in each industry, where supply (structural change) and demand (final consumption) interact to bring out a flying-wild-geese development pattern. To that end, we relax the assumption of homothetic preferences that neutralizes demand in the long-run. We discuss the implications of income distribution for growth in a set up where a society of mass consumption arises as a consequence of horizontal demand complementarities and technological spillovers across industries. Inequalities must be neither too high nor too low to let the time to each industry to exhaust its learning potential and benefit the productivity gains associated with it. This yields an inverted-U relationship between inequality and growth. The rate of growth ultimately depending on the size of the middle class which creates the conditions for both mass consumption and increasing productivity.

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1 Introduction

The twenty-first century marks the end of two centuries of hegemony of the Western economies. A decentering of the world has already started from West to East that disrupts the economic world balances. The growth in developing Asian countries has so far found its origin mostly in the growth of factors of production. However, just like China, India with 1,2 billions of people has an internal market without equivalent in the Western world. Combined with the emergence of a large middle class, its double-digit growth makes it a potential area of substantial mass consumption. A recent report from McKinsey Global Institute (2007) suggests that the middle class in India is expected to increase from 5% of the population today to approximately 40% in the next two decades, thus becoming the fifth most important market of the world. Moreover, the share of consumption in the Indian GDP, around 60%, is already close to the levels found in the U.S. and other industrialized nations, being substantially higher than that in other emerging countries, including China.

The study of economic growth traditionally comes in a long-run perspective. Thus, the supply side has so far been given the priority over the demand side. However, this view is not designed to highlight the mechanisms induced by mass consumerism on economic growth. This issue is nearly absent from both neoclassical and new (endogenous) theories of growth. In his introduction of the Handbook of Economic Growth (2005), Robert Solow regrets the lack of interest of the profession for interactions between supply and demand in the medium run. A proposed runway to investigate such interactions is to relax the traditional assumption of homothetic preferences which neutralizes demand on long-run growth; that is, to take into account the impact of household income on the composition of his consumption basket, and consequently, the role of the size of the market on those industries in which an economy will tend to specialize. This orientation builds on the work of Kevin Murphy, Andrei Shleifer & Robert Vishny (1989, henceforth MSV) who have formalized works on demand linkages of early development economists such as Paul Rosenstein-Rodan (1943), Albert Hirschman (1958), and Walt Rostow (1960) for whom a society of mass consumption must be the final stage of economic development of a nation.

It was not until recently that models of growth have relaxed the assumption of homothetic preferences in models of economic growth and structural change. The adoption of a hierarchic structure of preferences where poor consumers devote most of their expenditures toward low income elasticity goods and its impact on economic growth is discussed, for instance, in Zweimüller (2000) and Föllmi & Zweimüller (2006 and 2008). We follow in their footsteps. Our setup allows for hierarchies of needs in consumption and economic growth is an endogenous outcome of the economic system. It differs from them though in two ways. On the one hand, technical progress is not driven by innovations. There is no R&D sector as in Romer (1990) that generates blue prints for new inputs ('new methods to satisfy wants' in Zweimüller 2000) as a result of voluntary profit-motivated horizontal innovations. Instead, Technical progress is a by-product of the economic activity, learning-by-doing being the source of productivity gains. It is assumed to take place only in those industries where demand is high enough so that a firm becomes able to take advantage of internal economies of scale, i.e. to lower unit costs of production, thanks to mass consumption. On the other hand, our modelling involves no saving/investment. The present paper builds on the works of MSV and Matsuyama (2002). First, we follow MSV static setup by introducing pecuniary externalities working via the buying power of a middle class to eventually determine the extent of horizontal complementarity across all industries of the economy. The implementation of increasing returns technologies that are substituted for constant returns technologies is used as a metaphor for structural change. The kind of externalities at work can be described using Matsuyama's words (1995, 703):

> "... Suppose the [middle class] increases its demand for monopolistically competitive goods... Because prices exceed marginal costs, such a shift in demand would increase the level of monopoly profits in the economy and thus national income. This increased income would generate additional demand for monopolistically competitive goods, which further raises profits and income and so on..."

Secondly, this kind of argument which captures how one thing leads to another is central to Matsuyama (2002) who shows what characteristics of the distribution of income can lead to the emergence of an economy of mass consumption. This economic development requires gains in productivity through learning-by-doing which, by lowering prices, gives access to a consumption basket consisting of different goods depending on the household income, and not necessarily to the consumption of a greater quantity of the same goods. Demand spillovers between sectors then lead to a development process that is akin to a flight of wild geese in which the industries take off one after the other via the lower prices resulting from the learning process. With hierarchical preferences which rank goods in order of priority, a greater variety of goods becomes available to households, and the income effect of lower prices leads new industries to develop.

Our model shares with that of Matsuyama growth dynamics that result from learning-by-doing. Recent empirical evidence by Wolff (2011) suggests a strengthening of technological spillover effects in the US economy over the period 1958-2007. One characteristic of our model is that, similarly to Stokey (1988) and besides demand complementarities, there are technological spillovers across sectors that are ruled out in Matsuyama (2002). Our development process is then similar to a flying geese pattern where the increasing returns technology is implemented in industries one after the other. The flying geese model of economic development was first coined by Kaname Akamatsu in the 1930s, and gained popularity in the 1960s. It was initially based on the basis of Japan's experiences in catching up with the West with demand linkages being the driving force of development. Still, note that there is no international linkage in the present model which remains a closed-economy growth model.

In our setup as in Matsuyama's, learning-by-doing is bounded at the industry level. It is therefore also appropriate to put our model in correspondence with the bounded learning model by Alwyn Young (1991). In Young, all goods are not produced on a given date because they would be too expensive to produce. As long as knowledge accumulates in the different industries, the increase of knowledge reduces the labour unit cost of all goods including those goods whose cost was previously prohibitive. There comes a time when these goods can be produced at reasonable cost. In our model, all goods may be available as soon as there exists a consumer for it. It is only in those sectors where demand is strong enough to cover the fixed costs involved in the implementation of the increasing returns technology that the learning takes place. Therefore, it is a consequence of the substantial economic activity caused by mass consumption in these industries. The Indian pharmaceutical industry, in particular in the field of generic drugs, or its telecommunications industry provide examples of the importance of the size of the internal market in the area of apprenticeship, which is itself a source of increased knowledge-based productivity.

The paper is organized as follows. In Section 2, we present our model. Section 3 characterizes the relationship between mass consumption, learning-by-doing spillovers, and the steady-state rate of growth. The last section is devoted to analyzing the conditions with regard to technological progress, which underlie sustainable economic development when mass consumption and not population growth harms the environment.

2 The model

2.1 Households' non-homothetic preferences, wealth, and budget constraint

The preference side is modeled via a utility function which is defined over a continuum of indivisible goods $q \in (0, \infty)$ such that, at each date t,

$$V_t = \int_0^\infty \frac{1}{q} x_t^q dq, \tag{1}$$

where x_t^q as an indicator function which takes in values of either one or zero according to:

$$x_t^q = \begin{cases} 1 \text{ if the agent consumes } q \\ 0 \text{ otherwise} \end{cases}.$$

Thus, a household's utility increases with the range of goods $(0, q)$ it consumes and not with the consumption of a single good q. Consumption is hierarchically structured; that is, needs are ordered so that the proportion of income that households spend on lower-indexed goods or, equivalently, on goods with lower income elasticities of demand, decreases with a household's income. Different goods have different priorities in consumption and richer households can consume more than the bundle of goods available to poorer households (Bertola, Föllmi, and Zweimüller 2006, chapter 12).

Human capital is the only input, and the economy is endowed with an amount $\overline{h}L$, where L is the entire population and $\overline{h}$ is the average level of human capital in the economy, that we normalize to one. Moreover, h_γ denotes a household's human capital endowment which is assumed to be constant over time. It is given by

$$h_\gamma = \gamma \overline{h} L = \gamma L,$$

where γ is the share of human capital of a type-γ household, and $\underline{\gamma} \leq \gamma < \infty$. The total stock of human capital in the economy is distributed according to the cumulative distribution function $G(\gamma)$ which is assumed to be exogenous and constant over time. Therefore, each household is identified by its type γ. At each date t, the labor income of a type-γ household is given by:

$$w_t h_\gamma = w_t \gamma L,$$

where w_t is the wage per unit of human capital.

The nominal income of a type-γ household is defined as

$$Y_t^\gamma = \gamma(w_t L + \Pi_t),$$

where Π_t is the aggregate amount of profits realized by all firms from all industries q in the economy. Profits are redistributed to households up to their type γ.

