Overcoming Zero Lower Bound on Interest Rate

without any Inflation or Inflationary Expectations

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Abstract

If aggregate demand is low, negative interest rate may be required to ensure full employment. There is, however, a zero lower bound (ZLB) on interest rate, given that currency yields zero interest. With a simple model, three policies are considered (policy makers are assumed to have lexicographic preferences). These policies are labelled as (1) implicit tax-subsidy scheme by the central bank, (2) explicit and mimicked tax-subsidy scheme by the treasury, and (3) explicit and non-mimicked tax-subsidy scheme by the treasury. Only the first scheme requires inflation (or inflationary expectations). We find conditions under which the other two schemes work. The third scheme is 'robust'. It involves subsidy on investment and zero tax on savings in 'ZLB years' (and small tax on investment in each of the 'non-ZLB years'). We compare the subsidy rates in the first and the third schemes. The paper ends with a discussion on their other features.

Key words: Zero lower bound on interest rate, inflation, fiscal policy, monetary policy, inter-temporal government budget constraint.

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Contents

'The difficulty lies, not in the new ideas, but in escaping from the old ones ....'
(John Maynard Keynes, 1936)

1 Introduction

Sometimes investment is very low[2] compared to savings. This warrants a low market clearing interest rate - sometimes lower than zero. However, there is a zero lower bound (ZLB) on the market interest rate.[3] This is because an agent can always hold currency which yields zero interest. So there is an arbitrage opportunity when there are negative interest rates. Such an opportunity may be ruled out in competitive markets. This in turn rules out negative interest rates. Hence, we have the ZLB on interest rate. This can be problematic as it can lead to less than full employment. Keynes had advocated the use of fiscal policy to take care of the problem of inadequate aggregate demand as usual monetary policy fails in such a situation.

Post-Keynes, a new monetary policy was suggested in lieu of or in addition to the standard Keynesian fiscal policy to overcome the ZLB problem (see, for example, Summers (1991) and a recent paper like Yehoue (2012)). The key element is to relax the assumption of price level stability or deflation. The idea is that the central bank can plan for and implement some minimum inflation in the economy in normal times so that the nominal interest rate is adequately above the real rate of interest. In this scenario, if there is a fall in private aggregate demand, then there is scope for the nominal interest rate to fall and yet stay positive. However, the real interest rate can become negative when aggregate demand is low. This can help bridge the gap between investment and savings. In this way, policy makers can overcome the problem of ZLB (though this policy may need to be supplemented by the standard Keynesian fiscal policy to take the economy closer to full employment).

A weaker version of the just stated argument relies on credible inflationary expectations if the prevailing inflation rate is zero or low (Krugman, 1998). This too can bring about a gap between nominal and real interest rate. In fact, as papers like Dhami and al-Nowaihi (2011) show, there is really no need for persistent inflation. Instead, there can be inflationary

[2]From 1929 to 1933, the U.S. economy experienced the largest percentage decline in investment spending ever recorded ... - a decline of over 80%.' (Mishkin, 2004, p. 545)

[3]More generally, we may have ZLB or near-ZLB problem. The near-ZLB bound can be positive (as in the case of the Great Depression in 1930s) or it can be negative (as in the case of a recent auction of government securities in Germany in January, 2012). In this paper, we will, for simplicity, consider the ZLB problem and not near-ZLB problem.

expectations at the relevant time. This may require coordination with fiscal policy which can, in this setting, change government expenditure and/or taxes to directly affect aggregate demand in the economy. In this solution, though persistent inflation is avoided, inflationary expectations are not. These need to be credible, and so there is a need for inflation in future. It is interesting that the average rate of inflation over a long period of time is brought down in this case as compared to the case of on-going inflation. However, the inflation rate is still positive in the refined models in order to overcome the problem of ZLB. This paper will attempt to show how the ZLB problem can be overcome without any inflation. Before we do so, let us take a closer look at how inflation helps overcome the ZLB problem.

Inflation acts like a tax on currency holdings and simultaneously provides a 'subsidy' to the issuer of currency i.e. to the central bank. The basic reason for this is that the interest rate on currency is fixed at zero, and is invariant to the inflation rate. This is important. To see this, consider the effect in the case of other debt instruments on which the interest rate is not fixed. Consider the usual debt instruments like bonds or deposits. Assume for the moment that there is no ZLB problem. Then the real interest rate is the same before and after inflation (assuming, for simplicity, that the nominal interest rate adjusts quickly and exactly to accommodate inflation). So there is effectively no tax on interest received by debt holders. Similarly, there is effectively no subsidy on interest paid by debt issuers. In such a case, there is effectively no effect of inflation in this context.

Consider now a case in which there is a floor on nominal interest rate. In this case, the equivalence between before-inflation situation and after-inflation situation breaks down. Without inflation, the interest rate (both nominal and real) has a lower bound. With inflation, the lower bound applies to only the nominal interest rate and not to the real interest rate. Now inflation has a bite. It acts as a tax on savers as the real interest on debt instruments is now negative compared to the case of zero interest rate in the absence of inflation. Similarly, inflation acts as a subsidy for investors compared to the case of zero interest rate. So there is effectively an implicit tax on debt holders and effectively an implicit subsidy for debt issuers when the ZLB problem exists. It is precisely this implicit tax and the implicit subsidy that are at the heart of the solution to the problem of achieving a real negative interest rate.

All this raises an interesting question.

Why does the tax-subsidy scheme have to be implicit and through the central bank rather than explicit and through the treasury (or the Ministry of Finance)?

To answer this question, we consider an explicit tax-subsidy scheme by the treasury. This will attempt to mimic the implicit tax-subsidy scheme with one important difference. While the implicit tax-subsidy scheme involves inflation, the explicit tax-subsidy scheme does not involve any inflation or inflationary expectations. We will see that this explicit tax-subsidy scheme cannot in general ensure full employment if aggregate demand is low. We will refer to this as *explicit and mimicked tax-subsidy scheme by the treasury* and we will refer to the more familiar scheme as *implicit tax-subsidy scheme by the central bank*. The main reason why the latter scheme does and the former does not work lies in implementation difficulties.

Before we explain this, note that holders of debt instruments can be past savers or present savers. On the other hand, the issuers of such instruments finance their real investment. These can be past investments or current investments. In what follows, we will confine ourselves to debt instruments related to current savings and current investment. Furthermore, we will assume that all savings are used to buy debt instruments and that all investment is financed by debt instruments. So we will simply consider tax on return on savings and subsidy on return from investment. We return to implementaion difficulties now.

Implementation of implicit tax-subsidy scheme is familiar. This involves inflation. This is inescapable by economic agents. Therefore, the implicit tax[4] and the implicit subsidy get implemented easily. This is true even if the real interest rate is negative. Let us now consider implementation of explicit and mimicked tax-subsidy scheme.

Consider a hypothetical explicit tax on return on savings by the treasury. In case savings are completely interest inelastic, then of course, an explicit tax on return on savings will not have an effect. Indeed this will be irrelevant.[5] However, if savings are interest elastic, then a tax on return on savings can be implemented only if the net or after-tax interest rate for savers is non-negative. The reason is simple. An economic agent can always hold currency (and pretend that she has consumed rather than saved). There is an arbitrage and tax-evasion opportunity in our scheme since the interest rate on currency is zero. Hence, an explicit after-tax interest rate on savings below zero is very difficult, if not impossible, to implement.

Next consider the implementation of an explicit subsidy on return from investment.

[4]If savings are interest inelastic, then of course the implicit tax on savings is irrelevant.

[5]Even if savings are interest inelastic in practice, it is not clear that savers will not in practice react to a tax on return on savings if we go by the lessons from Behavioural Economics. But we will not and, as we will see later, we need not pursue this.

This is not problematic in so far as there is, unlike in the case of a tax, no incentive to evade a subsidy! So we can have negative after-subsidy interest rate for investment.[6]

Let us now see if the explicit and mimicked tax-subsidy scheme can be implemented. This scheme requires, as we will see formally, a negative after-tax interest rate on savings and a negative after-subsidy interest on investment. While a negative after-subsidy interest rate on investment can be implemented, it is not possible in general to implement a negative after-tax interest rate on savings. Hence, the explicit and mimicked tax-subsidy scheme does not work in general. Given this result, there is indeed a good reason for policy makers to use implicit tax-subsidy scheme to overcome the ZLB problem.

