An adjustment cost model of social mobility

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Abstract

We analyze the effects of human capital adjustment cost on social mobility in an environment with incomplete capital market. We find that a higher adjustment cost for human capital acquisition slows down the social mobility and results in a persistent inequality across generations. A low depreciation cost of human capital by contributing to longer life of the capital could further aggravate this process of slower social mobility. A capital income tax could have a regressive effects in economies with longer lived human capital. The quantitative analysis of our model suggests that the human capital adjustment cost is nontrivial to reproduce the observed persistence of inequality and social mobility. A welfare analysis suggests that economies with higher adjustment cost experience lower steady state welfare. This loss of welfare in high adjustment cost economies could be mitigated by human capital subsidy in the form of a lower capital income tax.

Key words:
Intergenerational mobility, inequality persistence, adjustment cost of capital
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1. Introduction

It is an open question whether the son of a poor farmer will become a high paid executive manager. The evidence during the last two decades points to the direction that such social mobility is slow (Machin, 2004). Clark and Cummins (2012) establish that there is considerable persistence in the wealth status of households in England from 1800 to 2012. They predict that it will take another 200 years to complete the process of social mobility. Our paper aims to understand the determinants of such mobility in an environment with incomplete markets where human capital is the driver of growth and it is costly to adjust human capital. The seminal paper of Becker and Tomes (1979) draws the conclusion that a stable distribution of income could be explained by individual and market lucks. Their crucial assumption is that the credit market is perfect implying that individuals with low wealth and high marginal product of capital could borrow from individuals with the opposite trait. This tends to equalize the differences in wealth. The residual inequality is then mostly attributed to luck. Since then a considerable literature (e.g., Loury, 1981, Banerjee and Newman, 1993, Galore and Zeira, 1993, Benabou, 1996, Mulligan, 1997, Bandyopadhyay and Basu, 2005, Bandyopadhyay and Tang, 2011) has evolved emphasizing the role of credit market imperfection in perpetuating the inequality.

In this paper, we explore how some factors are important to the investment climate such as adjustment costs could affect social mobility. As in Loury (1981), Galor and Zeira (1993) and Benabou (2000, 2002), we develop a scenario with missing credit and insurance markets. Individuals differ in terms of initial distribution of human capital and abilities. The differences in abilities are due to idiosyncratic shocks to productivity which cannot be hedged using an insurance market. We demonstrate that in such a scenario the presence of adjustment cost of human capital could impede the process of social mobility.

The human capital adjustment cost is modeled as a rising marginal cost schedule for investment in human capital. Such an adjustment cost can arise due to a number of reasons. First, there could be basic human inertia to respond to change and adjust to new opportunities or environment. An example of such inertia is that an adult finds a better job opportunity with a higher pay in a region different from her home town but due to friends and family ties, she is reluctant to move (Alesina and Giuliano, 2010). Second, this adjustment cost could be attributed to market based factors such as higher cost of advanced education compared to primary schooling or a higher employment adjustment cost as in Hansen and Sargent (1980).

Why does a higher adjustment cost impede social mobility? When the credit market is missing, investment opportunities (which is investment in human capital in our model) facing individuals are limited to the resources that they have in hand. Given a production function with private diminishing returns to reproducible human capital, poor with lower human capital have a higher marginal product than rich. Thus, their relative growth potential is higher compared to rich. However, if capital adjustment cost is present, this growth of the poor will be impeded because poor face a higher marginal cost of investment when they try to grow. Thus adjustment cost will slowdown the process of social mobility leading to a higher persistence of human capital inequality in the aggregate. The central point of our paper is to demonstrate that a society facing such a costly adjustment of human capital could experience persistent inequality and low social mobility measured by the serial correlation between the wealth of current and future generations. The role of this type of human capital adjustment cost has been ignored in the inequality and social mobility literature and this is precisely where our paper contributes.

In addition to adjustment cost, we also look at the role of the depreciation cost of human capital in determining social mobility. A lower depreciation of human capital makes a generation inherit a lot of capital from its predecessors. This lowers the marginal return to investment further when adjustment cost is already present. Thus, it particularly hurts poor's incentive to invest in education and could lower social mobility even more. An example of such low depreciation of human capital is the transfer of knowledge of a primitive farming technology in a less developed country from one generation to another which could provide little incentive to the current generation to learn new technology of farming.

We set up an incomplete market model in which households are heterogenous in terms of initial human capital and ability. They receive a warm-glow utility from investing in child's education in the spirit of Galor and Zeira (1993). As in Loury (1981) human capital is the only form of reproducible capital in the economy. Idiosyncratic productivity shocks together with initial difference in human capital could give rise to current cross-sectional inequality and such inequality transmits from one generation to another. The absence of credit and insurance markets prevents agents from mitigating negative idiosyncratic shocks. An unlucky agent suffering a bad productivity shock invests less resources to child's education which means that the child inherits less human capital. How quickly the offspring gets over this disadvantage depends on how costly it is to adjust the human capital. We develop closed form formula for the endogenous law of motion of inequality. The key theoretical result based on this closed form solution is that the persistence of inequality is higher in economies with a higher adjustment cost either in terms of a steeper curvature of the marginal cost of investment or a lower depreciation cost of capital. To the best of our knowledge, our closed form solution for the law of motion of growth and cross sectional variance in the presence of incomplete depreciation of human capital is novel and new to the literature.

Our calibration exercise suggests that the human capital adjustment cost has to be kept at a nontrivial level to reproduce the observed degree of social mobility, long-run inequality and growth. The calibrated social mobility parameter accords well with the slow social mobility predicted by Clark and Cummins (2012) and others. An adult's response to luck matches well with the well known move to opportunity (MTO) programme in North America (reported by Katz et al., 2001). The sensitivity analysis with key parameters such as adjustment cost and depreciation rate suggests that the persistence of inequality is consistently higher in economies with higher adjustment cost and lower depreciation of capital. Impulse responses of human capital with respect to initial luck differences suggest that the social mobility is slower in economies with higher adjustment cost, higher share of capital and a lower total factor productivity. These are all consistent with our key theoretical results.

Higher adjustment cost not only slows down the social mobility but it also adversely affects societal welfare. We demonstrate this by deriving the steady state social welfare function and simulating it for alternative adjustment cost parameters.

A subsidy to human capital in the form of a lower capital income tax could mitigate this loss of welfare in high adjustment cost economies.

The paper is organized as follows. Section 2 presents the model and the dynamics and equilibrium of individual wealth accumulation. Section 3 reports the quantitative analysis of the model and the welfare analysis. Section 4 concludes.

2. The model

2.1. Preference and technology

Consider a continuum heterogeneous households $i \in [0, 1]$ in overlapping generations. Each household i consists of an adult of generation t attached to a child. A child only inherits human capital from her parents and does not make any decision as her consumption is already included in that of her parents. Adult, at date t employs a unit raw labour into the production process which translates into h_{it} efficiency units (human capital) for the production of final goods and services to earn income (y_{it}) using the following Cobb-Douglas production function:

$$y_{it} = a_1 \varphi_{it} (h_t)^{1-\alpha} (h_{it})^{\alpha} \tag{1}$$

where $a_1 > 0$ is simply an exogenous productivity parameter, $\alpha \in (0, 1)$, h_t represents the aggregate stock of knowledge in the spirit of Arrow (1962) and Romer (1986) which is given to the adult.[1] Individuals are subject to an i.i.d. idiosyncratic productivity shocks (φ_{it}) which drive their total marginal productivity. The idiosyncratic shock φ_{it} follows the process: $\ln \varphi_{it} \sim N(-\upsilon^2/2, \upsilon^2)$. The child at date t behaves as an adult at $t + 1$.

2.1.1. Utility function and budget constraint

Agents care about their own consumption (c_{it}) and the human capital stock of their children (h_{it+1}), which can be justified by "joy of giving". In other words, the

[1] Such a technology basically means that there is private diminishing returns but social constant returns to human capital.

utility of the adult at date t is given by:[2]

$$u\left(c_{it}, h_{it+1}\right) = \ln c_{it} + \beta \ln h_{it+1} \tag{2}$$

where $0 < \beta < 1$ is the degree of parental altruism, h_{it+1} represents the human capital of the offspring of agent i. At the end of the period, parents allocate income between current consumption (c_{it}) and saving (s_{it}). The latter is used for investment in human capital accumulation of the offspring as shown in (4). The budget constraint is thus given by:

$$c_{it} + s_{it} = y_{it} \tag{3}$$

2.1.2. Technology of human capital production

The human capital is the only form of reproducible input in our model. The stock of human capital inherited by the current adult from her predecessors determines her state of technological knowledge which she can modify to advance the technological frontier. This modification can be done by investing in education or R&D. The production of the next period human capital (h_{it+1}) takes place with the aid of two factors: (i) past human capital (h_{it}), (ii) investment in schooling (s_{it}):

$$h_{it+1} = a_2 h_{it}^{1-\theta}\left((1-\delta)h_{it} + s_{it}\right)^{\theta} \tag{4}$$

where $\theta \in (0, 1)$, $\delta \in (0, 1)$ and $a_2 > 0$. The human capital production function is in spirit to Benabou (2002) except for the inclusion of the depreciation parameter δ. The parameter θ determines the curvature of the marginal return to investment ($\partial h_{it+1}/\partial s_{it}$) which we ascribe to adjustment cost. The marginal return to investment is given by:

[2] The choice of a logarithmic utility function and altruistic agents with a "joy of giving" motive is merely for simplicity. Also see Glomm and Ravikumar (1992), Galor and Zeira (1993), Saint-Paul and Verdier (1993) and Benabou, 2000) for similar settings. A version of dynastic altruism model as in Barro (1974) (with complete depreciation of human capital) with physical capital and labour are worked out in the Appendix C of this paper.

$$\partial h_{it+1}/\partial s_{it} = a_2\theta/\left(1-\delta+s_{it}/h_{it}\right)^{1-\theta} \tag{5}$$

Inverse of this marginal return to investment is the marginal cost of investment which is therefore rising in investment per unit of human capital (s_{it}/h_{it}). The fact that θ is a fraction is fundamental for a rising marginal cost of investment. Such a rising marginal cost reflects the adjustment cost of investment in human capital. Hereafter, we call θ as the adjustment cost parameter.

