Trade in Goods and Services and Economic Growth

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Abstract

This paper differentiates between trade in commodities and trade in services, and examines the impact of such trades on growth of trading economies. Within the framework of the model, growth rate of an economy in the long run is unaffected by any change in trade regime. But it is affected during periods of transitions. Transitional dynamics arises from the feature that services are non-essential in the utility function, which implies a variable rate of intertemporal substitution in consuming services. Through transitional dynamics, the static (or level) effects of trade regime changes affects the growth rate of the services sector. However, the sub-utility from consuming manufacturing is assumed such that the intertemporal rate of substitution in consuming manufactures is constant. This implies no growth effects in this sector, although there are level effects. In general (qualitative) growth effects are shown to depend on a country's comparative advantage in manufacturing and services.

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1 Introduction

The world economy has experienced 'phenomenal' growth of the services sector in the post-war era. In many major economies its share in GDP stands well above 50%. From the demand side, this phenomenon has been attributed commonly to the 'relative demand shift' hypothesis, stating that income elasticity of demand for services exceeds one and thus as real income grows, the sector producing services would grow faster. From the supply side, the boom of the IT sector has been cited as a major source of growth for the services sector. Indeed, Eichengreen and Gupta (2009) ascribe the second growth wave of the services sector to the rise of the IT sector. Baumol's disease (Baumol (1967)), which states that the lack of productivity increase in the services sector (because it is labor intensive) is likely to pull down the overall growth of an economy, certainly has not struck; instead, as Triplett and Bosworth (2003) have shown, productivity in the services sector has increased substantially in relatively recent years.

At the same time, the volume of trade in services has also significantly grown, thanks to the IT 'revolution,' which has considerably reduced the transactions or 'transport' costs of providing cross-border services. The volume of service imports tripled from 1994 to 2004 (Hoekman and Mattoo (2008)). Francois and Hoekman (2010) report that, according to WTO, the global value of cross-border export of services became 20% of world trade in commodities and services by the year 2007.

The existing trade theory has paid scant attention to trade in services, although there is a growing analytical literature on it; see, for example, Francois (1990a), Francois (1990b), Hoekman and Mattoo (2008), Mattoo et al. (2008) and Francois and Hoekman (2010). The purpose of this paper is to differentiate between commodities trade and trade in services, and consider the impact of each kind of trade on growth of a trading economy.

There are general equilibrium models that feature the manufacturing and the services sectors (plus agriculture in some). But almost all of them are *static*, e.g. Eswaran and Kotwal (2002) and Buera and Kaboski (2009) and Matsuyama (2009). There are a few papers, which analyze disaggregated growth in a *closed* economy, e.g., Kongsamut et al. (2001) and Klyuev (2004).[1]

Our search yielded only one paper, namely, Xu (1993), which considers growth of manufacturing and the services in the context of an open economy. It builds a capital accumulation based Solovian growth model with a constant savings rate. It has three sectors: a consumable manufacturing, an investment good which is also a manufacturing product and a service good. The objective is to explain the rise in the relative price of (consumer) services and the employment share of the services sector. It considers international trade in (manufacturing) commodities only. Trade in services is not analyzed.

As said earlier, in this paper we analyze how trade in commodities and trade

[1]Both papers develop growth models based on physical capital accumulation and allow two mobile factors of production, labor and capital. The former attempts reconcile differential growth rates across sectors (i.e. 'structural' changes) along with overall balanced growth for the entire economy (defined as the situation where the real interest rate is constant over time). The latter's aim is to explain the rise in the relative price of services vis-a-vis manufacturing, which results from the assumption that manufacturing is more capital intensive sector.

in services may affect the growth rates within an economy. Thus it falls in the tradition of the seminal work by Grossman and Helpman (1992), which was the first major attempt to integrate international trade with the modern theory of economic growth. There are three important differences however. First, in their work, international trade presumes trade in commodities only; there is no distinction between commodities and services trade. Second, their focus is on aggregate and balanced growth, whereas in this paper manufacturing and the services sectors grow (rather are 'allowed' to grow) at different rates and we ask how (free) international trade impacts on the growth performance of these sectors individually. Third, whereas in Grossman-Helpman's work, technological innovation is the source of long term growth and their endeavor is aimed towards understanding how international trade, via resource reallocation within an open economy, may affect the sector 'producing' innovation and thus influence the long-term growth of an economy, our focus is on sectoral growth rates *during transition periods* - which may be interpreted as short or the medium run. In our analysis, long-run or asymptotic growth rate is not affected by international trade. However, needless to say, long term *levels* of consumption and welfare – which ultimately matter – can be substantially affected growth rates during transition.

In standard growth models transitional dynamics results typically from adjustment costs of investment or diminishing returns to capital. In our model it stems from a different source, namely the structure of preferences. As trade is opened there are static effects on resource allocation and output, which, in a dynamic framework can seen as initial, one-period level effects. During transition, level effects lead to growth effects (having convergence properties).

In section 2, we develop our model of growth for a closed economy. This serves as our reference economy. Free trade in commodities is introduced in section 3. In section 4, we analyze free trade in both commodities and services. Section 5 concludes the paper.

2 Closed Economy

Our framework closely follows that of Das and Saha (2011), who analyze growth of manufacturing and services sectors in the context of a closed economy.

In our model, long-term growth is not driven by physical capital accumulation, technology innovation or learning-by-doing. Instead, a simple process of human capital accumulation is presumed and there is one primary factor of production, namely, effective labor. The reason for choosing this source of growth, as opposed to accumulation of physical capital or technology innovation as in Grossman and Helpman (1992) is that it allows to focus on differences in growth rates across sectors in a simplified one-primary-factor framework. This factor, which we call labor, can of course be broadly interpreted as a composite input.[2]

[2]In this sense, it is akin to the use of a Ricardian model, rather than a Heckscher-Ohlin model for analyzing certain international trade issues. Copeland and Taylor (1994) is a prominent example of how a one-primary-factor-based general equilibrium model can provide useful insights into the complex issue of trade and environment.

An economy has three sectors – manufacturing, services, and, following Matsuyama (2009) a "numeraire" sector. We shall call it food, although, by design, it is not meant to capture agriculture either in terms of diminishing-return-to-scale technology or less-than-unitary-income-elasticity of demand for it. Matsuyama (2009) does interpret his numeraire good as agriculture in that the income elasticity of demand for it is less than one. Kongsamut et al. (2001) also allow for a third sector (besides manufacturing and services) and interpret it as agriculture for the same reason. We refrain from doing so on the ground that since 1970s, agriculture's share in total output and employment has remained small *and* relatively invariant in prominent developed economies, whereas free trade in commodities or in both commodities and services is a more recent phenomenon.[3,4] But the presence of a third sector allows us to consider trade in commodities (manufacturing), independently of trade in services. We assume that the economy obtains a fixed endowment of food, say equal to E at each instant of time.

To gain algebraic convenience however, we will treat manufacturing as the numeraire good in the sense that its price is equal to unity.

The representative household consumes all three goods: manufacturing (M), services (S) and food (D); M and D are homogeneous, while S is differentiated.[5] At any instant of time t, let

$$U_t \equiv \lambda_1 \ln C_{mt} + \lambda_2 \ln(C_{st} + \delta) + (1 - \lambda_1 - \lambda_2) \ln C_{dt}, \; 0 < \lambda_1, \lambda_2, \lambda_1 + \lambda_2 < 1, \; \delta > 0 \quad (1)$$

be the felicity function, where C_{mt} and C_{dt} are consumptions levels of M and F, and C_{st} is a composite of individual services. Denoting the demand for individual services by C_{it}, let

$$C_{st} \equiv \left(\int_0^{N_t} C_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1, \quad (2)$$

where σ is the elasticity of substitution between two service varieties and N_t is the total number of service varieties available. Let p_{st} be the price of this composite (such that $p_{st}C_{st} = \int_0^{N_t} p_{it}C_{it}di$). Note importantly,

1. The specification of sub-utility from consuming M implies constant intertemporal rate of substitution of good M (equal to 1). This would imply constant growth rate of manufacturing, not just in a closed economy but also when the economy is open. There will thus be no transitional dynamics and hence no growth effects on the M sector.

2. The presence of the parameter δ implies two things: (a) a variable intertemporal rate of substitution of the service basket and hence transitional dynamics

[3] For example, in the period 1970-2007, agricultural output in the U.S. fell from 4% of GDP to 1% of GDP; manufacturing output fell from 35% of GDP to 22% of GDP; and services rose from 61% of GDP 77% of GDP. Similar magnitudes of sectoral output changes were seen in developed countries like UK, Japan, etc. *Source:* World Development Indicators, World Bank Database.

[4] If the purpose of an analysis were to consider 'long-term' growth over several decades extending to a century or half a century, the reason for incorporating agriculture would have been more compelling.

[5] See Eswaran and Kotwal (1993) among others who also depict the services sector as producing a differentiated product.

of output and employment in this sector and (b) services are not an essential 'good' and hence income elasticity of demand for services exceeds unity.[6]

3. Thus it is the non-essentiality parameter δ which delivers growth effects.
4. As the economy grows, while δ is fixed, services become less and less non-essential overtime and hence service sector growth rate would monotonically decline over time.
5. Per se, (b) will imply a higher growth rate of the service sector compared to manufacturing.