We can now ask another question. While an explicit and mimicked tax-subsidy scheme cannot be used in lieu of an implicit tax-subsidy scheme, can some other explicit tax-subsidy scheme be used to overcome the ZLB difficulty? It is interesting that the answer to this question is yes. There exists, what we call, *explicit and non-mimicked tax-subsidy scheme by the treasury*, which can overcome the ZLB problem without any inflation or inflationary expectation. The new explicit tax-subsidy scheme works as follows. Consider a situation of low investment demand in the economy. The treasury can explicitly subsidize investment and hence ensure an increase in aggregate demand to get closer to the full employment level of output. This can take care of the main problem of ZLB which is to find a way to have an effective negative interest rate for investing firms so that they are induced to invest in a recession. In this new explicit scheme, though there is scope for imposing some tax on return on savings so long as the after-tax interest rate on savings is non-negative, it is, as we will show, optimal to have zero tax on return on savings. So we need not worry about the possible political and psychological discomfort with the idea of a tax on savings.

The explicit and non-mimicked tax-subsidy scheme seemingly has one difficulty. The subsidy on return from investment needs to be financed. However, this is, as we will explain, easy to handle. The treasury can impose a small tax on investment in all years other than the (occasional) year(s) in which the problem of ZLB is faced.[7] There can also be, as we will discuss, scope for international sharing of the 'ZLB risks' across countries.

[6]One way to implement the proposed explicit subsidy on return from investment is as follows. Firms anyway file tax returns. These usual tax returns can include the information on not only income but also on investment. Thus the firms need only pay the net tax (which may be positive or negative). Something like this is done already in the case of *investment tax credits*. We will, for simplicity, assume that the leakages due to exaggerated subsidy claims in administration are zero.

[7]Note that Ricardian Equivalence (RE) is not relevant here. If RE holds, there is, under some conditions, an equivalence between taxes now and taxes in future. Here, we do not consider tax now *or* tax in future. Instead we consider subsidy in 'ZLB year(s)' *and* tax in 'non-ZLB years'.

We will discuss why there is merit in using the explicit and non-mimicked tax-subsidy scheme (fiscal policy) in lieu of the implicit tax-subsidy scheme (monetary policy). This is, in a sense, not surprising considering that tax-subsidy schemes are more naturally in the domain of fiscal policy than in the domain of monetary policy.

The plan of the paper is as follows. We will first present the literature review (section 2). Thereafter, we will present the formal analysis of the ZLB problem, the implicit tax-subsidy scheme, the explicit and mimicked tax-subsidy scheme, and finally, explicit and non-mimicked tax-subsidy scheme (section 3). The latter scheme leads to a fiscal deficit in the 'ZLB year'. We will discuss how this deficit can be met - domestically and internationally (section 4). This will be followed by a discussion on how the explicit and non-mimicked tax-subsidy scheme compares with the implicit tax-subsidy scheme on grounds other than whether or not inflation is used and whether or not there is a deficit in the 'ZLB years' (section 5). We will then conclude (section 6).

2 Literature review

Correia, et al. (2011) have a model with a similar theme as this paper.[8] Other than this paper, the theme in this paper is, as far as this author is aware, missing in the literature.

Though there is a large literature on, what we have called, the implicit tax-subsidy scheme to take care of the ZLB problem, there is hardly any work (except just the above mentioned paper) on the explicit tax-subsidy solution to the problem. This is also a reflection of what we have in practice. There is, in practice, usually hardly any explicit subsidy on interest paid by investing firms or households (hereafter, investors), though there is often a subsidy on return from investment in some other form such as investment tax credit or allowance for bonus depreciation. This is discussed in House and Shapiro (2006), or in an early paper like Sen and Turnovsky (1990). This literature is, however, more in the context of the economics of growth or in the context of the pure theory of investment. It is not in the context of economics of business cycle or more specifically in the context of the ZLB on nominal interest rate, which is the issue under consideration here. There is, as far as we are aware, one exception viz., Svensson (2003). This paper does mention investment tax credit in the context of the ZLB problem. However, even in this paper, the focus is on inflation or inflationary expectations (or depreciation of currency) as a way to overcome the ZLB problem. There is, in Svensson (2003), only a minor qualification to 'the inflation solution'

[8] I thank Robert Hall for drawing my attention to this paper on August 30, 2012.

by a mention of temporary investment tax credits. In our paper, an explicit subsidy that reduces the interest rate for investors in times of ZLB is at centre-stage, and there is zero inflation in explicit tax-subsidy schemes in our model.

Feldstein (2002) writes, 'Because tax rules do not distinguish between nominal and real interest rates, *a fall in inflation* with a constant real interest rate causes the real net-of-tax interest rate to rise. ... One way to offset this and maintain the same incentive to invest is to modify the depreciation rules or the investment tax credit.' (p. 7, emphasis ours) Though Feldstein (2002) does consider the effect of fiscal policy on the real net-of-tax interest rate, observe that this argument is in the context of taxation rules that are based on nominal values and not on real values. The point is that taxation rules can have (additional) effects in the presence of inflation (and in the absence of indexation). In this context, 'a fall in inflation' can have a positive effect when there is a ZLB problem. So it may seem that Feldstein does consider a policy similar to the explicit and non-mimicked tax-subsidy scheme proposed here in the context of the ZLB problem. However, this is not true. In this paper, we study the role of a subsidy paid by the treasury to the investors in times of ZLB in the absence of any inflation at all. Accordingly, there is no scope in our model for any effect of taxation rules due to the fact that nominal values differ from real values. The role of taxes and subsidies in our model is to act in lieu of inflation.

Much of the literature emphasises the role of inflation or inflationary expectations in overcoming the ZLB problem. However, some authors have also advocated depreciation of the currency in this context (McCallum, 2000). Klose (2011) suggests a modified Taylor rule to accommodate the issue of ZLB on nominal interest rate. These suggestions are all in the domain of monetary policy. This paper, on the other hand, considers the role of (non-Keynesian) fiscal policy in overcoming the ZLB problem.

This paper carries the argument in Singh (2012, Chapter 11) further. The latter considers, what it calls, *a narrow monetary policy and an extended fiscal policy*. It shows that some functions that are usually associated with monetary policy can instead be looked after by fiscal policy. This paper now carries the argument in Singh (2012) further. Fiscal policy can take care of the ZLB problem. So far a solution to this policy has been associated primarily with the monetary policy. So there can be, as this paper shows, a narrow monetary policy and an extended fiscal policy even in the context of the ZLB problem. This was not considered in Singh (2012).

There has been a somewhat widely held view for quite some time until recently that the optimal rate of inflation is 2% or so. Schmitt-Grohe and Uribe (2011) have recently

argued that the optimal rate of inflation is much less. They arrive at this conclusion by considering various costs and benefits of inflation. The latter include the beneficial role of inflation in helping policy makers overcome the problem of ZLB. The authors attach a low probability to an event such as the occurrence of the ZLB problem.[9] It is not surprising then that they arrive at a low figure for the optimal rate of inflation. We consider zero rate of inflation in the context of the explicit tax-subsidy schemes in this paper. The reason is that the scope of our paper is limited. We are not considering all the benefits of inflation. In fact, we consider one benefit only viz., inflation helps overcome the ZLB problem. In this context, the optimal rate of inflation is zero, given that the treasury can take care of the ZLB problem without any inflation. So this paper does not rely on a small probability of ZLB to conclude that the optimal inflation rate is zero. Instead, this paper relies on using an explicit and non-mimicked tax-subsidy scheme to overcome the ZLB problem.

There can be strategic considerations in policy implementation and in allocation of responsibility between the treasury and the central bank in the context of the familiar solutions to the ZLB problem. For a recent paper on these aspects, see Dhami and al-Nowaihi (2011). In this paper, we will abstract from these issues. The reason is that in our paper we are exploring the possibility that the treasury alone can take care of the ZLB problem through an explicit and non-mimicked tax-subsidy scheme.

