

# Leadership by Gradual Persuasion\*

September 30, 2012 (Preliminary & incomplete)

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## Abstract

Consider an agent (the leader) with private information who has to convince a group of agents to exert costly effort for a common-cause. How might he credibly convey that information? Hermalin (1998) argues that the leader does this by example – he undertakes costly effort that is *publicly observed*. We complement this observation by noting that the leader does even better by *secretly* convincing just one agent, the two of them convincing yet another agent and so on. We call this *gradual persuasion*, a process in which only the up-to-date progress made by early movers should be disclosed to the follower next in queue without revealing who contributed how much. **JEL Classification:** D21; L29, D29. **Key Words:** Leadership, voluntary contribution, signaling, gradualism.

## 1 INTRODUCTION

A leader persuades a group of individuals to strive towards a common goal (or project). When only the leader knows the project’s merit but cannot directly communicate the information,

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\*The main idea is conveyed in a simple example in section 2. A more complete analysis is being worked out.

<sup>1</sup>**Acknowledgment:** The work has received support from the Singapore Ministry of Education in the form of a grant to the second author.

the followers try to infer the value from the leader’s costly and productive effort. Benjamin E. Hermalin (1998) has argued about the importance of *leading by example* when the leader may lack formal authority. In his model, a continuum of leadership efforts helps the leader to convincingly reveal the project’s true type, following which the followers simultaneously exert their efforts. The combined team efforts under incomplete information and leadership exceed total efforts under symmetric information, resulting in strict improvements for the followers. Thus the leader indeed leads (rather than mislead).

In this paper, we raise the leadership question once more by directing attention to what might be the “right” approach to leadership. The leader can speak to his followers publicly – the *public speech* mode – as often political leaders and well-known figures in the public life tend to do.<sup>1</sup> Alternatively, the leader may take into confidence his followers one-by-one to gradually work towards the team objective. This sequential approach echoes widely used practices in committees and lobbies where the leader has to persuade other members in small groups to join him in stages, behind closed doors, before announcing it publicly. In a select group of key personnel, getting the endorsement and real backing (in terms of effort commitment) of a few individuals is quite important. When more than a few contributors are involved, even there getting a critical number of endorsement may be a more practical approach before going public with the team’s big plan. The sequential approach to gathering support to a common cause, we call *gradual persuasion*. We will argue why gradual persuasion may yield higher benefits for the team.

While the important ground-work has been done in the public speech protocol of Hermalin’s construct, the reasoning behind why a full-fledged sequential protocol of signalling should be better is rather novel. The leader, by keeping his own contribution secret from all but one follower, is passing on the responsibility of conveying that the underlying project is of high value to the immediate follower to whom he has revealed his contribution. Since the remaining followers do not yet have any signal of the project’s merit, the leader’s deputy (i.e.,

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<sup>1</sup>Speaking is figurative, rather the leader makes his action observable.

the first follower) will have to “up” his contribution to improve the leader’s and the deputy’s collective contribution and thus convince the follower next in the chain about the high value of the project. By assuming a system of skeptical beliefs on the part of the followers, the leader thus induces all the followers intermediate in the chain to each contribute more. This will have a cascading effect resulting in overall contributions going up.<sup>2</sup>

The above mechanism works because in the public speech game the followers all end up free-riding as in standard public good models by ignoring the positive externalities of their individual contributions. Thus each follower will have some slack that can be taken advantage of by positioning them sequentially. So long as at least one follower has a slack, the gradual persuasion chain can be prolonged by placing the agent ahead of some other agents and applying skeptical belief to the total contribution up to this agent.

The literature on voluntary contributions has studied the positive role of gradualism (Leslie M. Marx and Steven A. Matthews (2000), Chaim Fershtman and Samuel Nitzan (1991)). Hal R. Varian (1994) had argued, paradoxically, that sequential contribution may aggravate the free-rider problem. Some recent works have studied the role of observability of peer contributions in team projects and its incentive implications (Eyal Winter (2004; 2006), Parimal Kanti Bag and Nona Pepito (2012)). Our paper differs from these works in identifying a unique role of beliefs about the project’s type in the leadership signaling model of Hermalin (1998), and how gradualism can be helpful by *delaying revelation* of leader’s information.

In the next section, we construct an example to illustrate our basic idea. A more general analysis will be developed in section 3.

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<sup>2</sup>This cascading effect is a favorable one, in contrast to the negative informational cascades one encounters in the herd behavior literature (Sushil Bikhchandani, David Hirshleifer and Ivo Welch (1992)).