At each t the household maximizes U_t subject to the budget constraint $C_{mt} + \int_0^{N_t} p_{it}C_{it}di + p_{dt}C_{dt} \leq B_t$, where B_t is the sum of wage earnings, profit income and the value of the endowment E.

Eqs. (3) and (4) below are the first-order conditions with respect to C_{mt}, C_{st} and C_{dt}:

$$\frac{\lambda_1}{1-\lambda_1-\lambda_2} \cdot \frac{C_{dt}}{C_{mt}} = \frac{1}{p_{dt}} \tag{3}$$

$$\frac{\lambda_1}{\lambda_2} \cdot \frac{C_{st}+\delta}{C_{mt}} = \frac{1}{p_{st}}. \tag{4}$$

The demand function for individual varieties with the services basket is of the form:

$$C_{it} = C_{st}\left(\frac{p_{it}}{p_{st}}\right)^{-\sigma}, \text{ where } p_{st} \equiv \left(\int_0^{N_t} {p_{it}}^{-(\sigma-1)}di\right)^{-\frac{1}{\sigma-1}}. \tag{5}$$

The static household optimization problem implies the following indirect felicity function:

$$V_t = A + \ln(B_t + \delta p_{st}) - \lambda_2 \ln p_{st} - (1-\lambda_1-\lambda_2)\ln p_{dt}, \tag{6}$$

$$\text{where } A \equiv \lambda_1 \ln \lambda_1 + \lambda_2 \ln \lambda_2 + (1-\lambda_1-\lambda_2)\ln(1-\lambda_1-\lambda_2).$$

Note that through the parameter δ, the price of the service composite, p_{st} has a *positive* effect on utility, although its overall effect on indirect utility is negative. It is like a positive income effect. The reason is that δ measures the degree of how less essential services are compared to other consumption goods (M and D). Therefore, δp_{st} can be interpreted as *quasi real income* which is not spent on services because they are not essential but can be spent on other goods. Indeed it can be verified that expenditure on M or D is a constant fraction of the sum of actual income B_t and quasi income δp_{st}. As will be seen, the concept of quasi income just outlined will help us to intuitively understand the growth effects of trade in services in particular.

We now characterize the production side of the economy.

[6]Service goods have been represented as much in preferences by many, including Kongsamut et al. (2001) and Matsuyama (2009).

2.1 Manufacturing

Following Matsuyama (1992), manufacturing technology is assumed to satisfy diminishing returns.[7] Let the production function be

$$Q_{mt} = ML_{mt}^{\alpha},\ 0 < \alpha < 1, \tag{7}$$

where L_{mt} is the level of effective labor used in the M sector and M is a factor-neutral productivity parameter.[8] The market for manufacturing is perfectly competitive. The standard profit-maximizing condition with respect to employment can be equivalently stated as:

$$\alpha Q_{mt} = w_t L_{mt}. \tag{8}$$

2.2 The Services Sector

A service firm i has increasing returns to scale technology and faces a monopolistically competitive market. It produces a distinct variety i. Technology is specified by $Q_{it} = SL_{it} - 1$, where Q_{it} is the output of variety i, L_{it} is the amount of effective labor employed and S is a productivity parameter. It maximizes profit, taking as given the demand function for its product, (5). We obtain the familiar mark-up condition:

$$\frac{p_{it}}{w_t} = \frac{1}{S} \cdot \frac{\sigma}{\sigma - 1}. \tag{9}$$

This condition as well as the zero-profit condition imply

$$L_{it} = \frac{\sigma}{S}; \quad q_{it} = \sigma - 1. \tag{10}$$

That is, firm-level employment and output are time invariant. Total employment and output in the services sector are thus equal to

$$L_{st} = \frac{\sigma N_t}{S};\ Q_{st} = (\sigma - 1)N_t.$$

N_t represents the total number of varieties available as well as total employment and output in the services sector.

Notice that in the aggregate this sector exhibits constant returns to scale with respect to labor since each firm employs a given amount of labor under the competitive, zero-profit condition.

Hence returns to scale manufacturing are less than those in the services sector at both firm and industry levels. This accords with Basu et al. (2006) who report scale elasticities for various manufacturing and service industries in the U.S., wherein those for service industries are generally higher than those for manufacturing industries. Das and Saha (2011) have shown that such difference in scale elasticities across the sectors is a factor behind higher growth of the services sector. This feature carries to our model for both closed and open economies.

[7]Technically, we will 'need' diminishing returns in manufacturing in order to ensure stability in the labor market.

[8]The term 'effective' is used to indicate that labor embodies human capital.

2.3 Static General Equilibrium

In view of (10) we can write the full-employment condition as:

$$L_{mt} + \frac{\sigma}{S} N_t = \bar{L}_t, \tag{11}$$

where $\bar{L}_t$ is the total effective labor supply available for production. Using (9),

$$C_{st} = (\sigma - 1) N_t^{\frac{\sigma}{\sigma-1}}; \; p_{st} = N_t^{-\frac{1}{\sigma-1}} \frac{\sigma}{S(\sigma-1)} w_t. \tag{12}$$

Substituting these into the first-order condition (4), we obtain

$$\frac{\lambda_1}{\lambda_2} \frac{\sigma}{S} \left(N_t + \frac{\delta}{\sigma - 1} N_t^{-\frac{1}{\sigma-1}} \right) = \frac{C_{mt}}{w_t}. \tag{13}$$

Next, by substituting the household budget constraint,

$$C_{mt} + p_{ot} C_{dt} + p_{st} C_{st} = w_t \bar{L}_t + (1 - \alpha) Q_{mt} + p_{ot} E, \tag{14}$$

eq. (13) can re-expressed as:

$$\frac{(1 - \lambda_2) \sigma w_t}{\lambda_2 S} \left(N_t + \frac{\delta}{\sigma - 1} N_t^{-\frac{1}{\sigma-1}} \right) = Q_{mt} + p_{dt} E. \tag{15}$$

Using the market-clearing conditions in autarky, $C_{mt} = Q_{mt}$ and $C_{dt} = E$, the first-order condition (3) can be stated as

$$\frac{\lambda_1}{1 - \lambda_1 - \lambda_2} \frac{E}{Q_{mt}} = \frac{1}{p_{dt}}. \tag{16}$$

Eqs. (7) and (8) together with (11), (15) and (16) constitute five equations in five variables, namely, L_{mt}, Q_{mt}, N_t, w_t and p_{dt}. The parameters are $\bar{L}_t$ (the evolution of which will be analyzed later) and those of technology and preference.

In particular, note that the l.h.s. of (15), or equivalently that of (13), is non-monotonic with respect to N_t. This is an artifact of non-essentiality of the service basket in the utility function. It implies that the solution of N_t^h may not be unique.

Given non-essentiality, the marginal utility of purchasing power from consuming the services basket (the ratio of marginal utility to p_{st}^h) is a decreasing function with respect to the sum of total expenditure on it *and* the quasi income. Under symmetry, the former increases linearly with respect to the number of varieties available, whereas the latter decreases with the same since, due to positive valuation of varieties per se, an increase in N_t lowers the price the service basket. Overall then, an increase in the number of varieties have a non-monotonic effect on the marginal utility of purchasing power from consuming services. This is the source of multiple solutions of N_t^h.

Let us define the l.h.s. of (13) as $G(N_t)$. The analytical shape of this function is depicted in Figure 1. We see that for any C_{mt}/w_t, there are at most two solutions of N_t.

It is shown in Appendix 1 that for any given w_t and $\bar{L}_t$, an exogenous increase in N_t, would tend to increase (respectively decrease) an individual service firm's profit, according as the initial value of N_t lies on the falling arm (respectively rising arm) of the $G(N_t)$ function. Thus, the solution of N_t along the rising arm of the $G(N_t)$ function is consistent with stability in terms of free entry and exit in the services sector. We assume that this is the market solution of N_t.

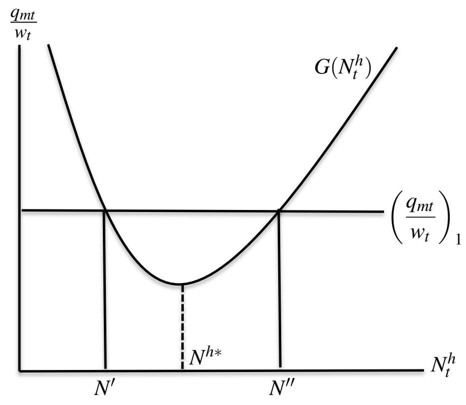

Figure 1: Solution of N_t^h

2.4 Comparative Statics

To set the stage for introducing international trade, we present autarky equilibrium in the familiar demand-supply diagram and conduct comparative statics with respect to increases in $\bar{L}_t$, M and S.