Define $(0, q_t^\gamma)$ as the set of goods purchased by a type-γ household. The budget constraint which describes the consumption options available to this household with income Y_t^γ can be written as:

$$\int_0^{q_t^\gamma} p_t^q x_t^q dq = \gamma(w_t L + \Pi_t). \tag{2}$$

2.2 Production technology and the equilibrium price

We assume that each good q can be produced with two production functions. The former exhibits constant returns to scale (CRS). One unit of good q requires α/A_t units of human capital, with $\alpha > 1$ and A_t is knowledge-based productivity at time t. The alternative production technology exhibits increasing returns to scale (IRS). Formally, $1/A_t$ units of human capital are required to produce one unit of good q. Nevertheless, in order to produce at such a marginal cost, a firm must also be able to cover a fixed cost equal to F/A_t units of human capital.

On the one hand, each good q may be produced by a competitive fringe of firms with the CRS technology. Then, the free-entry equilibrium number of firms satisfies the zero-profit condition, and the equilibrium price is equal to the average cost; that is,

$$p_t^q = p_t = \alpha w_t / A_t. \tag{3}$$

On the other hand, we show that if the distribution function $G(\gamma)$ is smooth enough which rules out perfect equality, there is a unique Nash equilibrium for a monopoly implementing the IRS technology, which consists in setting the price at the same level of the competitive fringe (see Appendix 1).

2.3 Market demand and the output multiplier

Whenever the demand is high enough to cover the fixed cost, a good q will be produced by a monopolist which will implement the IRS technology. Note that if $L > F/(\alpha - 1)$, a good q is produced using the IRS technology if and only if the demand for this good at time t, denoted D_t^q, is such that the following minimum efficient scale is satisfied:

$$\frac{(\alpha-1)w_t}{A_t}D_t^q - \frac{Fw_t}{A_t} \geq 0 \Leftrightarrow D_t^q \geq \frac{F}{\alpha-1}. \tag{4}$$

At each time t, there is a marginal good q_t^* such that the break-even condition $D^{q_t^*} = F/(\alpha-1)$ holds true. Note that $D^{q_t^*}$ is exogenous and constant over time. Then, industries which produce goods $q \leq q_t^*$, respectively $q > q_t^*$, use the IRS, respectively the CRS, production technology. Following MSV, we define γ_t^* as the share of income held by this marginal household whose purchasing power allows it to exactly purchase the range of goods $(0, q_t^*)$, where

$$q_t^* = \frac{w_t\gamma_t^*(L + \Pi_t/w_t)}{p_t} = \frac{A_t}{\alpha}\gamma_t^*\left(L + \frac{\Pi_t}{w_t}\right), \text{ and } \frac{w_t}{p_t} = \frac{A_t}{\alpha}. \tag{5}$$

We also define the upper class to be the set of households of type greater than γ_t^*. There is an amount N_t^* of such households, where

$$N_t^* = (1 - G(\gamma_t^*))L. \tag{6}$$

Their purchasing power allows them to buy goods produced with the IRS technology as well as goods with higher income elasticity of demand which are produced using the CRS technology. We therefore have the following break-even condition which is time-independent. We thus get rid of the t notation in both variables N^* and γ^*.

$$D^{q_t^*} = N^* = (1 - G(\gamma^*))L = \frac{F}{\alpha - 1}. \tag{7}$$

New models of economic growth along the lines of Romer (1990) emphasize the increase in available varieties of goods as a metaphor of economic growth. Overall, what matters in the new theories of growth is the nature of imperfect competition. Monopolistic competition with its zero-profit condition in equilibrium prevails extensively in new growth theories. Into our model, the variable of interest is not the number of varieties produced in equilibrium, but the number of goods produced with the IRS technology and, as a result, the equilibrium profits generated by industries which are able to implement the IRS technology allowing firms to achieve internal to the firm economies of scale.

Aggregate profits in the economy are the sum of profits realized by those industries which produce goods q in the range $(0, q_t^*)$:

$$\begin{aligned}\Pi_t &= p_t\int_0^{q_t^*} D_t^q dq - \int_0^{q_t^*}\frac{w_t}{A_t}(D_t^q - F)dq \\ &= (\alpha-1)\frac{w_t}{A_t}\int_0^{q_t^*} D_t^q dq - \frac{w_t}{A_t}\int_0^{q_t^*} F dq,\end{aligned}$$

where the demand for each good q_t at time t is given by

$$D_t^q = (1 - G(\gamma_t^q))L, \tag{8}$$

and where $\gamma_t^q = p_t q_t/(w_t L + \Pi_t)$ is the share of income of the poorest household whose purchasing power is high enough to exactly purchase $(0, q_t)$.

Now combining the above profit expression with (5) and (6) yields

$$\begin{aligned}
\frac{\Pi_t}{p_t} &= \frac{A_t}{\alpha w_t}\left((\alpha-1)\frac{w_t}{A_t}\int_0^{q_t^*} D^{q_t} dq - \frac{w_t}{A_t}\int_0^{q_t^*} F dq\right) \\
&= \frac{\alpha-1}{\alpha}\int_0^{q_t^*} D^{q_t} dq - \frac{1}{\alpha}\int_0^{q_t^*} F dq \\
&= \frac{\alpha-1}{\alpha}\frac{(w_t L + \Pi_t)}{p_t} T,
\end{aligned} \tag{9}$$

where $T = L\int_{\underline{\gamma}}^{\gamma^*} \gamma dG(\gamma)$ is defined as the share of income held by those households of type smaller than γ^* whose income is entirely devoted to purchase goods of mass consumption, i.e. in the range $(0, q_t^*)$. From this definition, we deduce:

$$\frac{\Pi_t}{p_t} = \frac{\alpha-1}{\alpha}\frac{1}{(1-\frac{\alpha-1}{\alpha}T)}\frac{w_t L}{p_t},$$

where the multiplier is defined by

$$M = 1/(1 - \frac{\alpha-1}{\alpha}T),$$

which is independent of time.

The average real income per capita (y_t) of the economy is therefore proportional to the multiplier and the knowledge-based productivity at time t. It takes the form

$$y_t = \frac{Y_t}{p_t L} = \frac{w_t L + \Pi_t}{p_t L} = \frac{1}{1-\frac{\alpha-1}{\alpha}T}\frac{A_t}{\alpha}. \tag{10}$$

The higher T, the higher is y_t. In the next section, we specify the rate of growth of A_t so that we become able to study the implications of inequality for growth and patterns of industrialization.

2.4 Accumulated experience at the industry level

It is assumed that learning by doing may occur only in those industries which produce goods that are generated from the IRS technology. At each date t, such learning leads to an accumulation of experience at the industry level denoted by E_t^q. We also assume that this accumulated experience diffuses instantaneously to all other firms, i.e. there are intersectoral spillovers. Accordingly, the level of knowledge-based productivity in the economy A_t is the same in all industries and is equal to the sum of experiences gathered over time by all industries producing goods q and having implemented the IRS technology. More specifically, we have:

$$A_t = \int_0^{q_t^*} E_t^q dq. \tag{11}$$

We follow Thompson (2010) and adopt the following functional form for the experience accumulated in industry q at each date t:

$$E_t^q = \frac{\varepsilon + \lambda \widetilde{E} \displaystyle\int_{t_q^*}^{t} X_v^q dv}{1 + \lambda \displaystyle\int_{t_q^*}^{t} X_v^q dv}, \tag{12}$$

with $\lambda > 0$ that describes the learning rate. We assume that learning by doing in every industry is bounded, the upper bound being defined by $\widetilde{E}$ which is assumed to be exogenous and constant both across industries and over time. Finally, t_q^* denotes the date at which the IRS technology has been adopted for the first time by the industry which produces good q, and X_v^q denotes the level of output achieved in this industry at time v. Thus, like in Melitz (2005), learning-by-doing is bounded in each industry and the limit is represented by the upper bound $\widetilde{E}$. However, it is not bounded at the aggregate level; the learning which occurs in one industry spills over across industries.

First, let us assume $\varepsilon = 0$, i.e. no learning occurred before a good q has been produced with the IRS technology. We thus rewrite

$$E_t^q = \frac{\lambda \widetilde{E} \displaystyle\int_{t_q^*}^{t} X_v^q dv}{1 + \lambda \displaystyle\int_{t_q^*}^{t} X_v^q dv} \Rightarrow \frac{\dot{E}_t^q}{E_t^q} = \frac{X_t^q}{\left(1 + \lambda \displaystyle\int_{t_q^*}^{t} X_v^q dv\right) \displaystyle\int_{t_q^*}^{t} X_v^q dv}.$$

As a result, E_t^q is monotonically increasing and concave with an asymptote equal to $\widetilde{E}$. The above specification of learning reproduces the stylized facts on the empirical functions of learning (Thompson 2010).