At the heart of the ZLB problem is the difficulty in realizing a negative interest rate. In this context, Goodfriend (2000) suggests a fiscal policy in the form of carry tax on money (a stock) so that the effective interest rate on money is negative.[10] In our model, we consider possible taxes on savings (a flow) and/or subsidies on investment (a flow) with the aim to have effective negative interest rates.

This completes the literature review though we will cite some other writings as and when they are relevant to the argument being made. In the next section, we will carry out a formal analysis of the ZLB problem and its solutions.

[9] Consider an example. The US economy has been hit by the ZLB or near-ZLB problem in the early 1930s and then in and around the year 2008. So we may say that the US economy has been hit by a ZLB problem once in about seventy five years.

[10] Goodfriend also suggests open market operations in long bonds, and monetary transfers.

3 Model

Though we will use the standard macroeconomic model which may now be termed Old Keynesian Model, we will incorporate the interest rate rigidity, and do so in novel ways.[11]

This section is divided into four subsections. First, we will explain briefly the ZLB problem in the absence of any policy intervention (subsection 3.1). Second, we will formally show that the implicit tax-subsidy scheme by the central bank can overcome the ZLB problem (subsection 3.2). Third, we will show that the explicit and mimicked tax-subsidy scheme cannot help overcome the ZLB problem (subsection 3.3). Finally, we will show that the explicit and non-mimicked tax-subsidy scheme can overcome the ZLB problem (subsection 3.4).

3.1 The zero lower bound (ZLB) problem

In this subsection, we assume that the inflation rate is zero. Accordingly, the nominal rate of interest is the same as the real rate of interest in this subsection. So there is no implicit tax-subsidy scheme in place. Furthermore, there is no explicit tax-subsidy scheme so that the gross interest rate is the same as the net interest rate, which is in turn the same for savers and for investors. So there is one interest rate in the economy. Let us denote this by r. Accordingly, the non-negativity constraint on interest rate in the economy is

$$r \geq 0. \tag{1}$$

The reasons for this have been given already in section 1. We will not include these in the formal model which will focus on implications of interest rate rigidity and on policy solutions to the problem. To retain this focus, we assume that wages and prices are flexible.[12] The

[11] Many new models include some variant of Phillips curve. However, the empirical evidence for Phillips curve is not strong. Uhlig (2012) writes,

> '... there is in fact no Phillips curve. It is hard to imagine that we would arrive at any other conclusion, if someone were to present figure 1 [showing US data on inflation and unemployment for 1948-2008] afresh for the first time today.' (p. 35, figure 1 not reproduced here)

[12] We would like to make two observations here. First, wage-price flexibility is sometimes viewed as a 'classical assumption' (or non-Keynesian assumption). This is not entirely correct. '... in 'Book V: Money-Wages and Prices', Keynes drops the assumption of a constant money-wage rate' (Patinkin, 2008, p. 697).

Second, in our model, *Pigou Effect* is absent. So wage-price flexibility may or may not ensure full employment. In a more general model that allows for Pigou Effect, complete wage-price flexibility can rule out unemployment. However, this may not be realistic. In our model, we have simplified by abstracting from Pigou Effect and allowing for complete wage-price flexibility.

rest of the model is as follows:

$$S(Y,r) = \overline{I} + I(r), \tag{2}$$

$$\overline{M} = P.L(Y,r), \tag{3}$$

$$Y = f(N), \tag{4}$$

$$\frac{w}{P} = f'(N), \tag{5}$$

and

$$N = \phi\left(\frac{w}{P}\right), \tag{6}$$

where S, Y, I, P, N, w denote savings, output, investment, price level, employment and nominal wage respectively. Further $\overline{M}$ and $\overline{I}$ denote exogenous money and exogenous investment respectively.

This is a one period model. $\overline{I}$ can take one of the two values $\overline{I}_1$ and $\overline{I}_2$, where

$$\overline{I}_1 > \overline{I}_2.$$

We will consider each case separately. The latter case is the problematic case of low aggregate demand which can lead to less than full employment level of output, as we will see.

We abstract from cost of intermediation. So the interest rate for savers and for investors is the same unless there are differential tax/subsidies (we will come to these later).

Let us describe the model. Equation (2) is the condition that savings are equal to investment in equilibrium. Equation (3) is the equilibrium condition that the supply of money is equal to the demand for the same. Equation (4) is simply the production function that relates output to employment. Equation (5) says that in equilibrium real wage is equal to the marginal product of labour. Finally, equation (6) is the equilibrium condition that workers are on their supply curve (supply of labour depends on real wage). As is clear from the structure, we have model of a closed economy.

Following features are implicit in the above model. Savings depend on income and interest rate. Investment consists of two parts - exogenous investment and endogenous investment. Exogenous investment incorporates the role of factors like animal spirits (Keynes, 1936), Tobin's q ratio, and the cost of financial intermediation (Bernanke, 1983). The endogenous investment depends on the interest rate. Demand for money increases proportionately with the price level. There is no money illusion. $L(Y,r)$ denotes the demand for real balances which are a function of income and interest rate. We assume that there is no government spending or taxes (we will come to these later). The central bank does not

intervene in this model if there is less than full employment (it only issues money which is of a fixed amount). We use the following restrictions on the functions used above:

$$S_1 > 0,\ S_2 \geq 0,\ I' < 0,\ L_1 > 0,\ L_2 < 0,\ f' > 0,\ f'' < 0 \text{ and } \phi' > 0. \tag{7}$$

These are fairly standard in the literature. Observe that we have allowed for $S_2 = 0$. This is because there is some evidence that savings do not respond to interest rate changes. We will see the significance of this later (see Proposition 1 in subsection (3.3)).

We have five equations (2) - (6) in five variables Y, r, P, N and w. Observe that equations (4) - (6) have three variables Y, N and w/P. Let the solution be

$$Y = Y^*,\ N = N^*, \text{ and } \left(\frac{w}{P}\right) = \left(\frac{w}{P}\right)^*. \tag{8}$$

Indeed, Y^* is the full employment level of output (Y^f). But these three equations constitute partial equilibrium. This may or may not hold in general equilibrium depending on whether or not conditions (1) -(3) hold for $Y = Y^f$. If all conditions are satisfied, then indeed we have $Y = Y^* = Y^f$ in general equilibrium. In general, we have

$$Y \leq Y^f.$$

Let us denote the solution to the remaining variables as (r^*, P^*, w^*). Assume that all variables other than interest rate are positive. We will make the following assumption on the interest rate, which will play an important role in the model.

Assumption 1. $r^* = \begin{cases} \overline{r} > 0, & \text{if } \overline{I} = \overline{I}_1, \\ \underline{r} < 0, & \text{if } \overline{I} = \overline{I}_2. \end{cases}$

If investment is large in the economy i.e. $\overline{I} = \overline{I}_1$, then $r^* = \overline{r} > 0$, and the mathematical solution (Y^*, r^*, P^*, N^*, w^*) is economically feasible. Furthermore, in this case, $Y^* = Y^f$. On the other hand, if investment is small in the economy i.e. $\overline{I} = \overline{I}_2$, then $r^* = \underline{r} < 0$, and the mathematical solution (Y^*, r^*, P^*, N^*, w^*) is not economically feasible. In this case, $Y < Y^f$. We will next determine the exact Y in this case.

Given that the savings and investment functions are well-behaved except for the fact that the market clearing rate of interest is negative when $\overline{I} = \overline{I}_2$, it follows that $S(Y^f, r) > \overline{I} + I(r)$ for all non-negative interest rates. The equilibrium then is given as follows. The

rate of interest is zero and output falls such that savings at the new (reduced) level of output equal the amount of investment. Let us denote this solution as ($\hat{Y}$, $\hat{r}$, $\hat{N}$,$\hat{P}$,$\hat{w}$).

The interest rate is flexible so long as $r > 0$. At $r = 0$, there is a downward rigidity. When investment is low, a negative interest rate is warranted. This is, however, not possible. Instead, we have $\hat{r} = 0$. Formally, the remaining four values $\hat{Y}$, $\hat{N}$,$\hat{P}$ and $\hat{w}$ are determined by considering equations (2) - (5). We have now four equations in four variables. This gives the solution to the four variables. Of course, this solution will not satisfy the remaining equation i.e. equation (6). In the new 'equilibrium', we have

$$\hat{N} < \phi\left(\frac{\hat{w}}{\hat{P}}\right).$$

The gap between the left hand side and the right hand side is the (involuntary) unemployment given that the real wage is $\left(\frac{\hat{w}}{\hat{P}}\right)$. There is some debate in the literature on whether or not to treat this as a state of equilibrium (in the short run and in the long run). But we will not get into this issue here. Following Keynes (1936), we will call this a state of equilibrium here for our purposes.