Eqs. (7), (8) and (11) implicitly yield

$$L_{mt} = L_m(\underset{-}{w_t}, \underset{+}{M}); \quad Q_{mt} = \tilde{Q}_m(\underset{-}{w_t}, \underset{+}{M}); \quad N_t = N(\underset{+}{w_t}, \underset{+}{\bar{L}_t}, \underset{-}{M}, \underset{+}{S}). \tag{17}$$

If we substitute the function $N(\cdot)$ into (15), we implicitly obtain,

$$Q_{mt} = \tilde{\tilde{Q}}_m(\underset{+}{w_t}, \underset{-}{p_{dt}}, \underset{+}{\bar{L}_t}, \underset{-}{M}, \underset{-}{S}).\text{[9]} \tag{18}$$

Figure 2 depicts the loci between Q_{mt} and w_t, defined by (17) and (18), and the solution of these variables at given p_{dt}, $\bar{L}_t$, M and S. We obtain the following comparative statics results:

$$\begin{aligned} Q_{mt} &= Q_m^s(\underset{-}{p_{dt}}, \underset{+}{\bar{L}_t}, \underset{+}{M}, \underset{-}{S});\text{[10]} \quad w_t = w(\underset{+}{p_{dt}}, \underset{-}{\bar{L}_t}, \underset{+}{M}, \underset{+}{S}) \\ L_{mt} &= \bar{L}_m(\underset{-}{p_{dt}}, \underset{+}{\bar{L}_t}, \underset{+}{M}, \underset{-}{S}); \quad N_t = \bar{N}(\underset{+}{p_{dt}}, \underset{+}{\bar{L}_t}, \underset{-}{M}, \underset{+}{S}). \end{aligned} \tag{19}$$

As Q_{mt} is negatively related to p_{dt} from the supply side, the ratio E/Q_{mt} varies positively with respect to p_{dt}. This is indicated in Figure 3. The demand-side

[9] The negative effect of S on Q_{mt} uses that, at given w_t, a change in S leaves N_t/S ratio unchanged.

[10] Although the positive effect of an increase in M on Q_{mt}^s is intuitive, it is not clear from Figure 2, since both curves shift. To see this we express (15) as

$$\frac{1-\lambda_2}{\lambda_1} w(\underset{-}{Q_{mt}}, \underset{+}{M}) G(\underset{+}{N}(L_m(\underset{-}{Q_{mt}}, \underset{+}{M}), \underset{-}{S})) = Q_{mt} + p_{dt}E,$$

where $w(\cdot)$ is the inverse of the $\tilde{Q}_m(\cdot)$ function and the negative relationship between N and M follows from the full-employment equation. The above equation implies $\partial Q_m^s/\partial M > 0$.

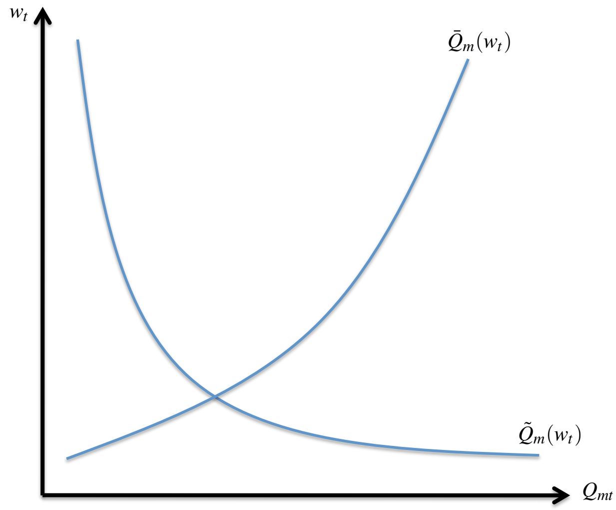

Figure 2: Determination of the Wage Rate and Manufacturing Output

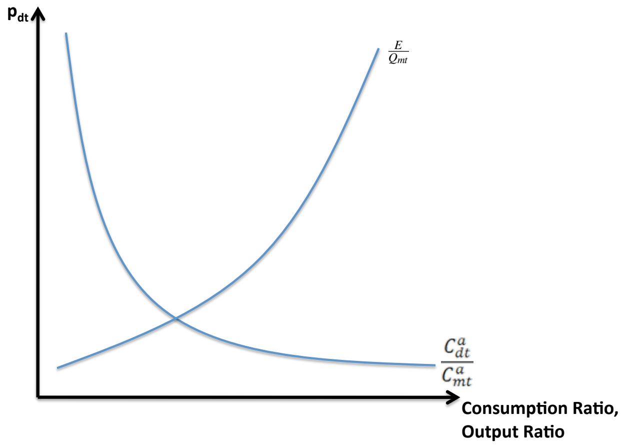

Figure 3: Autarky Equilibrium

equation (16) is shown as the downward sloping curve. Intersection of the two curves in Figure 3 defines the autarky equilibrium.

We will consider three sources of comparative advantage: technology difference in manufacturing (M), that in services production (S) and size of an economy ($\bar{L}_t$) or the level of development. We suppose that the world economy consists of two countries: Home (h) and Foreign (f).

From the supply side, at given p_{dt},

1. If $M^h > M^f$, while $S^h = S^f$ and $\bar{L}^h_t = \bar{L}^f_t$,

$$Q^h_{mt} > Q^f_{mt};\ N^h_t < N^f_t;\ w^h_t > w^f_t;\ p^h_{it} > p^f_{it}. \tag{20a}$$

2. If $S^h > M^f$, while $M^h = M^f$ and $\bar{L}^h_t = \bar{L}^f_t$,

$$Q^h_{mt} < Q^f_{mt};\ N^h_t > N^f_t;\ \frac{w^h_t}{S^h} < \frac{w^f_t}{S^f};\ p^h_{it} < p^f_{it}. \tag{20b}$$

3. If $M^h = M^f$, $S^h = S^f$ and $\bar{L}_t^h > \bar{L}_t^f$,

$$Q_{mt}^h > Q_{mt}^f;\ N_t^h > N_t^f;\ w_t^h < w_t^f;\ p_{it}^h < p_{it}^f. \tag{20c}$$

The pattern of comparative advantage is clear. In a two-country world economy, (a) the country with better manufacturing production technology will tend to have comparative advantage in manufacturing and comparative disadvantage in services, (b) the country with better services production technology will tend to have comparative advantage in services and comparative disadvantage in manufacturing and (c) the country at a higher initial level of development (i.e. which is initially larger) will have comparative advantage in both manufacturing and services.

We now turn to characterize the dynamics of our model economy.

2.5 Intertemporal Household Problem

At any given t, the representative household possesses one unit of time which is worth L_t units of effective labor or human capital. A part of this stock is used up in augmenting its human capital and the rest is supplied to the production sectors. Let $H_t \in (0,1)$ denote investment in human capital in term of time. Within a time period, the household first commits to allocate such investment and then offers the remaining stock of effective labor to the production sectors. Let

$$L_{t+1} = a_L H_t L_t, \quad a_L > 1, \tag{21}$$

describe the growth process of human capital. The tradeoff is that the higher the investment in human capital, the greater will be the stock of effective labor and hence the higher will be the total wage earnings in the future, but the less will be the total earnings in the current period.

It $t = 0$ is the initial period, given L_0, the household chooses $\{H_t\}_0^\infty$ and $\{L_t\}_1^\infty$ to maximize $\sum_0^\infty \rho^t V_t$ (where $\rho < 1$ is the discount factor and V_t is the indirect utility whose expression is given in (6)), subject to the human capital accumulation equation (21) and

$$B_t = w_t(1 - H_t)L_t + \pi_{mt} + p_{dt}E. \tag{22}$$

Here π_{mt} is the profit income from producing manufacturing. We obtain the following Euler equation:

$$\frac{B_{t+1} + \delta p_{st+1}}{w_{t+1}} \Big/ \frac{B_t + \delta p_{st}}{w_t} = \rho a_L.$$

A marginal increase in investment in human capital entails a current period loss of w_t, which translates into a marginal loss of current utility equal to $w_t/(B_t + \delta p_{st})$. It also entails an increase in future utility equal to $a_L w_{t+1}/(B_{t+1} + \delta p_{st+1})$. The discounted value of the marginal gain is $\rho[a_L w_{t+1}/(B_{t+1} + \delta p_{st+1})]$. Euler equation is a statement that the marginal loss in terms of current utility is equal to the discounted value of the next-period marginal gain. Using the first-order conditions of static household optimization problem, the Euler equation can be more simply stated as

$$\frac{C_{mt+1}/w_{t+1}}{C_{mt}/w_t} = \rho a_L.\text{[11]} \tag{23}$$

We assume that $\rho a_L > 1$, so that the economy's growth rate is positive. Eq. (33) says that the ratio of manufacturing consumption to the wage rate grows at the (gross) rate ρa_L.

2.6 Dynamics of the Economy

In autarky equilibrium $C_{mt} = Q_{mt}$. Hence, in view of (8),

$$\frac{C_{mt}}{w_t} = \frac{Q_{mt}}{w_t} = \frac{L_{mt}}{\alpha}.$$

Thus manufacturing employment grows at the rate ρa_L. Given the manufacturing production function, manufacturing output grows at the rate $(\rho a_L)^\alpha$, which is less than ρa_L.