Secondly, and for ease of use, we adopt a linear approximation of (12). More specifically, we define the experience accumulated in industry q at time t based on the cumulative level of production since it has adopted the IRS technology. This yields

$$E_t^q = \begin{cases} \lambda \widetilde{E} \displaystyle\int_{t_q^*}^{t} X_v^q dv \text{ if } \displaystyle\int_{t_q^*}^{t} X_v^q dv < 1/\lambda \\ \widetilde{E} \text{ otherwise} \end{cases}. \tag{13}$$

Note that as soon as $\int_{t_q^*}^{t} X_v^q dv \geq 1/\lambda$, learning in industry q has reached the upper bound. Thus, at each time t, among those industries which use the IRS technology, we can distinguish between two groups of industries. The former group includes those industries where learning-by-doing is exhausted and has reached the upper bound $\widetilde{E}$, whereas, in the latter group, we find these industries where the accumulated experience has not reached yet the upper bound. This group of industries lies within the range $(\widetilde{q}_t, q_t^*)$, where $\widetilde{q}_t$ is defined by

$$E^{\widetilde{q}_t} = \widetilde{E} \Leftrightarrow \int_{t^*_{\widetilde{q}_t}}^{t} X_v^{\widetilde{q}_t} dv = \frac{1}{\lambda},$$

and where $t^*_{\widetilde{q}_t}$ denotes the time at which good $\widetilde{q}_t$ was first produced using the IRS technology. In addition to q_t^*, there exists another marginal industry $\widetilde{q}_t$ which is defined as the most recent industry at time t where learning was exhausted. Similarly to the type-γ^* household, there is therefore another key marginal household of type $\widetilde{\gamma}_t$, with $\underline{\gamma} \leq \widetilde{\gamma}_t \leq \gamma^*$, whose purchasing power allows him to buy exactly the range of goods $(0, \widetilde{q}_t)$. Therefore, if households of type γ^* and above constitute the upper income class, mass consumers may now be divided into a low income class which includes households of type between $\underline{\gamma}$ and $\widetilde{\gamma}_t$, and a middle income class where we find households of type ranking from $\widetilde{\gamma}_t$ to γ^*.

3 Knowledge-based productivity growth

Knowledge-based productivity A_t is a function of the experience accumulated by all industries in which learning occurs and already took place. We thus rewrite (11) as

$$A_t = \int_0^{q_t^*} E_t^q dq = \widetilde{E}\widetilde{q}_t + \int_{\widetilde{q}_t}^{q_t^*} E_t^q dq.$$

Changes in A_t are the result of the experience accumulated in the economy at time t; that is,

$$\begin{aligned} \dot{A}_t &= \widetilde{E}\dot{\widetilde{q}}_t + \int_{\widetilde{q}_t}^{q_t^*} \dot{E}_t^q dq + E_t^{q_t^*}\dot{q}_t^* - E_t^{\widetilde{q}_t}\dot{\widetilde{q}}_t \\ &= \int_{\widetilde{q}_t}^{q_t^*} \dot{E}_t^q dq = \lambda\widetilde{E}\int_{\widetilde{q}_t}^{q_t^*} X_t^q dq, \end{aligned}$$

with

$$\dot{E}_t^q = \begin{cases} \lambda\widetilde{E}X_t^q \text{ if } E_t^q < \widetilde{E} \text{ and } \widetilde{q}_t < q \leq q_t^* \\ 0 \text{ otherwise} \end{cases} \Leftrightarrow$$

$$\frac{\dot{E}_t^q}{E_t^q} = \begin{cases} X_t^q / \int_{t_q^*}^{t} X_v^q dv \text{ if } E_t^q < \widetilde{E} \text{ and } \widetilde{q}_t < q \leq q_t^* \\ 0 \text{ otherwise} \end{cases}.$$

The accumulated experience is therefore a by-product of the economic activity in those industries where there is mass consumption. Given that $X_t^q =$

$(1-G(\gamma_t^q))L$, the evolution of demand addressed to industry q is at the origin of its learning curve. This yields

$$g_t = \frac{\dot{A}_t}{A_t} = \lambda\widetilde{E}\int_{\widetilde{q}_t}^{q_t^*}\frac{X_t^q}{A_t}dq = \lambda\widetilde{E}\int_{\widetilde{q}_t}^{q_t^*}\frac{(1-G(\gamma_t^q))L}{A_t}dq. \tag{14}$$

At each time t, the rate of growth equals the amount of human capital (excluding the fixed costs) required to produce the quantity X_t^q in industries using the IRS technology and for which learning takes place.

Whereas q_t^* evolves with time, recall that γ^* is constant over time and, using (2) and (10), is equal to q_t^*/y_tL. It is useful to rewrite (14) to obtain:

$$g_t = \lambda\widetilde{E}\frac{y_tL}{A_t}\int_{\widetilde{\gamma}_t}^{\gamma^*}(1-G(\gamma))Ld\gamma. \tag{15}$$

An integration by parts shows that:

$$\begin{aligned}\int_{\widetilde{\gamma}}^{\gamma^*}(1-G(\gamma))Ld\gamma &= \left(\gamma(1-G(\gamma))|_{\widetilde{\gamma}_t}^{\gamma^*} + \int_{\widetilde{\gamma}_t}^{\gamma^*}\gamma g(\gamma)d\gamma\right)L \\ &= \gamma^*(1-G(\gamma^*))L - \widetilde{\gamma}(1-G(\widetilde{\gamma}_t))L + T - \widetilde{T}_t,\end{aligned}$$

where $\widetilde{T}_t = \displaystyle\int_{\underline{\gamma}}^{\widetilde{\gamma}_t}\gamma g(\gamma)Ld\gamma$. The growth rate of A_t thus becomes:

$$g_t = \lambda\widetilde{E}L\frac{M}{\alpha}\left[\gamma^*N^* + T - \left(\widetilde{\gamma}_t\widetilde{N}_t + \widetilde{T}_t\right)\right], \tag{16}$$

where $M/\alpha = y_t/A_t$. Note that $\widetilde{N}$ cannot exceed L. Therefore, there is a maximum for g_t which is equal to:

$$g_{\max} = \lambda\widetilde{E}L\frac{M}{\alpha}\left[\gamma^*N^* + T - \underline{\gamma}L\right]. \tag{17}$$

In our model, similarly to the static model of MSV without learning-by-doing, γ^*N^*+T is the proportion of total income being spent in those industries having implemented the IRS technology. This income share is time-independent. The rate of growth at time t, therefore, depends positively on both the output multiplier and the proportion of income which is spent in industries where learning-by-doing is not yet exhausted. Put differently, (16) provides us with a relationship between growth and mass consumption, where growth depends on the share of aggregate real income held by those households of type between $\widetilde{\gamma}_t$ and γ^*. (See Figure 1 below which summarizes where expenditures of various consumers go and depicts those industries where learning takes place.) One more step is required to solve for the steady-state rate of growth. In the next section, we identify the marginal household of type $\widetilde{\gamma}_t$ in the steady state.

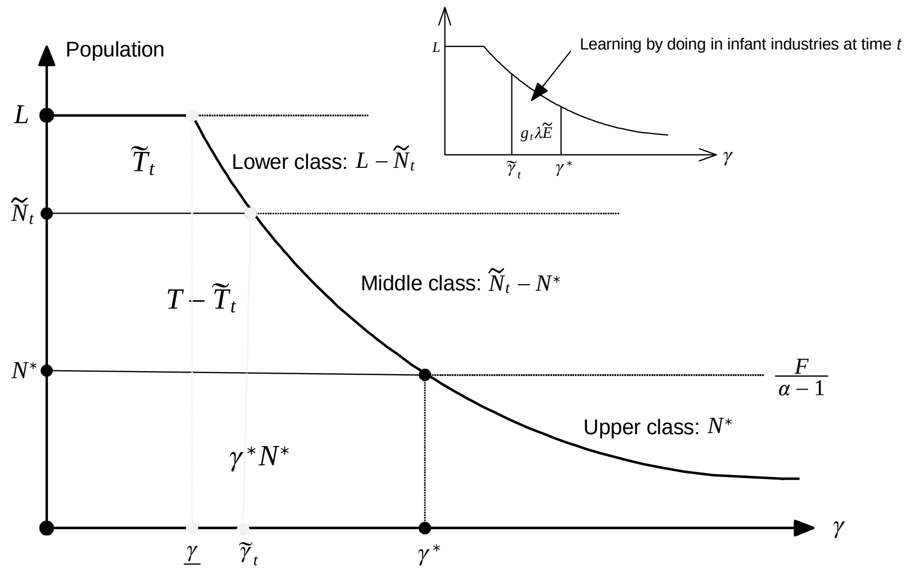

Figure 1: Implications of bounded learning and inequality for growth and structural change.

