Subgame-Perfect Cooperation in a Dynamic Game of Climate Change

Parkash Chander*

September 2012

Abstract

This paper formulates the climate change problem as a dynamic game and shows that it admits a unique open-loop Nash equilibrium as well as a unique closed-loop Nash equilibrium if the production and damage functions are quadratic. However, neither of these equilibriums satisfies inter-temporal efficiency. This motivates introducing a solution concept for the dynamic game, labeled a self-enforcing agreement, which satisfies inter-temporal efficiency and subgame perfection analogously to a subgame perfect Nash equilbrium in a dynamic game.

Keywords: Climate change, dynamic game, open-loop and closed-loop Nash equilibriums, inter-temporal efficiency, self-enforcing agreements, subgame-perfection.

JEL classification: C71-73, F53,H87, Q54

* Department of Economics, 1 Arts Link, AS2, Level 6, National University of Singapore, Singapore 117570. Email: ecsparka@nus.edu.sg.

1. Introduction

The root cause of the climate change problem is the fact that emission of greenhouse gases (GHGs) brings immediate benefits to the emitting country, but increases the stock of GHGs in the atmosphere which affects the present and future welfare of all countries. In the absence of any cooperation among the countries, each country when deciding its emissions takes account of only its own benefits and costs. As a result, the total emissions from all countries are too high compared to the emissions that are efficient from a global point of view.[1] In the absence of international institutions that can enforce environmental agreements among sovereign nations, the only way in which the countries can overcome this inefficiency, if at all, is by negotiating appropriate transfers among them in return for reducing their emissions.[2] Since the countries differ in costs and benefits regarding climate change mitigation as well as in their emissions of GHGs, such transfers should balance the costs and benefits of reducing emissions and induce voluntary cooperation.[3]

This paper addresses the problem of designing international agreements involving transfers among the sovereign countries that induce efficiency and are self-enforcing. It formulates the climate change problem as a dynamic game in discrete time in which the strategy sets of the players consist of sequences of emissions.[4] It shows that the dynamic game admits a unique open-loop Nash equilibrium as well as a unique closed-loop Nash equilibrium if the production and damage functions are quadratic. The open-loop Nash equilibrium is also closed–loop if the damage functions are linear. However, neither of these equilibriums is efficient. Therefore, appropriate transfers among the countries that can induce efficiency and cooperation among them are needed.[5]

[1] Other examples of similar international problems include ozone layer depletion, acid rain, and sea and ocean pollution, to name a few.

[2] Indeed, the current international negotiations following the United Nations Framework Convention on Climate Change (UNFCC) can be seen as an attempt to overcome this inefficiency.

[3] However, as Dockner and Long (1993) and Chander (2007) show, transfers may not be required if the countries are identical.

[4] Similar dynamic games have been studied previously by Harstad (2012), Dutta and Radner (2009), and Dockner and Long (1993) among others. Reinganum and Stokey (1985) study a dynamic game of resource extraction with a similar structure.

[5] Such transfers have been proposed in the current international negotiations on climate change and are implicit in the Kyoto Protocol via the instruments of differential caps and trade in emissions and the Clean Development Mechanism (see e.g. Chander, 2003). The Montreal Protocol on Substances that Deplete the Ozone Layer which has been hailed as an example of successful international cooperation explicitly proposes transfers.

In this paper, we identify a unique strategy profile that leads to an inter-temporally efficient outcome and streams of transfers among the countries such that no country or coalition of countries will have an incentive to choose a strategy which is different from the one in the strategy profile and forgo the agreed upon transfers. We interpret the unique strategy profile and such streams of transfers as self-enforcing agreements. A self-enforcing agreement has the property that, if countries in a coalition expect the other countries to adhere to the agreement *only if* they themselves do,[6] then it is in their self-interest to adhere to the agreement.[7]

Furthermore, an agreement, if it is to be effective, must be self-enforcing in *every* subgame and not only in the original game. To understand why, consider a stream of transfers which implies a payoff (summed over all periods) for a coalition such that the coalition will have no incentive to deviate in the beginning of the game. But the stream of transfers is such that it generates "too high" payoffs for the coalition in the early periods but "too low" payoffs in the later periods. For such a stream of transfers, the coalition may not have an incentive to deviate in the early periods, but may have it in later periods. An illustrative example below demonstrates this fact. Thus, to be effective an agreement involving transfers must be self-enforcing in every subgame along the history generated by the strategy profile leading to an efficient outcome. In other words, a self-forcing agreement must also satisfy subgame-perfection.[8]

Dockner et al. (1996) and Dutta and Radner (2009) propose cooperative solutions in dynamic games which can be supported as subgame-perfect equilibria. They rely on the use of a trigger strategy – once some country deviates from the agreement, punishment begins and continues forever, resulting in over accumulation of GHGs which may hurt all countries, and not merely the deviant country. A legitimate criticism of such strategies is that they are not robust against renegotiation as the countries can do far better by returning to the

---

[6] In contrast, in the other two concepts of cooperation, namely the strong and the coalition proof Nash equilibria, if a coalition deviates, the other players continue to adhere to their equilibrium strategies.

[7] There does not seem to be a universally accepted usage of the term "a self-enforcing agreement". The notion of a self-enforcing agreement in the present paper differs significantly from that in Barrett (1994) and Dutta and Radner (2004).

[8] This issue does not seem to have been addressed in the extant literature as most studies focus on self-enforcing environmental agreements without transfers.

agreement after deviation by a country triggers punishment. Furthermore, these studies allow deviations by a single country alone, and not by coalitions of many countries. In contrast, the self-enforcing agreements proposed in the present paper have an inbuilt punishment strategy which lasts only for the period in which the deviation occurs, but is sufficient to deter coalitional deviations.

As noted above, the dynamic game admits both open- and closed- loop Nash equilibria. An issue that arises is which type of strategies should be considered for studying cooperation in the dynamic game. A familiar criticism of equilibrium in open-loop strategies is that the entire sequence of emissions is decided simultaneously by all players at the outset of the game and the equilibrium is not subgame-perfect. Open-loop strategies do not allow coalitions to evaluate the impact of their deviations on their future payoffs, since the stock of GHGs which impacts their future payoffs depends on whether a coalition deviates or not. Thus, open-loop strategies are not suitable for studying cooperation in the dynamic game. Accordingly, this paper focuses on closed-loop or feedback strategies to evaluate the payoffs that coalitions can obtain from deviations.

Dynamic models are more intricate than static ones, in terms of both economics and game theory. In order to avoid that complexities of the economics blur the game theoretic argument, the model in this paper does not include accumulation of capital. Such an extension would permit us to explore the connection between climate change and economic growth - an important topic that the model in this paper does not allow us to handle. But as our main present objective is game theoretic, we prefer to leave it for another occasion.

The contents of the remaining part of the paper are as follows: Section 2 describes a dynamic model of climate change and characterizes the optimal solution and the unique efficient emission strategy. Section 3 states the dynamic game of climate change. Section 4 shows that the game admits unique open-loop Nash equilibrium as well as a unique closed-loop Nash equilibrium if the production functions are quadratic, but neither of these equilibriums is inter-temporally efficient. This section also shows that if the damage functions are linear, then the open-loop Nash equilibrium is also closed-loop and in fact a dominant strategy equilibrium. Section 5 introduces the concept of a self-enforcing agreement in a strategic game. Section 6 uses an example to illustrate that in a self-enforcing agreement to be effective must be self-enforcing in every subgame, i.e. satisfy subgame-perfection. Section 7 introduces the concept of a self-enforcing agreement in the dynamic game and shows that the dynamic game admits a self-enforcing agreement if the production and damage functions are quadratic. Section 8 draws the conclusion. Section 9 provides the proofs.

2. The dynamic model

There are n countries, indexed by $i = 1, \dots, n$. Time is treated as discrete and indexed $t = 1, \dots, T$, where T is finite but may approach infinity. The variables $x_{it} \geq 0$ and $y_{it} \geq 0$ denote the consumption and production (resp.) of a composite private good in country i at time t. Similarly, $e_{it} \geq 0$ and $z_t \geq 0$ denote (resp.) the amounts of the polluting input used by country i for producing the composite good and stock of pollution at time t. While x_{it}, y_{it}, and e_{it} are flow variables, z_t is a stock variable as formally defined below. We assume that using each unit of the polluting input emits one unit of GHGs. Therefore, the variable e_{it} also denotes the amount of GHGs emitted by country i at time t.

Production and utility of country i at time t are specified as $y_{it} = g_i(e_{it})$ and $u_i(x_{it}, z_t) = x_{it} - v_i(z_t)$, respectively. The function $g_i(e_{it})$ is the production function and $v_i(z_t)$ is the damage function. Given $z_0 \geq 0$, a consumption stream $(x_{1t}, \dots, x_{nt}; z_t)_{t=1}^T$ is *feasible* if there exists an emissions profile $(e_{1t}, \dots, e_{nt})_{t=1}^T$ such that for every $t = 1, \dots, T$,

$$\sum_{i \in N} x_{it} = \sum_{i \in N} g_i(e_{it}) \quad (1)$$

$$z_t = (1 - \delta) z_{t-1} + \sum_{i \in N} e_{it}, \quad (2)$$

Here $0 \leq \delta < 1$ is the natural rate of decay of the stock z_t. Notice that transfers of the composite private good are allowed across the countries in each period t, but not across the periods. Given the quasi-linearity of the utility functions $u_i(x_{it}, z_t)$, this is not really an assumption as there is no gain from postponing consumption and there is no possibility of borrowing against future consumption. A feasible consumption stream $(x_{1t}, \dots, x_{nt}; z_t)_{t=1}^T$ uniquely generates the aggregate utility $\sum_{t=1}^T \beta^{t-1} u_i(x_{it}, z_t) = \sum_{t=1}^T \beta^{t-1} [x_{it} - v_i(z_t)]$ for each country i where $0 < \beta \leq 1$ is the discount factor.

In the optimal control literature, the emissions $(e_{it})_{t=1}^{T}$ are called *control variables* and the resulting stocks $z_{t-1}, t = 1, \dots, T$, *state variables*. While the latter are not strategies in the dynamic game introduced below, they are generated by the former and appear in the payoff functions of the countries. In game theoretic terms, $z_{t-1}, t = 1, \dots, T$ are the decision nodes in the dynamic game and $(e_{it})_{t=1}^{T}$ is a strategy of player i.

In what follows, each production function, $g_i(e_{it})$, is assumed to be strictly increasing and strictly concave, and each damage function, $v_i(z_t)$, strictly increasing and convex, i.e., $g_i'(e_{it}) > 0, g_i''(e_{it}) < 0, v_i'(z_t) > 0$, and $v_i''(z_t) \geq 0$. We shall refer to $g_i'(e_{it})$ as the marginal abatement cost or the marginal benefit of emissions and $v_i'(z_t)$ as the marginal damages due to climate change of country i. We assume that $v_i'(z), z \geq 0$, is bounded above for $i = 1, \dots, n$, there exists an $e^0 > 0$ such that $g_i'(e^0) < v_i'(e^0)$ for each $i \in N$, and $\lim_{e_i \to 0} g_i'(e_i) > v_i'(z)$ for all $z \geq 0$. These assumptions are sufficient (but not necessary) to ensure that the emissions e_{it} chosen by each utility maximizing country i are such that $0 < e_{it} < e^0, t = 1, \dots, T$.