Considering both cases $\overline{I} = \overline{I}_1$ and $\overline{I} = \overline{I}_2$, the bottom line is that in the absence of policy intervention, in equilibrium we have

$$r = \begin{cases} r^* = \overline{r} > 0, & \text{if } \overline{I} = \overline{I}_1, \\ \hat{r} = 0, & \text{if } \overline{I} = \overline{I}_2. \end{cases}$$

The crucial part of the model is that it is not possible to realize $r = \underline{r} < 0$ in equilibrium when $\overline{I} = \overline{I}_2$. This in turn leads to

$$Y = \begin{cases} Y^* = Y^f, & \text{if } \overline{I} = \overline{I}_1, \\ \hat{Y} < Y^f, & \text{if } \overline{I} = \overline{I}_2. \end{cases}$$

We may say that the above analysis is consistent with what Keynes himself believed to be the essence. As Keynes (1937) observed a year after the publication of his *General Theory*,

> 'The novelty in my treatment of saving and investment consists, not in my maintaining their necessary aggregate equality, but in the proposition that it is, not the rate of interest, but the level of incomes which (in conjunction with certain other factors) ensures this equality.' (p. 249)

Summing up, less than full employment is possible if investment demand is low, there is a ZLB on interest rate, and there is no policy intervention.[13] In this subsection, we assumed that there is no implicit or explicit tax-subsidy scheme in place to ensure that investment is equal to savings at the full employment level of output. In the next subsection, we will allow for an implicit tax-subsidy scheme through inflation. Thereafter, we will allow for an explicit tax-subsidy scheme.

3.2 An implicit tax-subsidy scheme by the central bank

In the previous subsection, there is no policy intervention to ensure full employment. In this subsection, we will consider central bank intervention to use inflation to take care of the ZLB problem. As we have seen in the previous subsection, policy intervention is required in periods when exogenous investment is low i.e. $\overline{I} = \overline{I}_2$. However, it is assumed here that it is not possible to engineer inflation during only those periods and then revert back to price stability in other periods. Accordingly, the policy makers choose to maintain inflation in all periods. It is assumed that the inflation rate is constant and that actual inflation rate (π) is equal to the expected rate of inflation (π^e).

The model in this subsection is a multi-period model (unlike the one-period model in the previous subsection). So subscript t for date has been attached to the variables as we will see in the equations later. The basic model structure in this subsection is, however, the same as that in the previous subsection. In some periods, we have $\overline{I} = \overline{I}_1$. In other periods, we have $\overline{I} = \overline{I}_2$. The realization of $\overline{I}$ is uncertain in any period.

We assume that the preferences of policy makers are lexicographic. The first preference is for large output (Y) and the next preference is for large real money balances (M/P). The policy instrument for public authorities is π. The lower this rate, the lower is the nominal interest rate and the higher are the real money balances (see equation (16) below). Policy makers choose the least inflation rate that is compatible with $Y = Y^f$.

The role of monetary policy here is to maintain inflation to create a wedge between the nominal rate of interest (i^N) and the real rate of interest ($i^N - \pi$). The gap between the two interest rates π can be viewed as an implicit tax on return on savings and an implicit subsidy on return from investment. If investment demand is low, then the central bank can induce an increase in demand in two ways by

1. encouraging consumption through an implicit tax on return on savings, and

[13] The central bank only issues money. The quantity of money is exogenously given. Beyond this issue of money, the public authorities do not intervene in the economy in this subsection.

2. encouraging investment by paying an implicit subsidy on return from investment.

This policy leads to an increase in aggregate demand which in turn leads to an increase in output to the full employment level.

In the previous subsection, there was only one interest rate and this had to be non-negative. In this subsection, we have two interest rates - the nominal rate of interest and the real interest rate. The non-negativity constraint applies to only the nominal interest rate. The real interest rate can be negative. So the ZLB constraint takes the following form:

$$i^N \geq 0. \tag{9}$$

Given that both savers and investors are rational, it follows that savings and investment depend on the real rate of interest rather than on the nominal rate of interest. Accordingly, the equilibrium condition that savings are equal to investment takes the following form:

$$S(Y_t, i_t^N - \pi) = \overline{I} + I(i_t^N - \pi), \tag{10}$$

where t is, as mentioned already, the subscript for time period. The demand for real money balances depend on the nominal interest rate as this is the opportunity cost of holding real money balances. Hence, we have

$$M_t = P_t.L(Y_t, i_t^N). \tag{11}$$

The rest of the model resembles the model in the previous subsection except that subscript t has been attached to variables.[14]

$$Y_t = f(N_t) \tag{12}$$

$$\frac{w_t}{P_t} = f'(N_t) \tag{13}$$

$$N_t = \phi\left(\frac{w_t}{P_t}\right) \tag{14}$$

Let g_M denote the rate of growth of money. We assume that

$$\pi = \pi^e = g_M.$$

[14]We have implicitly assumed that the capital stock in the economy is constant which is why the production function is the same in all periods. This may be justified as follows. Assume that investment is made in long term projects. So the production function changes and output rises for a given amount of labour after a long time T. We will confine our analysis here to $t < T$.

Given the structure of the model, it follows that

$$X_t = X_0 e^{g_M t}, \text{ where } X \epsilon \{M, P, w\}.$$

Let $M_0 = \overline{M}$, which is exogenous. In contrast, P_0 and W_0 are endogenous. It is easy to check now that the subscript t can be removed. This step makes the model in this subsection comparable to the model in the previous subsection. The model (equations (10) - (14)) reduces to a system that includes

$$S(Y, i^N - \pi) = \overline{I} + I(i^N - \pi) \tag{15}$$

$$\overline{M} = P.L(Y, i^N) \tag{16}$$

and equations (4), (5) and (6) in the previous subsection. The last three equations have three variables Y, N and $\left(\frac{w}{P}\right)$. The solution is $Y = Y^*$, $N = N^*$ and $\left(\frac{w}{P}\right) = \left(\frac{w}{P}\right)^*$ (see (8)).

Taking $Y = Y^*$ in equation (15), we can solve for $i^N - \pi$. Note that equation (15) is the same as equation (2) in the previous subsection except that $i^N - \pi$ has replaced r. Hence, it follows that

$$i^N - \pi = r^* = \begin{cases} \overline{r} > 0, & \text{if } \overline{I} = \overline{I}_1, \\ \underline{r} < 0, & \text{if } \overline{I} = \overline{I}_2. \end{cases} \tag{17}$$

See Assumption 1. Further note that we have two values of i^N. Let us denote these as $\bar{i}^N$ and $\underline{i}^N$, where $\bar{i}^N > \underline{i}^N$.

Given lexicographic social preferences of the public authorities, we target the least value for $\underline{i}^N$ i.e. 0 and so policy makers choose

$$\pi = -\underline{r} \ (> 0). \tag{18}$$

Now this gives $\bar{i}^N = \overline{r} + \pi = \overline{r} - \underline{r} > 0$. It follows now that

$$i^N \begin{cases} > r^* > 0, & \text{if } \overline{I} = \overline{I}_1, \\ = 0 > r^*, & \text{if } \overline{I} = \overline{I}_2. \end{cases}$$

Note that this does not violate the ZLB condition (9) regardless of the value of $\overline{I}$. So in this case, $Y = Y^f$ even if $\overline{I} = \overline{I}_2$. The crucial assumption is that we have some minimum inflation.

