Lending to Uncreditworthy Borrowers

August 2012

Abstract

We study optimal lending behavior under adverse selection in environments with heterogeneous borrowers—specifically, where the borrower's reservation payoffs (outside options) increase with quality (creditworthiness). Our results show that factors affecting credit supply can also affect lending standards either directly through lending costs or indirectly through borrower reservation payoffs. Lending to uncreditworthy borrowers can be prevented by lowering reservation payoffs, by raising lending costs, or both. Lenders seeking to attract creditworthy borrowers with high reservation payoffs would have to lower rates and, consequently, increase collateral requirements on offers that screen out uncreditworthy types. This leads to higher screening costs, thereby increasing the profitability of offers that pool uncreditworthy borrowers—a veritable lowering of credit standards. In addition, equilibria in a competition version of the model can also explain the phenomenon of "cream-skimming" by outside (foreign) lenders. Surprisingly, we find that the presence of an informed rival actually aids "cream-skimming" behavior.

Keywords: Bank competition; Credit allocation, Lending standards *JEL*: G21; D43.

1 Introduction

Traditional theories of financial intermediation typically assume that borrowers are homogeneous, especially in terms of opportunities outside the lending relationship. If one considers the "period of quiescence in banking" in the U.S. from the mid-1930s up to the 1990s, such an assumption appears fairly innocuous. Indeed, banking in this period was characterized by limited entry and local deposit monopolies so that bank charters enjoyed a significant degree of monopoly value (Gorton, 2009). As a result, borrowers' outside options were fairly limited. Conditions have changed fairly rapidly since the 1990s as branching deregulation, technological advances, and cross-border entry increased lender competition and borrower poaching. Therefore, it is no surprise that deregulation and competition have improved outside options for borrowers (Jayaratne and Strahan, 1996; Black and Strahan, 2002).

This paper studies optimal lending behavior in environments with heterogeneous borrowers specifically, where borrowers' reservation payoffs (outside options) depend on quality (creditworthiness). We assume that reservation payoffs increase with borrower quality in that more efficient (less risky) borrowers have better outside options. Under incomplete information, the lender faces an adverse selection problem in that the borrower's type (creditworthiness or quality) is private information of the borrower.¹ Lenders use collateral as a screening mechanism to address this adverse selection problem and to sort borrowers of different quality (Besanko and Thakor, 1987).² Screening is both costly and inefficient because we assume that lenders can recover only a fraction of the collateral posted—a salvage rate strictly less than unity (Barro, 1976).

The borrower reservation payoffs considered here are exogenous but non-random: the payoffs are the same as would occur if borrowers could obtain loans from a rival lender that has complete information about borrower quality. A growing literature examines how lenders acquire information about borrower creditworthiness in the course of a lending relationship (Boot and Thakor, 1994, 2000; Boot, 2000). Consequently, new and entrant lenders currently face borrowers whose creditworthiness is no longer just the private information of the borrowers but also information available to rival lenders (possibly from a prior lending relationship). This form of multiple informational disadvantages exacerbates the standard adverse selection problem faced by lenders. Moreover, an informed rival's knowledge about creditworthiness often implies that lending rates to creditworthy borrowers are adjusted according to the borrower's creditworthiness.³ Accordingly, borrowers of higher quality (lower risk) have higher reservation

¹Under adverse selection, riskiness is an exogenous and unobservable characteristic of agents. Accordingly, the characterization of risk throughout this article refers to unobservable risk (i.e., risk conditional on observables). We use the terms "creditworthiness", "quality", "risk" and "type" interchangeably.

 $^{^{2}}$ Jimenez et al. (2006) present evidence consistent with such adverse selection theories that, conditional on observable risk, there exists a negative association between collateral and a borrower's risk.

³Accordingly, we distinguish between two types of good-risk or creditworthy borrowers: high-risk and low-risk borrowers. A third category of borrowers will be classified as bad-risk or uncreditworthy borrowers, given below.

payoffs.

Informed rivals are likely to have identified, in addition to creditworthy clients, a section of the borrower population that is uncreditworthy: potential borrowers whose likelihood of default is so high that it is not profitable to lend to them at any rate. As such these borrowers are likely to be denied loans from informed lenders. Nevertheless, they may choose to apply for loans from new (uninformed) lenders (Sharpe 1990, von Thadden 2004). So, not only does a lender have to sort creditworthy borrowers of different risk quality, but it also has to avoid lending to uncreditworthy borrowers. Therefore, while outside options of creditworthy types vary according to offers from rival lenders, the options of uncreditworthy borrowers are invariant in that they can borrow only from new (uninformed) lenders.

Our results show that factors affecting the supply of credit can also affect lending standards either directly through lending costs or indirectly through borrower reservation payoffs. Lending to uncreditworthy borrowers can be prevented by raising lending costs, by lowering reservation payoffs (of creditworthy borrowers), or both. Importantly, this result holds even in markets that are conducive to pooling, such as markets with a sufficiently low proportion of uncreditworthy types. An uninformed lender's ability to attract creditworthy borrowers depends on the magnitude of its (second-best) surplus, net of screening and pooling costs.⁴ Raising lending costs shrinks this loan surplus and consequently the lender's ability to secure creditworthy borrowers.⁵ Additionally, it increases pooling costs by increasing the losses from lending to uncreditworthy borrowers. The converse of this result has important implications: lower lending costs make pooling equilibria more profitable than screening.⁶

More importantly, the novelty in our setup arises from how the mix of contract offers changes with borrowers' reservation payoffs. Lowering reservation payoffs increases the surplus and, therefore, the lender's ability to attract creditworthy types. In addition, the greater surplus from lower reservation payoffs allows the lender to raise repayment requirements (to extract the full surplus). Raising repayment requirements reduces collateral requirements on offers that screen out uncreditworthy borrowers. Reduced collateral requirements lower screening costs, thereby favoring screening equilibria. The converse is also true: Higher reservation payoffs imply that the lender has to lower rates significantly to attract creditworthy types. On offers that are intended to screen out uncreditworthy borrowers, a lower repayment entails a higher

 $^{^{4}}$ The first-best (social) surplus—i.e., gains from trade under complete information—is obtained by subtracting lending costs and borrowers' reservation payoffs from the expected return on the loan. Under incomplete information, screening and pooling costs reduce this social surplus and, therefore, the second-best surplus is strictly smaller than the first-best.

 $^{^{5}}$ The model allows for a broader interpretation of lending costs than suggested by deposit rates. Asymmetries in lending costs can arise from differences in operating cost, interest expenses, or even the cost of inefficiencies that arise due to deviations from best practices. See Berger and Mester (2003) for a more formal treatment of the lending costs of banks.

⁶An active mechanism for screening or selection along the lines of borrower quality is germane to the discussion in this paper. In this sense, the result here is different from models in which the overlending problem occurs in the absence of a screening mechanism (DeMeza and Webb, 1987). See Section 5.3 for details.

collateral requirement. In this way, higher reservation payoffs make screening less profitable by increasing collateral requirements on offers than screen out uncreditworthy types. Of course, the greater the proportion of uncreditworthy types, the lower the borrower payoffs would have to be for lenders to switch from pooling to screening them.

We present two versions of our model: A baseline version that uses exogenous changes in borrower's type-dependent payoffs as described above and a competition version where we explicitly include an informed lender making competing offers. The rationale in presenting both versions is twofold: First, the competition version serves to motivate changes in borrowers' reservation payoffs in terms of changes in lending costs of the informed lender. We show that the (duopolistic) competition version is isomorphic to the baseline version and the optimization problem of the uninformed lender is similar in both versions.⁷ In short, the competition version provides an alternative interpretation for exogenous changes in borrower payoffs: namely, exogenous changes to the cost advantage of the uninformed lender over its informed rival. The second reason is to demonstrate that the presence of an informed rival would allow for equilibria in which the (uninformed) lender "cream-skims" borrowers of the highest quality. In essence, this makes explicit the result that such equilibria do not exist in the absence of informed rivals.

The competition version provides us with the counterintuitive result that the presence of a rival with an informational advantage actually aids the uninformed lender in cream-skimming the lowest-risk borrowers. Under certain conditions, the uninformed lender is unable to sort among creditworthy types without attracting uncreditworthy borrowers.⁸ Consequently, there is no equilibrium in which it can attract creditworthy borrowers. In this situation, the ability of the informed lender to attract away the intermediate type (high-risks) allows the uninformed lender to attract borrowers of the highest quality. This cream-skimming result finds support in empirical studies on foreign lenders in emerging markets (see Detragiache et al, 2008 and references therein). While the literature has attributed this phenomenon to differences in lending technologies between foreign and domestic lenders, this model shows that such a result can be derived from a general model of asymmetrically informed lenders (see Section 5 for more details).

The work perhaps most closely related to this paper is Dell'Ariccia and Marquez (2006), in which an uninformed lender is unable to distinguish between "lemons" rejected by the incumbent and new borrowers shopping around for lower interest rates (Dell'Ariccia et. al, 1999, Dell'Ariccia and Marquez, 2004).⁹ An interesting feature of all these models is that the informed

⁷Sengupta (2007) examines a problem of entry in which lenders compete over the incumbent's clients only. The question addressed in this paper takes on greater relevance because it relaxes the restrictive assumption that all borrowers are known to be creditworthy.

⁸Such conditions can arise with the uninformed lender's efforts to combine low repayment rates with high collateral requirements on its screening offers. On the one hand, a significantly high collateral requirement is needed to screen out uncreditworthy borrowers and attract only creditworthy ones. But doing so may reduce rates to sufficiently low levels—so low that the uninformed lender breaks even if, among creditworthy types, only the lowest-risk (best-quality) types accept.

⁹An alternative way of modeling lender asymmetry allows for differences in their abilities to protect themselves

lender successfully retains all of its creditworthy clients, and therefore lenders effectively compete for new borrowers only. In this setting, equilibrium behavior depends on the proportion of new (unknown) borrowers in the population. Yet, at any given time, the number of new entrepreneurs seeking credit could be small when compared with the number of existing firms in the market. In contrast, this paper models the information problems faced by banks in lending to existing borrowers, such as that described in the conventional bank lending channel (Bernanke and Blinder, 1988).¹⁰

An alternative interpretation of the results is that factors affecting credit supply can also affect lending standards.¹¹ In related studies, Ruckes (2004) and Dell'Ariccia and Marquez (2006) attribute changes in bank lending standards to exogenous changes in the demand for credit during the upward phase of the credit cycle. In contrast, our model takes credit demand conditions as given and attributes such changes to factors affecting credit supply. These supplyside effects can operate directly through lending costs or indirectly through borrower reservation pavoffs.¹² An increase in borrowers' reservation payoffs may arise both from an increase in lending costs of rivals (as in the competition version) or, alternatively, a greater number of rival lenders with heterogeneous lending costs. To the extent that increased competition leads to higher reservation payoffs, the model's results are related to the literature studying the effects of competition policy on lending standards (see Gorton and He, 2008 and references therein). In this regard, the findings deviate from Dell'Ariccia and Marquez (2006), which predicts a higher incidence of screening with an increase in the number of lenders but are consistent with Broecker (1990) and Marquez (2002) in that increased competition reduces loan quality. As mentioned previously, the mechanism described in our model is different in that it allows for borrower poaching. As a result, competing offers must have lower rates to attract creditworthy types.¹³

Borrowers' reservation payoffs are also known to increase during the upward phase of the

against contractual enforceability and, therefore, have different salvage values for posted collateral (Iacoviello and Minetti, 2006; Ferraris and Minetti, 2007). In an environment where the incumbent has superior liquidation skills, Ferraris and Minetti, (2007) study how credit quality deteriorates with lender entry and competition.

¹⁰Bernanke and Gertler (1995, p. 40) describe this channel as follows. "Banks, which remain the dominant source of intermediated credit in most countries, specialize in overcoming informational problems and other frictions in credit markets. If the supply of bank loans is disrupted for some reason, bank-dependent borrowers (small and medium-sized businesses, for example) may not be literally shut off from credit, but they are virtually certain to incur costs associated with finding a new lender, establishing a credit relationship and so on."

¹¹In the competition version of the model, the overall quality of loans is strictly worse under equilibria that pools uncreditworthy borrowers. In all other equilibria, offers are accepted by creditworthy borrowers only. Notably, both pooling and screening uncreditworthy borrowers are welfare reducing because both reduce the first-best surplus.

 $^{^{12}}$ Rajan (1994) also studies how supply-side factors, such as reputational concerns of bank managers, affect lending behavior of banks. Although the mechanism outlined in this paper is significantly different, our results provide support for Rajan's hypothesis. For example, low lending costs assist a liberal credit policy by allowing bank managers to absorb realized and expected losses, whereas higher lending costs would prompt more managers to screen borrowers.

¹³This feature is at odds with the other studies noted above, which predict that break-even rates must rise with increased competition. Unlike our model, which allows for lender heterogeneity in terms of lending costs, these models assume symmetric lenders.

credit cycle. Our model predicts that high reservation payoffs imply higher costs of screening out uncreditworthy types—leading to more pooling equilibria. In this sense, the model explores another mechanism by which loan quality can be adversely affected at the upward phase of the credit cycle—a feature that finds strong empirical support (Asea and Blomberg, 1998; Lown and Morgan, 2003; Berger and Udell, 2004). Finally, there is a growing empirical literature on how low lending costs affect lending standards (see Maddaloni and Peydro, 2011, and references therein). In Section 5, we discuss how the model's results find support in recent empirical work on risk-taking by lenders (Ioannidou et al., 2009, Jiménez et al., 2011, and Maddaloni and Peydro, 2011).

An important concern here is whether the results emphasized in this paper can be established in a more parsimonious model. In Section 5.3, we explain why this concern is misplaced. The rest of this paper is organized as follows. The baseline version of the model and the set of candidate equilibria are presented in Section 2. Section 3 describes the competition version of the model. Details on the model intuition and the solution to the model are provided in Section 4. Section 5 provides a discussion of the results. Section 6 concludes.