2.1 *The efficient emission strategy*

It will be useful later to characterize Pareto efficient or simply efficient, for brevity, consumption streams.

***Definition* 1** Given $z_0 \geq 0$, a feasible consumption stream $(x_{1t}^*, \dots, x_{nt}^*; z_t^*)_{t=1}^{T}$ is efficient if there is no other feasible consumption stream $(x_{1t}', \dots, x_{nt}'; z_t')_{t=1}^{T}$ such that $\sum_{t=1}^{T} \beta^{t-1} u_i(x_{it}', z_t') \geq \sum_{t=1}^{T} \beta^{t-1} u_i(x_{it}^*, z_t^*)$ for all $i \in N$ with strict inequality for at least one i.

Since $u_i(x_{it}, z_t) = x_{it} - v_i(z_t)$, an efficient consumption stream is a solution of the optimization problem

$$\max_{(x_{1t}, \dots, x_{nt}; z_t)_{t=1}^{T}} \left(\sum_{t=1}^{T} \beta^{t-1} \sum_{i \in N} [x_{it} - v_i(z_t)] \right)$$

subject to (1) and (2). After substituting from (1), this problem is equivalent to

$$\max_{(e_{1t},\ldots,e_{nt};z_t)_{t=1}^{T}} W = \sum_{t=1}^{T} \beta^{t-1} \sum_{i \in N} [g_i(e_{it}) - v_i(z_t)]$$

subject to (2). The Lagrangian for this optimization problem is

$$L = \sum_{t=1}^{T} \beta^{t-1} \sum_{i \in N} [g_i(e_{it}) - v_i(z_t)] + \sum_{t=1}^{T} \lambda_t [z_t - (1-\delta) z_{t-1} - \sum_{i \in N} e_{it}],$$

where the variables λ_t are the Lagrange multipliers associated with the T constraints. Limiting ourselves to an interior optimum for the reasons stated above, the first order conditions (FOCs) for $(e_{1t}^* \ldots, e_{nt}^*)_{t=1}^{T}$, $(z_t^*)_{t=1}^{T}$, and $(\lambda_t^*)_{t=1}^{T}$ to be an optimum are:

$$\frac{\partial L}{\partial e_{it}} = \beta^{t-1} g_i'(e_{it}^*) - \lambda_t^* = 0, t = 1, \ldots, T, i \in N,$$

$$\frac{\partial L}{\partial z_t} = -\beta^{t-1} \sum_{i \in N} v_i'(z_t^*) + \lambda_t^* - \lambda_{t+1}^*(1-\delta) = 0, t = 1, \ldots, T-1,$$

$$\frac{\partial L}{\partial z_T} = -\beta^{T-1} \sum_{i \in N} v_i'(z_T^*) + \lambda_T^* = 0,$$

$$z_t^* = (1-\delta) z_{t-1}^* + \sum_{i \in N} e_{it}^*, t = 1, \ldots, T. \quad (3)$$

The FOCs imply $g_i'(e_{it}^*) - \beta(1-\delta) g_i'(e_{it+1}^*) = \sum_{j \in N} v_j'(z_t^*), t = 1, \ldots T-1, i \in N$, and are called the Euler equations in dynamic optimization. They show a link between the marginal abatement costs of emissions of each country i in any two consecutive periods and the *sum* of the marginal damages in the first of these periods. Multiplying each of the equalities, except the first one, by $\beta(1-\delta)$ and then adding them, after cancelling the second term on the left of an equality with the first term on the left of the subsequent equality, the Euler equations can be rewritten as

$$g_i'(e_{it}^*) = \sum_{\tau=t}^{T} [\beta(1-\delta)]^{\tau-t} \sum_{j \in N} v_j'(z_t^*), t = 1, \ldots, T, i \in N. \quad (4)$$

Equalities (3) and (4) form a system of $(N+1)T$ equations in $(N+1)T$ variables. Let $(e_{1t}^* \dots, e_{nt}^*)_{t=1}^T$, $(z_t^*)\,_{t=1}^T$ be their solution. The solution is unique as can be seen from the following simple argument: Suppose contrary to the assertion that the equalities admit two solutions. Then, there exists another solution, namely a convex combination of the two, which is feasible, because constraints (2) are linear, and dominates the two solutions, since W is strictly concave. But that contradicts the optimality of the two solutions. Hence, (3) and (4) admit a unique solution $(e_{1t}^*, \dots, e_{nt}^*)_{t=1}^T$, $(z_t^*)\,_{t=1}^T$.

Since (3) and (4) are independent of $(x_{it}^*)_{t=1}^T, i \in N$, all consumption streams $(x_{1t}^*, \dots, x_{nt}^*; z_t^*)_{t=1}^T$ such that $\sum_{i\in N} x_{it}^* = \sum_{i\in N} g_i(e_{it}^*)\,, t = 1, \dots, T$, are efficient.[9] Therefore, an efficient consumption stream involves side transfers among the countries unless $x_{it}^* = g_i(e_{it}^*), t = 1, \dots, T. i \in N.$ Equalities (4) imply that in an efficient scheme the marginal abatement costs of all countries are equal and equal to the sum of discounted marginal damages of all countries that would be avoided over the time remaining up to the horizon T.[10] For this reason, we refer to $e^* = (e_{1t}^*, \dots, e_{nt}^*)_{t=1}^T$ as the efficient emissions strategy in the dynamic game to be introduced below.

Furthermore, equalities (3) and (4) characterizing the efficient emissions strategy show that the restricted stream of emissions $(e_{1\tau}^*, \dots, e_{n\tau}^*)_{\tau=t}^T, t = 1, \dots, T$, is the unique solution of the restricted optimization problem starting at time t with initial stock z_{t-1}^* . In other words, the efficient emissions strategy $(e_{1t}^*, \dots, e_{nt}^*)_{t=1}^T$satisfies time consistency or subgame-perfection in the induced game to be introduced below.

If the time horizon $T = \infty$, the FOCs for the variable z_T vanish, but equalities (4) still hold. The question may be raised whether the sum on the right hand side of (4), which now has an infinite number of terms, indeed converges to a finite number. Given that the multiple $[\beta(1-\delta)]^{\tau-t}$ gets smaller with $\tau \to \infty$, a sufficient condition for convergence is that the derivatives $v_i'(z_t)$ are bounded. Since the marginal damage functions $v_i'(z_t)$, by assumption, are indeed bounded, the game (N, E, u) admits an efficient strategy $e^* = (e_{1t}^*, \dots, e_{nt}^*)_{t=1}^T$

[9] If a consumption stream $(x_{1t}, \dots, x_{nt}; z_t^*)_{t=1}^T$ is feasible for $((e_{1t}^*)_{t=1}^T, \dots, (e_{nt}^*)_{t=1}^T)$, then it is efficient and $z_1^*, \dots, z_T^*$ is the efficient stream of a global public bad, i.e., the stocks of GHGs.

[10] This is a dynamic version of the Samuelson condition for efficient public good provision.

even for $T = \infty$. Moreover, since the efficient strategy is unique for each T however large, it is also unique for $T = \infty$.

3. The dynamic game

Since a strategic game provides a natural framework for analyzing interactive decision problems in which decisions of the agents affect each other, it is useful to begin with a strategic form of the dynamic game of climate change.

***Definition* 2** Given an initial stock $z_0 \geq 0$ and $T > 1$, the strategic form of the dynamic game of climate change is denoted (N, E, u) where

- $N = \{i = 1, 2, \dots, n\}$ is the set of players,
- $E = E_1 \times E_2 \times \cdots \times E_n$ is the set of joint strategies and $E_i = \{e_i \equiv (e_{it})_{t=1}^{T}: 0 \leq e_{it} \leq e^0\}$ is the set of strategies of player i.
- $u = (u_1, \dots, u_n)$ is the vector of payoff functions such that for each $e = (e_1, \dots, e_n) = ((e_{1t})_{t=1}^{T}, \dots, (e_{nt})_{t=1}^{T}) \in E$, $u_i(e) = \sum_{t=1}^{T} \beta^{t-1}[g_i(e_{it}) - v_i(z_t)]$, where $z_t = (1-\delta)z_{t-1} + \sum_{j \in N} e_{jt}, t = 1, \dots, T$.

Given an initial stock $z_0 \geq 0$ and $T > 1$, we denote the dynamic game of climate change by Γ_{z_0} and the resulting strategic form game, (N, E, u), by Ω_{z_0}. Similarly, we shall denote the subgames of the dynamic game Γ_{z_0} by $\Gamma_{z_{t-1}}, t = 1, \dots T$, and the resulting strategic form by $\Omega_{z_{t-1}}$ in which the strategy sets of the players are $\{(e_{i\tau})_{\tau=t}^{T}: 0 \leq e_{i\tau} \leq e^0\}$ and the payoffs are $\sum_{\tau=t}^{T} \beta^{\tau-1}[g_i(e_{i\tau}) - v_i(z_\tau)]$ where $z_\tau = (1-\delta)z_{\tau-1} + \sum_{j \in N} e_{j\tau}, \tau = t, \dots, T$. Notice that the subgame $\Gamma_{z_{t-1}}$ and the resulting strategic form game $\Omega_{z_{t-1}}, t = 1, \dots, T$, depend only on z_{t-1} and not on how the game reached the state z_{t-1}. The "statistic" z_{t-1} summarizes all that has happened before the dynamic game reaches the state z_{t-1}.

4. Non-cooperative solutions in the dynamic game

We show that the dynamic game admits a unique open-loop Nash equilibrium as well as a unique closed-loop Nash equilibrium if the production and damage functions are quadratic.

Furthermore, the unique open-loop Nash equilibrium is also closed-loop if the damage functions are linear. However, neither of these equilibriums satisfies inter-temporal efficiency.

.

***Definition* 2** Given $z_0 \geq 0$, A strategy profile $\bar{e} = ((\bar{e}_{1t})_{t=1}^{T}, \ldots, (\bar{e}_{nt})_{t=1}^{T})$ is a Nash equilibrium of the strategic game $\Omega_{z_0} = (N, E, u)$ if for each $i \in N$,

$$(\bar{e}_{it})_{t=1}^{T} = \arg\max_{(e_{it})_{t=1}^{T}} \sum_{t=1}^{T} \beta^{t-1} \left(g_i(e_{it}) - v_i(z_t)\right)$$

where

$$z_t = (1-\delta)z_{t-1} + e_{it} + \sum_{j \in N \setminus i} \bar{e}_{jt}, t = 1, \ldots, T.$$

***Proposition* 1** Given $z_0 \geq 0$, the strategic game $\Omega_{z_0} = (N, E, u)$ admits a unique Nash equilibrium.

Notice that in the Nash equilibrium there are no transfers or cooperation among the countries and in each period each country consumes only what it produces. The emissions of each country are such that its marginal abatement cost is equal to the sum of its *own* discounted marginal damages that would be avoided over the time remaining up to the horizon T, i.e., each country's own marginal damages, present as well as future, determine its emissions. Therefore, the Nash equilibrium is "nationalistic" - the countries give no consideration whatsoever to the damages their emissions inflict on other countries. A comparison of equalities (6) characterizing the Nash equilibrium with equalities (4) characterizing the efficient emissions strategy shows that the Nash equilibrium does not generate an efficient consumption stream.