4 Mass consumption and sustained growth in the long run

4.1 The steady-state growth rate

In the steady state and as long as there is positive long-run growth, an industry q goes through several stages. It begins with producing the good q with the CRS technology. Then, it goes through that stage where it becomes at some point in time the marginal industry q_t^*, i.e. the highest indexed industry at time t using the IRS technology. It produces q in an amount $X_t^{q^*} = N^*$. From then on, it begins to accumulate experience. Its apprenticeship will continue until exhaustion, that is, until it will have reached the upper bound $\widetilde{E}$. At the time t the industry just exhausted all its potential for learning, it becomes the marginal industry $\widetilde{q}_t$ whose good is purchased in quantity $X_t^{\widetilde{q}} = (1 - G(\widetilde{\gamma}_t))L$. One understands here that the experience accumulated by one industry before it reaches $\widetilde{E}$ not only depends on the size and the rate of growth of its market, but also of the time that elapsed between the moment when the industry has adopted technology IRS and when it has exhausted its potential for learning. Let us first note that in the steady state:

$$g = \frac{\dot{A}_t}{A_t} = \frac{\dot{y}_t}{y_t} = \frac{\dot{q}_t^*}{q_t^*} = \text{constant}.$$

Secondly, recall that at time t, an industry q such that $\widetilde{q}_t < q < q_t^*$, is an industry which has substituted the IRS technology for the CRS technology at a time we denoted by t_q^* anterior to time t. In the steady state, there exists a relationship between this industry, indexed by q, and the marginal industry q_t^* which depends on the steady-state rate of growth. Formally, we have:

$$\begin{aligned} q &\equiv q^*_{t^*_q} \text{ and } q^*_t = q^*_{t^*_q} \exp\left(g\left(t - t^*_q\right)\right) \\ &\Rightarrow q^*_t = q \exp\left(g\left(t - t^*_q\right)\right), \ \forall q \end{aligned}$$

$$\Rightarrow q_t^*/\widetilde{q}_t = \exp(g(t - t^*_{\widetilde{q}_t})), \tag{18}$$

where $t^*_{\widetilde{q}_t}$ is the time when the marginal industry $\widetilde{q}_t$ first implemented the IRS technology. Thus, we are able to convert the time interval $(t^*_{\widetilde{q}_t}, t)$ into a range of industries $(\widetilde{q}_t, q_t^*)$ in which learning takes place at time t. The number of industries in which there is learning in the steady state is an increasing function of both the growth rate g and the elapsed time between $t^*_{\widetilde{q}_t}$ and t.

Thirdly, using $\gamma_t^q = q/y_t L$, the rate of growth of demand for a good $q > \underline{q}_t$ is given by:

$$\frac{\dot{X}_t^q}{X_t^q} = \frac{g(\gamma_t^q)\gamma_t^q}{1 - G(\gamma_t^q)} \frac{\dot{y}_t}{y_t}, \tag{19}$$

where $X_t^q = (1 - G(\gamma_t^q))\, L$ and $g(\gamma_t^q)/(1 - G(\gamma_t^q))$ is the number of households of type γ_t^q relative to the number of households whose income is greater thanγ_t^q. Therefore, the income elasticity of demand for a good q is either zero for industries indexed by $q \leq \underline{q}_t$ or equal to $g(\gamma_t^q)\gamma_t^q/\left(1 - G(\gamma_t^q)\right)$ for all $q > \underline{q}_t$. We face here a technical difficulty, the income elasticity of demand for a good $q > \underline{q}_t$ evolves over time and depends on $G(\gamma_t^q)$. At this stage, we choose to specify the distribution of income $G(\gamma)$ and adopt the Pareto distribution which exhibits useful properties as a functional form for income distribution. We specify $G(\gamma) = 1 - (\underline{\gamma}/\gamma)^\beta$ with $\beta > 1$ and $\gamma \geq \underline{\gamma} > 0$. The larger the value of parameter β, the more equal the distribution of income. Note that $\int_{\underline{\gamma}}^\infty \gamma dG(\gamma) = 1/L$ which implies $\underline{\gamma} = (\beta - 1)/\beta L$, and that $\beta = g(\gamma_t^q)\gamma_t^q/\left(1 - G(\gamma_t^q)\right)$. We therefore have:

$$\frac{\dot{X}_t^q}{X_t^q} = g\beta, \tag{20}$$

which is constant both over time and across industries $q > \underline{q}_t$[1]. When β increases, i.e. inequalities decrease, the growth rate of demand for a good q

[1] Note that in the Pareto case, we have:

$$\gamma^* N^* = \gamma^* \left(\frac{\beta - 1}{\beta L \gamma^*}\right)^\beta Land T = 1 - \left(\frac{\beta - 1}{\beta L \gamma^*}\right)^{\beta - 1}.$$

One interesting property of the Pareto distribution in our framework is that:

$$\frac{\beta - 1}{\beta} = \frac{\gamma^* N^*}{1 - T} \Rightarrow \beta = \frac{1 - T}{1 - (\gamma^* N^* + T)},$$

increases more than proportionally with the rate of income growth. Indeed, the higher β, the higher are $\underline{\gamma}$ and $\underline{q}_t$. Each extra unit of income then carries over a smaller number of industries, leading each of them to grow at a higher rate.

On the one hand, using (2) and (10), we have:

$$\gamma_t^q = \begin{cases} q/y_t L \text{ for } q > \underline{q}_t \\ \underline{\gamma} \text{ otherwise} \end{cases},$$

where $(0, \underline{q}_t)$ is the range of goods that all households are able to purchase at time t.

Thus, in the particular case of $G(\gamma)$ being a Pareto distribution, the level of output in industry q at time t is:

$$X_t^q = (1 - G(\gamma_t^q))L = \begin{cases} \left(\frac{(\beta-1)y_t}{\beta q}\right)^\beta L \text{ for } q > \underline{q}_t \\ L \text{ otherwise} \end{cases}.$$

In the steady state $y_t = y_{t_q^*} \exp(g(t - t_q^*))$ which yields:

$$X_t^q = \begin{cases} \left(\frac{(\beta-1)L}{\beta q L}\right)^\beta \left(y_{t_q^*} \exp(g(t - t_q^*))\right)^\beta L \text{ for } \underline{q}_t < q < q_t^*, \\ L \text{ otherwise} \end{cases}.$$

On the other hand, as long as experience accumulated in industry q has not reached the upper bound $\widetilde{E}$, i.e., $\int_{t_q^*}^t X_v^q dv < 1/\lambda$, we can write:

$$\frac{E_t^q}{\widetilde{E}} = \lambda \int_{t_q^*}^t X_v^q dv = \lambda X^{q_{t_q^*}} \frac{1}{g\beta} \left(\exp(g\beta(t - t_q^*)) - 1\right), \tag{21}$$

where $X^{q_{t_q^*}}$ is the demand addressed to industry q when it has first implemented the IRS technology, i.e. N^*. We now make use of (18) in the Pareto case:

$$q_t^* = q \exp(g(t - t_q^*)) \Rightarrow \exp(g\beta(t - t_q^*)) = \left(\frac{q_t^*}{q}\right)^\beta.$$

Replacing in (21) and using $q = \gamma_t^q y_t L$, we obtain:

$$\frac{E_t^q}{\widetilde{E}} = \frac{\lambda}{g\beta} \left(X_t^q - N^*\right).$$

First, the experience accumulated in industry q at time t is proportional to the difference between the demand for q and the demand for q_t^*. Secondly, $E_t^q/\widetilde{E}$

where $\gamma^* N^*/(1-T)$ is the share of income spent by the upper class in goods produced with the IRS technology relative to their income share in the economy. This property being valid $\forall \gamma$, we also have:

$$\frac{\beta - 1}{\beta} = \frac{\widetilde{\gamma}\widetilde{N}}{1 - \widetilde{T}} \Rightarrow \beta = \frac{1 - \widetilde{T}}{1 - (\widetilde{\gamma}\widetilde{N} + \widetilde{T})}.$$

depends positively on the learning rate. Thirdly, using (20), the experience accumulated in industry q at time t is negatively related to the rate of growth of demand for good q at time t. On the one hand, low inequality causes a high rate of growth of demand. On the other hand, the lower inequality the less time an industry has to accumulate experience before reaching a given level of production X_t^q. Indeed, we have

$$\begin{aligned} X_t^q &= N^* \exp(g\beta(t - t_q^*)) \\ &\Rightarrow \;\; t - t_q^* = \frac{1}{g\beta} \ln\left(\frac{X_t^q}{X^{q_t^*}}\right) \\ &\Rightarrow \;\; t - t_q^* \simeq \frac{1}{g\beta}\left(\frac{X_t^q}{X^{q_t^*}} - 1\right) \\ &\Rightarrow \;\; t - t_q^* \simeq \frac{1}{g\beta}\left(\frac{X_t^q}{N^*} - 1\right). \end{aligned}$$

Let us consider similar amounts of good q produced at time t in two economies that differ only in $g\beta$, i.e. $\dot{X}_t^q / X_t^q$. The above equality holds only if the economy with the higher $g\beta$ exhibits a lower t_q^*. In other words, the more equal economy had more time to learn since good q has been produced with the IRS technology, i.e. over the period (t_q^*, t).