2 Baseline model

The basic setup of this paper is similar to that of Besanko and Thakor (1987). Entrepreneurs (borrowers) can borrow a dollar from a lender and invest in a project. The project returns xif it succeeds (with probability $1 - \theta$) and zero if it fails (with probability θ). Lenders' loan contracts consist of a repayment R and a collateral requirement C. Borrowers' reservation utility and lenders' lending cost are denoted by V^0 and ρ , respectively. If a borrower defaults, the lender can recover only a fraction, β , of the collateral ($0 < \beta < 1$). Thus, the *salvage rate* of collateral, β , is a measure of the disparity in the borrower and lender valuation of collateral. Both lenders and borrowers are risk neutral. Lenders' profits from the loan contract (R, C) are given by $\pi(R, C, \theta) = (1 - \theta)R + \beta\theta C - \rho$, while a borrower's payoff under the same contract is $V(R, C, \theta) = (1 - \theta)(x - R) - \theta C$. Therefore, a loan contract (R, C) generates a social surplus of $[(1 - \theta)x - \rho - V^0] - (1 - \beta)\theta C$. Notably, a strictly positive collateral requirement entails a deadweight loss of $(1 - \beta)\theta C$, implying that, *ceteris paribus*, zero-collateral loan contracts are first-best.

The model assumes a fixed pool of borrowers indexed by their risk parameter, θ , the probability of default. The fraction ν_l of entrepreneurs are low-risk ($\theta = \theta_l$), the fraction ν_h of borrowers are high-risk ($\theta = \theta_h$), and the fraction ν_b are bad-risk ($\theta = \theta_b$), with $0 < \theta_l < \theta_h$ $< \theta_b < 1$ and $\nu_h + \nu_l + \nu_b = 1$. Bad-risk borrowers are uncreditworthy in that the surplus generated on loans to them is strictly negative throughout (i.e., $(1 - \theta_b)x < \rho + V^0$, for all ρ). Both high-risk and low-risk borrowers are creditworthy (or "good" risk) in that all loan contracts always generate gains from trade-a positive first-best surplus (i.e., $(1 - \theta_g)x > \rho + V^0$, where k = g = h, l). Stated differently, a lender with complete information would always extend loans to good risks and deny them to bad risks. In addition, we assume a first boundary condition $[(1-\theta_g)(1-\theta_b)x-(1-\theta_g)V^0-(1-\theta_b)\rho] > 0$ for g = h, l. This ensures that uncreditworthy types do not find the lenders' full information competitive offers to creditworthy types unattractive.

2.1 Type-dependent Reservation Payoffs

In this setting, we study how changes in borrower's type-dependent reservation payoffs determine lender's contract offers—under both complete and incomplete information. Under incomplete information, the lender faces an adverse selection problem in that the borrower's type (creditworthiness) is private information of the borrower. The exogenous variations in borrower's reservation payoffs considered here are non-random. The reservation payoff of the uncreditworthy borrower is fixed at V^0 . In contrast, variations in type-dependent reservation payoffs for creditworthy borrowers are given by

$$V_q^0 = (1 - \theta_g)x - \lambda, \quad g = h, l$$

where $\lambda \ge \rho > 0$. In effect, we assume that exogenous variations in the parameter λ lead to equal and opposite changes in both V_h^0 and V_l^0 .

In addition to the boundary condition given above, we note that two other boundary conditions are satisfied. First, changes in λ satisfy the condition $(\theta_h - \theta_l)x \geq V_l^0 - V_h^0$. This condition ensures that lenders' offers are not overcollateralized (Besanko and Thakor, 1987). Second, these changes in λ also imply *countervailing incentives*, that is $(1 - \theta_h)V_l^0 > (1 - \theta_l)V_h^{0.14}$ In section 3, we demonstrate that the pattern of variations in type-dependent payoffs considered here are identical to the payoffs that borrowers would receive in transactions with an informed lender (a lender with complete information on borrower types). In that version, variations in the lending costs of the informed lender play a role identical to that of the variations in λ considered here.

The timing of the game can be described as follows: Nature selects borrower types. Under complete information, an (informed) lender can distinguish borrower types and offers one contract for each type. Under incomplete information, the (uninformed) lender cannot distinguish between types and therefore offers a menu of contracts. Finally, borrowers either accept or reject contracts. We focus exclusively on pure strategy equilibria. As is standard in the principal

¹⁴Under monopolistic screening, the lender (a monopolist principal) extracts the entire surplus. Since low-risk (efficient) types generate a greater surplus from the loan than high-risk (inefficient) types, the low-risk types are charged the higher rate. Naturally, under monopolistic screening, the efficient agent typically has the incentive to mimic the inefficient agent. *Countervailing incentives* refers to a case of type-dependent reservation payoffs wherein the efficient agent's reservation payoffs are sufficiently higher than that of an inefficient agent so as to make offers to the efficient agent attractive to the inefficient one. In this scenario, in order to attract low-risk types, who have such better outside opportunities, the monopolist has to lower rates significantly. As a result, these offers turn out to be attractive to high-risk types as well—and they mimic low risks to get the lower rate (see Laffont and Martimort, pp.104-105).

agent literature, we assume that if the borrower is indifferent between two offers, the contract that the lender prefers is chosen. We begin by describing the optimal contracts under complete information in lemma 1. Proofs for all results given below are provided in Appendix A.

Lemma 1 Under complete information, the lender denies credit to b-types. For borrowers of types g = h, l an informed lender offers a contract from the set $Z_g(\rho) = \{(R_g, 0) : R_g \in [\underline{R}_g(\rho), \overline{R}_g]\}$, where $\underline{R}_g(\rho) = \frac{\rho}{1-\theta_g}$ and $\overline{R}_g = x - \frac{V^0}{1-\theta_g}$, g = h, l are the first-best (zero-collateral) minimum and maximum repayments, respectively.

Under monopoly, a lender's full information offer is the first-best (zero-collateral) maximum, obtained by setting $V_g(R_g, 0) = V_g^0$, so that $R_g^0(\lambda) = \frac{\lambda}{1-\theta_g}$, g = h, l. Notably, $R_g^0 = \bar{R}_g$ if $V_g^0 = V^0$. Under perfect competition, fully informed lenders offer the first-best (zero-collateral) minimum repayment, $\underline{\mathbb{R}}_g(\rho)$, by setting $\pi_g = 0$, g = h, l. An informed (monopolist) lender's offers include a repayment rate that binds the borrower's participation constraint. This yields a creditworthy borrower the reservation payoff, V_g^0 , while the lender extracts the entire (first-best) surplus generated from the loan, $(1 - \theta_g)x - \rho - V_g^0$, g = h, l.

2.2 Lender's Optimization Problem under Incomplete Information

Under incomplete information, the revelation principle ensures that there is no loss of generality in restricting the principal to offer simple menus having at most as many options as the cardinality of the type space. This implies that an uninformed lender's menu contains at most three offers, one for each borrower-type, denoted by (R_k, C_k) where k = b, h, l. Ideally, the lender would like to avoid uncreditworthy b-types whose reservation payoffs are fixed at V^0 . This modifies the lender's standard optimization problem to one where it maximizes $\Pi \equiv \nu_b \pi_b + \nu_h \pi_h + \nu_l \pi_l$ where $\pi_k = (1 - \theta_k)R_k + \beta \theta_k C_k - \rho$, k = b, h, l subject to the following type-dependent participation constraints

$$V_h(R_h, C_h) \geq V_h^0 \tag{1}$$

$$V_l(R_l, C_l) \geq V_l^0 \tag{2}$$

and the following incentive compatibility constraints

$$V^0 \geq V_b(R_h, C_h) \tag{3}$$

$$V^0 \geq V_b(R_l, C_l) \tag{4}$$

$$V_h(R_h, C_h) \geq V_h(R_l, C_l) \tag{5}$$

$$V_l(R_l, C_l) \geq V_l(R_h, C_h), \tag{6}$$

where $V_g^0 = (1 - \theta_g)x - \lambda$, g = h, l and $\lambda \in [\rho, (1 - \theta_h)x - V^0]$. Since $\pi_b < 0$, the lender does not offer contract (R_b, C_b) in equilibrium and we replace $V_b(R_b, C_b)$ with the b-type's reservation

utility, V^0 , in (3)-(4).

Candidate	Profit	Customer types	Menu of contracts	Breakeven
equilibria		accepting offers	offered	cutoff
Screen-1	Π_1^S	h;l	$(R_h^S, C_h^S); (R_l^S, C_l^S)$	$\hat{\lambda}_{b,h}^{S}(ho),\hat{\lambda}_{h,l}^{S}(ho)$
Screen-2	Π_2^S	h	(R_h^S, C_h^S)	$\hat{\lambda}^{S}_{b,h}(ho)$
Pool-1	Π_1^P	(b,h,l)	$(\mathbf{R}_l^P, 0)$	$\hat{\lambda}_{1}^{P}(ho, u_{b}, u_{l})$
Pool-2	Π_2^P	(b,h)	$(\mathbf{R}_{h}^{P},0)$	$\hat{\lambda}_2^P(ho, u_b, u_h)$
Hybrid-1	Π_1^Y	(b,h);l	$(\mathbf{\bar{R}}_{h}^{P},0);(R_{l}^{S},C_{l}^{S})$	$\hat{\lambda}_1^P(ho, u_b, u_h), \hat{\lambda}_{h,l}^S(ho)$
Hybrid-2	Π_2^Y	(h,l)	$\left(R_g^Y, C_g^Y\right)$	${\hat \lambda}^Y(ho, { u}_h, { u}_l)$

Table 1: Uninformed lender's menu offers under different candidate equilibria

Three categories of equilibria are characterized in terms of the lender's offers: (1) screening equilibria, in which the lender's offers successfully sorts borrower types,¹⁵ (2) pooling equilibria, wherein the lender's offer of a single contract is accepted by two or more borrower types, and (3) hybrid equilibria, which involves the bunching (or pooling) of adjacent borrower types while screening the third type. This occurs if, for example, the lender bunches the creditworthy borrowers (*h*- and *l*-types) while screening the uncreditworthy ones (*b*-types). We characterize this category as hybrid because its offers involve both pooling and screening. In each case, superscripts S, P, and Y are used for screening, pooling, and hybrid equilibria, respectively.

After eliminating loss-making offers to the *b*-types, we obtain a set of six candidate equilibria as summarized in Table 1. Details of this process of elimination are provided in Appendix A. These candidate equilibria emerge as the final equilibria of the model for different values of the model parameters (as shown below). Within each category, candidate-1 has a greater number of borrower types accepting offers than candidate-2. For example, in candidate equilibrium Hybrid-2, the lender screens out the *b*-type, but in Hybrid-1 it pools them with *h*-types. If the lender can screen the *b*-type from the *h*-type, but not sort between the *h*-type and the *l*-type, then its offers in *Screen-2* would be accepted by the *h*-types. However, if the lender can sort between all borrower types, it can offer *Screen-1* whose profits dominate those of *Screen-2*. Similarly, for a given distribution of borrower types, the lender's offers in *Hybrid-1* dominate those in *Pool-2*. Finally, there is no equilibrium in which the monopolist lender bunches the non-adjacent, *b*- and *l*-types or is able to attract only the *l*-types away.¹⁶

¹⁵The terms "sorting", "screening" and "separating" are used interchangeably. Also, the terms "bunching" and "pooling" are used interchangeably.

¹⁶However, such equilibria do exist under competition between an informed and uninformed lender (see Section 3.2).

2.3 Candidate Equilibria under Incomplete Information

Screening and pooling equilibria impose costs that reduce the first-best surplus. Consequently, the second-best surplus in incomplete information settings is strictly smaller that the first-best gains from trade. In particular, all screening contract offers have a positive collateral requirement. Expected losses from liquidation of collateral in case of default reduce the first-best surplus to its second-best value of $[(1-\theta)x - \rho - V^0] - (1-\beta)\theta C$. Likewise, pooling offers reduce expected surplus because *b*-types accept such offers and their contribution to the loan surplus is negative: $(1 - \theta_b)x - \rho - V^0 < 0$. A non-negative (second-best) surplus indicates that screening or pooling is *feasible*: that is, an uninformed lender's offers can satisfy borrower's (participation and incentive) constraints and still break even.

2.3.1 Screening Equilibria

For screening equilibria to be feasible, either lending costs or borrower reservation payoffs have to be sufficiently low (the second-best surplus sufficiently high). Therefore, for a given lending cost screening is feasible if reservation payoffs satisfy a threshold or alternatively, if $\lambda \geq \hat{\lambda}$. There are two such screening thresholds, one for sorting each pair of adjacent types. The first threshold is $\hat{\lambda}_{h,l}^S$ for screening the *h*-types from the *l*-types and the second is $\hat{\lambda}_{b,h}^S$ for screening the *h*-types.

$$\hat{\lambda}_{h,l}^{S}(\rho) = \frac{1}{1 - (1 - \beta)\theta_l}\rho \tag{7}$$

$$\hat{\lambda}_{b,h}^{S}(\rho) = \frac{\theta_b - \theta_h}{\theta_b(1 - \theta_h) - \beta\theta_h(1 - \theta_b)}\rho + \frac{(1 - \beta)\theta_h(1 - \theta_h)}{\theta_b(1 - \theta_h) - \beta\theta_h(1 - \theta_b)}[(1 - \theta_b)x - V^0].$$
(8)

For both cutoffs, $\hat{\lambda}'(\rho) > 0$ and $\hat{\lambda}''(\rho) = 0$. Moreover, because screening costs depend only on collateral use and the likelihood of default, screening thresholds are independent of the distribution of borrower types in the population. When both thresholds are satisfied, the lender can sort all borrower types under *Screen-1* (see Table 1) where its offers (R_h^S, C_h^S) and (R_l^S, C_l^S) are given as

$$R_l^S(\lambda) = C_l^S(\lambda) = \lambda, \tag{9}$$

$$R_h^S(\lambda) = \frac{\theta_b}{\theta_b - \theta_h} \lambda - \frac{\theta_h}{\theta_b - \theta_h} [(1 - \theta_b)x - V^0]$$
(10)

$$C_h^S(\lambda) = -\frac{1-\theta_b}{\theta_b-\theta_h}\lambda + \frac{1-\theta_h}{\theta_b-\theta_h}[(1-\theta_b)x - V^0].$$
 (11)

Note that to sort creditworthy types, $C_l^{S'}(\lambda) > 0$, but to screen out *b*-types, collateral requirements are decreasing in λ , $C_h^{S'}(\lambda) < 0$. This is because increases in λ reduce both V_h^0 and V_l^0 by equal amounts in the former but leave $V_b^0 = V^0$ unchanged in the latter. Therefore, in

screening out b-types, increases in λ reduce screening costs by reducing collateral requirements. So, higher reservation payoffs reduce costs for screening b-types. We start with the propositions that describe the screening equilibria (proofs for all results are in Appendix A).