If the time horizon $T = \infty$, the FOCs for the variable z_T vanish, but equalities (6) still hold. By the same argument as in the preceding section, the game (N, E, u) admits a unique Nash equilibrium even for $T = \infty$.

4.1 *Linear damage functions*

In the special case of *linear damage functions*, that is, $v_i'(z) = \pi_i, \pi_i > 0$, for all z, equalities (6) become $g_i'(\bar{e}_{it}) = \pi_i(1+\beta(1-\delta)+(\beta(1-\delta))^2+\cdots+(\beta(1-\delta)^{T-t})$. That is, the Nash equilibrium strategies do not depend on the state or on the strategies of the other players. Therefore, the Nash equilibrium is, in fact, a dominant strategy equilibrium if the damage functions are linear. Furthermore, if $T = \infty$, (6) becomes

$$g_i'(\bar{e}_{it}) = \pi_i \frac{1}{1-\beta(1-\delta)}, t = 1, \ldots, \infty, i \in N,$$

implying that emissions are constant from the start and indefinitely.[11] They are also uniformly lower than if the time horizon T were finite. The stock z_t will either always increase or always decrease depending on the initial stock z_0. If $\sum_{i\in N} \bar{e}_{i1} > \delta z_0$, the stock z_t will be rising, and falling if the inequality holds in the opposite. In either case the stock will stabilize at a steady state level z_∞ such that $\delta z_\infty = \sum_{i\in N} \bar{e}_{i1}$.

4.2 *Subgame-perfect Nash equilibrium*

Equalities (5) and (6) characterizing the Nash equilibrium also show that the restricted strategy ($(\bar{e}_{1\tau})_{\tau=t}^T, \ldots, (\bar{e}_{n\tau})_{\tau=t}^T$) is the Nash equilibrium of the strategic form $\Omega_{\bar{z}_{t-1}}$ of the subgame $\Gamma_{\bar{z}_{t-1}}$ starting at time t. In other words, the Nash equilibrium strategy $((\bar{e}_{1t})_{t=1}^T, \ldots, (\bar{e}_{nt})_{t=1}^T)$ satisfies *time consistency* and, therefore, it is actually the open-loop Nash equilibrium of the dynamic game Γ_{z_0}. Though the strategy of each player in this open-loop Nash equilibrium is aptly formulated in terms of its emissions for each period, it is determined essentially by the initial state z_0. Thus, the open-loop Nash equilibrium requires the countries to commit to future emissions at the outset of the game, and, therefore, it may not satisfy subgame perfection.[12] If the state prevailing at time *t* belongs to the time path specified by the open-loop Nash equilibrium, then the actions prescribed by the equilibrium strategies remain the best and no player will have incentive to not respect the commitment. However, if at any time t the actual state of the system happens not to belong to the time path of the open-loop Nash equilibrium (for some reason exogenous to the model, such as a

[11] Dutta and Radner (2009) focus especially on a dynamic model with linear damage functions and obtain a similar characterization of the Nash equilibrium.

[12] A view originally presented in Reinganum and Stokey (1985).

random shock for instance, or more endogenously, due to actions of other countries that were not anticipated) then the actions prescribed by the open-loop Nash equilibrium may not be the best to take from that point onwards. The open-loop equilibrium strategies may not be equilibrium strategies anymore for the rest of the game, and the commitment they represent on the part of the countries loses its credibility. What is needed instead is a solution concept that would have the equilibrium property not only relative to the initial state z_0 but also relative to other possible values z_t of the state of the system, at any time t.

To make precise the argument, which essentially comes from allowing the possibility of cooperation, it is important to state explicitly what payoffs are possible for each country or coalition of countries both from cooperation and non-cooperation in each period. For that purpose one might propose at each period t the payoffs corresponding to the open loop Nash equilibrium defined at period 1 as the payoffs from non-cooperation in period t. However, this may not be appropriate as an open loop Nash equilibrium, by definition, assumes that no cooperation will occur in the future, no matter what the circumstances might be and how distant that future is. This is probably too pessimistic and too rigid a view to be attributed realistically to the countries. The countries which are unwilling to cooperate today may change their attitude later on, especially when they learn or are getting convinced of the feasibility of being made better off by moving to the efficient emission path - a move accompanied, if necessary, by appropriate transfers. If the countries indeed think that cooperation will occur in the future, then they must take into account the impact of their non-cooperative behavior today on the payoffs from such cooperation in the future.

Therefore, we consider below an alternative concept of Nash equilibrium, namely, closed-loop or subgame-perfect Nash equilibrium of the dynamic game which is more suited for studying cooperation in the dynamic game. However, existence of a subgame-perfect Nash equilibrium is less likely and requires additional conditions on the production and damage functions.

Proposition 2 The dynamic game Γ_{z_0} admits a unique subgame-perfect Nash equilibrium if $g_i''' = v_i''' = 0$ for each $i = 1, \dots n$.

If $\beta = 1$ and $\delta = 0$, equalities (4) characterizing the efficient strategy reduce to $g_i'(e_{iT}^*) = \sum_{j\in N} v_j'(z_T^*)$, $z_T^* = z_{T-1}^* + \sum_{i\in N} e_{iT}^*, i = 1, \dots, n$. Comparing them with (7) after substituting $e_{iT} = \bar{e}_{iT}$, implies that either $\bar{z}_{T-1} \neq z_{T-1}^*$ or $\bar{e}_{iT} \neq e_{iT}^*, i = 1, \dots, n$, which proves that the SPNE outcome does not imply an efficient outcome. The proof for $\beta \leq 1$ and $\delta \geq 0$ is analogous.

Since, as shown above, the open-loop Nash equilibrium strategies are independent of the states $z_t, t = 1, \dots, T$, if the damage functions are linear, it follows that the open-loop Nash equilbrium is also a SPNE of the dynamic game Γ_{z_0}if the damage functions are linear. It was also noted above that the open-loop Nash equilibrium of the dynamic game Γ_{z_0} is a dominant strategy equilibrium if the damage functions are linear. Thus, the dynamic model of climate change with linear damage functions has rather strong properties.[13] For this reason, Dutta and Radner (2009) seem to focus on a dynamic game with linear damage functions.

If the time horizon $T = \infty$, the existence of a unique SPNE of the dynamic game Γ_{z_0}can be proved by showing that the functional equations $q_i(z_0) = g_i(\bar{e}_{i1}) - (v_i\big(z_0 + \sum_{j\in N} \bar{e}_{j1}\big) - \beta q_i\big(z_0 + \sum_{j\in N} \bar{e}_{j1}\big)), i \in N$, with $g_i''' = 0, i \in N$, admits a unique solution $q_i, i \in N$, such that each q_i is an increasing and concave function with $q_i''' = 0$, where $(\bar{e}_{11}, \dots, \bar{e}_{n1})$ is a Nash equilibrium of the strategic game with payoffs functions given by $g_i(e_{i1}) - (v_i\big(z_0 + \sum_{j\in N} e_{j1}\big) - q_i\big(z_0 + \sum_{j\in N} e_{j1}\big)), i \in N$. One can start with guessing the solution to be $q_i(z) = -(a_i z + b_i)^2$ and find the values of the parameters a_i and b_i such that the required conditions are satisfied.

Finally, since in an SPNE each country gives no consideration whatsoever to the damages its emissions inflict on other countries, we shall refer to it as the *status quo*, i.e., the situation that may prevail in the absence of any cooperation among the countries.

5. Cooperation in a strategic game

In order to study cooperation in the dynamic game, it is useful to first take note of the notion of a self-enforcing agreement in a general strategic game.

[13] See Dutta and Radner (2009) for a forceful argument in favor of a model with linear damage functions.

We denote a general strategic game with transferable utility by (N, A, u) where $N = \{1, \dots, n\}$ is the player set, $A = A_1 \times \dots \times A_n$ is the set of strategy profiles, A_i is the strategy set of player i, $u = (u_1, \dots, u_n)$ is the vector of payoff functions, and u_i is the payoff function of player i. A strategy profile is denoted by $a = (a_1, \dots, a_n) \in A$, a coalition by S and its complement by $N \backslash S$. Given $a = (a_1, \dots, a_n) \in A$, let $a_S \equiv (a_i)_{i \in S}, a_{-S} \equiv (a_j)_{j \in N \backslash S}$, and $(a_S, a_{-S}) \equiv a = (a_1, \dots, a_n)$. Given a coalition $S \subset N$, the *induced* strategic game (N^S, A^S, u^S) is defined as follows:

- The player set is $N^S = \{S, (j)_{j \in N \backslash S}\}$, i.e., coalition S and all $j \in N \backslash S$ are the players (thus the game has $n - s + 1$ players);[14]
- The set of strategy profiles is $A^S = A_S \times_{j \in N \backslash S} A_j$ where $A_S = \times_{i \in S} A_i$ is the strategy set of player S and A_j is the strategy set of player $j \in N \backslash S$;
- The vector of payoff functions is $u^S = (u_S^S, (u_j^S)_{j \in N \backslash S})$ where $u_S^S(a_S, a_{-S}) = \sum_{i \in S} u_i(a_S, a_{-S})$ is the payoff function of player S and $u_j^S(a_S, a_{-S}) = u_j(a_S, a_{-S})$ is the payoff function of player $j \in N \backslash S$, for all $a_S \in A_S$ and $a_{-S} \in \times_{j \in N \backslash S} A_j$.

Observe that if $(\tilde{a}_S, \tilde{a}_{-S})$ is a Nash equilibrium of the induced game (N^S, A^S, u^S), then $u_S^S(\tilde{a}_S, \tilde{a}_{-S}) = \sum_{i \in S} u_i(\tilde{a}_S, \tilde{a}_{-S}) \geq \sum_{i \in S} u_i(a_S, \tilde{a}_{-S})$ for all $a_S \in A_S$. Thus, for each $S \subset N$, a Nash equilibrium of the induced game (N^S, A^S, u^S) assigns a payoff to S that it can obtain without cooperation from the remaining players. If the induced game (N^S, A^S, u^S) has multiple Nash equilibria, then any Nash equilibrium with highest payoff for S is selected.[15] In this way, a unique payoff can be assigned to each coalition.[16] Let a^* denote a Nash equilibrium of the induced game (N^S, A^S, u^S) for $S = N$.

Definition **3** A self-enforcing agreement in a strategic game (N, A, u) is a vector of transfers $x = (x_1, \dots, x_n)$ such that (1) for each $S \subset N$, $\sum_{i \in S} u_i(\tilde{a}_S, \tilde{a}_{-S}) \leq \sum_{i \in S} u_i(a^*) + \sum_{i \in S} x_i$

[14] The small letters n and s denote the cardinality of sets N and S, respectively.

