

Talent, Labor Quality and Economic Development

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Abstract

We develop a quantitative theory of labor quality based on the division of the labor force between unskilled and skilled workers, and investments in the quality of skilled labor. We use the theory to quantify the importance labor quality and Total Factor Productivity for cross-country income differences. A central ingredient in our analysis is the observed achievement levels (talent) constructed from PISA scores in a sample of 59 countries. Our findings imply that the cross-country differences in the quality of labor are almost twice as large as the conventional measures using Mincer returns. Thus, the implied disparities in TFP levels are smaller. The elasticity of output per worker with respect to TFP is about 2.3. We find no support for the hypothesis of skill-biased technology differences across countries.

KEYWORDS: PISA, Labor Quality, TFP, Economic Development.

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1 Introduction

It is well known by now that the observed cross-country differences in output per worker are large. For example, the richest countries in the world economy are about 30-40 times more productive than their poorest counterparts; see Hall and Jones (1999), McGrattan and Schmitz (2000), Prescott and Parente (2000), among others.

One potential reason why countries differ in their output per worker might be differences in labor input per worker, or *labor quality*. As direct measures of achievement of individuals entering a nation's labor force have been unavailable, the literature has inferred workers' skills by human capital levels constructed using Mincerian returns to schooling. Standard development accounting procedures impute cross-country differences in the quality of labor to differences in such human capital levels. These studies attribute roughly 10 percent of the output per worker differences to differences in the quality of labor across countries; see Caselli (2005) for a review.

In this paper, we take an alternative, more direct approach to the measurement of labor quality. We use the observed test scores from the Programme for International Student Assessment (PISA) in conjunction with a dynamic model to compute the labor quality in each country. The PISA is an internationally standardized assessment that is organized and conducted by OECD. The tests are administered to 15-year old individuals in schools and provide an alternative quantification of skills during the schooling period. The first advantage of PISA is the assessment of young people near the end of compulsory schooling. It captures students of the same age in each country independently of the structure of national school systems. By contrast, other studies focus on testing students in specific grades which may be distorted by the fact that countries may differ in their grade-entry ages and grade-repetition rules. The second advantage of PISA is that the tests are constructed to evaluate a range of relevant skills that capture how well young adults are prepared to meet future work challenges, by being able to analyze, reason, and communicate their ideas effectively. While previous studies are curriculum-based, PISA tests the young adults' ability to "use their knowledge and skills in order meet real-life challenges" (OECD 2001, p. 16). By design, PISA provides single comparable measure of skills for each country that can be used to index skills of individuals prior to their entry into the labor force.

We develop a parsimonious model where countries differ in two key dimensions – talent

heterogeneity and Total Factor Productivity (TFP). We use the PISA achievement score and our model to construct a measure of labor quality, and quantify its role in accounting for the cross-country differences in output per worker. Our main questions are: how large are the differences in implied labor quality across countries? What are the resulting magnitudes of TFP differences?

Our model has a representative household with a continuum of members. The members are heterogeneous and are born with some innate efficiency units of labor or *talent*. The household divides each cohort in two groups: *skilled* workers and *unskilled* workers. Converting a member of the household into a skilled worker is costly: it requires time (foregone earnings) and goods. As more goods are invested, the higher is the resulting quality of each skilled worker. Quality an unskilled worker is just the talent he was born with. The production technology uses capital, unskilled labor and skilled labor as factors of production. Labor of both types includes the fraction of each worker type and the quality of each worker type. Thus, while the distribution of talent is exogenous in our model, labor quality is *endogenous*. When the elasticity of substitution between labor types is in the empirically plausible range, we show that an increase in TFP increases the fraction of skilled workers as well as investments in their quality. In this sense, differences in TFP not only have a direct effect on the output per worker, but also an *indirect* effect via labor quality.

In a sample of 59 countries, we measure talent by the observed PISA score. We calibrate two other critical parameters in the model – the importance of goods in the enhancement of the quality of skilled labor and the share of unskilled labor in the aggregate technology – using only the U.S. data to deliver the observed expenditures per tertiary student as a fraction of GDP per worker, and the fraction of the population with secondary education or less. We then examine the quantitative implications of the observed cross-country differences in PISA scores for the differences in output per worker and for the differences in the fraction of labor that is unskilled. We subsequently quantify the role of TFP, along with the PISA scores, in accounting for the output and the division of labor across countries.

Our findings indicate that the use of PISA scores leads to substantially *larger* differences in labor quality across countries than in standard analyses based on Mincerian returns. Using Mincerian returns, labor quality in the poorest 10 percent of the countries in our sample is about 86 percent of the quality in the richest 10 percent of the countries. In our model, labor quality in the poorest 10 percent is only about 46 percent of the quality in the richest

10 percent. Our finding on labor quality differences is similar to that in Schoellman (2011) who uses data on immigrants in the U.S. to infer variation in labor quality, exploiting the fact that immigrants from richer countries earn higher returns to schooling than immigrants from poorer countries. He concludes that labor quality differences between poor and rich countries are twice as large as those under conventional measures using Mincer returns; e.g. Hall and Jones (1999), Hendricks (2002). Hence, the TFP differences emerging from our model are smaller than those emerging from standard analyses. In terms of elasticities, the elasticity of output per worker with respect to TFP is 2.35 in our model. The TFP elasticity is 1.5 in the standard growth model, and roughly 1.7 under a measure of labor quality based on Mincer returns, as this variable is correlated with output per worker.

Our paper is closely related to Manuelli and Seshadri (2010) and Erosa, Koreshkova, and Restuccia (2010), who examine the role of human capital in explaining cross-country income differences. Manuelli and Seshadri (2010) develop a model of human capital acquisition in early childhood, via schooling and over the life cycle, where investments of goods in human capital formation play a central role. They argue that TFP differences are quantitatively small and that measured differences in output largely reflect differences in unmeasured labor quality. The TFP elasticity of output per worker in their model is roughly 6.5. Erosa et al. (2010) develop a model where heterogenous households invest in the quantity and quality of schooling. Using data on earnings inequality and its persistence across generations to calibrate their model, these authors find TFP differences similar in magnitude to ours (around 2). Indeed, our results are quite close to those in Erosa et al. (2010), as a modest variation of the key parameter in their model would yield our elasticity estimates (see Erosa et al. (2010), Table 5). Overall, our findings contribute to estimates of potential variation of labor quality across countries. We argue that *observable* variation in talent via PISA scores combined with our model of division of labor and investment in the quality of skilled workers goes a long way towards capturing the variation of labor quality and its interplay with TFP differences.

Our paper is organized as follows. Section 2 presents in detail the data we use in the paper, with particular emphasis on data from the PISA program. Section 3 develops the theoretical framework. Section 7 discusses the parameterization of the model and its mapping to data. In section 5, we perform a number of hypothetical experiments to highlight how the model works. Section 6 presents the basic findings from our experiments. In section

7 we evaluate in detail the quantitative importance of TFP and how our findings compare with those emerging from standard analyses. In section 8 issues, we study the implications our framework for related issues of interest: skill premia and the possibility of skill-biased technology differences across countries. Finally, section 10 concludes.

2 Data

We summarize below the central aspects of data that pertain to our study. We concentrate on PISA scores, enrollment rates, the division of labor between unskilled and skilled workers, and corresponding observations on output per worker.

2.1 PISA

The PISA is an assessment that was jointly developed by participating economies. In this paper, we use the last assessments by PISA which were carried out in 2009 and include 65 economies. PISA tests abilities in *mathematics*, *reading* and *science*. The study is organized and conducted by the OECD, ensuring as much comparability among participants as possible.¹

The PISA sampled students between 15 years and 3 months and 16 years and 2 months old. Only students that are enrolled in an educational institution, regardless of the grade level or type of institution in which they were enrolled are sampled. The average age of OECD country students participating in PISA was 15 years and 8 months, varying by a maximum of only 2 months among the participating countries.²

The performance tests lasted for two hours and were taken using paper and pencil. They include both multiple-choice items and questions that require students to construct their own responses. The PISA aims to test not merely the mastery of the school curriculum, but also important knowledge and skills needed in adult life (OECD 2000, pp. 8). Each subject was tested at differing levels of difficulty in order to represent a coherent and comprehensive

¹For detailed information on the PISA study and its database, see the PISA homepage at www.pisa.oecd.org.