Proposition 1 If $\lambda \geq \max(\hat{\lambda}_{h,l}^S, \hat{\lambda}_{b,h}^S)$, where $\hat{\lambda}_{h,l}^S$ and $\hat{\lambda}_{b,h}^S$ are given in (7) and (8), a pure strategy equilibrium wherein the lender sorts all borrower types, is characterized as follows: (a) lender offers menu $\{(R_h^S, C_h^S), (R_l^S, C_l^S)\}$ as given in (9)-(11)

(b) both creditworthy types accept their respective offers but b-types types reject both offers (c) lender's expected profits are $\Pi_1^S = \nu_h \left[\frac{\theta_b(1-\theta_h)-\beta\theta_h(1-\theta_b)}{\theta_b-\theta_h}\lambda - \frac{(1-\beta)\theta_h(1-\theta_h)}{\theta_b-\theta_h}\{(1-\theta_b)x - V^0\}\right] + \frac{1}{2} \left[\frac{\theta_b(1-\theta_h)-\beta\theta_h(1-\theta_h)}{\theta_b-\theta_h}\lambda - \frac{(1-\beta)\theta_h(1-\theta_h)}{\theta_b-\theta_h}(1-\theta_b)x - V^0\right]$ $\nu_l [1 - (1 - \beta)\theta_l]\lambda - (\nu_h + \nu_l)\rho$

In Screen-1, the incentive constraints (3) and (5) bind. In addition, both participation constraints (1) and (2) bind as the lender extracts the entire surplus. Lastly, $\lambda \geq \max(\hat{\lambda}_{h,l}^{S}, \hat{\lambda}_{b,h}^{S})$ implies that both offers in the menu yield non-negative profits. However, if $\hat{\lambda}_{h,l}^S > \lambda \ge \hat{\lambda}_{b,h}^S$, the lender still has the option to just screen one creditworthy type, as shown in Screen-2. Interestingly, with $\hat{\lambda}_{h,l}^{S} > \lambda$, the lender's offer (R_{l}^{S}, C_{l}^{S}) in Screen-1 does not satisfy the participation constraint of *l*-types in Screen-2, $V_l(R_l^S, C_l^S) < V_l^0$. As a result, this offer would be rejected by *l*-types, giving us the *Screen-2* equilibrium as follows.

Proposition 2 If $\hat{\lambda}_{h,l}^S > \lambda \ge \hat{\lambda}_{b,h}^S$, where $\hat{\lambda}_{h,l}^S$ and $\hat{\lambda}_{b,h}^S$ are given in (7) and (8), a pure strategy equilibrium wherein the lender screens out the b-type and lends to the h-types only, is characterized as follows:

- (a) lender offers contract (R_h^S, C_h^S) ,

(b) b-types and l-types reject this offer, h-types accept (c) lender's expected profits are $\Pi_2^S = \nu_h \left[\frac{\theta_b(1-\theta_h)-\beta\theta_h(1-\theta_b)}{\theta_b-\theta_h}\lambda - \frac{(1-\beta)\theta_h(1-\theta_h)}{\theta_b-\theta_h}\{(1-\theta_b)x - V^0\}\right] - \nu_h \rho$

However, a separating equilibrium wherein the lender can screen out both *b*-types and *h*-types, and only lend to the *l*-types, does not exist. With $\hat{\lambda}_{b,h}^S > \lambda \geq \hat{\lambda}_{h,l}^S$, if the lender offers menu $\{(R_h^S, C_h^S), (R_l^S, C_l^S)\}$, h-types would reject the offer (R_h^S, C_h^S) because it doesn't satisfy their participation constraint $V_h(R_h^S, C_h^S) < V_h^0$. Instead, they would accept the offer intended for *l*-types, (R_l^S, C_l^S) essentially rendering such an offer unprofitable. Therefore, if $\hat{\lambda}_{b,h}^S > \lambda \geq 1$ $\hat{\lambda}_{h,l}^{S}$, an uninformed lender does not make a screening offer. Importantly, such equilibria are possible in the presence of an informed lender (see Section 3.2 for details).

2.3.2**Pooling Equilibria**

Strictly speaking, pooling equilibria here refers to pooling the b-types. First, for such pooling offers it is optimal for the lender to set the collateral requirement to zero (see Lemma 7 in Appendix A). Second, with b-types yielding a negative surplus pooling occurs only if ν_b is sufficiently small. Of course, pooling *b*-types reduces lender profits, and the first-best surplus to second-best. There are two such pooling equilibria: (1) where *b*-types are pooled with *h*-types only and (2) where all types are pooled together.

Proposition 3 If $\lambda \geq \hat{\lambda}_1^P \equiv (\frac{1-\theta_l}{1-\mathbb{E}(\theta)})\rho$, a pure strategy equilibrium wherein the lender pools all borrowers is characterized as follows:

- (a) lender offers contract $(R_l^0, 0)$, such that $R_l^0 = x \frac{V_l^0}{1 \theta_l} = \frac{\lambda}{1 \theta_l}$.
- (b) all borrowers accept this offer.

(c) lender's profits are $\Pi_1^P = \frac{1 - \mathbb{E}(\theta)}{1 - \theta_l} \lambda - \rho$ where $\mathbb{E}(\theta) \equiv \nu_b \theta_b + \nu_h \theta_h + \nu_l \theta_l$.

The proposition above describes *Pool-1*, the equilibrium where the lender pools all borrowers offering $(R_l^0, 0)$, so that (2) binds. This pooling offer covers losses from *b*-types and *h*-types with profits from *l*-types. Clearly, *Pool-1* is feasible (yields non-negative profits) only when ν_l is sufficiently large. The other pooling equilibrium offer is $(R_h^0, 0)$, so that (1) binds. This is given by *Pool-2*, wherein the lender covers losses from *b*-types with profits from *h*-types. Naturally, *Pool-2* is feasible (yields non-negative profits) only when ν_h is sufficiently large as given below.

Proposition 4 If $\lambda \geq \hat{\lambda}_2^P \equiv (\frac{(\nu_b + \nu_h)(1 - \theta_h)}{\nu_b(1 - \theta_b) + \nu_h(1 - \theta_h)})\rho$, a pure strategy equilibrium wherein the lender pools h-types and b-types only, is characterized as follows: (a) lender offers contract $(R_h^0, 0)$, where $R_h^0 = x - \frac{V_h^0}{1 - \theta_h} = \frac{\lambda}{1 - \theta_h}$. (b) b-types and h-types accept this offer but l-types reject this offer (c) lender's expected profits are $\Pi_2^P = \frac{\nu_b(1 - \theta_b) + \nu_h(1 - \theta_h)}{1 - \theta_h}\lambda - (\nu_b + \nu_h)\rho$

2.3.3 Hybrid Equilibria

Hybrid equilibria have elements of both pooling and screening. This occurs when the lender pools or bunches adjacent types and screens the third type.

Proposition 5 If $\hat{\lambda}_{b,h}^S > \lambda \ge \hat{\lambda}_{h,l}^S$ and $\lambda \ge \hat{\lambda}_2^P$, a pure strategy equilibrium wherein the lender separates only the l-types and bunches (pools) b-types and h-types is characterized as follows: (a) lender offers menu $\{(R_h^0, 0); (R_l^S, C_l^S)\}$ and $R_h^0 = x - \frac{V_h^0}{1-\theta_h} = \frac{\lambda}{1-\theta_h}$ (b) l-types accepts the offer (R_l^S, C_l^S) , while b-types and h-types accept the offer $(R_h^0, 0)$. (c) lender's expected profits are $\Pi_1^Y = \frac{\nu_b(1-\theta_b)+\nu_h(1-\theta_h)}{1-\theta_h}\lambda + \nu_l[1-(1-\beta)\theta_l]\lambda - \rho$

The proposition above describes Hybrid-1, which is a combination of lender's offers in *Pool-*2 and a screening offer to *l*-types, so that (5) binds. This allows the lender to pool *b*-types and *h*-types and sort out *l*-types. Unlike Hybrid-1 where all types accept loan offers, Hybrid-2screens out the *b*-types. This second hybrid menu, involves bunching of creditworthy types so that (4) binds. Lender offers $(R_g^Y, C_g^Y), g = h, l$, under Hybrid-2 as given by

$$R_g^Y(\lambda) = \frac{\theta_b}{\theta_b - \theta_l} \lambda - \frac{\theta_l}{\theta_b - \theta_l} [(1 - \theta_b)x - V^0]$$
(12)

$$C_g^Y(\lambda) = -\frac{1-\theta_b}{\theta_b-\theta_l}\lambda + \frac{1-\theta_l}{\theta_b-\theta_l}[(1-\theta_b)x - V^0]$$
(13)

Proposition 6 A pure strategy equilibrium wherein the lender screens out the b-types and bunches the l-type is characterized as follows:

- (a) the lender offers contract (R_g^Y, C_g^Y) given by (12) and (13)
- (b) h-types and l-types accept the offer (R_g^Y, C_g^Y) , but b-types reject this offer.

(c) lender's expected profits are
$$\Pi_2^Y = [\nu_h \{\theta_b(1-\theta_h) - \beta \theta_h(1-\theta_b)\} + \nu_l \{\theta_b(1-\theta_l) - \beta \theta_l(1-\theta_l)\}] \frac{\lambda}{\theta_b - \theta_l} - [\nu_h \{\theta_l(1-\theta_h) - \beta \theta_h(1-\theta_h)\} + \nu_l \{\theta_b(1-\beta)\theta_l(1-\theta_l)\}] \frac{\{(1-\theta_b)x - V^0\}}{\theta_b - \theta_l} - (\nu_h + \nu_l)\rho$$

In summary, there are 3 categories of candidate equilibria: pooling, screening, and hybrid. Within each category, candidate-1 has a greater number of customer types accepting the lender's offers than candidate-2. For example, in candidate equilibrium *Hybrid-2*, the uninformed lender screens out *b*-types but in *Hybrid-1* it pools them with *h*-types. In fact, if an uninformed lender can screen *l*-types borrowers (i.e., if $\lambda \geq \hat{\lambda}_{h,l}^S$), then its profits from offers in *Screen-1* dominate those from offers in *Screen-2*. Similarly, the lender's offers in *Hybrid-1* dominate those in *Pool-2*.

3 Competition between Informed and Uninformed Lender

3.1 An alternative formulation: The Duopoly model

Solutions to the lender's optimization problem with type-dependent reservation payoffs, V_k^0 , k = b, h, l, are given above. Payoffs of the uncreditworthy types are fixed at $V_b^0 = V^0$. In contrast, variations in type-dependent payoffs for creditworthy types V_h^0 and V_l^0 arise exogenously from changes to the parameter λ . Naturally, this raises important questions as to the motivation behind such variations in reservation payoffs. In this section, we show that exogenous variations in payoffs (in the baseline version) described above are identical to payoffs that borrowers would receive in transactions with an informed lender–a lender that has complete information on borrower types. Stated differently, the optimization problem of the previous section is identical to that of an uninformed lender (Lender-U) facing this adverse selection problem in competing with an informed lender (Lender-I).

This section describes duopolistic competition between an informed (incumbent) lender that has complete information about borrower creditworthiness and an uninformed (new or entrant) lender that is unable to distinguish between borrowers' risk types. The informed lender (or Lender-I) is (pre-entry) a price-setting monopolist whose lending cost is ρ^{I} and has information about borrower creditworthiness (type) from prior lending relationships. The uninformed lender (or Lender-U) is a new or outside lender whose lending cost is ρ^U . Lender-I's private information here extends not only to its existing (and therefore) creditworthy clients but also to other "prospective" uncreditworthy borrowers that Lender-U would like to avoid. Lender j's offer to borrower k is denoted by (R_k^j, C_k^j) , where j = I, U and k = b, h, l. The lender's profits from this offer are given by $\pi_k^j = (1 - \theta_k)R_k^j + \beta\theta_k C_k^j - \rho^j$ if the borrower accepts the loan contract and zero otherwise. Also, V_k^j denotes borrower k's payoff from contract (R_k^j, C_k^j) , where j = I, U and k = b, h, l.

The timing of this game is the same as in the baseline version; except that here both lenders making offers simultaneously. As is standard in the principal agent literature, we assume that if the borrower is indifferent between two offers of *the same lender*, the contract that the lender prefers is chosen. Also, if a borrower is indifferent between offers by either lender, the offer that yields higher profits for the lender is chosen.¹⁷

Evidently, Lender-*I*'s contractual offers in the competition version are the same as those obtained under complete information (in the baseline version) by replacing parameter λ with the Lender-*I*'s cost of funds ρ^I . For borrowers of type k = b, Lender-*I* denies credit. For borrowers of type k = g = h, l, Lender-*I* offers a contract from the set $Z_g^I(\rho^I) = \{(R_g^I, 0) : R_g^I \in [\mathbb{R}_g^I(\rho^I), \bar{R}_g]\}$, where $\mathbb{R}_g^I(\rho^I) = \frac{\rho^I}{1-\theta_g}$ and $\bar{R}_g = x - \frac{V^0}{1-\theta_g}$ are the first-best (zero-collateral) minimum and maximum repayments, respectively. These results imply that in competing with Lender-*I*, Lender-*U* can get borrowers to accept its offers only if its incentive scheme yields each borrower at least as much payoff as that from contracts offered by Lender-*I*. Stated differently, Lender-*U* faces borrowers whose reservation payoffs are determined by the first-best offers from Lender-*I*. As a result, Lender-*U*'s optimization problem is the same as that described under incomplete information (in the baseline version) and subject to the same incentive constraints as given by (3)-(6). However, type-dependent participation constraints (1)-(2) are now given as

$$V_h(R_h, C_h) \geq \bar{V}_h^I \tag{14}$$

$$V_l(R_l, C_l) \geq \bar{V}_l^I \tag{15}$$

where $\bar{V}_g^I = (1 - \theta_g)x - \rho^I, \ g = h, l.$

Clearly, there is a strong similarity in the equilibria between the baseline (Table 1) and competition versions (Table 2). For the most part, one may substitute ρ^I for λ , ρ^U for ρ , and \bar{V}_g^I , for V_g^0 , g = h, l, in the baseline version (Table 1) to derive the candidate equilibria for the competition version in Table 2. It turns out that the candidate equilibria in the competition version are the same as that for the baseline version, with an additional candidate (see Proposi-

¹⁷Suppose the borrower is indifferent between (feasible) offers from different lenders. If both contracts yield the same profit, the market is split. If a first lender makes zero profits while the second lender's profits are strictly positive, the borrower accepts offers of the second. The second lender can increase borrower payoff by reducing its profits. The lender making zero profits cannot do so and still break even.

tion 7). Exogenous variations in borrower's type-dependent reservation payoffs are now proxied by exogenous changes to Lender-*I*'s cost of funds ρ^I . Analogously, all the thresholds described in Table 1 are replaced by thresholds in Lender-*I*'s cost of funds, as shown in Table 2.

