Sensitivity of growth rate estimates to long memory process.[1]

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[1]Many thanks to Jonathan Temple and Parantap Basu for suggesting improvents to the paper. Needless to say any error is my responsibility

Abstract

Long run growth is estimated either by averaging log output difference (STM) or fitting a trendline to log output (DTM). Also there is evidence that log-per-capita GDP has long memory $I(d)$ characteristics and very often the numerical value of the average growth estimate differs across these models. In this paper, we show the growth estimates are sensitive to the pre-estimated value of d. It also suggests a correction method for the sensitivity.of the growth estimates. Using 100 yrs of Maddison Data, we find that across 30 countries, the FGLS and OLS estimates of STM are less sensitive than the estimates from the DTM. STM also produces a significantly lower growth rate estimates than fitting a deterministic trend line.

1 Introduction

To analyse long run growth (rate) one has to describe how to measure it. A standard approach is to take the average of the annual growth rates that in essence is the log-difference between of the initial and the final output. This procedure works best for a when the log-output process is a stochastic trend model (STM). Alternatively since the initial and final output may be some distance from the trend path of output, it may be preferable to use the growth rate obtained by regressing the log percapita GDP series against a constant and a linear trend (DTM). This performs well when the residuals are stationary. A growing literature points to the fact that the true log-output process could be a long memory process ($I(d)$) of some order d[1] (which can be estimated). The traditional procedure of computing long run growth rate works fine if the differencing parameter d takes watershed values such as 0 (in case DTM) or 1 (in case of STM). However, due to the low power of many long memory tests, in principle it may be difficult to difficult to distinguish between $I(0)$ process and an $I(d)$ process with d close to zero or between a $I(1)$ process and an $I(d)$ with d close to 1. The aim of this paper is to demonstrate that the estimate of growth rate is highly sensitive to a small change in the differencing parameter d. Thus if the true process for GDP is driven by long memory, this sensitivity of the long run growth estimate to a minute change in d makes the conventional estimate of the growth rate very unreliable. Indeed, very often the numerical value of the average growth estimate differs across these models even for large samples.

This paper investigates the whether the growth estimates from these models changes dramatically when d differs from 0 (or 1) by a small magnitude. We show that this deviation is more significant when d is near 0 rather than 1. Often a practitioner can use a Feasible Generalised Least Squares (FGLS) estimates for the STM and the DTM obtained by plugging in a pre-estimate of the long memory parameter $\widehat{d_0}$. We show that the FGLS growth rate estimates are also sensitive when $\widehat{d_0}$ differs from actual d by a small magnitude. The problem is more

[1]Michelacci and Zaffaroni (2000) (MZ) proposed that the log-output precess is a DTM with a long-memory fractionally integrated ($I(d)$) error process. They use a log-periodogram regression estimate, by Geweke and Porter-Hudak (GPH) (1983) and could not reject the hypothesis that all the OECD countries are non-stationary. Silverberg and Verspagen (2001) using STM also finds long memory in the log-PCGDP process. Further analysis in the literature shows that there is strong evidence in favour of an integration order between 0 and 1 in most of the countries in the sample. (Dolado, Gonzalo and Mayoral (2002a), Dolado et al (2002b) etc).

when $\widehat{d}_0$ underestimates d.

More precisely, if $\widehat{\beta}_r\left(\widehat{d}_0\right)$ and $\widehat{\beta}_r(d)$ are the estimated growth rates at the pre-estimated value $\widehat{d}_0$ and the true value d respectively, we ask the question how large is the difference $\widehat{\beta}_r\left(\widehat{d}_0\right) - \widehat{\beta}_r(d)$ compared to $\widehat{d}_0 - d$. A measure of such deviation, $B_r\left(\widehat{d}_0\right)$ is constructed by normalising the derivative of $\widehat{\beta}_r(d)$ at $\widehat{d}_0$. We call the estimate sensitive when the probability of such deviation is 'large'. We derive the distributional properties of $B_r\left(\widehat{d}_0\right)$ which provides a criterion of being 'large'. This in the spirit of Banerjee and Magnus (1999) who develops a similar statistic for AR(1) process[2]. This analysis is different from Canjels and Watson (1997) or Boswijk and Franses (2006) which focus on confidence bounds for different growth rate estimates under AR(1) process. They do not address the issue if the point estimate of growth itself is sensitive to small changes to the memory parameter (the AR(1) coefficient in their case).

The contribution of this paper is intended to go beyond the derivation of the sensitivity statistic and its application to the issue of growth estimate. We ask the question: if $\widehat{\beta}_r\left(\widehat{d}_0\right)$ turns out to be sensitive, is there a point $\widehat{d}_1$ near $\widehat{d}_0$ where the estimate $\widehat{\beta}_r\left(\widehat{d}_1\right)$ is less sensitive, in the sense of $B_r\left(\widehat{d}_1\right)$ being 'small'. We define the least sensitive estimate $\widehat{d}_1$ by using a Stochastic Newton-Raphson type method with a correction factor $CF(\widehat{d}_0)$. This correction factor then can be used iteratively (i.e. $\widehat{d}_j = \widehat{d}_{j-1} - CF_r\left(\widehat{d}_{j-1}\right)$) to produce the least sensitive growth estimates. We provide the distribution of the correction factor $CF_r\left(\widehat{d}_j\right)$ which helps us to formulate a stopping rule for this iteration procedure, although convergence is not guaranteed because of the irregular changes in shape of the distribution of $CF_r\left(\widehat{d}_j\right)$ with respect to $d_{j.}$

We make use of the GDP per capita (PCGDP) data (in USD) which is obtained from Maddison's Total Economy Database (http://www.ggdc.net) website for with 100 observations approximately (1902-2003). We estimate the long run growth rates using two different models, a linear trend model and a stochastic difference model. Comparing the STM and DTM, we observe that growth estimates (OLS and FGLS) from the STM are generally lower than the DTM (24/23 of the 30 countries studied) with a significant median difference of 16 basis points. Recall, the OLS estimate of long run growth from STM is an average of annual growth rates.

[2]Banerjee and Magnus (1999) gives the sensitivity measures for the predictor against ARMA disturbances.

Using our methodology, we show that the long-run PCGDP growth rates (OLS and FGLS) estimated from the DTM are very 'sensitive' to misspecifiication of the long memory parameter d. In comparison, OLS estimates of STM are less-sensitive to deviations from $\widehat{d}_0$, than the estimates of DTM (with exception of small economies like Venezuela and Portugal). The sensitive growth rates are then corrected using our proposed method of correction. In some cases the correction procedure fails to converge. Some countries like India has widely varying growth-rate estimates, depending on how and which model the estimate is arrived at.

Therefore less sensitive growth rates are produced by models which also lower growth rates than the more sensitive estimates.

The paper is organised as follows: section 2 gives the preliminaries; section 3 defines a sensitivity statistic, and develops the least sensitive estimate along with its statistical properties; section 4, analyses the growth estimates for sensitivity and corrects for them; section 5 concludes.

2 Preliminaries

Let us consider the following model with data $(\boldsymbol{y}_0, \boldsymbol{X}_0)$ such that:

$$y_{0,t} = x_{0,t}'\beta + \varepsilon_{0,t} \quad (t = 1, ..., T), \tag{1}$$

where $\varepsilon_{0,t}$ are distributed as distributed as $I(d)$ process:

$$\varepsilon_{0,t} = \Delta^{-d} u_t \tag{2}$$

with innovations $u_1, \ldots, u_T \sim$ i.i.d. $(0, \sigma^2)$.