Figure 1 plots the marginal return to investment, $\partial h_{it+1}/\partial s_{it}$ against s_{it}/h_{it} for $\theta = 0.8$ (our baseline value in the calibration later) and $\theta = 0.6$.[3] Lower θ makes the investment return schedule shift downward with a steeper curvature. This steep decrease in marginal return to investment due to lower θ is ascribed to a higher adjustment cost of human capital. If θ reaches the upper bound of unity, there is zero adjustment cost and the investment technology reverts to a standard linear depreciation rule. This notion of θ as the degree of human capital adjustment cost is borrowed from the standard capital adjustment cost technology used in Lucas and Prescott, 1971, Basu, 1987, and Basu et al., 2012).

The inclusion of the depreciation cost parameter δ in the human capital production function is novel in our paper, and in this respect it differs from Benabou's (2002) human capital technology. This depreciation cost determines how much human capital the child inherits from her parents. Thus even if parents undertake zero investment in child's education, unlike Benabou (2000) the child still inherits some human capital in proportion to $(1-\delta)h_{it}$. Viewed from this perspective, one may think of $1-\delta$ as the degree of intergenerational spillover knowledge as in Mankiw, Romer and Weil (1992) and Bandyopadhyay and Basu (2005).

The lower rate of depreciation of human capital makes the capital last longer which contributes to a lower marginal return to investment and thus higher marginal cost as seen from Figure 2. Unlike a change in θ, the marginal return to investment is less sensitive to a change in δ. A lower rate of depreciation lowers the intercept of

[3] The other two parameters a_2 and δ are fixed at their baseline levels of 0.03 and 1.655 respectively.

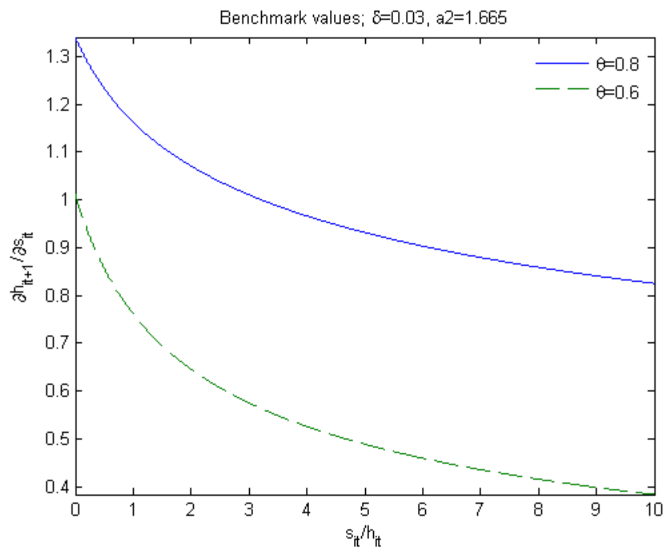

Figure 1: Effect of a change in θ on the marginal return to investment

the marginal return to investment schedule (which is $a_2\theta/\left(1-\delta\right)^{1-\theta}$) and it flattens its slope. The marginal rate of return to investment $(\partial h_{it+1}/\partial s_{it})$ thus undergoes a downward shift in response to a lower δ. A lower depreciation cost makes the current generation inherit a lot of human capital from ancestors which provides them less incentive to investment because of its negative effect on the marginal return to investment.

Lower θ and lower δ thus drive down the return to investment, raise the marginal cost of investment and thus discourage the adult's incentive to invest in schooling or learn a new technology. The central point of this paper is that both these factors contribute to less social mobility and persistent inequality.

2.2. Initial distribution of human capital

At the beginning, each adult of the initial generation is endowed with human capital h_{i0}. The distribution of h_{i0} takes a known probability distribution,

$$\ln h_{i0} \sim N(\mu_0, \sigma_0^2) \tag{6}$$

and it evolves over time along an equilibrium trajectory.

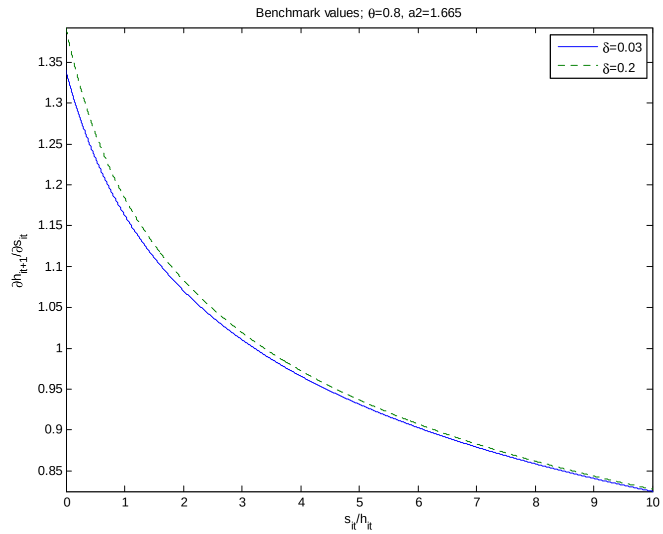

Figure 2: Effect of a change in δ on the marginal return to investment

2.3. Equilibrium

In equilibrium, all individuals behave optimally and the aggregate consistency conditions hold.

Optimality: An adult of cohort t solves the following problem, obtained by substituting (3), (4) and (2),

$$\max_{s_{it}} \ln\left(y_{it} - s_{it}\right) + \beta \ln\left(\left(1-\delta\right) h_{it} + s_{it}\right)^{\theta} \tag{7}$$

taking as given h_{it}. The optimization yields the following optimal investment functions,

$$s_{it} = \left(y_{it}\theta\beta - \left(1-\delta\right) h_{it}\right) / \left(1+\theta\beta\right) \tag{8}$$

An adult's optimal investment decision constitutes both new investment plus a replacement of depreciated capital. Note that a lower rate of depreciation depresses current investment because it lowers the marginal return to investment.[4]

[4]To see it, check from (4) that $\partial h_{it+1}/\partial s_{it}$ is positively related to δ.

Aggregate Consistency: (i) $c_t \equiv \int c_{it}di$, $s_t \equiv \int s_{it}di$, $y_t \equiv \int y_{it}di$, $h_t \equiv \int h_{it}di$ where the left hand side variable in each of them means the aggregate. (ii) The aggregate budget constraint is thus given by:[5]

$$c_t + s_t = y_t \tag{9}$$

2.4. Individual optimal human capital accumulation

The ith individual optimal human capital accumulation is given by, from (1), (4) and (8),

$$h_{it+1} = \phi h_{it} \left(1 - \delta + a_1 \varphi_{it} h_t^{1-\alpha} h_{it}^{\alpha-1}\right)^{\theta} \tag{10}$$

where

$$\phi \equiv a_2 \left(\theta\beta / \left(1 + \theta\beta\right)\right)^{\theta}$$

Thus, the ith individual offspring's human capital is determined by both the depreciation and adjustment cost of human capital and her parent's income.

2.5. Are children poor due to bad luck or poor parents?

We start with the age-old question: How does inequality transmit through generations through past lucks and initial conditions? To see this clearly, loglinearize (10) around a balanced growth path where all agents are identical in terms of luck and human capital $\varphi_{it} = \varphi = 1$ and $h_{it} = h_t$, respectively. One gets[6]:

$$\ln h_{it+1} \simeq \xi + (1-\alpha)\chi \ln h_t + \varrho \ln h_{it} + \chi \ln \varphi_{it} \tag{11}$$

[5] We use the operators $\int$ and E interchangeably in the text to denote aggregation across individuals.

[6] See Appendix A for details on the derivation.

where

$$\xi \equiv \ln \phi + \theta \ln (1 - \delta + a_1) \tag{12}$$

$$\varrho \equiv 1 - \chi(1 - \alpha) \in (0, 1) \tag{13}$$

$$\chi \equiv \theta a_1 / (1 - \delta + a_1) \in (0, 1) \tag{14}$$

Since we can do the same for the jth individual,

$$\Delta h_{it+1} = \varrho (\Delta \ln h_{it}) + \chi (\Delta \ln \varphi_{it}) \tag{15}$$

where $\Delta h_{it+1} \equiv \ln h_{it+1} - \ln h_{jt+1}$, $\Delta h_{it} \equiv \ln h_{it} - \ln h_{jt}$ and $\Delta \varphi_{it} \equiv \ln \varphi_{it} - \ln \varphi_{jt}$.

When the capital market is incomplete, difference in initial human capital of the first generation as well as difference in lucks play a central role in transmitting initial inequality through generations. The first term in (15) shows the effect of difference in human capital of parents while the second term captures the effect of parent's luck on the wealth inequality of their kids. Both differences in initial human capital and lucks transmit through generations. How they impact future generations depend on the parameters ϱ and χ, which in turn depend on the structural parameters. The initial difference in wealth has a decaying effect on the wealth difference of successive generations. The rate of decay is determined by $\varrho \in (0, 1)$. A larger ϱ makes the initial inequality have a persistent effect and thus slows down social mobility.[7] It is easy to verify that ϱ is larger if adjustment cost is higher (lower θ) or depreciation cost (δ) is lower.

2.5.1. Depreciation, TFP and Social Mobility

If depreciation rate is 100% ($\delta = 1$), the TFP parameter a_1 has no effect on the transmission of inequality. To see this clearly, note that a_1 influences the social mobility through the term χ in (14); when $\delta = 1$, χ is independent of a_1. However, in case of incomplete depreciation ($0 < \delta < 1$), a lower a_1 slows down social mobility.

[7] The social mobility is purely determined by the inverse of ϱ (see Benabou, 2002), which is also the focal point of this paper.

To see the intuition, think of a lower a_1 as a higher tax on capital.[8] Such a tax discourages investment. It hurts poor households more than the rich because poor have a higher marginal product of capital to start with due to diminishing returns to investment and incomplete markets. Start from a scenario, where rich (i with $h_t/h_{it} < 1$) and poor (j with $h_t/h_{jt} > 1$) have the same luck, the relative marginal investment returns of rich to poor (based on (1), (5) and (8)) is given by,

$$\frac{\partial h_{it+1}/\partial s_{it}}{\partial h_{jt+1}/\partial s_{jt}} = \left(\frac{a_1 \varphi \left(h_t/h_{it} \right)^{1-\alpha} + 1 - \delta}{a_1 \varphi \left(h_t/h_{jt} \right)^{1-\alpha} + 1 - \delta} \right)^{1-\theta} \tag{16}$$

which is less than unity**.** If a capital income tax is in place (lower a_1), it will boost the above relative marginal investment returns of rich further. Thus the convergence will be slower. Note that if there is complete depreciation ($\delta = 1$) this relative marginal product is independent of a_1.

