Prisoners of Strategy:

Costly Ties and Dueling Party Machines*

Eddie Camp, Avinash Dixit, and Susan Stokes

Princeton University and Yale University

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Abstract

Electoral competition pushes political parties to invest in effective new strategies. Even innovations that force a party to spend more may be worthwhile if they give it an electoral edge. But the edge can quickly become blunted as competors adopt the same strategies. Parties may find themselves in a prisoner's dilemma, and be stuck spending a lot with no relative electoral gains. We analyze a probabilistic voting model of choices of distributive strategies that explains this dynamic. Parties choose whether to distribute directly to voters or to hire agents to do so. Agents solve information problems for parties, but using them imposes costs. When two competitors both use agents the electoral advantages may be neutralized and the costs high; but unilateral shifts may bring electoral peril. The experiences of agent-mediated or machine politics in Britain and the U.S. offer insights into these dilemmas of distributive politics.

1 Introduction

Electoral competition encourages parties to adopt new strategies in their efforts to eke out victories. Increasingly effective strategies often impose new expenses on parties. But they are worth the added expense if they improve the chances of winning. Recent examples of innovations in the U.S. include parties' acquisition of large databases with detailed information about voters, and the use of social media and electronic communications to generate votes and donations.[1] In Mexico in recent years, candidates also deployed new techniques, giving out ATM-style cards to voters. The voters could redeem the cards for merchandise at department stores or for access to social programs, if the candidate won.

The hitch is that innovations may be mimicked by opponents within a few electoral cycles. Once this happens, the innovative strategy loses its electoral advantage. And yet parties may be compelled to continue investing in it out of fear of ceding an edge to the other party. In Mexico, PRI candidates introduced the conditional benefit cards but candidates from the leftist PRD soon mimicked them. Efforts at campaign finance reform in the United States draw on parties' joint interest in reducing costs; but these efforts often founder in part on fears that the other side will cheat and thus regain an electoral edge.[2]

The use of electoral agents or political machines to distribute benefits to voters displays just such dynamics, as we will show in this paper.[3] In many developing democracies today, just as in some advanced democracies in earlier eras, political parties engage in machine politics. They rely on agents to distribute targeted benefits directly to voters. Agents provide valuable information to parties about individual voters' party affinities, turnout propensities, and material needs. They also can monitor voters' actions at the polls.

But agents also impose costs. Their efforts cannot be perfectly monitored by the

1 . See Hersh 2011, x on social media.
2 . Heard, Metch.
3 . We use the terms "agents" and "machines" interchangeably.

parties and their interests do not coincide exactly with those of party leaders. Nineteenth-century British Liberal and Conservative politicians viewed their agents as "treacherous" and as "electioneering parasites;" and Democratic and Republican party leaders viewed the local machine as "a source of insubordination and untrustworthiness."[4]

Party leader in both countries were long aware of the agency losses that mediated distribution entailed, just as they were long aware that their common deployment of this strategy neutralized its electoral advantage, in national if not always in local terms. Nevertheless, party leaders were for decades wary of sloughing off their agents. The prisoner's dilemma in which British politicians found themselves is captured eloquently by Seymour:

> The average member [of the House of Commons] might really prefer a free election; bribery meant expense, and it meant that the skill of the election agent was trusted as more efficacious than the candidate's native powers, an admission that few members liked to make. But there was always a modicum of candidates who preferred to insure their seats by a liberal scattering of gold; in self-protection the others must place themselves in the hands of their agents, thus tacitly accepting, if not approving, corrupt work.[5]

If competitive innovations can quickly lead to costly ties – and if a unilateral reversal of the innovation would yield electoral advantages to one's opponent – how can parties ever abandon these innovations? Today, party leaders no longer deploy machines in the United States. Some vestiges survived until the 1960s, but they are today basically a thing of the past.[6] And though British parties use electoral agents to book meeting halls and contact constituency organizations, their role as purveyors of treats and bribes ended more than a century ago. What explains these changes?

4 . O'Leary, p. xx; Reynolds and McCormick 1986, p. 851.
5 . Seymour, p. 199.
6 . Banfield and Wilson; Mayhew 1986.

Our theoretical model and historical analyses suggest answers to this question. In general terms, exogenous changes may reduce the payoffs from what was once an attractive (if collectively sub-optimal) strategy. More specifically, in this setting we identify two comparative statics of importance in shaping parties' preferences for agent-mediated strategies: the effectiveness of agents in making voters responsive to party largess, and the value parties place on attaining office in relation to the costs they pay to attain it. Under some circumstances, parties will retain even fairly ineffective agents, as we show; but in large parts of the parameter space, they will prefer to fire them and engage in more centralized or direct distribution. And parties that face higher costs, without a correspondingly higher value of office, will eventually find that firing their agents is a dominant strategy.

Our historical analysis underscores the problem of costs. A key development that kept machine politics alive in the U.S. much longer than in Britain was another 19th-century strategic innovation: corporate financing of campaigns. British candidates, by contrast, paid for their own campaigns, giving them strong incentives to fire their electoral agents and reduce campaign costs. Ironically, from the perspective of our own day, the late 19th-century influx of vast sums of financial resources for American state parties and political campaigns averted the drive toward reforms, which in turn encouraged long and costly campaigns.

Hence, exogenous changes can make unilateral abandonment of the strategies leading to costly ties a dominant strategy. Yet if it remains beneficial to be the only party deploying them, both sides might wait for the other to desist, with the result that neither side does. Our theoretical findings point toward just such coordination problems. In such settings, we can look to institutional fixes, such as legislative actions, to ease both parties out of the costly tie. In our historical cases, we will observe one such an institutional fix.

The experiences against which we test our model are historical. Yet our findings are by no means irrelevant to 21st-century democracies. Machine politics remains prevalent in today's developing democracies, and has not been fully abandoned even in some advanced ones. As recently as 2004, the Italian parliament prohibited the introduction of mobile phones into voting booths; voters were taking pictures of their ballots, to prove to party operatives that they had complied with their end of a vote-buying arrangement. As just mentioned, our explanation for the endurance of U.S. machines well into the 20th century will emphasize the ready availability of campaign funding from corporate interests, a phenomenon that should resonate with observers of American elections today. Our theoretical model and historical cases underline the possibility that parties will retain costly strategies even though they do not derive any clear electoral advantage from them and would be better off if both sides abandoned them. Perhaps vast expenditures on television advertising and on enormous databases about voters obey a similar logic.

2 Related Literature

We contribute to a formal literature on distributive politics, exemplified by Cox and McCubbins, Lindbeck and Weibull, and Dixit and Lodregan, and Stokes.[7] Our model is closely related to formal theories of political parties as internally differentiated into actors who pursue conflicting goals, in contrast, most classically, to Downs.[8] The heterogeneous-party theorists include Hirschman, May (both authors model parties with leaders and activists), Roemer (opportunists and militants), and Alesina and Spear (party leaders from the current and the next generation).[9]

Closer still to the model introduced in the next sections are ones that distinguish party leaders, on one side, and brokers or agents, on the other; the role of agents being to monitor voters and target benefits in a fine-grained way.[10] Our model shares with that of

7 . Cox and McCubbins 1986, Lindbeck and Weibull 1987, Dixit and Lodregan 1996, and Stokes 2005.
8 . Downs 1957.
9 . Hirschman 1970, May 1973, Roemer 2001, Alesina and Spear 1987.
10 . See Stokes et. al. forthcoming, Camp 2010, 2012. Keefer 2007, and Keefer and Vlaicu 2008 also distinguish candidates from patrons in models of clientelism, though their patrons are not strategic agents.

Stokes and co-authors the feature of political machines overinvesting in core supporters.[11] In the current model, by assumption, parties reap some spillover benefits when they spend on their core supporters, though their agents will be tempted to spend more than optimal amounts in this way; whereas in earlier studies, a dollar spent on core supporters was, from the standpoint of party leaders, a dollar wasted.

Another important difference is that the model here focuses on inter-party competition in agent-mediated distribution; whereas the common assumption of earlier models was that only one party could use targeted distribution. For instance, the purveyor of agent-mediated distribution was the incumbent, and the opponent – who remained in the background – would have to rely on promises of programmatic benefits to come.

Another contrast is that earlier studies posited a budget constraint within which parties had to remain, but they were indifferent between spending all or just a part of this budget. In the current model, by contrast, other things being equal the less party leaders spend in their attempts to gain office the greater their utility.

A final, crucial difference is that previous agency models of machine politics only analyzed settings in which parties hired brokers or agents. In this paper we analyze four subgames, ones in which both agents hire agents, one in which neither does, and (implicitly) two in which one or the other party hires agents and the other does not. The comparison across subgames allows us to explore more explicitly the changing conditions that would encourage party leaders to shift between hiring agents and forgoing them. This modeling approach allows us to identify plausible reasons why historical actors shift from mediated to unmediated distributive strategies.

Our paper also contributes to discussions of political development and the pre-history of the welfare state in the U.S. and Britain.[12] Earlier studies describe the

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11 . A similar kind of overinvestment, but within the confines of models with homogeneous parties as teams, results from one party having administrative advantages (Lindbeck and Weibull) or efficiency (Dixit and Londregan) in targeting core constituencies, or from candidates being risk-averse (Cox and McCubbins).

12 . Eggers and Spirling (2011), Kam (2009), Skowronek (1982), Bensel (2004), Carpenter, Mayhew (1986), Banfield and Wilson (1966) . . .

tensions between party leaders and agents and the decline of machine politics, but they do not link the dynamics of decline to the agency losses that mediated distributed imposed on parties.[13] No one, to our knowledge, has identified the puzzling differences between the British and American experiences of agent-mediated distributive politics, much less attempted to explain these differences.[14] The explaination for the transition from party agents to programmatic politics in Britain has emphasized the crucial role played by key pieces of legislation, without asking why Parliamentary leaders were able to pass legislation when they did and why earlier attempts failed.[15]

3 The Model

The timing of our model is as follows. First, party leaders choose whether to hire agents or pay uniform benefits to all voters in an unmediated way. If they choose unmediated distribution, they then decide a level of transfers. If they opt for agent-mediated distribution, they choose how much to transfer to voters through agents and how much to offer agents as a bonus. On the path where agents are hired, the agents choose how much to allocate to core constituents versus swing voters. Nature then delivers a shock that influences voter opinion. Finally, voters observe their party affinities, their transfers, and the shock and decide which party to vote for. The party that wins a majority of votes is victorious in the election and pays a bonus to any agents it has employed. In the background is the idea that the process then repeats itself, though we confine our analysis to a single iteration. The model we analyze focuses on the most strategic part of this story: parties' choices of whether to hire or forgo agents and the welfare they derive from these choices.

We consider a two-party polity, and label the parties L and R. In this paper we

13 . A few exceptions are discussed below.
14 . With the exception of a brief comparative analysis in Sikes 1928, p. 125.
15 . The force of legislation in reducing the role of agents and of electoral bribery is emphasized by Seymour (1970[1915]) and by O'Leary (1962).

assume symmetry between the parties, in the sense that they have equal numbers of core supporters – voters whose partisan affiniites or ideological preferences leave them predisposed to support the party. We also assume the parties have access to identical methods of campaigning and vote-winning.[16]

There are three groups of voters, labeled L, R and S. The first two types are core supporters of the respective parties, while the S are swing voters – those whose lack of partisan attachment leaves them more responsive to distributive goods. There are N_c core supporters of each party, and N_S swing voters; the total population is $N = 2\,N_c + N_s$; these numbers are exogenous to the model.

3.1 Specification of Win Probabilities

To increase their chances of winning elections, the parties give transfers to the various types of voters. For party L, denote the amount given to each of its core supporters by l_c and that to each swing voter by l_s; similarly r_c and r_s for party R. With this notation, we assume that the probability π_L that party L will win the election is given by

$$\pi_L = \frac{f(l_c, l_s)}{f(l_c, l_s) + f(r_c, r_s)} \tag{1}$$

where $f(c, s)$ is a function specified and explained below. The R party's victory probability is given by $\pi_R = 1 - \pi_L$. [17]

Contest success functions of this form are used in many applications including R&D competition, rent-seeking, and political campaigns. Skaperdas reviews this literature and shows in his Theorem 2 that the only form satisfying certain desirable axioms is that when players 1 and 2 expend scalar efforts x_1 and x_2 respectively, the probability of winning for

16 . Dixit (2013) develops the asymmetric case.

17 . The probabilities could be alternatively interpreted as vote shares in a deterministic model. The objective functions stipulated below can then be interpreted as the value the parties place on vote shares, net of the cost of acquiring them. However, that entails assuming that the objective is a linear function of the vote share, which does not seem realistic.

the first player should take the form

$$\pi_1 = \frac{x_1^\theta}{x_1^\theta + x_2^\theta}\,,$$

and of course $\pi_2 = 1 - \pi_1$ is the probability that player 2 wins.[18] The parameter θ captures the marginal (incremental) returns to expending effort. This is more easily understood by considering the odds ratio

$$\frac{\pi_1}{\pi_2} = \left(\frac{x_1}{x_2}\right)^\theta .$$

Taking logarithms of both sides and differentiating,

$$\frac{d\,\ln(\pi_1/\pi_2)}{d\,\ln(x_1/x_2)} = \theta\,.$$

Thus θ is the elasticity of the odds ratio with respect to the effort ratio: increasing x_1 by 1% relative to x_2 will shift the odds ratio by θ% in player 1's favor. Second-order conditions of maximization impose limits on θ; for our purpose $\theta \leq 1$ will suffice.

In our application, the "effort" is two-dimensional: parties or their agents can transfer to core voters and to swing voters. Therefore we use the obvious generalization where the function $f(c, s)$ takes the Cobb-Douglas form

$$f(c, s) = A\; c^{\theta\,\alpha}\; s^{\theta(1-\alpha)}\,. \tag{2}$$

The constant A multiplies the effect of transfers to both the core and the swing voters, l_c and l_s, on the odds ratio π_l/π_r by the same factor. The α measures the relative importance of core supporters toward victory, and θ and α combine to determine the marginal returns

18 . Skaperdas (1996).

to various kinds of transfers. More precisely, from (1) and (2) we have

$$\frac{\pi_L}{\pi_R} = \left(\frac{l_c}{r_c} \right)^{\theta\,\alpha} \left(\frac{l_s}{r_s} \right)^{\theta(1-\alpha)} .$$

Therefore

$$\frac{d\,\ln(\pi_L/\pi_R)}{d\,\ln(l_c/r_c)} = \theta\,\alpha\,, \quad \frac{d\,\ln(\pi_L/\pi_R)}{d\,\ln(l_s/r_s)} = \theta\,(1-\alpha)\,.$$

That is, a 1% relative shift in the transfers given by each party to its own core supporters shifts the odds ratio of victory by $\theta\,\alpha$%; the corresponding effect of transfers to swing voters is $\theta\,(1-\alpha)$%.

The intuition behind the specification in (1) and (2) is as follows. The swing voters are not committed to either party, and consider targeted transfers from both parties as one consideration among many when making their decision. But swing voters are heterogenous in their preferences over other issues, and these preferences are also subject to idiosyncratic random shocks. When one party increases its transfers, that induces some swing voters to turn out and to vote for it rather than the other party. But the magnitude of this effect is uncertain; therefore we can only speak of the effect of transfers on the probability of victory.