The key differences in equilibria are largely on the borrower side: In the baseline version, a creditworthy borrower rejects the lender's offers because they do not satisfy the borrower's participation constraint. In the competition version, a creditworthy borrower rejects Lender-U's contract offers because Lender-I's offers yield higher payoff. This difference is central to understanding why the competition version includes an additional candidate in *Screen-3* that does not exist in the baseline version.

Candidate	Profit	Customer types	Menu of contracts	Breakeven			
equilibria		$accepting \ Lender-U$'s offer	offered by Lender-U	cutoff			
Screen-1	Π_1^S	h;l	$(R_h^U, C_h^U); (R_l^U, C_l^U)$	$\tilde{ ho}_{b,h}^{S}(ho^{U}), \tilde{ ho}_{h,l}^{S}(ho^{U})$			
Screen-2	Π_2^S	h	$(R_h^U,C_h^U);(R_l^0,C_l^0)$	$ ilde{ ho}^{S}_{b,h}(ho^{U})$			
Screen-3	Π_S^3	l	$\left(R_{l}^{U},C_{l}^{U} ight)$	$ ilde{ ho}_{h,l}^{S}(ho^{U})$			
Pool-1	Π_1^P	(b,h,l)	$(\underline{R}_{l}^{I},0)$	$ ilde{ ho}_1^P(ho^U, u_b, u_l)$			
Pool-2	Π_2^P	(b,h)	$(\underline{R}_{h}^{I},0)$	$\widetilde{ ho}_2^P(ho^U, u_b, u_h)$			
Hybrid-1	Π_1^Y	(b,h);l	$(\underline{R}_{h}^{I},0);(R_{l}^{U},C_{l}^{U})$	$\tilde{\rho}_1^P(\rho^U, \nu_b, \nu_h), \tilde{\rho}_{h,l}^S(\rho^U)$			
Hybrid-2	Π_2^Y	(h,l)	(R_g^U, C_g^U)	$\tilde{ ho}^Y(ho^U, u_h, u_l),$			

TABLE 2. Lender-U's offers under different candidate equilibria

3.2 Cream-skimming equilibria

The (candidate) equilibrium in *Screen-3* is best illustrated in terms of Figure 1 that shows borrower payoffs (bold lines of V) lender profits (dotted lines of π) in (R, C) space. Borrowers' payoffs increase as one moves southwest, while lenders' profits increase to the northeast. Because the informed lender denies credit to *b*-types, their reservation utility is $V_b = V^0$. Figure 1 illustrates the baseline version for $\hat{\lambda}_{b,h}^S > \lambda \geq \hat{\lambda}_{h,l}^S$, or analogously, the competition version for $\tilde{\rho}_{b,h}^S > \rho^I \geq \tilde{\rho}_{h,l}^S$. The uninformed lender can break even with offers of (R_l^S, C_l^S) that screen *h*-types from *l*-types but cannot break even with (R_h^S, C_h^S) that screen *b*-types, However, if offered, contract (R_l^S, C_l^S) will not only attract *l*-types but also attract *h*-types, essentially breaking down the equilibrium. This illustrates that there does not exist equilibria in the baseline version where the uninformed lender screens out only the *l*-types.

The scenario changes in the competition version. Now, *h*-types accept Lender-*I*'s offers $(\underline{R}_{h}^{I}, 0)$ that is equivalent to $(R_{h}^{0}, 0)$ in the baseline version. This allows Lender-*U* to break even with offers of (R_{l}^{U}, C_{l}^{U}) (or equivalently (R_{l}^{S}, C_{l}^{S}) in the baseline version) that screen the *h*-types from *l*-types. Therefore, the competition version yields a third screening equilibrium



Figure 1: Lender's contract offers under different equilibria in (R, C) space shown for the case $\hat{\lambda}_{b,h}^S > \lambda \ge \hat{\lambda}_{h,l}^S$ in the baseline version, or analogously, the case $\tilde{\rho}_{b,h}^S > \rho^I \ge \tilde{\rho}_{h,l}^S$ in the competition version. Borrowers' payoffs (shown in bold lines of V) increase as one moves southwest, while lenders' profits (shown in dotted lines of π) increase to the northeast. In the baseline model, the uninformed lender can break even with offers of (R_l^S, C_l^S) that screen the *h*-types from *l*-types but cannot break even with (R_h^S, C_h^S) that screen the *b*-types from *h*-types. In the competition model, *l*-types accept Lender-*U* offers $(R_l^U, C_l^U) = (R_l^S, C_l^S)$ and *h*-types accept Lender-*I*'s offers $(\underline{R}_h^I, 0) = (R_h^0, 0)$ under a screening equilibrium Screen-3.

in Screen-3 that does not exist in the baseline version. Substituting $\rho^I = \lambda$ and $\rho^U = \rho$ in (7) and (8), we obtain the screening thresholds $\tilde{\rho}_{h,l}^S$ and $\tilde{\rho}_{b,h}^S$, respectively.

$$\tilde{\rho}_{h,l}^S(\rho^U) = \frac{1}{1 - (1 - \beta)\theta_l} \rho^U \tag{16}$$

$$\tilde{\rho}_{b,h}^{S}(\rho^{U}) = \frac{\theta_{b} - \theta_{h}}{\theta_{b}(1 - \theta_{h}) - \beta\theta_{h}(1 - \theta_{b})}\rho^{U} + \frac{(1 - \beta)\theta_{h}(1 - \theta_{h})}{\theta_{b}(1 - \theta_{h}) - \beta\theta_{h}(1 - \theta_{b})}[(1 - \theta_{b})x - V^{0}].$$
(17)

We can now present Screen-3 in terms of the following proposition.

Proposition 7 If $\tilde{\rho}_{b,h}^S > \rho^I \ge \tilde{\rho}_{h,l}^S$, a pure strategy equilibrium wherein Lender-U separates only *l*-types, is characterized as follows:

- (a) Lender-U offers (R_l^U, C_l^U) where $R_l^U = C_l^U = \rho^I$ (similar to (9)).
- (b) Lender-I offers $(\underline{R}_{h}^{I}, 0)$ to h-types and $(\underline{R}_{l}^{I}, 0)$ to l-types, where $\underline{R}_{g}^{I}(\rho^{I}) = \frac{\rho^{I}}{1-\theta_{q}}, g = h, l.$
- (c) The h-types accept offers from Lender-I, but l-types accepts offers from Lender-U. The b-types reject all offers.

(d) Lender-U's expected profits are $\Pi_S^3 = \nu_l [1 - (1 - \beta)\theta_l]\rho^I - \nu_l \rho^U$.

We term *Screen-3* as a cream-skimming equilibrium, where an uninformed lender succeeds in attracting the most efficient (lowest-risk) type despite its information disadvantage. Interestingly, this equilibrium requires the presence of an informed lender that attracts away the intermediate type. The market for creditworthy types is now split between informed and uninformed lender.

In Table 3, we compute the changes in the profits under each candidate equilibrium with respect to changes in λ (ρ^I) and ρ (ρ^U), respectively. It is not difficult to see that the candidates with most derivative with respect to lending costs, ρ ,— namely, *Pool-1* and *Hybrid-1* are the ones that will arise as the final solution to the model in environments with low lending costs.

Candidate	Profit	w.r.t reservation payoffs (λ)	w.r.t lending costs (ρ)
equilibria		or lending cost of Lender- $I(\rho^I)$	or lending cost of Lender- $U(\rho^U)$
Screen-1	Π_1^S	$\nu_h [\frac{\theta_b (1-\theta_h) - \beta \theta_h (1-\theta_b)}{\theta_b - \theta_h}] + \nu_l [1 - (1-\beta)\theta_l]$	$-(u_h+ u_l)$
Screen-2	Π_2^S	$ u_h[rac{ heta_b(1- heta_h)-eta heta_h(1- heta_b)}{ heta_b- heta_h}]$	$- u_h$
Screen-3	Π_S^3	$ u_l [1 - (1 - \beta)\theta_l] $	$- u_l$
Pool-1	Π_1^P	$rac{1 - \mathbb{E}(heta)}{1 - heta_i}$	-1
Pool-2	Π_2^P	$rac{ u_b(1- heta_b)+ u_h(1- heta_h)}{1- heta_h}$	$-(u_b+ u_h)$
Hybrid-1	Π_1^Y	$\frac{\nu_b(1-\theta_b)+\nu_h(1-\theta_h)}{1-\theta_b} + \nu_l[1-(1-\beta)\theta_l]$	-1
Hybrid-2	Π_2^Y	$\frac{[\nu_h\{\theta_b(1-\theta_h)-\beta\ddot{\theta}_h(1-\theta_b)\}+\nu_l\{\theta_b(1-\theta_l)-\beta\theta_l(1-\theta_h)\}]}{\theta_b-\theta_l}$	$-(u_h+ u_l)$

TABLE 3. Comparative Statics

4 Model Solution

4.1 Intuition behind the solution

Tables 1 and 2 reveal strong similarities in the baseline and competition versions of our model. From the point of view of the uninformed lender, candidate equilibria in both versions are identical, except for the candidate Screen-3.¹⁸ The uninformed lender's (Lender-U's) profits

¹⁸For the most part, one may substitute ρ^{I} for λ , ρ^{U} for ρ , and \bar{V}_{g}^{I} , for V_{g}^{0} , g = h, l, in the baseline version (Table 1) to derive the candidate equilibria for the competition version in Table 2. In an extended appendix, we demonstrate that, with the exception of Screen-3, the candidate offers in the baseline and competition versions of the model are the same.

under each candidate equilibrium are described in Propositions 1-7. Among them, Lender-U selects one that yields the maximum (non-negative) profits for a given set of parameter values. Since the maximum is obtained over a finite set of values, each a function of the parameters, it provides us with a solution to the model. Before presenting the solution, we provide the intuition behind the optimal choice of the lender and equilibrium lending behavior.

A simple way to describe the intuition is to determine how various factors change the gains from trade: the expected surplus generated by the loan transaction. As shown above, we denote the *expected loan surplus* under complete information, $[(1 - \theta)x - \rho - V^0]$, as the *first-best surplus* and that under incomplete information as the *second-best surplus*. The first-best surplus is obtained by subtracting lending costs and borrowers' reservation payoffs from the expected return on the project. Under incomplete information, contract offers deviate from first-best because the lender gives up information rent to borrowers in order to get them to accept its offers. However, while the provision of information rent to borrowers typically involves a simple redistribution of the surplus, it does not change the size of surplus generated from the loan.

In contrast, screening and pooling equilibria impose costs that reduce the first-best surplus. Consequently, the second-best surplus in incomplete information settings is strictly smaller that the first-best gains from trade. In particular, all screening contract offers have a positive collateral requirement. Expected losses from liquidation of collateral in case of default reduce the first-best surplus to its second-best value of $[(1-\theta)x-\rho-V^0]-(1-\beta)\theta C$. Likewise, pooling offers reduce expected surplus because b-types accept such offers and the surplus generated from such loans is negative: $(1-\theta_b)x-\rho-V^0 < 0$. A non-negative (second-best) surplus indicates that an uninformed lender's offers can satisfy borrower's (participation and incentive) constraints and still break even. Among them, the lender's optimal offer is given by one that yields maximum profits—the model solution for a given set of parameter values.

Comparative statics for some of the model parameters is fairly simple. A higher salvage rate of collateral, β , implies lower screening costs and therefore a larger second-best surplus. A greater proportion of bad-risks in the borrower population, ν_b , implies higher pooling costs and therefore a smaller second-best surplus. Notably, both these parameters do not affect the firstbest surplus: Under complete information, (informed) lender's contract offers do not include collateral requirements. Neither do they extend credit to *b*-types. Numerical examples given below demonstrate how higher β and ν_b favor screening equilibria in the final solution of the model.

Next, we consider comparative statics in lending costs (changes in ρ) and borrower reservation payoffs (changes in λ). Importantly, changes in either of these variables affect both the first- as well as the second-best surplus. We denote the effect of lending costs and reservation payoffs on the first-best surplus as the *primary effect*—this effect operates under both complete and incomplete information settings. In contrast, the *secondary effect* operates only under incomplete information and shows how these variables affect the second-best surplus. For changes in lending costs, ρ , the primary effect on first-best surplus is fairly obvious. On the other hand, higher λ in the baseline version (or equivalently a higher lending costs ρ^I in the competition version) implies lower reservation payoffs V_g^0 , (or equivalently \bar{V}_g^I) g = h, l. Stated differently, the competition version of the model contrasts between changes in Lender-U's (absolute) lending costs, ρ^U , and Lender-U's cost advantage, over its informed rival: changes in ρ^I for a fixed ρ^U . From the point of view of Lender-U, an increase in the reservation payoffs of its borrowers is equivalent to a reduced cost advantage over its informed rival. To summarize, the primary effect of higher λ (equivalently, a greater cost advantage) or lower borrower reservation payoffs is to increase the first-best surplus by an equal amount.

The secondary effect of higher reservation payoffs (lower cost advantage) occurs in equilibria that involve the screening of *b*-types. In contrast, the secondary effect of higher lending costs occurs in equilibria that involve the pooling of *b*-types. As mentioned before, secondary effects are only observed under incomplete information. Therefore, the total effect of variations in reservation payoffs and lending costs under incomplete information is the sum of primary and secondary effects.