[15] If the strategy sets are compact (or finite) and the payoff functions are continuous, such a payoff will exist.

[16] Other selections in the case of multiple equilibria are possible. However, each induced strategic game in the application below admits a unique Nash equilibrium, and, therefore, this assumption does not really come into play in the current paper.

where $(\tilde{a}_S, \tilde{a}_{-S}) \in A$ is a Nash equilibrium of the induced game (N^S, A^S, u^S) with highest payoff for coalition S, and (2) $\sum_{i \in N} x_i = 0$.

A self-enforcing agreement ensures that no coalition can gain by not adhering to the agreement if each coalition thinks that if it does not adhere to the agreement then the remaining players also will not and will follow instead their individually best reply strategies. Note that in this formulation if a coalition deviates it does not take as given the strategies of its complement: instead it looks ahead to the new equilibrium that would be established as a result of its deviation. By contrast, in the two other concepts of cooperation in strategic games, namely the strong and the coalition proof Nash equilibria, a deviating coalition expects that the other players will continue to adhere to their individual equilibrium strategies even after its deviation. In the classical cooperative concepts of the α- and β- cores, a deviating coalition expects that the remaining players will abandon the agreement but will act to max-min or min-max its payoffs even if that would reduce their own payoffs. In these concepts, the deviating coalition and the players move sequentially: either the deviating coalition moves first and the remaining players next after seeing the strategies of the deviating coalition or the remaining players move first and the deviating coalition moves next after seeing the strategies of the remaining players. In contrast, the notion of a self-enforcing agreement in this paper is based on a more plausible behavioral assumption in that both the deviating coalition and the remaining players choose their strategies simultaneously and each coalition thinks that its deviation will result in the remaining players going their separate ways in pursuit of their own individual interests – any loss in its payoff is incidental and not the intention of the remaining players. Such behavior amounts to non-cooperation, but not to war on the deviating coalition by the remaining players, unlike the notions of α- and β-cores.[17]

However, one question still remains: Why does the deviating coalition think that the remaining players will act individually and not form a coalition of their own?[18] This question has been dealt with at length in Chander (2007, 2010). More specifically, it is shown that if a

[17] To highlight this contrast between self-enforcing agreements and the α- and β- cores, the set of payoff vectors generated by the set of self-enforcing agreements in a general strategic game is referred to as the γ-core of the strategic game in Chander (2010).

[18] This question does not arise if we restrict the number of players to two or the deviations to only single players.

coalition forms, then forming singletons is a subgame perfect Nash equilibrium strategy of the remaining players in a repeated game of coalition formation in which the players first decide which coalitions to form and then choose their emissions. Furthermore, forming the grand coalition is an equilibrium coalition structure and the self-enforcing agreements are solutions of the repeated game of coalition formation. Thus, the self-enforcing agreements can be justified as solutions of a non-cooperative game.

Given this justification for the self-enforcing agreements and their intuitive appeal, it is natural to consider extending them to the dynamic game Γ_{z_0}. A straightforward extension would be to define a self-enforcing agreement in the strategic form Ω_{z_0} as a self-enforcing agreement in the dynamic game Γ_{z_0}. However, such an extension suffers from a serious limitation in that it implicitly assumes that if a coalition does not deviate in some period, then it does not deviate ever thereafter even if it can be better-off by deviating later on. As a result, the notion of a self-enforcing agreement when extended in this way to a dynamic game does not satisfy subgame perfection. Let us illustrate this limitation of such a straightforward notion of a self-enforcing agreement for a dynamic game by an example.

6. An illustrative example

Let $\Omega_{z_0} = (N, E, u)$ denote the strategic game representation of the dynamic game Γ_{z_0} in which $z_0 = 0,\ \beta = 1, \delta = 0, N = \{1,2\}, T = 2, g_i(e_{it}) = 2e_{it}^{\frac{1}{2}}, i = 1,2, v_1(z_t) = \frac{1}{2}z_t,$, and $v_2(z_t) = z_t$. Thus, $E = E_1 \times E_2, E_i = \{e_i = (e_{i1}, e_{i2}) \geq 0\}, i = 1,2,\ u_1(e_1, e_2) = 2e_{11}^{\frac{1}{2}} - \frac{1}{2}(e_{11} + e_{21}) + 2e_{12}^{\frac{1}{2}} - \frac{1}{2}(e_{11} + e_{21} + e_{12} + e_{22})$ and $u_2(e_1, e_2) = 2e_{21}^{\frac{1}{2}} - (e_{11} + e_{21}) + 2e_{22}^{\frac{1}{2}} - (e_{11} + e_{21} + e_{12} + e_{22})$.

The efficient emissions strategy:

It is easily seen that $e^* = \big((e_{11}^*, e_{12}^*), (e_{21}^*, e_{22}^*)\big)$, where $e_{i1}^* = \frac{1}{9}, e_{i2}^* = \frac{4}{9}, i = 1,2,$ is the unique efficient strategy. Therefore, $z_1^* = \frac{2}{9}$, $z_2^* = \frac{2}{9} + \frac{8}{9} = \frac{10}{9}$, and the maximum total payoff of the two players summed over the two periods is $u_1(e^*) + u_2(e^*) = \frac{2}{3} - \frac{1}{9} + \frac{4}{3} - \frac{5}{9} + \frac{2}{3} - \frac{2}{9} + \frac{4}{3} - \frac{10}{9} = 2$.

The Nash equilibrium strategy:

Furthermore, $\bar{e}_{11} = 1, \bar{e}_{21} = \frac{1}{4}, \bar{e}_{12} = 4$, and $\bar{e}_{22} = 1$ is the unique Nash equilibrium strategy. Therefore, $\bar{z}_1 = 1 + \frac{1}{4} = \frac{5}{4}$ and $\bar{z}_2 = \frac{5}{4} + 4 + 1 = \frac{25}{4}$, and the Nash equilibrium payoffs of the two players are $u_1(\bar{e}) = 2 - \frac{5}{8} + 4 - \frac{25}{8} = \frac{9}{4}$ and $u_2(\bar{e}) = 1 - \frac{5}{4} + 2 - \frac{25}{4} = -\frac{9}{2}$.

A self-enforcing agreement:

The consumption stream $((x_{11}^*, x_{21}^*; z_1^*), (x_{12}^*, x_{22}^*; z_2^*)) = ((-\frac{2}{3}, 2, \frac{2}{9}), (5, -\frac{7}{3}, \frac{10}{9}))$ is feasible and generates payoffs of $q_1 = -\frac{2}{3} - \frac{1}{9} + 5 - \frac{5}{9} = \frac{11}{3} > \frac{9}{4}$ $(= u_1(\bar{e}))$ and $q_2 = 2 - \frac{2}{9} - \frac{7}{3} - \frac{10}{9} = -\frac{5}{3} > -\frac{9}{2}$ $(= u_2(\bar{e}))$. Since $q_1 + q_2 = 2$ $(= u_1(e^*) + u_2(e^*))$, $((-\frac{2}{3}, 2, \frac{2}{9}), (5, -\frac{7}{3}, \frac{10}{9}))$ is a self-enforcing agreement in the strategic game Ω_{z_0}.

A subgame-perfect self-enforcing agreement:

However, the consumption vector $(x_{12}^*, x_{22}^*; z_2^*) = (5, -\frac{7}{3}, \frac{10}{9})$ is not a self-enforcing agreement in the strategic form $\Omega_{z_1^*}$ of the subgame $\Gamma_{z_1^*}$, and, therefore, the consumption stream $((-\frac{2}{3}, 2, \frac{2}{9}), (5, -\frac{7}{3}, \frac{10}{9}))$ does not satisfy subgame perfection. That is so because $\bar{e}^T = (\bar{e}_{12}, \bar{e}_{22}) = (4, 1)$ is the Nash equilibrium of $\Omega_{z_1^*}$ and the payoffs of the two players are $4 - \frac{1}{2}(\frac{2}{9} + 5) = \frac{25}{18}$ and $2 - (\frac{2}{9} + 5) = -\frac{29}{9}$. But the consumption stream $(5, -\frac{7}{3}, \frac{10}{9})$ implies for player 2 a lower payoff of $-\frac{7}{3} - \frac{10}{9} = -\frac{31}{9}$ in period 2. Therefore, player 2 will deviate in period 2, since its payoff in the resulting Nash equilibrium $(-\frac{29}{9})$ is higher than the payoff $(-\frac{31}{9})$ implied by the consumption stream $(5, -\frac{7}{3}, \frac{10}{9})$. This shows that the consumption stream $((-\frac{2}{3}, 2, \frac{2}{9}), (5, -\frac{7}{3}, \frac{10}{9}))$ is self-enforcing in the strategic form Ω_{z_0} of the dynamic game Γ_{z_0}, but not in the strategic form $\Omega_{z_1^*}$ of the subgame $\Gamma_{z_1^*}$.

A feasible consumption stream $((x_{11}, x_{21}; z_1^*), (x_{12}, x_{22}; z_2^*))$ must satisfy the following inequalities if it is to be self-enforcing not only in the strategic form Ω_{z_0} of the dynamic game Γ_{z_0} but also in the strategic form $\Omega_{z_1^*}$ of the subgame Γ_{z^*}:

$x_{12} - \frac{1}{2}z_2^* \geq \frac{25}{18} (= u_1^T(\bar{e}^T)), x_{22} - z_2^* \geq -\frac{29}{9} (= u_2^T(\bar{e}^T)), x_{12} + x_{22} = 2(e_{12}^*)^{\frac{1}{2}} + 2(e_{22}^*)^{\frac{1}{2}} = \frac{8}{3}$.

$x_{11} - \frac{1}{2}z_1^* + x_{12} - \frac{1}{2}z_2^* \geq \frac{9}{4} (= u_1(\bar{e})), x_{21} - z_1^* + x_{22} - z_2^* \geq -\frac{9}{2} (= u_2(\bar{e})),$

$x_{11} + x_{21} = 2(e_{11}^*)^{\frac{1}{2}} + 2(e_{21}^*)^{\frac{1}{2}} = \frac{4}{3}$.

Substituting from above, it is seen that the consumption stream $((\frac{1}{3}, 1, \frac{2}{9}), (4, -\frac{4}{3}, \frac{10}{9}))$ is self-enforcing not only in the strategic form $\Omega_{z_0}$of the dynamic game Γ_{z_0}but also in the strategic form $\Omega_{z_1^*}$ of the subgame $\Gamma_{z_1^*}$. Notice that the total payoff of player 2 summed over the two periods is the same under both the consumption streams $((-\frac{2}{3}, 2, \frac{2}{9}), (5, -\frac{7}{3}, \frac{10}{9}))$ and $((\frac{1}{3}, 1, \frac{2}{9}), (4, -\frac{4}{3}, \frac{10}{9}))$ yet player 2 has incentive to deviate under the former, but not under the latter because in the former player 2 receives "too much" in period 1 and "too little" in period 2. Subgame perfection requires that the inter-temporal distribution of the transfers should be such that no player will have incentive to deviate in any period.