Whereas $\gamma^* N^* + T$ is exogenous depending only on the distribution of income, $\widetilde{\gamma}\widetilde{N}$ and $\widetilde{T}$ are determined simultaneously with the rate of growth g[2]. Let us use the outcome of the accumulated experience in the last sector for which learning has been exhausted. We have:

$$\widetilde{E} = \frac{\lambda \widetilde{E}}{g\beta}\left(G(\gamma^*) - G(\widetilde{\gamma})\right) L \Rightarrow$$

$$\widetilde{N} = \frac{g\beta}{\lambda} + N^*. \tag{22}$$

An increase in g, ceteris paribus, is associated with an increase in $\widetilde{N}$ and therefore in $\widetilde{N} - N^*$, i.e. of the number of mass consumers who purchase goods for which learning-by-doing takes place at time t and is not yet exhausted. Now, using the break-even condition (7) in (16) and (22), we obtain the following system which allows us to determine the steady state rate of growth g and $\widetilde{\gamma}\widetilde{N}$ (and $\widetilde{T}$):

$$\begin{cases} g = \frac{\lambda}{\beta} \frac{\widetilde{E}}{1+(\alpha-1)(1-T)} (T - \widetilde{T}) L \\ g = \frac{\lambda}{\beta}\left(\left(1 - \frac{N^*}{L}\right) - \left(1 - \frac{\widetilde{N}}{L}\right)\right) L \end{cases} . \tag{23}$$

The ratio of these two equations gives:

$$\frac{L}{\widetilde{E}} = \frac{T - \widetilde{T}}{\widetilde{N} - N^*} \frac{y_t L}{A_t},$$

[2] As g is constant in the steady state, it is also true for $\widetilde{N}$ and $\widetilde{T}$. Henceforth, we get rid of the t notation in these variables.

where the right-hand side term of this equation is the average income of this class of households whose type lies in the range $(\widetilde{\gamma}, \gamma^*)$.

In Figure 2, we depict one possible solution of (23). The northeast quadrant of Figure 2 depicts the Lorenz curve which describes the level of inequality in the economy. On the x-axis, one finds the cumulative share of households from lowest to highest incomes and the cumulative share of aggregate income is plotted on the y-axis. Recall that $\widetilde{N} - N^*$ represent the size of the middle class whose demand is responsible for the experience accumulated in the sectors $(\widetilde{q}_t, q_t^*)$, and thereby growth in equilibrium. The northwest quadrant is the depiction of the first equation of (23) where the long-run rate of growth is an increasing function of both the multiplier M and the income share held by households whose type lies in the range $(\widetilde{\gamma}, \gamma^*)$, i.e. $T - \widetilde{T}$. In the southeast quadrant, we depict the second equation of (23). Using the 45°-line in the southwest quadrant allows us to determine the steady state rate of growth. Permanent changes in the level of inequality (β) may be represented by shifts of the Lorenz curve which will have effects on the steady state rate of growth.

4.2 Inequality and sustained growth in the long run

However, for there to be a positive growth in the long run requires that certain conditions be met. Let us rewrite $\widetilde{N}$ as $\min(N^* + g\beta/\lambda, L)$ into the first equation of (23), we obtain:

$$\Rightarrow g = \frac{1}{\beta}\left(\left(\frac{\min(\frac{g\beta}{\lambda} + N^*, L)}{L}\right)^{(\beta-1)/\beta} - \left(\frac{N^*}{L}\right)^{(\beta-1)/\beta}\right)\frac{\widetilde{E}\lambda L}{1 + (\alpha - 1)\left(\frac{N^*}{L}\right)^{(\beta-1)/\beta}}. \tag{24}$$

Let us perform the following change in variables: $n^* = N^*/L$ and $b = (\beta - 1)/\beta$. Keep in mind that with the Pareto distribution, $(\beta - 1)/\beta$ is equal to $\gamma^* N^*/(1 - T)$. The higher the ratio, the more the society can be regarded as egalitarian. Equation (24) can be written as:

$$\begin{aligned} & F\left(g, b, \widetilde{E}\right) \\ = \; & \widetilde{E}\left(\left(\begin{array}{c} \min\left(g, (1-n^*)(1-b)\lambda L\right) \\ +n^*\left(1-b\right)\lambda L \end{array}\right)^b \begin{array}{c} \\ \\ -\left(n^*\left(1-b\right)\lambda L\right)^b \end{array}\right)(1-b)\lambda L - g\left(\left(1-b\right)\lambda L\right)^b\left(1+(\alpha-1)\left(n^*\right)^b\right) \\ = \; & 0, \end{aligned}$$

where

$$\frac{g}{(1-b)\lambda L} + n_t^* \leq 1 \Leftrightarrow g \leq \left(1 - n_t^*\right)(1-b)\lambda L.$$

Thus, for $g = \left(1 - n_t^*\right)(1-b)\lambda L$, we have

$$F\left(\left(1 - n_t^*\right)(1-b)\lambda L\right) = (1-b)^{1+b}\lambda L^{1+b}\left(\widetilde{E}(1 - (n^*)^b) - \left(\left(1-n_t^*\right) + (\alpha-1)\left(n^*\right)^b\right)\right)$$

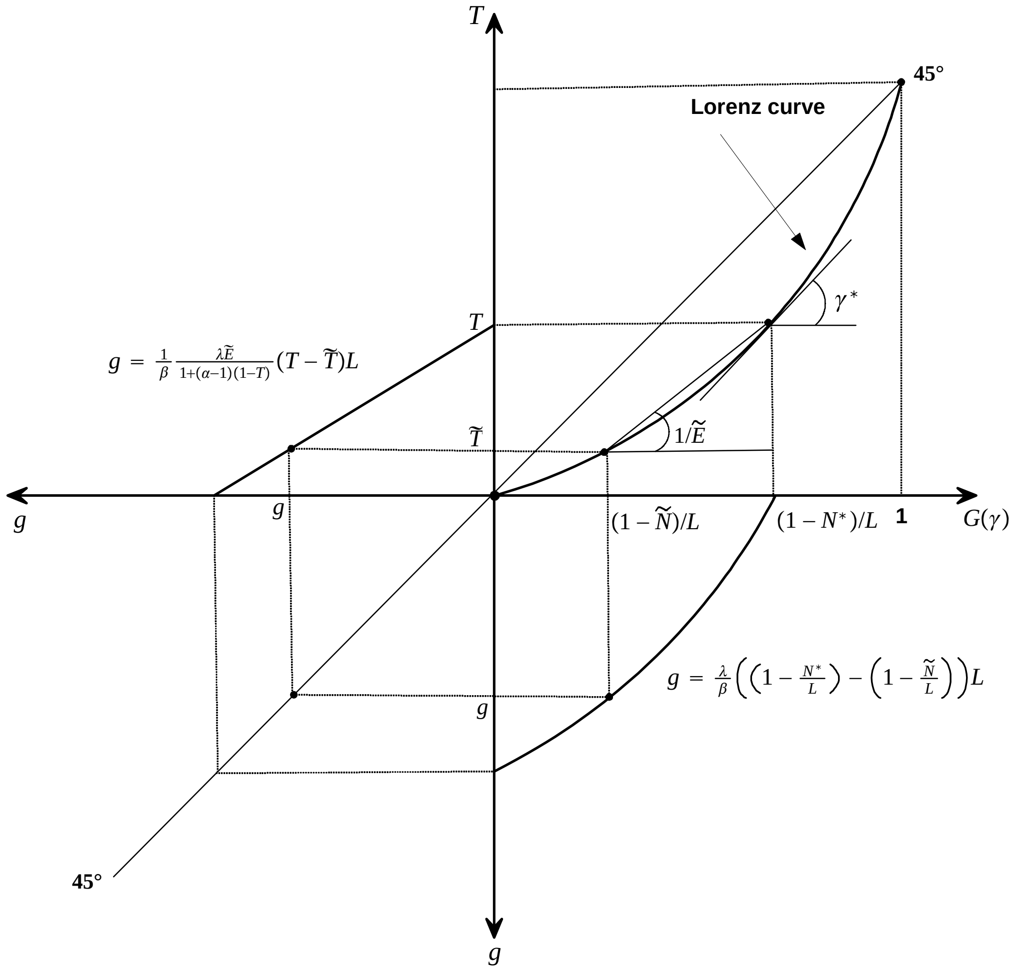

Figure 2: Inequality and the rate of growth in the steady state.