²The PISA sampling procedure ensured that a representative sample of the target population was tested in each country. Most PISA countries employed a two-stage stratified sampling technique. The first step consists on drawing a random sample of schools, and in the second stage students are randomly sampled in each of these schools making sure that each 15-year-old student in a school has equal probability of selection. This procedure typically yield a sample of between 4,500 and 10,000 students per country.

indicator of the continuum of students' abilities. Using item response theory, PISA mapped performance in each subject on a scale with an international mean of 500 test-score points across the OECD countries and an international standard deviation of 100 test-score points.

We consider fifty nine countries participating in the PISA 2009 study. These are: Albania*, Argentina*, Australia, Austria, Belgium, Brazil*, Bulgaria*, Canada, Chile, Colombia*, Croatia*, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hong Kong-China*, Hungary, Iceland, Indonesia*, Ireland, Israel, Italy, Japan, Jordan*, Kazakhstan*, Korea, Kyrgyz Republic*, Latvia*, Lithuania*, Luxembourg, Macao-China*, Mexico, The Netherlands, New Zealand, Norway, Panama*, Peru*, Poland, Portugal, Romania*, Russian Federation*, Republic of Serbia*, Singapore*, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Thailand*, Trinidad and Tobago*, Tunisia*, Turkey, UAE-Dubai*, United Kingdom, United States, Uruguay*. Those marked with a "*" are non-OECD countries.³ We report results using the Math test score. We note, however, that the correlation of student performance between the three subjects is substantial; 0.95 between reading and math, 0.98 between reading and science, and 0.97 between math and science.

2.2 Related Data

To characterize facts on cross-country relationship between test scores and economic development, we also use country level data on the countries' GDP per worker in 2007 (PPP) taken from the Penn World Tables 7.0. Since the the tests are administered to 15 year-old individuals, and in some countries a non-trivial fraction of them are not at school, we also document and subsequently use the enrollment rates at school at that age provided by the PISA study.

Our framework has central implications for the division of the labor force between *skilled* and *unskilled* workers. For the data counterpart of these concepts, we also consider data from Barro and Lee (2001). We define skilled individuals as those with strictly more than high school education, whereas unskilled people are their complement.

³We exclude from the total list of PISA participating countries China-Shanghai and China-Taipei, given the focus on only one urban area. We also exclude Liechtenstein, Montenegro and Azerbaijan, as we lack data for these countries on the composition of the labor force. Finally, we also exclude Qatar, due to its outlier character in terms of output per worker.

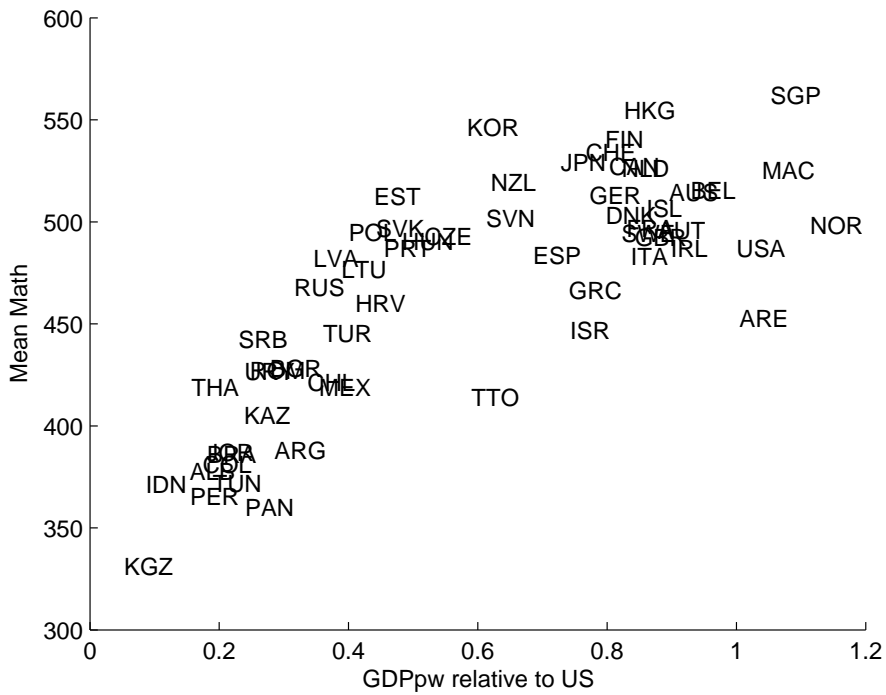


Figure 1: Mean PISA Scores (Math) and GDP per worker.

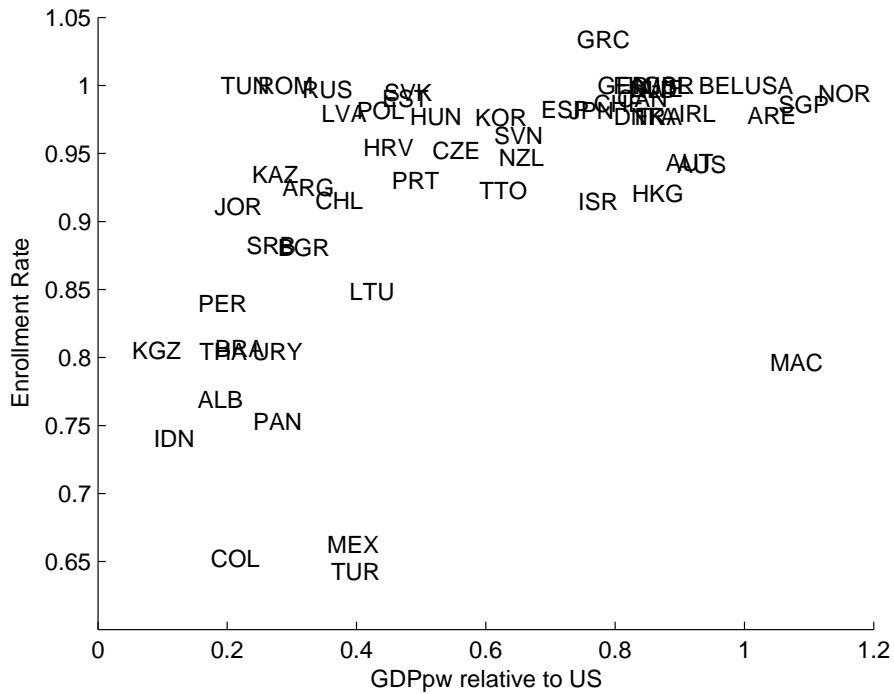


Figure 2: Enrollment Rates (15 years old) and GDP per worker.

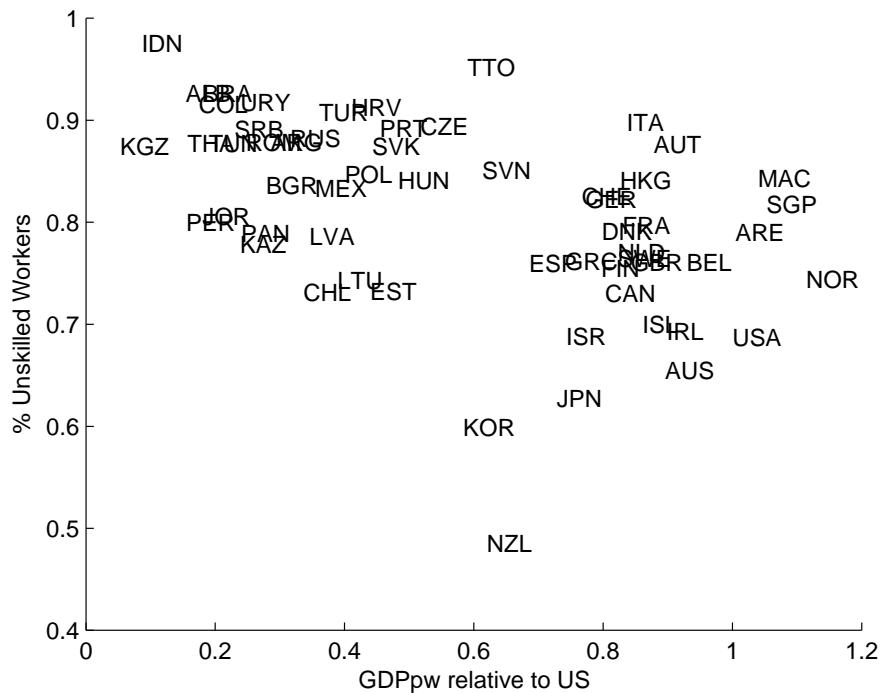


Figure 3: Fraction Unskilled and GDP per worker.