As long as N_t is determined along the rising part of the $G(N_t)$ function, from eq. (13), a higher Q_{mt}/w_t implies a higher value of N_t. Thus, N_t, which proportional to aggregate employment and output in the services sector, grows over time. Let $g_{Nt} \equiv N_{t+1}/N_t$, be the gross growth rate of N_t. Eq. (13) can expressed as

$$g_{Nt} \frac{1 + \frac{\delta}{\sigma-1}(g_{Nt}N_t)^{-\frac{\sigma}{\sigma-1}}}{1 + \frac{\delta}{\sigma-1}N_t^{-\frac{\sigma}{\sigma-1}}} = \rho a_L. \tag{24}$$

Hence the growth rate of N_t is dependent on its initial value and therefore time-variant. Let it be called the **growth function** of N_t. This is an important concept. As we shall see, the growth effects of international trade stem from (a) a movement along the growth function or (b) a shift of the growth function.

It is evident from (24) that $g_{Nt} > \rho a_L$, while $\lim_{t\to\infty} g_{Nt} = \rho a_L$.[12] Further, Appendix 2 shows that g_{Nt} declines monotonically with t.

In summary, we have

Proposition 1 *Employment in manufacturing grows at a constant (gross) rate ρa_L, while output grows grows at constant rate which is less than ρa_L The total employment and output in the services sector grow at a common time-varying rate, which is greater than ρa_L and which monotonically declines over time and becomes asymptotic to ρa_L as $t \to \infty$.*

Figure 4 depicts sectoral employment and growth rates as functions of time. Note the following.

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[11] More completely we can write

$$\frac{p_{st+1}(C_{st+1}+\delta)}{w_{t+1}} \Big/ \frac{p_{st}(C_{st}+\delta)}{w_t} = \frac{C_{mt+1}/w_{t+1}}{C_{mt}/w_t} = \frac{p_{dt+1}C_{dt+1}/w_{t+1}}{p_{dt}C_{dt}/w_t}$$
$$= \rho a_L$$

[12] As $t \to \infty$, so does Q_{mt}/w_t since it grows at a constant rate. From Figure **??**, $\lim_{t\to\infty} N_t = \infty$. As $\lim_{t\to\infty} N_t = \infty$, it is clear from (24) that $g_{Nt} \to \rho a_L$.

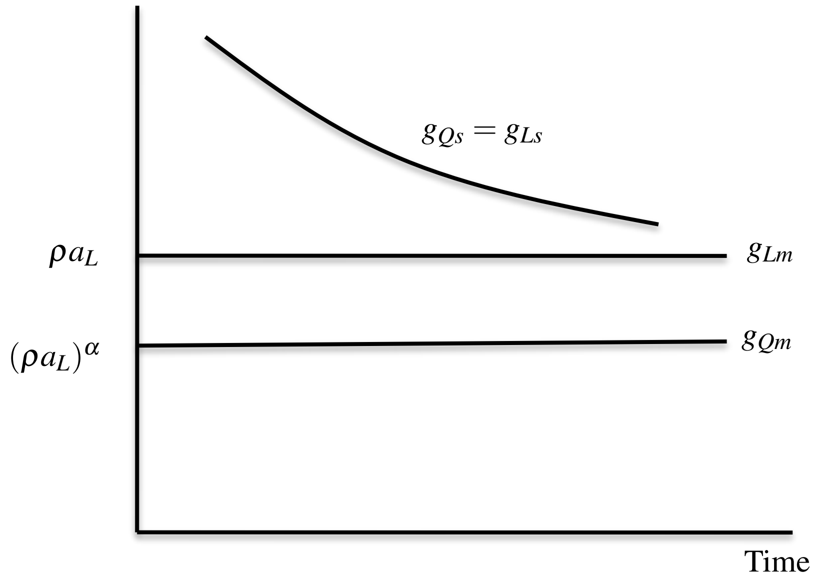

Figure 4: Sectoral Employment and Output Growth Rates in Autarky

1. Faster employment growth in the services sector stems from the demand pull effect due to the income elasticity of the demand for the services basket exceeding unity. Otherwise, (if $\delta = 0$), employment in the services sector would grow at the rate ρa_L.
2. We have $g_{Qs} > \rho a_L > g_{Qm}$, where g_{Qs} and g_{Qm} denote the growth rates of the services output and manufacturing output respectively. There are two reasons behind why output growth is higher in the services sector: (a) higher scale elasticity in services production compared to manufacturing and (b) income elasticity of demand for services being greater than unity.

2.7 Dynamics of Total Effective Labor Supply for Production and Human Capital Investment

Dynamics of sectoral employment levels implies the dynamics of effective labor supply for production, $\bar{L}_t$. Using the full-employment condition and that manufacturing employment grows at the rate ρa_L,

$$g_{\bar{L}} = \frac{\bar{L}_{t+1}}{\bar{L}_t} = \frac{L_{mt+1} + (\sigma/S)N_{t+1}}{L_{mt} + (\sigma/S)N_t} = \frac{\rho a_L L_{mt} + (\sigma/S)g_{N_t}N_t}{L_{mt} + (\sigma/S)N_t}$$

From the static equations (11) and (13) we implicitly obtain $L_{mt} = \tilde{L}_m(\bar{L}_t)$ and $N_t = \tilde{N}(\bar{L}_t)$, such that $\tilde{L}'_m(\bar{L}_t), \tilde{N}'(\bar{L}_t) > 0$. Totally differentiating, we get $dg_{\bar{L}}/d\bar{L} \gtrless 0$ as $\bar{L}_t \lesseqgtr A$, where A is some positive constant.

The remaining element is the dynamics of human capital investment. We use the definition of total effective labor supply, $\bar{L}_t = (1 - H_t)L_t$ and the learning equation (21) to get,

$$\Delta H_t \equiv H_{t+1} - H_t = (1 - H_t)\left(1 - \frac{g_{\bar{L}_t}/a_L}{H_t}\right). \quad (25)$$

Figure 5 plots the $\Delta H_t = 0$ locus in $(\bar{L}_t, H_t)$ space. The vertical arrows indicate the change in H_t as one moves away from $\Delta H_t = 0$ curve, while the horizontal arrows indicate that $\bar{L}_t$ always grows over time. It is clear that under perfect foresight no path can originate from region II, V or VI. So the path has to originate from $H_t > \rho$. If $\bar{L}_0 < A$ then multiple paths are possible. A path can originate from region III, where H_t can fall into region IV and asymptote to ρ; or another path can start at region I where H_t initially rises to region III and then asymptotes to ρ in region IV. So for unique path, we assume that $H_t < 1$ and $A < \bar{L}_0 = L_0(1 - H_t) < L_0(1 - \rho)$, which implies $L_0 > A/(1 - \rho)$. Hence there is unique path starting in region IV where H_t falls and asymptotes to ρ.

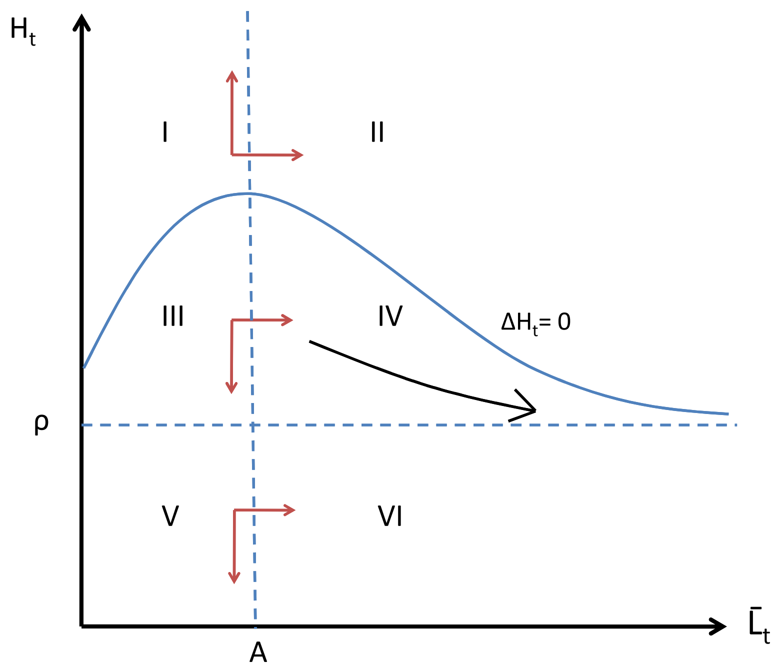

Figure 5: Dynamics of H_t in Autarky

3 Commodities Trade

Now suppose the two countries, h and f, open up free trade in commodities only, that is, in manufacturing and food, not services. It happens, say, in period 0, after labor is allocated to acquiring human capital, as a one-shot permanent regime change rather than something gradual. Hence changes in investment in human capital due to the regime change – which would imply a different time profile of future wages – begin to take place from period 1 onwards. Thus, L_0 as well as $\bar{L}_0$ remain unchanged.

Depending on relative magnitudes of technology parameters, M^k and S^k, and, initial level of development, L_0^k, $k = h, f$, one country will be a net exporter and the other will be a net importer of manufacturing. In order to understand the growth effects of trade qualitatively, it is enough to brand one country as manufacture-exporting and the other as manufacture-importing.