We are now left with the determination of two (nominal) variables P and w. P can be determined from 'money equation' (16). This has three variables Y, P and i^N. Note

that Y is the same whether $\overline{I} = \overline{I}_1$ or $\overline{I} = \overline{I}_2$ (since it is now possible to realize $\underline{r} < 0$ in equilibrium in the latter case). However, the interest rate can take two values. Accordingly we have two values of P. Let it be P_1 if $i^N = \bar{i}^N > 0$ and let it be P_2 if $i^N = \underline{i}^N = 0$. It is easy to check that

$$P = \begin{cases} P_1 > P^*, & \text{if } \overline{I} = \overline{I}_1, \\ P_2 < \hat{P}, & \text{if } \overline{I} = \overline{I}_2, \end{cases} \tag{19}$$

where P^* and $\hat{P}$ are the equilibrium values of P when $\overline{I} = \overline{I}_1$ and $\overline{I} = \overline{I}_2$ respectively in the previous subsection. Finally, it is easy now to determine w. Clearly, there will be two values of w. Let us for consistency denote these as w_1 and w_2. We have

$$w_j = P_j \left(\frac{w}{P}\right)^*, \text{ where } j = 1, 2.$$

In this subsection, we have seen how an 'inflation solution' can be used to get around the problem of ZLB and ensure full employment.[15] In the next two subsections, we will explore if there are 'non-inflation solutions' to take care of the problem of ZLB.

3.3 An explicit and mimicked tax-subsidy scheme by the treasury

In this subsection, we assume that the inflation rate is zero. So there is no implicit tax-subsidy scheme *by the central bank*. We will instead consider an explicit tax-subsidy policy *by the treasury*, which has the objective to mimic the implicit tax-subsidy scheme by the central bank (see the previous subsection). We will see that this *cannot* ensure full employment.

In the previous subsection, the implicit tax rate on savings was equal to the implicit subsidy rate on investment. Accordingly, to mimic this scheme, we will take the explicit tax rate on savings to be equal to the explicit subsidy rate on investment. Let this common rate be β (this common rate under the implicit tax-subsidy scheme was the inflation rate π in the previous subsection). The net interest rate is

$$i_G - \beta,$$

where i_G is the gross interest rate. We will later see the comparison between i_G and i^N, and that between β and π.

[15] The role of the central bank in the above model is to maintain some minimum inflation rate in the economy both in normal times and in times of the ZLB problem. In the above model, the central bank does not need to do anything extra in times of recession in the context of the model above.

There is an important difference between the implicit and the explicit tax-subsidy schemes. In the former case, the public authorities need to maintain inflation in all periods. As argued already, it is not possible to engineer inflation for only one period and revert back to price stability. In this subsection, we assume that the explicit tax-subsidy can be implemented quickly in the periods in which investment is low. So this scheme is temporary. Assume that the policy makers can observe the realization of $\overline{I}$ at the beginning of each period. They can accordingly decide the course of action immediately. If $\overline{I} = \overline{I}_1$, there is full employment anyway in the absence of any policy intervention. So they need not take any action. On the other hand, if $\overline{I} = \overline{I}_2$, they implement the explicit tax-subsidy scheme. It is assumed that there exists a constitutional mandate for this purpose so that there are no delays in Parliamentary approval for implementation. In what follows, we will discuss only the case in which investment demand is low i.e. $\overline{I} = \overline{I}_2$.

The explicit subsidy burden is exactly matched by the explicit tax collected, given that the tax rate is equal to the subsidy rate and that savings are equal to investment (we are considering the equilibrium). The government has no other expenditure or taxes.

The explicit tax-subsidy scheme here is a case of fiscal policy. However, observe that this is very different from the standard Keynesian fiscal policy.[16]

The net interest rate on savings is irrelevant for savers if savings are inelastic ($S_2 = 0$ (see condition (7)). If savings do respond to changes in interest rate (i.e. if $S_2 > 0$), then the net interest rate does become relevant for savings. For reasons given already in section 1, the net interest rate on savings cannot be negative. Accordingly, the non-negativity constraint or the ZLB constraint takes the form

$$i_G - \beta \geq 0, \text{ given that } S_2 > 0. \tag{20}$$

[16]In our model, the role of fiscal policy is to change the effective interest rate in the economy so that aggregate demand is induced to go up. In the standard Keynesian model, the role of fiscal policy is to directly increase aggregate demand in the economy. For lack of better terminology, we may say that ours is a *pricing policy* whereas the standard Keynesian fiscal policy is an *incomes policy*.

The standard Keynesian fiscal policy typically uses some variant of the following formulation:

$$\overline{I} + I(r) + \mathcal{G} = S((1-t)Y, r),$$

where $\mathcal{G}$ is government expenditure, r is the interest rate (both nominal and real, and both gross and net), and t is the tax rate applied to income.

The explicit tax-subsidy scheme may be viewed as an alternative to the implicit tax-subsidy scheme. It may be viewed as complimentary to the standard Keynesian fiscal policy. This is a fortiori true in the light of new work on the standard Keynesian fiscal policy, which shows that it can be quite effective when an economy faces a ZLB problem (Christiano, et al., 2011).

Assume that agents are rational. Accordingly, savings and investment are both functions of the after-tax interest rate and the after-subsidy interest rate respectively. Accordingly, the savings-investment equality takes the form

$$S(Y, i_G - \beta) = \overline{I} + I(i_G - \beta). \tag{21}$$

Next consider the 'money equation'. Demand for real money balances depends on the opportunity cost of holding money. Since there is no tax or subsidy on holding money, the opportunity cost of holding money is the gross interest rate i_G. Accordingly, we have

$$\overline{M} = P.L(Y, i_G). \tag{22}$$

The rest of the model consists of equations (4), (5) and (6) (as in the previous two subsections).

Is it possible to attain full employment under the explicit and mimicked tax-subsidy scheme? The next proposition answers this question.

Proposition 1. *Given $\overline{I} = \overline{I}_2$, Assumption 1, and 'the explicit and mimicked tax-subsidy scheme', we have in equilibrium (a) $Y < Y^f$ if $S_2 > 0$, and (b) $Y = Y^f$ if $S_2 = 0$.*

Proof: (a) We will compare the solutions in two models: (1) The model of implicit tax-subsidy scheme, and (2) the model of explicit and mimicked tax-subsidy scheme.

- Model 1: Equations (4), (5), (6), (15) and (16), and the ZLB constraint (9).
- Model 2: Equations (4), (5), (6), (21) and (22), and the ZLB constraint (20).

We know that $Y = Y^f$ in Model 1 (see subsection 3.2). Suppose that $Y = Y^f$ in Model 2 as well. We will prove by contradiction that this is not true.

Consider the five equations in each of the two models. Equations (4), (5) and (6) are common to the two models. Further observe that the set of equations (15) and (16) are mathematically the same as the set of equations (21) and (22) except that i_G has replaced i^N and that $i_G - \beta$ has replaced $i^N - \pi$. Given that the solution is unique, it follows that the solution of the two sets of five equations is the same. Hence,

$$i_G = i^N,$$

and

$$\begin{aligned} i_G - \beta &= i^N - \pi \\ &< 0, \end{aligned}$$

where the inequality follows from equation (17) (which gives the solution to interest rate in Model 1, given that $\overline{I} = \overline{I}_2$). We have arrived at $i_G - \beta < 0$. But this contradicts the ZLB constraint (20) which is relevant given that $S_2 > 0$ (even while $i^N - \pi < 0$ does not violate the ZLB constraint (9)). Hence, the supposition $Y = Y^f$ does not hold. It follows that $Y < Y^f$, given that $Y \leq Y^f$.

(b) If $S_2 = 0$, then the ZLB constraint (20) is not relevant. Hence, the contradiction seen in part (a) of the proof does not arise. It is easy to see that $Y = Y^f$ in this case. ||

The thrust of Proposition 1 is that the explicit and mimicked tax-subsidy scheme by the treasury can ensure full employment when investment is low, only if savings are interest inelastic. If savings are interest elastic, then the explicit and mimicked tax-subsidy scheme fails to ensure full employment if investment demand is low. It is interesting that not only does it fail to ensure full employment in the ZLB period, it can also lead to a fiscal burden on the government if we allow for administrative costs incurred in implementing the explicit scheme. These are reasonably absent (or negligible) under the implicit tax-subsidy scheme in the previous subsection. So there is indeed a good reason why the implicit tax-subsidy scheme should be used instead of the explicit and mimicked tax-subsidy scheme. This is indeed what we observe in practice.