2.6. Distributional Dynamics

We are now ready to characterize the dynamics of the cross sectional variance of human capital:

Proposition 1. *Given the initial cross sectional inequality characterized by (6) and (10), the dynamics of inequality and growth are given by the following laws of motion respectively,*

$$\sigma_{t+1}^2 = \theta^2 \ln \frac{\kappa^2 \exp\left(\theta^{-2}\sigma_t^2\right) + \left(a_1\right)^2 \exp\left(\left(0.5\omega + \lambda^2\right)\sigma_t^2 + \upsilon^2\right) + 2\kappa a_1 \exp\left(\left(0.5\omega + \lambda/\theta\right)\sigma_t^2\right)}{\left(\kappa + a_1 \exp\left(0.5\omega\sigma_t^2\right)\right)^2} \tag{17}$$

and

$$\gamma_{t+1} = \ln \phi + 0.5 \left(1/\theta - 1\right) \left(\sigma_t^2 - \sigma_{t+1}^2\right) + \theta \ln \left(\kappa + a_1 \exp\left(0.5\omega\sigma_t^2\right)\right) \tag{18}$$

[8]Define the capital income tax rate as τ. Re-parameterize a_1 as $1 - \tau$.

where

$$\gamma_{t+1} \equiv \ln h_{t+1} - \ln h_t$$
$$\kappa \equiv 1 - \delta,\ \lambda \equiv 1/\theta + \alpha - 1 > 0$$
$$\omega \equiv (\alpha - 1)(2/\theta + \alpha - 2) < 0$$

Proof. See Appendix B. ■

The social mobility and the dynamics of (the cross-sectional) wealth inequality are, therefore, characterized jointly by four crucial parameters, namely θ, δ, α and a_1. The dynamics of inequality is determined by its own history. It is not influenced by growth. On the other hand, the growth rate depends on the current and past inequality. This is evident by the fact that σ_{t+1}^2 is a function of σ_t^2 alone while γ_{t+1} depends on σ_{t+1}^2 and σ_t^2.

For comparative statics results, take the loglinear version in (11), which implies the following inequality dynamics (see Appendix A),[9]

$$\sigma_{t+1}^2 = \varrho^2 \sigma_t^2 + \chi^2 \upsilon^2 \tag{19}$$

The dynamics of income inequality ($\sigma_{t,y}^2$) can then be derived from (1) and (19),

$$\sigma_{t+1,y}^2 = \varrho^2 \sigma_{t,y}^2 + \upsilon^2 \left(1 - \varrho^2 + \alpha^2 \chi^2\right) \tag{20}$$

The capital share parameter, α, and the adjustment cost parameter, θ, have opposing effects on the persistence of inequality. When α is higher, the relative growth potential of the poor with respect to rich (due to poor's higher marginal product) is dampened which means that the process of convergence between rich and the poor will be slower. This explains why the initial inequality will tend to persist when α

[9] Because of its highly nonlinear nature, it is difficult to ascertain the comparative statics effects of these four underlying parameters. We study them numerically in the next section. The numerical comparative results accord with the loglinearized version.

is higher.[10] On the other hand, a higher adjustment cost (lower θ) will make the process of convergence between the poor and the rich slower because it is costly for the poor to invest. This technological disadvantage imposed by capital adjustment cost is compounded by the credit market imperfection which makes the social mobility slower and inequality more persistent.[11] These effects of θ and α on the social mobility are remarkably robust to alternative environments. In Appendix C, we outline a model with dynastic altruism as in Barro (1974) and derive the same results.

Why does a lower depreciation rate make the inequality process more persistent? When human capital depreciates slowly, it makes the current capital stock to have a stronger negative effect on investment as shown in Figure 2 and eq. (8). Such a negative effect on investment arises due to a higher marginal cost of investment when capital is long lived. Since investment is the only vehicle for social mobility, lower investment in human capital caused by low depreciation slows down this process of social mobility and makes the inequality more persistent. The following proposition summarizes the results for the social mobility.

Proposition 2. *A higher degree of adjustment cost (lower θ), lower depreciation cost (δ) and a higher capital share α make the social mobility slower and the inequality process more persistent.*

We next turn to the relationship between the short run dynamics of growth and inequality. The loglinear version of the growth equation is given by (see Appendix A),

$$\gamma_{t+1} = \ln \phi + \theta \left[\ln \left(1 - \delta + a_1 \right) \right] + 0.5 \upsilon^2 \chi \left(\chi - 1 \right) + 0.5 \sigma_t^2 \varrho \left(\varrho - 1 \right) \tag{21}$$

The growth rate (γ_{t+1}) at $t+1$ responds inversely to state of cross sectional inequality (σ_t^2). The strength of this negative association depends positively on the

[10]The inverse relationship between rate of convergence and the capital share parameter is well known in the convergence literature (see for example, Benabou, 2002).

[11]When α is close to unity, the adjustment cost ceases to play any role in determining the inequality persistence because the poor do not have any relative advantage in terms of higher marginal product.

degree of adjustment cost in human capital. This inverse relationship is not surprising in a model with imperfect credit market. In such models, due to diminishing returns to capital, $\alpha \in (0,1)$, the poor have a higher marginal product than the rich in the economy. When they cannot borrow from the rich who have a lower marginal product and invest due to the credit market imperfection, Pareto efficiency cannot be achieved. Therefore, in such an economy higher inequality corresponds to a greater inefficiency and thus lower growth.

2.7. Long-run inequality and growth

Until now we explored how initial differences in human capital, luck, depreciation and adjustment costs determine the transmission of inequality across generations. We found that initial inequality tends to lose importance while other parameters including θ, δ, α govern the intergenerational transmission of inequality. We now formally establish that the long-run inequality and growth are independent of initial wealth difference and determined crucially by these parameters.

The steady-state inequality and growth based on the closed form solutions (17) and (18) are given by the following expressions (derived in the appendix) respectively:

$$\sigma^2 = \theta^2 \ln \frac{\kappa^2 \exp\left(\theta^{-2}\sigma^2\right) + (a_1)^2 \exp\left(\left(0.5\omega + \lambda^2\right)\sigma^2 + \upsilon^2\right) + 2\kappa a_1 \exp\left(\left(0.5\omega + \lambda/\theta\right)\sigma^2\right)}{\left(\kappa + a_1 \exp\left(0.5\omega\sigma^2\right)\right)^2} \tag{22}$$

and

$$\gamma = \ln\phi + \theta \ln\left\{\kappa + a_1 \exp\left(0.5\omega\sigma^2\right)\right\} \tag{23}$$

The steady-state equivalents based on the log-linearized version of the model have simpler expressions. Considering (19),

$$\sigma^2 = \chi^2\upsilon^2 / \left(1 - \varrho^2\right) \tag{24}$$

The steady-state income inequality (σ_y^2) based on (20) is:[12]

[12]Note that when $\delta = 1$, all of the loglinearized and the exact solutions converge. For instance, the steady-state inequality in both (22) and (24) will reduce to $\sigma^2 = \upsilon^2\theta / \left(\left(1-\alpha\right)\left(2 - \left(1-\alpha\right)\theta\right)\right)$.

$$\sigma_y^2 = \upsilon^2 \left(1 - \varrho^2 + \alpha^2 \chi^2\right) / \left(1 - \varrho^2\right) \tag{25}$$

Finally, the long-run growth rate is given by, from (21) and (24):

$$\gamma = \ln \phi + \theta \left[\ln \left(1 - \delta + a_1\right)\right] + 0.5\upsilon^2 \chi \left(\chi - (1 + \varrho)\right) / \left(1 + \varrho\right) \tag{26}$$

Inequality in the long-run is thus mainly the result of individuals' differences in human capital investment decision as a response to differences in luck. We summarize these results in terms of the following proposition:

Proposition 3. *The long-run distribution of wealth (σ^2) is a function of initial distribution in luck (υ^2) and independent of the initial distribution of σ_0^2 whereas σ^2 increases in α and decreases with respect θ, δ and a_1.*

Proof. See (24) and (25). ■

Higher capital share (α) slows down the convergence between rich and poor and thus not surprisingly it promotes long run inequality. What is surprising is that θ, δ and a_1 have opposite effects on social mobility (ϱ) and long run inequality (σ^2). For example, a lower θ, δ, a_1 raise ϱ (slowing down social mobility) but it also lowers the long run inequality σ^2. To see the reasons for this asymmetry check the following two expressions for elasticities of child's human capital with respect to parent's human capital and parent's luck based on (11):

$$\partial \ln h_{it+1} / \partial \ln h_{it} = \varrho \tag{27}$$

$$\partial \ln h_{it+1} / \partial \ln \varphi_{it} = \chi \tag{28}$$

These two elasticities behave exactly opposite in response to change in θ, δ or a_1. Since the long run variance of inequality is determined by luck (but not by the initial level of inequality), the luck effect finally dominates and this explains the asymmetry of comparative statics behavior of σ^2 and ϱ.

3. Quantitative analysis

In this section, we numerically examine (17) and (22) to determine the role of θ and δ in social mobility, inequality dynamics and balanced growth respectively. We first construct parameter values which reasonably reflect actual economies. Assuming a psychological discount factor of 0.96, we set $\beta = 0.96^{30} \approx 0.3$, in a period of 30 years (de la Croix and Michel, 2002, p.255).[13]

There are five technology parameters, namely a_1, a_2, α, δ and θ. Following Barro, Mankiw and Sala-i-Martin, 1995 , we set $\alpha = 0.5$. The choices of $\upsilon^2 = 0.38$, $a_1 = 1.96$ and $a_2 = 1.655$ are made to target a steady state variance of the log wealth (σ^2) equal to 0.2422 and the variance of log of income (σ_y^2) equal to 0.441 and a long-run annual average growth rate of about 1.79 percent of the US economy for the last 125 years.[14] The remaining two parameters δ and θ are chosen to target the social mobility parameter ϱ as in (27) and the wealth elasticity with respect to luck, χ based on (28). Regarding ϱ the estimates for intergenerational persistence measured by wealth elasticity vary considerably in the literature.[15] Our baseline estimate of social mobility is 0.73 which is in line with the estimates of Mazumder and Clark and Cummins (2012).

The wealth elasticity with respect to luck, χ is an indicator of agent's response to luck or opportunity. We use the response rate of households from the well known move to opportunity programme (MTO) as reported by Katz et al., (2001) as a proxy for the agent's response to luck. Katz et al. (2001) report that about 48% to 62% households living in high poverty region in Boston move through the MTO programme. Fixing $\delta = 0.03$ and $\theta = 0.8$ as in Basu et al. (2012), we get an estimate for this response to luck around 0.53 which is in the range of Katz et al.'s (2001)

[13] A psychological discount factor of 0.96 matches a 4.17 percent rate of time preference ρ in an infinite lived agent model. That is, $\beta = 1/(1+\rho) = 1/(1+.0417) = 0.96$.