As for core supporters, those who side with party L are never going to vote for party R. But transfers to them increase the probability of L's victory in at least two ways. First, there may be unobserved heterogeneity within the core supporters as regards the strength of their support, which makes them more or less likely to turn out on the day despite competing claims on their time; transfers may tip some on the margin into voting. Second, core supporters who feel taken care of, and given some cash or appropriate in-kind transfers, are more likely to be energized and become activists who provide extra services such as holding meetings, going door-to-door before elections, volunteering as observers at polling stations, giving rides to others who need to get to and back from voting, which may help persuade some swing voters into supporting this party and turning out to vote. The Cobb-Douglas function $f(c, s)$ captures this interaction between activism of core supporters and turnout and voting from the swing group: the cross-partial derivative $\partial^2 f/\partial c\,\partial s$ is positive; therefore a larger transfer to core supporters raises their activism, which increases the marginal contribution to victory from promising transfers to the swing voters.

Our specification is a reduced form. The transfers increase the probability of winning or losing; they do not deterministically cause a win/loss outcome. The randomness could be due to some unobserved heterogeneity or random shocks to preferences of individual voters.

3.2 Agents

Transfers to core supporters and to swing voters have different effects on the probability of victory; therefore parties want freedom to choose unequal levels of the two. However, keeping $l_c \neq l_s$ requires them to identify core supporters and swing voters, and they usually lack the information. They can use local agents who have or acquire this expertise, and then channel the transfers through them in various forms of targeted benefits. The advantages of such agency appear in three ways in our model. The first two are in the form of the function $f(c, s)$:

$$f(c,s) = \begin{cases} A_p \; c^{\theta_p\,\alpha} \; s^{\theta_p(1-\alpha)} & \text{without agent,} \\ A_a \; c^{\theta_a\,\alpha} \; s^{\theta_a(1-\alpha)} & \text{with agent,} \end{cases} \tag{3}$$

where $A_a > A_p$ and $\theta_a > \theta_p$. Using the interpretations of A and θ following (2), this says that both the average and the marginal effects of transfers made through local agents are higher than those of transfers made directly by the party leaders. Thus voters are more responsive to resources distributed through agents.

There are several reasons why this might be true. Agents can deploy their detailed knowledge of constituents and neighborhoods to match distributive benefits to people's needs and leverage individual circumstances for votes. Agents can also monitor voters' actions – whether someone who received benefits actually went to the polls, and whether that voter is likely to have voted for the machine party. The extensive literature on clientelism has shown that, even when balloting is secret, party agents are often able to infer the voting behavior of individuals and many voters are aware of this ability.[19]

The third advantage of agency or machine politics appears in constraints on the parties' optimization. Without an agent, the party cannot distinguish between different types of voters, and can only make uniform transfers to all voters via programmatic policies. Thus party L can offer a uniform amount, say l, to all N voters. This not only imposes a constraint $l_c = l_s = l$, but also entails giving the same common per capita amount l to the core supporters of the R party, who are never going to vote for L. A similar restriction applies to party R when it does not use an agent.

3.3 Payoffs

We denote by V the value each party places on victory. This could be a monetary payoff in a kleptocratic polity, but is more likely to be the leaders' utility from implementing their desired policies when in power, or merely ego-rent. We assume that each party wants to maximize the expected value of victory net the costs of making the transfers, and also net of payments to agents when agents are used.

We denote by I_L and I_R the expenditures of the parties on the transfers to the electorate. When agents are used, the parties will have to promise them bonuses contingent on victory; we denote these by B_L and B_R. Thus party L's net payoff or utility is

$$U_L = \begin{cases} \pi_L \, V - I_L & \text{without agent,} \\ \pi_L \, (V - B_L) - I_L & \text{with agent,} \end{cases} \tag{4}$$

where π_L and I_L are to be expressed in terms of the choice variables l_c, l_s etc. A similar expression holds for party R.

19 . See Stokes et al. chapter 4.

Parties pay agents a bonus, contingent on the party's winning, as an incentive for the agents to work for victory. However, agents also get some private utility from cultivating, organizing, and leading a group of core voters who are loyal to the agent – regularly meeting with them, giving them instructions during election campaigns, being treated with respect by them, and so on. The party leaders cannot identify core supporters or observe how much of the budget is channeled toward them; therefore the agent has the temptation to favor the core supporters too much and build a larger group of these personal followers. That is the source of the agency problem in the model.

We express the expected payoff of the agent of party L as

$$A_L = \pi_L \; B_L + \beta \; l_c \; N_c \tag{5}$$

where the victory probability π_L is given by (1) as above. The term $\beta \; l_c \; N_c$ represents the local agent's private benefit. The idea is that as agents channel more resources to core voters, the agents are able to expand their personal power base; the linearity is for mathematical tractability. Of course a similar expression obtains for the expected payoff of party R's agent.

In what follows we compare subgames. We start with a *hypothetical* subgame that lies outside of our model. Here, party leaders have detailed information about voters: who is swing, who is one's own core, who is the opponent's core, what individuals need and what their voting behavior is. The leaders therefore don't need agents. The payoffs from this hypothetical subgame provide a baseline against which to compare the more realistic subgames that follow.

The first of these incomplete-information subgames begins with the assumption that neither party employs agents (*no agent-no agent*). The second one assumes that both employ agents (*agent-agent*). The third assumes that one party employs agents and the other does not (*agent-no agent*).[20] We use the payoffs of each of these subgames to

20 . Techically there are two symmetrical subgames of this kind, *agent-no agent* and *no agent-agent*.

generate a payoff matrix, which allows us to identify Nash equilibria.

3.4 Hypothetical Scenario: Targeted Direct Transfers with Full Information

Consider first a counterfactual situation in which party leaders are able to target benefits directly to voters, sending optimal amounts to core and swing voters. They are also able to replicate the monitoring and constituency-service functions of skilled political brokers. In effect this entails giving party leaders the benefits of agency with none of the costs. The situation is unrealistic: in mass electorates, centralized party elites or candidates with large constituencies cannot directly gather such fine-grained information or maintain the kinds of face-to-face relations with their constituents that would allow them to monitor the voters. Hence, according to a leading expert on information and campaigns in the contemporary U.S., "candidates - even experienced incumbents - rarely have knowledge such that they can simply mobilize their supporters on Election Day. Even veteran politicians target voters based on the simple characteristics available to them in public records, like their party registration, age, and race, rather than through the politicians private knowledge."[21]

To set this comparison standard, we suppose each party's leaders can directly observe the type of each individual voter and target transfers, in effect acting as its own local agent. So the L party leaders choose (l_c, l_s) to maximize

$$U_L = \frac{f(l_c, l_s)}{f(l_c, l_s) + f(r_c, r_s)}\, V - l_c\, N_c - l_s\, N_s$$

taking the R party's choices (r_c, r_s) as given (and vice versa). Each party's choices are characterized by two first-order conditions, and we solve these four equations simultaneously to find the transfers in the Nash equilibrium. The details are in Section A

21 . Hersh 2011, p. 2.

of the Mathematical Appendix.

The parties are symmetrically situated in terms of the numbers of their core supporters and the functions that determine how their transfers affect their probabilities of winning. Therefore we consider a symmetric Nash equilibrium. Each party wins with probability $\frac{1}{2}$. Label various entities by the subscript f; then each party's budget and transfer amounts are given by

$$I_f = \frac{1}{4}\,\theta_a\,V\,, \tag{6}$$

$$(l_c)_f = (r_c)_f = \frac{1}{4}\,\theta_a\,\alpha\,\frac{V}{N_c}, \quad (l_s)_f = (r_s)_f = \frac{1}{4}\,\theta_a\,(1-\alpha)\,\frac{V}{N_s} \tag{7}$$

and the resulting utilities are

$$U_f = \tfrac{1}{2}\,V - \tfrac{1}{4}\,\theta_a\,V = \frac{2-\theta_a}{4}\,V\,. \tag{8}$$

3.5 Choice of Whether to Use Agents

Now revert to the assumption that party leaders lack the information to implement targeted transfers, and must decide whether to use local agents who have this information, bearing in mind the agency cost – bonus payments and the distortion of transfers toward core supporters by the agent – as well as the benefit of more effective targeting. This is a two-stage game. At the first stage, each party decides whether to use an agent. If a party decides not to hire agents, it determines the total level of uniform transfers to voters that will maximize its payoffs, given the other party's strategy. If a party decides to hire agents, it chooses a level of transfers and bonuses to agents, again to maximize its payoffs, given the other party's strategy. Then the agent chooses levels of transfers to core and swing voters. We look for a symmetric subgame perfect Nash equilibrium. We begin by solving for the second-stage equilibria corresponding to each of the four available combinations of choices at the first stage (no agent-no agent, agent-agent, and the two symmetric versions of agent-no agent).

3.6 No Agent-No Agent Subgame

If the party leaders make direct transfers, but cannot identify the type of any individual voter, they have to give the same amount to each voter. Recall that this is suboptimal because (1) the party cannot give different per capita amounts to its core supporters and to swing voters, (2) it must be wasting some of the budget on giving to the other party's core supporters, even though they are not going to respond to this transfer, and (3) the party cannot address the particular needs of voters or monitor voters' actions and thus are less productive in their use of transfers. Denote the uniform per capita transfers of the two parties by l, r respectively. Then party L chooses l to maximize

$$U_L = \frac{f(l,l)}{f(l,l) + f(r,r)}\ V - l\ N$$

taking r as given, and similarly for party R, using the f functions without agents in (3).

The details are in Appendix B. In the symmetric Nash equilibrium of this subgame, label the entities by the subscript n; then the budget and transfer quantities for each party are

$$I_n = \frac{1}{4}\ \theta_p V\ , \tag{9}$$

$$l_n = r_n = \frac{1}{4}\ \theta_p\ \alpha\ \frac{V}{N}, \tag{10}$$

and the resulting utilities are

$$U_n = \tfrac{1}{2}\ V - \tfrac{1}{4}\ \theta_p\ V = \frac{2-\theta_p}{4}\ V\ . \tag{11}$$

We compare this result with the hypothetical full information case in Section 3.4. In the limiting case where θ_a in the hypothetical subgame is equal to θ_p in the no-agent subgame, the parties have identical total budgets and party utilities. Consider, more generally, when θ_a in the hypothetical case is greater than θ_p in the no-agent case – that is, when parties in the hypothetical case have higher marginal returns on expenditures than do parties in the no-agent case. Counterintuitively, both parties' payoffs are *greater* in the no-agent case than in the hypothetical, full-information case. The reason is that the higher marginal return in the hypothetical case causes parties spend more on transfer budgets, while the chances of victory remain identical.

3.7 Agent-Agent Subgame

This is itself a two-stage game: the first stage is a Nash game between the party leaders, who choose the budgets and bonuses (I_L, B_L), (I_R, B_R); at the second stage the agents choose the allocations (l_c, l_s), (r_c, r_s). We look for the symmetric subgame perfect equilibrium. The details of algebra are in Appendix C. In this subgame the equations defining the equilibrium do not have an explicit closed-form solution. We can characterize some qualitative properties, but further analysis requires numerical solutions, which we discuss in Section 7.

To state and discuss the qualitative properties, define the fraction of the budget each party's agent spends on core supporters as

$$z = l_c \, N_c / I_l = r_c \, N_c / I_R \, . \tag{12}$$

Combining (6) and (7) we see that if the party leaders could target transfers directly, they would set $z = \alpha$. But when targeted transfers must be channeled through agents, we find that in the resulting equilibrium $z > \alpha$. This confirms the obvious intuition: agents who get private utility from assembling and leading a group of core activists distort their choices to favor core supporters.

The calculation in Appendix C yields some additional properties: the agency bias of favoring core supporters will be smaller, other things equal, if (1) the bonus is larger, (2) the budget is smaller, (3) the number of core supporters N_c is larger, and (4) the coefficient β is smaller. Of particular interest for the comparative statics below, low values of β – meaning agents' intersts are well-aligned with those of party leaders – cause parties to retain agents even when agents are not especially efficient[22] and even when parties place a low value on electoral victory relative to campaign costs.[23]

All these results are quite intuitive; here are some further explanations and comments. (1) A higher bonus makes the agents value the party's victory more, and therefore reduces the distortion that would hurt those chances. A larger budget allows the agent to indulge more in his taste for cultivating his core club. Of course the leaders take these comparative statics into account when choosing their optimal budgets and bonuses in the first stage. (2) High bonuses are costly to the party leaders, so they will have to accept a second-best. In the full equilibrium the leaders are not going to give away the whole value of victory as bonus, so we find $B < V$, so the bias toward core voters will definitely exist. But even if $B = V$, some bias will remain. (3) If the number of core supporters N_c is large, giving them special favors is costly, even to the agent, so less of it will be done. (4) A small β means that the agent's interests are better aligned with those of the principals (the party leaders). The principals can deliberately try to select low-β agents, if they can find suitably competent as well as loyal and self-effacing people who have internalized the party's objective. Some parties at least try to develop such cadres to serve as local agents, instead of relying on purely self-interested ones. Some career concerns such as prospects of promotion to leadership positions may also serve to align the agents' interests with those of the principals.[24] Our model does not include such considerations explicitly, but they may be captured by exogenously lowering β.

Finally, the expression for utility of each party is

$$U_b = \tfrac{1}{2} \left[1 - \frac{\theta_a}{2 + \theta_a \, \Omega} \right] \, V \, , \tag{13}$$

where Ω is an endogenous variable that relates to the severity of the agency problem. It is

22 . They have low values of θ_a relative to θ_p.
23 . They have low values of V.
24 . Camp 2012 develops this idea formally.

defined in (C.4) in Appendix C; its exact form is not important here. It equals zero if the agent does not have divergent interests ($\beta = 0$), and positive otherwise.

With this, we can compare the utilities (common to the two parties) in the equilibria of two subgames, one where neither party uses agents (no agent-no agent) and the other where both do (agent-agent). Begin with the limiting case where $\theta_a = \theta_p$ – where transfers to voters have the same marginal productivity, whether they are carried out by the party directly or through agents. Here we find that both parties have higher utilities in the subgame where both use agents than in the one where neither does: using agents cannot be a prisoner's dilemma.

Recall the comparison we made earlier between the hypothetical subgame where both parties had full information and could target transfers directly, with the no agent-no agent subgame – in which the parties do not use agents and make untargeted uniform per capita transfers. In the limiting case where $\theta_a = \theta_p$, the equilibria of these two subgames had equal total budgets and expected utilities. Combining these two comparisons – hypothetical/no agent-no agent, and agent-agent/no agent-no agent – we see that even if the parties have full information, they would be better off using agents than making direct targeted transfers, even despite agency costs. This seemingly counterintuitive result arises because parties adopt symmetric strategies; therefore in neither setting does one party improve its chances of victory. Their payoffs therefore are driven by their expenditures. Agency costs impose sharper limits on what the party is willing to spend; therefore agent-based distribution is associated with lower overall expenditures than is direct (and appropriately targeted) distribution.

Now consider the case where $\theta_a > \theta_p$: agents' transfers have higher marginal productivity than the parties acting directly. From (13) and (11), we have

$$U_b - U_n = \frac{\theta_a \, \theta_p \, \Omega - 2 \, (\theta_a - \theta_p)}{4 \, (2 + \theta_a \, \Omega)} \; V, \tag{14}$$

where the difference on the left-hand side is between a party's utility in the agent-agent and the no agent-no agent subgames. Now, if θ_a is sufficiently larger than θ_p, we can have $U_b < U_n$. What this means is that it is possible for the parties to be trapped in a prisoner's dilemma: using agents is the dominant strategy even though the parties' utilities would be higher if neither used agents. Numerical solutions given below indicate the parameter space in which parties would prefer to shed their agents but are kept from doing so by this agent-agent PD. The intuition is that the higher marginal productivity of the agents makes it attractive for each party to hire them, but when both parties do so, the effects cancel out and neither gains an electoral advantage. And they are left with the increased expenditures associated with hiring agents.