We have seen in this subsection that the explicit and mimicked tax-subsidy scheme fails to ensure full employment only if $S_2 > 0$. In the next subsection, we will check if an explicit and non-mimicked tax-subsidy scheme by the treasury can help achieve full employment when investment is low and $S_2 > 0$.

3.4 An explicit and non-mimicked tax-subsidy scheme by the treasury

We will now make a departure from the analysis in the previous subsection. Instead of considering the special case in which the explicit tax rate on return on savings is equal to the explicit subsidy rate on return on investment, we will consider the more general case i.e. we will allow the two rates to be different from each other. Let τ and z denote the explicit tax rate on return on savings and the explicit subsidy rate on return on investment respectively. We have

$$\tau, z \geq 0.$$

(In the previous subsection, we had the special case $\tau = z(= \beta)$.) Observe that there is a possible deficit (surplus) if the subsidy is greater (less) than the tax rate. There was no

scope for such a deficit or surplus due to a difference in tax rate on return on savings and subsidy rate on return on investment in the previous subsections.

As mentioned earlier, we will henceforth deal with the case $S_2 > 0$.

Policy makers have two policy instruments: τ and z. We will again assume that the preferences of policy makers are lexicographic (as we did earlier). The first preference is for large output (Y). The second preference is for large real money balances (M/P). The third preference is for low deficit in the government budget (this is new as there was no need for this in this section so far).

As in the previous subsection and for reasons considered in the previous subsection, there is no policy intervention if investment demand is large i.e. if $\overline{I} = \overline{I}_1$. The treasury intervenes if $\overline{I} = \overline{I}_2$.

For reasons given already in section 1, the net interest rate on investment ($i_G - z$) can be positive, zero or negative. In contrast, the net interest rate on savings cannot be negative. Accordingly, the non-negativity constraint or the ZLB constraint now is that

$$i_G - \tau \geq 0, \text{ given that } S_2 > 0. \tag{23}$$

The rest of the model is now as follows. Since savings are a function of the net interest rate for savers i.e. $i_G - \tau$, and since investment is a function of the net interest rate paid by investors i.e. $i_G - z$, we have the following condition that savings are equal to investment in equilibrium:

$$S(Y, i_G - \tau) = \overline{I} + I(i_G - z). \tag{24}$$

There is no tax or subsidy on money holdings. So neither τ nor z is involved in this context. So the 'money equation' is the same as that in the previous subsection i.e. equation (22). The other three equations are equations (4), (5) and (6).

Is full employment possible now if $\overline{I} = \overline{I}_2$? Is the optimal τ more than or less than the optimal z? (We have already seen in the previous subsection that $\tau = z(= \beta)$ cannot ensure full employment.) Accordingly, does the treasury need to incur a deficit in this case? Before we answer these questions, let us state a definition. This is motivated by the consideration of finding the optimal value of z for policy makers.

Definition 1. *B is implicitly defined as the solution to the equation:*

$$S(Y^f, 0) = \overline{I}_2 + I(-B).$$

Given the model (equations (4), (5), (6), (22) and (24)), the ZLB constraint (23) and Definition 1), we can now state our next result as follows.

Proposition 2. *$\overline{I} = \overline{I}_2$, Assumption 1 holds, $S_2 > 0$, and 'explicit and non-mimicked tax-subsidy scheme' is used.*
(a) Given that $Y = Y^f$, it is optimal policy that $\tau = 0$ and $z = B$.
(b) Given the optimal policy, $Y = Y^* = Y^f$, $i_G = 0$, $N = N^*$, $P = P_2$ and $w = P_2\left(\frac{w}{P}\right)^*$.

Proof: (a) There are three interest rates in the model viz., i_G, $i_G - \tau$ and $i_G - z$. Policy makers have two policy instruments viz., τ and z. Once the values of τ and z are given, i_G is determined endogenously. However, the policy makers can equivalently in a mathematical sense choose τ and i_G, and then it is as if z is determined endogenously. We will follow this (technically more convenient) approach.

Given $Y = Y^f$ and the lexicographic social preferences of the policy makers, the optimal policy is to choose the least value of i_G in equation (22) to maximize real cash balances of agents in the economy. It follows from condition (23) and $\tau \geq 0$ that $i_G \geq 0$. It follows now that the optimal policy is $i_G = 0$. It further follows from $i_G = 0$, the ZLB constraint (23) and $\tau \geq 0$ that optimal policy is $\tau = 0$.

Taking $i_G = 0$ and $\tau = 0$ in equation (24), we get $S(Y^f, 0) = \overline{I}_2 + I(-z)$ after substituting $Y = Y^f$ and $\overline{I} = \overline{I}_2$. It follows from Definition 1 that $z = B$.

(b) We need to show that under the policy set $\tau = 0$ and $z = B$, we have $Y = Y^f$, $i_G = 0$, $N = N^*$, $P = P_2$ and $w = P_2\left(\frac{w}{P}\right)^*$. We may equivalently show that under 'the policy set' $\tau = 0$ and $i_G = 0$, we have $Y = Y^f$, $z = B$, $N = N^*$, $P = P_2$ and $w = P_2\left(\frac{w}{P}\right)^*$. We will follow this latter approach.

The model includes equations (4), (5), (6), (22) and (24). From the first three equations, we know that $Y = Y^*$, $N = N^*$ and $\left(\frac{w}{P}\right) = \left(\frac{w}{P}\right)^*$ (see (8)). Taking $\tau = 0$ and $i_G = 0$ in the last two equations, we get

$$\overline{M} = P.L(Y, 0) \tag{25}$$

and

$$S(Y, 0) = \overline{I}_2 + I(-z) \tag{26}$$

after substituting $\overline{I} = \overline{I}_2$. We know that $P = P_2$ satisfies equation (25) given that $Y = Y^* = Y^f$ (see equation (19)). It is obvious now from $P = P_2$ and $\left(\frac{w}{P}\right) = \left(\frac{w}{P}\right)^*$ that $w = P_2\left(\frac{w}{P}\right)^*$. Finally, it follows from equation (26) and Definition 1 that $z = B$ after using $Y = Y^f$. ||

The thrust of Proposition 2 is that the explicit and non-mimicked tax-subsidy scheme by the treasury can ensure full employment if investment demand is low, and it can do so with out any inflation or inflationary expectations.

So far, we have considered each scheme somewhat in isolation. Let us now carry out a comparison. In this section, we have seen that if investment demand is low, full employment can be attained under two schemes: the implicit tax-subsidy scheme (subsection 3.2) and the explicit and non-mimicked tax-subsidy scheme (this subsection). In subsection 3.2, we had seen that it is optimal policy that the implicit subsidy rate on return from investment is $\pi = -\underline{r}$ (see equation (18)). In this subsection, we have seen that the optimal policy is that explicit subsidy rate for investment is $z = B$ (see Proposition 2). Our next result compares the two subsidy rates.

Proposition 3. *Given $\overline{I} = \overline{I}_2$, Assumption 1 and $S_2 > 0$, the optimal subsidy rate under 'the explicit and non-mimicked tax-subsidy scheme' is greater than the optimal subsidy rate under 'the implicit tax-subsidy scheme' i.e. $B > (-\underline{r})$.*

Proof: We have $Y = Y^f$ under both schemes (Prior result shown in subsection 3.2, and Proposition 2). Further, under the implicit tax-subsidy scheme, we have $\pi = -\underline{r}$, and $i^N = \underline{i}^N = 0$ given that $\overline{I} = \overline{I}_2$. In what follows, we will use these.

We need to show that $B > (-\underline{r})$. We will prove by contradiction. Suppose not. Then $B \leq (-\underline{r})$.

