Horizontal mergers in the presence of vertical relationships

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(**Preliminary and Incomplete**)

We study the welfare implication of downstream horizontal mergers in the presence of vertical relationships. Even in the absence of exogenous synergies or reallocation efficiencies between merging firms, horizontal mergers may still increase welfare. Reduction in input price is necessary for welfare improvement. We provide a necessary and sufficient condition for input price reduction which depends on the curvatures of upstream and downstream demand functions. We focus on two sources of welfare improvement: cost asymmetry and entry/exit. If upstream firms have asymmetric costs, downstream mergers can improve welfare by shifting production towards more efficient upstream firms. In presence of excessive entry in the upstream sector, downstream merger can improve welfare by prompting exit of some upstream firms. Implications of downstream mergers on consumer surplus and profits are also explored. (*JEL* L13, L41, L42)

Can horizontal mergers improve social welfare? We address this important question by incorporating an important aspect of reality: vertical relationships. In many industries, firms procure intermediate products from other firms in vertically related

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upstream industries and/or sell intermediate products to firms in downstream industries. For example, automobile manufacturers purchase steel, tires, and a number of parts produced in other industries, and general constructors purchase cement, steel, and other construction materials produced by other firms. We demonstrate that the horizontal merger can increase social welfare even if downstream firms are symmetric and the horizontal merger has no synergy or learning effects.

We consider a successive oligopoly model in which M symmetric downstream firms can produce a homogeneous final product and faces a downward sloping inverse demand. Each downstream firm can transform one unit of an intermediate product into one unit of the final product with zero costs. Let N denote the number of upstream firms that can produce the homogeneous intermediate product with constant marginal costs (which may differ across firms). Upstream firms compete against each other by choosing quantity and downstream firms also engage in quantity competition, where the input price r is determined at the market-clearing level and taken as given by all firms.

Using the model outlined above, we study welfare effects of a merger between two downstream firms. We first show that the merger reduces the equilibrium input price under a range of parameterizations. We then demonstrate, under the two different scenario, that the lower input price may result in higher total surplus.

In the first scenario, we assume that the number of upstream firms N is fixed, and show that the lower input price may increase total surplus when upstream firms have asymmetric costs. To understand the logic, suppose that the upstream sector has only two firms, 1 and 2, with constant marginal costs c_1 and c_2, respectively, satisfying $c_1 < c_2$. The lower input price increases the cost efficient firm 1's competitive advantage. To see this, suppose that the downstream merger reduces the equilibrium input price from r^* to r^{**}, $r^* > r^{**}$. Then firm 1's competitive advantage in terms of price-cost margin increases from $(r^* - c_1)/(r^* - c_2)$ to $(r^{**} - c_1)/(r^{**} - c_2)$. The higher competitive advantage increases firm 1's equilibrium market share, implying that a larger fraction of the industry output is produced in the cost efficient firm when the input price is lower. This effect (referred to as the production reallocation effect) works in the direction of increasing total surplus under the downstream merger.

Even though the merger increases concentration in the downstream sector, it can still increase total surplus if the concentration effect is dominated by the production reallocation effect. We find that the downstream merger reduces the equilibrium aggregate output due to the concentration effect. Then a necessary condition for the merger to increase total surplus is that it increases not only firm 1's market share but also its output. We find that this in fact happens under a range of parameterizations.

It is important to notice that the production efficiency effect just mentioned is different from the standard production reshuffling effect associated with horizontal mergers. To see the difference, consider a standard Cournot oligopoly model (without vertical structure) consisting of firms A, B, and C, where firm A is more cost efficient than firm B. Suppose that firms A and B merge. The merged firm would then produce more output in A and less in firm B in equilibrium to minimize its overall production cost. Production reshuffling of this kind does not occur in our model because downstream firms are assumed to be symmetric. Downstream mergers change the equilibrium input prices, which in turn change the nature of competition in the upstream firms, leading to the production efficiency effect in our model.

In the second scenario, we rule out the above kind of production efficiency effect by assuming that upstream firms are symmetric. Instead, we endogenize the number of upstream firms. Assume that a large number of potential entrants exist for the upstream sector, where each potential entrant can enter by incurring a fixed entry cost. Once the entry process is over, upstream firms engage in quantity competition. The equilibrium number of upstream firms can be socially excessive in this setup as in Mankiw and Whinston (1986) and Suzumura and Kiyono (1987). We find that the downstream merger, if it reduces the equilibrium input price, can increase total surplus by mitigating the negative welfare effect of the excessive entry. This is because the lower input price makes upstream entry less attractive, leading to the smaller number of upstream entrants in equilibrium. Here, production efficiency improves as well as average cost declines with the exit of upstream firms.

Welfare effects of horizontal mergers have been previously investigated in the literature. Farrell and Shapiro (1990), an important contribution to the literature, analysed a Cournot oligopoly model with quite general cost and demand functions to study the output and welfare effects of horizontal mergers. Production reshuffling and synergy or learning associated with mergers play important roles in their analyses. They found, among other things, that a merger causes price to rise if a merger generates no synergies or learning.

We contribute to the literature by identifying input price as another important element to be considered when one analyses welfare effects of horizontal mergers. To focus on our point, our model rules out production reshuffling and synergy or learning associated with mergers. A necessary condition for a horizontal merger to increase social welfare is that it reduces the equilibrium input price. And, downstream merger is more likely to induce welfare improvement if the upstream sector is highly concentrated or the entry cost is high. Bhattacharyya and Nain (2011) use the data of United States company acquisitions between 1984 and 2003 to study the impact of downstream mergers on upstream suppliers. They find that, in those more concentrated industries or industries with high entry barrier, upstream suppliers experienced large input price decline after consolidation in the downstream sector. While a reduction in input price is not sufficient for welfare improvement in our framework, it is interesting to note that the two welfare-improvement conditions identified in the paper are large H (high concentration) in the first model and small N (high entry barriers) in the second model.

Recently, there is a small strand of literature exploring the market and welfare implications of horizontal merger with explicit modelling of upstream suppliers, such as Ziss (1995), Lommerud, Straume and Sorgard (2005, 2006) and Symeonidis (2010). These authors typically assume small number (one or two) of upstream firms, downstream firms producing differentiated goods and possible synergies in the merged firms. To highlight our mechanisms driving the welfare change, we assume away product differentiation and synergies to illustrate how mergers can have positive effect on total welfare. Moreover, our model either assumes arbitrary fixed number of upstream firms or use free-entry to endogenize this number. Snyder (1996) studies upstream collusion using a dynamic framework under which each downstream firm runs an auction to procure input. The input price goes down if two downstream firms merge due to the increased countervailing power. In our model, a merger can change input price in different directions, depending on the final demand curvatures. Snyder's paper focuses on impact of merger on the sustainability of collusion and firms' profit. Our main focus is instead on how downstream mergers affect social welfare.