$$\Rightarrow F((1-n_t^*)(1-b)\lambda L) \leq 0 \text{ iff } \widetilde{E} \leq \frac{(1-n_t^*)1+(\alpha-1)(n^*)^b}{1-(n^*)^b}.$$

Moreover,

$$\begin{aligned} F(0,b,\widetilde{E}) &= 0, \\ F_g(g,b,\widetilde{E}) &= \widetilde{E}b\left((g+n^*(1-b)\lambda L)^{b-1}\right)(1-b)\lambda L - ((1-b)\lambda L)^b\left(1+(\alpha-1)(n^*)^b\right), \\ F_{gg}(g,b,\widetilde{E}) &< 0, \forall \beta > 1. \end{aligned}$$

As long as $g \leq (1-n_t^*)(1-b)\lambda L$, for the rate of growth to be strictly positive, we therefore need the following condition to be satisfied:

$$F_g(0,b,\widetilde{E}) = \widetilde{E}b(n^*)^{b-1}((1-b)\lambda L)^b - ((1-b)\lambda L)^b\left(1+(\alpha-1)(n^*)^b\right) > 0 \Leftrightarrow$$

$$\widetilde{E}b(n^*)^{b-1} - 1 - (\alpha-1)(n^*)^b > 0 \Leftrightarrow$$

$$\Psi\left(b,\widetilde{E}\right) = b\widetilde{E} - (n^*)^{1-b} - (\alpha-1)n^* = b\widetilde{E} - \left(\left(\frac{F}{(\alpha-1)L}\right)^{1-b} + \frac{F}{L}\right)$$

First, let us define $H\left(b,\widetilde{E}\right)$ such that $H\left(b,\widetilde{E}\right) = b\widetilde{E}$ with $H\left(0,\widetilde{E}\right) = 0$, $H\left(1,\widetilde{E}\right) = \widetilde{E}$, and $H_b\left(b,\widetilde{E}\right) = \widetilde{E}$. Secondly, we also define $\Gamma(b)$ such that $\Gamma(b) = (n^*)^{1-b} + (\alpha-1)n^*$, with $\Gamma(0) = \alpha n^*$, $\Gamma(1) = 1+(\alpha-1)n^* = 1+F/L$, $\Gamma'(b) = -(n^*)^{1-b}\ln(n^*) > 0$, and $\Gamma''(b) = (n^*)^{1-b}(\ln(n^*))^2 > 0$.

Then, we have

$$\Psi(0,\widetilde{E}) = -\left(1+\frac{1}{\alpha-1}\right)\frac{F}{L} = -\frac{\alpha}{\alpha-1}\frac{F}{L}$$

$$\Psi(1,\widetilde{E}) = \widetilde{E} - 1 - \frac{F}{L} \gtreqless 0 \Leftrightarrow \widetilde{E} \gtreqless 1 + \frac{F}{L}$$

$$\begin{aligned} \Psi_b(b,\widetilde{E}) &= \widetilde{E} - \left(\frac{(\alpha-1)L}{F}\right)^{b-1}\ln\left(\frac{(\alpha-1)L}{F}\right) \gtreqless 0 \\ &\Leftrightarrow \widetilde{E} \gtreqless \left(\frac{(\alpha-1)L}{F}\right)^{b-1}\ln\left(\frac{(\alpha-1)L}{F}\right) \end{aligned}$$

$$\Psi_{bb}(b,\widetilde{E}) = -\left(\frac{(\alpha-1)L}{F}\right)^{b-1}\left(\ln\left(\frac{(\alpha-1)L}{F}\right)\right)^2 < 0$$

In a perfectly egalitarian allocation of human capital, i.e. $\beta \to \infty \Rightarrow b \to 1$, we define $\widetilde{E}^{**}$ such that

$$\begin{aligned} F'(1) &= 0 \Rightarrow \Psi(1) = 0 \Rightarrow \\ H\left(1,\widetilde{E}\right) &= \Gamma(1) \Leftrightarrow \widetilde{E}^{**} = 1+(\alpha-1)n^* = 1+\frac{F}{L}. \end{aligned}$$

Because $\Psi_{bb}(b,\widetilde{E})<0$, $\Psi_b(b,\widetilde{E})$ will be negative for all b, when

$$\begin{aligned}\Psi_b(0,\widetilde{E}) &= E-\left(\frac{(\alpha-1)\,L}{F}\right)^{-1}\ln\left(\frac{(\alpha-1)\,L}{F}\right)<0\Leftrightarrow\\ \widetilde{E} &< \left(\frac{(\alpha-1)\,L}{F}\right)^{-1}\ln\left(\frac{(\alpha-1)\,L}{F}\right).\end{aligned}$$

We can also deduce that when

$$\Psi_b(0,\widetilde{E})>0,$$

and

$$\Psi(1,\widetilde{E})=\widetilde{E}-1-\frac{F}{L}>0\Leftrightarrow\widetilde{E}>1+\frac{F}{L},$$

there is a b^c such that $b\le b^c\Rightarrow g=0$ and $b>b^c\Rightarrow g>0$.

On the other hand, the concavity of the function Ψ implies that if $\Psi_b(0,E)>0$ and

$$\begin{aligned}\Psi_b(1,\widetilde{E}) &= H_b\left(1,\widetilde{E}\right)-\Gamma'\left(1\right)=\\ &= \widetilde{E}-\ln\left(\frac{(\alpha-1)\,L}{F}\right)>0,\end{aligned}$$

then the function is increasing in the interval $(0,1)$ and the maximum of $\Psi(b,E)$ is on the boundary of this interval. However, when

$$\widetilde{E}<\ln\left(\frac{(\alpha-1)\,L}{F}\right),$$

there is an internal maximum ($b^{\widetilde{E}}$), that is defined by the following equation

$$\begin{aligned}\Psi_b\left(b^{\widetilde{E}},\widetilde{E}\right) &= 0\Longleftrightarrow H_b\left(b^{\widetilde{E}},\widetilde{E}\right)=\Gamma'\left(b^{\widetilde{E}}\right)\\ \Leftrightarrow &\ \ \widetilde{E}=\left(\frac{1}{n^*}\right)^{b^{\widetilde{E}}-1}\ln(1/n^*).\end{aligned}$$

As a consequence, if the function $\Psi\left(b,\widetilde{E}\right)$ evaluated at point $\left(b^{\widetilde{E}},\widetilde{E}\right)$ is negative, it will be negative in the interval $b\in(0,1)$ and $F_g(0,b,\widetilde{E})<0$. On the other hand, if

$$\Psi\left(b^{\widetilde{E}},\widetilde{E}\right)>0\Leftrightarrow H\left(b^{\widetilde{E}}\right)>\Gamma\left(b^{\widetilde{E}}\right),$$

and at the same time $\widetilde{E}<\ln\left(\frac{(\alpha-1)L}{F}\right)$ and $\widetilde{E}<\widetilde{E}^{**}$ there are an interval $b^{c1}<b<b^{c2}$, such that

$$\begin{aligned}\Psi\left(b^{\widetilde{E}},\widetilde{E}\right) &> 0,\ \ \forall b\in\left[b^{c1},b^{c2}\right],\\ \Psi\left(b^{\widetilde{E}},\widetilde{E}\right) &= 0,\ \ \forall b\in\left(0,b^{c1}\right)\cup\left(b^{c2},1\right)\end{aligned}$$

In this case, there exist a level $\widetilde{E}^*$ such that

$$\begin{cases} \Psi_b\left(b^m, \widetilde{E}^*\right) = 0 \Longleftrightarrow H_b\left(b^m, \widetilde{E}^*\right) = \Gamma'(b^m) \\ \Psi\left(b^m, \widetilde{E}^*\right) = 0 \Longleftrightarrow H\left(b^m, \widetilde{E}^*\right) = \Gamma(b^m) \\ b^m < 1 \end{cases} \Leftrightarrow$$

$$\begin{cases} \left(\frac{1}{n^*}\right)^{b^m-1} \frac{\ln(1/n^*)}{\widetilde{E}^*} = 1 \\ \frac{1}{n^*} = \frac{(1/n^*)^{b^m}}{b^m \widetilde{E}^*} + \frac{(\alpha-1)}{b^m \widetilde{E}^*} \\ b^m < 1 \end{cases}.$$