Note that there are, presumably, some specific factors in the manufacturing sector in each country, which gives rise to decreasing, rather than constant, returns to scale in that sector. Due to the presence of specific factors, in commodity-trading equilibrium no country will specialize in producing services; each will produce a positive amount of manufacturing.

Trade equalizes the prices of traded goods. Let p_{dt}^o denote the international relative price of food in commodity-trading equilibrium. Instead of market clearing within each country, there is global market clearing of both manufacturing and food. Accordingly, the system of equations characterizing the static equilibrium is the same, except (16).

More specifically, the manufacturing production function (7), the first-order condition with respect to labor employment in that sector (8), the full-employment condition (11) and consumption allocation equation (15) for both countries, and the international market clearing condition

$$\frac{\lambda_1}{1-\lambda_1-\lambda_2}\frac{2E}{Q_{mt}^h+Q_{mt}^f}=\frac{1}{p_{dt}}. \tag{26}$$

constitute a system of nine equations in nine variables: manufacturing output, manufacturing employment, employment in the services sector and wage rates in both countries – and p_{dt}.

As the trading regime changes, there are one-period static (i.e. level) effects, which are followed by dynamics of the system. The one-period, initial effects depend on comparative advantage, which, in turn, depends on relative magnitudes of technology difference and the initial level of development of trading countries. Depending on these parameters one country will be an exporter of manufactures and the other will be an importer of the same. The former country, say Home, will face a higher relative price of manufactures vis-a-vis autarky. Thus output and employment in its manufacturing sector will increase and output and employment in its services sector will fall. The opposite resource re-allocations will occur in the other country.

Turning to dynamics, the intertemporal household optimization problem in the two countries remains qualitatively unchanged from that in autarky, although household-level parameter values may be different. The same Euler equation follows. Thus, both C_{mt}^h/w_t^h and C_{mt}^f/w_t^f grow at the rate ρa_L.

Our first result is that

Proposition 2 *In the free-trade-in-commodities-only regime, manufacturing output and employment growth rates in both countries are same as in autarky. That is, manufacturing employment in both countries grows at the rate ρa_L and manufacturing output in both country grows at the rate $(\rho a_L)^\alpha$.*

The proof is provided in Appendix 3. As discussed earlier, no growth effect on the manufacturing sector stems from our assumption of log-linear sub-utility from consuming manufactures which implies constant intertemporal rate of substitution in consuming manufacturing.

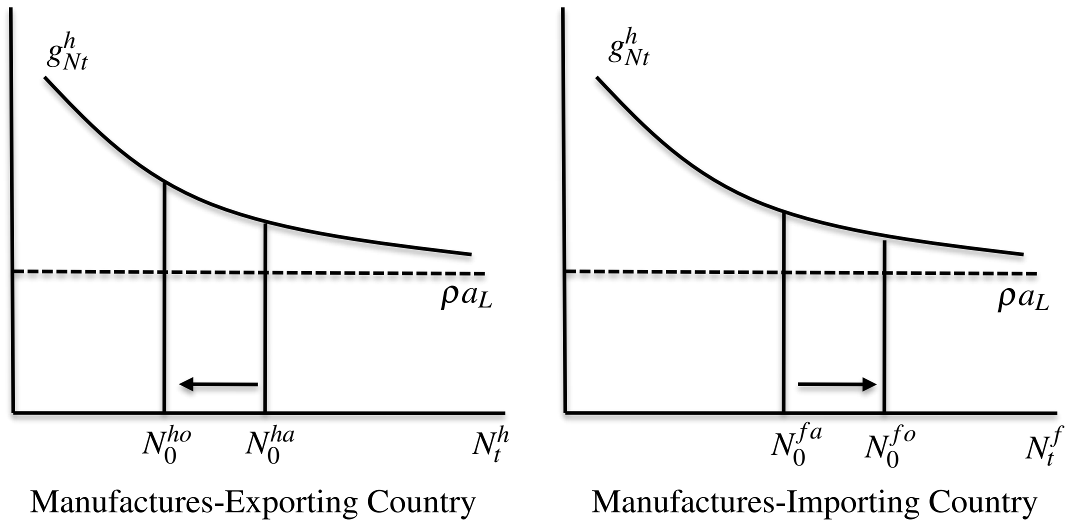

Figure 6: Growth Effects of Trade in Commodities Only

To determine any growth effect on the services sector, note that the relation (13) continues to hold in free-trade-in- commodities-only regime. Since C^k_{mt}/w^k_t grows at the rate ρa_L, the same growth equation holds for the services sector in both countries.

However, the same growth equation does *not* imply zero growth effect of the regime shift from autarky. There is an initial level effect on the services sector, and, it affects transition and hence growth rate over time. Consider the manufactures exporting country. As labor moves from the services sector to manufacturing, N_0 falls, and thus there is a movement along the growth function. There is an initial upward jump in the growth rate in the services sector. The opposite happens in the manufactures-importing country. These effects are illustrated in Figure 6.

Proposition 3 *As free trade in commodities is introduced, the manufactures-exporting country experiences an upward jump in the growth rate of employment and output in the services sector while the manufactures-importing country experiences a fall in employment and output growth in the services sector. After these initial effects, both economies traverse along the same growth function in the services sector.*

The last proposition outlines within-country growth effects of trade in commodities on the services sector. Growth rates across countries can be compared too. Suppose, initially, $N_0^h > N_0^f$ initially (which depends on the source comparative advantage). The growth functions being the same across the two countries, growth rates as functions of time exhibits the pattern shown in Figure 7.

Proposition 4 *In the commodities-trade-only regime, at each instant of time the growth rate of employment and output in the services sector is higher in the country in which initially N_0 is smaller.*

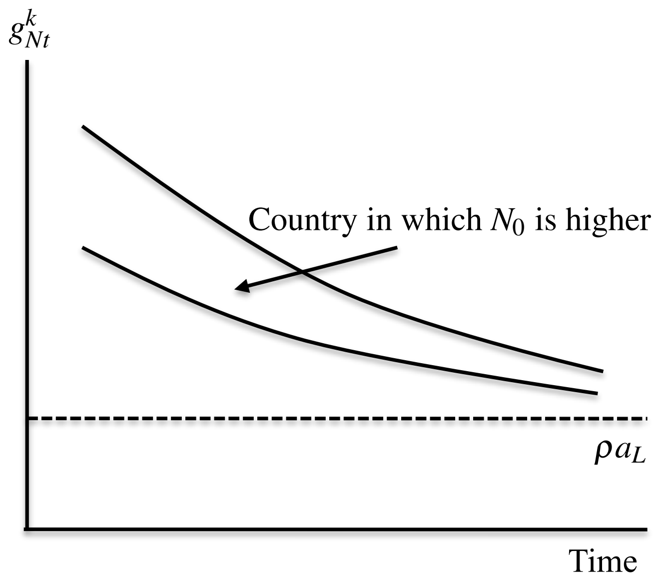

Figure 7: Cross-Country Comparison of Growth Rates in the Commodities Trade Regime

Note from (20a)-(20c) that as long as countries differ in one of the three parameters (technology in the manufacturing sector, technology in the services sector or initial level of development), the sign of $N_0^h - N_0^f$ is opposite of that of $p_{it}^h - p_{it}^f$, that is, a country which has comparative advantage in producing services produces a higher total output of services. It then follows that

Proposition 5 *Assuming that comparative advantage stems from differences in one of the three parameters, in the commodities-trade-only regime, at each instant of time the country having comparative advantage in producing services experiences a lower growth rate of this sector compared to the other country, although in the long run both growth rates approach ρa_L.*

Note the interesting contrast that within-country growth effects on the services sector are dependent on comparative advantage in producing manufacturing, whereas across-countries growth rates on the services sector depends on comparative advantage in producing services.

4 Trade in Commodities and Services: "Grand Free Trade"

Suppose the two economies, now, open up trade in services also. Let us call it the grand free trade regime. Assume that services are provided internationally in the cross-border mode. Consumers in each country now access service varieties produced in both countries just as commodities. Since services are not perfect substitutes, it amounts to a shift of the sub-utility function with regard to service varieties. This implies a *shift* of the growth functions of services, which constitutes the major difference compared to trade in commodities only.

4.1 Static Equilibrium

The service baskets of home and foreign consumers have the expressions:

$$C_{st}^h = \left[\int_0^{N_t^h} {q_{it}^{hh}}^{1-\frac{1}{\sigma}} di + \int_0^{N_t^f} {q_{it}^{fh}}^{1-\frac{1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}$$

$$C_{st}^f = \left[\int_0^{N_t^h} {q_{it}^{hf}}^{1-\frac{1}{\sigma}} di + \int_0^{N_t^f} {q_{it}^{ff}}^{1-\frac{1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}},$$

where q_i^{hh}, q_i^{fh}, q_i^{hf} and q_i^{ff} are respectively quantities provided by a (i) home producer to home consumers, (ii) foreign producer to home consumers, (iii) home producer to foreign consumers and (iv) foreign producer to foreign consumers. It is assumed that the elasticity of substitution between a home and a foreign variety is also equal to σ. The overall utility function remains unchanged as in (1) but the sub-utility function contains varieties produced in both countries.