(to be completed)

I. Vertical Oligopoly

We consider an industry with two sectors of production, upstream and downstream. In the upstream sector, a homogeneous intermediate product is produced by N upstream firms. The upstream industry structure, represented by the number of firms, N, is either fixed or endogenized by free-entry, which will be specified whenever necessary. Each upstream firm, k, produces at constant marginal cost c_k. Without loss of generality, $c_1 \leq c_2 \leq ... \leq c_N$. In the downstream sector, the intermediate products are transformed into homogeneous finals product with constant marginal cost, which is normalized to zero. Production of one unit of the final product requires one unit of the intermediate product. Before any merger taking place in the downstream sector, there is a fixed number of (denoted by $M > 1$) downstream firms. The downstream firms face a three-times continuously differentiable and strictly decreasing inverse demand function $P(Q)$, where $Q \geq 0$ denotes the aggregate output in the downstream sector. To ensure that finite quantity is produced in equilibrium, we also assume $P_0 \equiv \lim_{Q \to 0} P(Q) > \max_{k \in N} c_k > \lim_{Q \to \infty} P(Q) = P_\infty$.

The model has three stages. In Stage 0, a horizontal merger exogenously takes place in the downstream sector. In Stage 1, if the upstream market structure is fixed (no free-entry), the N incumbent firms compete in quantity (Cournot) to supply intermediate goods, taking rival upstream firms' outputs as given. If free-entry is present, then Stage 1 can be further divided into to sub-stages. In the first sub-stage, a large number of firms (who can produce the intermediate products) simultaneously consider whether to enter the upstream sector. If entry occurs, each entrant incurs entry cost $K > 0$. In the second sub-stage, the entrants engage in Cournot competition. In Stage 2, M downstream firms also compete in quantity to supply final products, taking the input price r and rival downstream firms' outputs as given. The input price is determined at the market-clearing level, which equates the demand of downstream firms to the total amount of the intermediate product supplied by the upstream firms. Note that the downstream firms have no oligopsony power over the upstream sector and take the input price as given (see also, Greenhut and Ohta, 1979, Salinger, 1988 and Ghosh and Morita, 2007).

We consider the subgame perfect Nash equilibrium (SPNE) in pure strategies of the game. As is well known, the following assumption guarantees the existence and uniqueness of the Cournot-Nash equilibrium in the downstream competition (see, for instance, Vives, 1999).

Assumption 1 $(M+1)P'(Q) + QP''(Q) < 0$ *for all* $Q > 0$.

We solve the game by backward induction. In Stage 2, each downstream firm, $i(= 1, 2, ..., M)$, chooses its output, $q_i (\geq 0)$, to maximize its profit, $\left[P\left(q_i + \sum_{j \neq i}^{M} q_j\right) - r\right] q_i$, taking other downstream firms' output and input price, r, as given. Under Assumption 1, there exists an unique interior solution to this maximization, which is characterized by the first-order condition

$$P\left(q_i + \sum_{j \neq i}^{M} q_j\right) - r + P'\left(q_i + \sum_{j \neq i}^{M} q_j\right) q_i = 0, \tag{1}$$

where $i = 1, 2, ..., M$. If $r \in (0, P_0)$, equation (1) yields the sole candidate for the equilibrium in stage 2 sub-game, $q_1^* = q_2^* = ... = q_M^* \equiv q^*$. On the other hand, if $r \in [P_0, \infty)$, each firm i chooses $q_i = 0$ in the equilibrium. Assume $r \in (0, P_0)$, adding the first-order condition and rearranging yields

$$r = P(Mq^*) + \frac{P'(Mq^*)Mq^*}{M}. \tag{2}$$

Noting that q^* is a function of r, let $Q(r) = Mq^*$ where $r \in (0, P_0)$.

Next we consider the Stage 1 game (or the second sub-stage if there is free entry in the first sub-stage) in which $N(\geq 1)$ upstream firms compete in the upstream sector. Let $x_k(\geq 0)$ denote the amount of intermediate product produced by upstream firm $k(=1,2,...,N)$ and let $X=\sum_k^N x_k$. The one-to-one transformation technology from intermediate product to final product implies $X=Mq^*$. Equation (2) can be written as $r=P(X)+(P'(X)X/M)\equiv g(X,M)$. Recall that the input price is determined at the market-clearing level. Then the inverse demand function faced by upstream firms at Stage 2 is given by $r=P_0$ if $X=0$, $g(X,M)$ if $X\in(0,Q_0)$, and 0 if $X\geq Q_0$, where $Q_0\equiv\lim_{r\to 0}Q(r)$. Note that we have $g_X(X,M)=\partial g/\partial X<0$ for all $X>0$ and M by Assumption 3.

Using the inverse demand function in upstream sector, we can derive upstream firm k's profit function, $\left[g(x_k+\sum_{l\neq k}^N x_l, M)-c_k\right]x_k$. Then firm k chooses its output, x_k, to maximize its profit, taking other upstream firms' outputs as given. To make sure that the solution to upstream firms' maximization exists and is unique, we make the following assumptions.

Assumption 2 $(N+1)g_X(X,M)+Xg_{XX}(X,M)<0$ *for all* $X>0$.

Assumption 2 is the counterpart of Assumption 1 in the upstream sector so that each upstream firm's profit function is strictly concave. Then, the following first-order condition

$$g(x_k+\sum_{l\neq k}^N x_l, M)-c_k+g_X(x_k+\sum_{l\neq k}^N x_l, M)x_k=0 \tag{3}$$

yields a unique solution $x_1^*\geq x_2^*\geq...\geq x_N^*$ to the upstream Cournot game. Note that x_k^* depends on M and N. Adding the first-order conditions in (3) together yields

$$Ng(X^*,M)-\sum_{k=1}^N c_k+g_X(X^*,M)X=0, \tag{4}$$

where $X^* = \sum_{k=1}^{N} x_k^*$.

If the number of upstream firms is determined by free-entry, the post-entry expected profit should be driven down to equal the entry cost. Especially, when marginal costs are symmetric and the equilibrium outputs for each upstream firm are $x_1^* = x_2^* = ... = x_N^* \equiv x^*$, the free-entry condition is

$$(g(X^*, M) - c)x^* = K. \tag{5}$$

This condition identifies the number of entrants in the upstream sector for given M, c and K.

II. Characterizing the Variation of Input Price

In the standard horizontal merger analysis, the input price is usually treated as constant both before and after mergers taking place. However, when the vertical interaction between upstream suppliers and downstream manufacturers is explicitly modeled as in ours, the input price can increase, decrease or stay constant if a downstream horizontal merger occurs. This is very important for our analysis because both the upstream and downstream firms' production incentives along with the entry incentive in the upstream sector will be altered by the change of input price. This section is thus devoted to characterize the variation of input price following the change of the number of downstream firms. To model the horizontal merger, we proceed by treating the number of firms as a continuous variable which is standard in a large body, if not most, of the literature. However, in the numerical examples we provide to help illustrate results, we take integer for the number of firms.

We first define the relative curvatures of the inverse demand function. For the downstream demand, let $\epsilon_d = QP''(Q)/P'(Q)$ and $\alpha = QP'''(Q)/P''(Q)$ represent the second-order and third-order elasticities of the inverse demand $P(Q)$. For the upstream demand, let $\epsilon_u = Xg_{XX}(X, M)/g_X(X, M)$ represents the second-order elasticity of the upstream inverse demand $g_X(X, M)$. In Proposition 1, we show that the variation of input price following a downstream merger van be completely characterized by using these demand curvatures.