Let us rewrite the first equation in the following way,

$$\left(\frac{1}{n^*}\right)^{b^m} = \frac{(1/n^*)\,\widetilde{E}^*}{\ln(1/n^*)},$$

and replace it in the second equation

$$\Rightarrow b^m = \frac{1}{\ln(1/n^*)} + \frac{(\alpha-1)}{\widetilde{E}^* \frac{1}{n^*}}.$$

we thus obtain the threshold value $\widetilde{E}^*$ such that:

$$\left(\frac{1}{n^*}\right)^{\left(\frac{1}{\ln(1/n^*)} + \frac{(\alpha-1)}{E^* \frac{1}{n^*}}\right)} \frac{\ln(1/n^*)}{(1/n^*)\,\widetilde{E}^*} - 1 = 0,$$

$$b^m = \frac{1}{\ln(1/n^*)} + \frac{(\alpha-1)}{\widetilde{E}^* \frac{1}{n^*}} < 1$$

But the condition $b^m < 1$ is compatible with $\widetilde{E}^* \leq \widetilde{E}^{**}$, if and only if

$$\begin{aligned} \Psi_b\left(1, \widetilde{E}^{**}\right) &< 0 \Leftrightarrow H_b\left(1, \widetilde{E}^{**}\right) - \Gamma'(1) < 0 \\ &\Leftrightarrow \widetilde{E}^{**} - \ln\left(\frac{(\alpha-1)\,L}{F}\right) < 0 \\ &\Leftrightarrow 1 + \frac{F}{L} < \ln\left(\frac{(\alpha-1)\,L}{F}\right) \end{aligned}$$

Thus, as long as the below inequality holds, three cases may arise:

$$1 + \frac{F}{L} < \ln\left(\frac{(\alpha-1)\,L}{F}\right),$$

1. Case #1

$$\widetilde{E} < \widetilde{E}^* \Rightarrow g = 0, \forall \beta > 1.$$

2. Case #2

$$\begin{aligned} \widetilde{E}^{*} &< \widetilde{E} \leq \widetilde{E}^{**}, \ni 1 < \beta^{c1} < \beta^{c2} \text{ such that} \\ \text{i) if } \beta &\leq \beta^{c1} \Rightarrow g = 0; \\ \text{ii) if } \beta^{c1} &< \beta < \beta^{c2} \Rightarrow g > 0; \\ \text{iii) if } \beta &\geq \beta^{c2} \Rightarrow g = 0. \end{aligned}$$

3. Case #3

$$\begin{aligned} \widetilde{E} &> \widetilde{E}^{**}, \ \ni \beta^{c} > 1 \text{ such that} \\ \text{i) if } \beta &\leq \beta^{c} \Rightarrow g = 0; \\ \text{ii) if } \beta &> \beta^{c} \Rightarrow g > 0. \end{aligned}$$

Otherwise, if $1 + F/L > \ln((\alpha - 1) L/F)$, only the cases #1 and #3 may arise.

As summarized in Figure 3, in Case #1, whatever the degree of equality, the potential for learning relatively to the size of the population (L), fixed costs (F), and to the marginal cost in competitive industries (α), is too weak to yield positive growth in the long-run. In Case #3 instead, as soon as inequality is not too high, the learning potential in those industries that implement the IRS technology is substantial enough to generate a positive steady state rate of growth. Case #2 is more sensitive since inequality must neither be too low nor too high so that there is sustained growth.

5 Mass consumption, output multiplier, and sustainable economic development

Let us start with an analysis of Fred Pearce[3], freelance journalist in England and entitled, "Consumption dwarfs population as main environmental threat":

> "Take carbon dioxide emissions - a measure of our impact on climate, [...] Stephen Pacala, director of the Princeton Environment Institute, calculates the the world's richest half-billion people -that's about 7 percent of the global population- are responsible for 50 percent of the world's carbon dioxide emissions. Meanwhile the poorest 50 percent are responsible for 7 percent of emissions... For a wider perspective of humanity's effects on the planet's life support systems, the best available measure is the "ecological footprint", which estimates the area of land required to provide each of us with food clothing, and other resources, as well as to soak up our pollution... They show that sustaining the lifestyle of the average American takes 9.5 hectares, ... and the Japanese, 4.9. The world average is 2.7

[3] http://e360.yale.edu/content/feature.msp?id=2140

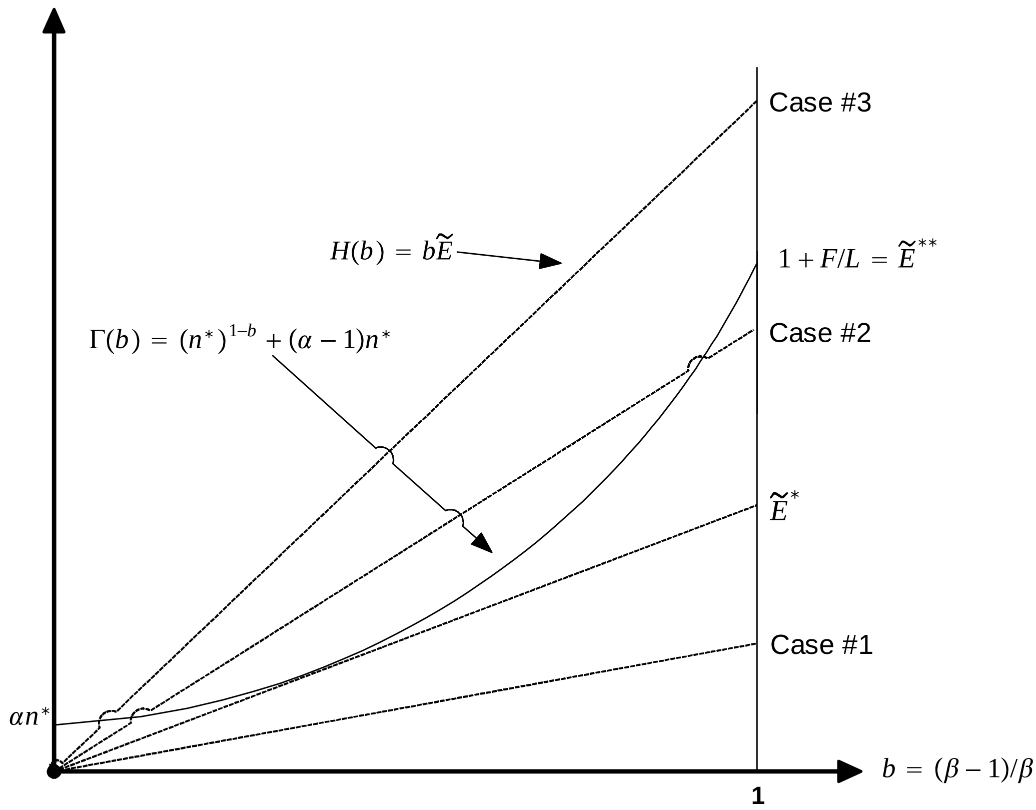

Figure 3: Income distribution and positive steady-state growth rates.

hectares. China is still below that figure at 2.1, while India and most of Africa are at or below 1.0... The carbon emissions of one American today are equivalent to those of four Chinese, 20 Indians, or 250 Ethiopians."

Following the example of Brock & Taylor (2010), it is assumed that the total emission of pollutants at time t by all industries is given by:

$$\epsilon_t = e(A_t)\int_0^{\infty} X_t^q dq = e(A_t)y_t L,$$

with $e(A_t)$, the emission of pollutants per unit of output, and $e_{A_t}(A_t) < 0$. The pace of change in the stock of pollution (P_t) is described by:

$$\dot{P}_t = \epsilon_t - \theta P_t,$$

and θ a parameter of regeneration of the environment which reflects a mechanism opposite to that of depreciation). On the one hand, as the economic activity increases, we observe a worsening of the quality of the environment. On the other hand, the higher the stock of knowledge, the more we are able to save the quality of the environment per unit of goods produced.