Similar effects do not arise from average productivity differences with and without agents ($A_a > A_p$). This is because in the situations being compared – one where both parties use agents and the other where neither does – both parties f functions have the same multiplicative factors – A_a when both use agents and A_p when neither does – so the factor cancels out from the numerator and denominator of the crucial ratio π_L. (This is illustrated in Appendix C in the derivation of equation (C.8).)

Of course the question whether or not there is a prisoner's dilemma presupposes that both parties using agents is an equilibrium of the full game. To answer that we need to find the consequences of deviations, that is, payoffs in the subgames where only one party uses an agent. This we do in the next subsection.

3.8 Agent-No Agent Subgame

We retain the assumption that the two parties are otherwise identical: they have the same number N_c of core supporters, the identical functional forms of the objective function (that of party L is shown in (4)). They have identical forms for the function $f(c, s)$ (which affects the probabilities of victory via (1)). And they have identical forms for the objective function of an agent if one is employed (for party L, is shown in (5)). But now one of the parties employs an agent to make targeted transfers to its own core supporters and to swing voters, while the other party makes untargeted uniform direct transfers to the whole population. Remember that we are not saying that the parties will in fact behave thus; it is merely a subgame of the full game where each party decides whether to employ an agent, and may well turn out to be a subgame off the equilibrium path of play.

We do the calculations assuming that party L uses an agent and party R does not; of course identical calculations hold for the opposite case, a no agent-agent subgame. The subgame we are considering itself has two stages. At Stage 1, party L chooses the budget I_L and the victory bonus B_L for its agent, and party R chooses its uniform transfer policy r. Stage 2 is only a one-player decision problem, where party L's agent chooses the targeted transfers (l_c, l_s) to party L's core supporters and swing voters respectively.

The details are in Appendix D. We derive a system of five equations that can be solved for the equilibrium levels of I_L, B_L, l_c, l_s and r. Little can be said in general about the solution. We cannot prove that employing an agent always increases the probability of victory, that is, $\pi_L > \frac{1}{2}$, nor can we obtain interpretable conditions for this. Therefore we turn to numerical solutions, and use them to obtain results about the equilibria of the full game where each party decides whether to choose an agent.

4 Numerical Solutions

Numerical solutions allow us to identify sets of parameter values that determine parties' equilibrium strategic choices – whether to distribute resources to voters through agents or to distribute them directly, without the mediation of agents. To compute numerical solutions, we fix a set of parameters at particular values. We then calculate the payoffs for each subgame, to generate a payoff matrix, and we use this payoff matrix to identify a pure strategy Nash equilibrium. A Nash equilibrium consists of one of four strategy profiles: {No Agent, No Agent}, {Agent, Agent}, {Agent, No Agent}, and {No

Agent, Agent}.

Over much of the parameter space, a decline in the value that a party places on victory relative to the money needed to win (V) induces it to abandon its agents and shift to agent-free distribution. And over much of the parameter space, a decline in the relative efficiency of agents – how effective their distributive work is in helping their party win (θ_a relative to θ_p) – also causes parties to abandon them.

Each party's choice of strategies is, of course, conditioned by the decisions made by the other party. Parties frequently find themselves caught in prisoner's dilemmas; and the nature of these dilemmas depends on the degree of agency loss. Consider the agent-agent equilibrium. When agents, interested in boosting their own local power, place a high priority on giving resources to core voters, both parties would be better off if they got rid of their agents: neither party would hurt its chances of winning and both would reduce expenditures. But the dilemma is that each party is better off retaining its agents when the other side retains them. By the same token, if neither side uses agents, either one of them would gain by hiring them – as long as the other side did not follow suit. For relatively high values of β – that is when both parties' agents squander a lot of resources on core voters – every equilibrium in which parties use agents is a PD.

A *no-agent* equilibrium can also be a prisoner's dilemma. But the parameter space giving rise to the no-agent PD is much smaller than the space giving rise to the agent-agent PD. The no-agent PD obtains only when agents have interests that coincide fairly closely with those of the party.

Consider Parties L and R, neither of which uses agents. If, off the equilibrium path, Party L hired agents, its overall expenditures, and in particular its expenditures on voters, would decline. The party would spend more efficiently: it would waste less on voters who strongly support Party R and who will never be moved in L's favor. But the general effect of L's hiring agents in these equilibria is that its chances of winning decline. The reduction of L's expenditures would not be sufficient to offset its loss of electoral strength.

Anticipating a net loss of utility, neither L nor R will, in equilibrium, hire agents. Yet were *both* to hire agents, they would be better off than they are when neither hires them – their relative chances of winning would remain unaltered at 50 percent and they would save money.

To discern the effects of different parameter values on equilibrium outcomes, we conduct two rounds of simulations. We set the agent's multiplicative return from distributing resources to core voters, β, to 0.5. Recall that β represents the degree to which agents prioritize growing their own local power base at the expense of winning more votes for the party. The ideal agent, from the party's perspective, has a $\beta = 0$. Between the first and second round of simulations, all parameters other than the βs remain unchanged.

In the simulations we vary the value that the parties place on victory relative to expenditures, V, as well as the marginal returns from resource expenditures when a party employs agents, θ_a.[25]

Figure (1) depicts equilibria as a function of V and of the relative efficiency of agents, as captured by $\frac{1}{\theta_p} - \frac{1}{\theta_a}$. The straight line at the bottom of the figures represents equilibria in which parties derive the same utility when they both hire agents and when neither does. In all equilibria above this line the parties derive higher payoffs when they both do not employ agents than when they both employ them. In all equilibria below this line the parties derive higher payoffs when they both employ agents than when they do not employ them.

The regions A, B, C, and D indicate whether parties use agents as an equilibrium strategy and whether the equilibrium is a prisoner's dilemma.

- **Region A**: equilibria in which both parties use agents. These equilibria are

25 . We vary V from 4 to 100 in increments of 1, and θ_a from 0.1 to 0.999 in increments of 0.001. All simulations assume the same mix of core and swing voters in the electorate (N_c and N_s).We also hold constant the multiplicative constants to the returns of resource expenditures, A_a and A_p, and the marginal returns of resource expenditure when a party does not employ an agent, θ_p. We set the number of core supporters, N_c, to 0.4 and the number of swing voters, N_s, to 0.2. The marginal returns of resource expenditure when a party does not employ an agent, θ_p, is set to 0.1. The multiplicative constants to the returns of resource expenditure, A_a and A_p, is set to 1 when a party uses an agent and when party does not use an agent. We excluded $\theta_a = 0.8$. 89,901 equilibria were calculated.

prisoner's dilemmas: parties would be better off when neither party employs agents but employing agents is a dominant strategy.

Sample Payoff Matrix: Region A

L \ R	*NoAgent*	*Agent*
NoAgent	16.63, 16.63	11.98, 18.91
Agent	18.91, 11.98	13.68, 13.68

$V = 35, \theta_a - \theta_p = .8$

- **Region B**: contains the equilibria in which one party uses and one party does not use an agent. These equilibria are not prisoner's dilemmas, but are chicken games as each party prefers to retain its agent, as long as the opposing party plays a no agent strategy.

Sample Payoff Matrix: Region B

L \ R	*NoAgent*	*Agent*
NoAgent	16.63, 16.63	15.16, 16.64
Agent	16.64, 15.16	15.11, 15.11

$V = 35, \theta_a - \theta_p = .2$

- **Region C**: contains equilibria in which neither party uses agents. These equilibria are not prisoner's dilemmas: parties are better off when neither employs agents than when both do.

Sample Payoff Matrix: Region C

L \ R	*NoAgent*	*Agent*
NoAgent	16.63, 16.63	15.93, 16.20
Agent	16.20, 15.93	15.49, 15.49

$V = 35, \theta_a - \theta_p = .15$

- **Region D**: contains equilibria in which neither party uses agents. These equilibria are prisoner's dilemmas: parties would be better off when both employ agents but not employing agents is a dominant strategy.

Sample Payoff Matrix: Region D

R

		NoAgent	*Agent*
L	*NoAgent*	16.63, 16.63	15.70, 17.55
	Agent	17.55, 15.70	16.66, 16.66

$V = 35, \theta_a - \theta_p = .004$

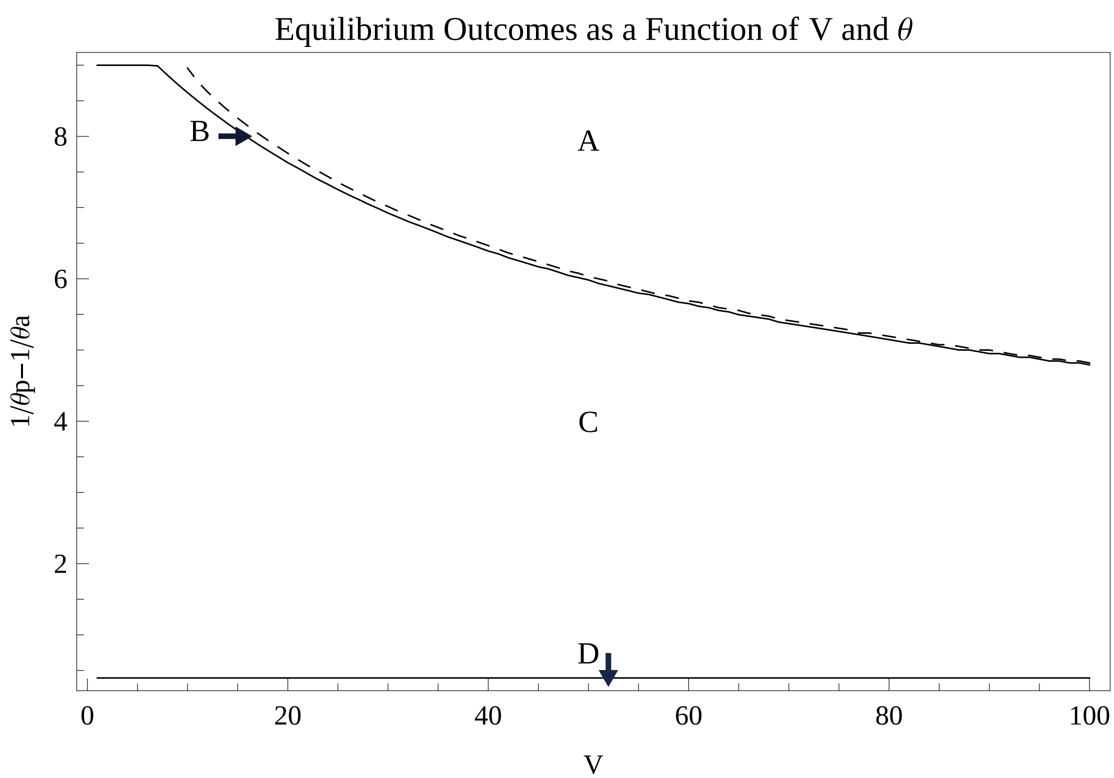

Figure 1: Equilibria Outcomes for $\beta = 0.5$

Figure (1) provides theoretical intuition into why parties abandon agents. Recall that the advantage of using agents is that they can target an appropriate mix of core and swing voters, while avoiding the waste entailed in targeting voters who are core supporters of the opposing party. The main *disadvantages* are that agents are prone to waste resources by spreading them among too many core voters and that they must be paid. One factor that will induce parties to abandon agents is an erosion of agents' efficiency. Holding V constant, a decline in the relative efficiency of agents eventually causes a shift from both parties using agents, to one party using them, and finally to neither party using them.

The same can be said of declines in the value of office relative to the cost of attaining it. Holding the efficiency of agents constant, a decline in the value of victory net expenses causes parties to make the same transition: from one party dropping agents to both of them dropping them.

In the transition from the agent-agent to the no agent-no agent equilibria, Region B identifies an important potential coordination problem between the parties. Within Region B, one party could increase its payoff by unilaterally firing agents. But doing so also increases the opposing party's payoffs by an even greater margin. So each side would gain from abandoning agents but would gain more if the other side abandoned them instead. Moreover, the party that abandons agents would derive even greater benefits if the opposing party also abandoned agents. These incentives suggest that parties might seek an institutional coordination device to assure a simultaneous shift to unmediated distribution. We shall see that, in Britain, parties used legislation as a coordination device in this way.

There is an additional reason to suspect legislation banning agents would become more feasible as agents become relatively less efficient. To see this consider figure (2), in which we hold V constant at 35, but allow $\theta_a - \theta_p$ to vary from 0 to .9. The curve labeled *Benefit of Deviation* measures the utility that a party gains by deviating from a (No Agent, No Agent) strategy profile to hiring an agent,and entering the (Agent, No Agent) strategy profile. The curve labeled *Cost of Non-Cooperation* measures the difference in a party's utility between the (No Agent, No Agent) strategy profile and the (Agent, Agent) strategy profile.

Figure (2) shows that as $\theta_a - \theta_p$ declines the benefit of deviating from cooperation by hiring an agent decline faster than the cost of non-cooperation. When $\theta_a - \theta_p$ is large the cost of non-cooperation takes on its largest value, but parties derive an even larger benefit from deviation. The benefit of deviation could be so large that legislation preventing the use of agents would be impossible to enforce. Yet as the $\theta_a - \theta_p$ decreases the cost of non-cooperation eventually exceeds the benefit from deviation. After this point the difference between the cost of non-cooperation and the benefit of deviation increases as $\theta_a - \theta_p$ decreases. As this difference grows, pressure for legislation banning agents should increase and such legislation should become more feasible. Finally, when $\theta_a - \theta_p$ equals about .2, the parties are in region B in figure (1). Figure (2) shows that when $\theta_a - \theta_p = .2$ the benefits from deviating from the (No Agent, No Agent) strategy profile by hiring agents are nearly zero. At this point, legislation banning agents should be easy to enforce. Ironically, it is also the point in which legislation becomes nearly obsolete as parties lose the all incentives to hire agents.

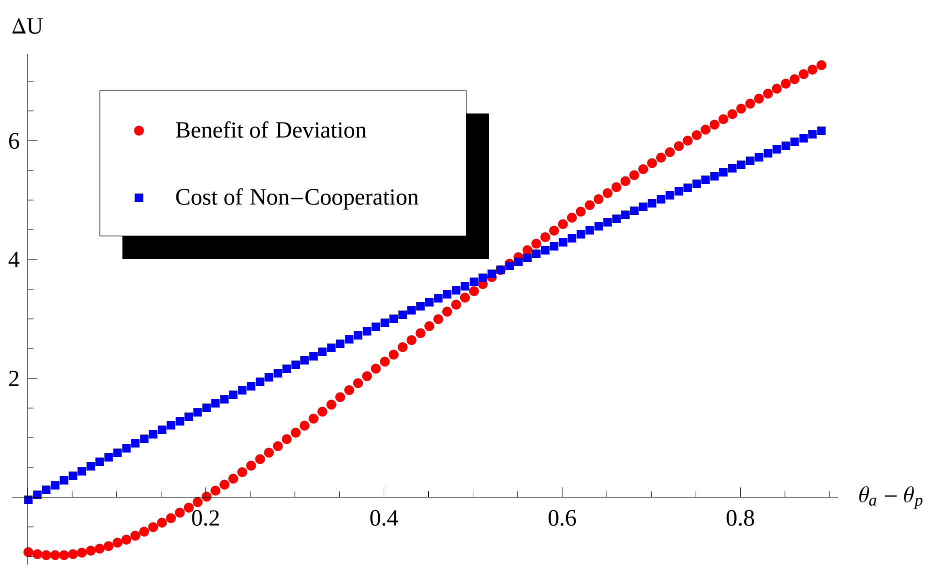

Figure 2: Equilibria Outcomes for $\beta = 0.5$

In sum, parties will shift to direct distribution to voters when agents fail to make resource expenditure substantially more efficient. And if the relative efficiency of agents declines over time, parties that place a lower value on electoral victory or face higher costs will abandon agents earlier than will parties that place a higher relative value on electoral victory or face lower costs.