Proposition 1 *When the upstream market structure is fixed, a downstream merger reduces input price if and only if*

$$(\epsilon_u - \epsilon_d)|_{Q=Q^*} > 0 \quad \textit{or equivalently} \quad \left.\frac{d\epsilon_d}{dQ}\right|_{Q=Q^*} > 0;$$

raises input price if and only if

$$(\epsilon_u - \epsilon_d)|_{Q=Q^*} < 0 \quad \textit{or equivalently} \quad \left.\frac{d\epsilon_d}{dQ}\right|_{Q=Q^*} < 0;$$

does not affect input price if and only if

$$(\epsilon_u - \epsilon_d)|_{Q=Q^*} = 0 \quad \textit{or equivalently} \quad \left.\frac{d\epsilon_d}{dQ}\right|_{Q=Q^*} = 0.$$

Proof:
See Appendix, part A.

Proposition 1 indicates that, to predict how a downstream merger affects input price, we only need to compare the second order elasticities of the upstream and downstream inverse demand functions, or, simply calculate the derivative of the second order elasticity of downstream inverse demand function at the equilibrium level. The implication carried by Proposition 1 is three-fold. First, the possible variation of input price following a downstream merger creates incentive to reschedule production for both upstream and downstream firms. Without this possibility, it would be pointless to go beyond the standard horizontal merger analysis by incorporating vertical structure. Second, the conditions given in Proposition 1 are rather general, which do not require specifying function form for the final demand. Finally, the conditions identified in Proposition 1 is convenient for checking. The downstream-merger induced input-price variation can be predicted by solely examining the model primitive, ϵ_d.

There is little direct intuition we can offer for Proposition 1 since high order elasticities are involved. Nevertheless, the direction of the change of input price depends on the elasticity of demand for upstream firms, $e_u = -(r/X)(\partial X/\partial r)$. Firstly, from the aggregated first-order condition for upstream firms, we have

$$g(X^*, M) + \frac{g_X(X^*, M)X^*}{N} = \frac{\sum_{k=1}^{N} c_k}{N}. \tag{6}$$

Take out $r = g(X^*, M)$ and use the expression of e_u, we obtain

$$r\left(1 - \frac{1}{Ne_u}\right) = \frac{\sum_{k=1}^{N} c_k}{N}. \tag{7}$$

This equation implies the following relation between the marginal change in r and the marginal change in e_u,

$$\text{sign}\left(\left.\frac{dr}{dM}\right|_{Q=Q^*}\right) = -\text{sign}\left(\left.\frac{de_u}{dM}\right|_{Q=Q^*}\right). \tag{8}$$

Remember that in this model downstream firms have no market power in the upstream sector. Then, as being pointed out by the famous doctrine for pricing, equation (8) says that optimal price should increase(decrease) when demand becomes less(more) elastic.

Proposition 1 shows that input price can go either way when downstream mergers take place. Then, a reasonable conjecture would be that it is possible to have mergers induce improvement in both total welfare and consumer surplus because a reduced input price might offset the anti-competitive effect caused by higher market concentration. In Section III and IV, we sequentially present two possible channels for downstream mergers to improve total welfare, and that contrary to total welfare, consumer surplus always goes down following a merger.

III. Asymmetric Upstream Firms

In this section, we consider a vertical market where the number of upstream firms is fixed (no entry) but the incumbent upstream firms may have asymmetric marginal cost of supplying the intermediate goods. We focus on the welfare consequence induced by a downstream horizontal merger.

As being assumed previously, the marginal production cost for upstream firms are $c_1 \leq c_2 \leq ... \leq c_N$. From equation (3), each upstream firm k produces $x_k = -[g(X, M) - c_k]/g_X(X, M)$ units in equilibrium. Then

$$\frac{dx_k}{dM} = -\left(1 + \frac{x_k g_{XX}}{g_X}\right)\frac{dX}{dM} - \left(\frac{g_M}{g_X} + \frac{x_k g_{XM}}{g_X}\right). \tag{9}$$

Let $s_k = x_k/X$ denote the market share of upstream firm k. Substitute in the expressions of g_M and g_{XM}, the above equation can be rewritten as

$$\frac{dx_k}{dM} = -(1 + s_k\epsilon_u)\frac{dX}{dM} + \left[\frac{X}{M(M + 1 + \epsilon_d)}\right][1 + s_k(1 + \epsilon_d)]. \tag{10}$$

Use the fact that $dX/dM = \sum_{k=1}^{N}(dx_k/dM)$ and $\sum_{k=1}^{N} s_k = 1$ and sum up all dx_k/dM, we obtain the expression of dX/dM as

$$\frac{dX}{dM} = \frac{X(N + 1 + \epsilon_d)}{M(M + 1 + \epsilon_d)(N + 1 + \epsilon_u)}. \tag{11}$$

Since the final products are homogeneous, the consumer surplus is solely measured by the total output. That is, consumer surplus increases if and only if the total output increases. From equation (11), it is clear that $dX/DM > 0$ as by assumption $N + 1 + \epsilon_d$, $M + 1 + \epsilon_d$ and $N + 1 + \epsilon_u$ are all strictly positive. Then, a downstream merger always reduces total output and therefore consumer surplus.

Proposition 2 *When N is fixed, a downstream horizontal merger reduces consumer surplus.*

When upstream firms have asymmetric production cost, the total welfare, W, is the gross surplus, $\int_0^X P(y)dy$, less the total cost, $\sum_{k=1}^{N} x_k c_k$. Differentiating W w.r.t M yields

$$\frac{dW}{dM} = \sum_{k=1}^{N}(P - c_k)\frac{dx_k}{dM} = (P - r)\frac{dX}{dM} + \sum_{k=1}^{N}(r - c_k)\frac{dx_k}{dM}. \tag{12}$$

By using the expressions of dX/dM and dx_k/dM and also the first-order conditions for both upstream and downstream firms, dW/dM can be eventually written in terms of ϵ_u, ϵ_d and the Herfindahl index $H = \sum_{k=1}^{N}(s_k)^2$. Proposition 3 shows the possibility for a downstream merger to improve total welfare.

Proposition 3 *When the upstream firms have asymmetric marginal costs of production, a downstream merger improves social welfare if and only the following condition holds,*

$$\frac{N + 1 + \epsilon_d}{M + 1 + \epsilon_d} + H(N + 1 + \epsilon_d) - (NH - 1)(\epsilon_u - \epsilon_d) < 0. \tag{13}$$

Proof:
See Appendix, part B.

For inequality (13) to hold, $\epsilon_u - \epsilon_d$ has to be strictly positive and large enough. From Proposition 1, this implies that input price must go down for a downstream merger to improve total welfare. Provided that $\epsilon_u - \epsilon_d > 0$ holds and is large enough (greater than $(N+1+\epsilon_d)/N$), the more concentrated (manifested by a great Herfindahl index H) the upstream sector is, the more likely a downstream horizontal merger improves total welfare.

A downstream merger changes the composition of total output because it affects upstream firms' output levels asymmetrically. The marginal change of each upstream firm k's output level is

$$\frac{dx_k}{dM} = \frac{X}{M(M + 1 + \epsilon_d)}\left[1 + s_k(1 + \epsilon_d) - \frac{(1 + s_k\epsilon_u)(N + 1 + \epsilon_d)}{N + 1 + \epsilon_u}\right]. \tag{14}$$

Thus, dx_k/dM and $[1+s_k(1+\epsilon_d)](N+1+\epsilon_u)-(1+s_k\epsilon_d)(N+1+\epsilon_d)$ share the same sign. Assume $\epsilon_u-\epsilon_d$ is positive and large enough. Then there exists a cut-off value of market share, $\hat{s}=(\epsilon_u-\epsilon_d)/[N(\epsilon_u-\epsilon_d-1)-1-\epsilon_d]$, such that $dx_k/dM>0$ when $s_k>\hat{s}$ and $dx_k/dM<0$ otherwise. This tells us that a downstream merger increases outputs of some relatively efficient upstream firms, because more efficient upstream firms have larger market shares. In other words, a downstream merger reallocates total output by increasing outputs of some relatively efficient upstream firms and decreasing outputs of other relatively inefficient firms. The intuition behind this result can be drawn from thinking of the standard Cournot competition where the equilibrium output ratio is proportional to the profit-margin ratio. When a downstream merger reduces input price, the profit-margin ratio gets larger for more efficient firms, who gain a greater fraction of the upstream industry output. When $\epsilon_u-\epsilon_d$ is large enough, a downstream merger increases not only more efficient firms' market share but also their outputs.