[14] Assuming a lognormal distribution of income, the mean-median ratio implies 0.44 average log-income variance for the United States for the years 1991, 1994, 1997, and 2000, based on Luxembourg Income Study (UNU-WIDER, 2007).

[15] Note that $\varrho^2 \simeq \rho \equiv \partial\sigma_{t+1}^2/\partial\sigma_t^2$ (Appendix B) . ϱ in each table is calculated based on the loglinearized version (11) which is close to the estimate based on the exact solution (17).

study.[16] Table 1 summarizes the baseline parameter values.

Tables 2, 3 and 4 show the effects of adjustment cost, depreciation costs and the TFP on social mobility, inequality and growth based on the exact closed form solutions (17), (18). Higher adjustment cost and lower depreciation cost (lower θ, lower δ, lower a_1), slow down social mobility (higher ϱ). On the other hand, the same factors contribute to a lower long-run growth rate (γ) and lower steady state inequality for the reasons mentioned earlier.

[16]To the best of our knowledge for the adjustment cost parameter there is no direct estimate available in the extant literature except Basu et al. (2012) . However, Basu et al. employ physical capital adjustment cost technology while in the present setting we have a human capital adjustment cost technology. However, the same calibrated value for θ generates a plausible estimate for the response to luck.

Table 1: Benchmark values

Preference and technology parameters:	$\beta = 0.3$, $a_1 = 1.96$, $a_2 = 1.655$
Production and policy parameter:	$\alpha = 0.5$, $\theta = 0.8$, $\delta = 0.03$
Inequality parameter	$\upsilon^2 = 0.38$

Table 2: Effects of adjustment cost on inequality, mobility and growth

Adjustment cost (θ)	ϱ	σ^2	γ
0.9	0.6990	0.2793	0.0477
0.85	0.7157	0.2605	0.0317
0.8	0.7324	0.2422	0.0179
0.75	0.7491	0.2243	0.0066
0.7	0.7659	0.2069	-0.0021
0.6	0.7993	0.1732	-0.0108

Figure 3 demonstrates the effects of changes in adjustment and depreciation costs respectively on the distributional dynamics. Given the baseline values of other parameters, a higher adjustment cost (from $\theta = 0.8$ to $\theta = 0.6$) slows down the social mobility by about 10 generations. A lower rate of depreciation by 2% slows down the convergence by about 4 generations.

Figure 4 compares growth dynamics with respect to changes in δ, θ and a_1. The growth dynamics is driven by the distributional dynamics shown in (18). The figure shows transitional dynamics following a 1% shock to inequality from its steady-state

Table 3: Effects of depreciation cost on inequality, mobility and growth

Depreciation cost (δ)	ϱ	σ^2	γ
0.2	0.7159	0.2581	-0.0341
0.15	0.7210	0.2532	-0.0184
0.13	0.7230	0.2513	-0.0122
0.10	0.7259	0.2485	-0.003
0.05	0.7306	0.244	0.012
0.03	0.7324	0.2422	0.0179
0.01	0.7342	0.2405	0.0238

Table 4: Effects of TFP on inequality, mobility and growth

TFP (a_1)	ϱ	σ^2	γ
1.68	0.7464	0.2290	-0.0591
1.82	0.7391	0.2359	-0.0196
1.96	0.7324	0.2422	0.0179
2.0	0.7306	0.2480	0.0538
2.1	0.7264	0.2533	0.0880
2.24	0.7209	0.2582	0.1208
2.52	0.7112	0.2627	0.1524

level. The sudden rise in inequality results in a sharp fall in growth, initially. But eventually it starts to pick up as inequality declines towards its steady-state. The speed of convergence is lower for higher adjustment cost (lower θ), lower depreciation and TFP (a_1).

3.1. Effect of luck on social mobility

Parent's luck impacts the difference in wealth of their children immediately through the optimal investment function (8). This inequality transmits to the future generations through adjustment of human capital stock. If it is costly to adjust human capital, luck effect of parents could persist over generations. To see this, start from a steady state where all agents are identical in terms of human capital and let the initial generations experience some lucks (say, ith family enjoys a good luck and jth family suffers a bad luck). Figures 3 and 4 plot (15) the impulse responses of 0.1 standard deviation difference in such luck on the time path of dynastic inequality for two values of the adjustment cost parameters ($\theta = 0.8$ and $\theta = 0.6$). When $\theta = 0.8$ convergence occurs after 20 generations whereas it takes more than 25 generations to converge when $\theta = 0.6$. This reinforces our earlier result that a higher adjustment cost slows down social mobility.

Figure 5 plots the same impulse responses when α is increased from its baseline value. Such an increase in α slows down convergence considerably for well known reasons mentioned earlier. The social mobility slows down for at least 10 generations in response to such an increase.

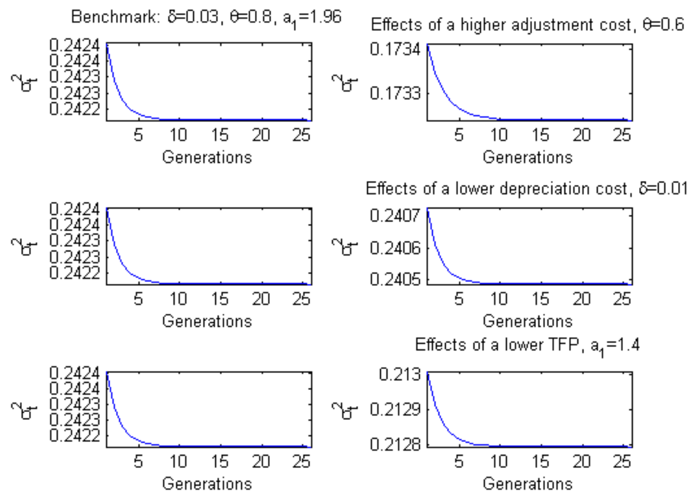

Figure 3: Adjustment, depreciation costs and TFP, with inequality dynamics.

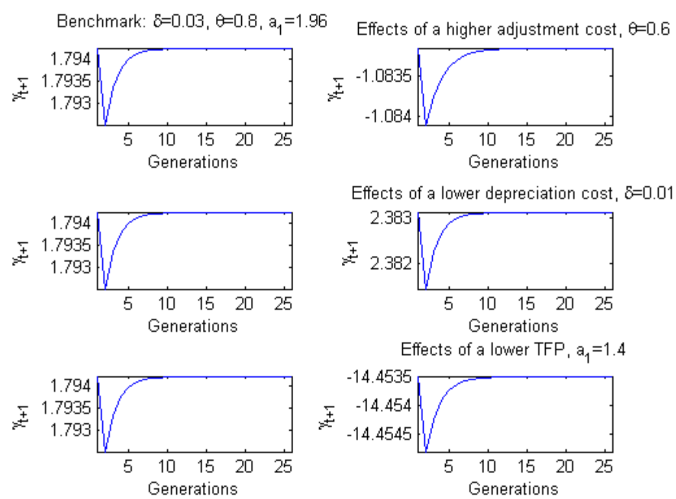

Figure 4: Adjustment, depreciation costs and TFP, with growth dynamics.

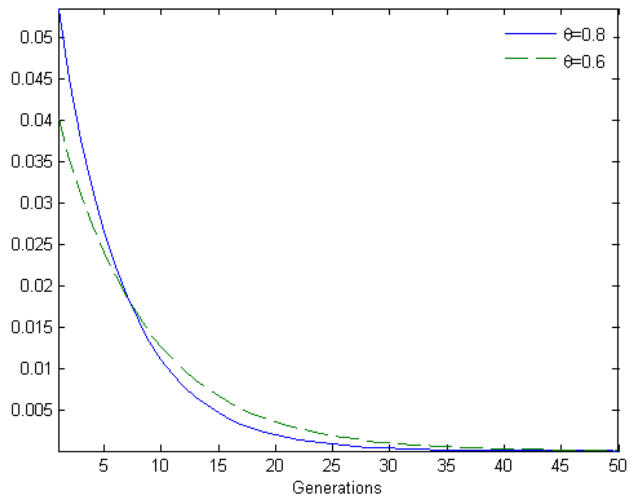

Figure 5: Effect of a difference in luck on human capital when $\theta = 0.6$ and $\theta = 0.8$

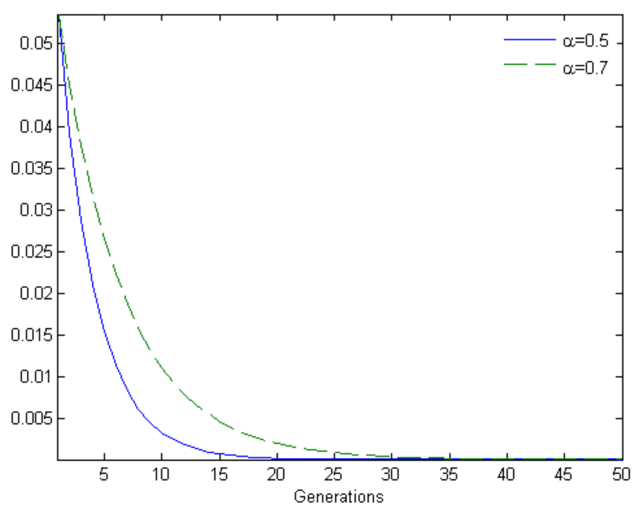

Figure 6: Effect of a difference in luck on human capital when $\alpha = 0.5$ and $\alpha = 0.7$

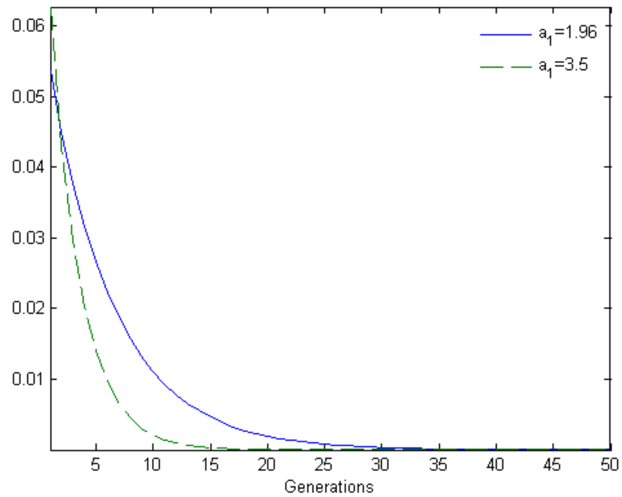

Figure 7: Effect of a difference in luck on human capital when $a_1 = 1.96$ and $a_1 = 3.5$

A change in depreciation alone has an insignificant effect on the transmission of inequality (plots of which we omit here for brevity). However, incomplete depreciation greatly influences the effect of TFP on social mobility for reasons mentioned earlier. Figure 7 demonstrates the effect of a lower a_1 on the impulse responses with respect to luck. In response to such a lower TFP, the social mobility is thwarted for at least 17 generations for the reasons mentioned earlier.