There are if we have a pre-estimator of d with $\widehat{d}_0$ we transform the model (1) to use a Feasible GLS estimator. Let the transformed data be

$$y_t\left(\widehat{d}_0\right) = \Delta^{\widehat{d}_0} y_{0,t} \text{ and } x_t'\left(\widehat{d}_0\right) = \Delta^{\widehat{d}_0} x_{0,t}'.$$

The estimator $\widehat{d_0}$ can be either be $\widehat{d_0} = 0$, in which case we use the OLS estimate or $\widehat{d_0} = 1$, in which case we can use the first difference estimate or if $\widehat{d_0} = \widehat{d_G}$ an usual estimator like the GPH estimator or the Whittle estimator ($\widehat{d_0} = \widehat{d_W}$), we can transform the model using fractional differencing. Therefore the transformed model is given by

$$\begin{aligned} \Delta^{\widehat{d_0}} y_{0,t} &= \Delta^{\widehat{d_0}} x'_{0,t}\beta + \Delta^{\widehat{d_0}} \varepsilon_{0,t} \\ y_t\left(\widehat{d_0}\right) &= x'_t\left(\widehat{d_0}\right)\beta + \varepsilon_t, \end{aligned} \tag{3}$$

where ε_t are distributed as $I(d - \widehat{d_0})$ process:

$$\varepsilon_t = \Delta^{-\left(d-\widehat{d_0}\right)} u_t \tag{4}$$

We can write the variance matrix of the process as,

$$\mathbf{\Omega}\left(\theta\right) = \sum_{h=0}^{T-1} \omega_h\left(\theta\right) \mathbf{T}^{(h)},\ i = 1, 2. \tag{5}$$

where we denote by $\mathbf{T}^{(h)}, 0 \leq h \leq T-1$, the $T \times T$ symmetric Toeplitz matrix with

$$\mathbf{T}^{(h)}_{(i,j)} = \begin{cases} 1 & \text{if } |i-j| = h, \\ 0 & \text{otherwise.} \end{cases},$$

such that $\theta = d - \widehat{d_0}$. The coefficients $\omega_h(\theta)$, are given by the autocovariance generating function:

$$g(\theta, z) = \left[(1-z)\left(1-z^{-1}\right)\right]^{-\theta} = \sum_{h=-\infty}^{\infty} \omega_h\left(\theta\right) z^h.$$

Letting $\mathbf{X} = \left(x'_t\left(\widehat{d_0}\right)\right)$ and $\mathbf{y} = \left(y_t\left(\widehat{d_0}\right)\right)$, we can write the GLS estimator using the transformed model (3) as:

$$\widehat{\beta}\left(d\right) = \left(\mathbf{X}'\left[\mathbf{\Omega}\left(\theta\right)\right]^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\left[\mathbf{\Omega}\left(\theta\right)\right]^{-1}\mathbf{y}.$$

When the unknown longmemory parameter d is exactly equal to its pre-estimated value $\widehat{d_0}$, i.e.

at $d = \widehat{d}_0$, we have the feasible GLS (FGLS) estimator

$$\widehat{\beta}\left(\widehat{d}_0\right) = \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{y}.$$

since $\boldsymbol{\Omega}(0) = \boldsymbol{I}$. Note that if $\widehat{d}_0 = 1$, the constant is not identified in the transformed model (3).

Theorem 1 *Let $\sigma^2\boldsymbol{\Omega}(\theta)$ be the covariance matrix of $u_1, \ldots, u_T$. Then the $T \times T$ symmetric matrix $\mathbf{A}^{(j)}$, $j = 1, 2$ as:*

$$\mathbf{A}^{(1)} = \left.\frac{\partial\boldsymbol{\Omega}(\theta)}{\partial\theta}\right|_{\theta=0} = \sum_{t=1}^{T-1}\frac{1}{t}\mathbf{T}^{(t)}, \tag{6}$$

$$\mathbf{A}^{(2)} = \left.\frac{\partial^2\boldsymbol{\Omega}(\theta)}{\partial\theta^2}\right|_{\theta=0} = \frac{\pi^2}{3}\mathbf{I} + \sum_{t=1}^{T-1}\frac{4\left(t\Psi(t)+\gamma\right)+2}{t^2}\mathbf{T}^{(t)}, \tag{7}$$

where $\Psi(t) = \frac{\Gamma'(t)}{\Gamma(t)}$ is the polygamma function and γ is the Euler's constant.

Proof of Theorem 1: See Appendix.

We shall consider general linear combinations of the slope estimates

$$\widehat{\beta}_r(d) = \mathbf{r}'\widehat{\beta}(d).$$

In particular individual β_k can be obtained by choosing appropriate $\mathbf{r}$ vectors. We also define the vectors:

$$\mathbf{c}_r^{(1)}\left(\widehat{d}_0\right)' = \mathbf{r}'\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{A}^{(1)}\mathbf{M},$$

$$\mathbf{c}_r^{(2)}\left(\widehat{d}_0\right)' = \mathbf{r}'\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\left(\mathbf{A}^{(2)} - 2\mathbf{A}^{(1)}\left(\mathbf{I}-\mathbf{M}\right)\mathbf{A}^{(1)}\right)\mathbf{M},$$

where $\mathbf{M} = \mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ is the usual idempotent matrix.

In order to derive our sensitivity measures and a method of correction we shall use the derivatives of the $\widehat{\boldsymbol{\beta}}(d)$ w.r.t. d. The following lemma will be useful in deriving some of the distributional properties:

Lemma 2 *Let the first and second derivatives of* $\widehat{\beta}_r\left(\widehat{d}_0\right)$ *w.r.t.* θ *be:*

$$b_r^{(j)}\left(\widehat{d}_0\right) = \left.\frac{\partial^j \widehat{\beta}_r(d)}{\partial d^j}\right|_{d=\widehat{d}_0}, \text{ j=1,2}$$

then

$$\begin{pmatrix} b_r^{(1)}\left(\widehat{d}_0\right) \\ b_r^{(2)}\left(\widehat{d}_0\right) \end{pmatrix} = \begin{pmatrix} \mathbf{c}_r^{(1)}\left(\widehat{d}_0\right)' \mathbf{y} \\ \mathbf{c}_r^{(2)}\left(\widehat{d}_0\right)' \mathbf{y} \end{pmatrix} \simeq \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \sigma^2 \begin{pmatrix} \sigma_1^2(\theta) & \sigma_{12}(\theta) \\ \sigma_{12}(\theta) & \sigma_2^2(\theta) \end{pmatrix} \right).$$

where $\sigma_1^2(\theta) = \mathbf{c}_r^{(1)}\left(\widehat{d}_0\right)' \mathbf{\Omega}(\theta)\mathbf{c}_r^{(1)}\left(\widehat{d}_0\right)$, $\sigma_2^2(\theta) = \mathbf{c}_r^{(2)}\left(\widehat{d}_0\right)' \mathbf{\Omega}(\theta)\mathbf{c}_r^{(2)}\left(\widehat{d}_0\right)$ *and*

$\sigma_{12}(\theta) = \mathbf{c}_r^{(1)}\left(\widehat{d}_0\right)' \mathbf{\Omega}(\theta)\mathbf{c}_r^{(2)}\left(\widehat{d}_0\right)$.

If the distribution of u is normal then the joint-distribution of $b_r^{(1)}\left(\widehat{d}_0\right)$ and $b_r^{(2)}\left(\widehat{d}_0\right)$ is also normal.

3 Sensitivity of the FGLS

If the true value of d is equal to the pre-estimated value $\widehat{d}_0$ then the FGLS estimator will be equal to the GLS estimator. Majority of the time this will not be the case, since either the pre-estimator is an estimator with statistical error or we can choose the wrong $\widehat{d}_0$ to transform the model. So we want to study the case when the true value of the long memory parameter d is away from $\widehat{d}_0$, and ask the question how different $\widehat{\beta}(d)$ from $\widehat{\beta}\left(\widehat{d}_0\right)$. To do this we use sensitivity analysis similar to the analysis by Banerjee and Magnus (1999) for OLS estimates against short memory processes like the ARMA. This article will go further and discuss a method of correction for the slope estimate if it is sensitive.

3.1 Measuring sensitivity

Developing $\widehat{\beta}_r(d)$ in a Taylor expansion gives:

$$\widehat{\beta}_r(d) - \widehat{\beta}_r\left(\widehat{d}_0\right) = \left(d - \widehat{d}_0\right) \left.\frac{\partial \widehat{\beta}_r(d)}{\partial d}\right|_{d=\widehat{d}_0} + \cdots . \tag{8}$$

We would consider $\widehat{\beta}_r(d)$ and $\widehat{\beta}_r\left(\widehat{d}_0\right)$ to be "almost equal" if

$$\left(d-\widehat{d}_0\right)\left.\frac{\partial\widehat{\beta}_r(d)}{\partial d}\right|_{d=\widehat{d}_0}\approx 0$$

and a sufficient condition for this is that

$$\left.\frac{\partial\widehat{\beta}_r(d)}{\partial d}\right|_{d=\widehat{d}_0}=b_r^{(1)}\left(\widehat{d}_0\right)=0. \tag{9}$$

We shall consider general linear combinations of the slope estimates $\widehat{\beta}_r(d)$. We define the sensitivity of the $\widehat{\beta}_r(d)$ with respect to the difference between d and $\widehat{d}_0$, i.e. $\theta=d-\widehat{d}_0$. as:

$$B_r\left(\widehat{d}_0\right)=\frac{b_r^{(1)}\left(\widehat{d}_0\right)}{\widehat{\sigma}\left(\widehat{d}_0\right)\sqrt{\mathbf{c}_r^{(1)}\left(\widehat{d}_0\right)'\mathbf{c}_r^{(1)}\left(\widehat{d}_0\right)}}, \tag{10}$$

where $\widehat{\sigma}^2\left(\widehat{d}_0\right)=\mathbf{y}'\mathbf{M}\mathbf{y}/(T-k)$ is the estimated variance of the modified model 3.