7. Cooperation in the dynamic game

Given the dynamic game Γ_{z_0}, let $\Gamma_{z_0}^S, S \subset N$, denote the induced dynamic game in which coalition S acts as one single player. Similarly, let $\Gamma_{z_{t-1}}^S$, $S \subset N$, denote an induced subgame in period t. Since each subgame $\Gamma_{z_{t-1}}^S$has exactly the same mathematical structure as the initial game Γ_{z_0}, it also admits a unique SPNE, if $g_i''' = v_i''' = 0$ for each $i = 1, \dots n$. Let $w(S, z_{t-1})$ denote the SPNE payoff of coalition S in the subgame $\Gamma_{z_{t-1}}^S$.[19]

[19] As the proof of Proposition 2 shows the SPNE and the SPNE payoffs can be found by backward induction.

Definition 4 A self-enforcing agreement in the dynamic game Γ_{z_0} is a feasible consumption stream $(x_{1t},\dots,x_{nt};z_t)_{t=1}^{T}$ such that for each coalition $S\subset N$, $w(S;z_{t-1})\leq\sum_{i\in S}\sum_{\tau=t}^{T}\beta^{\tau-1}(x_{i\tau}-v_i(z_\tau))$ for each $z_{t-1}, t=1,\dots T$.

Notice that if a consumption stream $(x_{1t},\dots,x_{nt};z_t)_{t=1}^{T}$ is feasible for a strategy profile $((e_{1t})_{t=1}^{T},\dots,(e_{nt})_{t=1}^{T})$ in the game Γ_{z_0}, then the restricted stream $(x_{1\tau},\dots,x_{n\tau};z_\tau)_{\tau=t}^{T}$ is feasible for the restricted strategy profile $((e_{1\tau})_{\tau=t}^{T},\dots,(e_{n\tau})_{\tau=t}^{T})$ in the subgame $\Gamma_{z_{t-1}}, t=t,\dots,T$. This means that if $(x_{1t},\dots,x_{nt};z_t)_{t=1}^{T}$ is feasible consumption stream in the game Γ_{z_0}, then $(x_{1\tau},\dots,x_{n\tau};z_\tau)_{\tau=t}^{T}$ is a feasible consumption stream in each subgame $\Gamma_{z_{t-1}}, t=1,\dots,T$. Definition 4, therefore, implies that if a feasible consumption stream $(x_{1t},\dots,x_{nt};z_t)_{t=1}^{T}$ is self-enforcing in Γ_{z_0}, then $(x_{1\tau},\dots,x_{n\tau};z_\tau)_{\tau=t}^{T}$ is a self-enforcing consumption stream in each subgame $\Gamma_{z_{t-1}}, t=1,\dots,T$.

Since the efficient strategy $((e_{1\tau}^*)_{\tau=t}^{T},\dots,(e_{n\tau}^*)_{\tau=t}^{T})$, by definition, is the unique SPNE in the induced game $\Gamma_{z_0}^{N}$, $w(N,z_0)=\sum_{t=1}^{T}\beta^{t-1}\sum_{i\in N}[g_i(e_{it}^*)-v_i(z_t^*)]$, where $(z_t^*)_{t=1}^{T}$ is the stream of stocks generated by $((e_{1\tau}^*)_{\tau=t}^{T},\dots,(e_{n\tau}^*)_{\tau=t}^{T})$ as in (3) above. Since, by definition, $\sum_{t=1}^{T}\beta^{t-1}\sum_{i\in N}[g_i(e_{it}^*)-v_i(z_t^*)] > \sum_{t=1}^{T}\beta^{t-1}\sum_{i\in N}[g_i(e_{it})-v_i(z_t)]$ for any feasible consumption stream $(x_{1t},\dots,x_{nt};z_t)_{t=1}^{T}$ generated by a strategy profile $((e_{1t})_{t=1}^{T},\dots,(e_{nt})_{t=1}^{T})\neq((e_{1\tau}^*)_{\tau=t}^{T},\dots,(e_{n\tau}^*)_{\tau=t}^{T})$, only those consumption streams $(x_{1t},\dots,x_{nt};z_t^*)_{t=1}^{T}$ which are feasible for the efficient strategy profile $((e_{1t}^*)_{t=1}^{T},\dots,(e_{nt}^*)_{t=1}^{T})$ can be self-enforcing in the game Γ_{z_0}.

As the example in Section 6 demonstrates, two alternative consumption streams$(x_{1t}',\dots,x_{nt}';z_t^*)_{t=1}^{T}$ and $(x_{1t}'',\dots,x_{nt}'';z_t^*)_{t=1}^{T}$ which are feasible for the strategy profile $((e_{1t}^*)_{t=1}^{T},\dots,(e_{nt}^*)_{t=1}^{T})$ can be such that they both lead to the same total payoff summed over all periods for each player, i.e.,, $\sum_{t=1}^{T}\beta^{t-1}(x_{1t}'-v_1(z_t^*)),\dots,\sum_{t=1}^{T}\beta^{t-1}\left(x_{nt}'-v_n(z_t^*)\right)=\sum_{t=1}^{T}\beta^{t-1}(x_{1t}''-v_1(z_t^*)),\dots,\sum_{t=1}^{T}\beta^{t-1}(x_{nt}''-v_n(z_t^*))$, but for some coalition $S\subset N$ and $t=\bar{t}$, $(\sum_{i\in S}\sum_{\tau=\bar{t}}^{T}\beta^{\tau-1}(x_{i\tau}'-v_i(z_\tau^*))<\sum_{i\in S}\sum_{\tau=\bar{t}}^{T}\beta^{\tau-1}(x_{i\tau}''-v_i(z_\tau^*))$. In words, the consumption stream $(x_{1t}',\dots,x_{nt}';z_t^*)_{t=1}^{T}$ assigns higher amounts of the private good to coalition S in the early periods, but lower amounts in the later periods though the total payoff of the coalition is the same under both streams. Therefore, coalition S may deviate in the later periods if the consumption stream is $(x'_{1t}, \dots, x'_{nt}; z^*_t)_{t=1}^T$ instead of $(x''_{1t}, \dots, x''_{nt}; z^*_t)_{t=1}^T$. The self-enforcing agreements in the dynamic game are such that no coalition will have incentive to deviate in any period.

.

The dynamic game can be shown to admit a self-enforcing agreement under a variety of conditions. To mention a few, the dynamic game generally admits a self-enforcing agreement if the countries are identical or if the countries are heterogeneous but the damage functions are linear.[20] Given restrictions on space, it is not possible to discuss all these cases here. But the one below illustrates how the existence of a self-enforcing agreement, like a SPNE of an extensive game, can be found by the method of backward induction. The fact that it can be found by the method of backward induction leads to an additional interpretation and insight regarding self-enforcing agreements in the dynamic game.

Assume that the production functions $g_i(e_{it})$ are quadratic,

$$g_i(e_{it}) = c_i e_{it} - \frac{1}{2} e_{it}^2, \qquad (10)$$

where $c_i > 0$ is sufficiently large, and the damage functions

$$v_i(z_t) = \frac{1}{2} z_t^2. \qquad (11)$$

For these specific production and damage functions, as Proposition 2 shows, the dynamic game Γ_{z_0} and the induced games $\Gamma^S_{z_{t-1}}, S \subset N, z_{t-1} \geq 0, t = 1, \dots, T$, each admit a unique SPNE. We first prove the following result.

Proposition 3 The one-period subgame $\Gamma_{z^*_{T-1}}$ admits a self-enforcing agreement if the production and damage functions are quadratic as in (10) and (11).

Note that if a coalition S deviates, then the resulting equilibrium strategies of the other countries are $\tilde{e}_{jT} > \bar{e}_{jT} > e^*_{jT}, j \in N \backslash S$. This means that if a coalition deviates, the other

[20] Dockner and Long (1993) study a dynamic game with quadratic production and damage functions, but with only two identical players. Dutta and Radner (2009) study a dynamic game with many players and linear damage functions, but restrict deviations to single countries only.

countries may emit more. Thus, the strategies of the other countries in the resulting equilibrium can be interpreted as a form of punishment that is imposed by them on the deviating country. This form of punishment is similar to that in Dutta and Radner (2009) except that it is imposed instantly and restricted to the period in which the deviation occurs. Also, as noted above, it is in the self-interest of the other countries to impose such a punishment.[21] The proof of the next result leads to an additional interpretation of the concept of a self-enforcing agreement and the implicit punishment strategy.

Theorem 4 The dynamic game Γ_{z_0} admits a self-enforcing agreement if the production and damage functions are quadratic as in (10) and (11).

Notice that definition of the payoffs in (13) assumes that each country expects full cooperation to prevail in the next period and, therefore, a payoff that is equal to the one generated by the self-enforcing agreement $(x_{1T}(z^*_{T-1}), \dots, x_{nT}(z^*_{T-1}); z^*_T)$ in the subgame $\Gamma_{z^*_{T-1}}$. This means that each coalition expects full cooperation to prevail in the next period irrespective of whether or not it deviates in the current period. Using this argument successively, it follows that coalitions in the current period expect full cooperation to prevail in all future periods. Put another way, it means that each deviating coalition expects the punishment imposed by the other countries to last only the period in which it may deviate, and not forever.

Finally, note that the above approach to solving the dynamic game with backward induction cannot be extended to the game with infinite time horizon. That is because if the time horizon is infinite, then there is no game from which to start the backward induction. Instead, the existence proof requires showing that a system of functional equations admits a unique solution. The argument is analogous to the one used for the existence of a SPNE for the dynamic game with $T = \infty$. First show that the functional equations given by $q_i(z_0) = c_i e^*_{i1} - \frac{1}{2}(e^*_{i1})^2 - \frac{1}{2}((1-\delta)z_0 + \sum_{j\in N} e^*_{i1})^2 + \beta q_i((1-\delta)z_0 + \sum_{j\in N} e^*_{i1}), i \in N,$ admit a

[21] If the damage functions are linear, then as shown above $\bar{e}_{jT}$ is a dominant strategy and, therefore, $\tilde{e}_{jT} = \bar{e}_{jT}, j \in N\backslash S$.. This means that if the damage functions are linear, the punishment strategy implicit in the notion of a self-enforcing agreement is identical to that in Dutta and Radner (2009), except that it is imposed instantly and restricted to the period in which the deviation occurs.

solution of the form[22] $q_i(z) = -a(z+b)^2 + d_i, i \in N,$ and then show that there are unique such functions for which $(e_{11}^*, \ldots, e_{n1}^*)$ is the unique solution of the optimization problem: $max \sum_{i \in N}[c_i e_{i1} - \frac{1}{2} e_{i1}^2, -\frac{1}{2}((1-\delta)z_0 + \sum_{j \in n} e_{j1})^2 + \beta q_i((1-\delta)z_0 + \sum_{j \in n} e_{j1})]$ and $x_{i1} = g_i(\bar{e}_{i1}) - \frac{v_i'(z_1^*) - q_i'(z_1^*)}{\sum_{j \in N} v_j'(z_1^*) - q_j'(z_1^*)} [\sum_{j \in N} g_j(\bar{e}_{j1}) - \sum_{j \in N} g_j(e_{j1}^*)], i \in N$, where $z_1^* = z_0 + \sum_{i \in N} e_{i1}^*$ and the functions g_i and v_i are as in (11) and (12), is a self-enforcing agreement in the strategic game with payoff functions $c_i e_{i1} - \frac{1}{2} e_{i1}^2, -\frac{1}{2}((1-\delta)z_0 + \sum_{j \in n} e_{j1})^2 + \beta q_i((1-\delta)z_0 + \sum_{j \in n} e_{j1}), i \in N$.