3.2. Welfare effect of adjustment cost and public policy implications

Our model suggests that a higher adjustment cost of human capital slows down social mobility. What is the implication for such slow mobility for social welfare? To properly address this issue, we need to have the right welfare metric. In the overlapping generations setting, adults born at different times attain different felicity levels. Thus the evaluation of social welfare also depends on the weights that the social planner assigns to various generations. For our present experiment, we assume that the social planner uses the same discount factor as the private agents to aggregate utilities across generations. It turns out that the comparative statics properties of social welfare is robust to the choice of the social planner's discount factor.

The social welfare is calculated in three steps. In the first step, the tth cohort's welfare ($w_{it} = \ln c_{it} + \beta \ln h_{it+1}$) is computed using the loglinearized decision rules for c_{it} and h_{it+1}. In the second step, we compute the aggregate welfare ($w_t = \int_i w_{it} di$). In the final step, we work out the social welfare (W_0) of all generations born since date 0 using a social discount rate ρ as follows:

$$W_0 = \textstyle\sum_{t=0}^{\infty} \rho^t w_t \tag{29}$$

In Appendix D, we have shown that the social welfare function (29) is given by:

$$W_0 = q_1 + \frac{(1+\beta)\,\rho}{(1-\rho)^2} \ln(1+\gamma) + \frac{1+\beta}{1-\rho} \ln h_0 + \frac{m_2}{(1-\varrho^2\rho)} \sigma_0^2 \tag{30}$$

where q_1 and m_2 are constants,

$$\begin{aligned} q_1 &\equiv \left(\ln \frac{a_1 + 1 - \delta}{1 + \theta\beta} + \beta\xi - 0.5\,(n_2 + \beta\chi)\,\upsilon^2 \right) (1-\rho)^{-1} \\ &\quad + \sigma^2 \rho \left(1 - \varrho^2\right) m_2 / \left((1-\rho)\left(1 - \varrho^2 \rho\right)\right) \\ m_2 &\equiv -0.5\,(1 + n_1 + \beta\varrho) \end{aligned}$$

An important observation based on the social welfare function (30) is that the steady state welfare depends positively on the steady state growth rate (γ) as well as the initial capital stock stock (h_0) but negatively on the initial inequality of human capital (σ_0^2). The negative relation between initial inequality and welfare reflects the usual efficiency loss due to the imperfection in the credit markets, which prevents the efficient amount of investment to be undertaken in the economy. A higher adjustment cost (lower θ) by slowing down social mobility makes this adverse effect of initial inequality stronger. Recall that a higher adjustment cost (lower θ) slows down the social mobility (higher ϱ). Figure 8 plots the social welfare against θ fixing the other parameters at the baseline levels. The relationship is clearly monotonic suggesting that a higher adjustment cost lowers the social welfare.

This loss of welfare in high adjustment cost economies can be mitigated by a possible subsidy to human capital which promotes the total factor productivity. The

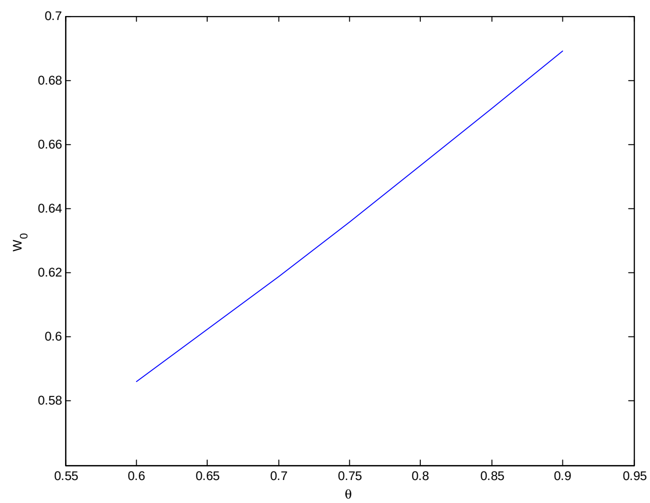

Figure 8: Adjustment cost and social welfare

government can lower the capital income tax, for instance, which boosts a_1. We have seen earlier that a lower capital income tax also promotes social mobility. Figure 9 shows that it also stimulates social welfare through the usual growth channel as seen from Table 4.

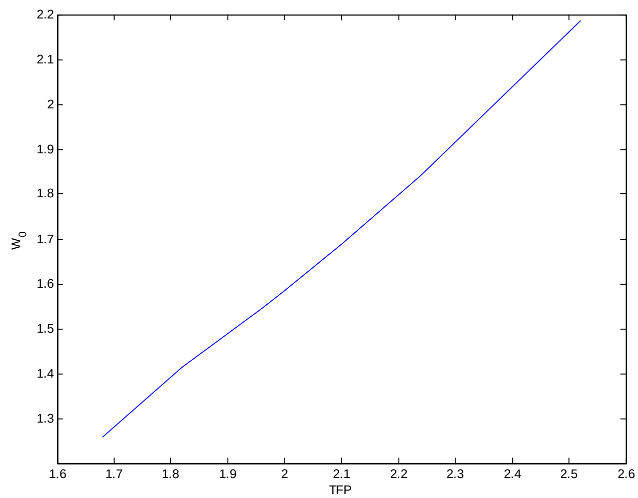

Figure 9: TFP and social welfare

4. Conclusion

This paper has developed models that analyze the distributional effects of human capital adjustment cost within an incomplete market and a heterogeneous economy. The source of endogenous inequality is missing credit and insurance markets. When individuals cannot perfectly insure themselves from future income uncertainty and, the credit market is imperfect, inequality persists. The dynamics of aggregate variables and inequality are jointly determined. The presence of a higher adjustment cost and lower depreciation cost for human capital slows down social mobility and results in a persistent inequality across generations. A lower TFP aggravates social immobility further when there is incomplete depreciation of capital. The quantitative analysis of our model suggests that the human capital adjustment cost has to be nontrivial to reconcile long run growth, inequality and social mobility. Social welfare is unambiguously lower in high adjustment cost economies with lower social mobility. A possible extension of our work is to examine the role of government redistributive policy in determining social mobility in an environment where it is costly to adjust human capital.

Appendix

A. Derivation of the loglinear form

To derive (11), first rewrite the ith individual optimal human capital accumulation (10) as

$$\ln h_{it+1} = \ln\phi + \ln h_{it} + \theta \ln\left(1 - \delta + a_1 \varphi_{it} \left(h_{it}/h_t\right)^{\alpha-1}\right) \tag{A.1}$$

Applying the first order Taylor expansion in (A.2) around $h_{it+1} = h_{t+1}$, $h_{it} = h_t$ and $\varphi_{it} = \varphi = 1$, one obtains

$$\ln h_{it+1} \simeq \ln\phi + \theta \ln\left(1 - \delta + a_1\right) + \ln h_{it} + (\alpha - 1)\,\chi \ln \widetilde{h}_{it} + \chi \ln\varphi_{it} \tag{A.2}$$

where $\ln\widetilde{h}_{it} \equiv \ln h_{it} - \ln h_t$, $\chi \equiv \theta a_1/\left(1 - \delta + a_1\right)$ and $\varrho \equiv 1 - \chi(1 - \alpha)$, which is rewritten as (11).

To derive the loglinearized version of the growth eqs. (21) and (26), aggregate (A.2),

$$\begin{aligned} \mathrm{E}\left[\ln h_{it+1}\right] &= \ln\phi + \theta\ln\left(1-\delta+a_1\right) + \mathrm{E}\left[\ln h_{it}\right] + (\alpha-1)\,\chi\,\mathrm{E}\left[\ln\widetilde{h}_{it}\right] + \chi\,\mathrm{E}\left[\ln\varphi_{it}\right] \\ \ln h_{t+1} - 0.5\sigma_{t+1}^2 &= \ln\phi + \theta\ln\left(1-\delta+a_1\right) + \left(\ln h_t - 0.5\sigma_t^2\right) - 0.5\left(\alpha-1\right)\chi\sigma_t^2 - 0.5\chi\upsilon^2 \end{aligned} \tag{A.3}$$

since $\mathrm{E}\left[\ln h_{it}\right] = \ln h_t - 0.5\sigma_t^2$. Rearranging and simplifying this, one obtains,

$$\gamma_{t+1} = \ln\phi + \theta\left[\ln\left(1-\delta+a_1\right)\right] - 0.5\left(1 + (\alpha-1)\,\chi\right)\sigma_t^2 - 0.5\chi\upsilon^2 + 0.5\sigma_{t+1}^2 \tag{A.4}$$

where $\gamma_{t+1} \equiv \ln h_{t+1} - \ln h_t$. Substituting (19) into the above yields:

$$\gamma_{t+1} = \ln\phi + \theta\left[\ln\left(1-\delta+a_1\right)\right] + 0.5\chi\left(\chi-1\right)\upsilon^2 + 0.5\varrho\left(\varrho-1\right)\sigma_t^2$$

B. Aggregation and distribution dynamics

In this section we derive (17) from (10). We can also rewrite (10) as

$$(h_{it+1})^{\varsigma} = \phi^{\varsigma}\left(h_{it}^{\varsigma}\kappa + \epsilon_t \varphi_{it} h_{it}^{\varkappa+\varsigma}\right) \tag{B.5}$$

where $\varsigma \equiv 1/\theta$, $\varkappa \equiv \alpha - 1$, $\kappa \equiv 1 - \delta$ and $\epsilon_t \equiv a_1 h_t^{1-\alpha}$.