Since the sensitivity measure $b_r^{(1)}\left(\widehat{d}_0\right)$ will generally be random variables therefore we shall normalise we study the following probabilities as a measure of "closeness" to zero,

$$\pi_r\left(d:\widehat{d}_0\right)=\Pr_{d-\widehat{d}_0}\left(\left|B_r\left(\widehat{d}_0\right)\right|\geq z_\alpha\left(\widehat{d}_0\right)\right), \tag{11}$$

where $\Pr_{d-\widehat{d}_0}$ is the probability measure associated with the random variable $u_t\sim I\left(d-\widehat{d}_0\right)$. The cutoff point $z_\alpha\left(\widehat{d}_0\right)$ is obtained assuming that $d=\widehat{d}_0$:

$$\pi_r\left(\widehat{d}_0:\widehat{d}_0\right)=\Pr_0\left(\left|B_r\left(\widehat{d}_0\right)\right|\geq z_\alpha\left(\widehat{d}_0\right)\right)=\alpha\ ,\ 0<\alpha<1. \tag{12}$$

where $\Pr_0$ is the probability measure associated with white noise. The next section we shall evaluate the distribution of $B_r\left(\widehat{d}_0\right)$. We shall usually take $\alpha=0.05$ for empirical purposes.

In large sample, since $\operatorname{plim}\widehat{\sigma}^2\left(\widehat{d}_0\right)=\sigma^2$, $B_r\left(\widehat{d}_0\right)$ is normally distributed in large sample.

That is

$$B_r\left(\widehat{d_0}\right) \simeq \; N(0,1), \quad \text{if } d = \widehat{d_0} \qquad .$$

In small sample, we should note that $B_r\left(\widehat{d_0}\right)$ is not a standard $t-dist$ since the numerator and the denominator are not independent. So when T is small we have to use IMHOF procedure to compute the distribution of $B_r\left(\widehat{d_0}\right)^2$ as a ratio of two quadratic forms. In our empirical application $\alpha = 0.05$, so $z_{0.95}\left(\widehat{d_0}\right) = 1.96$ in large samples. Therefore as a rule of thumb if

$$\left|B_r\left(\widehat{d_0}\right)\right| > 1.96$$

we shall conclude that the FGLS estimator $\widehat{\beta}_r\left(\widehat{d_0}\right)$ is sensitive. The probability curves are given by (asymptotically)

$$\pi_r\left(d : \widehat{d_0}\right) = \Pr_{d-\widehat{d_0}}\left(\left|B_r\left(\widehat{d_0}\right)\right| \geq 1.96\right).$$

3.2 Correcting sensitivity

The question remains, what should we do if the sensitivity is "large" and we must conclude that the FGLS estimator $\widehat{\beta}_r\left(\widehat{d_0}\right)$ is sensitive? One possible solution is to find $\widehat{d_1}$ as the solution to the equation

$$\left.\frac{\partial\widehat{\beta}_r(d)}{\partial d}\right|_{d=\widehat{d_1}} \equiv b_r^{(1)}\left(\widehat{d_1}\right) = 0, \tag{13}$$

which gives us the least sensitive statistic $\widehat{\beta}_r\left(\widehat{d_1}\right)$. Then $\widehat{d_1}$ is the new estimate for the long memory parameter and $\widehat{\beta}_r\left(\widehat{d_1}\right)$ the least sensitive FGLS estimator for β_r.To find the unknown point $\widehat{d_1}$, we solve the set of equations using a One-Step Newton-Raphson procedure by approximating at $\widehat{d_1}$. Using a Taylor's expansion $b_r^{(1)}\left(\widehat{d_1}\right)$ around $\widehat{d_0}$ and ignoring the higher orders we have,

$$b_r^{(1)}\left(\widehat{d_1}\right) = b_r^{(1)}\left(\widehat{d_0}\right) + \left(\widehat{d_1} - \widehat{d_0}\right) b_r^{(2)}\left(\widehat{d_0}\right) \tag{14}$$

Hence by (13) we have

$$\widehat{d}_1 = \widehat{d}_0 - \frac{b_r^{(1)}\left(\widehat{d}_0\right)}{b_r^{(2)}\left(\widehat{d}_0\right)} = \widehat{d}_0 - CF_r\left(\widehat{d}_0\right).$$

where $CF_r\left(\widehat{d}_0\right)$ is the *correction factor*. Therefore we can write the new estimate of the long-memory parameter d as:

$$\widehat{d}_1 = \widehat{d}_0 - CF_r\left(\widehat{d}_0\right). \tag{15}$$

Then the new estimate is calculated using $\widehat{\beta}_r\left(\widehat{d}_1\right)$. Further extension of the one-step method can be done by using a recursive procedure by computing $\widehat{\beta}_r\left(\widehat{d}_j\right)$,where $\widehat{d}_j$ the j^{th} iterate is obtained conditional on the $j-1^{th}$ iterate $\widehat{d}_{j-1}$. This iteration is computed using the following equation:

$$\widehat{d}_j = \widehat{d}_{j-1} - CF_r\left(\widehat{d}_j\right). \tag{16}$$

The stopping rule will be such that $\widehat{d}_j \approx \widehat{d}_{j-1}$.

Since the sensitivity measure $CF_r\left(\widehat{d}_j\right)$ will generally be random variables, we study the following probabilities as a measure of "closeness" to zero,

$$\Pr_0\left(cf_\alpha\left(\widehat{d}_{j-1}\right) < CF_r\left(\widehat{d}_j\right) < cf_{1-\alpha}\left(\widehat{d}_{j-1}\right)\right), \tag{17}$$

where $\Pr_0$ is the probability measure associated with the random variable $u_t \sim iid$ and $cf_\alpha\left(\widehat{d}_j\right)$ is cutoff from the distribution of $CF_r\left(\widehat{d}_j\right)$.

The following theorem gives us the distribution of $CF_r\left(\widehat{d}_0\right)$ under normality;

Theorem 3 *The distribution of $CF_r\left(\widehat{d}_0\right)$ is given by:*

$$F(cf) = \frac{1}{2} + \sqrt{\frac{2}{\pi}}\arctan\left(\frac{\sigma_2(\theta)\,cf - \rho(\theta)\,\sigma_1(\theta)}{\sigma_1(\theta)\sqrt{1-\rho(\theta)^2}}\right),$$

where $\rho(\theta) = \frac{\sigma_{12}(\theta)}{\sigma_1(\theta)\sigma_2(\theta)}$.

The theorem shows that CF is a symmetric random variable around zero, i.e. the median of $CF = 0$. Also CF does not have higher moments, so we cannot compute the standard deviations but we can compute the α and $1-\alpha$ percentiles as follows:

$$cf_{\alpha} = \frac{\sigma_1}{\sigma_2}\left(\rho + \sqrt{1-\rho^2}\right)\tan\sqrt{\frac{\pi}{2}}\left(\alpha - \frac{1}{2}\right) = -cf_{1-\alpha}.$$

Therefore the stopping rule for the iterative correction method is given by:

$$\widehat{d}_{j-1} + cf_{\alpha} < \widehat{d}_j < \widehat{d}_{j-1} + cf_{1-\alpha}.$$

Again in our empirical section we take $\alpha = 0.05$.

This condition gives us the stopping rule for the iteration, but the convergence is not guaranteed which depends on the data and the starting point $\widehat{d}_0$. Spall (2000) provides conditions for global convergence of such stochastic Newton-Raphson method, but it is difficult to ensure them as the higher moments of $CF(\widehat{d}_j)$ do not exist. Therefore the possibility of finding a locally insensitive point depending on the starting value $\widehat{d}_0$. This is less of a problem as we are looking for a locally insensitive point in the analysis. Though we can compare one or more 'insensitive' points by starting from different $\widehat{d}_0$. More problematically non-convergence might also be an issue.

3.3 Simulation

We study the properties of the sensitivity curve $\pi_r\left(d : \widehat{d}_0\right)$ and the sensitivity correction distribution $cf_{\alpha}\left(d : \widehat{d}_0\right)$ by using the following models

$$y_t = \beta_1 + \beta_2 t + \varepsilon_{0,t},. \tag{18}$$

$$y_t = \alpha_1 + \varepsilon_{0,t}, \quad t = 1, ..., 100. \tag{19}$$

where $\varepsilon_{0,t} \sim I(d)$ process with normal innovations.