2.3 Facts Summary

Table 1 and Figures 1, 2 and 3 summarize main features from the data. We note first that there are substantial differences in PISA scores across countries. The ratio of the 90th percentile to the 10th percentile is about 1.4, with a maximum gap of a factor of about 1.7, between Hong Kong and the Kyrgyz Republic. The gap between the United States and the Kyrgyz Republic is a factor of nearly 1.5.

Second, there is a positive relationship between enrollment rates and output per worker, with a correlation in the sample of about 0.50. About 80 percent of individuals at 15 are at school in Uruguay, whereas all of them are in school in Finland and the U.S. At the bottom of the distribution, only about 64 percent of 15-year old individuals are at school in Turkey.

Third, there is a strong and positive relationship between PISA scores and output per worker. Richer countries have, on average, high achievement scores. As Table 1 indicates, the correlation between mean math score and output per worker is almost 0.8. Differences in output per worker, however, dwarf observed differences in PISA scores. For instance, the gap between the U.S. and Argentina is a factor of nearly 1.3 in PISA scores, whereas the

gap in output per worker is about 3.5.

Finally, the data reveals a negative relationship between output per worker and the fraction of unskilled individuals. While about 98 percent of workers in Indonesia (the lowest observation in our sample) are unskilled at an output per worker level of about 9 percent of the U.S., only 69 percent of workers are unskilled in the U.S. The correlation between the fraction of unskilled workers and output per worker is about -0.47.

3 Theoretical Framework

There is a single representative household in the economy. The household comprises at time t a continuum of members of total size L_t , who value only consumption. The size of the household (population) grows at the constant rate (g_L). The household is infinitely lived and maximizes

$$\sum_{t=0}^{\infty} \beta^t L_t \log(C_t/L_t), \quad (1)$$

where $\beta \in (0, 1)$ and C_t denotes total household consumption at date t .

Endowments Each household member is born with z units of talent. The talent is distributed with support in $Z = [0, \bar{z}]$, with cdf $G(z)$ and density $g(z)$, which is invariant to population growth. Household members have one unit of time which he/she supplies inelastically. Depending upon his type, each household member can be a *skilled* or *unskilled* worker. We describe below this decision and the associated incomes in detail. The household is also endowed with an initial capital stock $K_0 > 0$.

Technology There is a representative firm that operates a constant returns to scale technology. This technology requires three inputs: capital K , and two types of labor services: skilled labor S and unskilled labor U . Output (Y) is given by

$$Y = F(K, U, S; A) = A K^\alpha [\mu U^\rho + (1 - \mu) S^\rho]^{(1-\alpha)/\rho} \quad (2)$$

where A stands for a Total Factor Productivity parameter, and $\rho \in (-\infty, 1)$. The elasticity of substitution between skilled and unskilled labor is $1/1 - \rho$. Output per worker, $y \equiv Y/L$, is given by

$$y = A k^\alpha l^{1-\alpha}, \quad (3)$$

with $l \equiv [\mu u^\rho + (1 - \mu)s^\rho]^{1/\rho}$, $u \equiv U/L$ and $s \equiv S/L$.

The representative firm behaves competitively and faces rental prices R , W_S , W_U for the use of capital, skilled and unskilled labor, respectively. Capital depreciates at the rate δ .

The Household Problem We assume that the segregation of individuals by skill is costly. This segregation only applies to newborns, and cannot be changed once a household member has been assigned to either pool. Converting one newborn into a skilled worker requires goods, and implies foregone earnings for a period. If a newborn household member is selected for the unskilled labor pool at t , her talent is contemporaneously transformed into efficiency units of unskilled labor, and her income is given by $W_{U,t}z$.

If she becomes part of the skilled pool instead, it takes one period to contribute to skilled labor. The household invests x_t consumption goods in the ‘quality’ of her efficiency units, an amount common to all household members. Investing x_t implies that her talent is amplified by the factor h_{t+1} , where $h_{t+1} = x_t^\phi$, $\phi \in (0, 1)$. Her contribution to household’s income is then given at $t + 1$ by $W_{S,t+1}zh_{t+1}$. It follows that only individuals with sufficiently high levels of talent become skilled. Given rental prices, there exists a unique threshold \hat{z}_t such that newborn household members with talent below this threshold become unskilled workers at t , and those with talent above it are skilled workers, from $t + 1$ on.

If there are $N_t \equiv g_L L_{t-1}$ newborns in each period, and $N_t(1 - G(\hat{z}_t))$ newborns become skilled, the household invests a total of $N_t x_t (1 - G(\hat{z}_t))$ goods at t to amplify their talent with quality h_{t+1} . Then, the aggregate quantities of unskilled and skilled labor evolve according to

$$U_t = U_{t-1} + \underbrace{N_t \int_0^{\hat{z}_t} z g(z) dz}_{\text{Additions to unskilled labor}} \quad (4)$$

$$S_t = S_{t-1} + \underbrace{N_{t-1} h_t \int_{\hat{z}_{t-1}}^{\bar{z}} z g(z) dz}_{\text{Additions to skilled labor}} \quad (5)$$

We also assume that the cost for the household of transforming one unit of consumption into investment, is potentially different from one. We represent these costs by an exogenous

barrier $p_t \geq 1$.

The problem of the household is to choose sequences of consumption, the fractions of household members who are skilled and unskilled, the quantity of goods allocated to affect the productivity of each skilled member, and the amount of capital to carry over to the next period. Formally the household problem is to select $\{C_t, I_t, \hat{z}_t, x_t\}_0^\infty$ to maximize (1) subject to (4), (5),

$$C_t + I_t + N_t(1 - G(\hat{z}_t))x_t = (W_{U,t}U_t + W_{S,t}S_t) + R_tK_t,$$

$$h_{t+1} = x_t^\phi,$$

$$K_{t+1} = (1 - \delta)K_t + \frac{I_t}{p_t}$$

with

$$N_0, S_0, U_{-1}, K_0 > 0.$$

The solution to the household problem is then characterized by the following First Order Conditions:

$$\frac{p_t}{C_t/L_t} = \beta \frac{[R_{t+1} + (1 - \delta)p_{t+1}]}{C_{t+1}/L_{t+1}} \quad (6)$$

$$\frac{W_{U,t} \hat{z}_t + x_t}{C_t/L_t} = \beta \frac{W_{S,t+1} \hat{z}_t x_t^\phi}{C_{t+1}/L_{t+1}} \quad (7)$$

$$\frac{1 - G(\hat{z}_t)}{C_t/L_t} = \beta \frac{W_{S,t+1} \left(\int_{\hat{z}_t}^{\bar{z}} z g(z) dz \right) \phi x_t^{\phi-1}}{C_{t+1}/L_{t+1}} \quad (8)$$

Condition (6) is the standard Euler equation for capital accumulation. Condition (7) states that the discounted compensation of the household member with marginal skill \hat{z}_t at t , weighted by the marginal utility of consumption at $t + 1$, must be equal to the compensation of an unskilled household member *plus* the cost of skill conversion, x_t , weighted by the marginal utility of consumption at t . Finally, condition (8) states that the marginal cost of investing one unit of the consumption good in the quality of a skilled worker must equal its discounted marginal benefit. This benefit depends on the rental price of skilled labor at $t + 1$ and the ‘raw’ addition member to the pool of skilled labor at $t + 1$, $\int_{\hat{z}_t}^{\bar{z}} z g(z) dz$.

Equilibrium In competitive equilibrium, the markets must clear and factor prices are paid their marginal products. Equilibrium in the markets for unskilled and skilled labor implies

$$U_t^* = U_{t-1}^* + N_t \int_0^{\hat{z}_t^*} z g(z) dz \quad (9)$$

$$S_t^* = S_{t-1}^* + N_{t-1} h_t^* \int_{\hat{z}_{t-1}^*}^{\bar{z}} z g(z) dz, \quad (10)$$

for all $t = 0, 1, 2, \dots$, where a ‘*’ over a variable denotes its equilibrium value. Equilibrium in the goods market implies

$$C_t^* + I_t^* + N_t(1 - G(\hat{z}_t^*))x_t^* = Y_t^*,$$

Factor prices equal

$$W_{U,t}^* = \frac{\partial F(K_t^*, S_t^*, U_t^*)}{\partial U_t} \quad (11)$$

$$W_{S,t}^* = \frac{\partial F(K_t^*, S_t^*, U_t^*)}{\partial S_t} \quad (12)$$

$$R_t^* = \frac{\partial F(K_t^*, S_t^*, U_t^*)}{\partial K_t} \quad (13)$$

for all $t = 0, 1, 2, \dots$. It is now possible to define a competitive equilibrium. A competitive equilibrium is a collection of sequences $\{C_t^*, K_t^*, \hat{z}_t^*, x_t^*, W_{U,t}^*, W_{S,t}^*, R_t^*\}_0^\infty$, such that (i) given $\{W_{U,t}^*, W_{S,t}^*, R_t^*\}_0^\infty$, the sequences $\{C_t^*, K_t^*, x_t^*, \hat{z}_t^*\}_0^\infty$ solve the household problem; (ii) factor prices are competitive for all t ; (iii) markets clear for all t .