Figure (1) also provides theoretical intuition into the prisoner's dilemmas that parties face. For every equilibrium in regions A, B, and C, the parties derive more utility when neither party uses agents than when both parties use them. This means that, in this region of the parameter space, every equilibrium in which both parties use agents is a prisoner's dilemma. Empirically, then, it should not be surprising that party leaders would abhor agents and view them as a drain on the party, even while they continued to employ them. For every equilibrium in region D, parties derive more utility when both parties use agents than when neither party uses them. This means that some of the equilibria in which neither party uses an agent are also prisoner's dilemmas. But these prisoner's dilemmas arise in a much smaller area of the parameter space. Over a much larger area, the equilibria in which neither party uses agents is not a prisoner's dilemma.

The findings reported in Figure (1) depend on our assumptions about the distributive preferences of agents. If we instead assumed that agents do not derive much utility from sending resources to core voters, then several key differences would emerge. First, the parameter space in which parties retain agents becomes much larger. And parties are willing to retain agents even when they are inefficient. Secondly, in a small parameter space, parties can employ agents and be better off than if they were both not using agents. These differences show that high agency costs, in addition to the sharp fall-offs in the efficiency of agents and in the value that parties place on victory would generally be the background against which parties to abandon agents in real-world settings.

5 Why Did Machine Politics Decline in Britain and (Eventually) in the U.S.?

In Britain in the decades following 1832, Liberal and Conservative parties sent agents out "through the boroughs to discover the private circumstances of the voter and make use of any embarrassment as a club to influence votes." Party agents carried ledgers with "a space for special circumstances which might give an opportunity for political blackmail, such as debts, mortgages, need of money in trade, commercial relations, and even the most private domestic matters."[26]

In the mid-19th century U.S., Bensel writes that for many men, "the act of voting was a social transaction in which they handed in a party ticket in return for a shot of whiskey, a pair of boots, or a small amount of money,"[27] transactions that required myriad party agents. Machine politics persisted longer in the U.S. than in Britain. The emerging welfare state in the 1930s was superimposed on a system of brokers and ward-heelers. In Pittsburgh, one-third of Democratic ward and precinct captains became project supervisors in the Works Progress Administration (WPA). In Jersey City, the Hague machine appropriated a percentage of WPA workers' salaries to pay for campaign expenses.[28] New York's Tammany Hall machine required party affiliation for applicants for another early New Deal program, the Civil Works Administration (CWA).[29]

Our model helps resolve the historical paradoxes mentioned at the outset. Why do parties employ agents whom they view as untrustworthy? Why was agent-mediated distributive politics prevalent in Britain and the U.S. in the 19th century, only to disappear later? Why did it persist longer in the U.S. than in Britain?

Figure (3) reproduces our simulated results from Figure (1), and superimposes a

26 . Seymour 1970[1915], p.184.
27 . Bensel 2004, p.
28 . Erie, p. 129-30.
29 . Erie, p. 131. Wright (1974) shows that distribution of public relief funds across states during the New Deal was partly a function of their "political productivity," which meant that more unemployment-ravaged states in the South received lower levels of relief than did electorally responsive states in the West.

stylized trajectory of distributive politics in both countries. In the first half of the 19th century, parties used agents as distributive intermediaries – hiring them was a dominant strategy for Liberals and Conservatives and for Democrats and Republicans.

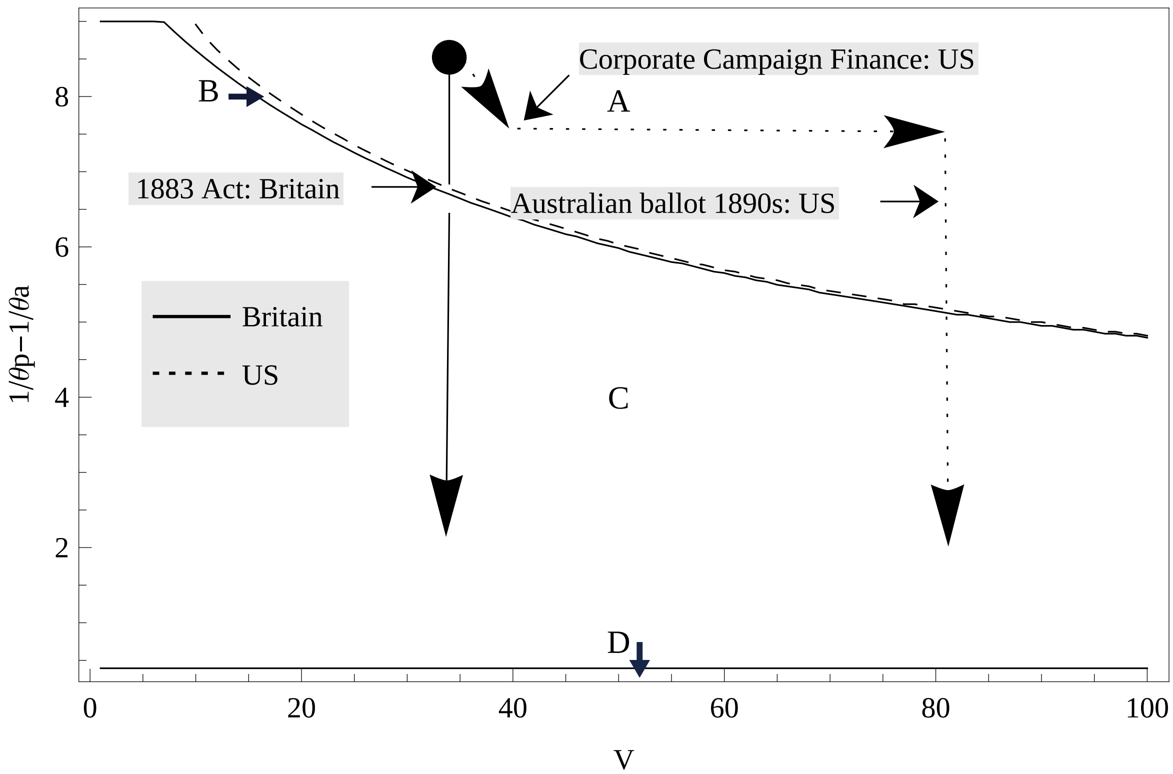

Figure 3: Stylized Trajectories of Party Strategies in Britain and the U.S.

Focusing first on Britain, we locate the situation of Liberals and Conservatives after the 1832 Great Reform Act in the "A" region of the figure: using agents was a dominant strategy, but both endemic agency losses and the prisoner's dilemma explain parties' less-than-enthusiastic view of their electoral agents.

Industrialization in the middle decades of the 19th century transformed the electorate and eroded the efficiency of agents. We represent this shift graphically in a downward descent in Figure (3). The Liberal and Conservative parties found themselves in a situation like Region B, in which employing agents was no longer a dominant strategy for either party. Recall the coordination problems that can arise in Region B. Both L and R want the other to be the one that shifts to unmediated distribution, though both would benefit individually – but less – from making this change themselves. And if L, say, unilaterally abandons agent-mediated distribution, R would have an incentive to retain its agents.[30]

The setting is ripe for an instituional fix, which is how we interpret anti-agent legislation adopted by the House of Commons. Parliament passed effective legislation, in the form of the Anti-Corrupt Practices Act of 1883. We interpret the 1883 Act as a coordination device that enforced a simultaneous departure from mediated distribution. But, ironically – if our model is right – a continuing decline in the efficiency of agents would in effect have moved the parties into Region B. Hence they might well have eventually unilateraly abandoned agent-mediated distribution, even absent legislation.

In the U.S. as well, industrialization eroded the efficiency of party agents in the later 19th century. The adoption of the Australian ballot by most states in the 1890s was parallel to the anti-agent legislation in the House of Commons: it represented party leaders moving against their agents and their machines. But another change discouraged an end to machine politics. This was the rise of state-level political parties, which in the early years of the 20th century increasingly organized, and financed, candidates' campaigns. The source of their funding was, increasingly, corporate interests, the much-maligned "trusts" against which Progressive Reformers raged. British politicians had chafed – before 1883 – under the burden of expensive campaigns which the candidates, or individual sponsors, had to bear. Many U.S. politicians were freed of such financial burdens. In the terms of our model, we see an increase in V, the value of office in relation to the costs of attaining it. The result is captured by a rightward shift in Figure (3), which delayed the drive to adopt anti-machine reforms.

30 . And symmetrically if R were to unilaterally abandon them.

5.1 The Declining Efficiency of Party Agents ($\frac{\theta_a}{\theta_p}$)

A fundamental explanation for the demise of agent-mediated distribution in both countries has to do with the declining effectiveness of agents, and, behind that, the changing nature of the electorates under the stimulus of the industrial revolution. In the terms of our model, the impact of industrialization in 19th-century Britain and the U.S. was to depress θ_a relative to θ_p.

The crucial changes, in both countries, were that the electorates became larger, more urban and thus more difficult to monitor, and wealthier.[31]

Agents' roles of providing individualized information about voters and monitoring their actions meant that each agent was responsible for a small number of voters – usually his neighbors. With growing electorates, ever more agents had to be hired. Though we go here somewhat beyond our model, it is not hard to see that parties facing ever-larger electorates would turn to programmatic campaigning, which scaled more easily.

Regarding rising incomes, our model does not deal with the impact of voters' incomes on the effectiveness of agents. Other related models do, several of them incorporating the assumption of diminishing marginal utility of incomes.[32] And this assumption enjoys some empirical support. Vote selling today is more common among poor people within countries and more widespread in poorer countries. It is more pervasive in Africa than in Latin America, more pervasive in Latin America than in Europe, and more pervasive in Eastern and Central Europe than in Western Europe.[33] And diminishing marginal utility of incomes, leading parties with limited budgets to favor the highly responsive poor, is likely to be the explanation.[34]

It should not be surprising, then, that as populations and (eventually) electorates

31 . For more details, see Stokes et al. forthcoming.
32 . Dixit and Londregan 1996, Stokes et al. forthcoming.
33 . See Stokes et al. forthcoming, and Kitschelt 2012.
34 . Stokes et al., chapter 7, show that risk-aversion among poor people, though it is implied by diminishing marginal utility of income, does not explain the propensity of the poor to sell their votes. In Argentina, neither poor people nor vote sellers, whatever their incomes, attributed greater risk to electoral promises.

became wealthier, the direct offers of material rewards by party agents became less effective. And electorates did get wealthier.[35] In Britain, real wages in manufacturing grew by more than 60% between 1850 and the turn of the century.[36] In the United States, per capita income grew about 20% between 1820 and 1850 and roughly doubled between the end of the Civil War and 1900.[37]

And, indeed, rising incomes were part of the story of the declining effectiveness of party agents – the reason why, even eventually in the U.S., minor campaign gifts became regarded as "a joke."[38]

5.2 The Declining Value of Office Relative to Campaign Expenses (V)

5.2.1 Persistently Expensive Campaigns in Britain

British politicians often complained that their agents were bleeding them dry. The vast sums that agents prodded candidates into spending often came out of the candidates' own pockets. Or it came out of the pockets of a local aristocrat or notable who sponsored the candidate.

Candidates and party leaders' unhappiness with electoral agents notwithstanding, they found themselves in a prisoner's dilemma. Charles Seymour, whose *Electoral Reform in England and Wales* remains, a century after its publication, the *locus classicus* on the electoral bribery there, captures well this dilemma. In the early decades after the Great Reform Act of 1832, MPs viewed themselves as in peril of losing office should they stop working through agents while others kept using them. Seymour wrote,

> The average member [of the House of Commons] might really prefer a free

35 . They did so despite the fact that successive waves of franchise reform, in particular in Britain, opened the franchise to poorer people and hence, over the short run, depressed the average income of the electorate.
36 . Hoppen 2000, see also Lindert 2000.
37 . Lindert 2000.
38 . Banfield and Wilson (1963).

> election; bribery meant expense, and it meant that the skill of the election agent was trusted as more efficacious than the candidate's native powers, an admission that few members liked to make. But there was always a modicum of candidates who preferred to insure their seats by a liberal scattering of gold; in self-protection the others must place themselves in the hands of their agents, thus tacitly accepting, if not approving, corrupt work.[39]

Effective anti-bribery legislation had to await a moment when the transformation of the electorate – outlined earlier – had undermined the effectiveness of the electoral agent. At that point, at least one party could profitably disband its agents. But both parties would be tempted to wait for the other to fire their agents first. To coordinate the transition to direct distributive politics, the House of Commons adopted legislation that basically eliminated electoral agents as they had operated for decades.

A first really significant legislative blow to electoral agents and to the market for votes came in 1872, with the introduction of the written ballot. Corruption receded definitively a decade later, in the wake of the Corrupt and Illegal Practices Act of 1883. Indeed, O'Leary holds that the 1883 act "eliminated" corruption.[40] This late-Victorian reform imposed strict regulations on campaign spending, barred the use of paid canvassers, and put in place procedures for investigating and punishing violators. Thus it became risky for election agents to spend funds illegally on bribes. Leaders of both major parties desired, in O'Leary's phrase, "to wipe out the tribe of electioneering parasites." Hence there was a "surprising degree of accord between the leaders of the [Liberal and Conservative] parties during the debates between 1880 and 1883" – surprising given the intensity of party conflict in this period.[41]

In the debates leading to the passage of the 1883 Act, some Conservative back-benchers objected to the bill's proposed campaign spending limits. Significantly it

39 . Seymour, p. 199.
40 . O'Leary 1962. Seymour concurs in seeing the 1883 Act as the key to ending electoral brivery.
41 . O'Leary 1962, p. 229.

was John Gorst, a former Tory head agent, who reassured them. Gorst countered that candidates would still be able to mount effective campaigns, at lower costs: "All that was really required was that the constituencies should have the means of amply being informed, or informing themselves, of the character, qualifications and political views of the candidates."[42]

As Gorst's words make clear, a central motivation for finally passing effective anti-agent reforms was to reduce the costs of campaigns. And in this sense, too, the reforms were effective. Figure (4) shows that per-voter costs were brought down with the introduction of the written ballot in 1872, and came down even more sharply, and irreversibly, after the 1883 Act. Of course, with a growing electorate one might well expect per-voter costs to fall. (Though this would not be the case in the United States.) But Figure (5) shows that not just the total sum but also the composition of expenditures shifted. Expenditures on agents declined after the 1883 Act, as they were intended to do, while expenditures on publicity increased – the latter reflecting the late-century shift to unmediated party appeals.