We study the properties of $\widehat{\beta}_2$ using $\pi_2\left(d : \widehat{d}_0\right)$ and $cf_{95}(d : \widehat{d}_0)$. Figure 1.1 and 1.2 shows the sensitivity curves $\pi_2\left(d : \widehat{d}_0\right)$ and Figure 2.1 and 2.2 shows the 5th and the 95th percentiles of the correction distribution of model (18) and (19) respectively. It is to be noted that the sensitivity increases as $d > \widehat{d}_0$, this implies that underestimation of the long memory parameter makes the model estimates more sensitive. The probability of sensitivity can be as high as 90%.

The correction distribution $cf_{95}(d : \widehat{d}_0)$ and $cf_{05}(d : \widehat{d}_0)$ shows that, the bounds are highly irregular implying that convergence of the correction procedure can be a problem. In our empirical models we do find the problem of convergence with the correction procedure.

[Insert Figure 1.1 and Figure 1.2]

[Insert Figure 2.1 and Figure 2.2]

4 Empirical model and analysis

We make use of the GDP per capita (PCGDP) data (in USD) for the years from 1902 to 2003, which is obtained from Maddison's Total Economy Database (http://www.ggdc.net) website.

We will make use of the existing work and model log of PCGDP (y_t) as a deterministic trend model (DTM):

$$y_t = \beta_1 + \beta_2 t + \varepsilon_{1,t}, \quad t = 1902, ..., 2003. \tag{20}$$

where β_2 is the deterministic long-run growth rate of the country. We assume that the error term $\varepsilon_{1,t}$ follows a $I(d)$ process in all the models as defined in 2.

We shall consider the OLS estimates $\widehat{\beta}_2(0)$ and the FGLS estimates $\widehat{\beta}_2\left(\widehat{d}_G\right)$ and $\widehat{\beta}_2\left(\widehat{d}_W\right)$ where $\widehat{d}_G$ is the usual GPH estimate and $\widehat{d}_W$ is the Whittle estimate (see Phillips and Shimotsu, 2005, Shimotsu 2010[3]) of the longmemory parameter d. We compute the corresponding sensitivity statistic $B_2(0)$, $B_2\left(\widehat{d}_G\right)$ and $B_2\left(\widehat{d}_W\right)$ to check for sensitivity for the growth estimates. If these are sensitive, as a next step we use the correction procedure to correct for the growth rates as described in the previous section.

The second model we use is the stochastic growth model (STM), i.e.

$$\Delta y_t = \alpha_1 + \varepsilon_{2,t}, \quad t = 1903, ..., 2003. \tag{21}$$

[3]Thanks to Katsumi Shimotsu for providing the matlab code for the proccedure (http://www.econ.hit-u.ac.jp/~shimotsu/Site/Matlab_Codes.html).

where $\Delta y_t = y_t - y_{t-1}$ is the annual growth rates of each country and α_1 is the long-run average growth rate. We assume that the error term $\varepsilon_{2,t}$ follows a $I(d)$ process in all the models as defined in (2).

As with the first model we shall consider the OLS estimates $\widehat{\alpha}_1(0)$ and the FGLS estimates $\widehat{\alpha}_1\left(\widehat{d}_G\right)$ and $\widehat{\alpha}_1\left(\widehat{d}_W\right)$ where $\widehat{d}_G$ is the usual GPH estimate and $\widehat{d}_W$ is the Whittle estimate of the long-memory parameter d. We analyse the sensitivity of the growth rates and correct them if needed as before.

Note that since model (21) can be obtained from (20) by differencing $u_t^{(2)} = \Delta u_t^{(1)}$ and $\alpha_1 = \beta_2$.

4.1 Results

Consider the results of the DTM (20) for the Unites States of America (USA). The OLS estimates in Table 1. i.e. $\widehat{d}_0 = 0$, which shows that $B_2(0) = 2.40$. This means the OLS estimator for long run growth $\widehat{\beta}_2(0) = 1.976\%$ is sensitive to long memory. As a next step, we use our correction method and we see that $d_{61} = 0.105$ (i.e. it took 61 steps to converge), and $B_2(d_{61}) =$ -1.558 with $\widehat{\beta}_2(d_{61}) = 1.9831\%$. Now if we consider the FGLS estimator with the GPH parameter (Table 2) $\widehat{d}_G = 0.462$, the growth rate is $\widehat{\beta}_2\left(\widehat{d}_G\right) = 1.9848\%$ with the sensitivity statistic $B_2\left(\widehat{d}_G\right) =$ -8.315 which makes it even more sensitive than the OLS estimated rate. Correcting for the long-memory parameter get $d_{27} =$ -0.012 with growth rate 1.9745% and $B_2(-0.012) = 0.3921$. Similarly the Whittle estimator (Table 3) is $\widehat{d}_W = 0.713$ with $\widehat{\beta}_2\left(\widehat{d}_G\right) =$ 1.971% and $B_2(\widehat{d}_W) =$ -8.910, correcting that in 13 steps gives us $d_{13} = -0.009$ with growth rate of 1.975% and corresponding $B_2(-0.009) =$0.975. This means that the United States PCGDP is modelled as a DTM has the least sensitive growth rate of 1.975% with sensitivity level of $B_2(-0.012) = 0.3921$. One might argue that the initial estimates (1.976%, 1.9848% and 1.987% under different estimated values of d) of the growth rates are not that different in case of USA, so practically it does not matter. But this might not be the case for all countries. For example, the initial estimates for United Kingdom are OLS: $\widehat{\beta}_2(0) = 1.601\%$, GPH: $\widehat{\beta}_2(0.737) = 1.547\%$ and Whittle $\widehat{\beta}_2(0.915) = 1.523\%$. After correction the most stable growth rate turns out to be

1.601% the OLS growth rate. The initial estimates of Portugal's long run growth OLS: $\widehat{\beta}_2(0) =$ 2.779%, GPH: $\widehat{\beta}_2(1.301) = 2.494\%$ and Whittle $\widehat{\beta}_2(1.188) = 2.539\%$. After correction the most stable growth rate turns out to be 2.792% at $d = 0.07$. Uruguay's OLS estimate (at 1.102%) is stable but the FGLS estimates are not. Indonesia with initial estimates of 1.494%, 1.123% and 1.133% and India with initial growth estimates of 0.887% 0.550% and 0.525% does not have any insensitive estimates even after 100 iterations. This implies long-run growth estimates for India are sensitive to stochastic shocks and we should be careful about interpreting India's growth trajectory.

Next we consider the stochastic growth model. Again take the example of USA. The average of the annual growth rate (OLS estimate) in Table 4 show that the long run growth $\widehat{\alpha}_1(0) =$ 1.8637% but is insensitive to long memory $B_1(0) = 0.3071$. The GPH estimator (in Table 5) $\widehat{d}_G =$ -0.4385 gives a growth rate of 1.7743% which is sensitive $B_1(\widehat{d}_G) = 2.1297$ but the correction method fails to converge (in 100 steps). The Whittle estimator (in Table 6) is insensitive, $\widehat{d}_W =$ -0.176, with $\widehat{\alpha}_1\left(\widehat{d}_W\right) = 1.824\%$ and $B_1(\widehat{d}_W) = 1.0177$. From the analysis of both the models $B_1(-0.015) =$ -0.214, is the least among the other sensitivity measure, so the best possible stable long run growth is 1.974%. Though it is to be noted that the growth estimates from the stochastic model are very different from the linear trend model.

Comparing the estimates of the two models, a important observation is that almost all the OLS estimates of stochastic growth model are insensitive to longmemory parameters than the DTM (with exception of small economies like Venezuela and Portugal). Table 7 summarises the sensitivity results. Secondly, LS growth estimates from STM are generally lower than the linear trend model (24 out of 30 for OLS, 23 out of 30 for FGLS with both GPH and Whittle Estimates). The median difference for the OLS estimate is -0.16 (-16 basis points) and is statistically significant using a signtest. The same is true for the FGLS estimate difference. Using the correction method on the OLS as proposed, 21 out of 26, growth estimates from STM are lower than the DTM. The same is true for the FGLS estimates. So even after correcting for sensitivity the stochastic model produces lower growth estimates than the trend model. Figure 3, demonstrates this using a kernel density estimate for the growth rates obtained from STM and DTM using OLS.

[Insert Figure 3]

[Insert Table 7]

Therefore less sensitive growth rates are produced by models which also have lower estimated growth rates than the more sensitive estimates.