8. Conclusion

This paper shows that dynamic game formulation of the climate change problem provides a useful framework for analyzing self-enforcing agreements to control climate change and a useful insight that may not be available from a strategic game formulation alone. Countries differ in their costs and benefits regarding climate-change mitigation as well as their GHGs emissions. The only way in which they can overcome inefficiency of the status quo characterized by the SPNE of the dynamic game is by negotiating appropriate transfers among them in return for reducing their emissions. The agreed upon transfers must be not only self enforcing but to be effective should also satisfy the property of subgame- perfection. It was shown that the method of backward induction leads to a self-enforcing agreement which satisfies inter-temporal efficiency and subgame perfection. Instead of a trigger strategy where all countries punish a deviating country forever, a simple rule of sanctions where sanctions last only the period in which the deviation occurs and the countries immediately return to cooperation thereafter is sufficient to ensure voluntary cooperation.

The notion of a self-enforcing agreement introduced in this paper assumes that deviating coalitions of countries can write binding agreements in the same sense as in the concept of a strong Nash equilibrium. However, this assumption does not come into play if there are only two countries or deviations are restricted to single countries as is often assumed in the related literature (see e.g. Dockner and Long, 1993 and Dutta and Radner, 2009). If deviating coalitions lack the ability to write binding agreements, then the status quo characterized by the unique SPNE of the dynamic game which does not satisfy inter-temporal

[22] This functional form is suggested by the solution of the dynamic game for finite T.

efficiency may prevail. Thus, an implication of our analysis is that inefficiency of the status quo cannot be overcome if deviating coalitions lack the ability to write binding agreements.

This paper lays down the foundations for a theory of cooperation in dynamic games of climate change. Future research should address a number of assumptions that were made to focus on the game theoretic aspects of the problem. We assumed no capital accumulation and no technological progress.[23] Another natural extension of our model would be to incorporate uncertainty regarding climate change.

9. Appendix

Proof of Proposition 1: Since each player's strategy space is a compact and convex set of a Euclidean space and each player's payoff function is continuous for all $e \in E$ and concave with respect to his own strategy $e_i \in E_i$. The existence of the Nash equilibrium follows from standard arguments (see e.g. Osborne and Rubinstein, 1994).

The proof for uniqueness uses FOCs for a Nash equilibrium. In view of Definition 2, the Lagrangian associated with the optimization problem of player $i \in N$ is:

$$L_i = \sum_{t=1}^{T} \beta^{t-1} \left(g_i(e_{it}) - v_i(z_t)\right) + \sum_{t=1}^{T} \lambda_{it}[z_t - (1-\delta)z_{t-1} - e_{it} - \sum_{j\in N\setminus i} \bar{e}_{jt}]$$

where the variables λ_{it} are non-negative Lagrange multipliers associated with the T constraints. Limiting to an interior optimum, for reasons noted above, the FOCs for $(\bar{e}_{it})_{t=1}^{T}, (\bar{z}_t)_{t=1}^{T}$ and $(\bar{\lambda}_{it})_{t=1}^{T}$ to maximize the Lagrangian imply:

$$\frac{\partial L_i}{\partial e_{it}} = \beta^{t-1} g_i^{'}(\bar{e}_{it}) - \bar{\lambda}_{it} = 0, t = 1, \ldots, T$$

$$\frac{\partial L_i}{\partial z_t} = -\beta^{t-1} v_i^{'}(\bar{z}_t) + \bar{\lambda}_{it} - \bar{\lambda}_{it+1}(1-\delta) = 0, t = 1, \ldots, T-1$$

$$\frac{\partial L_i}{\partial z_T} = -\beta^{T-1} v_i^{'}(\bar{z}_T) + \bar{\lambda}_{iT} = 0$$

[23] In an interesting paper, Harstad (2012) studies a model in which investments in green technologies are determined endogenously, but transfers to induce cooperation and efficiency are not permitted.

$$\bar{z}_t = (1-\delta)\bar{z}_{t-1} + \sum_{j\in N} \bar{e}_{jt}, t = 1, \dots, T, \tag{5}$$

$z_0 > 0$ given.

Substituting for $\bar{\lambda}_{it}$ and $\bar{\lambda}_{it+1}$ and dividing by β^{t-1} yields $g_i^{'}(\bar{e}_{it}) - \beta(1-\delta)g_i^{'}(\bar{e}_{it+1})= v_i^{'}(\bar{z}_t), t = 1, \dots, T-1$. These equalities, i.e. the Euler equations, show a link between the marginal abatement costs of emissions at any two successive periods and the marginal damage at the first of these periods, when the emissions of player i are individually optimal. As above, these equalities can be rewritten for each $i \in N$ as

$$g_i^{'}(\bar{e}_{it}) = \sum_{\tau=t}^{T} [\beta(1-\delta)]^{\tau-t} \; v_i^{'}(\bar{z}_\tau), t = 1, \dots, T. \tag{6}$$

Equalities (5) and (6) characterize a Nash equilibrium $\bar{e} = ((\bar{e}_{1t})_{t=1}^{T}, \dots, (\bar{e}_{nt})_{t=1}^{T})$ of the game (N, E, u).[24] Given (5) and (6), the proof for uniqueness goes as follows. Suppose contrary to the assertion that given the same initial stock z_0 there exist two Nash equilibria $\bar{e} = ((\bar{e}_{1t})_{t=1}^{T}, \dots, (\bar{e}_{nt})_{t=1}^{T})$ and $\hat{e} = ((\hat{e}_{1t})_{t=1}^{T}, \dots, (\hat{e}_{nt})_{t=1}^{T})$ and let $(\bar{z}_t)_{t=1}^{T}$ and $(\hat{z}_t)_{t=1}^{T}$ be the associated sequences of the stock. If $\bar{z}_T = \hat{z}_T$, then $\bar{e}_{iT} = \hat{e}_{iT}$ and, therefore, $\bar{z}_{T-1} = \hat{z}_{T-1}$. Repeating this argument one obtains $\bar{z}_t = \hat{z}_t$ and $\bar{e}_{it} = \hat{e}_{it}$ for all $t = 1, \dots, T$. This means that the game can have more than one Nash equilibria only if $\bar{z}_T \neq \hat{z}_T$.

Suppose without loss of generality that $\bar{z}_T > \hat{z}_T$. Since each v_i is an increasing and convex function, $v_i^{'}(\bar{z}_T) \geq v_i^{'}(\hat{z}_T)$ for all i. Therefore, $g_i^{'}(\bar{e}_{iT}) = v_i^{'}(\bar{z}_T)$ and $g_i^{'}(\hat{e}_{iT}) = v_i^{'}(\hat{z}_T)$ which implies $g_i^{'}(\bar{e}_{iT}) \geq g_i^{'}(\hat{e}_{iT})$ for all i. As a consequence, $\bar{e}_{iT} \leq \hat{e}_{iT}$ for all i because the function g_i is increasing and concave. Thus, $\sum_{i\in N} \bar{e}_{iT} \leq \sum_{i\in N} \hat{e}_{iT}$.

Combining this inequality with the fact that $\bar{z}_T = (1-\delta)\bar{z}_{T-1} + \sum_{i\in N} \bar{e}_{iT} > \hat{z}_T = (1-\delta)\hat{z}_{T-1} + \sum_{i\in N} \hat{e}_{iT}$, one obtains $\bar{z}_{T-1} > \hat{z}_{T-1}$. Using the Euler equations and (6), one has $g_i^{'}(\bar{e}_{iT-1}) = v_i^{'}(\bar{z}_{T-1}) + \beta(1-\delta)v_i^{'}(\bar{z}_T)$ and $g_i^{'}(\hat{e}_{iT-1}) = v_i^{'}(\hat{z}_{T-1}) + \beta(1-\delta)v_i^{'}(\hat{z}_T)$. Together, $\bar{z}_T > \hat{z}_T$ and $\bar{z}_{T-1} > \hat{z}_{T-1}$ just established imply $g_i^{'}(\bar{e}_{iT-1}) > g_i^{'}(\hat{e}_{iT-1})$. Thus, $\bar{e}_{iT-1} < \hat{e}_{iT-1}$ for all i. Repeating this argument for all $t < T-1$, we eventually have $\sum_{i\in N} \bar{e}_{it} < \sum_{i\in N} \hat{e}_{it}$ and $\bar{z}_t > \hat{z}_t$ for all $t = 1, \dots, T$. However, this is a contradiction since it

[24] The sequences $(\bar{e}_{1t}, \dots, \bar{e}_{nt})_{t=1}^{T}$, $\bar{x}_{it} = \bar{y}_{it} = g_i(\bar{e}_{it}), t = 1, \dots, T,$ and $(\bar{z}_t)_{t=1}^{T}$ constitute a non-cooperative solution of the dynamic model.

implies that at $t = 1, \bar{z}_t > \hat{z}_t$, but $\bar{z}_1 = (1-\delta)z_0 + \sum_{i\in N} \bar{e}_{i1}$, $\hat{z}_1 = (1-\delta)z_0 + \sum_{i\in N} \hat{e}_{i1}$, and $\sum_{i\in N} \bar{e}_{i1} < \sum_{i\in N} \hat{e}_{i1}$.Thus, we must have $\bar{z}_T = \hat{z}_T$ and hence the Nash equilibrium is unique. ■

Proof of Proposition 2: In order to keep the derivations simple, we assume henceforth that $\beta = 1$ and $\delta = 0$. The proof for $\beta \leq 1$ and $\delta \geq 0$ is analogous. We show that backward induction leads to a unique subgame-perfect Nash equilibrium (SPNE). Begin with a subgame in period T. A strategy profile $(e_{1T}, \dots, e_{nT})$ is a SPNE of a subgame $\Gamma_{z_{T-1}}$if each e_{iT} maximizes $g_i(e_{iT}) - v_i\big(z_{T-1} + \sum_{j\in N} e_{jT}\big)$, given $e_{jT}, j \neq i$. Therefore, by FOCs for optimization,

$$g_i'(e_{iT}) = v_i'\big(z_{T-1} + \textstyle\sum_{j\in N} e_{jT}\big), i = 1, \dots, n. \tag{7}$$