3.1 Balanced Growth

Along a balanced growth path, aggregate investment of both types as well as output, consumption, skilled and unskilled labor grow at the constant population growth rate g_L . Factor prices, capital per worker ($k \equiv K/L$), investment per new skilled worker x , and the threshold \hat{z} are constant.

Note that the laws of motion for unskilled labor imply that unskilled labor per worker ($u \equiv U/L$) equals $\int_0^{\hat{z}^*} zg(z)dz$ along the balanced growth path. Similarly, skilled labor per worker, ($s \equiv S/N$), equals

$$s^* = \frac{h^* \int_{\hat{z}^*}^{\bar{z}} zg(z)dz}{1 + g_L}$$

Given the properties of the aggregate technology, we can write $Y = Lf(k, s, u)$. Hence, along the balanced growth path, (6) and competitive factor prices imply

$$\frac{\partial f(k^*, s^*, u^*)}{\partial k} = p \left(\frac{1}{\beta} - (1 - \delta) \right). \quad (14)$$

Similarly, condition (7) and competitive prices imply

$$\frac{\partial f(k^*, s^*, u^*)}{\partial u} \hat{z} + x^* = \beta \frac{\partial f(k^*, s^*, u^*)}{\partial s} \hat{z}^* x^{*\phi} \quad (15)$$

Likewise,

$$(1 - G(\hat{z}^*)) = \beta \frac{\partial f(k^*, s^*, u^*)}{\partial s} \left[\int_{\hat{z}}^{\bar{z}} zg(z)dz \right] \phi x^{*\phi-1} \quad (16)$$

Hence, (14), (15) and (16) can be used to solve for a steady-state equilibrium. They determine the steady-state capital stocks per worker of both types, the threshold value \hat{z}^* and the investment in labor quality x^* .

How changes in TFP and investment barriers affect the allocation of talent and then, the segregation of individuals between skilled and unskilled workers? We show below that under empirically plausible conditions, economies with low TFP and high investment barriers are characterized by a lower fraction of skilled individuals than their counterparts.

Assumption 1 *The density $g(\cdot)$ is log-concave.*

Proposition 1 *Let $\rho \in [0, 1)$. Assume that assumption 1 holds. Then,*

1. *There is a unique steady state with $\hat{z}^* > 0$ and $x^* > 0$.*
2. *An increase in TFP (a reduction in investment barrier) implies:*
 - *a reduction in \hat{z}^* , if $\rho \in (0, 1)$;*

- *no effects on \hat{z}^* , if $\rho = 0$.*
(Cobb Douglas case)
- *No effects on \hat{z}^* if $\phi \rightarrow 0$.*

Proof. See Appendix. ■

The above proposition is important in many respects and some comments are in order. First, it provide conditions under which a unique steady state exists. This is naturally important for the quantitative analysis that we conduct later. These conditions are sufficient, and easily satisfied. Many widely used distributions satisfy the log-concavity requirement; this family includes the uniform, the exponential, the normal and the gamma distributions. Second, the proposition shows that the relative fraction of skilled workers and investment in their quality *always* increases with an increase in TFP (reduction of investment barrier) when the parameter restrictions are satisfied. Thus, whether or not these forces account for the magnitudes of changes empirically observed is a quantitative issue. It is worth emphasizing here that a host of empirical estimates of the elasticity of substitution between skilled and labor fall in the range required. For the limiting case of the Cobb-Douglas production function, changes in TFP or relative investment prices have no effects on the skill segregation. Finally, investments in goods are *essential* for any steady-state effects of TFP or relative investment prices on skill segregation. In the absence of such investments, changes in these parameters are neutral.

4 Parameter Values

We start by setting the model period equal to *four* years, a compromise regarding the time it takes to become a skilled worker. To calibrate parameter values we use the U.S. as a benchmark. We later explain what aspects of our parameterization change for the cross-country analysis.

Preferences and Demographics We choose the discount factor so that the steady state annual interest rate equals 6%. This implies a discount factor (β) equal to 0.9433, or about 0.792 at the four-year frequency. We set the growth rate in population equal to

the annual value of about 0.9%, calculated using the population figures in the Penn World Tables 7.0.

Technology We set the capital share (α) in the aggregate production function to standard value of 0.33. Empirical studies indicate an elasticity of substitution between skilled and unskilled labor of around 1.5 (Katz and Murphy (1992), Heckman, Lochner, and Taber (1998)). Then, we select the parameter ρ in the aggregate production technology to $\rho = 1/3$. The depreciation rate is set to 4% on an annual basis. We normalize the Total Factor Productivity level (A) and relative investment prices to 1.0.

From the technology side, the curvature parameter in the production of skills (ϕ) and the share parameter in the production function (μ) remain to be set. We choose these parameters so that in stationary equilibrium the model reproduces two empirical targets. The first target is the fraction of unskilled workers in the U.S. from Barro and Lee (2001). This amounts to about 68.7%, and corresponds to the fraction of the population with completed secondary education or less, aged 15 years or older. The second target are expenditures per tertiary student as a fraction of GDP per worker (at PPP values). This fraction amounted to 28.7% in 2004, and is taken from the UNESCO 2007 World Education Indicators report. This procedure implies $\phi = 0.30$ and $\mu = 0.54$.

Talent We calibrate the distribution of talent using PISA data. We assume that the distribution of talent is a Gamma distribution with parameters κ and θ . We choose κ and θ in order to reproduce the observed *mean* and *coefficient of variation* in the PISA test for Mathematics for the U.S.

Table 2 and 3 below shows the resulting parameter values. It is not problematic for the model to reproduce the data, as the Table illustrates. The table also shows that our parametric Gamma approximation to the distribution is very good, as it generates multiple percentiles of the distribution quite well.

5 Model Mechanics

We explore in this section the long-run effects of a number of changes in the calibrated parameters of the benchmark economy. In doing so, we highlight the role of different forces

at work, and help the reader understand the main results in subsequent sections.

Specifically, we consider the following empirically-motivated departures from the benchmark case. We consider a reduction in the mean talent, an increase in the dispersion of talent, as well as a reduction in TFP. We first entertain all these changes in isolation. We then combine all these changes.

The motivation for the experiments that follow is straightforward. We documented in section 2 substantial differences in PISA scores across countries and the correlation between the PISA scores and output per worker. In addition, since there are within-country variation in test scores that may matter in context of our environment, we also assess the effects of changes in dispersion. Finally, the variation in TFP, in conjunction with changes in the distribution of PISA scores, contributes to assessing the relative importance of TFP for output variation and the the division of labor across countries.

A Reduction in Mean Talent In our first experiment, we consider a reduction in the mean math score the level in the 10th percentile country in the cross-country distribution of PISA scores. Results are summarized in Table 4. Intuitively, all else remaining the same, a reduction in the mean talent leads to a reduction in the value of investing in the quality of the average skilled worker (see equations 8 or 16). Hence, as Table 4 shows, this reduction leads to a reduction in the investments in skilled workers x and to an increase in the threshold \hat{z} across steady states, which in turn implies an increase in the fraction of unskilled workers. Given these changes, the wage premium (W_S/W_U) increases as the ratio of skilled to unskilled labor declines.

Quantitatively, the effects of the reduction in mean talent on output per worker can be substantial. Reducing the mean from the benchmark case to the bottom 10th percentile implies a reduction of about 18%. The corresponding decline in output across steady states is about 26%.

An Increase in Dispersion of Talent Within-country dispersion in PISA scores varies inversely with the level of economic development. Hence, in our second experiment we consider an increase in the *coefficient of variation* of talent to the level in the 90th percentile country in the cross-country distribution of this variable. We increase the coefficient of variation from the benchmark value of 0.19 to 0.23. Table 4 shows that this change leads to

an increase in the number of unskilled individuals across steady states. This reduction also results in an *increase* in the investments on skilled workers. This is intuitive: an increase in dispersion, all else equal, implies an increase in the talent of those at top tail of the talent distribution; i.e. skilled workers. Quantitatively, as Table 4 shows, the most significant changes are in the fraction of unskilled workers. Output, however, increases mildly by about 1.4%.