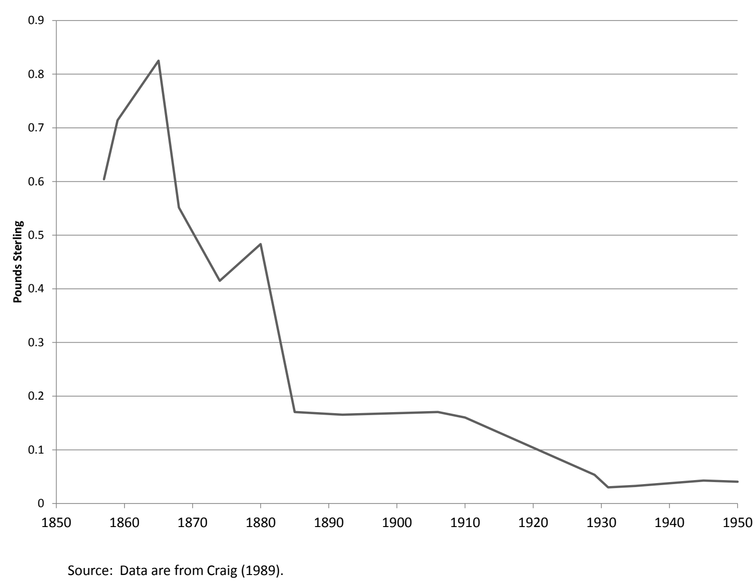

Figure 4: Campaign Expenditures Per Voter in Britain, 1857-1959

42 . Hansard April 27, 1882, cclxviii, cited in O'Leary, p. 165.

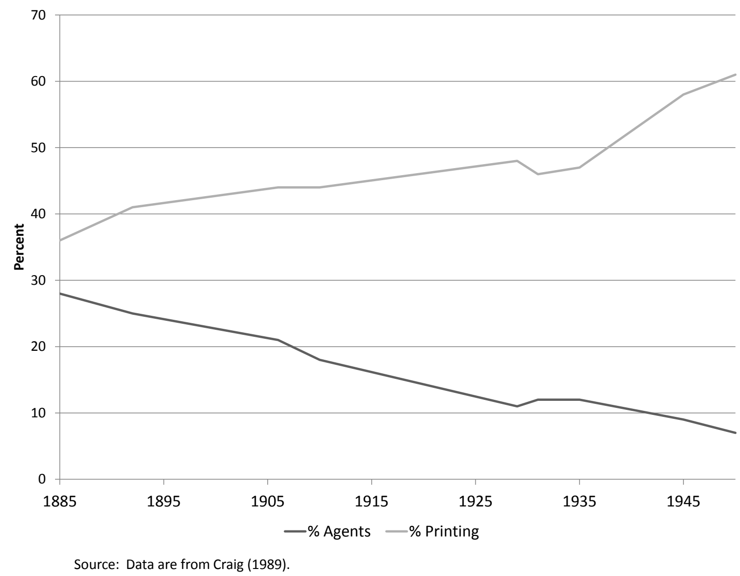

Figure 5: Trends in British Campaign Spending on Agents and Printing, 1885-1960

In sum, in the context of the declining effectiveness of party agents and financially costly campaigns, Parliamentary leaders passed legislation that eased their parties' transition to unmediated distributive competition.

5.2.2 The Rise of Externally Funded Campaigns in the United States

Democratic and Republican leaders in the 19th-century U.S. were no fonder of their agents than were their British counterparts. They saw them as unreliable and ineffective. About "treacherous" electoral agents in New York and New Jersey in the last two decades of the 19th century, Reynolds and McCormick write that, "To the partisan leaders the local machine was a source of insubordination and untrustworthiness."[43] And the machine's efforts were decreasingly effective: "Perhaps in an earlier day when the electorate was smaller and more deferential, the party organization had been able to deliver the vote with fewer hitches, but if that had ever been the case, it was no longer true by the 1880s."[44] Like their British counterparts, American political leaders undertook reforms aimed at dislodging their untrustworthy agents.

The most effective and widely enacted reform, as we have seen, was the adoption of the Australian or "official" ballot. In New Jersey and in other states, "the Democratic and Republican leadership used the official ballot to wrest control over the election from the hands of machine operatives."[45] The period between 1880 and 1920 saw the introduction of other regulatory measures over elections, such as voter registration laws and primaries. The Pendleton Act was also a product of this period (1883), its provisions including a ban on soliciting campaign contributions from federal employees.[46]

Yet despite these regulations and reforms, machine politics persisted – as we saw at the outset of this paper – into the early days of the welfare state and beyond. If at the end of the 19th century, America was on the same course toward eliminating machines as the one recently travelled by the British parties, their paths were soon to diverge. The crucial difference were new infusions of cash to finance American political campaigns in the early 20th century – money that came not from candidates or local sponsors but from large

43 . Reynolds and McCormick 1986, p. 851. The title of Reynolds and McCormick's 1986 essay is "Outlawing 'Treachery': Split Tickets and Ballot Laws in New York and New Jersey, 1880-1910."
44 . Reynolds and McCormick 1986, p. 848.
45 . Reynolds, p. 49.
46 . See Mutch 1988, Mayhew 1986, and Heard 1960.

business organizations. These were the "trusts": railroad and insurance companies, banks, and utilities. Their bankrolling of state party organizations reduced the urgency that politicians felt to cut out costly agents. The role of corporate money in politics was a source of scandal, at least since the muckrakers uncovered it in 1904-1908.[47] But because candidates were less in danger of being personally bankrupted by their agents, the latter were more irritant than threat to the candidates and party leaders.

Hence, rather than a step along the road to reform soon to be followed by a final blow to the machine, the introduction of the Australian ballot in the states represented a high point in anti-machine legislation. Despite active Progressive Era reforms on many fronts, no equivalent of the British Act of 1883 was to follow.

A contemporary academic and reformer, Earl Sikes, in 1928 posed the same question that we have asked here. Why did the U.S. fail to pass legislation that would have ended machine politics, as the Anti-Corrupt Practices Act of 1883 had 40 years earlier in Britain? His explanation was that the simple solution of limiting candidates' own expenditures was impotent in the American setting:

> To control by law a candidate for parliament who personally or by his agent manages his own campaign, and whose canvass is distinct by itself is a comparatively simple matter. To deal with a dozen or more candidates, all running for office at the same time on a party ticket and voted for within the same election district, none of whom may have anything to do with the actual conduct of the campaign, is a task of much greater complexity . . .[48]

6 Conclusion

Political parties constantly seek strategic advantages. But our model shows that these advantages can have adverse consequences. In 19th-century Britain and the U.S., the

47 . McCormick 1981.

48 . Sikes 1928, p. 125.

use of electoral agents exemplified the advantages but also the pitfalls of strategic innovation. Not infrequently, parties deployed agents even though they would have been collectively better off without them; this is the sense in which they were "prisoners of strategy."

But cutting out the machines ultimately became a dominant strategy for parties. We have shown theoretically that declining agent effectiveness relative to parties and persistent high costs relative to the value of winning office induce this transition. Historically, industrialization and the changes it wrought in the electorate made agents less effective in both Britain and the U.S. The flood of corporate money into American campaigns delayed anti-machine reforms in that country by making victory relatively inexpensive for candidates. Ironically, the long-term effect was to encourage escalation of campaign costs – borne, still, in large part by corporate donors – which continues to define American democracy today.

The model also points to coordination problems. Highly effective anti-agent legislation in late 19th-century Britain represented an institutional coordination device that helped ease the major parties from a world of political agents to a world of programmatic campaigns and direct distribution.

The prisoner's dilemma that may trap parties in agent-mediated distribution points to a more general dynamic. In other political and market settings, actors compete by investing in new technologies that induce them to spend more but can quickly be mimicked by the other side. In politics, the recent adoption of large-scale voter databases and highly focused turn-out-the-vote campaigns may, when adopted by both sides, become innovations that increase costs without changing the outcome of the competition.

Astute actors who anticipate these dilemmas might try to forestall them. For instance, they can try to keep new technologies out of the hands of their competitors. A case in point are the turn-out-the-vote techniques crafted by behavioral social scientists to aid the Democrats in the 2012 U.S. presidential campaign. The campaign required the social scientists to sign non-disclosure agreements.[49]

Along these lines, future theoretical work might extend the ideas developed here to settings in which parties are assumed to be asymmetric in some key respects, such as in the effectiveness of agents, the size of their core constituencies, or the responsiveness of the core to distributive benefits. A contemporary example of a strategy with asymmetric effectiveness would be voter mobilization efforts using communications media that are heavily used by one's core constituents but little used by one's opponents' core. Or such actors might focus on gaining an edge through strategies that rely on some fixed feature of their constituents. Without these longer-lasting advantages, what appears today as an optimal strategy may return in the next election as a prisoner's dilemma.

49 . New York Times, Nov 12, 2012.

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Mathematical Appendix

Here we present details of mathematical derivations of the results presented in the text. The broad ideas and intuitions are discussed there; therefore here we focus on the technical aspects.

A. Hypothetical targeted direct transfers with full information

Here we consider a hypothetical equilibrium to be used as a comparison standard, where each party's leaders can directly observe the type of each individual voter and target transfers. So the L party leaders choose (l_c, l_s) to maximize

$$U_L = \frac{f(l_c, l_s)}{f(l_c, l_s) + f(r_c, r_s)}\, V - l_c\, N_c - l_s\, N_s$$

taking the R party's choices (r_c, r_s) as given (and vice versa).

The differentiation is easier if we write

$$U_L = \left[1 - \frac{f(r_c, r_s)}{f(l_c, l_s) + f(r_c, r_s)} \right]\, V - l_c\, N_c - l_s\, N_s$$

The first-order conditions are

$$\begin{aligned} \frac{f(r_c, r_s)}{[f(l_c, l_s) + f(r_c, r_s)]^2}\, f_c(l_c, l_s)\, V - N_c &= 0 \\ \frac{f(r_c, r_s)}{[f(l_c, l_s) + f(r_c, r_s)]^2}\, f_s(l_c, l_s)\, V - N_s &= 0 \end{aligned}$$

where $f_c(c, s)$ and $f_s(c, s)$ denote the partial derivatives of f.

These imply

$$\frac{f_c(l_c, l_s)}{f_s(l_c, l_s)} = \frac{N_c}{N_s} \tag{A.1}$$

This has an obvious constrained maximization interpretation as a tangency condition: The marginal rate of substitution between l_c and l_s along a curve of equal $f(l_c, l_s)$ (and therefore a curve of equal political effectiveness for the L party) quals the marginal rate of transformation of the two types of transfers along an equal-expenditure line.

Using the Cobb-Douglas function (2), the tangency condition becomes

$$\frac{\theta\,\alpha\; l_c^{\theta\alpha-1}\; l_s^{\theta(1-\alpha)}}{\theta(1-\alpha)\; l_c^{\theta\alpha}\; l_s^{\theta(1-\alpha)-1}} = \frac{N_C}{N_S}$$

which simplifies to

$$\frac{l_c\; N_c}{\alpha} = \frac{l_s\; N_s}{1-\alpha} \tag{A.2}$$

so the total expenditure on each group is proportional to its importance as represented by the exponent in the Cobb-Douglas function.

Write the pair of first-order-condition equations as

$$\begin{aligned}
\frac{f(l_c,l_s)\; f(r_c,r_s)}{[f(l_c,l_s)+f(r_c,r_s)]^2}\;\frac{f_c(l_c,l_s)}{f(l_c,l_s)}\; V - N_c &= 0 \\
\frac{f(l_c,l_s)\; f(r_c,r_s)}{[f(l_c,l_s)+f(r_c,r_s)]^2}\;\frac{f_s(l_c,l_s)}{f(l_c,l_s)}\; V - N_s &= 0
\end{aligned}$$

Similar conditions hold for party R.

In view of the symmetry of the underlying structure of the parties' core support and ability to influence votes, we consider a symmetric Nash equilibrium where $l_c = r_c$, $l_s = r_s$, and $\pi_L = \pi_R = \frac{1}{2}$. Then we have

$$\frac{1}{4}\;\frac{f_c(l_c,l_s)}{f(l_c,l_s)}\; V = N_c, \qquad \frac{1}{4}\;\frac{f_s(l_c,l_s)}{f(l_c,l_s)}\; V = N_s$$

The Cobb-Douglas form (2) we are using makes $f(c,s)$ homogeneous of some degree θ_a.[50] Therefore, multiplying these equations by l_c, l_s respectively, adding, and using Euler's Theorem gives

$$\frac{1}{4}\;\theta_a\; V = l_c\, N_c + l_s\, N_s = I_L$$

50 . Recall that here we have the hypothetical situation where party leaders are fully informed and act as their own agents; therefore the agent value of θ as in (3) is appropriate.

Then, using the fact that in equilibrium the probability of each party's victory is $\frac{1}{2}$, the equilibrium value of each party's objective function becomes

$$U = \tfrac{1}{2}\, V - \tfrac{1}{4}\, \theta\, V = \tfrac{2-\theta_a}{4}\, V\,.$$

B. Subgame where neither party uses an agent

As explained in the text, in this case party L chooses l to maximize

$$U_L = \frac{f(l,l)}{f(l,l)+f(r,r)}\; V - l\; N$$

taking r as given. The first-order condition is

$$\frac{f(r,r)}{[f(l,l)+f(r,r)]^2}\; [\, f_c(l,l) + f_s(l,l)\,]\; V = N$$

or

$$\frac{f(l,l)\;\, f(r,r)}{[f(l,l)+f(r,r)]^2}\; \frac{f_c(l,l)+f_s(l,l)}{f(l,l)}\; V = N$$

In symmetric equilibrium this becomes

$$\frac{1}{4}\; \frac{f_c(l,l)+f_s(l,l)}{f(l,l)}\; V = N$$

Using the no-agent Cobb-Douglas form of f in (3), then multiplying both sides by l and using Euler's Theorem gives

$$\frac{1}{4}\theta_p\; V = l\,N = I_L$$

Similarly for party R. Then, with the victory probabilities of $\frac{1}{2}$ each in the symmetric equilibrium, the parties' objective function values are

$$U_n = \tfrac{1}{2}\, V - \tfrac{1}{4}\, \theta\, V = \tfrac{1}{2}\, \left[1 - \tfrac{1}{2}\, \theta \right]\, V\,, \tag{B.1}$$

where the subscript n on the utility indicates that neither party is using an agent.

C. Subgame where both parties use agents

Recall that we have a two-stage game: at the first stage the party leaders who choose the budgets and bonuses (I_L, B_L), (I_R, B_R), and at the second stage the agents choose the allocations (l_c, l_s), (r_c, r_s). We look for the symmetric subgame perfect equilibrium.

The L agent maximizes A_L defined in (5), subject to the budget constraint

$$l_c\, N_c + l_s\, N_s = I_L$$

We are assuming that the party keeps the agent's budget down to a level where he cannot steal directly, or gets no utility from such cash stealing. Then the first-order conditions are

$$\begin{aligned} \frac{f(r_c, r_s)}{[f(l_c, l_s) + f(r_c, r_s)]^2}\; f_c(l_c, l_s)\; B_L + \beta\; N_c &= \lambda\; N_c \\ \frac{f(r_c, r_s)}{[f(l_c, l_s) + f(r_c, r_s)]^2}\; f_s(l_c, l_s)\; B_L &= \lambda\; N_s \end{aligned}$$

where λ is the Lagrange multiplier.