A downstream merger increases market concentration in the downstream sector and decreases total output as Proposition 2 tells us. This effect works in the direction of reducing total welfare. However, if the merger reduces input price, the induced production reallocation in the upstream sector may countervail this negative impact and creates total welfare improvement. This trade-off can be seen from rewriting equation (12) as follows.

$$\frac{dW}{dM}=\underbrace{(P-r)\frac{dX}{dM}}_{\text{market power}}+\underbrace{\underbrace{(r-c_1)\frac{dx_1}{dM}}_{+}...+\underbrace{(r-c_k)\frac{dx_k}{dM}}_{0}+...+\underbrace{(r-c_N)\frac{dx_N}{dM}}_{-}}_{\text{reallocation}}. \tag{15}$$

Proposition 3 tells us that the sign of this entire expression is positive when $\epsilon_u-\epsilon_d$ is positive and large enough.

Example 1 *The inverse demand is $P(X)=(1-X)^b$ with $b>0$ (Malueg, IJIO 1992). This demand function is convex for $b>1$, linear for $b=1$ and concave for $0<b<1$. The equilibrium condition is give by each upstream firms k' first-order* *condition*

$$(1-X^*)^b - b(1-X^*)^{b-1}\frac{X^*}{M} + \Big[-b(1-X^*)^{b-1} + b(b-1)(1-X^*)^{b-2}\frac{X^*}{M} - b(1-X^*)^{b-1}\frac{1}{M}\Big]x_k^* = c_k,$$

and the aggregated first-order condition

$$N\Big[(1-X^*)^b - b(1-X^*)^{b-1}\frac{X^*}{M}\Big] + \Big[-b(1-X^*)^{b-1} + b(b-1)(1-X^*)^{b-2}\frac{X^*}{M} - b(1-X^*)^{b-1}\frac{1}{M}\Big]X^* = \sum_{k=1}^{N} c_k.$$

Let $N = 6$, $b = 0.05$, $c_1 = 0.1$ and $c_k = 0.8$ for $k \neq 1$. The table below presents the equilibrium levels of individual output, x_1 and x_k for $k \neq 1$, total output, X^*, input price, r^*, Herfindahl index, H^*, and social welfare, W^*.

M	x_1^*	x_k^*	X^*	r^*	H^*	W^*
2	0.755482	0.0131050	0.821007	0.812357	0.848022	0.667994
3	0.742104	0.0220346	0.852277	0.821420	0.761515	0.662173
4	0.733718	0.0273354	0.870395	0.827088	0.715532	0.658222

Under our parameter specification, clearly when downstream merger takes place, the production is shifting towards more efficient upstream firm, firm 1, input price is decreasing and the social welfare is increasing.

IV. Upstream Free-Entry

In this section, we suggest an alternative channel for downstream horizontal mergers to improve total welfare. To fix the idea, we assume symmetric marginal cost for upstream suppliers, but allow free entry to the upstream sector. The pre-entry expected profit for a representative upstream firm is $\pi_u(N) = (g(Nx^*, M) - c)x^* - K$ (where K is the entry cost). The standard free-entry condition, $\pi_u(N_f) = 0$, then determines the equilibrium number of firms entering the upstream sector, N_f. We assume $\pi_u(1) \geq 0$, which is equivalent to $K \leq \overline{K} \equiv [g(x^*, M) - c]x^*$. This condition ensures that at least one upstream firms will enter in the equilibrium of the entire game.

Assumption 3 $K \leq \overline{K}$.

Under the vertical market structure, input price is determined by both the number of downstream firms and the number of upstream firms. Thus, downstream mergers affect input price not only through directly changing M but also indirectly changing N through the free-entry condition. In particular, we can write

$$\begin{aligned} \frac{dr}{dM} &= g_X \left(\frac{\partial X}{\partial M} + \frac{\partial X}{\partial N} \frac{\partial N}{\partial M} \right) + g_M \\ &= \left(g_X \frac{\partial X}{\partial M} + g_M \right) + g_X \frac{\partial X}{\partial N} \frac{dN}{dM}. \end{aligned} \tag{16}$$

The first term in (16) shows the direct effect of downstream mergers on input price, which is similar as in previous section. The second term reflects the indirect effect due to free-entry. Since $q_X(\partial X/\partial N)(dN/dM)$ is in general negative, it is now harder for downstream mergers to reduce input price. We therefore should expect that downstream mergers are less likely to overturn the output-reducing outcome arising from market power increase. The next proposition formally verifies this intuition.

Proposition 4 *When N is determined by free-entry and $c_k = c, \forall k$, downstream mergers never improve consumer surplus.*

Proof:
Recall that consumer surplus becomes greater if and only if the total output becomes

larger after merger. According to the expression of dX/dM, we have

$$\begin{aligned}
\frac{dX}{dM} &= \frac{\partial X}{\partial M} + \frac{\partial X}{\partial N}\frac{\partial N}{\partial M} \\
&= \left[\frac{N+1+\epsilon_d}{(M+1+\epsilon_d)(N+1+\epsilon_u)}\right] + \left[\frac{N+1+\epsilon_d+N(\epsilon_u-\epsilon_d)}{(2N+\epsilon_u)(M+1+\epsilon_d)(N+1+\epsilon_u)}\right] \\
&= \frac{(N+1+\epsilon_d)(2N+\epsilon_u)+N+1+\epsilon_d+N(\epsilon_u-\epsilon_d)}{(M+1+\epsilon_d)(N+1+\epsilon_u)(2N+\epsilon_u)}
\end{aligned}$$

Recall $\epsilon_u, \epsilon_d > -2$, so the denominator of the last fraction is always positive. For the numerator, we have

$$\begin{aligned}
(N+1+\epsilon_d)(2N+\epsilon_u) + N{+}1+\epsilon_d+N(\epsilon_u-\epsilon_d)& \\
&= (N+1+\epsilon_d)(2N+\epsilon_u)+N+1+\epsilon_d+N(\epsilon_u-\epsilon_d) \\
&> (N+1+\epsilon_d)(2N-2)+N+1+\epsilon_d-2N-N\epsilon_d \\
&= N(N-1)+(N+1+\epsilon_d)(N-1) \\
&\geq 0
\end{aligned}$$

Thus we have $dX/dM \geq 0$. *Q.E.D.*

Despite the above impossibility result on consumer surplus, hereafter we focus on the impact of downstream mergers on total welfare. Intuitively, if downstream merger decreases input price, it discourages the entry of potential entrants of the upstream sector. This deterring effect may create a welfare improvement when there exists excessive entry in the upstream sector if downstream mergers do not take place.