To understand the secondary effect of changes in reservation payoffs, we have to remember that these changes, although exogenously determined, are essentially non-random. As demonstrated above, changes in reservation payoffs considered in our model are best motivated by changes in cost of funds of a rival with complete information over borrower types. Recall that higher λ (or higher ρ^I in the competition version) imply decreases in V_h^0 and V_l^0 by equal amounts but no change in reservation payoffs of the *b*-types, which is fixed at V^0 . Lower V_h^0 and V_l^0 implies an unambiguously higher surplus due to the primary effect. In offers that screen out *b*-types, this prompts the lender to switch to offers with a higher repayment but lower collateral requirement. The lower collateral requirement shown as $C'(\lambda) < 0$ in (11) and (13) generates the secondary effect.¹⁹ We know that screening costs increase with higher collateral requirements. Therefore, lower reservation payoffs (greater λ or ρ^I) imply a higher first-best surplus (primary effect) and lower cost of screening out *b*-types in terms of reduced collateral requirements (secondary effect). The secondary effect reinforces the primary effect and the total effect is a greater surplus.

The intuition behind the secondary effect of higher lending costs is somewhat simpler. In pooling equilibria that involves lending to *b*-types, lenders face expected losses. Higher lending costs increase expected losses from offers accepted by *b*-types and thereby reduce the secondbest loan surplus. Such costs are avoided in complete information settings because any informed lender denies credit to *b*-types. Therefore, the primary effect of higher lending costs reduces the

¹⁹Note that this effect is reversed for (9) with $C'(\lambda) > 0$. This is because both V_h^0 and V_l^0 change with λ , whereas $V_b^0 = V^0$ for all λ . Since (4) binds in all offers that sort *h*-types and *l*-types, we must have $V_h(R_l^0, C_l^0) - \lambda = V_h(R_l^1, C_l^1)$ and $V_l(R_l^0, C_l^0) - \lambda = V_l(R_l^1, C_l^1)$.

first-best surplus generated from lending to creditworthy h- and l-types whereas the secondary effect increases the losses from b-types in pooling equilibria. Again, both primary and secondary effects reinforce each other under incomplete information and the total effect of higher lending costs is negative.

An important caveat here relates to large reductions in surplus generated from large increases in ρ . Traditional models of investment demand focus on how large increases in ρ can switch positive NPV projects into negative NPV projects. To abstract from such considerations in our setting, we impose the boundary conditions $[(1-\theta_b)x - V^0 - \rho] < 0$ and $[(1-\theta_g)x - V^0 - \rho] > 0$ for g = h, l throughout. In our setting, therefore, variations in ρ are bounded to ensure that h- and l-types are always creditworthy and that b-types are always uncreditworthy. For the rest of this section, however, we work mostly with candidate equilibria from the competition version simply because the intuition is somewhat easier to explain. Nevertheless, we draw clear parallels wherever needed between the competition and the baseline versions of the model for clarity.

4.2 Numerical Solution

We define parameters x, V^0 and the three different values of θ , namely θ_b , θ_h , and θ_l to be the primitives of the model. Since there can be infinite variations in the set of parameter values, the exposition here is selective. The aim is to illustrate how institutional features and market conditions affect lending behavior. This is done by using a numerical example showing how the equilibria change with changes in parameter values of ν_b , β , and ρ^U respectively. We begin with a discussion of the equilibria for primitives of x = 3.99, $V^0 = 2$, $\theta_b = 0.25$, $\theta_h = 0.065$, $\theta_l = 0.052$. For the given set of primitives, Figures 2, 3 and 4 describe solutions to the model in (ν_h, ρ^I) space or (ν_h, λ) space. Increases along the vertical axis denote higher Lender-I's cost of funds, ρ^I , or equivalently, lower reservation payoffs for borrowers facing Lender-U (higher λ). As mentioned above, this change may also be interpreted in terms of a greater cost advantage of Lender-U over Lender-I. The four plots in Figures 2, 3 illustrate the comparative statics for ν_b and β respectively. Finally, Figure 4 describes the comparative statics for changes in lending costs of Lender-U, ρ^U .

The dotted lines in the graphs denote Lender-U's lending cost, ρ^U . The colored regions denote equilibria in which Lender-U is able to secure at least one creditworthy borrower type. Higher values of Lender-I's cost of funds, ρ^I , implies that Lender-U has a greater cost advantage and therefore, faces borrowers with lower reservation payoffs. Consequently, at smaller cost advantage of Lender-U, Lender-I dominates (i.e., all borrowers go to Lender-I for loans), as shown by the white region above the dotted line in all Figures.²⁰ Lender-I dominates in

 $^{^{20}}$ Domination below the dotted line is trivial since, in that case, the informed lender has both cost and information advantage.

this region because it faces borrowers with sufficiently high reservation payoffs so that the second-best surplus from creditworthy types is negative. On the other hand, if Lender-U's cost advantage is very large, borrower reservation payoffs are sufficiently low. As a result, the second-best surplus is non-negative; and at significantly higher cost advantage, Lender-U dominates.

For a given subplot, we fix ν_b , so that movements along the x-axis denote increases in ν_h . This way, Lender-U's informational disadvantage is lower at either end of the axis and higher in the middle region where the probability that a borrower selected at random being either creditworthy type is maximum. Interestingly, the choice between candidates *Hybrid-2* and *Screen-1* depends on ν_h . In the former, *h*-types are pooled with *l*-types and in the latter, they are screened. Notably, both candidate equilibria involve screening out *b*-types, and this typically occurs at high values of ρ^I . At high ρ^I , Lender-U faces borrowers with low reservation payoffs. From our discussion on secondary effects above, we learn that low reservation payoffs reduce screening costs by lowering collateral requirements on offers that screen out *b*-types. Consequently, at sufficiently high ρ^I , Lender-U favors equilibria that screen out *b*-types: *Pool-1* and *Hybrid-1* are replaced at high ρ^I by *Hybrid-2* and *Screen-1*, respectively. In sum, if Lender-U has a large cost advantage (significantly high ρ^I), it screens out *b*-types as shown by *Hybrid-2* for low ν_h (most borrowers are *l*-types) and *Screen-1* for high ν_h (most borrowers are *h*-types). Notably the switch between the two equilibria occurs at threshold of ν_h as seen by the vertical line between the two regions in all Figures.

As demonstrated below, most of the changes in the solution to the model are observed for a moderate cost advantage of Lender-U. We begin with four plots in Figure 2 that describe the solutions to the model for different values of ν_b for given values of $\beta(=0.5)$ and $\rho^U(=1.07)$. As described above, increases in ν_b raise pooling costs. Therefore, it is not surprising that equilibria for a given cost advantage is dominated by pooling for low ν_b and screening for higher ν_b . In Figure 2(a), equilibria *Pool-1*, *Pool-2* and *Hybrid-1* dominate with $\nu_b = 0.06$. However, for higher ν_b , Figure 2(b)-(c) show that *Pool-1*, *Pool-2* and *Hybrid-1* are replaced by *Hybrid-2*, *Screen-2* and *Screen-1* respectively. Interestingly, the screening thresholds in (16) and (17) are shown in 2(c)-(d) as the upper and lower boundaries of the region under *Screen-2*. Clearly, these thresholds are independent of ν_h .

Second, Figure 3 describe solutions to the model in (ν_h, ρ^I) space for $\nu_b = 0.08$ and $\rho^U = 1.07$. The various plots are drawn for different values of β . As mentioned above, screening costs decrease with increases in β . Naturally, for a given level of cost advantage, pooling equilibria dominate for low β whereas screening replaces pooling at higher values of β . With $\beta = 0.35$, the solution includes equilibria *Pool-1* and *Pool-2* at a moderate cost advantage, as shown in Figure 3(a). However with increasing β , these pooling equilibria are replaced by *Screen-1*, *Screen-2* and *Hybrid-2*.

Third, $\nu_b = 0.08$ and $\beta = 0.5$ yields Figure 4, which describe solutions to the model in



Figure 2: Solution in (ν_h, ρ^I) space with variations in ν_b . The dotted lines in the graphs denote the Lender-U's lending cost, ρ^U . The plots are drawn to parameter values x = 3.99, $V^0 = 2$, $\theta_b = 0.25$, $\theta_h = 0.065$, $\theta_l = 0.052$ for $\rho^U = 1.07$ and $\beta = 0.5$. The value of ν_b varies from 0.06 in (a) to 0.12 in (d).

 (ν_h, ρ^I) space for different values of ρ^U . Higher values of ρ^U (the vertical distance between the dotted line and the x-axis) in Figure 4 (a)-(d) denote higher lending costs for Lender-U. Recall that shaded areas denote regions in parameter space where Lender-U's offers are accepted by at least one creditworthy type. As explained above, the primary effect of higher lending costs is a lower surplus—this can be viewed in terms of the shrinking size of the shaded regions as ρ^U increases progressively from Figure 4(a) to Figure 4(d). While the primary effect determines the size of the shaded region, the secondary effect influences the nature of equilibria over these regions.

Recall that the secondary effect comes from the fact that a smaller ρ^U implies lower costs of pooling *b*-types. This leads to a greater incidence of pooling equilibria with *b*-types, such as *Pool-1* and *Pool-2* at very low ρ^U (Figure 4(a)). Interestingly, such pooling occurs even in markets where the distribution of types is not conducive for pooling such as intermediate values of ν_h . However, pooling costs increase with ρ^U and this is reflected in fewer regions for pooling



Figure 3: Solution in (ν_h, ρ^I) space with variations in β . The dotted lines in the graphs denote the Lender-U's lending cost, ρ^U . The plots are drawn to parameter values x = 3.99, $V^0 = 2$, $\theta_b = 0.25$, $\theta_h = 0.065$, $\theta_l = 0.052$ for $\rho^U = 1.07$ and $\nu_b = 0.08$. The value of β varies from 0.35 in (a) to 0.75 in (d).

as ρ^U increases progressively from Figure 4(a) to Figure 4(d). At high ρ^U in Figure 4(d), there is no scenario for which pooling *b*-types is an equilibrium.

In the numerical example above, the model does not describe a scenario in which *Screen-3* emerges as the equilibrium of the model. This is because for the numerical examples considered above, $\tilde{\rho}_{h,l}^S > \rho^I > \tilde{\rho}_{b,h}^S$ (or equivalently $\hat{\lambda}_{h,l}^S > \lambda \ge \hat{\lambda}_{b,h}^S$). However, for a different numerical example where $\tilde{\rho}_{b,h}^S > \rho^I > \tilde{\rho}_{h,l}^S$, *Screen-3* is shown an equilibrium of the model. Using primitives $x = 3.99, V^0 = 2, \theta_b = 0.25, \theta_h = 0.062, \theta_l = 0.04$, the set of parameter values $\nu_b = 0.06$ and $\beta = 0.5$, yields $\tilde{\rho}_{b,h}^S > \tilde{\rho}_{h,l}^S$ for low values of ρ^U , namely $\rho^U = 1.02^{.21}$ In this scenario, *Screen-3* emerges as an equilibrium in Figure 5(a). However, as ρ^U increases, the equilibrium in Figure 5(d) is similar to that shown in the numerical example discussed above. In what follows, I

²¹It follows from (17) and (16) that the thresholds are linear in ρ^U with (16) increasing at a faster rate (greater slope). As result, there exists a threshold $\hat{\rho}^U$ such that $\rho^U \gtrless \hat{\rho}^U \Leftrightarrow \tilde{\rho}^S_{h,l} \gtrless \tilde{\rho}^S_{h,l} \end{Bmatrix}$. The closed form solution is $\hat{\rho}^U \equiv \frac{(1-\beta)\theta_h(1-\theta_h)}{\theta_b(1-\theta_h)-\beta\theta_h(1-\theta_b)} [(1-\theta_b)x - V^0] (\frac{1}{1-(1-\beta)\theta_l} - \frac{\theta_b-\theta_h}{\theta_b(1-\theta_h)-\beta\theta_h(1-\theta_b)})^{-1}.$



Figure 4: Solution in (ν_h, ρ^I) space with variations in ρ^U . The dotted lines in the graphs denote the Lender-U's lending cost, β . The plots are drawn to parameter values x = 3.99, $V^0 = 2$, $\theta_b = 0.25$, $\theta_h = 0.065$, $\theta_l = 0.052$ for $\nu_b = 0.08$ and $\beta = 0.5$. The value of ρ^U varies from 1.02 in (a) to 1.12 in (d).

discuss features of the equilibrium for both numerical examples, in terms of the four cases given below.

Case (i) $\min\{\tilde{\rho}_{b,h}^S, \tilde{\rho}_{h,l}^S\} > \rho^I$ (or $\min\{\lambda_{b,h}^S, \lambda_{h,l}^S\} > \lambda$) Since Lender-U cannot screen adjacent types, we focus on pooling equilibria *Pool-1*, *Pool-2*, and *Hybrid-2*. Figures 2-5 show that Lender-I can dominate in regions even if Lender-U has the cost advantage. For a low cost advantage (low ρ^I or equivalently low λ) borrowers reservation payoffs are significantly large, so that Lender-U often cannot satisfy high reservation payoffs and still cover for the screening and pooling costs. However, there are some cases where despite the lack of a sufficiently high cost advantage, conditions are conducive for pooling. This is illustrated in terms of subplots (a) in Figures 2-5 with sufficiently low ν_b,β , or ρ^U so that Lender-U attracts creditworthy types by pooling.



Figure 5: Solution in (ν_h, ρ^I) space with variations in ρ^U . The dotted lines in the graphs denote the Lender-U's lending cost, β . The plots are drawn to parameter values x = 3.99, $V^0 = 2$, $\theta_b = 0.25$, $\theta_h = 0.062$, $\theta_l = 0.04$ for $\nu_b = 0.06$ and $\beta = 0.5$. The value of ρ^U varies from 1.02 in (a) to 1.12 in (d).

For low ν_h Lender-U either pools all types under *Pool-1* or pools *h*- and *l*-types only under *Hybrid-2*. The cutoff for *Hybrid-2*, $\tilde{\rho}^Y(\nu_h, \nu_l)$, is increasing and convex in ν_h ; a higher cost advantage is needed for pooling a larger proportion of high risks in the population. Moreover, *Hybrid-2* dominates *Pool-1* as pooling costs increase with higher ν_b .

For high ν_h , Lender-U pools b-types with h-types under Pool-2. An interesting feature of the solution for $\min\{\tilde{\rho}_{b,h}^S, \tilde{\rho}_{h,l}^S\} > \rho^I$ is that while pooling occurs for high or low ν_h , Lender-I dominates for intermediate values of ν_h . This happens because the proportion of h-types is neither too large to be pooled with b-types under Pool-2 nor too small to be pooled with l-types under Pool-1 or Hybrid-2. In these regions Lender-U would ideally like to screen out b-types, but is unable to do so since $\min\{\tilde{\rho}_{b,h}^S, \tilde{\rho}_{h,l}^S\} > \rho^I$.