A Reduction in Total Factor Productivity Reducing the value of TFP leads to the results discussed in Proposition 1. As goods are an input for the quality of skills and since the elasticity of substitution between labor inputs is higher than 1, the reduction in TFP leads to an increase in the threshold \hat{z} and to a reduction in the quality of skilled labor. This translates into an increase in the steady-state value of unskilled labor, and an increase in the wage premium.

Quantitatively, the magnitude of the effects in Table 4 are substantial. Reducing the value of TFP by 50 percent implies an increase in the fraction of unskilled workers of about 5%, a reduction in the quality of skilled labor of about 29% and a reduction in output of nearly 70%. As we elaborate later, reductions of TFP of these magnitudes are indeed required to account for the output per worker gaps in our sample.

All Together Now The last row in Table 4 shows the combined effects of all changes simultaneously. As a poor country in our sample is similar to this hypothetical scenario, the combined effects are illustrative of the quantitative implications of our model. The table shows that when all changes are combined, the fraction of unskilled labor increases by 8%, the quality of skilled labor drops by about a third, and output drops by about almost 78%. The table also demonstrates that the changes in TFP capture the bulk of the effects on output and the composition of the labor force.

6 Findings

We investigate in this section the extent to which the forces in our model can account for the observed differences in the division of labor in our sample of countries, and the corresponding differences in output per worker and the quality of labor.

Our analysis explicitly takes into account the international heterogeneity in enrollment rates that we documented earlier. Only a fraction of $\gamma \in (0, 1)$ of the unskilled pool in the model can be converted into skilled workers – those who are at school at age 15 in the data. We assign to the rest of the cohort (fraction $1 - \gamma$) a common set of skills, z_{min} . This procedure implies that the stationary values of unskilled and skilled labor per worker obey, respectively,

$$u^* = (1 - \gamma)z_{min} + \gamma \int_{z_{min}}^{z^*} zg(z)dz,$$

$$s^* = \frac{\gamma}{1 + g_L} h^* \int_{\hat{z}^*}^{\infty} zg(z)dz.$$

For each country, γ is the fraction of the population that is 15 years old and is enrolled in the educational system, as reported in PISA data. We set z_{min} for each country equal to the 10th percentile of the PISA Math test score for that country. We subsequently investigate the sensitivity of our results to these choices.

The values of the country-specific population growth rates are those reported in the Penn World Tables 7.0.

We conduct two sets of experiments below. First, we assign to each country the distribution of talent parameterized from PISA data, and compute stationary equilibria. The results and their relation to data are reported in Table 5. Second, we add variation in relative investment prices and TFP across countries. We use the investment prices reported in the Penn World Tables 7.0, and find the relative TFP levels such that the output per worker in the model is the same as in the data. The second experiment shows the consequences of *all* of the driving forces in our model and the results are reported in Table 6. For both experiments, we organize our results according to the observed distribution of output per worker ; e.g. poorest 10% countries versus richest 10% countries.

Variation in PISA Scores Our results indicate that adding *only* the variation in PISA scores leads to lower investments in the quality of skilled labor and a higher fraction of unskilled labor in poorer countries. In turn, these changes lead to lower values of aggregate capital in poorer economies, and given the effects on labor inputs, result in lower values of output.

Quantitatively, variation in the PISA scores implies that the quality of skilled labor (h) in the poorest 10% of countries is about 88% of that in the richest 10% (see Table 5). Similarly, the fraction of unskilled labor in the poorest 10% is about 5.5% higher than that in the richest 10%. Output per worker, as a result, is about 60% in the poorest 10% relative to the richest 10%.

As Table 5 demonstrates, the model generates about 40% of the gap in the fraction of unskilled workers between the poorest 10% and the richest 10%. In terms of output per worker, the productivity in the richest 10% of the countries is about 1.7 times as high as that in the poorest 10%; the corresponding ratio in the data is more than 8. As in the previous literature, there is still an ample role for TFP differences across countries to account for observed income differences.

Variation in Technology and PISA Scores When we add technology variation (i.e., TFP variation and observed variation in relative price of investment) to the observed variation in PISA scores, we find that our framework can now account for a substantial fraction of the differences in the division of labor across countries (see Table 6). With TFP levels that match the output per worker levels *exactly*, the model captures about 80% of the differences in the fraction of unskilled workers between the poorest 10% and the richest 10%. Thus, with technology differences and their interplay with the PISA score differences, the model can go a long way toward understanding the cross-country differences in the division of the labor.

Table 6 illustrates how large TFP differences need to be in order to account of the output per worker differences. TFP at the poorest 10% is about four-tenths of that at the richest 10%. Not surprisingly, the quality of skilled workers is now substantially lower in poorer countries (only 56% relative to richer countries) since TFP has an indirect effect on the division of labor and the investment in skilled workers. These changes in quality and division of labor result in smaller differences in TFP than those that emerge from standard analyses. We elaborate on this issue in the next section.

7 TFP and Economic Development

We now use our model to assess the relative importance of Total Factor Productivity vis-a-vis other factors in accounting for the differences in output per worker across countries. We also provide a comparison of the findings from our model with standard findings in the development literature.

The Relative Importance of TFP Recall that differences in talent, relative prices and TFP are not enough to reproduce differences in the division of the labor force. We now force the model to reproduce *exactly* both the levels of output per worker and the fraction of unskilled workers. Specifically, we introduce an implicit tax (*wedge*) in the model, so that in conjunction with TFP variation and PISA scores, the model can jointly reproduce output per worker differences in our sample and the division of the labor force by skill in stationary equilibrium, country by country. The tax wedge affects the conversion of talent into skilled labor: if the household transfers an individual with talent z to the skilled pool at t , the contribution to household income at $t + 1$ is now

$$W_{S,t+1} z h_{t+1} (1 - \tau),$$

where τ is the tax wedge. Tax collections are returned to the household in a lump-sum manner.

We present the results in Table 7. As the table shows, differences in TFP are smaller relative to those in Table 6. The model requires *higher* tax wedges in poorer countries than in rich countries. These country-specific wedges lead to wider differences between rich and poor countries in the values of the aggregate labor input. These larger cross-country differences in aggregate labor, in turn, lead to TFP differences across countries that are smaller. In Table 7, TFP at the poorest 10% countries is about 43% of the richest 10% countries whereas in Table 6 the corresponding number is only 40% when the model was not forced to reproduce the cross-country variation in the division of labor.

Tables 5, 6 and 7 show the progression of the magnitude of the labor input across experiments. The first experiment (‘variation in PISA scores only’) indicates that the labor input at the poorest 10% countries is about 65% of the labor input at the richest 10% countries. When variation in technology is added in the second experiment, and TFP is chosen to

match output per worker differences, the corresponding figure for labor input is about 51%; matching also the division of labor by skill in the third experiment leads to a labor input of 46%. In our model, more than half of the rich to poor ratio of labor input per worker is delivered by (measured) differences in achievement as of age 15 (i.e., PISA scores) and the rest is due to the combined effects of TFP, relative prices, tax wedges and their interplay with achievement scores.

Summing up, matching the division of labor within countries leads to smaller TFP differences between rich and poor countries. Quantitatively, however, the bulk of the resulting TFP differences are already captured when we force the model to reproduce output per worker across countries. Matching the division of labor exactly contributes only modestly to reducing the implied TFP gaps across countries.

Comparison with Standard Exercises We now compare the predictions of our model with those resulting from the one-sector growth model with differences in labor quality across economies. These differences in labor quality are in turn driven by differences in years of schooling via Mincerian returns.

Recall that in our model, output per worker (y) can be written as

$$y = Ak^\alpha l^{1-\alpha},$$

where k is capital per worker and l is labor per worker. This provides the basis for a comparison with findings from the standard one-sector model; the central difference between our model and the standard one is in the notion of labor per worker.

As in Hall and Jones (1999), Caselli (2005) and others, suppose that labor per worker in each country is given by

$$l = \exp \psi(s),$$

where ψ is a function of years of schooling (s), determined by rates of return that vary with average years of schooling in consistency with empirical estimates; Psacharopoulos (2004). Specifically, we set $\psi(s) = 0.134 s$ for $s \in [0, 4]$, $\psi(s) = 0.134 \cdot 4 + 0.101 (s - 4)$ for $s \in (4, 8]$, and $\psi(s) = 0.134 \times 4 + 0.101 \times 4 + 0.068 \times (8 - s)$ for $s > 8$. We use the average years of schooling in each country as reported by Barro and Lee (2001). All other parameters in

the model are as set previously in Section . We input into the one-sector growth model the relative prices of investment from Penn World Tables 7.0 and find the levels of TFP that reproduce the observed relative levels of output per worker.