The static household optimization problem implies the prices of the services basket in the two countries having the same expression:

$$p_{st}^k \equiv \left(\int_0^{N_t^h} {p_{it}^h}^{-(\sigma-1)} di + \int_0^{N_t^f} {p_{it}^f}^{-(\sigma-1)} di\right)^{-\frac{1}{\sigma-1}}, \quad k = h, f.$$

The demand functions for individual varieties within a composite basket are:

$$q_{it}^{hh} = C_{st}^h \left(\frac{p_{it}^h}{p_{st}^h}\right)^{-\sigma}; \quad q_{it}^{fh} = C_{st}^h \left(\frac{p_{it}^f}{p_{st}^h}\right)^{-\sigma} \tag{27}$$

$$q_{it}^{hf} = C_{st}^f \left(\frac{p_{it}^h}{p_{st}^f}\right)^{-\sigma}; \quad q_{it}^{ff} = C_{st}^f \left(\frac{p_{it}^f}{p_{st}^f}\right)^{-\sigma}. \tag{28}$$

The optimization conditions with respect to food, manufacturing and the service basket remain unchanged. That is, (3) and (4) continue to hold.

The output of a service firm is allocated to consumers in both countries. Let $q_{it}^h \equiv q_{it}^{hh} + q_{it}^{hf}$ and $q_{it}^f \equiv q_{it}^{fh} + q_{it}^{ff}$. The total revenue expression, for say a home country service provider is

$$R_{it}^h = p_{it}^h(q_{it}^{hh} + q_{it}^{hf}) = p_{it}^h \left(A^h {p_{it}^h}^{-\sigma} + A^f {p_{it}^h}^{-\sigma}\right), \text{ where } A^k \equiv q_s^k {p_s^k}^{\sigma},\ k = h, f$$
$$= (A^h + A^f){p_{it}^h}^{-(\sigma-1)}.$$

The costs are equal to

$$w_t^h L_{it}^h = \frac{w_t^h}{S^h}(q_{it}^{hh} + q_{it}^{hf} - 1) = \frac{w_t^h}{S^h}\left[(A^h + A^f){p_{it}^h}^{-\sigma} - 1\right].$$

Analogous expressions hold for the foreign country. Profit maximization leads to same mark-up conditions:

$$\frac{p_{it}^h}{w_t^h} = \frac{\sigma}{\sigma-1}\cdot\frac{1}{S^h}; \quad \frac{p_{it}^f}{w_t^f} = \frac{\sigma}{\sigma-1}\cdot\frac{1}{S^f}. \tag{29}$$

Together with the zero-profit condition they imply

$$q_{it}^h = q_{it}^f = \sigma - 1;\ L_{it}^h = \frac{\sigma}{S^h};\ L_{it}^f = \frac{\sigma}{S^f}. \tag{30}$$

Notice that competition forces each service provider to produce the same amount as when trade in services was closed. With respect to allocation of a firm's output in the two markets and pricing,

Proposition 6 *In grand free trade, as long as both countries produce services*

$$q_i^{hh} = q_i^{fh},\ q_i^{hf} = q_i^{ff},\ p_i^h = p_i^f.$$

Proof: We have

$$\frac{q^{hh}}{q^{fh}} = \frac{q^{hf}}{q^{ff}} = \left(\frac{p_{it}^h}{p_{it}^f}\right)^{-\sigma}.$$

This is equal to

$$\frac{q^{hh} + q^{hf}}{q^{fh} + q^{ff}} = \frac{\sigma - 1}{\sigma - 1} = 1,$$

implying the three inequalities. ■

Proposition 6 says that home and foreign varieties are priced equally. In view of constant mark-up pricing, this implies that wages across two countries remain in constant proportion over time:

$$\frac{w_t^h}{S^h} = \frac{w_t^f}{S^f}. \tag{31}$$

The equations characterizing static equilibrium in the grand free trade regime are same as in free trade in commodities only, except the one pertaining to consumption allocation, namely, (15). Instead, we have

$$\frac{(1-\lambda_2)\sigma w_t^k}{\lambda_2 S^k}\left(\mathcal{N}_t + \frac{\delta}{\sigma - 1}\mathcal{N}_t^{-\frac{1}{\sigma-1}}\right) = Q_{mt}^h + Q_{mt}^f + 2p_{dt}E,\ k = h, f, \tag{32}$$

where $\mathcal{N}_t \equiv N_t^h + N_t^f$ is the total number of service varieties available in the global market.

This pair of equations, the pair of manufacturing production function, the pair of first-order conditions with respect to employment in manufacturing (8), the pair of full employment equations and the market clearing condition (26) solve Q_{mt}^h, Q_{mt}^f, L_{mt}^h, L_{mt}^f, N_t^h, N_t^f, w_t^h, w_t^f and p_{dt}.

Unfortunately, it is hard to generally compare the magnitudes of variables in grand free trade with those in free trade in commodities only, because trade in services has elements of trade among dissimilar and trade among similar countries.

But it is worth exploring the case of trade among similar countries since it reveals the symmetric effects (which are part of the overall effects) across the countries. In this case, we have

Proposition 7 *If the two countries are identical, a movement from free trade in commodities only to grand free trade leads, in each country, to more employment and output in the services sector and less employment and output in manufacturing.*

Intuitively, as households in either country are able to consume both home and foreign varieties, at given price of a service variety, the composite price of the services basket falls (due to more varieties being present in the basket). This implies less quasi income and hence less expenditure on manufacturing. In equilibrium there is less output and employment in manufacturing. Given full-employment, this implies higher employment and output in the services sector.[13]

In other words, in both countries there is a resource reallocation from the manufacturing sector to the services sector. These are one-period initial effects when a permanent shift to grand free trade occurs. On the top of it, there will be comparative-advantage led effects which are asymmetric.

4.2 Dynamics and Growth Effects

We now move back to the general case in which the two countries may be dissimilar. Euler equations however are same as before. Thus,

$$\frac{C^k_{mt+1}/w^k_{t+1}}{C^k_{mt}/w^k_t} = \rho a_L, \qquad k = h, f. \tag{33}$$

As in case of free trade in goods only,

Proposition 8 *Trade in services has no growth effect on output and employment in the manufacturing sector in either country.*

The underlying reason is the same: that is, constant intertemporal rate of substitution of consumption of the manufacturing good. Appendix 5 proves this proposition.

However, trade in services has growth effects in the services sector which are different from those of trade in commodities. Recall that in the commodities-trade-only regime, there are country-specific own growth functions of the number of service varieties (firms) in the sense that $g_{N^k_t}$ is dependent on N^k_t. In the grand free trade regime, there is, instead, a growth function of the total number of service varieties available globally, in the sense that the growth rate of $\mathcal{N}_t$ is dependent on the initial value of the same. Furthermore, the growth rate of N^h_t or N^f_t is function of *both* N^h_t and N^f_t.

Proposition 9 *There exists a growth function of $\mathcal{N}_t$, defined by*

$$\frac{\mathcal{N}_{t+1} + \frac{2\delta}{\sigma-1}\mathcal{N}_{t+1}^{-\frac{1}{\sigma-1}}}{\mathcal{N}_t + \frac{2\delta}{\sigma-1}\mathcal{N}_t^{-\frac{1}{\sigma-1}}} = \rho a_L. \tag{34}$$

Proof: Substitute (26) into (32) and eliminate $p_{dt}E$. Next, use the global market-clearing condition of manufacturing: $Q^h_{mt} + Q^f_{mt} = C^h_{mt} + C^f_{mt}$. By using (31) in the resultant equation, we get

$$\frac{\sigma\lambda_1}{\lambda_2}\left(\mathcal{N}_t + \frac{2\delta}{\sigma-1}\mathcal{N}_t^{\frac{-1}{\sigma-1}}\right) = \left(S^h\frac{C^h_{mt}}{w^h_t} + S^f\frac{C^f_{mt}}{w^f_t}\right).$$

[13] Proof of this proposition is given in Appendix 4.

The r.h.s. grows at the rate ρa_L, implying (34). ■

An immediate implication is that

Corollary 1 *The total number of global service varieties grows at a time-varying rate higher than ρa_L, which falls over time and becomes asymptotic to ρa_L.*

While the sum total of home and foreign varieties grows at a time-varying rate, the difference between them grows at a constant rate. Appendix 6 proves that

Proposition 10 *$N_t^h - N_t^f$ grows at the rate ρa_L.*

It then follows that

Corollary 2 *The growth rates of N_t^h and N_t^f satisfy*

$$\frac{g_{Nt}^h - \rho a_L}{g_{Nt}^f - \rho a_L} = \frac{N_t^f}{N_t^h}. \tag{35}$$

Note that eqs. (34) and (35) implicitly define the growth functions of N_t^h and N_t^f. It is clear that each growth function depends on initial values of both N_t^h and N_t^f. These equations also imply

Proposition 11 *In both counries, employment in the services sector growth faster than that in manufacturing.*

Proof: In view of (35), either both g_{Nt}^h and g_{Nt}^f are negative or both are positive. But if both are negative, g_{N_t}, which is weighted average of g_{Nt}^h and g_{Nt}^f, must be negative. This contradicts that g_{N_t} has positive growth path. ■

We are now in a position to characterize within-country growth effects and cross-country growth rate comparisons. Within-country effects are tractable in the special case of identical countries. In this case, eq. (34) reduces to

$$g_{Nt}\frac{1 + \frac{\delta}{2^{1/(\sigma-1)}(\sigma-1)}(g_{Nt}N_t)^{-\sigma/(\sigma-1)}}{1 + \frac{\delta}{2^{1/(\sigma-1)}(\sigma-1)}N_t^{-\sigma/(\sigma-1)}} = \rho a_L. \tag{36}$$

This implies a different growth function of N_t compared to autarky or free trade in commodities only, as indicated in (24). More specifically it lies *below* its counterpart in autarky or free trade in commodities.[14] This is illustrated in Figure 8.