Divide the first of these equations by N_c, the second by N_s, and subtract to eliminate λ:

$$\frac{f(r_c, r_s)}{[f(l_c, l_s) + f(r_c, r_s)]^2}\; \left[\frac{f_c(l_c, l_s)}{N_c} - \frac{f_s(l_c, l_s)}{N_s} \right]\; B_L + \beta = 0 \tag{C.1}$$

Therefore

$$\frac{f_c(l_c, l_s)}{N_c} - \frac{f_s(l_c, l_s)}{N_s} < 0, \quad \text{or} \quad \frac{f_c(l_c, l_s)}{f_s(l_c, l_s)} < \frac{N_c}{N_s} \tag{C.2}$$

Comparing this with the tangency condition (A.1) of optimality when the party directly chooses transfers with full information, we see that the agent (unsurprisingly) chooses l_c too high relative to l_s. In the text we discuss various sources of the bias in more detail.

To get further results, write (C.1) as

$$\frac{f(l_c, l_s)\ f(r_c, r_s)}{[f(l_c, l_s) + f(r_c, r_s)]^2} \left[\frac{l_c\ f_c(l_c, l_s)}{f(l_c, l_s)} \frac{1}{l_c\, N_c} - \frac{l_s\ f_s(l_c, l_s)}{f(l_c, l_s)} \frac{1}{l_s\, N_s} \right] B_L + \beta = 0$$

Using the Cobb-Douglas form (2), this becomes

$$\pi_L\ \pi_R\ \theta_a \left[\frac{\alpha}{l_c\ N_c} - \frac{1-\alpha}{l_s\ N_s} \right] B_L + \beta = 0$$

Define $z_l = l_c\ N_c / I_L$, that is, the fraction of the budget spent on core supporters. Then the conditions simplifies to

$$\frac{z_l - \alpha}{z_l\ (1 - z_l)} = \frac{\beta}{\theta_a} \frac{1}{\pi_L\ \pi_R} \frac{I_L}{B_L} \tag{C.3}$$

A similar equation governs the R agent's allocation.

Calculating (C.2) for the Cobb-Douglas case, we see that

$$\frac{\alpha\ l_s}{(1-\alpha)\ l_c} < \frac{N_c}{N_s}, \quad \text{or} \quad \frac{\alpha}{1-\alpha} < \frac{l_c\, N_c}{l_s\, N_s} = \frac{z_l}{1 - z_l}, \quad \text{so} \quad z_l > \alpha\,.$$

This is also consistent with (C.3).

Consider small changes around equilibrium. The logarithmic differential of the left hand side (omitting l subscripts because a similar equation is valid with r subscripts also) is

$$\begin{aligned}
\left[\frac{1}{z-\alpha} - \frac{1}{z} + \frac{1}{1-z} \right] dz &= \frac{z(1-z) - (z-\alpha)(1-z) + z(z-\alpha)}{z(1-z)(z-\alpha)}\, dz \\
&= \frac{z - z^2 - z + z^2 + \alpha - \alpha\, z + z^2 - \alpha\, z}{z(1-z)(z-\alpha)}\, dz \\
&= \frac{z^2 - 2\,\alpha\, z + \alpha}{z(1-z)(z-\alpha)}\, dz \\
&= \frac{(z-\alpha)^2 + \alpha(1-\alpha)}{z(1-z)(z-\alpha)}\, dz
\end{aligned}$$

$$= \frac{(z-\alpha)^2 + \alpha(1-\alpha)}{(z-\alpha)^2} \; \frac{z-\alpha}{z(1-z)} \; dz$$

Define

$$\Omega = \frac{(z-\alpha)^2}{(z-\alpha)^2 + \alpha(1-\alpha)} \tag{C.4}$$

Using this and (C.3), we have

$$\left[\frac{1}{z-\alpha} - \frac{1}{z} + \frac{1}{1-z} \right] \; dz = \frac{1}{\Omega} \; \frac{\beta}{\theta} \; \frac{1}{\pi_L \, \pi_R} \; \frac{I}{B} \; dz \tag{C.5}$$

If $z = \alpha$ (the party leaders' ideal), $\Omega = 0$, and as z increases to 1, Ω increases to $(1-\alpha)$. We can then regard the magnitude of Ω in this range as an indicator of the magnitude of the agency problem. Of course Ω is endogenous and determined by the party leaders' choices of I and B. This will emerge as a part of the solution below.

The logarithmic differential of $\pi_L \, \pi_R$ is

$$\begin{aligned} \frac{d(\pi_L \, \pi_R)}{\pi_L \, \pi_R} = \frac{d\pi_L}{\pi_L} + \frac{d\pi_R}{\pi_R} \quad &= \quad \frac{d\pi_L}{\pi_L} - \frac{d\pi_L}{1-\pi_L} \\ &= \quad \frac{1 - 2 \, \pi_L}{\pi_L \, (1-\pi_L)} \; d\pi_L \end{aligned} \tag{C.6}$$

which vanishes at a symmetric equilibrium where $\pi_L = \frac{1}{2}$.

This property simplifies the algebra of the first-stage calculation. In principle, the first-stage choices (I_L, B_L), (I_R, B_R) of the leaders of both parties will affect the second-stage choices (l_c, l_s), (r_c, r_s) of both agents. The party leaders' first stage choices will look ahead to this in the subgame perfect equilibrium. But as (C.3) shows, the R-party leaders' choice affects z_l only via π_R (and of course $\pi_L = 1 - \pi_R$). But (C.6) shows that this effect fortunately vanishes at the symmetric equilibrium.

Therefore the comparative statics of the agent's choice at the symmetric equilibrium (again omitting l subscripts) are given by the effects only of the budget and bonus set by that party's leaders:

$$\frac{1}{\Omega}\ \frac{\beta}{\theta_a}\ \frac{1}{\pi_L\,\pi_R}\ \frac{I_L}{B_L}\ dz_l = \frac{dI_L}{I_L} - \frac{dB_L}{B_L}\,, \tag{C.7}$$

and similarly for dz_r.

Now consider the first-stage symmetric equilibrium of the party leaders' choices. Start with

$$\begin{aligned}
\frac{\pi_L}{1-\pi_L} &= \frac{f(l_c,l_s)}{f(r_c,r_s)} = \frac{A_a\ l_c^{\theta_a\,\alpha}\ l_s^{\theta_a(1-\alpha)}}{A_a\ r_c^{\theta_a\,\alpha}\ r_s^{\theta_a(1-\alpha)}} \\
&= \frac{l_c^{\theta_a\,\alpha}\ l_s^{\theta_a(1-\alpha)}}{r_c^{\theta_a\,\alpha}\ r_s^{\theta_a(1-\alpha)}} \quad \text{observe how } A_a \text{ cancels} \\
&= \frac{z_l^{\theta_a\,\alpha}\ (1-z_l)^{\theta_a(1-\alpha)}\ {I_L}^{\theta_a}}{N_c^{\theta_a\,\alpha}\ N_s^{\theta_a(1-\alpha)}}\ \frac{1}{r_c^{\theta_a\,\alpha}\ r_s^{\theta_a(1-\alpha)}}
\end{aligned} \tag{C.8}$$

Party L's leaders choose their (I_L,B_L) taking the other party leaders' choice of (I_R,B_R) and therefore the R-party agent's choice of (r_c,r_s) as given, because those have zero first-order effect on π_L as seen above. Logarithmic differentiation gives

$$\frac{d\pi_L}{\pi_L} + \frac{d\pi_L}{1-\pi_L} = \theta_a\,\alpha\ \frac{dz_l}{z_l} - \theta_a\,(1-\alpha)\ \frac{dz_l}{1-z_l} + \theta_a\ \frac{dI_L}{I_L}$$

or

$$\begin{aligned}
\frac{d\pi_L}{\pi_L\,\pi_R} &= \theta_a\ \left[\frac{\alpha}{z_l} - \frac{1-\alpha}{1-z_l}\right]\ dz_l + \theta_a\ \frac{dI_L}{I_L} \\
&= -\ \theta_a\ \frac{z_l-\alpha}{z_l\,(1-z_l)}\ dz_l + \theta_a\ \frac{dI_L}{I_L} && \text{(C.9)} \\
&= -\ \theta_a\ \frac{\beta}{\theta_a}\ \frac{1}{\pi_L\ \pi_R}\ \frac{I_L}{B_L} + \theta_a\ \frac{dI_L}{I_L} && \text{using (C.3)} \\
&= -\ \theta_a\,\Omega_L\ \left[\frac{dI_L}{I_L} - \frac{dB_L}{B_L}\right] + \theta_a\ \frac{dI_L}{I_L} && \text{using (C.7) for party } L \\
&= \theta_a\ \left[(1-\Omega_L)\ \frac{dI_L}{I_L} + \Omega_L\ \frac{dB_L}{B_L}\right] && \text{(C.10)}
\end{aligned}$$

The line (C.9) in this calculation illustrates another aspect of the agency distortion:

an increase in z_l when it is already above α reduces π_l and therefore goes against the party leaders' interest. But there is also the beneficial direct effect of an increase in I_L. When everything is added together, the final result (C.10) shows that the net effect of a larger budget is beneficial for the victory probability.

Now we can calculate the effects of variations in (I_L, B_L) around the symmetric equilibrium on the objective function (4) of L-party leaders.

$$\begin{aligned} dU_L &= (V - B_L)\, d\pi_L - \pi_L\, dB_L - dI_L \\ &= (V - B_L)\, \pi_L\, \pi_R\, \theta_a \left[(1 - \Omega_L) \frac{dI_L}{I_L} + \Omega_L \frac{dB_L}{B_L} \right] - \pi_L\, dB_L - dI_L \\ &= [\, (V - B_L)\, \pi_L\, \pi_R\, \theta_a\, (1 - \Omega_L) - I_L\,] \frac{dI_L}{I_L} + [\, (V - B_L)\, \pi_L\, \pi_R\, \theta_a\, \Omega_L - \pi_L\, B_L\,] \frac{dB_L}{B_L} \end{aligned}$$

Therefore the first-order conditions for the optimum choice of (I_L, B_L) are

$$\begin{aligned} (V - B_L)\, \pi_L\, \pi_R\, \theta_a\, (1 - \Omega_L) &= I_L \\ (V - B_L)\, \pi_L\, \pi_R\, \theta_a\, \Omega_L &= \pi_L\, B_L \end{aligned}$$

or, using $\pi_L = \pi_R = \frac{1}{2}$, and dropping subscripts since the same condition holds for both parties,

$$(V - B)\, \theta_a\, (1 - \Omega) = 4\, I \tag{C.11}$$

$$(V - B)\, \theta_a\, \Omega = 2\, B \tag{C.12}$$

Divide these to write

$$\frac{\Omega}{1 - \Omega} = \frac{1}{2} \frac{B}{I} \tag{C.13}$$

or

$$\frac{(z - \alpha)^2}{\alpha\, (1 - \alpha)} = \frac{1}{2} \frac{B}{I} \tag{C.14}$$

We know from (C.3) and (C.7) that z is an increasing function of I/B, and $z > \alpha$; therefore the left hand side of (C.14) increases as I/B increases. The right hand side decreases as I/B increases, and spans the whole range from ∞ to 0. Therefore this equation yields a unique solution for I/B. Then z and Ω can be calculated.

Next, (C.12) gives

$$B = \frac{\theta_a \, \Omega}{2 + \theta_a \, \Omega} \, V \tag{C.15}$$

This completes the solution. Note that $B < V$, and the ratio B/V is higher when θ_a is higher (the agent has higher marginal productivity) and when Ω is higher (when the agency problem is more severe).

Finally, using (C.13), we get the size of each party's budget assigned to its agent transfers to the electorate:

$$I = \frac{1}{2} \, \frac{1 - \Omega}{\Omega} \, B = \tfrac{1}{2} \, \tfrac{\theta_a \, (1 - \Omega)}{2 + \theta_a \, \Omega} \, V \, .$$

Therefore each party's utility in equilibrium is

$$U_b = \tfrac{1}{2} \, (V - B) - I = \tfrac{1}{2} \, \left[1 - \frac{\theta_a}{2 + \theta_a \, \Omega} \right] \, V \, , \tag{C.16}$$

where the subscript b on the utility indicates that both parties are using agents.

Now we can compare utilities in the equilibria of the subgames where neither party is using an agent and where both are using agents. From (**??**) and (C.16), we have

$$U_b - U_n = \frac{\theta_a \, \theta_p \, \Omega - 2 \, (\theta_a - \theta_p)}{4 \, (2 + \theta_a \, \Omega)} \, V \, .$$

In the limiting case where $\theta_a = \theta_p$, this is positive. If the equilibrium of the full game is one where both parties use agents, it cannot be a prisoner's dilemma. But if θ_a is sufficiently greater than θ_p, such a dilemma is possible. In the text we discuss this in the context of numerical results and historical applications.

C. Both parties use agents

The notation for budgets, bonuses etc. is the same, and the parameter β now gets party subscripts because the agents' private benefits could differ between the parties.

Party L's agent maximizes

$$A_L = \pi_L \, B_L + \beta_l \; l_c \, N_l$$

subject to

$$l_c \, N_l + l_s \, N_s = I_L \, .$$

Using the total differential (D.12), the first-order conditions are

$$\begin{aligned} B_L \;\pi_L \, \pi_R \, \theta_l \, \alpha_l \; \frac{1}{l_c} + \beta_l \, N_l \quad &= \quad \lambda_l \; N_l \\ B_L \;\pi_L \, \pi_R \, \theta_l \, (1 - \alpha_l) \; \frac{1}{l_s} \quad &= \quad \lambda_l \; N_s \, . \end{aligned}$$

Eliminating the Lagrange multiplier λ_l between the two gives

$$B_L \;\pi_L \, \pi_R \, \theta_l \, \alpha_l \; \frac{1}{l_c \, N_l} + \beta_l = B_L \;\pi_L \, \pi_R \, \theta_l \, (1 - \alpha_l) \; \frac{1}{l_s \, N_s} \, ,$$

or

$$\begin{aligned} \beta_l \quad &= \quad \theta_l \; B_l \; \pi_L \, \pi_R \; \left[\; \frac{1 - \alpha_l}{l_s \, N_s} - \frac{\alpha_l}{l_c \, N_l} \; \right] \\ &= \quad \theta_l \; B_l \; \pi_L \, \pi_R \; \frac{(1 - \alpha_l) \, l_c \, N_l - \alpha_l \, l_s \, N_s}{l_c \, N_l \; l_s \, N_s} \\ &= \quad \theta_l \; B_l \; \pi_L \, \pi_R \; \frac{l_c \, N_l - \alpha_l \, (l_c \, N_l + l_s \, N_s)}{l_c \, N_l \; l_s \, N_s} \\ &= \quad \theta_l \; B_l \; \pi_L \, \pi_R \; \frac{l_c \, N_l - \alpha_l \, I_L}{l_c \, N_l \; l_s \, N_s} \\ &= \quad \pi_L \, \pi_R \; \theta_l \; \frac{B_l}{I_L} \; \frac{z_l - \alpha_l}{z_l \, (1 - z_l)} \qquad \text{(C.17)} \\ &= \quad \pi_L \, \pi_R \; \theta_l \; \frac{B_l}{I_L} \; \phi_l(z) \, , \qquad \text{(C.18)} \end{aligned}$$

where $z_l = l_c\, N_l / I_L$ is the fraction of the budget the agent spends on core supporters, and the function ϕ_l is defined as

$$\phi_l(z) = \frac{z - \alpha_l}{z(1-z)} \,.$$

Then

$$\begin{aligned}
\phi_l'(z) &= \frac{z(1-z) - (z-\alpha_l)\,(1-2\,z)}{z^2\,(1-z)^2} = \frac{z - z^2 - z + 2\,z^2 + \alpha_l - 2\,\alpha_l\,z}{z^2\,(1-z)^2} \\
&= \frac{z^2 - 2\,z\,\alpha_l + \alpha_l}{z^2\,(1-z)^2} = \frac{z^2 - 2\,z\,\alpha_l + \alpha_l^2 + \alpha_l - \alpha_l^2}{z^2\,(1-z)^2} \\
&= \frac{(z-\alpha_l)^2 + \alpha_l\,(1-\alpha_l)}{z^2\,(1-z)^2} > 0\,.
\end{aligned}$$

Also $\phi_l(z) = 0$ when $z = \alpha_l$ and $\phi_l(z) \to \infty$ as $z \to 1$; therefore (C.18) has a unique solution for z_l given the other magnitudes. (This is not yet a complete solution because the probabilities are endogenous.)