Case (ii) $\tilde{\rho}_{h,l}^S > \rho^I > \tilde{\rho}_{b,h}^S$ (or $\hat{\lambda}_{h,l}^S > \lambda > \hat{\lambda}_{b,h}^S$) This case is best illustrated in terms of Figure 2. With low ν_b , the solution is similar to that in the previous case: the equilibrium is *Pool-1* or *Hybrid-2* at low ν_h , but *Pool-2* at high ν_h . But whereas Lender-I dominated at intermediate

values of ν_h in the previous case, Lender-U attracts the h-types by screening them from b-types under *Screen-2* in Figure 2(a). With increases in ν_b over the plots in 2(a)-(d), the area under *Screen-2* increases, at the expense of *Pool-2*. Because increases in ν_b imply increases in pooling costs, pooling equilibria are replaced by *Screen-2* (at high ν_h) and *Hybrid-2* (at low ν_h) in Figures 2(a)-(d).

Case (iii) $\tilde{\rho}_{b,h}^S > \rho^I > \tilde{\rho}_{h,l}^S$ (or $\hat{\lambda}_{b,h}^S > \lambda > \hat{\lambda}_{h,l}^S$) Under this condition, Lender-U cannot screen b-types from h-types, but can sort h-types from l-types just as in Figure 1. The solution to this case is illustrated in Figure 5(a)-(b). For high ν_h , Hybrid-1 replaces Pool-2, because with $\rho^I > \tilde{\rho}_{h,l}^S$, Lender-U sorts l-types and bunches b-types with h-types. For low ν_h , the equilibrium is given by Pool-1 or Hybrid-2, just as in the previous case. However, unlike the previous case, Lender-U cannot screen b-types from h-types, for intermediate values of ν_h . Nor can it pool h-types with either adjacent types. Interestingly, because it's screening offer to l-types are rejected by b-types, we get Screen-3 when h-types accept offers from Lender-I.

Case (iv) $\rho^I > \max(\tilde{\rho}_{b,h}^S, \tilde{\rho}_{h,l}^S)$ (or $\lambda \ge \max(\hat{\lambda}_{h,l}^S, \hat{\lambda}_{b,h}^S)$) Under this condition, Lender-U can sort all borrower types. Stated differently, there exists a distribution of borrower types where the expected loan surplus net of either screening or pooling cost for all of the menus is nonnegative. This implies that the complete set of contracts listed in Table 2 (or equivalently in Table 1) is at the disposal of the uninformed lender. Among them, Lender-U's offers in *Pool-2* are dominated by those in *Hybrid-1*, and its offers in *Screen-2* and *Screen-3* by those in *Screen-1*. Consequently, Lender-U chooses among contract offers in the four alternatives: *Pool-1*, *Screen-1*, *Hybrid-1*, and *Hybrid-2*. The choice between *Screen-1* and *Hybrid-2* depends on ν_h as explained above.

The choice between Hybrid-2 and Pool-1 and that between Hybrid-1 and Screen-1 depends on the cost advantage of Lender-U (i.e., ρ^I or λ). This choice can be explained in terms of the secondary effect of borrower reservation payoffs explained in the previous subsection. In offers that screen out b-types, a lower reservation payoff (higher ρ^I or λ) allows Lender-U to switch to offers with a higher repayment but lower collateral requirement, thereby reducing screening costs. Therefore, for a given level of (low) ν_h where Lender-U is pooling all types under Pool-1, an increase in ρ^I reduces screening costs significantly, for it to start screening out the b-types under Hybrid-2. For fixed ν_b , as shown in Figures 3-5, switching from Pool-1 to Hybrid-2 becomes more profitable at higher ν_h and lower ρ^I than at lower ν_h and higher ρ^I . Under Pool-1, profits from l-types covers for both h- and b-types, therefore a higher ν_h implies that screening out b-types yields higher profits at lower ρ^I . The converse is true for lower ν_h . Similar considerations apply for the choice in switching from Hybrid-1 to Screen-1. Both equilibria sort h-types from l-types but under Screen-1, b-types are screened out as well. Again, this depends on ν_h : A higher ν_h implies that pooling h-types with b-types under Hybrid-1 is less costly, and



Figure 6: Three simplexes drawn to parameter values x = 3.99, $V^0 = 2$, $\theta_b = 0.25$, $\theta_h = 0.065$, $\theta_l = 0.052$. The simplexes show the solution to the model for $\rho^U = 1.02$ and $\rho^I = 1.042, 1.047$ and 1.05 respectively. The clear regions (in white) denote parameter values for which Lender-*I* dominates. Note that for the parameter values under consideration $\tilde{\rho}_{h,l}^S > \tilde{\rho}_{b,h}^S$.

therefore, screening out *b*-types under *Screen-1* yields greater profits only at significantly higher ρ^{I} . These factors help explain the curvature between the regions demarcated by *Screen-1* and *Hybrid-1* on the one hand and *Pool-1* and *Hybrid-2* on the other.

Another way of illustrating the full scope of possible equilibria is in terms of Figure 6, which replicates the equilibria in Figure 3 in terms of three simplex diagrams. These diagrams illustrate how the equilibria change with changes in the cost advantage of Lender-U.

5 Discussion of Results

5.1 Variations in Reservation Payoffs

Variations in borrower's type-dependent reservation payoffs in the baseline version are motivated in the competition version by equal (and opposite) changes in the uninformed lender's cost advantage over its informed rival. As described earlier, a lower reservation payoff (higher cost advantage) results in a direct increase in the first- and second-best surplus for informed borrowers. This direct effect, which operates under both complete and incomplete information, reduces the surplus by the same amount.

In addition, there is a secondary effect that operates only under incomplete information and only for equilibria screening out the worst borrower types (bad risks). With fixed reservation payoffs for bad risks, lower reservation payoffs for creditworthy types (greater cost advantage) imply a greater surplus. This prompts higher repayment and lower collateral requirements in offers to extract the greater surplus. In this way, lower reservation payoffs (baseline version) or equivalently higher cost advantage (competition version) reduces collateral requirements on offers that screen out uncreditworthy borrowers. Thus, the secondary effect of lower (higher) reservation payoffs affect screening costs through reduced (increased) collateral requirements.

Since the secondary effect of reservation payoffs occurs through equilibria that screen bad risks, it depends on the distribution of uncreditworthy types in the borrower population. This implies that in markets with a larger proportion of bad risks, screening uncreditworthy borrowers is optimal even for a small cost advantage of the lender (high reservation payoff of the borrower). In contrast, for markets that are well-suited for pooling (sufficiently small proportion of bad risks), uncreditworthy borrowers will be screened if the outside options of creditworthy borrowers are sufficiently low (or equivalently if the lender has a sufficiently large cost advantage).

Lastly, it is important to note that under incomplete information, the primary and secondary effects reinforce each other and the total effect of higher reservation payoffs (lower cost advantage) on the borrower surplus is negative. This implies reduced opportunities for (uninformed) lenders to attract creditworthy borrower types and a greater likelihood for screening uncreditworthy borrowers especially in markets where its cost advantage is sufficiently high.

5.2 Changes in Lending Costs

Just as in the case of reservation payoffs, variations in lending costs have primary and secondary effects on the loan surplus. Changes in lending costs have equal and opposite effects on the firstand second-best surplus. This is the primary effect that reduces the surplus generated from loans to creditworthy types and operates both under complete and incomplete information.

The secondary effect of increases in lending cost affects equilibria that pools uncreditworthy types. By definition, default rates on loans to uncreditworthy borrowers are sufficiently high so that the surplus generated from loans accepted by them is negative. Therefore, an increase in lending costs increases losses associated with lending to bad risks, thereby creating a secondary effect that reinforces the primary effect. Of course, this additional effect increases with the proportion of bad-risk types in the population.

In sum, under both complete and incomplete information, an increase in lending costs reduces lending opportunities for the lender by shrinking the size of the surplus. Moreover, the secondary effect of an increase in lending costs reduces the likelihood of pooling equilibria in markets, even for those markets where the distribution of borrower types is conducive to pooling. The converse is also true: pooling uncreditworthy borrowers can be optimal with sufficiently low lending costs, irrespective of reservation payoffs (or cost advantage).

5.3 The Overlending Problem

The pooling equilibria described above have similarities to the overlending result in de Meza and Webb (1987). However, there is one major difference: With a continuum of borrower types, de Meza and Webb show that pooling uncreditworthy types can occur at all values of lending costs as competition drives lender's profits to zero. In contrast, the uninformed lender resorts to pooling only when lending costs are significantly low. Even in markets that are conducive to pooling, lenders find it optimal to screen borrowers if lending costs are high. This feature of the equilibrium appears to find support in most empirical studies mentioned below.²²

Moreover, as Besanko and Thakor (1987) show, this overlending problem can be eliminated in settings where a screening technology is available to the lender. Separation can be induced as borrowers with lower risk of default choose contracts with lower interest rates and higher collateral requirements, whereas borrowers with higher risk of default choose contracts with higher interest rates and lower collateral requirements (Jimenez, et al. 2006).

Turning our attention to models that include the provision for borrower screening, we find that, for competition under asymmetric information, Nash equilibria are never pooling. This is a fairly well established result in the literature on competition under asymmetric information (Rothschild and Stiglitz, 1976, Wilson 1977). Modifying the framework in Besanko and Thakor (1987) to include type-dependent borrower payoffs (or, alternatively, competition between asymmetrically informed lenders), we derive conditions under which equilibria with pooling uncreditworthy borrowers are shown to exist. To conclude, our model does adhere to the dictum of Occam's razor: it is difficult to conceive of a simpler model that restores the overlending result in the presence of screening technologies.²³

 $^{^{22}}$ This does not in any way detract from the importance of the result in De Meza and Webb (1987). It is simply to suggest that an active mechanism for screening or selection along lines of borrower quality is germane to the discussion in this paper.

²³This does not imply that competitive pooling equilibria cannot exist under adverse selection. Existence of pooling equilibria in competitive screening models has been developed under more elaborate settings (see Dubey and Geanakoplos (2002) and Martin (2007) for further details).

5.4 Cream Skimming on Entry

The only provision in the model that does not satisfy the dictum of Occam's razor is the inclusion of two creditworthy borrower types. Here, the deviation from brevity allows us to illustrate an interesting result that has its parallels in observed lending patterns. In terms of the model, this result is illustrated as *Screen-3*, where the uninformed lender secures the low-risk types only. The lenders split the market, with the informed lender saddled with high-risk types despite its information advantage.

Recent studies suggest that the entry of outside lenders into credit markets can lead to "cream-skimming," whereby outside lenders obtain a safer loan portfolio on entry, leaving inside lenders with the riskier clients (see Detragiache et al., 2008, and references therein). Detragiache et al., (2008) provide evidence on this phenomenon for the entry of foreign banks into developing countries, also showing how this effect can be welfare reducing. Their model shows that cream skimming arises primarily out of differences in lending technologies between foreign and domestic lenders.

From an information perspective, cream-skimming appear counterintuitive. How might an outside lender compete with a better-informed inside lender and yet be successful in securing the most creditworthy clients in the borrower pool? Arguably, the inside lender should be able to use its information advantage to retain clients of the highest quality. The competition version shows that the uninformed lender's cost advantage allows it to offer a lower rate. When combined with a sufficiently high collateral requirement, this low rate helps it attract only the low-risk types while high-risks take up offers of the informed lender. Even though the high-risks are creditworthy, the uninformed lender doesn't include them in its portfolio because it cannot sort them from uncreditworthy types.

The intuition behind the cream skimming result discussed here is significantly different from that in Dell'Ariccia and Marquez (2004) or Sengupta (2007). In the former, the result requires a high correlation between quality of borrowers and the degree of information asymmetries about them, while the latter shows that cream-skimming is an attribute of uninformed lenders concentrating on market segments where adverse selection problems are less acute. In contrast, this paper captures cream-skimming equilibria where the uninformed lender succeeds in capturing borrowers of highest quality by making the collateral requirements significantly high so as to make such contracts unattractive to borrowers of poorer quality.

5.5 Empirical Implications

Recent empirical studies have provided significant empirical evidence on how factors affecting credit supply can affect lending patterns.²⁴ In broad support of the results of this model, they find that low lending costs prior to origination create excessive risk taking by lenders (Ioannidou et al. 2009; Maddaloni and Peydró, 2011; Jimenez et al. 2011). For the United States, it is not inconceivable that a low-rate environment, as the one that prevailed in the early part of this decade, contributes significantly to the lax lending standards in the years that followed.²⁵ Admittedly, the sustained low-rate environment that prevailed in the early part of this decade was unprecedented in recent monetary history. However, the fallout in terms of institutional lending was no less remarkable. For the first time in recent economic history, mainstream lenders penetrated "subprime" markets, making loans to borrowers who were until now denied conventional sources of funds. The best example of this phenomenon is the entry of mainstream lenders to the subprime mortgage market in the U.S.²⁶ I discuss this example in Appendix B.

More relevant to this study, Maddaloni and Peydró (2011) find that increased competition is one of the important determinants of increased risk-taking by lenders. This result provides important support for the model's prediction about how borrower reservation payoffs affect lending standards. Increased lender competition is most likely to increase borrowers' outside options and thereby favor pooling uncreditworthy types as described above. It augurs well with more general macro-evidence on how financial deregulation affects risk-taking by banks. Lastly, Ioannidou et al. (2009) find that risk pricing is inadequate in times of lax lending because spreads do not reflect the additional risk taken. This phenomenon can be explained in terms of the model: Lending to uncreditworthy types involves pooling equilibria, which, unlike borrower screening, does not allow for adequate risk pricing of individual loans.

Another feature of this model implies that lenders that increase market share by lending aggressively during booms should ex post display higher default rates. This prediction seems at odds with Ruckes (2004) and Dell'Ariccia and Marquez (2006), where portfolio quality is similar across banks and each bank has similar acceptance of default risk when extending loans. Interestingly, the model's predictions are more in line with Rajan (1994) even though the mechanism described in this model is different. In the vein of Rajan (1994), differences in loan quality across banks is most pronounced for those that successfully poach borrowers

 $^{^{24}}$ These studies exploit different institutional arrangements for econometric identification. To establish the exogeneity of monetary policy, and thereby lending costs, Jimenez et al. (2011) utilize Spain's membership to the European Monetary Union while Ioannidou et al. (2009) exploit the "dollarization" of Bolivia's banking system.