Given that $\underline{r} < 0$ (Assumption 1) and $S_2 > 0$, we have

$$\begin{aligned} S(Y^f, \underline{r}) &< S(Y^f, 0) \\ &= \overline{I}_2 + I(-B) \\ &\leq \overline{I}_2 + I(\underline{r}) \\ &= S(Y^f, \underline{r}) \end{aligned}$$

where the first equality follows from Definition 1, the weak inequality follows from $B \leq -\underline{r}$ (which we have now supposed) and the assumption $I' < 0$ (condition (7)), and the last equality follows from equation (15) after taking $Y = Y^f$, $i^N = \underline{i}^N = 0$, and $\pi = -\underline{r}$. So we have arrived at the inequality $S(Y^f, \underline{r}) < S(Y^f, \underline{r})$. But this is a contradiction. Hence, we get the required result. ||

Intuition for Proposition 3 is straightforward. Under the implicit tax-subsidy scheme, $(-\underline{r})$ is not only the optimal implicit subsidy on return from investment, but it is also the implicit tax on return on savings. So aggregate demand is increased due to both the implicit subsidy on return from investment and due to the implicit tax on return on savings (see subsection 3.2). In contrast, B is the optimal subsidy rate on investment and there is zero optimal tax on return on savings under the explicit and non-mimicked tax-subsidy scheme. So the burden of adjustment is through one instrument only viz., the explicit subsidy on return from investment. So this is more than the subsidy rate on investment under the implicit tax-subsidy scheme.

Let us sum up this section and re-capitulate two well known results in the context of the ZLB problem in the literature alongside three new results in this paper. First, in the absence of policy intervention, there can be less than full employment (Keynes, 1936). Second, the implicit tax-subsidy scheme can ensure full employment (writings such as Summers (1991)). Third, the explicit and mimicked tax-subsidy scheme cannot ensure full employment (Proposition 1). Fourth, the explicit and non-mimicked tax-subsidy scheme can ensure full employment (Proposition 2). Fifth, the optimal explicit subsidy on return from investment under the explicit and non-mimicked tax-subsidy scheme is greater than the optimal implicit subsidy on return from investment under the implicit tax-subsidy scheme (Proposition 3).

Though the explicit and non-mimicked tax-subsidy scheme proposed here ensures full employment, there is some minimum deficit for the government in the 'ZLB year'. In the next section, we will see how this deficit can be financed.

4 Government's budget, and international pooling of the 'ZLB risks'

We have seen in the previous section that the treasury needs to run a deficit in its budget. This deficit needs to be financed. This financing can be quite simple. The treasury can

impose a tax on investment in normal years. Since the ZLB problem arises occasionally (once in about seventy years in the US, as mentioned earlier), this implies that there can be a small tax over a large number of years. Furthermore, these taxes can be earmarked (possibly by legislation) for use in a ZLB year. This ensures credibility of the financing arrangement. In this way, we have an inter-temporal balance in the government's budget (and a deficit in the ZLB year and a surplus in non-ZLB years).

If there is certainty about the frequency and the timing of the ZLB problem, the above financing arrangement can work smoothly. However, there can be uncertainty. In this case, there can be a need for a cushion in, what we may call *the stabilization budget* in the form of an ex-ante reserve of funds or an ex-post support from *the usual budget* of the government.

We have just considered a financing arrangement for subsidy in the ZLB years in the context of one economy. However, we may consider several economies so that the risks can be shared at the international level. Though there is often a correlation between the economic downturns at the international level, it is interesting that the ZLB risk is not highly correlated internationally given the past experience. Consider some examples. In the aftermath of the crash in the stock market and in the real estate market in 1989-90 in Japan, there was a problem of ZLB (or near-ZLB) for Japan at a time when hardly any other country faced such a problem. Consider another example. In the more recent financial crisis, though much of the developed world in North America and Europe has faced the ZLB problem or near-ZLB problem somewhat simultaneously, the rest of the world (emerging economies, developing economies and oil producing countries) does not face this problem (and not all have high inflation which can make it easy to overcome ZLB problem). This suggests that there is scope for international risk sharing in the context of the ZLB problem.

International risk sharing in the context of the ZLB problem may happen as follows. At the international level, we may consider the involvement of an international public body such as the IMF (or possibly the BIS) which may function not only as a financial institution but also as an insurance institution (though this may require a change in the mandate of the IMF). Consider a case in which many countries adopt the proposed scheme to take care of the ZLB problem. Each country buys insurance from the IMF. Each country pays an insurance premium to the IMF every year. It finances this premium by a small tax on investment in normal years. The funds thus collected can be used to pay the countries that face the ZLB problem. In the unlikely case that too many countries face the ZLB problem, the IMF may need to pay more than was initially anticipated. In such a case, the IMF may use up its own resources and/or increase the insurance premium in future years.

The insurance solution can work best if the risks of hitting the ZLB on nominal interest rate across countries are not perfectly correlated. The less is the correlation, the greater is the scope for the proposed scheme at the international level.

5 Two solutions to the ZLB problem: A further comparison

We have considered three schemes in this paper. These are (1) implicit tax-subsidy scheme, (2) explicit and mimicked tax-subsidy scheme, and (3) explicit and non-mimicked tax-subsidy scheme. We have seen that only the first and the third schemes can help overcome the ZLB problem. In what follows, we will focus on these two schemes.

So far any comparison between the schemes has been on two grounds viz., whether or not inflation is needed, and whether or not there is a budget deficit in the years of low investment. We will now discuss a comparison on other grounds.

First, in the formal model in section 3, we considered aggregate savings and aggregate investment. Let us now consider a more disaggregated model.

In the case of an implicit tax and implicit subsidy, the gap between the nominal interest rate and the real interest rate is effectively the rate of tax and also the rate of subsidy. So we have a common rate of tax and subsidy. Recall that the investment function was

$$I = \overline{I} + I(i^N - \pi).$$

In contrast, in the context of the explicit and non-mimicked tax-subsidy scheme, recall that the investment function was given by

$$I = \overline{I} + I(i_G - z),$$

where i_G is the gross interest rate and z is the common subsidy rate for all investment in the economy. However, we can more generally think of a refined version of the explicit subsidy scheme in which the policy makers use differential subsidy rates.

Consider two sectors in an economy - sector A and sector B (say). Assume that investment in sector A is more desirable than investment in sector B of the economy. More desirable investment can be subsidized at a higher rate than less desirable investment. Let us use the superscripts A and B for investment in sector A and in sector B respectively. Let z^A and z^B be the subsidy rates for sector A and for sector B respectively. The investment function is now given by

$$I = \overline{I}^A + I^A(i_G - z^A) + \overline{I}^B + I^B(i_G - z^B).$$

Note that a set of differentiated subsidy rates in the occasional ZLB years across different sectors of the economy is not possible under the implicit tax-subsidy scheme given the common inflation rate for all sectors. But different subsidy rates are possible under the explicit and non-mimicked tax-subsidy scheme. So the explicit and non-mimicked tax-subsidy scheme can be superior to the implicit tax-subsidy scheme implemented through the monetary policy.

Second, in the formal model in this paper, we considered the real sector and the monetary sector only. Let us now informally consider an economy that includes the financial sector as well. When monetary policy is used to engineer negative real rates of interest in the context of the problem of ZLB a la Summers (1991), such rates of interest apply not only to the real and the monetary sector of the economy but they apply more generally to the economy as a whole including the financial sector. In contrast, the changes in interest rates under the explicit and non-mimicked tax-subsidy scheme need not apply to the financial sector. So the possible side-effects under the implicit tax-subsidy scheme can be minimized, if not eliminated completely, under an explicit tax-subsidy scheme. The side-effects can be quite bad for the real economy (Feldstein, 2002). These may lead to asset price bubbles which can in turn have adverse effects on the real sector. A good example of this is the boom or bubble in the housing market in the post-2000 period upto about 2007, which has been attributed by several economists to the low interest rates *in general*. A scheme like the explicit and non-mimicked tax-subsidy scheme can avoid or reduce such effects. For a recent discussion on side-effects of the (general) low interest policy since 2007-08, see Bank for International Settlements (2012).

Third, in the formal model in this paper, we abstracted from distribution effects. However, when the central bank imposes an implicit tax on savers and provides an implicit subsidy on return from investment, a redistribution from savers to investors is involved (Goodfriend, 2000). In contrast, the explicit and non-mimicked tax-subsidy scheme in this paper does not involve any such redistribution from savers to investors. All that is involved is an inter-temporal redistribution amongst the investors. In the ZLB year, the treasury pays a subsidy to investors. In the normal years, the treasury receives taxes paid by the investors.