In the context with upstream free-entry, the total welfare, W, is defined by

$$W = \int_0^{X^*} P(y)dy - X^*c - N_f K. \tag{17}$$

That is, the total welfare is the gross benefit generated by the total output less the sum of production costs and entry costs. Adding the free-entry conditions over entering firms yields $X^*r - X^*c = N_fK$. The total welfare can then be written as

$$W = \int_0^{X^*} P(y)dy - X^*r^*. \tag{18}$$

Notice that the equilibrium input price, $r^* = c^* + \frac{N_fK}{X^*}$, is the social per-unit cost for producing the total output X^* $(= Q^*)$.

Can downstream merger increase total surplus? In other words, can a reduction in M increase W? Proposition 4 tells us that a reduction in M decreases X^* and hence decreases the gross benefit generated by the total output, $\int_0^{X^*} P(y)dy$. Then, a reduction in M must decrease the social per-unit cost, r^*, for producing X^*, in order for downstream merger to increase total surplus. Figure 1 depicts this relationship. Suppose that a reduction in M decreases the equilibrium total output from Q^* to $\bar{Q}^*$. Area $DGHIJ$ captures the corresponding loss of total surplus, holding the input price constant at r^*. If the reduction in M increases the input price, it unambiguously decreases total surplus. Suppose the reduction in M decreases the equilibrium input price from r^* to $\bar{r}^*$. Then area $AEFD$ captures the corresponding gain of total surplus associated with the reduction in the social per-unit cost. The decrease in M increases total surplus if area $AEFD$ is greater than area $DGHIJ$.

Can area $AEFD$ be in fact greater than area $DGHIJ$? Algebraically, this happens if and only if dW/dM is negative. Proposition 5 tells us that this condition holds under a range of parameterizations.

Proposition 5 *When N is determined by free-enty, downstream mergers improve total welfare if and only if the following condition holds,*

$$\frac{(N + \epsilon_u)(\epsilon_u - \epsilon_d) - (N + 1 + \epsilon_d)}{(N + 1 + \epsilon_u)(2N + \epsilon_u)} > \frac{1}{M + 2 + \epsilon_d}, \tag{19}$$

where all expressions are evaluated at $N = N_f$.

Proof:
See Appendix, part C.

To better understand the logic behind the possibility of merger-induced welfare improvement, it is useful to decompose dW/dM as follows. Note that social welfare can be written as $W = \int_0^X P(y)dy - Xc - NK$ and differentiating it w.r.t M yields

$$\frac{dW}{dM} = (P-c)\frac{dX}{dM} - K\frac{dN}{dM}. \tag{20}$$

Rewrite equation (20) as

$$\frac{dW}{dM} = (P-c)\left(\frac{\partial X}{\partial M} + \frac{\partial X}{\partial N}\frac{dN}{dM}\right) - K\frac{dN}{dM}. \tag{21}$$

Adding and subtracting the same term, $r(\partial X/\partial N)(dN/dM)$, on the R.H.S yields

$$\begin{aligned} \frac{dW}{dM} &= (P-c)\frac{\partial X}{\partial M} + (P-r)\frac{\partial X}{\partial N}\frac{dN}{dM} + (r-c)\frac{\partial X}{\partial N}\frac{dN}{dM} - K\frac{dN}{dM} \\ &= (P-c)\frac{\partial X}{\partial M} + \left[(P-r)\frac{\partial X}{\partial N} + (r-c)\left(x + N\frac{\partial x}{\partial N}\right) - K\right]\frac{dN}{dM} \end{aligned} \tag{22}$$

Using the fact that $(r-c)x = K$, we get

$$\frac{dW}{dM} = \underbrace{(P-c)\frac{\partial X}{\partial M}}_{\text{market power}} + \left[\underbrace{(P-r)M\frac{\partial q}{\partial N}}_{\text{business creating}} + \underbrace{(r-c)N\frac{\partial x}{\partial N}}_{\text{business stealing}}\right]\frac{dN}{dM} \tag{23}$$

Suppose that $dW/dM < 0$ so that downstream merger increases total surplus. Recall that a reduction in M decreases the equilibrium total output $Q^* = X^*$. Also, $dW/dM < 0$ implies that the reduction in M decreases r^* as explained above. Then the reduction in M increases the equilibrium output of each upstream firm x^* because the zero profit condition $r^*x^* - K = 0$ holds in the equilibrium. Then, the reduction in M decreases X^* and increases x^*, and hence decreases the equilibrium number of upstream firms. Hence we have $dN/dM > 0$ if $dW/dM < 0$. In equation (23) we have that the market power effect is positive. Then, if $dW/dM < 0$, the term in the square bracket must be negative. That is,

$$(P-r)M\frac{\partial q}{\partial N}+(r-c)N\frac{\partial x}{\partial N}<0 \tag{24}$$

must hold. But, this is exactly the same condition for having upstream excessive entry in successive Cournot model, as identified by Ghosh and Morita (2007). Hence, downstream merger increases total surplus if excessive entry is present in the upstream sector, and the merger reduces the input price and discourages upstream entry, thereby mitigating excessive entry in the upstream sector.

Proposition 6 *With symmetric production costs and free-entry in the upstream sector, a necessary condition for downstream merger to improve total welfare is that there exists excessive entry in the upstream sector.*

Example 2 *Consider again the inverse demand function $P(X) = (1-X)^b$. Let $N = 6$ and $c = 0.1$. Notice that we can arbitrarily choose N by changing K. The table below shows the equilibrium levels of total output, X^*, input price, r^*, and social welfare W^*.*

M	b	X^*	r^*	W^*
1	0.1	0.827851	0.435359	0.334642
2	0.1	0.882591	0.503791	0.290036
3	0.1	0.905259	0.538415	0.263115
1	0.2	0.741905	0.324220	0.354561
2	0.2	0.823016	0.378375	0.335351
3	0.2	0.857009	0.406937	0.318125

V. Discussion

A. Downstream Mergers and Industry Profitability

We consider the downstream industry profit in this section. The profit in the downstream sector is

$$\Pi_d = (P - r)X = -\left(\frac{P'X^2}{M^2}\right). \tag{25}$$

We first fix the number of upstream firms. When the number of downstream firms changes, we have

$$\begin{aligned}
\frac{d\Pi_D}{dM} &= -\left(\frac{2XP' + X^2P''}{M}\right)\frac{dX}{dM} + \frac{X^2P'}{M^2} \\
&= -\left(\frac{2XP' + X^2P''}{M}\right)\left[\frac{X(N+1+\epsilon_d)}{M(M+1+\epsilon_d)(N+1+\epsilon_u])}\right] + \frac{X^2P'}{M^2} \\
&= \frac{X^2P'}{M^2}\left[1 - \frac{(2+\epsilon_d)(N+1+\epsilon_d)}{(M+1+\epsilon_d)(N+1+\epsilon_u)}\right].
\end{aligned}$$

Since $2 + \epsilon_d \leq M + 1 + \epsilon_d$, $[(2+\epsilon_d)(N+1+\epsilon_d)]/[(M+1+\epsilon_d)(N+1+\epsilon_u)] \leq 1$ when $\epsilon_d \leq \epsilon_u$. This implies $d\Pi_d/dM < 0$ when $\epsilon_d \leq \epsilon_u$. This is standard in the horizontal merger analysis. However, when $\epsilon_d < \epsilon_u$, the reduction of M may reduce industry profits.