Intuitively, it is the shift of the sub-utility function with respect to service varieties that causes the shift of the growth function. As the number of varieties are, per se, valued, service trade tends to lower the price of the service basket. Quasi real income falls. The service basket becomes less unessential and more like the manufacturing good. Income elasticity of demand for services gets closer to one. As a result, its growth rate becomes closer to that of manufacturing. This amounts to a downward shift of the growth function.

We already know the initial level effect. Combining the two effects, the overall growth effect is then evident from Figure 8.

[14] Instead of $\frac{\delta}{\sigma-1}$ in the l.h.s., now it is $\frac{\delta}{2^{1/(\sigma-1)}(\sigma-1)}$, which is less than $\frac{\delta}{\sigma-1}$. Hence the directional impact of this on g_{Nt} at given N_t is same as that of an increase in N_t on g_{Nt} – which is negative.

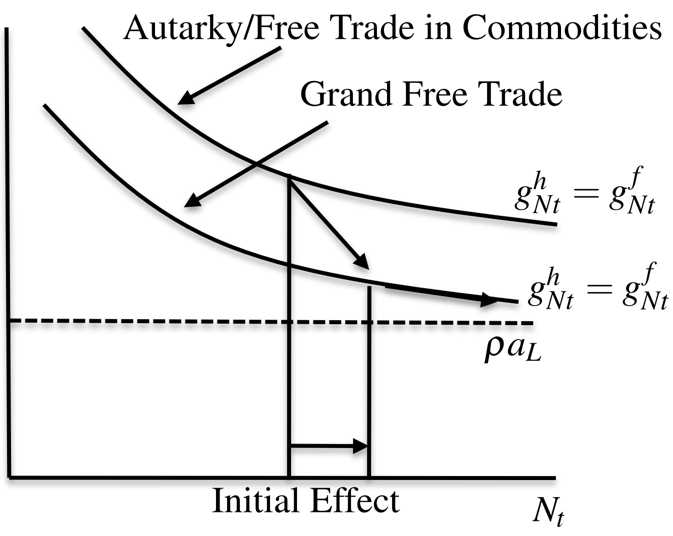

Figure 8: Growth Effects of Trade in Services when the Two Economies are Identical

Proposition 12 *If the two countries are identical, a movement from commodities-trade only (or from autarky) to grand free trade entails a downward jump in the growth rate of employment and output in the services sector, after which it gradually declines and becomes asymptotic to ρa_L.*

Finally, growth rates across countries can be compared via the relation (35). It implies that $g^h_{Nt} \gtreqless g^f_{Nt}$ as $N^h_t \lesseqgtr N^f_t$. Thus,

Proposition 13 *As in the regime of commodities trade only, the country in which the initial production of the servi*

5 Concluding Remarks

In early stages of development, agriculture dominated an economy. Then manufacturing became the sector that defined an economy's state of development. Now it is the service sector that is becoming ubiquitous in the world economy. At the same time, the IT sector revolution has dramatically lowered the transaction costs associated with exporting or importing services. Trade in services is a growing phenomenon too.

The present paper is an attempt to link the two: trade in services and growth of the services sector. The central question the paper addresses is, how openness in trade in commodities and services may affect the growth of an economy, especially that of the services sector. The paper explores a particular channel through which trade may affect economic growth, i.e., the nature of consumer services in preferences. Our analysis features that, compared to manufactures, services are a 'luxury' good, less essential – such that the income elasticity of demand for services is greater than unity.

It is shown that the impact of free trade in commodities and services on growth depends on the pattern of comparative advantage in producing manufactures and services.

A major limitation our analysis lies in its specifications which imply no growth effect of trade regime changes on the manufacturing sector; all growth effects are confined to the services sector. One straightforward way to accommodate it would be assume that manufactures have become such a necessity that a minimum positive level of consumption of it is 'required' (as in Matsuyama (2009)). Another important extension will be to incorporate business services which are the fastest growing component within the basket of services. Business services serving as production inputs in the manufacturing sector, freer trade in business services would affect growth of the manufacturing sector. A model based on physical capital accumulation in which investment goods are a part of manufactures only would also imply growth effects of trade policy changes in manufacturing.

Exploring these extensions so that a more comprehensive growth effects of trade policy changes emerge is uncovered is on our current research agenda.

Appendix 1

Free Entry and Exit Stability analysis for the Services Sector in Autarky

Assume for notational simplicity that $M = S = 1$. By substituting the manufacturing goods market clearing $C_{mt} = Q_{mt}$ and (12) into (4) we obtain,

$$Q_{it} = Q_i(\underset{-}{w_t}, \underset{?}{N_t}) \equiv \frac{\sigma - 1}{\sigma N_t} \cdot \frac{Q_{mt}(\underset{-}{w_t})}{w_t} - \frac{\delta}{N_t^{\frac{\sigma}{\sigma-1}}}.$$

Using the above expressions and the price-markup condition for services, the equilibrium profit of a service firm i can be expressed as

$$\pi_{it} \equiv \pi_{it}(\underset{-}{w_t}, \underset{?}{N_t}) = \frac{w_t Q_i(w_t, N_t)}{\sigma - 1} - w_t,$$

where we have used that $Q_{it} = L_{it} - 1$.

Stability of entry and exit processes requires $\partial \pi_{it}/\partial N_t < 0$. By using Samuelson's correspondence principle, we shall prove that stability ensured if and only if in Figure 1, the solution of N_t lies in the rising part of the $G(N_t)$ function.

In equilibrium, $\pi_{it}(w_t, N_t) = 0$. Differentiating it,

$$\frac{dN_t}{dw_t} = -\frac{\partial \pi_{it}/\partial w_t}{\partial \pi_{it}/\partial N_t}.$$

We know that $\partial \pi_{it}/\partial w_t < 0$. Hence the signs of dN_t/dw_t and $\partial \pi_{it}/\partial N_t$ must be the same. Now turn to Figure 1. If the solution of N_t is at a point such as N' (respectively N''), $dN_t/dw_t >\ (<)\ 0$. It implies $\partial \pi_{it}/\partial N_t >\ (<)\ 0$ and thus free entry-exit equilibrium is unstable (respectively stable).

Therefore, stability-consistent solution of N_t lies on the rising arm of $G(N_t)$ in Figure 1. ■

Appendix 2: Proof that $dg_{Nt}/dN_t < 0$ from eq. (24)

Define $y \equiv \frac{\delta}{\sigma-1} N^{-\frac{\sigma}{\sigma-1}}$ and $x \equiv g_{Nt}$. Eq. (24) can be expressed as

$$y = \frac{x - \rho a_L}{\rho a_L - 1/x^{\frac{1}{\sigma-1}}},$$

where $x > \rho a_L > 1$. To show $dg_{Nt}/dN_t < 0$, it is equivalent to prove $dy/dx > 0$. Totally differentiating,

$$\frac{dy}{dx} = \frac{J_1}{(\rho a_L - 1/x^{\frac{1}{\sigma-1}})^2},$$

where

$$J_1 \equiv \rho a_L - \frac{1}{x^{1/(\sigma-1)}} - \frac{1}{\sigma - 1} \cdot \frac{x - \rho a_L}{x^{\sigma/(\sigma-1)}}.$$

$dy/dx > 0$ if and only if $J_1 > 0$. Note that J_1 increases with ρa_L. Hence

$$J_1 > J_2, \text{ where } J_2 \equiv 1 - \frac{1}{x^{1/(\sigma-1)}} - \frac{1}{\sigma - 1} \frac{x - 1}{x^{\sigma/(\sigma-1)}}.$$

It is sufficient to prove $J_2 > 0$. We write J_2 as

$$J_2 = \frac{1}{(\sigma-1)x^{1/(\sigma-1)}} J_3, \text{ where } J_3 \equiv (\sigma-1)x^{\frac{1}{\sigma-1}} + \frac{1}{x} - \sigma.$$

It is straightforward to derive that for any given σ, J_3 attains global minimum at $x = 1$. Substituting $x = 1$ into J_3, for any given σ, we obtain min $J_3 = 0$. Hence $J_2 \geq 0$. In turn, $J_1 > 0$, implying $dy/dx > 0$. ■