Fourth, though the formal model in the paper abstracted from issues related to *transparency* and *accountability*, these can be important. First consider the issue of transparency. The use of the explicit and non-mimicked tax-subsidy scheme is transparent. The public knows that all or some investors are being subsidized in the ZLB year(s), and that investors

are being taxed in normal years. The use of implicit tax-subsidy scheme is not transparent. The subsidy for investors and the tax on savers are implicit and hence not transparent. To the extent that we value transparency,[17] the explicit tax-subsidy scheme is preferable to the implicit tax-subsidy scheme.

Next consider the issue of accountability. As mentioned already, when the central bank imposes an implicit tax on savers and provides an implicit subsidy on return from investment, redistribution from savers to investors is involved. Ideally any redistribution should receive the approval of the Parliament. However, this does not happen in practice in the case of the implicit tax-subsidy scheme. So there is little accountability in the implicit tax-subsidy scheme. In contrast under the explicit and non-mimicked tax-subsidy scheme, there is a need for Parliamentary approval and so the treasury is accountable.

Fifth, it may be argued that the ZLB problem is being currently faced by many developed countries and that if such countries were to adopt the explicit and non-mimicked tax-subsidy scheme proposed here, they would need to incur the expenditure on subsidy on return from investment now and collect taxes in future (given they had not built any reserve for this purpose in the past). So they would need to borrow (in addition to their current borrowings) at present. This can further aggravate the public debt problem faced by these countries. Hence the proposed new fiscal policy to take care of the ZLB problem is, it may be argued, not pragmatic. This is particularly true given that there exists an alternative implicit tax-subsidy scheme to overcome the ZLB problem without creating any budgetary imbalance. While there is merit in this argument, we would like to make a few observations.

1. It is not clear that the fiscal burden due to the proposed new fiscal policy can be large in a ZLB year. Consider a *back of the envelope* calculation on the possible fiscal burden. Assume that the prevailing interest rate is zero. Investment as percentage of GDP is low and needs to be increased by another, say, 8% of GDP. Suppose that the treasury needs to provide a subsidy such that the effective or net or after-subsidy interest rate for investing firms is -5% (one figure mentioned in Buiter (2009)). This implies that the explicit subsidy required for raising investment is 0.4% (= 0.05 x 8) of GDP. This is not a large figure relative to the size of fiscal stimulus in many countries in the aftermath of the financial crisis and the recession beginning in 2007-08.
2. Consider the financing of this subsidy by a tax on investment in non-ZLB years. As-

[17] It is true that under some conditions, transparency beyond a point can be undesirable. See Hermalin and Weisbach (2012). However, in the context of the problem at hand, there is very little transparency at present.

	Gross debt	Net debt
Advanced economies	103.5	72.4
Emerging economies	37.6	27.0
Low-income countries	38.2	Not available
Oil producers	22.6	Not available

Table 1: Debt as per cent of GDP in 2011
(Data from Table 5, p. 9, International Monetary Fund, 2012)

sume that the ZLB problem occurs after 50 years (though in the US it has, as mentioned already, occurred after about 75 years). Ignoring economic growth and discounting for time, there is a need for a tax of 0.008% (= 0.4/50) of GDP in each of the non-ZLB years. This is a small figure.

3. The subsidy in the occasional ZLB periods is used to increase investment which is productive. This can be better than the fiscal stimulus seen in practice in the years around 2008-09 which was not entirely for productive purposes (or redistributive purposes).
4. Taxes on investment in future in normal years can be earmarked (possibly by legislation) for the purpose of paying subsidies in the occasional ZLB years. Therefore, there can be credible repayment of the public debt used for this purpose.
5. The proposed scheme is not only to tackle the ZLB problem at present but also in future. Since it is not the case that even the advanced countries will forever continue to have serious debt problems in future, there is a need to consider the proposed scheme.
6. The proposed scheme here is not only for advanced countries that are highly indebted but also for other countries that are not highly indebted. Such countries need not have any worries if the proposed scheme is adopted. It may help to consider some data on the public debt at the international level. See Table 1. The gross debt as per cent of GDP in 2011 was high for advanced countries only. For the others, the debt problem on an average does not seem to be serious. So there can be a case for putting in the proposed solution in place in a large number of countries.

All this suggests that though the implicit tax-subsidy scheme is quite convenient, the explicit tax-subsidy scheme too can be pragmatic.

Sixth, the implicit tax-subsidy scheme implemented through monetary policy involves inflation. We had assumed that the expected inflation is equal to the actual inflation in

our model. So we have abstracted from any adverse effects for the economy due to any gap between the actual inflation and the expected inflation. However, it is hard to achieve this equality in practice. We have also assumed that agents are rational and that they clearly distinguish between the nominal and the real interest rate. However, as behavioural economics has shown, this assumption is not always realistic. So there can be adverse effects of the implicit tax-subsidy scheme. Such effects are absent in the case of the explicit tax-subsidy scheme proposed here since there is no inflation involved and the distinction between gross and net interest rates can be much clearer than the distinction between nominal and real interest rate *for the general public* (people are anyway familiar with usual taxes and subsidies). So on this count the explicit and non-mimicked tax-subsidy scheme can be superior to the implicit tax-subsidy scheme.

Consider an example of a divergence between expected and actual inflation. In the early 1980s, the FED and the Bank of England embarked on an anti-inflationary policy. This policy was indeed successful in bringing about a significant and apparently lasting reduction in the inflation rate. However, there were other effects such as high interest rates for quite some time. One reason for such high nominal interest rates was the belief that high inflation was likely to return. Though this belief turned out to be incorrect and the nominal interest rates did fall over time, there were adverse effects in these economies. This was also due to the fact that agents did not always realize exactly the distinction between nominal and real interest rates (see Shiller (2008) for the US case). It is also sugggested that the boom or the irrational exuberance in the stock market before the year 2000 can be attributed to the belief that interest rates had fallen considerably over time, which was not entirely true in real terms.

Summing up, we have informally discussed that there is a *prima facie* case that the explicit and non-mimicked tax-subsidy scheme on the whole may be superior to the use of an implicit tax-subsidy scheme.

6 Conclusion

The standard policy recommended in the literature to deal with the problem of zero lower bound (ZLB) on nominal interest rate is to have inflation or credible inflationary expectations. This creates a wedge between nominal and real rate of interest. This helps realize a negative real rate of interest which in turn helps in macroeconomic stability. The standard solution through inflation involves an implicit tax on savers and an implicit subsidy for

investors. We called it the *implicit tax-subsidy scheme by the central bank*.

In this paper, we began by asking why the tax-subsidy scheme has to be implicit and not explicit. Related to this question, we asked why the tax-subsidy scheme has to be by the central bank and not by the treasury. To answer this question, we considered an explicit tax-subsidy scheme that mimics the implicit tax-subsidy with one important difference. There is no inflation associated with the explicit tax-subsidy scheme. We called it the *explicit and mimicked tax-subsidy scheme by the treasury*. We found that such a scheme can ensure full employment only if savings are interest inelastic. In case savings are interest elastic, then this scheme cannot take care of the ZLB problem. So there is indeed a good reason why the tax-subsidy scheme has to be implicit rather than explicit and why it has to be by the central bank rather than by the treasury.

We went on to ask another question in this paper. Though an explicit and mimicked tax-subsidy scheme cannot overcome the ZLB problem if savings are interest elastic, does there exist any other scheme by the treasury that can help overcome this ZLB problem without inflation or inflationary expectations? We found that the answer is yes. We called it the *explicit and non-mimicked tax-subsidy scheme by the treasury*. In this scheme, there is no inflation, the tax on return on savings is zero, and the subsidy on return from investment makes the interest rate for investment effectively negative. Full employment is possible under this scheme even if exogenous investment is low.

Under the explicit and non-mimicked tax-subsidy scheme, the treasury needs to incur a deficit in the ZLB years. We discussed how this deficit can be financed by a small tax on investors in non-ZLB years. So there need not be any inter-temporal deficit for the government on this account. There can also be sharing of the ZLB risks at the international level.

Finally, we informally compared the implicit tax-subsidy scheme by the central bank (with inflation), and the explicit and non-mimicked tax-subsidy scheme by the treasury (without inflation). Our discussion suggests that the latter scheme can be superior to the former scheme. Formal research on this in future can be very useful.

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