5 Conclusion

We develop a method of sensitivity of slope estimates when the error process is a long memory process. we argue that the long run growth estimates when the log-output process is an I(d) process is sensitive to the estimates of the long memory parameters. The implication of a sensitive numerical estimate of the growth has implications to the evaluation of the long run economic trajectory of countries. We also provide a methodology to correct for the sensitivity of the growth estimates. One of the major finding is that a stochastic growth model, which averages the annual growth rates are less sensitive to long memory parameter misspecification than the deterministic trend model. But the growth estimates from the stochastic growth model are significantly lower than the growth estimates of the deterministic trend model.

Appendix

Proof of Lemma 1: Let $z = \exp(-\boldsymbol{i}x)$, then $(1-z)(1-z^{-1}) = 2\sin^2\left(\frac{x}{2}\right)$ where $\boldsymbol{i}=\sqrt{-1}$. Consider the function $\left[(1-z)(1-z^{-1})\right]^{-d}$. Let the power series expansion be

$$\left[(1-z)\left(1-z^{-1}\right)\right]^{-d} = \sum_{h=0}^{\infty} \omega_h(d) \exp(\boldsymbol{i}xh).$$

Note that $\omega_h(d)$ can be obtained from by using the Integral transform:

$$\frac{1}{2\pi}\int_0^{2\pi}\left[(1-z)\left(1-z^{-1}\right)\right]^{-d}\exp(-\boldsymbol{i}xh)\,dx=\omega_h(d)$$
$$\frac{4^{-d}}{\pi}\int_0^{\pi}\sin^{-2d}(x)\exp(-\boldsymbol{i}xh)\,dx=\omega_h(d)\,, \tag{22}$$

since the Fourier coefficients of the sine part of the transform is zero, then

$$\frac{4^{-d}}{\pi}\int_0^{\pi}\sin^{-2d}(x)\cos(2hx)=\omega_h(d)\,.$$

Then by (Erdélyi et.al (1953, p. 12) we have

$$\omega_h(d)=\frac{(-1)^h\Gamma(1-2d)}{\Gamma(-d-h+1)\Gamma(-d+h+1)}. \tag{23}$$

Now note that

$$\left.\frac{\partial^j\left[(1-z)\left(1-z^{-1}\right)\right]^{-d}}{\partial d^j}\right|_{d=0}=\left[\log(1-z)\left(1-z^{-1}\right)\right]^j,\ j=,1,2.$$

Differentiation under the integral sign in (22) on both sides gives

$$\left.\frac{\partial^j\omega_h(d)}{\partial d^j}\right|_{d=0}=\frac{1}{2\pi}\int_0^{2\pi}\left[\log(1-z)\left(1-z^{-1}\right)\right]^j\exp(-\boldsymbol{i}xh)\,dx,$$
$$=\frac{1}{2\pi}\int_0^{2\pi}\left[\log 2\sin^2\left(\frac{x}{2}\right)\right]^j\exp(-\boldsymbol{i}xh)\,dx_{d=0},\ j=1,2.$$

From (23) we get

$$\left.\frac{\partial\omega_h(d)}{\partial d}\right|_{d=0}=\begin{cases}0, & \text{if } h=0\\ \frac{1}{h}, & h>0\end{cases}$$

and

$$\left.\frac{\partial^2\omega_h(d)}{\partial d^2}\right|_{d=0}=\begin{cases}\frac{\pi^2}{3}, & \text{if } h=0\\ \frac{2+4h(\Psi(h)+\gamma)}{h^2} & h>0\end{cases}\,.$$

QED.

Proof of Theorem 2:

Define $\frac{\partial^j \mathbf{\Omega}(\theta)^{-1}}{\partial \theta^j} = \mathbf{A}^{(j)}(\theta)$ and $\widehat{\mathbf{e}}(\theta) = \left(\mathbf{y} - \mathbf{X}\widehat{\beta}(\theta)\right)$. Then,

$$\frac{\partial \widehat{\beta}(\theta)}{\partial \theta} = -\left(\mathbf{X}'\mathbf{\Omega}(\theta)^{-1}\mathbf{X}\right)^{-1}\mathbf{X}\mathbf{A}^{(1)}(\theta)\,\widehat{\mathbf{e}}(\theta).$$

Hence, at $\theta = 0$

$$\mathbf{b}_r^{(1)} = -\mathbf{r}'\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{A}^{(1)}\widehat{\mathbf{e}} = -\mathbf{r}'\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{A}^{(1)}\mathbf{M}\mathbf{y} = \mathbf{c}_r^{(1)}\left(\widehat{d}_0\right)\mathbf{y} = \mathbf{c}_r^{(1)}\left(\widehat{d}_0\right)\mathbf{u}.$$

$$\begin{aligned}
\frac{\partial^2 \widehat{\beta}(\theta)}{\partial \theta^2} &= -\left[\frac{\partial\left(\mathbf{X}'\mathbf{\Omega}(\theta)^{-1}\mathbf{X}\right)^{-1}}{\partial \theta}\mathbf{X}'\mathbf{A}^{(1)}(\theta) + \left(\mathbf{X}'\mathbf{\Omega}(\theta)^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{A}^{(2)}(\theta)\right]\widehat{\mathbf{e}}(\theta) \\
&\quad + \left(\mathbf{X}'\mathbf{\Omega}(\theta)^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{A}^{(1)}(\theta)\,\mathbf{X}\frac{\partial \widehat{\beta}(\theta)}{\partial \theta} \\
&= -\left[\begin{array}{c} \frac{\partial\left(\mathbf{X}'\mathbf{\Omega}(\theta)^{-1}\mathbf{X}\right)^{-1}}{\partial \theta}\mathbf{X}'\mathbf{A}^{(1)}(\theta) + \left(\mathbf{X}'\mathbf{\Omega}(\theta)^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{A}^{(2)}(\theta) \\ + \left(\mathbf{X}'\mathbf{\Omega}(\theta)^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{A}^{(1)}(\theta)\,\mathbf{X}\left(\mathbf{X}'\mathbf{\Omega}(\theta)^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{A}^{(1)}(\theta) \end{array}\right]\widehat{\mathbf{e}}(\theta) \\
&= -\left[\begin{array}{c} -\left(\mathbf{X}'\mathbf{\Omega}(\theta)^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{A}^{(1)}(\theta)\,\mathbf{X}\left(\mathbf{X}'\mathbf{\Omega}(\theta)^{-1}\mathbf{X}\right)^{-1}\mathbf{X}\mathbf{A}^{(1)}(\theta) \\ + \left(\mathbf{X}'\mathbf{\Omega}(\theta)^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{A}^{(2)}(\theta) \\ -\left(\mathbf{X}'\mathbf{\Omega}(\theta)^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{A}^{(1)}(\theta)\,\mathbf{X}\left(\mathbf{X}'\mathbf{\Omega}(\theta)^{-1}\mathbf{X}\right)^{-1}\mathbf{X}\mathbf{A}^{(1)}(\theta) \end{array}\right]\widehat{\mathbf{e}}(\theta)
\end{aligned}$$

then at $\theta = 0$

$$\begin{aligned}
\mathbf{b}_r^{(2)} &= -\mathbf{r}'\left[\begin{array}{c} -\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{A}^{(1)}\mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}\mathbf{A}^{(1)} + \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}\mathbf{A}^{(2)} \\ -\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}\mathbf{A}^{(1)}\mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}\mathbf{A}^{(1)} \end{array}\right]\widehat{\mathbf{e}} \\
&= \mathbf{r}'\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\left[\mathbf{A}^{(2)} - 2\mathbf{A}^{(1)}\mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{A}^{(1)}\right]\widehat{\mathbf{e}} \\
&= \mathbf{r}'\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\left[\mathbf{A}^{(2)} - 2\mathbf{A}^{(1)}\mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{A}^{(1)}\right]\mathbf{M}\mathbf{y} = \mathbf{c}_r^{(2)}\left(\widehat{d}_0\right)\mathbf{y} = \mathbf{c}_r^{(2)}\left(\widehat{d}_0\right)\mathbf{u}.
\end{aligned}$$

Since $\varepsilon_t = \Delta^{-\left(d-\widehat{d}_0\right)}u_t$ the result follows.

Proof of Theorem 3: See Appendix.