Recall that first φ_{it} and h_{it} are assumed to have lognormal distributions:

$$\ln \varphi_{it} \sim N(-\upsilon^2/2, \upsilon^2) \tag{B.6}$$

$$\ln h_{it} \sim N(\mu_t, \sigma_t^2) \tag{B.7}$$

And, from a normal-lognormal relationship, we have:

$$\mathrm{E}\left[h_{it}\right] \equiv h_t = e^{\mu_t + 0.5\sigma_t^2} \tag{B.8}$$

$$\mathrm{var}\left[h_{it}\right] = \left(e^{\sigma_t^2} - 1\right) e^{2\mu_t + \sigma_t^2} \tag{B.9}$$

If h_{it}, then $(h_{it})^x$ is also lognormal for any constant x.

$$\ln (h_{it})^x \sim N(x\mu_t, x^2\sigma_t^2) \tag{B.10}$$

Thus, considering (B.10), (B.8) and (B.9), we have:

$$\mathrm{E}\left[(h_{it})^x\right] = (h_t)^x\, e^{0.5\sigma_t^2 x(x-1)} \tag{B.11}$$

$$\mathrm{var}\left[(h_{it})^x\right] = (h_t)^{2x}\, e^{\sigma_t^2 x(x-1)} \left(e^{x^2\sigma_t^2} - 1\right) \tag{B.12}$$

We now apply (B.11) and (B.12) to derive the following important relations that we use latter on:

$$\mathrm{E}\left[(h_{it+1})^{\varsigma}\right] = h_{t+1}^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_{t+1}^2} \tag{B.13}$$
$$\mathrm{E}\left[h_{it}^{\varsigma}\right] = h_t^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_t^2} \tag{B.14}$$
$$\mathrm{E}\left[h_{it}^{\varsigma+\varkappa}\right] = h_t^{\varsigma+\varkappa} e^{0.5(\varsigma+\varkappa)(\varsigma+\varkappa-1)\sigma_t^2} \tag{B.15}$$
$$\mathrm{E}\left[h_{it}^{2\varsigma+\varkappa}\right] = h_t^{2\varsigma+\varkappa} e^{0.5(2\varsigma+\varkappa)(2\varsigma+\varkappa-1)\sigma_t^2} \tag{B.16}$$
$$\mathrm{E}\left[\varphi_{it}\right] = 1 \tag{B.17}$$
$$\mathrm{var}\left[h_{it+1}^{\varsigma}\right] = h_{t+1}^{2\varsigma} e^{\varsigma(\varsigma-1)\sigma_{t+1}^2}\left(e^{\varsigma^2\sigma_{t+1}^2} - 1\right) \tag{B.18}$$
$$\mathrm{var}\left[\varphi_{it}\right] = \left(e^{\upsilon^2} - 1\right) \tag{B.19}$$
$$\mathrm{var}\left[h_{it}^{\varsigma}\right] = h_t^{2\varsigma} e^{\varsigma(\varsigma-1)\sigma_t^2}\left(e^{\varsigma^2\sigma_t^2} - 1\right) \tag{B.20}$$
$$\mathrm{var}\left[h_{it}^{\varsigma+\varkappa}\right] = h_t^{2(\varsigma+\varkappa)} e^{(\varsigma+\varkappa)(\varsigma+\varkappa-1)\sigma_t^2}\left(e^{(\varsigma+\varkappa)^2\sigma_t^2} - 1\right) \tag{B.21}$$

Then, aggregate (B.5) from both side to derive the aggregate human capital:

$$\mathrm{E}\left[(h_{it+1})^{\varsigma}\right] = \phi^{1/\theta}\,\mathrm{E}\left[h_{it}^{\varsigma}\kappa + \epsilon_t\varphi_{it}h_{it}^{\varsigma+\varkappa}\right] = \phi^{1/\theta}\left\{\kappa\,\mathrm{E}\left[h_{it}^{\varsigma}\right] + \epsilon_t\,\mathrm{E}\left[h_{it}^{\varsigma+\varkappa}\right]\right\}$$

Substituting (B.13), (B.14) and (B.15) into the above,

$$\begin{aligned} h_{t+1}^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_{t+1}^2} &= \phi^{\varsigma}\left\{\kappa h_t^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_t^2} + \epsilon_t h_t^{\varsigma+\varkappa} e^{0.5(\varsigma+\varkappa)(\varsigma+\varkappa-1)\sigma_t^2}\right\} \\ &= \phi^{\varsigma} h_t^{\varsigma}\left\{\kappa e^{0.5\varsigma(\varsigma-1)\sigma_t^2} + a_1 e^{0.5[\varsigma(\varsigma-1)+\varsigma\varkappa+\varkappa(\varsigma+\varkappa-1)]\sigma_t^2}\right\} \end{aligned}$$

Thus, the aggregate human capital is given by:

$$h_{t+1}^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_{t+1}^2} = \phi^{\varsigma} h_t^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_t^2}\left\{\kappa + a_1 e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\right\} \tag{B.22}$$

To derive the distributional dynamics, take the variance from both sides of (B.5),

$$
\begin{aligned}
\operatorname{var}\left[(h_{i,t+1})^{\varsigma}\right] &= \phi^{2\varsigma} \operatorname{var}\left[h_{it}^{\varsigma}\kappa + \epsilon_t \varphi_{it} h_{it}^{\varsigma+\varkappa}\right] \\
&= \phi^{2\varsigma}\left[\kappa^2 \operatorname{var}\left[h_{it}^{\varsigma}\right] + \epsilon_t^2 \operatorname{var}\left[\varphi_{it} h_{it}^{\varsigma+\varkappa}\right] + 2\kappa\epsilon_t \operatorname{cov}\left(h_{it}^{\varsigma}, \varphi_{it} h_{it}^{\varsigma+\varkappa}\right)\right] \qquad \text{(B.23)}
\end{aligned}
$$

Using (B.14), (B.15), (B.16) and (B.17), the cov term is computed as follows:

$$
\begin{aligned}
\operatorname{cov}\left(h_{it}^{\varsigma}, \varphi_{it} h_{it}^{\varsigma+\varkappa}\right) &= \mathrm{E}\left[h_{it}^{\varsigma}\varphi_{it} h_{it}^{\varsigma+\varkappa}\right] - \mathrm{E}\left[h_{it}^{\varsigma}\right]\mathrm{E}\left[\varphi_{it} h_{it}^{\varsigma+\varkappa}\right] \\
&= \mathrm{E}\left[h_{it}^{2\varsigma+\kappa}\right] - \mathrm{E}\left[h_{it}^{\varsigma}\right]\mathrm{E}\left[h_{it}^{\varsigma+\varkappa}\right] \\
&= h_t^{2\varsigma+\varkappa} e^{0.5(2\varsigma+\varkappa)(2\varsigma+\varkappa-1)\sigma_t^2} - h_t^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_t^2} h_t^{\varsigma+\varkappa} e^{0.5(\varsigma+\varkappa)(\varsigma+\varkappa-1)\sigma_t^2} \\
&= h_t^{2\varsigma+\varkappa} e^{0.5(\varkappa(2\varsigma+\varkappa-1)+2\varsigma(\varsigma-1))\sigma_t^2}\left(e^{\varsigma(\varsigma+\varkappa)\sigma_t^2} - 1\right) \qquad \text{(B.24)}
\end{aligned}
$$

The second term in the right hand side of (B.23) is computed as,[17]

$$
\begin{aligned}
\operatorname{var}\left[\varphi_{it} h_{it}^{\varsigma+\varkappa}\right] &= \left(\mathrm{E}\left[\varphi_{it}\right]\right)^2 \operatorname{var}\left[h_{it}^{\varsigma+\varkappa}\right] + \operatorname{var}\left[\varphi_{it}\right]\left(\left(\mathrm{E}\left[h_{it}^{\varsigma+\varkappa}\right]\right)^2 + \operatorname{var}\left[h_{it}^{\varsigma+\varkappa}\right]\right) \\
&= \operatorname{var}\left[h_{it}^{\varsigma+\varkappa}\right]\left(1 + \operatorname{var}\left[\varphi_{it}\right]\right) + \operatorname{var}\left[\varphi_{it}\right]\left(\mathrm{E}\left[h_{it}^{\varsigma+\varkappa}\right]\right)^2
\end{aligned}
$$

since $\mathrm{E}\left[\varphi_{it}\right] = 1$.

Substituting (B.15), (B.19) and (B.21) into the above yields:

[17] If x and y are independent, the variance of their product is:

$$
\operatorname{var}[xy] = \left(\mathrm{E}[x]\right)^2 \operatorname{var}[y] + \left(\mathrm{E}[y]\right)^2 \operatorname{var}[x] + \operatorname{var}[y]\operatorname{var}[x]
$$

$$
\begin{aligned}
\operatorname{var}\left[\varphi_{it} h_{it}^{\varsigma+\varkappa}\right] &= h_t^{2(\varsigma+\varkappa)} e^{(\varsigma+\varkappa)(\varsigma+\varkappa-1)\sigma_t^2} \left(e^{(\varsigma+\varkappa)^2\sigma_t^2} - 1\right)\left(1 + \left(e^{\upsilon^2} - 1\right)\right) \\
&+ \left(e^{\upsilon^2} - 1\right) h_t^{2(\varsigma+\varkappa)} e^{(\varsigma+\varkappa)(\varsigma+\varkappa-1)\sigma_t^2} \\
&= h_t^{2(\varsigma+\varkappa)} e^{(\varsigma+\varkappa)(\varsigma+\varkappa-1)\sigma_t^2} \left(\left(e^{(\varsigma+\varkappa)^2\sigma_t^2} - 1\right) e^{\upsilon^2} + \left(e^{\upsilon^2} - 1\right)\right) \\
&= h_t^{2(\varsigma+\varkappa)} e^{(\varkappa(2\varsigma+\varkappa-1)+\varsigma(\varsigma-1))\sigma_t^2} \left(e^{(\varsigma+\varkappa)^2\sigma_t^2} e^{\upsilon^2} - 1\right) \qquad \text{(B.25)}
\end{aligned}
$$

Then, substituting, (B.18), (B.20), (B.24) and (B.25) into (B.23) yields:

$$
\begin{aligned}
& h_{t+1}^{2\varsigma} e^{\varsigma(\varsigma-1)\sigma_{t+1}^2} \left(e^{\varsigma^2\sigma_{t+1}^2} - 1\right) \\
&= \phi^{2\varsigma} \left[\begin{array}{c} h_t^{2\varsigma\kappa 2} e^{\varsigma(\varsigma-1)\sigma_t^2} \left(e^{\varsigma^2\sigma_t^2} - 1\right) \\ +\epsilon_t^2 \left\{ h_t^{2(\varsigma+\varkappa)} e^{[\varkappa(2\varsigma+\varkappa-1)+\varsigma(\varsigma-1)]\sigma_t^2} \left(e^{(\varsigma+\varkappa)^2\sigma_t^2} e^{\upsilon^2} - 1\right)\right\} \\ +2\kappa\epsilon_t \left\{ h_t^{2\varsigma+\varkappa} e^{0.5[\varkappa(2\varsigma+\varkappa-1)+2\varsigma(\varsigma-1)]\sigma_t^2} \left(e^{\varsigma(\varsigma+\varkappa)\sigma_t^2} - 1\right)\right\} \end{array} \right] \qquad \text{(B.26)}
\end{aligned}
$$

Finally, substituting (B.22) into the above, we get :

$$
\begin{aligned}
& \phi^{2\varsigma} h_t^{2\varsigma} e^{\varsigma(\varsigma-1)\sigma_t^2} \left\{\kappa + a_1 e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\right\}^2 \left(e^{\varsigma^2\sigma_{t+1}^2} - 1\right) \\
&= \phi^{2\varsigma} h_t^{2\varsigma} \left[\begin{array}{c} \kappa^2 e^{\varsigma(\varsigma-1)\sigma_t^2} \left(e^{\varsigma^2\sigma_t^2} - 1\right) \\ +(a_1)^2 \left\{ e^{[\varkappa(2\varsigma+\varkappa-1)+\varsigma(\varsigma-1)]\sigma_t^2} \left(e^{(\varsigma+\varkappa)^2\sigma_t^2} e^{\upsilon^2} - 1\right)\right\} \\ +2\kappa a_1 \left\{ e^{0.5[\varkappa(2\varsigma+\varkappa-1)+2\varsigma(\varsigma-1)]\sigma_t^2} \left(e^{\varsigma\varkappa\sigma_t^2} - 1\right)\right\} \end{array} \right] \qquad \text{(B.27)}
\end{aligned}
$$

since $\epsilon_t \equiv a_1 h_t^{-\varkappa}$.