A similar calculation holds for Party R's agent. Express the probabilities in terms of the agents' choices:

$$\begin{aligned}
\pi_L &= A_l\;(l_c)^{\theta_l\,\alpha_l}\;(l_s)^{\theta_l\,(1-\alpha_l)}\,/\,K \\
&= \frac{A_l\;(z_l)^{\theta_l\,\alpha_l}\;(1-z_l)^{\theta_l\,(1-\alpha_l)}}{(N_l)^{\theta_l\,\alpha_l}\;(N_s)^{\theta_l\,(1-\alpha_l)}}\;\frac{(I_l)^{\theta_l}}{K}
\end{aligned} \tag{C.19}$$

and similarly for π_R, where

$$K = \frac{A_l\;(z_l)^{\theta_l\,\alpha_l}\;(1-z_l)^{\theta_l\,(1-\alpha_l)}\;(I_l)^{\theta_l}}{(N_l)^{\theta_l\,\alpha_l}\;(N_s)^{\theta_l\,(1-\alpha_l)}} + \frac{A_r\;(z_r)^{\theta_r\,\alpha_r}\;(1-z_r)^{\theta_r\,(1-\alpha_r)}\;(I_r)^{\theta_r}}{(N_r)^{\theta_r\,\alpha_r}\;(N_s)^{\theta_r\,(1-\alpha_r)}}\,. \tag{C.20}$$

This enables us to express the L-agent's condition (C.18) and the similar condition for the R-agent in terms of just the two choice variables z_l and similarly z_r. The solution of this pair of equations then gives the agents' responses to (I_L, B_L), (I_R, B_R); however, here we don't have a guarantee of existence or uniqueness. Assuming that is not a problem, the stage is set for finding the first-stage Nash equilibrium of the parties' choices.

Without obtaining the complete solution, we can make some comparisons of the choices of the two parties' agents. First suppose that the two have the same parameter reflecting the relative importance of per capital transfers to core and swing voters in the probability-generating functions: $\alpha = \alpha_l = \alpha_r$. Then they have the same functional forms $\phi(z) = \phi_l(z) = \phi_r(z) = (z - \alpha)/[z(1 - z)]$. Using (C.18) and the similar equation for the R-agent, we see that

$$\frac{\phi(z_l)}{\phi(z_r)} = \frac{\beta_l}{\beta_r} \, \frac{\theta_r}{\theta_l} \, \frac{I_L}{I_R} \, \frac{B_R}{B_L} \, .$$

Since the function ϕ is increasing, this says that the agency problem is worse for Party L (higher z_l) if (i) its agent has a higher private benefit parameter (higher β_l), (ii) it has a lower parameter reflecting returns to scale of overall effort (lower θ_l), (iii) it offers the agent a larger budget (larger I_L), (iv) it offers its agent a smaller bonus (smaller B_L). These are intuitively obvious except for the effect of the scale parameter θ.

Next suppose the α parameters differ between the parties, but the parameters β and θ are equal for the two, and their budges and bonuses are held equal for purposes of the comparison. Then the agents' choices will satisfy

$$\phi_l(z_l) = \phi_r(z_r) \, . \tag{C.21}$$

Suppose $\alpha_l > \alpha_r$, that is, transfers to core supporters are relatively more effective in the probability-generating function for Party L than for Party R. Then, for any given $z \in (0, 1)$,

$$z - \alpha_l < z - \alpha_r, \quad \text{or} \quad \frac{z - \alpha_l}{z(1 - z)} < \frac{z - \alpha_r}{z(1 - z)}, \quad \text{or} \quad \phi_l(z) < \phi_r(z) \, .$$

Since the ϕ functions are increasing, to attain the equality (C.21) we must have $z_l > z_r$. However, we cannot in general say whether the agency problem is worse, that is whether the departure from the party leaders' desired level is greater for Party L or for Party R:

$z_l - \alpha_l >$ or $< z_r - \alpha_r$.

Differences in the multiplicative "productivity" parameters (that is, $A_l \neq A_r$) do not affect the comparison of z_l and z_r. They do affect the actual solution because they affect π_L and π_R as we see from (C.19) and (C.20), but the factor $\pi_L \, \pi_R$ cancels when we take the ratio of the L and R conditions. This is similar to what we were finding earlier, where differences in θs with and without agents mattered, but differences in As did not.

D. Subgame where only party L has an agent

Here we have a two-stage game. At the first stage, party L chooses the budget I_L and bonus B_L for its agent while party R chooses its uniform per capita transfer amount r. In the second stage, L's agent chooses the targeted transfers l_c and l_s. As usual this is solved by backward induction, starting with the second-stage decision problem given (I_L, B_L) and r.

The agent wants to maximize A_L subject to the given budget I_L. This is the same problem as in Appendix C, and leads to the same condition (C.3), which I rewrite as

$$\pi_L \; (1 - \pi_L) \; \frac{z_l - \alpha}{z_l \, (1 - z_l)} = \frac{\beta}{\theta_a} \; \frac{I_L}{B_L} \, , \tag{D.1}$$

where $z_l = l_c \, N_c / I_L$ is the fraction of the budget the agent allocates to the core supporters.

Also, the same calculation that led to (C.8), but now remembering $r_c = r_s = r$, yields

$$\begin{aligned} \frac{\pi_L}{1 - \pi_L} &= \frac{f(l_c, l_s)}{f(r_c, r_s)} = \frac{A_a \, l_c^{\theta_a \, \alpha} \; l_s^{\theta_a (1-\alpha)}}{A_p \, r^{\theta_p}} \\ &= \frac{A_a}{A_p} \; \frac{z_l^{\theta_a \, \alpha} \; (1 - z_l)^{\theta_a (1-\alpha)} \; I_L^{\theta_a}}{N_c^{\theta_a \, \alpha} \; N_s^{\theta_a (1-\alpha)}} \; \frac{1}{r^{\theta_p}} \end{aligned} \tag{D.2}$$

These two equations define z_l and π_L as functions of (I_L, B_L) and r.

Consider how z_l and π_L change as (I_L, B_L) and r change. Logarithmic differentiation of (D.1) yields

$$\frac{d\pi_L}{\pi_L} - \frac{d\pi_L}{1-\pi_L} + \left[\frac{1}{z_l - \alpha} - \frac{1}{z_l} + \frac{1}{1-z_l} \right] \, dz_l = \frac{dI_L}{I_L} - \frac{dB_L}{B_L} \, ,$$

or, using (C.5), which remains valid because the L agent's optimality conditions thus far are the same,

$$\frac{1-2\,\pi_L}{\pi_L\,(1-\pi_L)} \, d\pi_l + \frac{1}{\Omega} \, \frac{\beta}{\theta_a} \, \frac{1}{\pi_L\,(1-\pi_L)} \, \frac{I_L}{B_L} \, dz_l = \frac{dI_L}{I_L} - \frac{dB_L}{B_L} \, .$$

This simplifies to

$$(1-2\,\pi_L) \, d\pi_l + \frac{1}{\Omega} \, \frac{\beta}{\theta_a} \, \frac{I_L}{B_L} \, dz_l = \pi_L \, \pi_R \, \left[\frac{dI_L}{I_L} - \frac{dB_L}{B_L} \right] \, . \tag{D.3}$$

Next, logarithmic differentiation of (D.2) yields

$$\frac{d\pi_L}{\pi_L} + \frac{d\pi_L}{1-\pi_L} = \theta_a \, \frac{dI_L}{I_L} + \theta_a \, \left[\alpha \, \frac{dz_l}{z_l} - (1-\alpha) \, \frac{dz_l}{1-z_l} \right] - \theta_p \, \frac{dr}{r} \, ,$$

or

$$\frac{1}{\pi_L\,(1-\pi_L)} \, d\pi_L = \theta_a \, \frac{dI_L}{I_L} - \theta_a \, \frac{z_l - \alpha}{z_l\,(1-z_l)} \, dz_l - \theta_p \, \frac{dr}{r} \, ,$$

or, using (D.1),

$$\frac{1}{\pi_L\,(1-\pi_L)} \, d\pi_L = \theta_a \, \frac{dI_L}{I_L} - \frac{\beta}{\pi_L\,(1-\pi_L)} \, \frac{I_L}{B_L} \, dz_l - \theta_p \, \frac{dr}{r} \, .$$

This simplifies to

$$d\pi_L + \beta \, \frac{I_L}{B_L} \, dz_l = \pi_L \, \pi_R \, \left[\theta_a \, \frac{dI_L}{I_L} - \theta_p \frac{dr}{r} \right] \tag{D.4}$$

The two comparative statics equations (D.3) and (D.4) can be solved for dz_l and

$d\pi_L$ to get

$$dz_l = \frac{1}{\Delta} \frac{\pi_L \pi_R}{\beta} \frac{B_L}{I_L} \left\{ [1 + \theta_a (2\pi_L - 1)] \frac{dI_L}{I_L} - \frac{dB_L}{B_L} - \theta_p (2\pi_L - 1) \frac{dr}{r} \right\} \quad \text{(D.5)}$$

$$d\pi_L = \frac{1}{\Delta} \pi_L \pi_R \left\{ \frac{1 - \Omega}{\Omega} \frac{dI_L}{I_L} + \frac{dB_L}{B_L} - \frac{\theta_p / \theta_a}{\Omega} \frac{dr}{r} \right\} \quad \text{(D.6)}$$

where (D.4):

$$\Delta = \frac{1}{\theta \, \Omega} + 2 \, \pi_L - 1 \, . \quad \text{(D.7)}$$

If $\pi_L > \frac{1}{2}$, which in turn ensures $\Delta > 0$, all comparative static effects have the intuitive signs. (1) An increase in I_L increases z_L, the fraction the agent spends on core supporters: the more relaxed budget enables him to indulge more in his preference. (2) An increase in B_L decreases z_l: the incentive works to align the agent's choice more closely with the party leaders' preferred level $z_l = \alpha$. (3) An increase in r decreases z_L: greater pressure of competition from the other party's transfers forces the agent to reduce his spending to indulge his own preference for a larger core club. (4) An increase in I_L increases π_L: worsening of the agent's moral hazard (higher z_l) is not so severe as the reduce the party's probability of victory. (5) An increase in B_L increases π_L and an increase in r reduces π_L: these are obvious.

The property $\pi_L > \frac{1}{2}$ is intuitively appealing: an important reason to employ the agent is to use his ability to make transfers with better targeting and higher productivity, which should increase the probability of winning. But the general theory does not allow us to prove this definitively. We will examine the issue using numerical solutions.

The comparative static results for stage 2 are needed for analyzing the stage 1 Nash game between the party leaders. The L leaders choose (I_L, B_L) for given r to maximize

$$U_L = \pi_L \, (V - B_L) - I_L \, ,$$

and the R leaders choose r for given (I_L, B_L) to maximize

$$U_R = (1 - \pi_L)\, V - r\, N\,.$$

We can use the comparative statics results of (D.6) to find the parties' calculation of effects of changes in their strategies (I_L, B_L) and r respectively, taking into account the L agent's response at the second stage. We have total differentials of the objective functions:

$$\begin{aligned} dU_L &= (V - B_L)\, d\pi_L - \pi_L\, dB_L - dI_L \\ &= (V - B_L)\, \frac{\pi_L\, \pi_R}{\Delta} \left\{ \frac{1-\Omega}{\Omega}\, \frac{dI_L}{I_L} + \frac{dB_L}{B_L} \right\} - \pi_L\, dB_L - dI_L \end{aligned}$$

and

$$\begin{aligned} dU_R &= -V\; d\pi_L - N\, dr \\ &= V\, \frac{\pi_L\, \pi_R}{\Delta\, \Omega}\, \frac{\theta_p}{\theta_a}\, \frac{dr}{r} - N\, dr \end{aligned}$$

Note the absence of dr in the expression for dU_L and of (dI_L, dB_L) in the expression for dU_R, reflecting the Nash noncooperative assumption where each party takes the other's strategy as given.

Now party L's first-order conditions can be found by setting the coefficients of dI_L and dB_L separately equal to zero in the expression for dU_L:

$$(V - B_L)\, \frac{\pi_L\, \pi_R}{\Delta}\, \frac{1-\Omega}{\Omega}\, \frac{1}{I_L} - 1 = 0\,, \tag{D.8}$$

$$(V - B_L)\, \frac{\pi_L\, \pi_R}{\Delta}\, \frac{1}{B_L} - \pi_L = 0\,. \tag{D.9}$$

The R party's first-order condition is found by setting the coefficient of dr equal to zero in the expression for dU_R:

$$V\, \frac{\pi_L\, \pi_R}{\Delta\, \Omega}\, \frac{\theta_p}{\theta_a}\, \frac{1}{r} - N = 0\,. \tag{D.10}$$

The complete solution for the two stages together – for all five endogenous variables I_L, B_L, r, z_l and π_L – is then implicitly defined by the five equations (D.1), (D.2), (D.8), (D.9) and (D.34). No general inferences can be drawn from the algebra, so we resort to numerical solution.

7 Asymmetric Cases

Let the values of victory for the two parties be V_L, V_R. Let the numbers of core supporters of the two parties be N_l, N_r respectively, and let the number of swing voters be N_s; the total population is $N = N_l + N_s + N_r$. (In the symmetric case earlier we had $N_l = N_r$ and the common value was labelled N_c.)

Let the odds ratio be given by

$$\frac{\pi_L}{\pi_R} = \frac{A_l\ (l_c)^{\theta_l\,\alpha_l}\ (l_s)^{\theta_l(1-\alpha_l)}}{A_r\ (r_c)^{\theta_r\,\alpha_r}\ (r_s)^{\theta_r(1-\alpha_r)}}\,. \tag{D.11}$$

The notation for the variables is as before; the parameters A, θ, α can now differ for the two parties so they have party label subscripts.