Results are presented in Table 8, where we also present for comparison the corresponding results from our model when we match the division of labor by skill as well as when we do not. As the table indicates, the implied differences in TFP are much *larger* in the one-sector growth model with labor quality measured via Mincerian returns. In the one-sector growth model, TFP at the poorest 10% countries is about 28% of the richest 10% countries (Panel A); to contrast, the ratio in our model is 42.5% when we match the division of labor by skill (Panel C). Clearly, the larger TFP differences are due to the differences in the value of labor input in the two cases. In the one-sector growth model, labor quality is about 86% at the poorest 10% relative to the richest 10% whereas our model generates differences 46%.

A related way to illustrate our findings is to calculate the elasticity of output per worker across countries with respect to TFP. As it is well known, the one-sector growth model without labor quality adjustments implies an elasticity of 1.5. With labor quality measured via Mincerian returns, we estimate an elasticity of 1.69 since labor quality is correlated with output per worker. Our model implies elasticities substantially larger, of about 2.31 when we do not match the division of labor in each country, and of about 2.35 when we do.

Summary In our model, TFP not only has the standard effect on output but also an indirect effect on our measure of labor input through the division of labor by skill and the augmenting of skilled labor. In other words, labor in our setup is a function of TFP. This important feature is reflected in our finding of an elasticity of output per worker with respect to TFP that is larger than the one obtained using the standard growth model. We conclude from our findings that the explicit consideration of achievement scores considerably reduces the importance of TFP in our understanding of cross-country income differences. We should emphasize, however, that our findings still attribute a central role to TFP differences. As Table 8 shows, TFP differences between the richest 10% and the poorest 10% countries are in excess of a factor of two.

8 Skill Premia

In this section, we investigate and discuss the implications of our model for the relative earnings of skilled to unskilled workers, or *skill premia*. This is important, as it reflects on the importance of the economic mechanisms that our paper highlights for data that we do not explicitly target.

We take measures of skill premia from Fernandez, Guner, and Knowles (2005). The data used by the authors are calculated directly from national surveys from 1990 to 1997. The countries included are Argentina, Australia, Belgium, Bolivia, Brazil, Canada, Chile, Colombia, Costa Rica, Czech Republic, Denmark, Ecuador, Finland, France, Germany, Hungary, Israel, Italy, Luxembourg, Mexico, Netherlands, Norway, Panama, Paraguay, Peru, Poland, Slovakia, Spain, Sweden, Taiwan, United Kingdom, Uruguay, United States and Venezuela. Strictly, this is a subset of the broad set of countries of our benchmark group. The skill premium is defined as the ratio of *earnings (labor income) per worker*, for skilled male workers to unskilled ones.⁴ A skilled worker is an individual who has more years of education than those required to complete secondary school. Therefore, this definition is consistent with the data we use on educational attainment. Figure 4 plots skill premia (relative to the U.S. value) against output per worker for each country (relative to the U.S.). As the figure shows, there is a *negative* correlation between these two variables, with a correlation coefficient of -0.75 . The skill premium in the poorer countries of this subsample is substantially higher than in richer countries. Table 9 indicates that the skill premium in the poorest ten percent of countries is about 2.4 times the skill premium in the richest ten percent.

We now construct a notion of skill premia within our model that is consistent with the data we focus on. Recall that earnings of an unskilled worker are given by $W_U^* z$, whereas the earnings of a skilled worker are given by $W_S^* z h^*$. Then, the skill premium (SP) defined as the ratio of per-worker earnings of skilled workers to per-worker earnings of unskilled workers, is given by

$$SP = \frac{(W_S^* h^* \int_{\hat{z}^*}^{\bar{z}} z g(z) dz) / (1 - G(\hat{z}^*))}{(W_U^* \int_0^{\hat{z}^*} z g(z) dz) / G(\hat{z}^*)}$$

We can rewrite the above as an expression that involves *relative wages*, augmented skills

⁴In their sample, workers are *husbands* of age between 36 and 45 years old.

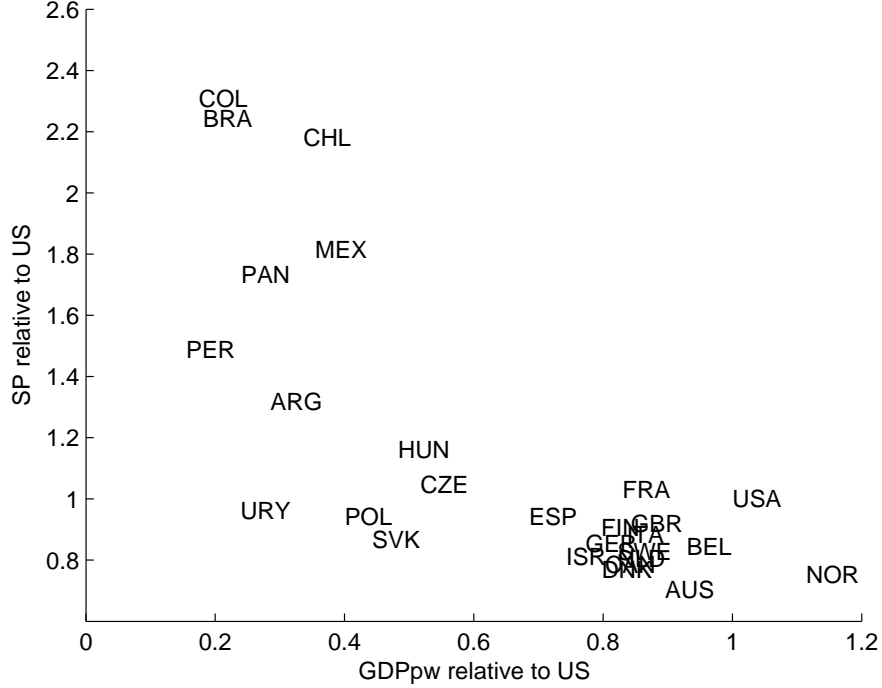


Figure 4: Skill Premia (relative to U.S.) and GDP per worker.

(h), and conditional expectations of talent:

$$SP = \left(\frac{W_S^*}{W_U^*} \right) h^* \left(\frac{E(z|z \geq \hat{z}^*)}{E(z|z < \hat{z}^*)} \right)$$

We note that the notion of skill premia consistent with the data is *not* just relative wages. Hence, changes in the division of the labor force by skill will generate movements in all components components above. Notice that an increase in the fraction of unskilled workers (i.e. an increase in \hat{z}^*) reduces the ratio of the conditional expectations of talent, but increases relative wages. Henceforth, changes in skill premia implied by the model are the result of countervailing forces.

Findings In line with our benchmark analysis, we conduct two types of experiments using only data from the restricted sample of countries. First, along with differences in PISA scores, we consider technology variation, forcing the model to reproduce differences in output per worker via TFP differences. In our second experiment, in addition to technology variation, we introduce tax wedges in order to reproduce the division of labor by skill.

Table 9 shows the performance of the model in terms of skill premia and its components. When we consider only technology variation within our restricted sample, the model does not perform well in terms of the observed differences in the division of the labor force by skill. As Table 9 demonstrates, this performance is effectively worse than in our benchmark case. In terms of skill premia, the model does not generate substantial differences across countries. In the data, skill premia at the top 10% poorest countries is about 144% higher than at the richest 10%; in the model it is just 11.8%.

When the model is forced to reproduce the division of labor by skill along output per worker via productivity differences, it generates a higher skill premia at the top 10% poorest countries that is about 107% higher than the richest 10%, and about 59.3 higher at the richest 20% versus the bottom 20%. We conclude that forcing the model to reproduce the division of labor by skill goes a long way towards accounting for skill premia. As we discussed earlier, accounting for the observed division of labor is not critical in quantifying the importance of productivity for income differences in our model. Nevertheless, reproducing the division of labor appears key in accounting for variation in skill premia.

8.1 Skill-Biased Technology Differences?

In recent work, Caselli and Coleman (2006) and Hendricks (2008) conclude that differences in technology across countries are *skill biased* in a cross-section of countries. To arrive to this conclusion, these authors used measures of skilled and unskilled labor inputs constructed using Mincerian returns, and implied skill premia determined by skill prices from an aggregate production function with imperfect labor substitutability (CES).