Now consider the case where free-entry is present in the upstream sector. The zero-profit condition under free-entry, $(r - c)x = K$, implies

$$-g_X x^2 = -\left(\frac{g_X X^2}{N^2}\right) = K.$$

Since $g_X = [P'(M+1+\epsilon_d)]/M$, we have $-[X^2P'(M+1+\epsilon_d)]/(MN^2) = K$. The industry profit can then be written as

$$\Pi_d = \frac{N^2K}{M+1+\epsilon_d}. \tag{26}$$

Differentiating Π_d w.r.t M yields

$$\frac{d\Pi_d}{dM} = \frac{(M+1+\epsilon_d)2N\frac{dN}{dM} - N^2\left(1+\frac{d\epsilon_d}{dX}\frac{dX}{dM}\right)}{(M+1+\epsilon_d)^2}. \tag{27}$$

Hereafter, we assume $\epsilon_d(Q) = \epsilon_u(Q) = \epsilon$ for any $Q \leq 0$ where ϵ is a constant. The following proposition describe the peak vale of industry profit with respect to the number of downstream firms.

Proposition 7 *For any $\epsilon > 2$, there exists an threshold number of upstream firms, $\hat{N}$, such that for any $N > \hat{N}$, $\arg\max_{M \in \mathbf{Z}} \Pi_D(M) \neq 1$.*

Proof:
See Appendix, part D.

B. Firm-Specific Input

In the model in the above sections, the input price is determined by an unified intermediate product market. All upstream and downstream firms are participants in this market. In this section, we consider a different setting where the intermediate products each downstream firm uses is firm-specific. In other words, a downstream firm can only use the intermediate goods supplied by a fraction of upstream firms.

There are still M downstream firms producing homogeneous products. But each downstream firm i is supplied by N_i upstream firms which produce firm-specific inputs for firm i. Each upstream firm can product at most one kind of firm-specific intermediate products.

To guarantee the uniqueness of equilibrium outcome, we impose the following assumption which ensues the aggregated first-order condition is strictly downward sloping so that the equilibrium total output is unique.

Assumption 4 $M^2N + MN + 2M + (MN + 2M + 2 + \alpha)\epsilon_d > 0.$

Let the input price faced by downstream firm i be r_i. Downstream firm i's maximization problem is

$$\max_{q_i} \left[P(q_i + \sum_{j\neq i}^{M} q_j) - r_i\right] q_i. \tag{28}$$

The first-order condition yields

$$P(q_i + \sum_{j\neq i}^{M} q_j) - r_i + P'(q_i + \sum_{j\neq i}^{M} q_j)q_i = 0. \tag{29}$$

In each sub-market i, input demand $X - i = q_i$. Then, the inverse demand curve faced by the suppliers to firm i is represented by

$$r_i = P(X_i + \sum_{j\neq i}^{M} X_j) + P'(X_i + \sum_{j\neq i}^{M} X_j)X_i = g^i(X_i, \sum_{j\neq i}^{M} X_j). \tag{30}$$

A representative suppler to firm i, firm ik, with marginal production cost c_i maximizes its profit

$$\max_{x_{ik}} \left[g^i(x_{ik} + \sum_{l\neq k}^{N_i} x_{il}, \sum_{j\neq i}^{M} X_j) - c_i\right] x_{ik} \tag{31}$$

The first-order condition yields

$$g^i(x_{ik} + \sum_{l\neq k}^{N_i} x_{il}, \sum_{j\neq i}^{M} X_j) - c_i + g_1^i(x_{ik} + \sum_{l\neq k}^{N_i} x_{il}, \sum_{j\neq i}^{M} X_j)x_{ik} = 0 \tag{32}$$

Add the N_i first-order conditions together, we have

$$N_i \left(g^i(X_i, \sum_{j\neq i}^{M} X_j) - c_i\right) + g_1^i(X_i, \sum_{j\neq i}^{M} X_j)X_i = 0. \tag{33}$$

We impose symmetry on upstream firms' cost and firm numbers across sub-markets.

That is $c_i = c$ and $N_i = N$, for all $i = 1, ..., M$. The aggregate fist-order condition is then

$$N\left(g^i(X_i, \sum_{j\neq i}^{M} X_j) - c\right) + g_1^i(X_i, \sum_{j\neq i}^{M} X_j)X_i = 0. \tag{34}$$

By definition of g and the symmetry,

$$g_1^i(X_i, \sum_{j\neq i}^{M} X_j) = 2P'(X) + P''(X)\frac{X}{M}, \tag{35}$$

and the aggregate first-order condition is

$$P(X) + P'(X)\frac{X}{M} - c + \left(P''(X)\frac{X}{M} + 2P'(X)\right)\frac{X}{MN} = 0. \tag{36}$$

Proposition 8 *When input is firm-specific, a merger improves consumer surplus only (i)when a downstream duopoly merges to a monopoly (M=2); (ii)each duopoly was supplied by a monopoly upstream firm (N=1).*

Proof:
Totally differentiate (36) w.r.t X and M and rearrange, we get

$$\frac{dX}{dM} = \frac{X\left(N + 2 + \frac{2\epsilon_d}{M}\right)}{M^2N + MN + 2M + (MN + 2M + 2 + \alpha)\epsilon_d}. \tag{37}$$

By assumption (5), the denominator is positive. Therefore, $dX/dM < 0$ can only happen when $X(N+2+2\epsilon_d/M) < 0$. This is only possible when $M = 2$ and $N = 1$. *Q.E.D.*

C. Matching and Bargaining (Incomplete)

In this section, the upstream and downstream firms are randomly and pairwisely matched and a matched pair determines input price through Nash bargaining. Denote $S(M, N)$ as the matching technology. When there are M upstream firms and N

downstream firms, $S(M,N)$ pairs will be formed. Assume $S(M,N) = \min\{M, N\}$ so that the number of matches is determined by the short side of the market. We further assume that there are more downstream firms than upstream firms so that $S(M,N) = M$. Thus, a downstream firm can always get matched but an upstream firm gets matched with probability M/N.[1] When all matches are formed, each pair will produce to maximize the profit of the match. Denote β and $1-\beta$ as the bargaining power of downstream and upstream firms respectively. For any given joint profit π, a downstream firm earns $\beta\pi$ while an upstream earns $(1-\beta)\pi$.

The total output is then determined by

$$MP(Q) + QP'(Q) = Mc. \tag{38}$$

This equation implicitly defines $Q(M)$. When free-entry is present in the upstream industry, the number of upstream firms can also be written as $N(M)$. The total welfare is given by

$$W = \int_0^{Q(M)} P(y)dy - cQ(M) - Kn(M). \tag{39}$$

Differentiate total welfare w.r.t M yields

$$\frac{dQ}{dM} = -\frac{P-c}{(M+1)P' + QP''} = \frac{q}{M+1+\epsilon_d}. \tag{40}$$

The free-entry condition for upstream firms is

$$M(1-\beta)(P-c)q = Nk. \tag{41}$$

Substitute in the first-order condition, it becomes

$$-\frac{(1-\beta)P'Q^2}{MN} = k. \tag{42}$$

[1]The same matching technology can be found on Osborne and Rubinstein (1990), page 125.