Furthermore, we assume that $\overline{P}$ is an upper limit that pollution shall not exceed without producing an environmental disaster that would be irreversible. Therefore, we must have:

$$0 \leq P_t \leq \overline{P},\ \forall t.$$

In the steady state, we have:

$$\dot{P}_t = 0 \Rightarrow P_t^* = \frac{\epsilon_t}{\theta} \text{ et } \frac{\dot{y}_t}{y_t} = -e_{A_t}(A_t)\frac{A_t}{e(A_t)}\frac{\dot{A}_t}{A_t},$$

and (see above),

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{A}_t}{A_t} \Rightarrow -\frac{e_{A_t}(A_t)A_t}{e(A_t)} = -1 \Rightarrow e(A_t) = \frac{\xi}{A_t}, \text{ avec } \xi > 0.$$

Thus, to the question, is a sustainable environment compatible with economic growth in a society of mass consumption? The answer is yes, but the steady state is contingent on neutralizing the effect of the increased production ("scale effect") created by mass consumerism on the quality of the environment, by technical progress ("technical effect ") source of more efficient and cleaner technologies per unit of output. Improving the environment occurs here as a by-product of technological progress.

Moreover, if we want to avoid an environmental catastrophe, the following condition must be met:

$$P_t^* = \frac{e(A_t)y_t L}{\theta} = \frac{\xi}{\theta}\frac{y_t L}{A_t} = \frac{\xi}{\theta}\frac{M}{\alpha}L \leq \overline{P}.$$

In other words, pollution is a nondecreasing function of the population size and of the multiplier. Despite the positive impact of mass consumption on the rate of long-term growth, too many mass consumers may produce powerful enough demand spillover effects so that the economy may experience an ecological disaster while being on a balanced growth path, i.e. such that $\dot{P}_t = 0$.

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7 Appendix 1: Price equilibrium

Proof. On the one hand, at time t, a monopolist entering the market for a particular good q cannot set a price higher than the competitive price without giving way to a competitive fringe of firms. On the other hand, could he seriously consider to increase its profits by lowering its price unilaterally below $p_t = \alpha w_t / A_t$, i.e., while all other firms keep their price unchanged? The answer is no, as long as the marginal profit satisfies the following condition:

$$\frac{\partial \pi_t^q}{\partial \widehat{p}_t^q} = \frac{\partial \widehat{D}_t^q}{\partial \widehat{p}_t^q}\left(\widehat{p}_t^q - \frac{w_t}{A_t}\right) + \widehat{D}_t^q > 0 \Leftrightarrow -\frac{\partial \widehat{D}_t^q}{\partial \widehat{p}_t^q}\frac{\widehat{p}_t^q}{\widehat{D}_t^q}\left(\frac{\widehat{p}_t^q - w_t/A_t}{\widehat{p}_t^q}\right) < 1, \quad (25)$$

with $\widehat{p}_t^q \leq p_t$, and where $\widehat{D}_t^q$ is the effective demand for good q produced at $\widehat{p}_t^q$. In other words, the price elasticity of demand multiplied by the price-cost margin should not exceed unity.

Let us define $\widehat{q} \leq q$ such that

$$V_t = \int_0^\infty \frac{1}{q} x_t^q dq, \quad (26)$$

$$\frac{1}{q}\frac{1}{\widehat{p}_t^q} = \frac{1}{\widehat{q}}\frac{1}{p_t} \Rightarrow \widehat{q} = \frac{\widehat{p}_t^q}{p_t} q.$$

Among the first category of households, customers for the variety of good q include all those which are rich enough to buy $\widehat{q}$, i.e., households of type $\gamma \geq \gamma_t^{\widehat{q}}$, with

$$\gamma_t^{\widehat{q}}\left(w_t \overline{h}_t L_t + \pi_t\right) =$$

$$\gamma_t^{\widehat{q}} = \frac{p_t \widehat{q}}{w_t \overline{h}_t L_t + \pi_t} = \frac{\widehat{p}_t^q q}{w_t \overline{h}_t L_t + \pi_t},.$$

Therefore, the effective demand for good q produced at price $\widehat{p}_t^q$, is

$$\widehat{D}_t^q = (1 - G_j(\gamma_t^{\widehat{q}}))L.$$

Let $g(\gamma)$ be the density of type-γ households and $\beta(\gamma) = g(\gamma)\gamma/(1 - G(\gamma))$. The price elasticity of demand for good q can be written as follows

$$\begin{aligned} -\frac{\partial \widehat{D}_t^q}{\partial \widehat{p}_t^q}\frac{\widehat{p}_t^q}{\widehat{D}_t^q} &= \frac{g(\gamma_t^{\widehat{q}})\widehat{p}_t^q q L}{\left(w_t \overline{h}_t L_t + \pi_t\right)(1 - G(\gamma_t^{\widehat{q}}))L} \\ &= \frac{\gamma_t^{\widehat{q}} g(\gamma_t^{\widehat{q}})}{(1 - G(\gamma_t^{\widehat{q}}))} \\ &= \beta(\gamma_t^{\widehat{q}}). \end{aligned} \quad (27)$$

First, we can show that using (25) and (27), we have:

$$-\frac{\partial \widehat{D}_t^q}{\partial \widehat{p}_t^q}\frac{\widehat{p}_t^q}{\widehat{D}_t^q}\left(\frac{\widehat{p}_t^q - w_t/A_t}{\widehat{p}_t^q}\right) = \beta\left(\gamma_t^{\widehat{q}}\right)\left(\frac{\widehat{p}_t^q - w_t/A_t}{\widehat{p}_t^q}\right) < \beta\left(\gamma_t^{\widehat{q}}\right)\left(\frac{p_t - w_t/A_t}{p_t^q}\right) = \beta\left(\gamma_t^{\widehat{q}}\right)\left(\frac{\alpha - 1}{\alpha}\right).$$

Therefore the following inequality provides a sufficient condition for ruling out price-cutting equilibria:

$$\beta\left(\gamma_t^{\widehat{q}}\right)\left(\frac{\alpha-1}{\alpha}\right)<1 \Rightarrow -\frac{\partial \widehat{D}_t^q}{\partial \widehat{p}_t^q}\frac{\widehat{p}_t^q}{\widehat{D}_t^q}\left(\frac{\widehat{p}_t^q-w_t/A_t}{\widehat{p}_t^q}\right)<1 \tag{28}$$

Indeed, as long as (25) is satisfied, when a firm with access to IRS technology in industry q aims to cut the price below $\alpha w_t/A_t$, it is not able to expand its customer base to such an extent as to compensate the loss in the rate of profit per customer, thus discouraging price-cutting. In our framework, such condition results in (28). The income distribution should not degenerate around any type-γ.

Let us define $G(\gamma)$ to be the Pareto distribution which has the following useful properties: (i) $\beta(\gamma)=\beta\ \forall\gamma$, and (ii) $\underline{\gamma}=(\beta-1)/(\beta L)$. Then, the condition becomes

$$\beta\left(\frac{\alpha-1}{\alpha}\right)<1 \Longleftrightarrow \beta<\frac{\alpha}{\alpha-1} \Longleftrightarrow \frac{\beta-1}{\beta}<\frac{1}{\alpha} \Longleftrightarrow b<\frac{1}{\alpha}. \tag{29}$$

where, in the text, we have defined $b=(\beta-1)/\beta$

On the other hand, when $\beta(\alpha-1)/\alpha>1 \Leftrightarrow b>1/\alpha$, the price equilibrium is equal to

$$-\frac{\partial \widehat{D}_t^q}{\partial \widehat{p}_t^q}\frac{\widehat{p}_t^q}{\widehat{D}_t^q}\left(\frac{\widehat{p}_t^q-w_t/A_t}{\widehat{p}_t^q}\right)=1 \Rightarrow \beta\left(\frac{\widehat{p}_t^q-w_t/A_t}{\widehat{p}_t^q}\right)=1 \Rightarrow$$

$$\widetilde{p}_t^q=\frac{\beta}{\beta-1}w_t/A_t=\frac{1}{b}w_t/A_t<p_t=\alpha w_t/A_t.$$

In the case of a Pareto distribution, the Gini coefficient is equal to $1/(2\beta-1)$. Thus, the inequality $(\beta-1)/\beta>1/\alpha$ yields

$$Gini<\frac{\alpha-1}{\alpha+1}.$$

Let us consider a mark-up $(\alpha-1)/\alpha$ which is equal to 0.2 which means $\alpha=1.25$. In this particular case for α, the above inequality is such that $Gini<0.11$. As soon as $Gini<0.11$, the equilibrium price will differ from $\alpha w_t/A_t$ being equal to

$$p_t=\frac{\beta}{\beta-1}\frac{w_t}{A_t}.$$

Gini coefficients across countries reveal that the assumption $(\beta-1)/\beta<1/\alpha$ is more realistic. In this article, we therefore work with this assumption and the price in equilibrium is determined by $p_t=\alpha w_t/A_t$. ■