 $^{^{25}}$ The Federal Reserve lowered the target federal funds rate from a high of 6.5 percent in early January 2001 to just one percent in January 2002. The FOMC statement released on August 2003 announced that "policy accommodation can be maintained for a considerable period" and the low rate environment continued well into 2004.

²⁶A lesser known phenomenon has been the entry of private banks into the microfinance market in developing countries during this period. Reille and Forster (2008) record that, between 2004 and 2006, the stock of foreign capital investment—covering both debt and equity—more than tripled to US\$4 billion.

during times of expansion.

6 Conclusion

This paper analyzes a simple stylized setup of how type-dependent reservation payoffs and lending costs affect adverse selection problems faced by lenders. In summary, our results show that lending to uncreditworthy borrowers can be prevented by raising lending costs or by lowering reservation payoffs of (creditworthy) borrowers—or both. We find that this result holds even in markets that are conducive to pooling such as markets with a significantly low proportion of uncreditworthy types. Higher lending costs favor screening by increasing the losses of pooling uncreditworthy borrowers. In addition, lowering reservation payoffs increase opportunities for screening bad-risks through reduced collateral requirements. These results have important implications for monetary policy as well as competition policy in credit markets. While the lending cost results have significant empirical support in recent studies on bank risk-taking, more research is needed to understand the role of competition policy in this regard.

References

Asea, P. K. and B. Blomberg, (1998). "Lending cycles." Journal of Econometrics, Elsevier, 83(1-2), 89-128.

Barro, R., (1976), "The Loan Market, Collateral, and Rates of Interest," Journal of Money, Credit, and Banking, 8, 439-456.

Besanko, D. and A.V. Thakor, (1987). "Collateral and Rationing: Sorting Equilibria in Monopolistic and Competitive Credit Markets." International Economic Review, 28(3), 671-89.

Bernanke, B.S. and A.S. Blinder, (1988). "Credit, Money, and Aggregate Demand." American Economic Review, American Economic Association, 78(2), 435-39.

Bernanke, Ben S & Gertler, Mark, 1995. "Inside the Black Box: The Credit Channel of Monetary Policy Transmission," Journal of Economic Perspectives, American Economic Association, vol. 9(4), 27-48.

Berger, A.N. and L.J. Mester, (2003). "Explaining the dramatic changes in performance of US banks: technological change, deregulation, and dynamic changes in competition." Journal of Financial Intermediation, Elsevier, 12(1), 57-95.

Berger, A.N. and G.F. Udell, (2004). "The institutional memory hypothesis and the procyclicality of bank lending behavior." Journal of Financial Intermediation, Elsevier, 13(4), 458-495.

Black, S.E. and P.E. Strahan (2002), "Entrepreneurship and bank credit availability," Journal of Finance, 57, 2807-2833.

Boot, A.W.A., (2000). "Relationship Banking: What Do We Know?" Journal of Financial Intermediation 9(1), 7-25. Boot A., and A. Thakor (1994), "Moral hazard and secured lending in an infinitely repeated credit market game," International Economic Review, 35, 899-920.

Boot A., and A. Thakor (2000), "Can relationship banking survive competition?," Journal of Finance, 45, 679-713.

Broecker, T., (1990), "Credit-Worthiness Tests and Interbank Competition", Econometrica, 58, pp. 429-52.

Brueckner, J.K. (2000). "Mortgage Default with Asymmetric Information." Journal of Real Estate Finance and Economics 20(3), 251-74.

De Meza, D. and D.C. Webb, (1987). "Too much investment: a problem of asymmetric information." Quarterly Journal of Economics 102, 281-292.

DellAriccia, G., E. Friedman, and R. Marquez (1999). "Adverse Selection as a Barrier to Entry in the Banking Industry." RAND Journal of Economics 30(3) (Autumn), 515-534.

DellAriccia, G. and R. Marquez, (2004). "Information and bank credit allocation." Journal of Financial Economics, 72, 185-214.

DellAriccia, G. and R. Marquez, (2006). "Lending booms and lending standards." Journal of Finance 61, 2511-2546.

Dell'Ariccia, G., and R. Marquez, (2008). "Can cost increases increase competition? Asymmetric information and equilibrium prices." RAND Journal of Economics, 39(1), 144-162.

Detragiache, E.,T. Tressel, and P. Gupta, (2008)."Foreign Banks in Poor Countries: Theory and Evidence." Journal of Finance, 63(5), 2123-2160.

Dubey, P. and J. Geanakoplos, (2002) "Competitive Pooling: Rothschild-Stiglitz Reconsidered." The Quarterly Journal of Economics (2002), 117(4), 1529-1570.

Ferraris L. and Minetti R., 2007. "Foreign Lenders and the Real Sector," Journal of Money, Credit and Banking, Blackwell Publishing, vol. 39(4), pages 945-964, 06.

Gorton, G., (2009) "Slapped in the Face by the Invisible Hand: Banking and the Panic of 2007" prepared for the Federal Reserve Bank of Atlanta's 2009 Financial Markets Conference: Financial Innovation and Crisis, May 11-13, 2009

Gorton, G. and P. He, (2008) "Bank Credit Cycles." Review of Economic Studies, 75(4), 1181-1214.

Ioannidou, V.P., S. Ongena, and J.L. Peydro, (2009). "Monetary Policy, Risk-Taking and Pricing: Evidence from a Quasi-Natural Experiment", Center - Tilburg University / European Central Bank, Tilburg.

Iacoviello, M. and Minetti, R., (2006). "International business cycles with domestic and foreign lenders," Journal of Monetary Economics, Elsevier, vol. 53(8), pages 2267-2282,

Jayaratne, J. and P. E. Strahan (1996), "The Finance-Growth Nexus: Evidence from Bank Branch Deregulation," Quarterly Journal of Economics, 111, 639-670.

Jimenez, G., Salas, V. and J. Saurina, (2006). "Determinants of collateral," Journal of Financial Economics, Elsevier, vol. 81(2), 255-281.

Jimenez, G., S. Ongena, J.L. Peydro, and J. Saurina, (2011). "Hazardous times for monetary policy: What do twenty-three million bank loans say about the effects of monetary policy on credit risk-taking?" Banco de España Working Papers 0833.

Laffont, J. J. & Martimort, D. The Theory of Incentives (Princeton Univ. Press, Princeton, 2002).

Lown, C. and D. Morgan, (2003). "The credit cycle and the business cycle: New findings using the loan officer opinion survey." Journal of Money, Credit, and Banking, 38(6), 1575-1597.

Maddaloni, A. and J.-L. Peydro (2011). Bank risk-taking, securitization, supervision, and low interest rates: Evidence from lending standards. Review of Financial Studies.

Marquez R., 2002. "Competition, Adverse Selection, and Information Dispersion in the Banking Industry," Review of Financial Studies, Oxford University Press for Society for Financial Studies, vol. 15(3), pages 901-926.

Martin, A., (2007). "On Rothschild-Stiglitz as competitive pooling" Economic Theory, 31, 371-386.

Rajan, R. J., (1994). "Why Bank Credit Policies Fluctuate; A Theory and Some Evidence." Quarterly Journal of Economics, 109(2), 399-411.

Reille, X. and S. Forster, (2008), "Foreign Capital Investment in Microfinance: Balancing Social and Financial Returns." Focus Report No 44., The Consultative Group to Assist the Poor (CGAP), World Bank.

Rothschild, M. and J.E. Stiglitz, (1976). "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information." Quarterly Journal of Economics 90(4), 630-649.

Ruckes, M., (2004). "Bank competition and credit standards." Review of Financial Studies 17, 1073-1102.

Sengupta, R., (2007). "Foreign Entry and Lender Competition." Journal of Financial Economics, 84(2), 502-528.

Sharpe, S.A., (1990). "Asymmetric Information, Lender Lending, and Implicit Contracts: A Stylized Model of Customer Relationships." Journal of Finance, 45(4), 1069-87.

von Thadden, Ernst-Ludwig, 2004. "Asymmetric information, bank lending and implicit contracts: the winner's curse," Finance Research Letters, Elsevier, vol. 1(1), pages 11-23, March.

Wilson, C., (1977). "A Model of Insurance Markets with Incomplete Information." Journal of Economic Theory, 16, 167-207.

Appendix A: Proofs

Monopolist's offers under Complete Information (Baseline Version)

Proof of Lemma 1: Since $\pi_b < 0$, we restrict attention, without loss of generality, to either creditworthy type g = h, l. First, an informed lender will always offer zero-collateral contracts. We prove by contradiction. Suppose not, that is, $\pi_g(R_g^1, C_g^1) \ge 0$. Consider offer (R_g^2, C_g^2) with $R_g^2 > R_g^1, C_g^2 < C_g^1$ such that $V_g(R_g^1, C_g^1) = V_g(R_g^2, C_g^2), g = h, l$. This deviation satisfies all constraints and provides higher profits. Therefore, we must have $C_g = 0, g = h, l$. A monopolist

lender's full information offer is the first-best (zero-collateral) maximum, obtained by setting $V_g(R_g, 0) = V_g^0$. Notably, $R_g^0 = \bar{R}_g$ when $V_g^0 = V^0$, g = h, l. Under perfect competition, fully informed lenders offer the first-best (zero-collateral) minimum repayment, by setting $\pi_g = 0$, g = h, l.

Monopolist's offers under Incomplete Information (Baseline Version)

The *revelation principle* ensures that, without loss of generality, an uninformed principal may restrict offers to the cardinality of the type space. This implies that the lender makes at most three offers, one for each borrower type. Accordingly, there would be 14 possible permutations as shown in the Table A.1 below. The offers are denoted by the borrower types that would accept the lender's offers in equilibrium (offers in parentheses denotes pooling or bunching):

Table A.	1: Lende	er's menu	comp	orises of
# 3 of	fers #	2 offers	#	1 offer

π	0 011015	π	2 011015	TT	1 Onor
1	b; h; l	2	(b,h);l	8	b
		3	(b, l); h	9	h
		4	(l,h);b	10	l
		5	b;h	11	(b, h, l)
		6	b;l	12	(b,h)
		7	h;l	13	(b,l)
				14	(l,h)

With $\pi_b < 0$ for any offer, an uninformed lender can easily replace a menu that sorts the *b*-type with a menu that does not include offers to *b*-types. Therefore, replacing menu 1 with menu 7 unambiguously increases profits. Likewise, menus 4, 5 and 6 can be replaced with menus 14, 9 and 10 respectively. Also, offer 8 is redundant. Moreover, as will be demonstrated below, any offer that is accepted by the *b*-type and the *l*-type is also always accepted by the *b*-type. Stated differently, an uninformed lender is unable to sort the *h*-type while continuing to make offers to the *b*- and *l*-types. Therefore, menus 3 and 13 are never offered in equilibrium. Lastly, as shown below, there is no equilibrium wherein the uninformed lender attracts only *l*-types as in (10). This reduces the number of menus offered in equilibrium to six as shown in Table 1 in the paper.

For equilibrium offers of the uninformed lender, the following results are shown to hold.

Lemma 2 Lender's expected profits from loans to l-types are non-negative, $\pi_l(R_l, C_l) \ge 0$.

Lemma 3 The IR constraint of the l-type, (2), must bind.

Lemma 4 In any equilibrium that screens out the b-type, the IC constraint of the b-type w.r.t the h-type, (3), must bind.

Lemma 5 In any equilibrium that sorts the h-types from the l-type, the IC constraint of the h-type w.r.t the l-type,(5), must bind.

Lemma 6 In any equilibrium that sorts the h-types from the l-type, the IR constraint of the h-type, (1) must bind.

Readers are referred to the Extended (Web) Appendix for proofs of lemmas 2-6. They are omitted here for the sake of brevity. From Lemmas 2-6, we obtain expressions for (R_h, C_h) and (R_l, C_l) as: $R_l \in [R_l^S, R_g^Y]$, $R_h \in [R_g^Y, R_h^S]$, $C_l \in [C_g^Y, C_l^S]$, and $C_h \in [C_h^S, C_g^Y]$ where using boundary conditions and $\lambda > \rho > 0$, all of the offers given by (9)-(13) are strictly positive.

From Lemmas 2-6, it follows that the screening offers to *h*-types and *l*-types are (R_h^S, C_h^S) and (R_l^S, C_l^S) respectively. Likewise, (R_g^Y, C_g^Y) is an offer satisfying Lemmas 2-6 in which the lender bunches both *h*-types and *l*-types while screening out the *b*-types.

Candidate screening equilibria are feasible only if the borrower's (exogenous) reservation payoff is significantly low-that is one of two screening cutoffs (one for each pair of adjacent types) are satisfied. The first cutoff is $\hat{\lambda}_{h,l}^S$ for screening the *h*-types from the *l*-types and the second is $\hat{\lambda}_{b,h}^S$ for screening the *h*-types from the *b*-types. Clearly for both cutoffs, $\hat{\lambda}'(\rho) > 0$ and $\hat{\lambda}''(\rho) = 0$. Note that there is a one-to-one correspondence between λ and V_g^0 , g = h, l.

Candidate Equilibria Under Incomplete Information (Baseline Version)

Proof of Proposition 1: In any screening equilibrium wherein the lender separates all borrower types, Lemmas 2-6 must hold. This implies that the lender offers menu $\{(R_h^S, C_h^S), (R_l^S, C_l^S)\}$. For the lender to screen the *l*-type from the *h*-type, profits $\pi_l(R_l^S, C_l^S) \ge 0$. This occurs when $\lambda \ge \hat{\lambda}_{h,l}^S(\rho) = \frac{1}{1-(1-\beta)\theta_l}\rho$. Similarly, the lender can screen the *h*-type from the *b*-type, if $\pi_h(R_h^S, C_h^S) \ge 0$. That is if $\lambda \ge \hat{\lambda}_{b,h}^S(\rho)$, which, using (10) and (11), gives $\hat{\lambda}_{b,h}^S(\rho)$ as in (8).

Proof of Proposition 2: The maximization problem for *Screen-2* is the same as that of *Screen-1*, except that $\hat{\lambda}_{h,l}^S > \lambda \geq \hat{\lambda}_{b,h}^S$. Therefore, lender's screening offer (R_l^S, C_l^S) to *l*-types does not satisfy (2). Stated differently, *l*-types would reject offer (R_l^S, C_l^S) because $V_l(R_l^S, C_l^S) < V_l^0$. Accordingly, all the results in *Screen-1* hold for *Screen-2* except for the lenders' offers to the *l*-type.