Considering,

$$
\left(\kappa + a_1 e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\right)^2 = \kappa^2 + 2\kappa a_1 e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2} + (a_1)^2 e^{(\varkappa(2\varsigma+\varkappa-1))\sigma_t^2}
$$

further simplifying (B.27) gives

$$e^{\varsigma^2\sigma_{t+1}^2} = \frac{\kappa^2 e^{\varsigma^2\sigma_t^2} + (a_1)^2\left(e^{\varkappa(2\varsigma+\varkappa-1)\sigma_t^2} e^{(\varsigma+\varkappa)^2\sigma_t^2} e^{\upsilon^2}\right) + 2\kappa a_1\left(e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2} e^{\varsigma(\varsigma+\varkappa)\sigma_t^2}\right)}{\left(\kappa + a_1 e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\right)^2}$$

Alternatively,

$$e^{\theta^{-2}\sigma_{t+1}^2} = \frac{\kappa^2 e^{\theta^{-2}\sigma_t^2} + (a_1)^2\left(e^{\left[(\alpha-1)(2/\theta+\alpha-2)+(1/\theta+\alpha-1)^2\right]\sigma_t^2+\upsilon^2}\right) + 2\kappa a_1\left(e^{[0.5(\alpha-1)(2/\theta+\alpha-2)+(1/\theta+\alpha-1)/\theta]\sigma_t^2}\right)}{\left(\kappa + a_1 e^{0.5(\alpha-1)(2/\theta+\alpha-2)\sigma_t^2}\right)^2}$$

after substituting $\varsigma \equiv 1/\theta$, $\varkappa \equiv \alpha - 1$. Or,

$$e^{\theta^{-2}\sigma_{t+1}^2} = \frac{\kappa^2 e^{\theta^{-2}\sigma_t^2} + (a_1)^2\left(e^{\left(\omega+\lambda^2\right)\sigma_t^2+\upsilon^2}\right) + 2\kappa a_1\left(e^{(0.5\omega+\lambda/\theta)\sigma_t^2}\right)}{\left(\kappa + a_1 e^{0.5\omega\sigma_t^2}\right)^2} \tag{B.28}$$

where

$$\omega \equiv (\alpha - 1)(2/\theta + \alpha - 2) < 0,\ \lambda \equiv 1/\theta + \alpha - 1 > 0$$

as given by (17).

B.1. Steady State

The steady-state inequality and growth are given by the following equations, considering (B.22) and (B.28), respectively:

$$\sigma^2 = \theta^2 \ln \frac{\kappa^2 \exp\left(\theta^{-2}\sigma^2\right) + (a_1)^2\upsilon^2 \exp\left(\left(\omega + \lambda^2\right)\sigma^2\right) + 2\kappa a_1 \exp\left((0.5\omega + \lambda/\theta)\,\sigma^2\right)}{\left(\kappa + a_1 \exp\left(0.5\omega\sigma^2\right)\right)^2} \tag{B.29}$$

and

$$\gamma = \ln\phi + \theta \ln\left\{1 - \delta + a_1 \exp\left(0.5\omega\sigma^2\right)\right\} \tag{B.30}$$

where $\gamma \equiv \ln(h_{t+1}/h)$ and $\sigma^2 = \sigma_{t+1}^2 = \sigma_t^2$.

B.2. Social mobility

The social mobility parameter, ψ, is derived by simply taking the first derivative of (B.28):

$$\begin{aligned}\psi &\equiv \partial\sigma_{t+1}^2/\partial\sigma_t^2 \\ &= \left(\frac{\kappa^2\theta^{-2}\exp(\theta^{-2}\sigma_t^2)+(a_1)^2 b_1 b_2 \exp(b_2\sigma_t^2)+2\kappa a_1 b_3 \exp(b_3\sigma_t^2)}{\kappa^2\exp(\theta^{-2}\sigma_t^2)+(a_1)^2 b_1\exp(b_2\sigma_t^2)+2\kappa a_1\exp(b_3\sigma_t^2)} - \frac{a_1\omega\exp(0.5\omega\sigma_t^2)}{\kappa+a_1\exp(0.5\omega\sigma_t^2)}\right)\theta^2\end{aligned}$$

where

$$b_1 \equiv \exp\left(\upsilon^2\right),\ b_2 \equiv \omega+\lambda^2,\ b_3 \equiv 0.5\omega+\lambda/\theta$$

C. A model of dynastic altruism with labour and capital

In this appendix, we show that the key result that a higher adjustment cost slows down social mobility continues to hold in a model with dynastic altruism as in Barro (1974) with labour and physical capital. Each generation lives one period and discounts the future generation's utility by β. The ith agent born at date t has the utility function:

$$v_{it} = \ln c_{it} + b\ln(1-l_{g,it}-l_{h,it}) + \beta E_t v_{it+1} \tag{C.31}$$

where $l_{g,it}$ is labour time spend on good production, $l_{h,it}$ is labour time spent on child's education and $b \in (0,1)$ is the relative importance of leisure in utility. The total time is normalized at unity. We assume for simplicity that the technology of goods production requires a fixed amount of raw labour time (say 8 hours a day) and the adult has no choice to allocate more or less time to it. Thus fix $l_{g,it} = l$. The remaining time can be allocated freely between leisure and child's education. The goods production function is thus:

$$x_{it} = \vartheta_{it}\left(lh_{it}\right)^{\varpi}\left(k_{it}\right)^{\eta}\left(h_t\right)^{\varepsilon} \tag{C.32}$$

where ϑ_{it} is idiosyncratic shock; k_{it} is physical capital and the production function obeys constant returns to scale property meaning $\varpi + \eta + \varepsilon = 1$.

The human capital production function is specified as (assuming $\delta = 1$ for analytical tractability).

$$h_{it+1} = h_{it}^{1-\theta}(s_{it} l_{h,it})^{\theta} \tag{C.33}$$

The effective investment in human capital is the raw labour ($l_{h,it}$) spent on children times resources (s_{it}) spent on schooling.

Although parents cannot borrow against their offspring's human capital because of the immutable moral hazard and adverse selection issues, they have an access to an international credit market to finance their purchase of physical capital, k_{it} at a fixed interest rate r^*. The adult fully pays off the loan with interest before the end of their life. All physical capital is used up in the production process and nothing is left for the upcoming generation. The optimal purchase of capital is thus given by the equality between the marginal product of physical capital and the user cost of capital, which means,

$$\partial x_{it} / \partial k_{it} = r^* + \delta_k \tag{C.34}$$

where δ_k is the rate of depreciation of physical capital.[18] The adult's value added (y_{it}) after paying off the loan servicing and the user costs of capital is given by,

$$y_{it} = x_{it} - (r^* + \delta_k) k_{it} \tag{C.35}$$

Substituting out k_{it} using (C.34) and (C.32), equation (C.35) can be rewritten as:

[18] To see this arbitrage condition, note that the adult's choice of physical capital solves the static maximization problem:

$$\max_{k_{it}} \left[x_{it} + (1 - \delta_k) k_{it} - (1 + r^*) k_{it} \right]$$

$$y_{it} = f\varphi_{it}\left(h_{it}\right)^{\alpha}\left(h_t\right)^{1-\alpha}$$

where $\varphi_{it} \equiv \left(\vartheta_{it}\right)^{1/(1-\eta)}$, $f \equiv l^{\varpi/(1-\eta)}\left(\eta/\left(r^{*}+\delta_k\right)\right)^{\eta/(1-\eta)}(1-\eta)$ and $\alpha \equiv \varpi/(1-\eta)$.

The ith adult is thus subject to the budget constraint:

$$c_{it} + s_{it} = y_{it} \tag{C.36}$$

and she maximizes (C.31), subject to (C.33) and (C.36), taking φ_{it}, l and h_t as given.

The value function for this problem can be written as:

$$v(h_{it}, z_{it}) = \max_{h_{it+1}}\left[\ln\left\{y_{it} - \frac{\left(h_{it+1}\right)^{1/\theta}}{l_{h,it}\left(h_{it}\right)^{(1-\theta)/\theta}}\right\} + b\ln(1-l-l_{h,it}) + \beta\,\mathrm{E}_t\,v(h_{it+1}, z_{it+1})\right] \tag{C.37}$$

The proof mimics Basu (1987). Conjecture that the value function is loglinear in state variables as follows:

$$v(h_{it}, z_{it}) = \pi_0 + \pi_1\ln h_{it} + \pi_2\ln z_{it} \tag{C.38}$$

where $z_{it} \equiv f\varphi_{it}h_t^{1-\alpha}$. Plugging (C.38) into the value function (C.37),

$$\begin{aligned} &\pi_0 + \pi_1\ln h_{it} + \pi_2\ln z_{it} \qquad \text{(C.39)} \\ &= \max_{h_{it+1}}\left[\ln\left\{y_{it} - \frac{\left(h_{it+1}\right)^{1/\theta}}{l_{h,it}\left(h_{it}\right)^{(1-\theta)/\theta}}\right\} + b\ln(1-l-l_{h,it}) + \beta\left\{\pi_0 + \pi_1\ln h_{it+1} + \ln z_{it+1}\right\}\right] \end{aligned}$$

We will use the method of undetermined coefficients to solve for π_i which only matters for determining the decision rule of investment. Conjecture that $l_{h,it}$ is time invariant and is equal to $l_{h,i}$.