## 2 AN EXAMPLE

We build on Hermalin's model and analysis. Our simple argument can be conveyed using a three-player team example. Player 1 (the leader) has private information about the project's common value  $\theta \in \{\theta_\ell, \theta_h\}$ . The project output is the sum of efforts by the leader and followers (players 2 and 3), i.e.,  $y = e_1 + e_2 + e_3$ . Effort cost is increasing and convex:  $c_i(e_i) = \frac{e_i^2}{2}$ . Thus each player's payoff is given by

$$u_i = \theta[s_i(e_1 + e_2 + e_3)] - \frac{e_i^2}{2}, \quad i = 1, 2, 3,$$

where  $0 \leq s_i \leq 1$  is player  $i$ 's share of total output so that  $s_1 + s_2 + s_3 = 1$ .

*Public speech:* First consider the public speech game. Player 1, knowing the realization of  $\theta$ , exerts an effort  $e_1(\theta)$  that is publicly observed by players 1 and 2, who then assign common beliefs  $\mu_\ell(e_1) = \text{pr}(\theta = \theta_\ell | e_1)$ . More specifically, the beliefs are as follows:

$$\mu_\ell(e_1) = \begin{cases} 1 & \text{if } e_1 < e^*, \\ 0 & \text{if } e_1 \geq e^*, \end{cases} \quad (1)$$

where  $e^*$  is such that

$$\begin{aligned} & \theta_\ell \left[ s_1 \left( e_1(\theta_\ell) + e_2(e_1(\theta_\ell), \theta_\ell) + e_3(e_1(\theta_\ell), \theta_\ell) \right) \right] - \frac{(e_1(\theta_\ell))^2}{2} \\ & \geq \theta_\ell \left[ s_1 \left( e^* + e_2(e^*, \theta_h) + e_3(e^*, \theta_h) \right) \right] - \frac{(e^*)^2}{2}, \end{aligned} \quad (2)$$

$$e^* = \operatorname{argmax}_{e_1 \geq e^*} \theta_h \left[ s_1 \left( e_1 + e_2(e_1, \theta_h) + e_3(e_1, \theta_h) \right) \right] - \frac{(e_1)^2}{2}, \quad (3)$$

where

$$(e_1(\theta_\ell), e_2(e_1(\theta_\ell), \theta_\ell), e_3(e_1(\theta_\ell), \theta_\ell)),$$

are the subgame-perfect equilibrium efforts under symmetric information that the project is of type  $\theta_\ell$ , and

$$(e_2(e_1, \theta_h), e_3(e_1, \theta_h))$$

are the Nash equilibrium efforts by players 2 and 3 in the continuation games following effort  $e_1 \geq e^*$ , and the (common) belief that the project is of type  $\theta_h$ .<sup>3</sup>

Intuitively, conditions (2) and (3) guarantee that the leader who observes that the project valuation is ‘low’ will not benefit by mimicking that the type is ‘high’, and the high-type leader will optimally exert effort  $e^*$  in anticipation of the followers’ equilibrium best responses (based on Bayesian updated beliefs).

We will thus denote a (revealing) perfect Bayesian equilibrium in the public speech game (or, *PBE-PS*) as a pair of beliefs, given in (1), and efforts (see footnote 3) as follows:

$$(e_1(\theta_\ell), e_2(\theta_\ell), e_3(\theta_\ell)) \tag{4}$$

$$(e_1(\theta_h) = e^*, e_2(\theta_h), e_3(\theta_h)), \tag{5}$$

satisfying conditions (2) and (3). Such an equilibrium will exist if  $\theta_h - \theta_\ell$  is sufficiently large, which we assume to be the case.

**Remark.** It may be noted that the high-type leader’s effort  $e^*$  in the revealing equilibrium may well exceed  $e_1(\theta_h)$ , the Nash (and subgame perfect) equilibrium effort by player 1 under symmetric information. That is, under asymmetric information the leader may have to exert an effort level exceeding  $e_1(\theta_h)$  to separate from her low-type self.

Now let us see how gradual persuasion by the leader can improve the leader’s payoff and in the process lift overall team efforts. Suppose the leader makes his action, i.e. effort  $e_1$ , observable to only player 2, who then puts in effort  $e_2$ . Player 3 moves last after observing the combined efforts  $e_1 + e_2$  but without knowing the break-up. Since only player 1 has the information about  $\theta$ , when player 3 moves she has to second-guess  $\theta$  from the combined message  $e_1 + e_2$  whereas player 2 can make a first-hand inference based on  $e_1$ .