Our model and the skill premia data in this section are ideal to review these findings. Does the aggregate technology vary in a systematic way across countries? Is the importance of unskilled labor (μ) negatively correlated with output per worker? To answer these questions, we allow μ to change across countries in order to reproduce the division of the labor force, country by country, alongside the observed differences in talent and in technology (TFP) to generate observed levels of output per worker. When we do so, we find that the weight of unskilled labor in technology is indeed higher in poorer countries: it is about 0.56 in the richest 10% of countries whereas it is about 0.69 in the poorest 10%. However, the model does *not* generate in this case differences in skill premia of the same magnitudes of those

observed in our sample. The model implies that skill premia in the poorest 10% is only 14% higher than in richest 10%. This is not surprising: a higher level of μ in poorer countries implies that the wage ratio component of skill premia is *lower* there.

If instead we force the model to reproduce both the division of labor by skill and skill premia, via wedges as before and variation in μ , we find that the weight of unskilled labor in technology is slightly *higher* in richer countries than in poorer countries: about 0.61 in the richest 10% of countries versus 0.69 in the poorest 10%. These findings lead us to conclude that our model in conjunction with evidence on skill premia do *not* support the conclusion of skill-biased differences in technology across countries. In a nutshell, this conclusion follows from the effects that our model features have on the size of skilled and unskilled labor across countries. Cross-country differences in talent, in conjunction with the effects on the division of labor by skill driven by technology differences and wedges plus the corresponding investments in skill quality, can go a long way in accounting for variation in skill premia. No skill-biased technology differences seem critical.

9 Discussion

9.1 A Counterfactual Exercise

One way to assess the impact of differences in talent across countries is to conduct a “policy” exercise where we assign to the poorest country in the sample the PISA distribution of a rich one, the United States.

The poorest country in our sample is the Kirgыз Republic (Kyrgyzstan), which incidentally has the lowest mean value of PISA (math) in the sample. This country has an output per-worker of just about 5.6 % of the United States in our data (the U.S. is richer by a factor of of nearly 17). Our model implies that assigning Kyrgyzstan the PISA distribution of the U.S., leaving all else (e.g. TFP) the same, would lead to a a level of output per worker that is about 63.5% higher than in the benchmark situation. This results in a level of output per worker of about 9.1% of the U.S. (the U.S. would be richer by a factor of of nearly 17). Since the capital to output ratio is unchanged across steady-states, the experiment implies a change in the labor input of Kyrgyzstan of also about 63.5%, which stems from an increase in both the unskilled and skilled labor inputs. The fraction of unskilled workers drops by about 5%.

9.2 PISA as Labor Quality

Suppose that instead of using our theory of labor quality, we just simply use the *mean* levels of PISA scores as a measure of each country's labor input. What would be the consequences for relative TFP's, etc, in this case? To answer this question, we assign the mean levels of PISA scores as each country's labor input, and then proceed to find the level of TFP in each country that reproduces the empirical level of output per worker. Results are displayed in Table 10. The elasticity of output with respect to TFP now becomes 1.83

9.3 Amplification Effects: No Investments in Quality

What are the implications in terms of the amplifying role of TFP differences associated to investments in the quality of skilled workers? To answer this question, we set the parameter ϕ to zero, eliminating as a result any investment incentive for the representative household in each economy. We then proceed to find the relative levels of TFP that reproduce the empirical levels of output per worker. Results are displayed in Table 10. The elasticity of output with respect to TFP now becomes 1.88

9.4 High Elasticity of Substitution

Recently, authors such as Gancia, Mueller, and Zilibotti (2011) and others, have argued that the elasticity of substitution between skilled and unskilled labor is higher than the conventional value that we use. They estimate elasticities in the range of 1.4-2.5. We ask: what are the consequences for our findings of assuming a high value for this elasticity? We proceed to set such elasticity to the highest value in the range mentioned (2.5), which corresponds to a value of $\rho = 0.6$. We recalibrate the model under this assumption, and proceed to find the relative levels of TFP that reproduce the empirical levels of output per worker. Results are displayed in Table 10. The elasticity of output with respect to TFP now slightly increases to 2.34

10 Conclusion

11 Appendix

Proof of Proposition 1

We start by defining the marginal products for our specification of the production technology. Output per worker is given by

$$y = F(K, U, S; A)/L = A k^\alpha l^{1-\alpha}$$

with $l \equiv [\mu u^\rho + (1 - \mu)s^\rho]^{1/\rho}$.

Hence, the marginal products of capital (MPK), skilled labor (MPS) and unskilled labor (MPU) are, respectively:

$$MPK = A\alpha\tilde{k}^{\alpha-1}, \tag{17}$$

$$MPS = A(1 - \alpha)\tilde{k}^\alpha[\mu + (1 - \mu)\tilde{s}^\rho]^{\frac{1-\rho}{\rho}} \tilde{s}^{\rho-1}(1 - \mu), \tag{18}$$

$$MPU = A(1 - \alpha)\tilde{k}^\alpha[\mu + (1 - \mu)\tilde{s}^\rho]^{\frac{1-\rho}{\rho}} \mu, \tag{19}$$

with $\tilde{k} \equiv k/l$ and $\tilde{s} \equiv s/u$.

The model implies in steady state (balanced growth path):⁵

$$MPK = p \left(\frac{1}{\beta} - (1 - \delta) \right). \tag{20}$$

$$MPU \hat{z} + x = \beta MPS \hat{z} x^\phi \tag{21}$$

$$1 = \beta MPS E(z|z \geq \hat{z}) \phi x^{\phi-1} \tag{22}$$

Notice that equation (20) sets the stationary capital to labor ratio as a function of preference and technology parameters, as well as TFP and relative prices. Hence, $\tilde{k} \equiv B(A, p)$. Hence, we can rewrite (21) and (22) as the marginal products of skilled and unskilled labor in steady state as

⁵To simplify notation, we omit from now on the (*) notation for equilibrium values.

$$MPS = T(A, P)[\mu + (1 - \mu)\tilde{s}^\rho]^{\frac{1-\rho}{\rho}} \tilde{s}^{\rho-1}(1 - \mu), \quad (23)$$

$$MPU = T(a, p)[\mu + (1 - \mu)\tilde{s}^\rho]^{\frac{1-\rho}{\rho}} \mu, \quad (24)$$

with $T(A, p) \equiv A(1 - \alpha)B(A, p)^\alpha$. Clearly, $T(A, p)$ is strictly decreasing in p and strictly increasing in A .

Using equations (21) and (22), after manipulations we can write:

$$1 = \beta x^\phi \left(\frac{MPS}{MPU} \right) \left[1 - \phi \frac{E(z|z \geq \hat{z})}{\hat{z}} \right] \quad (25)$$

Given the CES technology, we have:

$$\frac{MPS}{MPU} = \frac{1 - \mu}{\mu} [f(\hat{z})]^{\rho-1} x^{\phi(\rho-1)} \quad (26)$$

where $f(\hat{z})$ is defined as

$$f(\hat{z}) \equiv \frac{\int_{\hat{z}}^{\bar{z}} z dG(z)}{\int_0^{\hat{z}} z dG(z)}$$

Note that the function $f(\hat{z})$ satisfies important properties: $f' < 0$, $\lim_{\hat{z} \rightarrow 0} f(\hat{z}) = +\infty$, and $\lim_{\hat{z} \rightarrow \infty} f(\hat{z}) = 0$.

Using (26) and (22) in equation (25), after manipulation, we get

$$[f(\hat{z})]^{1-\rho} = \frac{\beta(1 - \mu)}{\mu} T(A, p)^{\frac{\rho\phi}{1-\phi}} D(\hat{z})^{\rho\phi} \left[1 - \phi \frac{E(z|z \geq \hat{z})}{\hat{z}} \right] \quad (27)$$

where $D(\cdot)$ is a strictly increasing function of \hat{z} .

Equation (27) allows us to establish the uniqueness and properties of the threshold \hat{z} . First, for $\rho \in [0, 1)$ and from the properties of $f(\hat{z})$, the left-hand side is a non increasing function of \hat{z} ; for $\rho \in (0, 1)$ is strictly decreasing, approaching 0 as $\hat{z} \rightarrow \infty$, and ∞ as $\hat{z} \rightarrow 0$. The right-hand side of (27) is monotonically increasing and eventually positive if two conditions are met. First, $\frac{E(z|z \geq \hat{z})}{\hat{z}}$ must be a decreasing function of \hat{z} . Lemma 1 below shows that this is indeed the case under log-concavity of the density $g(\cdot)$. Second, the right-hand side is eventually positive if

$$\lim_{\hat{z} \rightarrow \bar{z}} \frac{E(z|z \geq \hat{z})}{\hat{z}} < \frac{1}{\phi},$$

which is satisfied as the limit above equals 1. Hence, by the intermediate value theorem, there is a unique value $\hat{z} > 0$ that solves (27). The fact that x is strictly positive follows from this, and the fact that marginal products are positive, using either (21) or (22). Figure A1 depicts the solution for the threshold \hat{z} .