Differentiating with respect to h_{it+1} and rearranging terms one gets:

$$h_{it+1} = \left(\pi_1\beta\theta/\left(1+\pi_1\beta\theta\right)\right)^{\theta}\left(h_{it}\right)^{\alpha\theta+1-\theta}\left(l_{h,it}\right)^{\theta}\left(z_{it}\right)^{\theta} \tag{C.40}$$

Plugging (C.40) into (C.39) and comparing the left hand side and right hand side coefficients of the value function we can uniquely solve π_1 as follows:

$$\pi_1 = \frac{\alpha}{1 - \beta(\alpha\theta + 1 - \theta)}$$

which after plugging into (C.40) we get,

$$h_{it+1} = \left(\alpha\beta\theta / \left(1 - \beta(1-\theta)\right)\right)^{\theta} \left(h_{it}\right)^{\alpha\theta+1-\theta} \left(l_{h,i}\right)^{\theta} \left(z_{it}\right)^{\theta} \tag{C.41}$$

Next solve $l_{h,i}$ by noting the fact that

$$l_{h,i} = \arg\max[b \ln(1 - l - l_{h,i}) + \beta\pi_1\theta \ln l_{h,i}]$$

which gives

$$l_{h,i} = \beta\pi_1\theta(1-l) / \left(b + \beta\pi_1\theta\right) = \frac{\beta\alpha\theta(1-l)}{b - b\beta(\alpha\theta + 1 - \theta) + \beta\alpha\theta} \tag{C.42}$$

Time devoted to education is thus a constant confirming our conjecture. Note that $l_{h,i}$ is also increasing in θ.

C.1. Distributional dynamics

Based on (C.41) the social mobility equation is given by,

$$\ln h_{it+1} = \overline{p} + (\alpha\theta + 1 - \theta) \ln h_{it} + \theta(\ln z_{it}) \tag{C.43}$$

where

$$\overline{p} \equiv \theta \ln\left(\frac{\beta\alpha\theta(1-l)}{b - b\beta(\alpha\theta + 1 - \theta) + \beta\alpha\theta}\right) + \theta \ln\left(\alpha\beta\theta / \left(1 - \beta(1-\theta)\right)\right)$$

Note that (C.43) takes the same form as (11) where the social mobility parameter is the same as ϱ when $\delta = 1$. The dynamics of cross sectional variance of wealth based on (C.43) is given by,

$$\sigma_{t+1}^2 = (\alpha\theta + 1 - \theta)^2 \sigma_t^2 + \theta^2 \upsilon^2 \tag{C.44}$$

which display similar properties as in our baseline "joy of giving" utility function. Higher adjustment cost (lower θ) and a higher human capital share parameter (α) slows down social mobility and raises the persistence of inequality as before.

C.1.1. Long run inequality and growth

Long run inequality based on (C.44) is given by:

$$\sigma^2 = \frac{\theta^2 \upsilon^2}{1 - (\alpha\theta + 1 - \theta)^2} \tag{C.45}$$

To get the long run growth, aggregate (C.43) :

$$\begin{aligned}
\mathrm{E}\left[\ln h_{it+1}\right] &= \overline{p} + (\alpha\theta + 1 - \theta)\,\mathrm{E}\left[\ln h_{it}\right] + \theta\,\mathrm{E}\left[\ln f \varphi_{it} h_t^{1-\alpha}\right] \\
&= \overline{p} + \ln f + (\alpha\theta + 1 - \theta)\,\mathrm{E}\left[\ln h_{it}\right] + \theta(1-\alpha)\ln h_t + \theta\,\mathrm{E}\left[\ln \varphi_{it}\right] \\
\ln h_{t+1} - 0.5\sigma_{t+1}^2 &= \overline{p} + \ln f + (\alpha\theta + 1 - \theta)(\ln h_t - 0.5\sigma_t^2) + \theta(1-\alpha)\ln h_t - 0.5\upsilon^2\theta
\end{aligned}$$

Along a balanced growth path, $\sigma_{t+1}^2 = \sigma_t^2 = \sigma^2$ as in (C.45) and $\gamma \equiv \ln h_{t+1} - \ln h_t$, this means,

$$\gamma = \overline{p} + \ln f + 0.5\theta(1-\alpha)\sigma^2 - 0.5\upsilon^2\theta$$

D. Steady state welfare

The date t welfare of the ith adult is given by:

$$w_{it} = \ln c_{it} + \beta \ln h_{it+1} \tag{D.46}$$

The term $\ln h_{it+1}$ is given by the loglinearized optimal decision rule for human capital accumulation (11).The optimal consumption rule ($\ln c_{it}$) is derived from (3) and (8),

$$c_{it} = y_{it} - \left(\left(y_{it}\theta\beta - (1-\delta)\, h_{it} \right) / \left(1+\theta\beta \right) \right) \tag{D.47}$$

which after loglinearization will yield

$$\ln c_{it} = n_0 + (1+n_1) \ln h_{it} - n_1 \ln h_t + n_2 \ln \varphi_{it} \tag{D.48}$$

where[19]

$$\begin{aligned} n_0 &\equiv \ln \frac{(a_1 + (1-\delta))}{1+\theta\beta} \\ n_1 &\equiv \frac{1-\delta+\alpha a_1}{a_1+1-\delta} \\ n_2 &\equiv \frac{a_1}{a_1+1-\delta} \end{aligned}$$

Plugging (11) and (D.48) on (D.46), the date t welfare of the ith household is given by,

$$w_{it} = n_0 + \beta\xi + (\beta(1-\alpha)\chi - n_1) \ln h_t + (1+n_1+\beta\varrho) \ln h_{it} + (\beta\chi + n_2) \ln \varphi_{it} \tag{D.49}$$

The social welfare at t is then given by:

$$\begin{aligned} w_t &= \textstyle\int_i w_{it} = \mathrm{E}\left[w_{it} \right] \\ &= n_0 + \beta\xi + (\beta(1-\alpha)\chi - n_1) \ln h_t + (1+n_1+\beta\varrho)\, \mathrm{E}\left[\ln h_{it} \right] + (\beta\chi + n_2)\, \mathrm{E}\left[\ln \varphi_{it} \right] \\ &= n_0 + \beta\xi + (\beta(1-\alpha)\chi - n_1) \ln h_t + (1+n_1+\beta\varrho) \left(\ln h_t - 0.5\sigma_t^2 \right) - 0.5\,(\beta\chi + n_2)\, \upsilon^2 \\ &= m_0 + m_1 \ln h_t + m_2 \sigma_t^2 \end{aligned}$$

[19] To obtain (D.48) first rewrite (D.47) after substituting for y_{it}

$$\ln (c_{it}/h_t) = \ln (1+\theta\beta)^{-1} + \ln (h_{it}/h_t) + \ln \left(a_1 \varphi_{it} \left(h_{it}/h_t \right)^{\alpha-1} + (1-\delta) \right)$$

Then, loglinearize this at constant points c_t/h_t, and $h_t/h_t = \varphi_t = 1$.

where

$$
\begin{aligned}
m_0 &\equiv n_0 + \beta\xi - 0.5\left(n_2 + \beta\chi\right)\upsilon^2 \\
m_1 &\equiv 1 + \beta \\
m_2 &\equiv -0.5\left(1 + n_1 + \beta\varrho\right)
\end{aligned}
$$

Now the social planner aggregates the welfare of all generations using own discount factor (ρ). The social welfare at date 0 is, then,

$$
\begin{aligned}
W_0 &= \textstyle\sum_{t=0}^{\infty} \rho^t w_t = \sum_{t=0}^{\infty} \rho^t \left(m_0 + m_1 \ln h_t + m_2 \sigma_t^2\right) \\
&= \frac{m_0}{1-\rho} + m_1 \textstyle\sum_{t=0}^{\infty} \rho^t \ln h_t + m_2 \sum_{t=0}^{\infty} \rho^t \sigma_t^2 \\
&= \frac{m_0}{1-\rho} + m_1 \textstyle\sum_{t=0}^{\infty} \rho^t \ln h_0 (1+\gamma)^t + m_2 \sum_{t=0}^{\infty} \rho^t \sigma_t^2 \\
&= \frac{m_0}{1-\rho} + m_1 \ln h_0 \textstyle\sum_{t=0}^{\infty} \rho^t + m_1 \ln(1+\gamma) \left(\sum_{t=0}^{\infty} \rho^t t\right) + m_2 \sum_{t=0}^{\infty} \rho^t \sigma_t^2 \\
&= \frac{m_0}{1-\rho} + \frac{m_1 \ln h_0}{1-\rho} + \frac{m_1 \rho}{(1-\rho)^2} \ln(1+\gamma) + m_2 \textstyle\sum_{t=0}^{\infty} \rho^t \left(\varrho^{2t}\left(\sigma_0^2 - \sigma^2\right) + \sigma^2\right) \\
&= \frac{m_0}{1-\rho} + \frac{m_1 \ln h_0}{1-\rho} + \frac{m_1 \rho}{(1-\rho)^2} \ln(1+\gamma) + \frac{m_2 \sigma^2}{(1-\rho)} + \frac{m_2}{(1-\varrho^2 \rho)} \left(\sigma_0^2 - \sigma^2\right) \\
&= q_1 + \frac{m_1 \rho}{(1-\rho)^2} \ln(1+\gamma) + \frac{m_1 \ln h_0}{1-\rho} + \frac{m_2}{(1-\varrho^2\rho)} \sigma_0^2
\end{aligned}
$$

since

$$
\sigma_t^2 = \varrho^2 \sigma_{t-1}^2 + \chi^2 \upsilon^2 \Rightarrow \sigma_t^2 = \varrho^{2t}\left(\sigma_0^2 - \sigma^2\right) + \sigma^2
$$

where

$$
\begin{aligned}
q_1 \equiv\ & \left(\ln \frac{a_1 + 1 - \delta}{1 + \theta\beta} + \beta\xi - 0.5\left(n_2 + \beta\chi\right)\upsilon^2\right) / (1-\rho) \\
& + \sigma^2 \rho \left(1 - \varrho^2\right) m_2 / \left((1-\rho)\left(1 - \varrho^2 \rho\right)\right)
\end{aligned}
$$

and $\sigma^2 = \chi^2 \upsilon^2 / (1 - \varrho^2)$.

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