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<sup>3</sup>Note that, under symmetric information each player’s equilibrium effort is independent of the other team members’ efforts irrespective of whether the leader moves first or all move simultaneously. So, these efforts are also dominant strategies and we will write  $e_2(e_1(\theta_\ell), \theta_\ell) \equiv e_2(\theta_\ell)$ ,  $e_3(e_1(\theta_\ell), \theta_\ell) \equiv e_3(\theta_\ell)$ ,  $e_2(e_1, \theta_h) \equiv e_2(\theta_h)$ , and  $e_3(e_1, \theta_h) \equiv e_3(\theta_h)$ . However, under asymmetric/incomplete information the followers’ efforts will depend on the leader’s effort *indirectly* as the latter determines followers’ beliefs about the project’s type.

Suppose now that player 2's beliefs are as in (1), while player 3 applies the following skeptical belief:

$$\mu_\ell(e_1 + e_2) = \begin{cases} 1 & \text{if } e_1 + e_2 < \hat{e}, \\ 0 & \text{if } e_1 + e_2 \geq \hat{e}, \end{cases} \quad (6)$$

where  $\hat{e} > e^* + e_2(\theta_h)$  such that

$$\begin{aligned} & \theta_\ell \left[ s_2 \left( e_1(\theta_\ell) + e_2(\theta_\ell) + e_3(\theta_\ell) \right) \right] - \frac{(e_2(\theta_\ell))^2}{2} \\ \geq & \theta_\ell \left[ s_2 \left( e_1(\theta_\ell) + [\hat{e} - e_1(\theta_\ell)] + e_3(\theta_h) \right) \right] - \frac{(\hat{e} - e_1(\theta_\ell))^2}{2}, \end{aligned} \quad (7)$$

$$\begin{aligned} & \theta_h \left[ s_2 \left( e^* + [\hat{e} - e^*] + e_3(\theta_h) \right) \right] - \frac{(\hat{e} - e^*)^2}{2} \\ = & \max_{e_2 < \hat{e} - e^*} \theta_h \left[ s_2 \left( e^* + e_2 + e_3(\theta_h) \right) \right] - \frac{(e_2)^2}{2}. \end{aligned} \quad (8)$$

We now claim that an  $\hat{e}$  as specified above exists.

In the *PBE-PS* given by (1)–(5), player 2 is enjoying a strictly positive surplus when the leader's effort of  $e^* = \theta_h s_1$  reveals  $\theta = \theta_h$  (compared to his payoff when  $\theta = \theta_\ell$ ) to which players 2 and 3 respond by exerting  $e_2 = \theta_h s_2$  and  $e_3 = \theta_h s_3$ :

$$\begin{aligned} & \max_{e_2} \theta_h \left[ s_2 \left( e^* + e_2 + e_3(\theta_h) \right) \right] - \frac{(e_2)^2}{2} \\ = & (\theta_h)^2 \left[ s_2 \left( s_1 + s_2 + s_3 \right) \right] - \frac{(\theta_h s_2)^2}{2} \\ = & (\theta_h)^2 \left[ s_2 \left( s_1 + \frac{s_2}{2} + s_3 \right) \right] \\ > & (\theta_\ell)^2 \left[ s_2 \left( s_1 + \frac{s_2}{2} + s_3 \right) \right] \\ \equiv & \text{Leader's maximum payoff when he is of type } \theta_\ell. \end{aligned}$$

Thus, there is a slack in player 2's surplus for  $\theta = \theta_h$  in the public speech game.

Now player 3, by placing a heavier burden on player 2 (by requiring some  $\hat{e} > e^* + e_2(\theta_h)$  for the skeptical belief), will force player 2 to increase her effort in the gradual persuasion game above  $e_2(\theta_h)$ , when she sees an effort of  $e^*$  by the leader. Given that player 3's equilibrium effort (for belief  $\theta = \theta_h$ ) is independent of the other players' efforts, the total

team efforts in equilibrium in the gradual persuasion game when  $\theta = \theta_h$  is strictly higher. On the other hand, equilibrium efforts in the two game forms when  $\theta = \theta_\ell$  are identical.

**PROPOSITION 1 (Benefit of gradual persuasion).** (i) *Corresponding to any revealing PBE-PS in a three-player public speech leadership game, there is a revealing equilibrium in the gradual persuasion leadership game that generates strictly higher overall contribution when  $\theta = \theta_h$ .*

(ii) *Compared to the public speech game, the high-type leader's payoff in the gradual persuasion game is strictly higher while the low-type leader is indifferent.*

### 3 GENERAL RESULT

(This section will be written up.)

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