Following Hinkley (1969) if $(u_1, u_2) \sim BN(0, 0, \sigma_1^2, \sigma_2^2, \rho)$ and let $u = \frac{u_1}{u_2}$, the the distribution of u is given by

$$F(u) = 2\mathcal{L}\left(\frac{\sigma_2 u - \rho\sigma_1}{\sqrt{u^2\sigma_2^2 - 2u\sigma_2\rho\sigma_1 + \sigma_1^2}}\right),$$

where

$$\mathcal{L}(\theta) = \frac{1}{4} + \frac{1}{\sqrt{2\pi}}\arctan\left(\sqrt{\frac{\theta^2}{1-\theta^2}}\right).$$

Therefore,

$$F(u) = \frac{1}{2} + \sqrt{\frac{2}{\pi}}\arctan\left(\sqrt{\frac{(\sigma_2 u - \rho\sigma_1)^2}{(1-\rho^2)\sigma_1^2}}\right).$$

QED.

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Figures

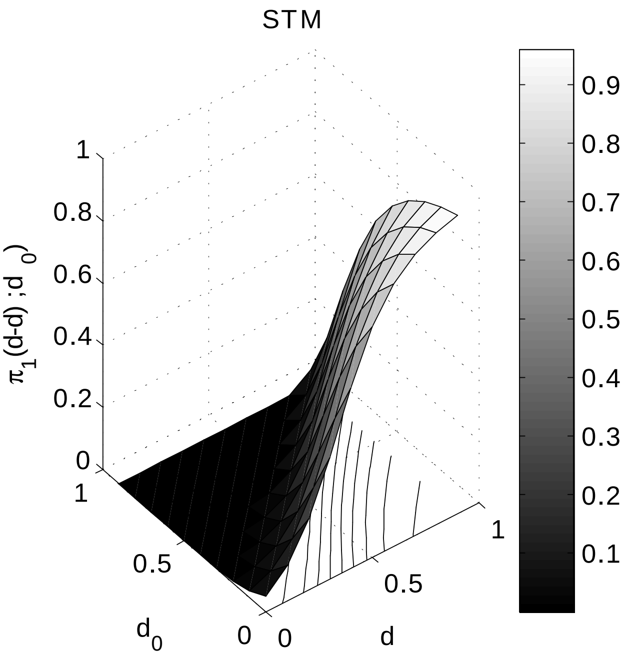

Figure 1.1: Sensitivity curves model 19.

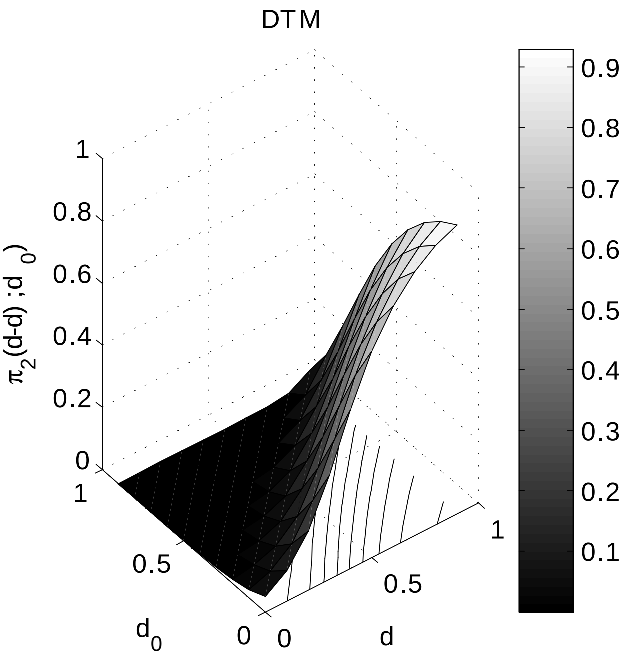

Figure 1.2: Sensitivity curves model 18.

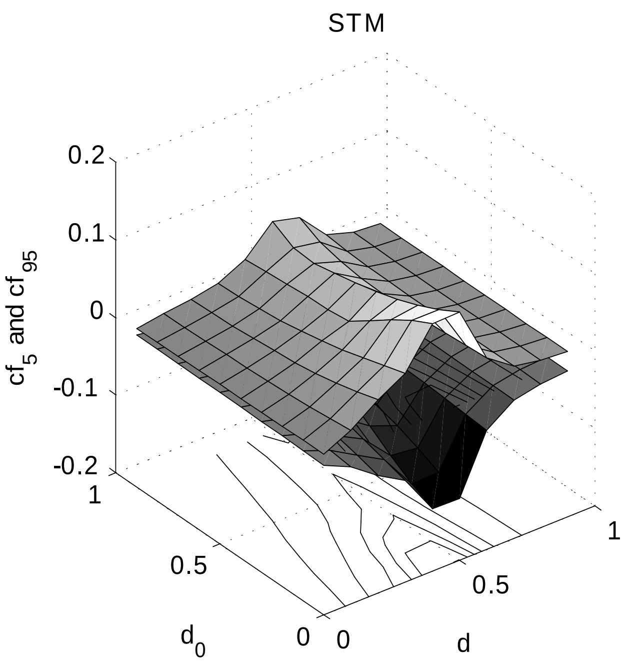

Figure 2.1: The 95th percentile of $CF(d, \widehat{d_0})$ of model 19.

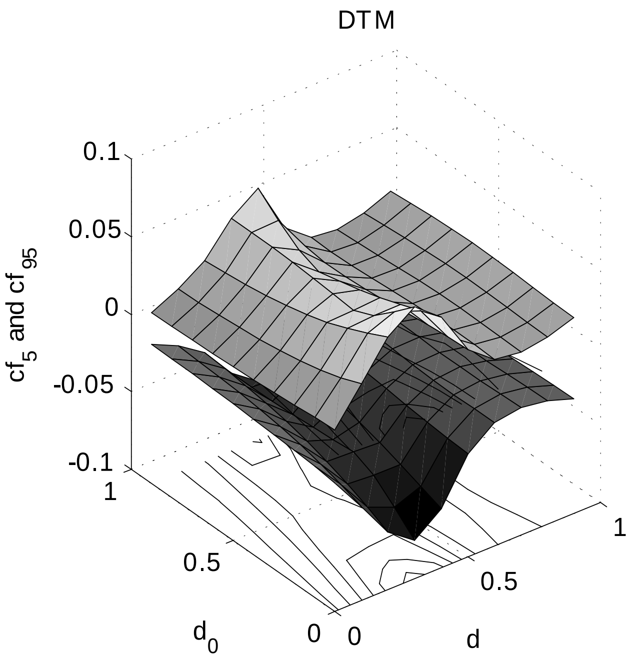

Figure 2.2: The 95th and the 5th percentile of $CF(d, \widehat{d_0})$ of model 18.

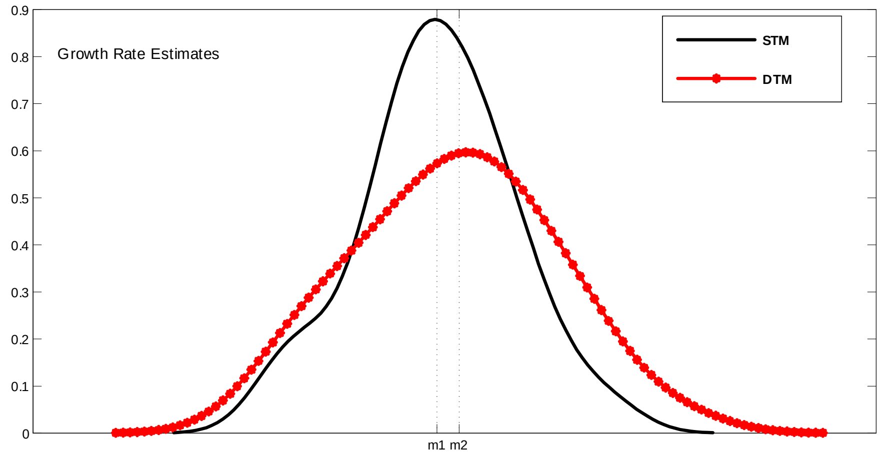

Figure 3: Kernel Density of OLS growth rate estimates of STM and DTM.