Appendix 3: Proof of Proposition 2

Consider the budget constraint (14) of the representative household. Since services are not traded, $p_{st}C_{st}$, which is the revenue generated in the services sector, is equal to labor income generated in that sector. Using this as well as the first-order conditions (3) and (8), the budget equation gives:

$$\frac{1-\lambda_2}{\lambda_1} C_{mt} = \frac{w_t L_{mt}}{\alpha} + p^o_{dt} E.$$

This holds for each country. Substituting into it the international market-clearing condition (26) and relation (8) for each country gives rise to

$$(1+\lambda_1-\lambda_2)w^h_t L^h_{mt} + (1-\lambda_1-\lambda_2)w^f_t L^f_{mt} = 2\alpha(1-\lambda_2)C^h_{mt}$$
$$(1-\lambda_1-\lambda_2)w^h_t L^h_{mt} + (1+\lambda_1-\lambda_2)w^f_t L^f_{mt} = 2\alpha(1-\lambda_2)C^f_{mt}.$$

Next, we divide the first and the second equation respectively by w^h_t and w^f_t and use that $w^h_t/w^f_t = (M^h/M^f)(L^h_{mt}/L^f_{mt})^{-(1-\alpha)}$. This gives

$$L^{h\ 1-\alpha}_{mt} \left(k_1 L^{h\ \alpha}_{mt} + \frac{k_2}{\zeta} L^{f\ \alpha}_{mt} \right) = \frac{C^h_{mt}}{w^h_t} \tag{A.1}$$

$$L^{f\ 1-\alpha}_{mt} \left(k_2 \zeta L^{f\ \alpha}_{mt} + \frac{k_1}{\zeta} L^{f\ \alpha}_{mt} \right) = \frac{C^f_{mt}}{w^f_t}. \tag{A.2}$$

where $k_1, k_2 and \zeta$ are some positive constants.[15] Since the r.h.s.s of the above equations are growing at the rate ρa_L, their l.h.s.s must be proportionate to each other. That is,

$$\frac{L^h_{mt}}{L^f_{mt}} \cdot \frac{k_1 + \frac{k_2}{\zeta}\left(\frac{L^h_{mt}}{L^f_{mt}}\right)^{-\alpha}}{k_2\zeta\left(\frac{L^h_{mt}}{L^f_{mt}}\right)^{\alpha} + k_1} = K,$$

where K, a function of initial values, is time-invariant. It is easy to show that the l.h.s. of the above equation increases monotonically with the ratio, L^h_{mt}/L^f_{mt}. Moreover, l.h.s. approaches 0 or ∞ as this ratio approaches 0 or ∞. Hence there exists a unique solution to the above equation and therefore the ratio of employment remains constant over time. Using this, the l.h.s. of (A.1) is linear in L^h_{mt}; similarly, the l.h.s. of (A.2) is linear in L^f_{mt}. Because C^h_{mt}/w^h_t and C^f_{mt}/w^f_t grow at the rate ρa_L, so do L^h_{mt} and L^f_{mt}. ■

[15]

$$k_1 = \frac{1+\lambda_1-\lambda_2}{2\alpha(1-\lambda_2)}; \qquad k_2 = \frac{1-\lambda_1-\lambda_2}{2\alpha(1-\lambda_2)}; \qquad \zeta = \frac{M^h}{M^f}$$

Appendix 4: Proof of Proposition 7

Proof: Consider eqs. (8), (11), (15) and (26) pertaining to free trade in commodities only. Eq. (11) yields a negative schedule between L_{mt} and N_t. This is shown in Figure 9 and marked as FF. If we substitute (8) and (26) into (15) and eliminate w_t and p_{dt}, we obtain

$$L_{mt} = \frac{\sigma\alpha\lambda_1}{\lambda_2}\left(N_t + \frac{\delta}{\sigma-1}N_t^{-\frac{1}{\sigma-1}}\right). \tag{A.3}$$

This posits a positive locus between L_{mt} and N_t, shown as DD in Figure 9 (as long as the solutions of N_t occurs on the rising portion of the $G(N)$ function).

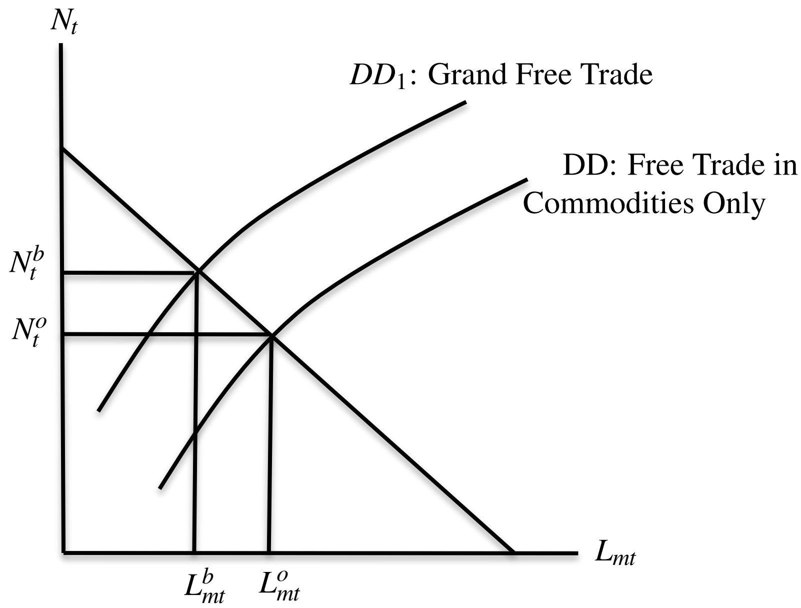

Figure 9: Initial Level Effects of Trade in Services when the Two Economies are Identical

Now turn to static general equilibrium in grand free trade. The schedule FF continues to hold. By substituting (8) and (26) into (32), we again eliminate p_{dt}. Next we use $\mathcal{N}_t = 2N_t$. The resultant relation is:

$$L_{mt} = \frac{\sigma\alpha\lambda_1}{\lambda_2}\left(N_t + \frac{\delta}{2^{\frac{\sigma}{\sigma-1}}(\sigma-1)}N_t^{-\frac{1}{\sigma-1}}\right). \tag{A.4}$$

It defines a positive schedule between L_{mt} and N_t and is shown as DD_1 in Figure 9. It is easy to see that DD_1 lies to the left of DD. Solutions in commodities-trade-only and grand free trade regimes are marked by superscripts ^{o} and ^{b} respectively. We see that $N_t^b > N_t^o$ and $L_{mt}^b < L_{mt}^o$. ■

Appendix 5: Proof of Proposition 8

Eqs. (7), (8) and (31) together imply

$$\frac{L_{mt}^h}{L_{mt}^f} = \left(\frac{M^f S^h}{M^h S^f}\right)^{\frac{-1}{1-\alpha}} \equiv \phi \tag{A.5}$$

Just an in case of trade in commodities only, we add the household budgets across two countries, and substitute in it (8), (11) and the market clearing condition $C^h_{dt} + C^f_{dt} = 2E$. We then obtain

$$\frac{w^h L^h_m + w^f L^f_m}{\alpha} = C^h_m + C^f_m. \tag{A.6}$$

Using (31) and (A.5), the above relation implies

$$L^f_{mt} = \frac{1}{1+\Phi}\left(\frac{S^h}{S^f}\cdot\frac{C^h_{mt}}{w^h_t} + \frac{C^f_{mt}}{w^f_t}\right)$$

where $\Phi = \phi S^h/S^f$.

Because C^k_m/w^k, $k = h, f$ grows at the rate ρa_L, it follows that L^f_{mt} and thus L^h_{mt} grow at the rate ρa_L. Further, the manufacturing output in either country grows at $(\rho a_L)^\alpha$. Hence there are no growth effects on the manufacturing sector. ■

Appendix 6: Proof of Proposition 10

The household's problem, the manufacturing sector's problem and the labor market clearing condition has not changed from autarky. To get the dynamics of N^k, we simplify the household budget by substituting the household optimization conditions (3)-(4), the manufacturing sector's profit maximisation condition (8) and full employment condition (11) into (14) and rewrite as

$$\frac{S^k}{\lambda_1}\frac{C^{kb}_{mt}}{w^k_t} - \delta S^k \frac{p^b_{st}}{w^{kb}_t} - \frac{S^k L^{kb}_{mt}}{\alpha} - \frac{S^k E p^b_{dt}}{w^{kb}_t} = \sigma N^{kb}_t. \tag{A.7}$$

The growth rate of C^h/w^h and C^f/w^f are equal. Due to factor price equalization, $S^h E p^b_{dt}/w^h = S^f E p^b_{dt}/w^f$. From the expression of composite price of services (**??**) we get that $S^h p^b_S/w^h = S^f p^b_S/w^f$. We already know that manufacturing employment in both countrie grows at ρa_L. Hence on taking a difference of the above budget equation for home and foreign country, we get that $N^h - N^f$ grows at the rate ρa_L. Hence

$$\frac{N^h_{t+1} - N^f_{t+1}}{N^h_t - N^f_t} = \frac{g^h_{N_t} N^h_t - g^f_{N_t} N^f_t}{N^h_t - N^f_t} = \rho a_L.$$

Rearranging the terms in the above equation yields (35). ■

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