Take log on both sides and totally differentiate

$$-\frac{P''}{P'}dQ - \frac{2}{Q}dQ = \frac{dM}{N} + \frac{dN}{N} \tag{43}$$

Rearrange it and we get

$$\frac{dN}{dM}\frac{M}{N} = -\left[\frac{dQ}{dM}\frac{M}{Q}(2+\epsilon_d)+1\right] = -\left[\frac{2+\epsilon_d}{M+1+\epsilon_d}+1\right] = -\left(\frac{M+3+2\epsilon_d}{M+1+\epsilon_d}\right). \tag{44}$$

Use the free-entry condition and substitute in dN/dM,

$$\frac{dW}{dM} = \frac{(P-c)q}{M+1+\epsilon_d} - \frac{dN}{dM}\frac{M}{N}(1-\beta)(P-c)\epsilon_d = \frac{(P-c)q}{M+1+\epsilon}\left[1-(M+3+2\epsilon_d)(1-\beta)\right]. \tag{45}$$

Then, down stream merger increases social welfare if and only if

$$(1-\beta)(M+3+2\epsilon_d) > 1 \tag{46}$$

This result is more likely to hold when β is small (large profit share to upstream firms induces excessive entry), or when M is large, or when the demand curve is concave enough.

VI. Conclusion

To be completed..

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(To be completed)

Appendix

A. *Proof of Proposition 1*

Equation (2) implicitly defines r as a function of M. Totally differentiate r and M and rearrange, we get

$$\frac{dr}{dM} = g_X \frac{dX}{dM} + g_M. \tag{47}$$

We can derive the expression of dX/dM from equation (4). Totally differentiating equation (4) w.r.t X and M, we get

$$[(N+1)g_X + Xg_{XX}]\, dX + (Ng_M + Xg_{XM})\, dM = 0 \tag{48}$$

From equation (2), we have $g_M = -XP'/M^2$, $g_X = P'(M+1+\epsilon_d)/M$, $g_{XM} = -P'(1+\epsilon_d)/M^2$ and $g_{XX} = [(M+2)P'' + XP''']/M$. Substituting these partial derivatives into the total differentiation above and rearranging yields

$$\begin{aligned}\frac{dX}{dM} &= \frac{XP'(N+1+\epsilon_d)}{M^2[(N+1)g_X + Xg_{XX}]} = \left(\frac{X}{M}\right)\left(\frac{P'}{M}\right)\left[\frac{N+1+\epsilon_d}{(N+1)g_X + Xg_{XX}}\right] \\ &= \left(\frac{X}{M}\right)\left(\frac{g_X}{M+1+\epsilon_d}\right)\left[\frac{N+1+\epsilon_d}{g_X(N+1+\epsilon_u)}\right] = \left(\frac{X}{M}\right)\left[\frac{N+1+\epsilon_d}{(M+1+\epsilon_d)(N+1+\epsilon_u)}\right]\end{aligned}$$

Substitute dX/dM into the expression of dr/dM, we have

$$\frac{dr}{dM} = \left(\frac{-XP'}{M^2}\right)\left(\frac{\epsilon_u - \epsilon_d}{N+1+\epsilon_u}\right) \tag{49}$$

By Assumption 1 and 4, $-XP'/M^2$ and $N+1+\epsilon_u$ are both positive. Then, $\text{sign}(dr/dM) = \text{sign}(\epsilon_u - \epsilon_d)$. By definition, $\epsilon_u = Xg_{xx}(X,M)/g_x(X,M)$, we can write ϵ_u as function of ϵ_d,

$$\begin{aligned}\epsilon_u &= \frac{X\left[(M+2)P'' + XP'''\right]}{(M+1)P' + XP''} = \frac{XP''(M+2+\alpha)}{P'(M+1+\epsilon_d)} \\ &= \frac{\epsilon_d(M+2+\alpha)}{M+1+\epsilon_d} = \epsilon_d + \frac{\epsilon_d(1+\alpha-\epsilon_d)}{M+1+\epsilon_d}\end{aligned}$$

This gives $\epsilon_u - \epsilon_d = \epsilon_d(1+\alpha-\epsilon_d)/(M+1+\epsilon_d)$. Then, it must be true that $\text{sign}(dr/dM) = \text{sign}(\epsilon_d(1+\alpha-\epsilon_d))$ since $M+1+\epsilon_d > 0$. Differentiating ϵ_d w.r.t X (notice $X = Q$) yields

$$\begin{aligned}\frac{d\epsilon_d}{dX} = \frac{d}{dX}\left(\frac{XP''}{P'}\right) &= \frac{P'(P''+XP''') - X(P'')^2}{(P')^2} = \frac{P'P''\left[\left(1+\frac{XP'''}{P''}\right) - \frac{XP''}{P'}\right]}{(P')^2} \\ &= \frac{P''(1+\alpha-\epsilon_d)}{P'} = \frac{\epsilon_d(1+\alpha-\epsilon_d)}{X}\end{aligned}$$

That is $\text{sign}(dr/dM) = \text{sign}(d\epsilon_d/dX)$. *Q.E.D.*

B. *Proof of Proposition 3*

Substitute dX/dM and the first-order conditions of both upstream and downstream firms, equation (12) becomes

$$\frac{dW}{dM} = \left(-\frac{P'X}{M}\right)\left[\frac{X(N+1+\epsilon_d)}{M(M+1+\epsilon_d)(N+1+\epsilon_u)}\right] - \sum_{k=1}^{N} g_X x_k \frac{dx_k}{dM}.$$

Also, the change of each upstream firm k's output is

$$\frac{dx_k}{dM} = \frac{X}{M(M+1+\epsilon_d)}\left[1 + s_k(1+\epsilon_d) - \frac{(1+s_k\epsilon_u)(N+1+\epsilon_d)}{N+1+\epsilon_u}\right],$$

and we thus have

$$
\begin{aligned}
g_X x_k \frac{dx_k}{dM} &= \frac{g_X x_k X}{M^2(M+1+\epsilon_d)}\left[1 + s_k(1+\epsilon_d) - \frac{(1+s_k\epsilon_u)(N+1+\epsilon_d)}{N+1+\epsilon_u}\right] \\
&= \frac{P'X^2}{M^2}\left[s_k + s_k^2(1+\epsilon_d) - \frac{(s_k+s_k^2\epsilon_u)(N+1+\epsilon_d)}{N+1+\epsilon_u}\right].
\end{aligned}
$$

Aggregate $g_X x_k(dx_k/dM)$ over all k, we get

$$
\sum_{k=1}^{N} g_X x_k \frac{dx_k}{dM} = \frac{P'X^2}{M^2}\left[\left(1 - \frac{N+1+\epsilon_d}{N+1+\epsilon_u}\right) + \sum_{k=1}^{N} s_k^2 \left((1+\epsilon_d) - \frac{\epsilon_u(N+1+\epsilon_d)}{N+1+\epsilon_u}\right)\right].
$$

Substituting it into (12) yields

$$
\begin{aligned}
\frac{dW}{dM} = -\left(\frac{P'X^2}{M(N+1+\epsilon_u)}\right)\Bigg\{ & \frac{N+1+\epsilon_d}{M+1+\epsilon_d} + (\epsilon_U - \epsilon_d) + \\
& H[(N+1+\epsilon_u)(1+\epsilon_d) - \epsilon_u(N+1+\epsilon_d)]\Bigg\}.
\end{aligned}
$$