Tables

Table 1: Deterministic Trend Model: OLS

	$\beta_2(0)\%$	$B_2(0)$	$\widehat{d}_I$	$\beta_2(\widehat{d}_I)\%$	$B_2(\widehat{d}_I)$	I
Austria	2.3171	3.1533	-0.0368	2.3053	1.5316	3
Belgium	1.9142	2.4304	0.0763	1.9226	-0.3789	4
Denmark	2.1168	3.2342	0.1112	2.1265	-1.9388	12
Finland	2.7844	5.4000	0.0774	2.7998	1.2360	8
France	2.2970	2.9296	-0.0319	2.2901	0.2065	3
Germany	2.1755	3.2877	0.1100	2.1967	-1.5011	6
Italy	2.4902	2.2630	0.0629	2.4991	0.7676	4
Netherlands	1.9714	1.8908	0.0000	1.9714	1.8908	1
Norway	2.7681	2.9401	-0.0229	2.7658	-1.6391	3
Sweden	2.3802	6.6223	-0.0336	2.3721	-0.4683	11
Switzerland	2.0832	6.1103	0.1354	2.1127	-1.8748	22
United Kingdom	1.6009	1.3180	0.0000	1.6009	1.3180	1
Portugal	2.7790	3.3911	0.0688	2.7922	0.3030	6
Spain	2.3463	1.2383	0.0000	2.3463	1.2383	1
Australia	1.7300	1.2764	0.0000	1.7300	1.2764	1
New Zealand	1.4422	4.2694	-0.0192	1.4393	0.4206	11
Canada	2.0818	3.6145	0.1104	2.0973	-1.5883	4
United States	1.9756	2.4025	0.1047	1.9831	-1.5580	62
Argentina	1.0836	3.5511	-0.0108	1.0823	1.2125	41
Brazil	2.3954	5.8406	0.1241	2.4202	-1.4601	8
Chile	1.3479	-2.4072			-4.0595	100
Colombia	1.8016	5.2301	0.1152	1.8115	-1.7992	14
Mexico	1.7877	3.7227	0.0627	1.7987	1.4856	4
Peru	1.6451	1.5733	0.0000	1.6451	1.5733	1
Uruguay	1.1019	0.7317	0.0000	1.1019	0.7317	1
Venezuela	2.7468	2.2336			7.0760	100
India	0.8874	-3.0952			65535.0000	100
Indonesia	1.4938	-2.6293			-3.9698	100
Japan	3.2889	2.4898	-0.0427	3.2770	0.6349	9
Philippines	1.1800	-1.7705	0.0000	1.1800	-1.7705	1

Table 2: Deterministic Trend Model: FGLS with GPH estimates

	$\widehat{d}_G$	$\beta_2(\widehat{d}_G)\%$	$B_2(\widehat{d}_G)$	$\widehat{d}_I$	$\beta_2(\widehat{d}_I)\%$	$B_r(\widehat{d}_I)$	I
Austria	0.7767	2.2419	-8.8507	0.0681	2.3339	0.5008	4
Belgium	1.1637	1.7254	-9.2531	0.0619	1.9214	0.5034	9
Denmark	0.9859	2.0774	-9.1614	0.1034	2.1260	-1.4726	28
Finland	0.7447	2.8176	-8.8243	0.1244	2.8073	-1.9120	6
France	0.8288	2.2612	-8.9521	0.1144	2.3153	-1.9518	8
Germany	0.6386	2.1851	-8.6536	0.1002	2.1952	-0.9141	2
Italy	0.8254	2.4277	-8.9652	0.0981	2.5030	-1.3252	4
Netherlands	0.7630	1.9235	-8.9410	-0.0164	1.9694	0.4722	7
Norway	0.6937	2.7444	-8.8634	-0.0219	2.7659	-1.3579	3
Sweden	0.9789	2.4594	-9.0978	0.1320	2.4072	-1.9564	66
Switzerland	1.0664	2.2038	-9.1727	0.1358	2.1128	-1.8971	23
United Kingdom	0.7366	1.5474	-8.9771	0.0017	1.6010	1.5186	9
Portugal	1.3013	2.4936	-9.2774	0.0929	2.7953	-1.1830	4
Spain	1.3302	1.9243	-9.3112	-0.0538	2.3385	-1.9032	5
Australia	1.1829	1.6112	-9.3124	0.0040	1.7302	1.6429	8
New Zealand	0.9872	1.4581	-9.1526	0.0804	1.4529	0.4431	10
Canada	0.5971	2.1069	-8.6351	0.0983	2.0959	-0.8326	45
United States	0.4616	1.9848	-8.3155	-0.0122	1.9745	0.3921	28
Argentina	0.5405	1.1299	-8.5189	-0.0121	1.0822	0.9261	10
Brazil	0.9780	2.4262	-9.0601	0.1279	2.4208	-1.6852	6
Chile	0.6611	1.2403	-8.9877	0.0653	1.3394	-1.7094	2
Colombia	0.5740	1.8176	-8.6147	0.1155	1.8115	-1.8198	17
Mexico	1.1504	1.7102	-9.2163	0.1154	1.8061	-1.9426	11
Peru	1.0050	1.8830	-9.1748	0.0023	1.6453	1.9163	5
Uruguay	-0.2272	1.0934	-9.2765			-4.5993	100
Venezuela	0.8176	3.2350	-8.6860	-0.0053	2.7451	1.7467	3
India	1.1556	0.5499	-9.3847			9.3527	100
Indonesia	1.1125	1.1229	-8.9875			-8.5446	100
Japan	1.0387	3.1003	-9.0607	-0.0223	3.2830	1.8445	4
Philippines	0.4154	1.1680	-7.9953			-9.0530	100

Table 3: Deterministic Trend Model: FGLS with Whittle estimates

	$\widehat{d}_W$	$\beta_2(\widehat{d}_W)\%$	$B_2(\widehat{d}_W)$	$\widehat{d}_I$	$\beta_2(\widehat{d}_I)\%$	$B_r(\widehat{d}_I)$	I
Austria	0.9295	2.1849	-9.0039	-0.0454	2.3023	0.7277	3
Belgium	1.1530	1.7286	-9.2472	0.0729	1.9223	-0.1708	9
Denmark	1.0022	2.0752	-9.1735	0.1112	2.1265	-1.9393	24
Finland	0.9704	2.7951	-9.0832	0.0990	2.8034	-0.3111	12
France	1.2585	2.1523	-9.2738	-0.0177	2.2933	1.7783	4
Germany	0.8608	2.1449	-8.9632	0.1160	2.1976	-1.8459	6
Italy	0.9904	2.3828	-9.1085	0.0641	2.4993	0.6994	3
Netherlands	0.9141	1.8941	-9.0900	-0.0177	1.9692	0.3298	11
Norway	0.9426	2.7126	-9.1355	0.1061	2.7749	-1.8130	18
Sweden	1.2050	2.4503	-9.2720	0.1319	2.4072	-1.9533	61
Switzerland	1.0493	2.2036	-9.1591	0.1367	2.1130	-1.9475	13
United Kingdom	0.9148	1.5225	-9.1491	0.0013	1.6010	1.4664	7
Portugal	1.1881	2.5390	-9.2152	0.0692	2.7922	0.2785	5
Spain	1.2513	1.9628	-9.2766	-0.0426	2.3404	-0.9032	4
Australia	1.0008	1.6420	-9.2045	-0.0119	1.7294	-0.6885	5
New Zealand	0.9838	1.4583	-9.1498	-0.0221	1.4389	-0.3399	19
Canada	0.9101	2.0790	-9.0679			0.0274	100
United States	0.7133	1.9720	-8.8949	0.1114	1.9834	-1.9559	20
Argentina	0.9920	1.1376	-9.1602	-0.0220	1.0810	-1.2502	45
Brazil	1.0581	2.4172	-9.1282	0.1319	2.4215	-1.9171	9
Chile	0.7216	1.2299	-9.0589			-4.0595	100
Colombia	0.8499	1.8050	-9.0377	0.1174	1.8116	-1.9312	12
Mexico	1.2571	1.6848	-9.2781	0.1152	1.8061	-1.9325	17
Peru	0.9233	1.8622	-9.0945	-0.0180	1.6436	-0.9397	7
Uruguay	0.4705	1.1040	-8.3960			-4.6220	100
Venezuela	1.3207	3.5947	-9.3746			7.0760	100
India	1.2448	0.5253	-9.4150	2.7781	0.2447	-0.4986	9
Indonesia	1.1334	1.1161	-8.9949			-3.9698	100
Japan	1.1919	3.0319	-9.1629	-0.0635	3.2705	-0.9568	4
Philippines	0.6873	1.1732	-8.7004	0.0263	1.1776	0.3769	3