We claim these equations have a unique solution. Suppose not, and let $(\bar{e}_{1T}, \dots, \bar{e}_{nT})$ and $(\bar{\bar{e}}_{1T}, \dots \bar{\bar{e}}_{nT})$ be two different solutions such that $\sum_{i\in N} \bar{e}_{iT} = (>) \sum_{i\in N} \bar{\bar{e}}_{iT}$. Then, since each v_i is convex and g_i is strictly concave, (7) implies $\bar{e}_{iT} = (<)\bar{\bar{e}}_{iT}$ for $i = 1, \dots, n$, which contradicts our supposition. Hence, $\Gamma_{z_{T-1}}$ admits a unique SPNE for $z_{t-1} \geq 0$. Let $(e_{1T}(z_{T-1}), \dots, e_{nT}(z_{T-1}))$ denote the unique SPNE of $\Gamma_{z_{T-1}}$. By differentiating (7),

$$g_i''\big(e_{iT}(z_{T-1})\big)e_{iT}'(z_{T-1}) = v_i''(z_{T-1} + \textstyle\sum_{j\in N} e_{jT}(z_{T-1}))(1 + \sum_{j\in N} e_{jT}'(z_{T-1}), i \in N. \tag{8}$$

Since $g_i'' < 0$ and $v_i'' \geq 0$, equations (8) imply $e_{iT}'(z_{T-1}) \leq 0$ and $(1 + \sum_{j\in N} e_{jT}'(z_{T-1}) \geq 0$. By differentiating (8), $g_i'''\big(e_{iT}(z_{T-1})\big)(e_{iT}'(z_{T-1})^2 + g_i''\big(e_{iT}(z_{T-1})\big)e_{iT}''(z_{T-1}) =$ $v_i'''(z_{T-1} + \sum_{j\in N} e_{jT}(z_{T-1}))(1 + \sum_{j\in N} e_{jT}')^2 + v_i''(z_{T-1} + \sum_{j\in N} e_{jT}(z_{T-1})) \sum_{j\in N} e_{jT}''(z_{T-1})$, $i = 1, \dots, n$. Therefore,

$$g_i''\big(e_{iT}(z_{T-1})\big)e_{iT}''(z_{T-1}) = v_i''(z_{T-1} + \textstyle\sum_{j\in N} e_{jT}(z_{T-1})) \sum_{j\in N} e_{jT}''(z_{T-1}), i \in N, \tag{9}$$

since $g_i''' = v_i''' = 0, i = 1, \dots n$. Since $g_i'' < 0$ and $v_i'' \geq 0$, equations (9) imply $e_{iT}''(z_{T-1}) = 0$. Let $q_{iT}(z_{T-1}) \equiv g_i\big(e_{iT}(z_{T-1})\big) - v_i\big(z_{T-1} + \sum_{j\in N} e_{jT}(z_{T-1})\big), i = 1, \dots, n$. Then, $q_{iT}'(z_{T-1}) = g_i'\big(e_{iT}(z_{T-1})\big)e_{jT}'(z_{T-1}) - v_i'(z_{T-1} + \sum_{j\in N} e_{jT}(z_{T-1}))(1 + \sum_{j\in N} e_{jT}'(z_{T-1}) \leq 0$, since $g_i' > 0, v_i' > 0$, and, as shown, $e_{jT}'(z_{T-1}) \leq 0$ and $(1 + \sum_{j\in N} e_{jT}'(z_{T-1}) \geq 0$.

Furthermore, $q''_{iT}(z_{T-1}) = g''_i\big(e_{iT}(z_{T-1})\big)(e'_{iT}(z_{T-1}))^2 - v''_i(z_{T-1} + \sum_{j\in N} e_{jT}(z_{T-1}))(1 + \sum_{j\in N} e'_{jT}(z_{T-1}))^2 + g'_i\big(e_{iT}(z_{T-1})\big)e''_{iT}(z_{T-1}) - v'_i(z_{T-1} + \sum_{j\in N} e_{jT}(z_{T-1}))\sum_{j\in N} e''_{jT}(z_{T-1})$ $= g''_i\big(e_{iT}(z_{T-1})\big)(e'_{iT}(z_{T-1}))^2 - v''_i(z_{T-1} + \sum_{j\in N} e_{jT}(z_{T-1}))(1 + \sum_{j\in N} e'_{jT}(z_{T-1}))^2 \leq 0$, since, as shown, $e''_{iT}(z_{T-1}) = 0, i = 1, \dots, n$.

Thus, each $q_{iT}(z_{T-1}), i = 1, \dots, n$, is a non-increasing concave function of z_{T-1}. In fact, by differentiating the expression above and using $g'''_i = v'''_i = 0$ for each $i = 1, \dots n$, it is seen that $q'''_{iT}(z_{T-1}) = 0$. Using backward induction, a strategy profile $((e_{1t})_{t=T-1}^{T}, \dots, (e_{nt})_{t=T-1}^{T})$ is a SPNE of the subgame $\Gamma_{z_{T-2}}$ if each e_{iT-1} maximizes $g_i(e_{iT-1}) - [v_i\big(z_{T-2} + \sum_{j\in N} e_{jT-1}\big) - q_{iT}\big(z_{T-2} + \sum_{j\in N} e_{jT-1}\big)]$. Since $q_{iT}(z_{T-1}), i = 1, \dots, n$, is a non-increasing concave function of z_{T-1}, the subgame $\Gamma_{z_{T-2}}$ has essentially the same structure as the game $\Gamma_{z_{T-1}}$. Therefore, $\Gamma_{z_{T-2}}$ admits a unique SPNE and the SPNE payoffs $q_{iT-1}(z_{T-2}), i = 1, \dots, n$, are similarly non-increasing and concave functions of z_{T-2}. Continuing in this manner, the backward induction would lead to a unique SPNE of the extensive game Γ. ■

Proof of Proposition 3: The game $\Gamma_{z^*_{T-1}}$ is essentially a strategic game. Using the FOCs for a Nash equilibrium in the games $\Gamma_{z^*_{T-1}}$, $\Gamma^S_{z^*_{T-1}}$, and $\Gamma^N_{z^*_{T-1}}$, respectively, $g'_i(e_{iT}) = v'_i(z^*_{T-1} + \sum_{j\in N} e_{jT}), i \in N$, $g'_i(e_{iT}) = \sum_{j\in S} v'_j(z^*_{T-1} + \sum_{k\in N} e_{kT}), i \in S$, $g'_j\big(e_{jT}\big) = v'_j(z^*_{T-1} + \sum_{i\in N} e_{iT}), j \in N\backslash S$, and $g'_i(e_{iT}) = \sum_{j\in N} v'_j(z^*_{T-1} + \sum_{k\in N} e_{kT}), i \in N$. Substituting from (10) and (11), it is seen that the strategies $\bar{e}_{iT} = c_i - \frac{1}{1+n}\big(z^*_{T-1} + \sum_{j\in N} c_j\big), i \in N$, $\tilde{e}_{iT} = c_i - \frac{s}{s^2+n-s+1}(z^*_{T-1} + \sum_{j\in N} c_j), i \in S$, $\tilde{e}_{jT} = c_i - \frac{1}{s^2+n-s+1}(z^*_{T-1} + \sum_{k\in N} c_k), j \in N\backslash S$, and $e^*_{iT} = c_i - \frac{n}{1+n^2}\big(z^*_{T-1} + \sum_{j\in N} c_j\big), i \in N$, are the Nash equilibrium strategies in the games $\Gamma_{z^*_{T-1}}$, $\Gamma^S_{z^*_{T-1}}$, and $\Gamma^N_{z^*_{T-1}}$, respectively. Clearly, the Nash equilibrium strategies $e^*_{iT}, i = 1, \dots, n$, in the game $\Gamma^N_{z^*_{T-1}}$ are the same as the actions in period T in the unique SPNE $((e^*_{1t})_{t=1}^{T}, \dots, (e^*_{nt})_{t=1}^{T})$ of the dynamic game Γ_{z_0}.

We show that the specific consumption vector $(x_{1T}, \dots, x_{nT}; z^*_T)$ in which $x_{iT} = g_i(\bar{e}_{iT}) - \frac{v'_i(z^*_T)}{\sum_{j\in N} v'_j(z^*_T)}[\sum_{j\in N} g_j\big(\bar{e}_{jT}\big) - \sum_{j\in N} g_j\,(e^*_{jT})]$, $i = 1, \dots, n$, and $z^*_T = z^*_{T-1} + \sum_{i\in N} e^*_{iT}(z^*_{T-1})$ is self-enforcing. The proof is by contradiction. Suppose contrary to the assertion that $(x_{1T}, \dots, x_{nT}; z_T^*)$ is not self-enforcing. Then, for some coalition $S \subset N$ and the Nash equilibrium $\tilde{e}_{iT} = c_i - \frac{s}{s^2+n-s+1}(z_{T-1}^* + \sum_{j\in N} c_j), i \in S$, $\tilde{e}_{jT} = c_i - \frac{1}{s^2+n-s+1}(z_{T-1}^* + \sum_{k\in N} c_k), j \in N\backslash S,$ $\sum_{i\in S} g_i\,(\tilde{e}_{iT}) - \sum_{i\in S} v_i\,(z_{T-1}^* + \sum_{j\in N} \tilde{e}_{jT}) > \sum_{i\in S} x_{iT} - \sum_{i\in S} v_i\,(z_T^*)$. Now consider an alternative feasible consumption vector $(\hat{x}_{1T}, \dots, \hat{x}_{nT}; z_T^*)$ in which $\hat{x}_{iT} = g_i(\tilde{e}_{iT}) - \frac{v_i'(z_T^*)}{\sum_{j\in N} v_j'(z_T^*)}[\sum_{j\in N} g_j(\tilde{e}_{jT}) - \sum_{j\in N} g_j\,(e_{jT}^*)],\ i = 1, \dots, n$. We show that $\sum_{i\in S} \hat{x}_{iT} - \sum_{i\in S} v_i\,(z_T^*) > \sum_{i\in S} g_i(\tilde{e}_{iT}) - \sum_{i\in S} v_i\,(z_{T-1}^* + \sum_{j\in N} \tilde{e}_{jT})$ ($> \sum_{i\in S} x_{iT} - \sum_{i\in S} v_i\,(z_T^*)$) and $\sum_{j\in N\backslash S} \hat{x}_{jT} - \sum_{j\in N\backslash S} v_j\,(z_T^*) > \sum_{j\in N\backslash S} x_{jT} - \sum_{j\in N\backslash S} v_j\,(z_T^*)$ contradicting that $(x_{1T}, \dots, x_{nT}; z_T^*)$ is efficient. The first of these inequalities follows from $\sum_{i\in S} \hat{x}_{iT} = \sum_{i\in S} g_i(\tilde{e}_{iT}) - \frac{\sum_{i\in S} v_i'(z_T^*)}{\sum_{j\in N} v_j'(z_T^*)}[\sum_{j\in N} g_j(\tilde{e}_{jT}) - \sum_{j\in N} g_j\,(e_{jT}^*)] \geq$

$\sum_{i\in S} g_i(\tilde{e}_{iT}) - \frac{\sum_{i\in S} v_i'(z_T^*)}{\sum_{j\in N} v_j'(z_T^*)}\sum_{j\in N} v_j'\,(z_T^*)\,(\sum_{j\in N} \tilde{e}_{jT} - \sum_{j\in N} e_{jT}^*)$, using the concavity of the production functions g_i and the FOCs for the strategy $(e_{1T}^*, \dots, e_{nT}^*)$ to be the Nash equilbrium in $\Gamma_{z_{T-1}^*}^{N}$. This implies $\sum_{i\in S} \hat{x}_{iT} - \sum_{i\in S} v_i'\,(z_T^*)\, z_T^*$
$\geq\ \sum_{i\in S} g_i(\tilde{e}_{iT}) - \sum_{i\in S} v_i'\,(z_T^*)(z_{T-1}^* + \sum_{j\in N} \tilde{e}_{jT}) \Longrightarrow \sum_{i\in S} \hat{x}_{iT} - \sum_{i\in S} v_i\,(z_T^*) \geq \sum_{i\in S} g_i(\tilde{e}_{iT}) - \sum_{i\in S} v_i\,(z_{T-1}^* + \sum_{j\in N} \tilde{e}_{jT})$, since $v_i((z_T^*) - v_i\big((z_{T-1}^* +\ \sum_{j\in N} \tilde{e}_{jT})\big) \geq \sum_{i\in S} v_i'\,(z_T^*)\,(z_T^* - (z_{T-1}^* + \sum_{j\in N} \tilde{e}_{jT})) = \sum_{i\in S} v_i'\,(z_T^*)\,(\sum_{j\in N} e_{jT}^* -\ \sum_{j\in N} \tilde{e}_{jT}))$ using convexity of v_i.