Totally log-differentiating (D.11) gives

$$\frac{d\pi_L}{\pi_L} - \frac{d\pi_R}{\pi_R} = \theta_l\,\alpha_l\ \frac{dl_c}{l_c} + \theta_l(1-\alpha_l)\ \frac{dl_s}{l_s} - \theta_r\,\alpha_r\ \frac{dr_c}{r_c} - \theta_r(1-\alpha_r)\ \frac{dr_s}{r_s}\,.$$

Using $\pi_l + \pi_R = 1$, we have $d\pi_R = -d\pi_L$, so the left hand side of the above equation becomes

$$\frac{d\pi_L}{\pi_L} + \frac{d\pi_L}{\pi_R} = \frac{\pi_L + \pi_R}{\pi_L\ \pi_R}\ d\pi_L = \frac{d\pi_L}{\pi_L\ \pi_R}\,.$$

Then

$$d\pi_L = \pi_L\,\pi_R\ \left[\ \theta_l\,\alpha_l\ \frac{dl_c}{l_c} + \theta_l(1-\alpha_l)\ \frac{dl_s}{l_s} - \theta_r\,\alpha_r\ \frac{dr_c}{r_c} - \theta_r(1-\alpha_r)\ \frac{dr_s}{r_s}\ \right]\,. \tag{D.12}$$

Parties directly choose fully targeted transfers

This is the hypothetical comparison standard. Party L chooses l_c, l_s to maximize

$$U_L = \pi_L \; V_L - l_c \; N_l - l_s \; N_s \tag{D.13}$$

taking Party R's choices r_c, r_s as given. Using (D.12), we can write the total differential of U_L:

$$\begin{aligned} dU_L &= V_L \; d\pi_L - N_l \; dl_c - N_s \; dl_s \\ &= V_L \; \pi_L \, \pi_R \left[\theta_l \, \alpha_l \, \frac{dl_c}{l_c} + \theta_l (1 - \alpha_l) \, \frac{dl_s}{l_s} \right] - N_l \; dl_c - N_s \; dl_s \, . \end{aligned}$$

Therefore the first-order conditions of Party L's maximization are

$$V_L \; \pi_L \, \pi_R \; \theta_l \, \alpha_l \, / \, l_c \; = \; N_l \tag{D.14}$$

$$V_L \; \pi_L \, \pi_R \; \theta_l \, (1 - \alpha_l) \, / \, l_s \; = \; N_s \tag{D.15}$$

Similarly Party R's conditions are

$$V_R \; \pi_L \, \pi_R \; \theta_r \, \alpha_r \, / \, r_c \; = \; N_r \tag{D.16}$$

$$V_R \; \pi_L \, \pi_R \; \theta_r \, (1 - \alpha_r) \, / \, r_s \; = \; N_s \tag{D.17}$$

Solving for π_L, π_R from (D.11) and $\pi_L + \pi_R = 1$, we can express them as functions of l_c, l_s, r_c, r_s. Then (D.14), (D.15), (D.16) and (D.17) constitute a system of four equations that yields the Nash equilibrium values of l_c, l_s, r_c, r_s .

A full analytical solution is infeasible. But we can get some simple results on the cheap. Compare the giveaways of the two parties (i) each to its core supporters (dividing (D.14) by (D.16)):

$$\frac{l_c}{r_c} = \frac{N_r}{N_l} \; \frac{V_L}{V_R} \; \frac{\theta_l}{\theta_r} \; \frac{\alpha_l}{\alpha_r} \, , \tag{D.18}$$

and (ii) to swing voters (dividing (D.15) by (D.17)):

$$\frac{l_s}{r_s} = \frac{V_L}{V_R} \frac{\theta_l}{\theta_r} \frac{1-\alpha_l}{1-\alpha_r}. \tag{D.19}$$

Most of these results are quite intuitive. In (D.18) the per capital transfers depend inversely on the numbers of the two parties' core supporters because of the cost of giving to a larger number of core supporters. The exact inverse proportionality is a result of the Cobb-Douglas specification; with a more general form, the total expenditures would depend on the numbers. Other things equal, we expect the ratio $(l_c\, N_l)/(r_c\, N_r)$ to be an increasing function of the ratio N_l/N_r if the elasticity of substitution in the function f is less than one. In (D.19) the number of swing voters, which is common to both parties even with other asymmetries, nicely cancels out in the ratio.

Here is a possible way to proceed with the solution. Using (D.14), (D.15) and (D.16), (D.17), define

$$X_l = \frac{l_c,\, N_l}{\alpha_l} = \frac{l_s\, N_s}{1-\alpha_l}), \qquad X_r = \frac{r_c,\, N_r}{\alpha_r} = \frac{r_s\, N_s}{1-\alpha_r}). \tag{D.20}$$

Then we can substitute for l_c and l_s in terms of x to write

$$\begin{aligned} \pi_L &= A_l \left(\frac{\alpha_l\, X_l}{N_l}\right)^{\theta_l\, \alpha_l} \left(\frac{(1-\alpha_l)\, X_l}{N_s}\right)^{\theta_l(1-\alpha_l)} / \, K \\ &= A_l \left(\frac{\alpha_l}{N_l}\right)^{\theta_l\, \alpha_l} \left(\frac{1-\alpha_l}{N_s}\right)^{\theta_l(1-\alpha_l)} (X_l)^{\theta_l} / \, K \end{aligned} \tag{D.21}$$

where

$$K = A_l \left(\frac{\alpha_l}{N_l}\right)^{\theta_l\, \alpha_l} \left(\frac{1-\alpha_l}{N_s}\right)^{\theta_l(1-\alpha_l)} (X_l)^{\theta_l} + A_r \left(\frac{\alpha_r}{N_r}\right)^{\theta_r\, \alpha_r} \left(\frac{1-\alpha_r}{N_s}\right)^{\theta_r(1-\alpha_r)} (X_r)^{\theta_r} \tag{D.22}$$

To avoid clutter of notation, introduce the abbreviation

$$B_l = A_l \, \left(\frac{\alpha_l}{N_l}\right)^{\theta_l \, \alpha_l} \, \left(\frac{1-\alpha_l}{N_s}\right)^{\theta_l (1-\alpha_l)} \tag{D.23}$$

and similarly for B_r. Then

$$\pi_L \, \pi_R = B_l \, B_r \, (X_l)^{\theta_l} \, (X_r)^{\theta_r} \, / \, K^2 \,,$$

and (D.14) and (D.16) can be written as

$$X_l \;=\; \theta_l \, V_l \, B_l \, B_r \, (X_l)^{\theta_l} \, (X_r)^{\theta_r} \, / \, K^2 \,, \tag{D.24}$$

$$X_r \;=\; \theta_r \, V_r \, B_l \, B_r \, (X_l)^{\theta_l} \, (X_r)^{\theta_r} \, / \, K^2 \,, \tag{D.25}$$

Taking logs and collecting terms:

$$(1-\theta_l) \, \ln(X_l) - \theta_r \, \ln(X_r) \;=\; \ln(\theta_l \, V_l) + \ln(B_l \, B_r) - 2 \, \ln(K) \tag{D.26}$$

$$-\, \theta_l \, \ln(X_l) + (1-\theta_r) \, \ln(X_r) \;=\; \ln(\theta_r \, V_r) + \ln(B_l \, B_r) - 2 \, \ln(K) \tag{D.27}$$

Write the equations as

$$\begin{bmatrix} 1-\theta_l & -\,\theta_r \\ -\,\theta_l & 1-\theta_r \end{bmatrix} \begin{bmatrix} \ln(X_l) \\ \ln(X_r) \end{bmatrix} = \begin{bmatrix} \ln(\theta_l \, V_l) + \ln(B_l \, B_r) - 2 \, \ln(K) \\ \ln(\theta_r \, V_r) + \ln(B_l \, B_r) - 2 \, \ln(K) \end{bmatrix}$$

The solution is

$$\begin{aligned} \begin{bmatrix} \ln(l) \\ \ln(r) \end{bmatrix} &= \begin{bmatrix} 1-\theta_l & -\,\theta_r \\ -\,\theta_l & 1-\theta_r \end{bmatrix}^{-1} \begin{bmatrix} \ln(\theta_l \, V_l) + \ln(B_l \, B_r) - 2 \, \ln(K) \\ \ln(\theta_r \, V_r) + \ln(B_l \, B_r) - 2 \, \ln(K) \end{bmatrix} \\ &= \frac{1}{(1-\theta_l)\,(1-\theta_r) - \theta_l \, \theta_r} \begin{bmatrix} 1-\theta_r & \theta_r \\ \theta_l & 1-\theta_l \end{bmatrix} \begin{bmatrix} \ln(\theta_l \, V_l) + \ln(B_l \, B_r) - 2 \, \ln(K) \\ \ln(\theta_r \, V_r) + \ln(B_l \, B_r) - 2 \, \ln(K) \end{bmatrix} \end{aligned}$$

$$= \frac{1}{1-\theta_l-\theta_r} \left[\begin{array}{c} (1-\theta_r)\ \ln(\theta_l\, V_l) + \theta_r\ \ln(\theta_r\, V_r) + \ln(B_l\, B_r) - 2\ \ln(K) \\ \theta_l\ \ln(\theta_l\, V_l) + (1-\theta_l)\ \ln(\theta_r\, V_r) + \ln(B_l\, B_r) - 2\ \ln(K) \end{array} \right] \tag{D.28}$$

There is no guarantee that $(1-\theta_l-\theta_r)$ is positive. But except in the unlikely case where $\theta_l+\theta_r$ is precisely equal to 1, we have a unique solution.

Then

$$X_l = \left[\ (\theta_l\, V_l)^{1-\theta_r}\ (\theta_r\, V_r)^{\theta_r}\ B_l\, B_r\, K^{-2}\ \right]^{1/(1-\theta_l-\theta_r)} \tag{D.29}$$

$$X_r = \left[\ (\theta_l\, V_l)^{\theta_l}\ (\theta_r\, V_r)^{1-\theta_l}\ \ B_l\, B_r\, K^{-2}\ \right]^{1/(1-\theta_l-\theta_r)} \tag{D.30}$$

These can then be substituted into (D.22) to get an equation in one unknown, K.

Parties give direct non-targeted uniform transfers

Now Party L chooses l and Party R chooses r, leading to the odds ratio

$$\frac{\pi_l}{\pi_R} = \frac{A_l\ l^{\theta_l}}{A_r\ r^{\theta_r}} \tag{D.31}$$

Then, proceeding as before, we have

$$d\pi_L = \pi_L\, \pi_R\ \left[\theta_L\ \frac{dl}{l} - \theta_R\ \frac{dr}{r} \right] \tag{D.32}$$

Party L chooses l, for given r, to maximize

$$U_L = \pi_L\ V_L - l\ N\,.$$

(Remember that non-targeted transfers must be given to the whole population, including core supporters of the other party.) Similarly for Party R. This yields the first-order conditions

$$l = V_L \, \pi_L \, \pi_R \, \theta_L \, / \, N \tag{D.33}$$

$$r = V_R \, \pi_L \, \pi_R \, \theta_R \, / \, N \tag{D.34}$$

The ratio works out very nicely

$$\frac{l}{r} = \frac{V_L}{V_R} \, \frac{\theta_L}{\theta_R} \, .$$

To solve the equations, use a method similar to that used above.

Numerical Appendix

The two tables below provide more information about some of the equilibria that figure (1) depicts. The tables contain all of the endogenous outcomes of the model, the values of θ_a and V, and the four possible payoffs for each party. Table (1) contains the endogenous outcomes of the model for both the case when only one party employs an agent and when both parties employ an agent. Table (2) contains the payoffs for the parties for all of the subgames in the model.

Table 1: Equilibria Outcomes for $\beta = 0.5$

V	θ_a-θ_p	B_L1	I_L1	l_c1	$r1$	π_L1	B_L2	I_L2	l_c2
100	0.8	6.526	2.731	3.579	0.974	0.837	17.718	9.655	9.35
100	0.6	7.333	2.926	3.809	1.338	0.784	15.501	7.037	7.298
100	0.4	7.996	2.829	3.699	1.838	0.703	12.721	4.55	5.172
100	0.2	7.621	2.005	2.749	2.339	0.587	9.054	2.294	2.99
100	0.	3.988	0.517	0.87	2.503	0.473	3.796	0.507	0.862
80	0.8	5.569	2.405	3.094	0.836	0.825	14.174	7.724	7.48
80	0.6	6.194	2.526	3.237	1.131	0.771	12.401	5.629	5.838
80	0.4	6.655	2.377	3.072	1.52	0.69	10.177	3.64	4.137
80	0.2	6.221	1.634	2.227	1.89	0.578	7.243	1.835	2.392
80	0.	3.19	0.414	0.696	2.003	0.473	3.037	0.406	0.69
60	0.8	4.538	2.039	2.561	0.686	0.808	10.631	5.793	5.61
60	0.6	4.979	2.086	2.619	0.909	0.753	9.301	4.222	4.379
60	0.4	5.25	1.895	2.413	1.189	0.672	7.632	2.73	3.103
60	0.2	4.787	1.253	1.695	1.435	0.566	5.433	1.376	1.794
60	0.	2.393	0.31	0.522	1.502	0.473	2.278	0.304	0.517
40	0.8	3.399	1.61	1.954	0.518	0.782	7.087	3.862	3.74
40	0.6	3.658	1.585	1.934	0.666	0.725	6.2	2.815	2.919
40	0.4	3.754	1.371	1.709	0.837	0.646	5.088	1.82	2.069
40	0.2	3.307	0.86	1.152	0.972	0.548	3.622	0.918	1.196
40	0.	1.595	0.207	0.348	1.001	0.473	1.519	0.203	0.345
20	0.8	2.07	1.061	1.214	0.318	0.729	3.544	1.931	1.87
20	0.6	2.152	0.976	1.134	0.387	0.67	3.1	1.407	1.46
20	0.4	2.108	0.776	0.935	0.454	0.596	2.544	0.91	1.034
20	0.2	1.753	0.449	0.591	0.496	0.518	1.811	0.459	0.598
20	0.	0.798	0.103	0.174	0.501	0.473	0.759	0.101	0.172
4	0.8	0.646	0.359	0.358	0.095	0.551	0.709	0.386	0.374
4	0.6	0.615	0.28	0.291	0.1	0.504	0.62	0.281	0.292
4	0.4	0.54	0.188	0.21	0.102	0.464	0.509	0.182	0.207
4	0.2	0.398	0.096	0.122	0.101	0.444	0.362	0.092	0.12
4	0.	0.16	0.021	0.035	0.1	0.473	0.152	0.02	0.034

The number after the outcome variables indicates the number of agents.

Table 2: Party Utilities for $\beta = 0.5$

V	θ_a-θ_p	1 No Agent	1 Agent	No Agent	2 Agent
100	0.8	15.365	75.47	47.5	31.486
100	0.6	20.22	69.764	47.5	35.213
100	0.4	27.822	61.888	47.5	39.09
100	0.2	38.92	52.259	47.5	43.179
100	0	50.198	44.895	47.5	47.595
80	0.8	13.186	58.98	38.	25.189
80	0.6	17.174	54.393	38.	28.17
80	0.4	23.272	48.238	38.	31.272
80	0.2	31.865	41.015	38.	34.543
80	0	40.159	35.916	38.	38.076
60	0.8	10.828	42.78	28.5	18.892
60	0.6	13.911	39.344	28.5	21.128
60	0.4	18.478	34.909	28.5	23.454
60	0.2	24.614	29.989	28.5	25.907
60	0	30.119	26.937	28.5	28.557
40	0.8	8.204	27.01	19.	12.595
40	0.6	10.334	24.762	19.	14.085
40	0.4	13.339	22.03	19.	15.636
40	0.2	17.094	19.261	19.	17.272
40	0	20.079	17.958	19.	19.038
20	0.8	5.107	12.005	9.5	6.297
20	0.6	6.21	10.985	9.5	7.043
20	0.4	7.621	9.892	9.5	7.818
20	0.2	9.149	8.998	9.5	8.636
20	0	10.04	8.979	9.5	9.519
4	0.8	1.702	1.489	1.9	1.259
4	0.6	1.883	1.427	1.9	1.409
4	0.4	2.041	1.419	1.9	1.564
4	0.2	2.121	1.505	1.9	1.727
4	0	2.008	1.796	1.9	1.904

"1" indicates the payoff is when 1 party uses an agent.
"No Agent" indicates that the payoff is for the party that is not using an agent.