Therefore, $dW/dM \gtreqless 0$ if and only if

$$
\frac{N+1+\epsilon_d}{M+1+\epsilon_d} + (\epsilon_U - \epsilon_d) + H[(N+1+\epsilon_u)(1+\epsilon_d) - \epsilon_u(N+1+\epsilon_d)] \gtreqless 0.
$$

The result then follows. *Q.E.D.*

C. *Proof of Proposition 4*

Differentiating W w.r.t M yields

(50)
$$
\frac{dW}{dM} = (P - r)\frac{dX}{dM} - X\frac{dr}{dM}
$$

Using the aggregated first-order condition (2) and the fact $r = g(X, M)$, dW/dM can be written as

(51)
$$\frac{dW}{dM} = -\frac{P'X}{M}\frac{dX}{dM} - Xg_x\frac{dX}{dM} - Xg_M = -\frac{P'X}{M}\left[\frac{dX}{dM} + (M+1+\epsilon_d)\frac{dX}{dM} - \frac{X}{M}\right]$$
$$= -\frac{P'X}{M}\left[\left(\frac{\partial X}{\partial M} + \frac{\partial X}{\partial N}\frac{dN}{dM}\right)(M+2+\epsilon_d) - \frac{X}{M}\right].$$

Totally differentiate (4) w.r.t X and N, we can derive the expression of $\partial X/\partial N$

$$\frac{\partial X}{\partial N} = \left(\frac{X}{N}\right)\left(\frac{1}{N+1+\epsilon_u}\right). \tag{52}$$

From the free-entry condition and the first-order condition (4), we have

$$-g_x(X, M)X^2 = N^2K. \tag{53}$$

Totally differentiate (53)w.r.t X, M and N, we get

$$-\left(g_{XX}X^2 + 2g_xX\right)dX - g_{XM}X^2 = 2NKdN \tag{54}$$

Substitute out g_{XX} and g_{XM} by using the expressions we derived in the proof of Proposition 1, we get

$$-Xg_X(2+\epsilon_u)dX + \frac{(1+\epsilon_d)X^2g_X}{(M+1+\epsilon_d)M}dM = -\frac{2g_XX^2}{N}dN. \tag{55}$$

Cancel $(-Xg_X)$ on both sides, we get

$$(2+\epsilon_u)\left(\frac{\partial X}{\partial M} + \frac{\partial X}{\partial N}\frac{dN}{dM}\right) - \frac{X(1+\epsilon_d)}{M(M+1+\epsilon_d)} = \frac{2X}{N}\frac{dN}{dM}. \tag{56}$$

Substitute in the expressions of $\partial X/\partial M$ (same as dX/dM in the case without free-entry) and $\partial X/\partial N$ (52) and rearrange, we get

$$\frac{dN}{dM} = \left(\frac{N}{M}\right)\left(\frac{1}{2N+\epsilon_u}\right)\left[\frac{N+1+\epsilon_d+N(\epsilon_u-\epsilon_d)}{M+1+\epsilon_d}\right]. \tag{57}$$

Thus, we have

$$\begin{aligned}\frac{dW}{dM} = -\frac{XP'}{M}\Bigg\{\frac{X}{M}\Bigg\{\Bigg[&\frac{N+1+\epsilon_d}{(M+1+\epsilon_d)(N+1+\epsilon_u)}+ \\ &\frac{N+1+\epsilon_d+N(\epsilon_u-\epsilon_d)}{(2N+\epsilon_u)(M+1+\epsilon_d)(N+1+\epsilon_u)}\Bigg](M+2+\epsilon_d)-1\Bigg\}\Bigg\}.\end{aligned} \tag{58}$$

Simplify the term in the bracket before $(M+2+\epsilon_d)$, we get

$$\frac{dW}{dM} = -\frac{X^2P'}{M^2}\left\{\left[\frac{(N+\epsilon_u)(\epsilon_u-\epsilon_d)-(N+1+\epsilon_d)}{(N+1+\epsilon_u)(2N+\epsilon_u)}\right](M+2+\epsilon_d)-1\right\}. \tag{59}$$

Since $-X^2P'/M^2 > 0$, $dW/dM < 0$ is equivalent to the following condition

$$\frac{(N+\epsilon_u)(\epsilon_u-\epsilon_d)-(N+1+\epsilon_d)}{(N+1+\epsilon_u)(2N+\epsilon_u)} > \frac{1}{M+2+\epsilon_d}. \tag{60}$$

The result then follows. *Q.E.D.*

D. *Proof of Proposition 7*

Let $\epsilon_u = \epsilon_d = \epsilon$ and substitute the expressions of $d\epsilon_d/dM$ and dX/dM into equation (27), we have

$$\frac{d\Pi_d}{dM} = \left[(M+1+\epsilon_d)\frac{2}{N}\frac{dN}{dM}-1\right]\frac{N^2}{(M+1+\epsilon_d)^2}.$$

Then,

$$\text{sign}\left(\frac{d\Pi_d}{dM}\right) = \text{sign}\left(\frac{2(M+1+\epsilon_d)}{N}\frac{dN}{dM}-1\right).$$

We first show that the downstream industry profit Π_d increases (decreases) in M if and only if $(2+\epsilon)\left[(1/M) - 1/(2(N+1+\epsilon))\right]$ is larger (smaller) than 1. Differentiate $-g_X X^2/N^2 = K$ w.r.t M, we have

$$
\begin{aligned}
-(g_{XX}X^2 + 2Xg_X)\left(\frac{\partial X}{\partial M} + \frac{\partial X}{\partial N}\frac{dN}{dM}\right) &= 2NK\frac{dN}{dM} \\
-Xg_X(2+\epsilon)\left[\frac{X}{M(M+1+\epsilon)} + \frac{X}{N(N+1+\epsilon)}\frac{dN}{dM}\right] &= -2N\frac{dN}{dM}\frac{g_X X^2}{N^2} \\
(2+\epsilon)\left[\frac{1}{M(M+1+\epsilon)} - \frac{1}{N(N+1+\epsilon)}\frac{dN}{dM}\right] &= \frac{2}{N}\frac{dN}{dM} \\
\left(\frac{M+1+\epsilon}{N}\right)\left(\frac{2N+\epsilon}{N+1+\epsilon}\right)\frac{dN}{dM} &= \frac{2+\epsilon}{M} \\
2\left(\frac{M+1+\epsilon}{N}\right)\frac{dN}{dM} - 1 &= \frac{2(2+\epsilon)(N+1+\epsilon)}{M(2N+\epsilon)} - 1 > 0
\end{aligned}
$$

This is equivalent to

$$
\begin{aligned}
2(2+\epsilon)(N+1+\epsilon) - M(2N+\epsilon) > 0 \\
\frac{2(2+\epsilon)}{M} &> \frac{2N+\epsilon}{N+1+\epsilon} \\
\frac{2(2+\epsilon)}{M} &> \frac{2N+\epsilon}{N+1+\epsilon} = 1 + \frac{N-1}{N+1+\epsilon}.
\end{aligned}
$$

For arbitrary fixed M and ϵ, it is clear we can always find a small enough N such that $(2+\epsilon)\left[(1/M) - 1/(2(N+1+\epsilon))\right] > 0$. In particular, when $M = 1$, we can let $\hat{N}$ be the smallest integer such that $\hat{N} > -(1/2) - \epsilon$. Thus, for any $N \geq \hat{N}$, dPi_d/dM is strictly positive at $M = 1$. This means the peak value of industry profit does not appear at the downstream monopoly case. *Q.E.D.*