Table 4: Stochastic Trend Model: OLS

	$\alpha_1(0)\%$	$B_1(0)$	$\widehat{d_I}$	$\alpha_1(\widehat{d_I})\%$	$B_1(\widehat{d_I})$	I
Austria	1.9557	0.5397	0.0000	1.9557	0.5397	1
Belgium	1.7182	0.7430	0.0000	1.7182	0.7430	1
Denmark	1.9769	0.5282	0.0000	1.9769	0.5282	1
Finland	2.5311	0.6044	0.0000	2.5311	0.6044	1
France	2.0435	0.5717	0.0000	2.0435	0.5717	1
Germany	1.8708	0.4761	0.0000	1.8708	0.4761	1
Italy	2.3294	0.3473	0.0000	2.3294	0.3473	1
Netherlands	1.7843	0.5679	0.0000	1.7843	0.5679	1
Norway	2.5841	1.2945	0.0000	2.5841	1.2945	1
Sweden	2.1345	0.7768	0.0000	2.1345	0.7768	1
Switzerland	1.7540	1.6197	0.0000	1.7540	1.6197	1
United Kingdom	1.5342	0.5618	0.0000	1.5342	0.5618	1
Portugal	2.3658	2.0410	0.1026	2.4146	-0.0912	2
Spain	2.2064	0.5576	0.0000	2.2064	0.5576	1
Australia	1.7891	-1.1833	0.0000	1.7891	-1.1833	1
New Zealand	1.3619	-0.2766	0.0000	1.3619	-0.2766	1
Canada	1.9232	0.3811	0.0000	1.9232	0.3811	1
United States	1.8637	0.3071	0.0000	1.8637	0.3071	1
Argentina	1.0272	-0.4305	0.0000	1.0272	-0.4305	1
Brazil	2.0317	1.9206	0.0000	2.0317	1.9206	1
Chile	1.5482	-0.3319	0.0000	1.5482	-0.3319	1
Colombia	1.6180	1.4060	0.0000	1.6180	1.4060	1
Mexico	1.6502	-0.1182	0.0000	1.6502	-0.1182	1
Peru	1.6499	-0.5485	0.0000	1.6499	-0.5485	1
Uruguay	0.9632	0.7217	0.0000	0.9632	0.7217	1
Venezuela	2.0788	2.1791			2.8000	100
India	1.1820	-1.5541	0.0000	1.1820	-1.5541	1
Indonesia	2.2254	0.0612	0.0000	2.2254	0.0612	1
Japan	2.9046	0.8647	0.0000	2.9046	0.8647	1
Philippines	2.2882	0.0889	0.0000	2.2882	0.0889	1

Table 5: Stochastic Trend Model: FGLS with GPH estimates

	$\widehat{d_G}$	$\alpha_1(\widehat{d_G})\%$	$B_1(\widehat{d_G})$	$\widehat{d_I}$	$\alpha_1(\widehat{d_I})\%$	$B_r(\widehat{d_I})$	I
Austria	-0.2902	1.6196	1.3439	-0.2902	1.6196	1.3439	1
Belgium	0.3365	1.7083	-0.3614	0.3365	1.7083	-0.3614	1
Denmark	0.1477	1.9963	0.3866	0.1477	1.9963	0.3866	1
Finland	-0.0793	2.5120	1.0601	-0.0793	2.5120	1.0601	1
France	-0.1237	1.9868	1.0017	-0.1237	1.9868	1.0017	1
Germany	-0.3293	1.7723	1.0991	-0.3293	1.7723	1.0991	1
Italy	-0.1257	2.3121	0.6912	-0.1257	2.3121	0.6912	1
Netherlands	-0.2111	1.6291	1.2050	-0.2111	1.6291	1.2050	1
Norway	-0.3498	2.0193	4.0031	0.4349	2.5983	1.2316	2
Sweden	0.1663	2.1629	0.7213	0.1663	2.1629	0.7213	1
Switzerland	-0.0028	1.7525	1.6138	-0.0028	1.7525	1.6138	1
United Kingdom	-0.1875	1.4335	2.5903	0.3229	1.4961	-0.6111	2
Portugal	0.5365	2.3155	0.0025	0.5365	2.3155	0.0025	1
Spain	0.3298	2.1359	-1.1007	0.3298	2.1359	-1.1007	1
Australia	0.1730	1.7527	-0.5249	0.1730	1.7527	-0.5249	1
New Zealand	-0.0048	1.3624	-0.2553	-0.0048	1.3624	-0.2553	1
Canada	-0.3456	1.7326	2.0340	0.9819	1.8510	1.3961	2
United States	-0.4385	1.7743	2.1297			3.1288	100
Argentina	-0.3456	1.9985	-0.6030	-0.3456	1.9985	-0.6030	1
Brazil	0.3557	2.1422	1.4812	0.3557	2.1422	1.4812	1
Chile	-0.2427	1.4384	0.9552	-0.2427	1.4384	0.9552	1
Colombia	0.0484	1.6312	1.3732	0.0484	1.6312	1.3732	1
Mexico	0.2388	1.6661	0.5963	0.2388	1.6661	0.5963	1
Peru	0.0076	1.6484	-0.4305	0.0076	1.6484	-0.4305	1
Uruguay	-0.5730	0.9220	1.2056	-0.5730	0.9220	1.2056	1
Venezuela	-0.0002	2.0785	2.1738			2.8002	100
India	0.5130	0.8412	-2.2149	5.8035	0.1896	0.8045	2
Indonesia	0.0654	2.2243	-0.5502	0.0654	2.2243	-0.5502	1
Japan	0.0497	2.9304	0.6442	0.0497	2.9304	0.6442	1
Philippines	0.3388	2.4008	1.9757	-0.2281	2.4806	0.1519	2

Table 6: Stochastic Trend Model: FGLS with Whittle estimates

	$\widehat{d_W}$	$\alpha_1(\widehat{d_W})\%$	$B_1(\widehat{d_W})$	$\widehat{d_I}$	$\alpha_1(\widehat{d_I})\%$	$B_r(\widehat{d_I})$	I
Austria	-0.0522	1.9270	0.8327	-0.0522	1.9270	0.8327	1
Belgium	0.1356	1.7349	-0.5076	0.1356	1.7349	-0.5076	1
Denmark	0.0024	1.9773	0.5194	0.0024	1.9773	0.5194	1
Finland	-0.0171	2.5271	0.7088	-0.0171	2.5271	0.7088	1
France	0.2738	2.0769	0.1260	0.2738	2.0769	0.1260	1
Germany	-0.1190	1.8198	0.7088	-0.1190	1.8198	0.7088	1
Italy	-0.0107	2.3276	0.3656	-0.0107	2.3276	0.3656	1
Netherlands	-0.0730	1.7509	0.8489	-0.0730	1.7509	0.8489	1
Norway	-0.0309	2.5701	1.5800	-0.0309	2.5701	1.5800	1
Sweden	0.1838	2.1653	0.7770	0.1838	2.1653	0.7770	1
Switzerland	0.0372	1.7718	1.6495	0.0372	1.7718	1.6495	1
United Kingdom	-0.0702	1.5166	1.6245	-0.0702	1.5166	1.6245	1
Portugal	0.1748	2.4241	-0.5921	0.1748	2.4241	-0.5921	1
Spain	0.2565	2.1671	-1.2761	0.2565	2.1671	-1.2761	1
Australia	0.0045	1.7878	-1.2181	0.0045	1.7878	-1.2181	1
New Zealand	-0.0431	1.3677	-0.0737	-0.0431	1.3677	-0.0737	1
Canada	-0.0766	1.9076	0.7510	-0.0766	1.9076	0.7510	1
United States	-0.2391	1.8039	1.2699	-0.2391	1.8039	1.2699	1
Argentina	-0.0719	1.0569	-0.9138	-0.0719	1.0569	-0.9138	1
Brazil	0.0698	2.0676	1.5964	0.0698	2.0676	1.5964	1
Chile	-0.2872	1.3417	1.1048	-0.2872	1.3417	1.1048	1
Colombia	-0.1193	1.5783	1.6524	-0.1193	1.5783	1.6524	1
Mexico	0.2483	1.6668	0.6168	0.2483	1.6668	0.6168	1
Peru	-0.0734	1.6790	-1.0666	-0.0734	1.6790	-1.0666	1
Uruguay	-0.4743	0.8450	1.0184	-0.4743	0.8450	1.0184	1
Venezuela	0.3257	2.4962	3.3685	-0.1017	1.9463	0.7272	16
India	0.2252	1.0402	-2.9253	0.7979	0.6746	-1.8103	2
Indonesia	0.2430	2.1997	-0.2876	0.2430	2.1997	-0.2876	1
Japan	0.1893	2.9765	0.3587	0.1893	2.9765	0.3587	1
Philippines	0.0886	2.3033	1.0599	0.0886	2.3033	1.0599	1

	# of sensitive estimates	# of failed corrections
STM using OLS	2/30	1/30
STM using GPH	7/30	2/30
STM using Whittle	2/30	0/30
DTM using OLS	23/30	4/30
DTM using GPH	30/30	4/30
DTM using Whittle	30/30	5/30

Table 7: Summary of sensitivity of growth estimates for STM and DTM using OLS and FGLS