We now establish the second inequality. By definition, $\sum_{i\in N\backslash S} \hat{x}_{iT} = \sum_{i\in N\backslash S} x_{iT} + \sum_{i\in N\backslash S} g_i\,(\tilde{e}_{iT}) - \sum_{i\in N\backslash S} g_i\,(\bar{e}_{iT}) + \frac{\sum_{i\in N\backslash S} v_i'(z_T^*)}{\sum_{j\in N} v_j'(z_T^*)}(\sum_{j\in N} g_j(\bar{e}_{jT}) - \sum_{j\in N} g_j(\tilde{e}_{jT})) = \sum_{i\in N\backslash S} x_{iT} + [\sum_{i\in N\backslash S} g_i\,(\tilde{e}_{iT}) - \sum_{i\in N\backslash S} g_i\,(\bar{e}_{iT}) - \frac{\sum_{i\in N\backslash S} v_i'(z_T^*)}{\sum_{j\in N} v_j'(z_T^*)}(\sum_{j\in N\backslash S} g_j(\tilde{e}_{jT}) - \sum_{j\in N\backslash S} g_j\big(\bar{e}_{jT}\big))] - \frac{\sum_{i\in N\backslash S} v_i'(z_T^*)}{\sum_{j\in N} v_j'(z_T^*)}(\sum_{j\in S} g_j(\tilde{e}_{jT}) - \sum_{j\in S} g_j\big(\bar{e}_{jT}\big)) \geq \sum_{i\in N\backslash S} x_{iT}$, since $\tilde{e}_{jT} \geq \bar{e}_{jT}$ for $j \in N\backslash S$ and $\tilde{e}_{jT} \leq \bar{e}_{jT}$ for all $j \in S$. ■

Proof of Theorem 4: We take $\beta = 1$ and $\delta = 0$. The proof for $\beta \leq 1$ and $\delta \geq 0$ is analogous. In view of our discussion of Definition 4, it is sufficient to show that a feasible consumption stream $(x_{1t}, \dots, x_{nt}; z_t^*)_{t=1}^T$ is self-enforcing in each subgame $\Gamma_{z_{t-1}^*}, t = 1, \dots, T$. We construct such a feasible consumption stream by backward induction. Let us begin with the subgame $\Gamma_{z_{T-1}^*}$ in the last period T.

As in the proof of Proposition 3, the unique SPNE of the subgame $\Gamma_{z^*_{T-1}}$ and the induced game $\Gamma^N_{z^*_{T-1}}$ are given by $\bar{e}_{iT}(z^*_{T-1}) = c_i - \frac{1}{1+n}\left(\sum_{j\in N} c_j + z^*_{T-1}\right)$ and $e^*{}_{iT}(z^*_{T-1}) = c_i - \frac{n}{1+n^2}\left(\sum_{j\in N} c_j + \; z^*_{T-1}\right)$, respectively.[25] Furthermore, since $((e^*_{1t})^T_{t=1}, \dots, (e^*_{nt})^T_{t=1})$ is the unique SPNE of the dynamic game $\Gamma^N_{z_0}$ and generates the stream of stocks $(z^*_t)^T_{t=1}$, $e^*_{iT}(z^*_{T-1}) = e^*_{iT}, i = 1, \dots, n$.

Let $x_{iT}(z^*_{T-1}) = g_i\left(\bar{e}_{iT}(z^*_{T-1})\right) - \frac{v_i'(z^*_T)}{\sum_{j\in N} v_j'(z^*_T)} \; [\sum_{j\in N} g_j\left(\bar{e}_{jT}(z^*_{T-1})\right) - \sum_{j\in N} g_j\,(e^*_{jT}(z^*_{T-1})]$, $i \in N$, where $z^*_T = z^*_{T-1} + \sum_{i\in N} e^*_{iT}(z^*_{T-1})$. Then, as Proposition 3 shows $(x_{1T}(z^*_{T-1}), \dots, x_{nT}(z^*_{T-1}); z^*_T)$ is a self-enforcing agreement in the subgame $\Gamma_{z^*_{T-1}}$. Let $q_{iT}(z^*_{T-1}) = x_{iT}(z^*_{T-1}) - v_i(z^*_{T-1} + \sum_{i\in N} e^*_{iT}(z^*_{T-1})), i = 1, \dots, n$. Using (10) and (11) and after substitution, we obtain

$$q_{iT}(z^*_{T-1}) = \frac{1}{2}c_i^2 - \frac{1}{1+n^2}\left(1 + \frac{1}{2}\frac{n^2}{1+n^2}\right)(z^*_{T-1} + \sum_{j\in N} c_j)^2, i \in N. \tag{12}$$

Now consider a reduced form of the subgame $\Gamma_{z^*_{T-2}}$ in which the payoff of country i is given by

$$c_i e_{iT-1} - \frac{1}{2}e^2_{iT-1} - \frac{1}{2}(z^*_{T-2} + \sum_{j\in N} e_{jT-1})^2 + q_{iT}(z^*_{T-2} + \sum_{j\in N} e_{jT-1}). \tag{13}$$

Let $\hat{\Gamma}_{z^*_{T-2}}$ denote the reduced form of the game $\Gamma_{z^*_{T-2}}$. Then, as is easily seen, $(e^*_{1T-1}, \dots, e^*_{nT-1})$ is the unique SPNE of the induced game $\hat{\Gamma}^N_{z^*_{T-2}}$ and $z^*_{T-2} + \sum_{j\in N} e^*_{jT-1} = z^*_{T-1}$. That is so because the sum of expressions (13) over all i is the same as the payoff of coalition N in the subgame $\Gamma^N_{z^*_{T-2}}$ and the unique strategy $((e^*_{1T-1})^T_{t=T-1}, \dots, (e^*_{nT-1})^T_{t=T-1})$ maximizes the payoff of coalition N.

Since each $q_{iT}(z^*_{T-1}), i \in N$, is quadratic in z^*_{T-1}, the payoff functions in the reduced form $\hat{\Gamma}_{z^*_{T-2}}$ have essentially the same functional form as the payoff functions in the game $\Gamma_{z^*_{T-1}}$. Therefore, the reduced form $\hat{\Gamma}_{z^*_{T-2}}$ also admits a self-enforcing agreement

[25] Notice that both are affine functions of the current stock z^*_{T-1}.

$(x_{1T}(z^*_{T-2}), \ldots, x_{nT}(z^*_{T-2}); z^*_{T-1})$, where $z^*_{T-1} = z^*_{T-2} + \sum_{i \in N} e^*_{iT-1}$. Thus, $(x_{1t}(z^*_{t-1}), \ldots, x_{nt}(z^*_{t-1}); z^*_t)^T_{t=T-1}$ is a self-enforcing agreement in the subgame $\Gamma_{z^*_{T-2}}$.

Continuing this process, leads to a self-enforcing agreement $((x_{1t}(z^*_{t-1}), \ldots, x_{nt}(z^*_{t-1}); z^*_t))^T_{t=1}$ for the dynamic game Γ_{z_0}. ■

.

References

1. Aumann, R. (1959), “Acceptable points in general cooperative *n*-person games”, in *Contributions to the Theory of Games* IV,” Princeton University Press, Princeton, N. J.

2. Barrett, S. (1994), “Self-enforcing international environmental agreement”, *Oxford Economic Papers*, 46, 878-94.

3. Basar, B. and G-J. Olsder (1995), *Dynamic Non-cooperative Game Theory*, 2nd edition, Academic Press, London and San Diego.

4. Bernheim, B. D., B. Peleg, and M. D. Whinston (1987), “Coalition-proof equilibria. 1. Concepts”, *Journal of Economic Theory*, 42, 1-12.

5. Chander, P. (2007), “The gamma-core and coalition formation”, *International Journal of Game Theory*, 2007: 539-556.

6. Chander, P. (2010), “Cores of games with positive externalities”, CORE DP No. 2010/4.

7. Chander, P. and M. Wooders (2010), “Subgame perfect cooperation in an extensive game”, Vanderbilt Working Paper No. 10-W08.

8. Dixit, A. (1990), *Optimization in Economic Theory*, 2nd edition, Oxford University Press.

9. Dockner, E. J. and N. van Long (1993), “International pollution control: cooperative versus nonocooperative strategies”, *Journal of Environmental Economics and Management*, 25, 13-29.

10. Dockner, E.J. N. van Long and G. Sorger (1996), “Analysis of Nash equilibria in a class of capital accumulation games”, *Journal of Economic Dynamics and Control*, 20, 1209-35.

11. Dutta, P. K. and R. Radner (2004), “Self-enforcing climate change treaties”, *Proceedings of the National Academy of Science*, 101, 5174-5179.

12. Dutta, P. K. and R. Radner (2009), “Strategic analysis of global warming: theory and some numbers”, *Journal of Economic Behavior and Organization*, 71, 187-209..

13. Harstad, B. (2012), “Climate contracts: A game of emissions, investments, negotiations, and renegotiations, *Review of Economic Studies*, forthcoming.

14. Nordhaus, W. D. and Z. Yang (1996). “A regional dynamic general-equilibrium model of alternative climate-change strategies”, *American Economic Review*, 96, 741-765.

15. Osborne, M. and A. Rubinstein (1994), *A Course in Game Theory*, MIT Press, Cambridge, Mass.

16. Reinganum, J. and N. Stokey (1985), “Oligopoly extraction of a commom property natural resource; the importance of the period of commitment in dynamic games”, *International Economic Review*, 26, 161-173.

17. Yang, Z. (2003), “Reevaluation and renegotiation of climate change coalitions – a sequential closed-loop game approach”, *Journal of Economic Dynamics and Control*, 27, 1563-1594.