Three central properties of the solution follow. First, from (27), in the limiting case of $\rho = 0$ (Cobb-Douglas case), the threshold \hat{z} is independent of TFP and relative investment prices. Second, as $\rho \in (0, 1)$, and increase in TFP (a reduction in p) implies a reduction in the threshold \hat{z} and viceversa. Figure A2 provides an illustration of this result. Finally, the threshold is independent of TFP and investment prices in the limiting case of $\phi = 0$. ■

Lemma 1: $\frac{E(z|z \geq \hat{z})}{\hat{z}}$ is decreasing in \hat{z} if the density $g(\cdot)$ is log-concave.

Proof. Let

$$\psi(\hat{z}) = \frac{E(z|z \geq \hat{z})}{\hat{z}}$$

Then, $\psi'(\hat{z}) < 0$ iff

$$\frac{\partial E(z|z \geq \hat{z})}{\partial \hat{z}} < \frac{E(z|z \geq \hat{z})}{\hat{z}}$$

The derivative above is less or equal than one by Proposition 2 in Burdett (1996) if and only if

$$\int_{\hat{z}}^{\infty} (1 - G(\hat{z})) dz$$

is log-concave. Log-concavity of the density $g(\cdot)$ ensures this. Furthermore, $\frac{E(z|z \geq \hat{z})}{\hat{z}} > 1$.

Hence,

$$\frac{\partial E(z|z \geq \hat{z})}{\partial \hat{z}} \leq 1 < \frac{E(z|z \geq \hat{z})}{\hat{z}}$$

and the result follows.

■

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Table 1: Summary Statistics

Statistic	PISA Score	Unskilled (%)	Enrollment (%)	Output Per-Worker
Mean	464	0.81	0.92	46,646
Median	483	0.82	0.98	42,988
St. Deviation	57	0.09	0.10	26,034
90th/10th Ratio	1.39	1.32	1.26	5.21
80th/20th Ratio	1.24	1.19	1.18	3.51
Corr (x, Y/N)	0.79	-0.47	0.50	1.00

Note: Entries show summary statistics for the data we consider. For the correlation in the last row, x stands for the countries' mean PISA score, fraction of unskilled workers, enrollment rates and output per worker.

Table 2: Parameter Values

Parameter	Value
Discount Factor (β)	0.96
Population Growth Rate (g_L)	0.009
Substitution Elasticity ($1/(1 - \rho)$)	1.50
Capital Share (α)	0.33
Depreciation Rate (δ)	0.04
Share of Unskilled Labor (μ)	0.54
Skill Curvature Parameter (ϕ)	0.30
Gamma Distribution (θ)	17.6
Gamma Distribution (κ)	27.7

Table 3: Empirical Targets: Model and Data

Statistic	Model	Data
Fraction of Unskilled workers	0.69	0.69
Expenditure per tertiary student	0.29	0.29
Mean Math score	487	487
Coeff. Variation Math score	0.19	0.19
<u>Talent Distribution</u>		
Median	481	488
10th percentile	373	368
25th percentile	422	425
75th percentile	546	551
90th percentile	607	609

Table 4: Model Mechanics

	Fraction Unskilled	Quality (h)	Wage Premium (W_s/W_u)	Output
Benchmark	100.0	100.0	100.0	100.0
Reduction in Means (A)	101.3	92.0	108.6	74.5
Increase in Dispersion (B)	102.5	101.8	99.3	101.4
Reduction in TFP (C)	105.2	70.7	141.2	29.1
A-C Together	108.1	66.3	151.2	22.1

Table 5: Differences in PISA Scores

	Poor (Bottom 10%) vs. Rich (Top 10%)	Poor (Bottom 20%) vs. Rich (Top 20%)	Poor (Bottom 25%) vs. Rich (Top 25%)
<u>% Unskilled</u>			
Model	105.5	104.3	104.3
Data	114.1	113.6	111.7
<u>Output per worker</u>			
Model	59.2	67.3	66.4
Data	12.1	17.3	19.7
Augmented Skill (h)	87.6	90.7	90.4
Labor Input	65.4	70.8	70.7
Capital (K)	49.8	61.9	59.3
TFP	100.0	100.0	100.0

Table 6: Differences in Technology and PISA Scores

	Poor (Bottom 10%) vs. Rich (Top 10%)	Poor (Bottom 20%) vs. Rich (Top 20%)	Poor (Bottom 25%) vs. Rich (Top 25%)
<u>% Unskilled</u>			
Model	111.7	109.7	109.2
Data	114.1	113.6	111.7
<u>Output per worker</u>			
Model	12.1	17.3	19.7
Data	12.1	17.3	19.7
Augmented Skill (h)	55.5	61.3	63.6
Labor Input	50.8	57.0	58.2
Capital (K)	10.6	16.1	17.6
TFP	39.6	45.5	49.2

Table 7: Matching the Division of Labor by Skill

	Poor (Bottom 10%) vs. Rich (Top 10%)	Poor (Bottom 20%) vs. Rich (Top 20%)	Poor (Bottom 25%) vs. Rich (Top 25%)
<u>% Unskilled</u>			
Model	114.1	113.6	111.7
Data	114.1	113.6	111.7
<u>Output per worker</u>			
Model	12.1	17.3	19.7
Data	12.1	17.3	19.7
Augmented Skill (h)	56.3	61.9	64.1
Labor Input	46.0	52.3	54.4
Capital (K)	10.6	16.1	17.6
TFP	42.5	48.2	51.4
Tax Wedge (Rich, %)	28.0	24.7	26.1
Tax Wedge (Poor, %)	47.0	44.1	41.3

Table 8: Comparison with One-Sector Growth Model

	Poor (Bottom 10%) vs. Rich (Top 10%)	Poor (Bottom 20%) vs. Rich (Top 20%)	Poor (Bottom 25%) vs. Rich (Top 25%)
<i>Panel A: Growth Model</i>			
Labor Input	85.9	86.9	88.6
Capital (K)	10.6	16.1	17.6
TFP	28.0	34.4	37.2
<i>Panel B: This paper (Matching Y/L only)</i>			
Labor Input	50.8	57.0	58.2
Capital (K)	10.6	16.1	17.6
TFP	39.6	45.5	49.2
<i>Panel C: This paper (Matching Division of Labor)</i>			
Labor Input	46.0	52.3	54.4
Capital (K)	10.6	16.1	17.6
TFP	42.5	48.2	51.4

Table 9: Implications for Skill Premia

	Poor (Bottom 10%) vs. Rich (Top 10%)	Poor (Bottom 20%) vs. Rich (Top 20%)	Poor (Bottom 25%) vs. Rich (Top 25%)
<i>Panel A: Data (Restricted Sample)</i>			
% Unskilled	123.7	115.8	112.1
Skill Premium	244.1	205.5	201.5
<i>Panel B: Matching Y/L only</i>			
% Unskilled	107.1	109.4	108.3
Wage Ratio	161.7	153.6	147.0
Augmented Skill (h)	62.4	65.9	68.9
Skill Premium	111.8	112.1	110.5
<i>Panel C: Matching Division of Labor</i>			
% Unskilled	123.7	115.8	112.1
Wage Ratio	288.2	212.9	190.3
Augmented Skill (h)	62.6	66.2	69.3
Skill Premium	207.0	159.3	146.1

Table 10: Sensitivity

	Poor (Bottom 10%) vs. Rich (Top 10%)	Poor (Bottom 20%) vs. Rich (Top 20%)	Poor (Bottom 25%) vs. Rich (Top 25%)
<i>Panel A: This paper (Matching Y/L only)</i>			
Labor Input	50.8	57.0	58.2
Capital (K)	10.6	16.1	17.6
TFP	39.6	45.5	49.2
<i>Panel B: PISA as Labor Input</i>			
Labor Input	74.2	77.6	77.5
Capital (K)	10.6	16.1	17.6
TFP	30.8	37.1	40.7
<i>Panel C: $\phi = 0$</i>			
Labor Input	70.2	74.5	74.6
Capital (K)	10.6	16.1	17.6
TFP	31.9	38.0	41.7
<i>Panel D: High Substitution Elasticity</i>			
Labor Input	50.0	56.7	58.6
Capital (K)	10.6	16.1	17.6
TFP	40.1	47.8	51.6