No equilibrium where lender gets the *l*-types only. Importantly, a separating equilibrium, wherein the uninformed lender can screen out both *b*-types and *h*-types, and only lend to the *l*-types does not exist. Even for $\hat{\lambda}_{b,h}^S > \lambda \geq \hat{\lambda}_{h,l}^S$. If the lender offers menu $\{(R_h^S, C_h^S), (R_l^S, C_l^S)\}$, *h*-types would reject this offer because it doesn't satisfy their participation constraint $V_h(R_h^S, C_h^S) < V_h^0$. Instead, they would accept the offer intended for *l*-types, (R_l^S, C_l^S) essentially rendering such an offer unprofitable. However, such equilibria are possible in the presence of an informed lender, as shown below. If the *h*-types accept offers made by the informed lender, then the uninformed lender's offers remain attractive only to *l*-types.

Lemma 7 In any equilibrium where the lender pools b-types, it offers a zero collateral contract.

Proof. The proof is by contradiction. Suppose not, that is, there exists a pooling equilibrium where the lender's offers (R_P^1, C_P^1) and pools all borrowers. Since *l*-types accept this contract, we must have from Lemma 3 that $V_l(R_P^1, C_P^1) = V_l^0$. Consider alternative offer (R_P^2, C_P^2) with $R_P^2 > R_P^1$ and $C_P^2 < C_P^1$, such that $V_l(R_P^2, C_P^2) = V_l(R_P^1, C_P^1)$. It follows that $V_h(R_P^2, C_P^2) > V_h(R_P^1, C_P^1)$ and $V_b(R_P^2, C_P^2) > V_b(R_P^1, C_P^1)$. Therefore, both *b*-types and *h*-types accept this new contract as it yields them higher payoff. But (R_P^2, C_P^2) yields the lender higher profits than (R_P^1, C_P^1) . Accordingly, in equilibria where the lender pools all borrowers, $C_P = 0$. Similarly, we can show that this result holds in equilibria where the lender pools just the *h*-and *b*-types, but not the *l*-types.

Proof of Proposition 3: From Lemma 7, it follows that the pooling offer is of the form $(R_P, 0)$. Since the pooling offer must yield non-negative profits, it must satisfy $[1-\mathbb{E}(\theta)]R_P \ge \rho$, where $E(\theta) \equiv \nu_b \theta_b + \nu_h \theta_h + \nu_l \theta_l$ is the expected value of θ . This implies that pooling is feasible for contracts of the form $(R_P, 0)$ such that $R_P \ge \rho/[1-E(\theta)]$. For pooling equilibria, the lender has to ensure that *l*-types accept its offer. Therefore, it must be true that $R_P \le R_l^0$, that is $\lambda \ge (\frac{1-\theta_l}{1-\mathbb{E}(\theta)})\rho \equiv \hat{\lambda}_1^P(\rho)$. Since increasing R_P increases profits, the lender offers $(R_l^0, 0)$ such that $R_l^0 = \frac{\lambda}{1-\theta_l}$ and the participation constraint of *l*-types just bind.

Proof of Proposition 4: Following the same procedure as above, we know that the pooling offer must yield non-negative profits. So it must satisfy $[\nu_b(1-\theta_b)+\nu_h(1-\theta_h)]R_P \geq (\nu_b+\nu_h)\rho$. Clearly, pooling is feasible for contracts of the form $(R_P, 0)$ such that $R_P \geq (\nu_b+\nu_h)\rho/[\nu_b(1-\theta_b)+\nu_h(1-\theta_h)]$. For this pooling contract $(R_P, 0)$ to hold, the entrant has to ensure that *h*-types accept its offer. Therefore, it must be true that $R_P \leq R_h^0$, where $R_h^0 = \frac{\lambda}{1-\theta_h}$. That is, $\lambda \geq (\frac{(\nu_b+\nu_h)(1-\theta_h)}{\nu_b(1-\theta_b)+\nu_h(1-\theta_h)})\rho \equiv \hat{\lambda}_2^P$. Since increasing R_P increases profits, the lender offers $(R_h^0, 0)$ such that the participation constraint of the *h*-types just bind.

Proof of Proposition 5: Since $\hat{\lambda}_{b,h}^S > \lambda \ge \hat{\lambda}_{h,l}^S$, the lender cannot sort the *h*-types from *b*-types but can sort *l*-types from *h*-types. The lender offers (R_l^S, C_l^S) as given in (9). Just as in *Screen-1*, this is accepted by *l*-types and rejected by *b*-types and the *h*-types. Also, as was the case for *Pool-2*, for a sufficiently low ν_b and $\lambda \ge \hat{\lambda}_2^P$, the lender offers $(R_h^0, 0)$, where $R_h^0 = \frac{\lambda}{1-\theta_h}$. This offer is accepted by the *b*-type and the *h*-type and yields non-negative profits overall.²⁷

Proof of Proposition 6: First, as *l*-types accept the lender's offer Lemma 3 must hold. Also, because the lender screens out the *b*-types, Lemma 4 must hold. Therefore, $V_l(R, C) = V_l^0$ and $V^0 = V_b(R, C)$. Solving these two equations for (R, C), we get the lender's offers to be (R_g^Y, C_g^Y) , given by (12) and (13). Note that $V_h(R_g^Y, C_g^Y) > V_h^0$, and therefore, *h*-types accept this offer as well.

Duopoly: Informed and Uninformed Lender (Competition version)

We interpret λ in the previous section to be the cost of funds for an informed lender, ρ^{I} .

²⁷Note that, since $\lambda > \rho$, the lender can make offers with $R_h < \mathbb{R}_h$, but $(R_h^0, 0)$ maximizes lender profits by bunching.

Lemma 8 For borrowers of type-b, the Lender-I denies credit. For borrowers of types g = h, l the Lender-I offers a contract from the set $Z_g^I(\rho^I) = \{(R_g^I, 0) : R_g^I \in [R_g^I(\rho^I), \bar{R}_g]\}$, where $R_g^I(\rho^I) = \frac{\rho^I}{1-\theta_g}$ and $\bar{R}_g = x - \frac{V^0}{1-\theta_g}$, g = h, l are the first-best (zero-collateral) minimum and maximum repayments, respectively.

Proof. Proof is similar to the proof for Lemma 1. See Extended Appendix for details. ■

In what follows, we solve the problem of competition between asymmetrically informed lenders by fixing the uninformed lender's cost of funds, ρ^U , and varying the informed lender's cost of funds, ρ^I . Needless to say, borrowers accept offers from the uninformed lender only in situations where $\rho^I \ge \rho^U$. The list of equilibria are provided in Table 2 in the paper (note the additional equilibria in *Screen-3*).

It is not difficult to see that the set of equilibria under competition (Table 2), are the same as that for the baseline version (Table 1). Therefore, the offers denoted in Table 2 are given by replacing λ with ρ^{I} in (9)-(13)

$$R_l^U = C_l^U = \rho^I, \tag{18}$$

$$R_h^U = \frac{\theta_b}{\theta_b - \theta_h} \rho^I - \frac{\theta_h}{\theta_b - \theta_h} [(1 - \theta_b)x - V^0]$$
(19)

$$C_h^U = -\frac{1-\theta_b}{\theta_b-\theta_h}\rho^I + \frac{1-\theta_h}{\theta_b-\theta_h}[(1-\theta_b)x - V^0].$$
(20)

$$R_g^U = \frac{\theta_b}{\theta_b - \theta_l} \rho^I - \frac{\theta_l}{\theta_b - \theta_l} [(1 - \theta_b)x - V^0]$$
(21)

$$C_g^U = -\frac{1-\theta_b}{\theta_b-\theta_l}\rho^I + \frac{1-\theta_l}{\theta_b-\theta_l}[(1-\theta_b)x - V^0]$$
(22)

However, there is one major difference in Tables 1 and 2. Unlike the monopoly case, there now exists an equilibrium under *Screen-3* wherein Lender-U cream-skims the *l*-types. This result is given in terms of proposition 7.

Proof of Proposition 7: The maximization problem for *Screen-3* is the same as that of *Screen-1*, except for the fact that now $\tilde{\rho}_{b,h}^S > \rho^I \ge \tilde{\rho}_{h,l}^S$. Lender-U can simply make and offer to (R_l^U, C_l^U) as given in (18). This implies that since $\tilde{\rho}_{b,h}^S > \rho^I \ge \tilde{\rho}_{h,l}^S$, (R_h^U, C_h^U) as given in (19)-(20), if offered, would yield negative profits for Lender-U. Therefore, Lender-U cannot screen out the *b*-types. Lender-*I* continues to offer $(\mathbb{R}_h^I, 0)$ as before. Note that while Lender-*U* cannot match this offer, Lender-*I* cannot raise R_h^I because the h-types are just indifferent between its offer of $(\mathbb{R}_h^I, 0)$ and Lender-*U*'s offer of (R_l^U, C_l^U) . It is important, therefore, that Lender-*U*'s profits from *h*-types are non-positive—a feature that is true given that Lender-*U*'s profits from (R_h^U, C_h^U) are negative.

Appendix B: Subprime Mortgage Market

The model can be used to shed light on recent events in the mortgage market, especially those related to subprime. Needless to say, some of the mechanisms are significantly more complex than those outlined in terms of this stylized model. Accordingly, the aim here is somewhat modest: we discuss the causal link in terms of a simple description of the intuition and some anecdotal evidence in support of the arguments. We begin by emphasizing features of the subprime market that are relevant to the model settings.

In terms of market structure there is strong evidence of entry of new (and uninformed) lenders. In the early years, a majority of subprime lenders were a combination of non-depository finance companies, specialized subprime mortgage lenders, and local depository institutions (Temkin et. al, 2002). Subsequently, subprime originations increased at a high rate of 25 percent per year from 1994 to 2003. It is important to point out that, of the largest and most notable subprime lenders in 2004 and 2005 such as Ameriquest, New Century, Countrywide, and Wells Fargo, only Countrywide ranked among the top 10 lenders in 2000.²⁸

On the borrower side, it is easy to argue that reservation payoffs were determined in terms of existing lending relationships—a feature that drives reservation payoffs in the model. Importantly, a significant majority of subprime mortgages are refinances, emphasizing the role of repeated interaction(s) between borrowers and lenders in this market. Studies using Corelogic-LoanPerformance data on more than 7 million securitized loans, find that between 60 to 72 percent of first-lien subprime originations between 1998 and 2007 were refinances. Therefore, poaching borrowers became increasingly relevant as the subprime market grew because major players increased market share at the expense of other informed lenders, including local lending companies.

Finally, the key insight comes from Brueckner (2000) who argues that competition can induce separation in the mortgage market because riskier borrowers agree to a price premium for high loan-to-value (LTV) mortgages. In the context of mortgage financing, the collateral requirement in the model turns out to be the inverse of the LTV ratio on mortgages. The turn of this century witnessed significant reductions in lending costs that have been documented elsewhere. Intense competition for volumes in the mortgage market meant that outside lenders could penetrate these markets simply by pooling borrowers. In terms of the model, they achieved this by lowering downpayment (collateral) requirements (i.e., allowing higher LTV in lieu of higher mortgage rates) on mortgage contracts. This would be especially true for borrowers seeking to refinance mortgages and/or extracting equity on their homes. It is interesting to note that the proportion of first-lien subprime mortgages in the (cumulative) LTV range of 90+ increased from 10 percent in 2000 to over 50 percent in 2006. Arguably, this is a simplistic view of the events. Nevertheless, it does provide insight into how reservation payoffs of borrowers can interact with lending costs to generate loans to uncreditworthy borrowers.

²⁸ Mortgage Market Statistical Annual, Inside Mortgage Finance Publications, various issues. Of course, other factors like changes in the regulatory structure, intense competition over profits in the prime market and the house price appreciation in the U.S. since 1996 significantly influenced the increase in subprime lending (Gramlich, 2004).

Extended (Web) Appendix: Not for Publication

The following (boundary) conditions hold throughout:

1. $[(1-\theta_b)x - V^0 - \rho] < 0$ and $[(1-\theta_g)x - V^0 - \rho] > 0$ for g = h, l.2. $[(1-\theta_g)(1-\theta_b)x - (1-\theta_g)V^0 - (1-\theta_b)\rho] > 0$ for g = h, l.3. $(\theta_h - \theta_l)x \ge V_l^0 - V_h^0$ 4. $\frac{V_l^0}{1-\theta_l} > \frac{V_h^0}{1-\theta_h}$

Condition 1 ensures that while h, l-types are always creditworthy, b-types are never creditworthy. Condition 2 ensures that uncreditworthy types do not find the lenders' competitive offers to creditworthy types unattractive. Finally, with $V_g^0 = (1 - \theta_g)x - \lambda$, g = h, l, conditions 3 and 4 are satisfied. Condition 3 follows from Besanko and Thakor (1987) and ensures that lenders' offers to screen out uncreditworthy borrowers are not overcollateralized (C > R). Condition 4 is the condition for countervailing incentives. The following lemma provides details on the lenders offers under complete information.

1 Equilibria under Monopoly

1.1 Contract offers under Complete Information

Lemma 1 Under complete information, the lender denies credit to b-types. For borrowers of types g = h, l an informed lender offers a contract from the set $Z_g(\rho) = \{(R_g, 0) : R_g \in [\underline{R}_g(\rho), \overline{R}_g]\}$, where $\underline{R}_g(\rho) = \frac{\rho}{1-\theta_g}$ and $\overline{R}_g = x - \frac{V^0}{1-\theta_g}$, g = h, l are the first-best (zero-collateral) minimum and maximum repayments, respectively.

Proof. Since $\pi_b < 0$, we can focus our attention without loss of generality to either creditworthy type g = h, l. We first show that an informed lender will always offer zero-collateral contracts. We prove by contradiction. Suppose not, that is, there exists an offer with a positive collateral requirement from the lender, (R_g^1, C_g^1) , that yields non-negative profits, $\pi_g(R_g^1, C_g^1) \ge 0$. Consider offer (R_g^2, C_g^2) with $R_g^2 > R_g^1$, $C_g^2 < C_g^1$ such that $V_g(R_g^1, C_g^1) = V_g(R_g^2, C_g^2)$, g = h, l. From $V_g(R_g^1, C_g^1) = V_g(R_g^2, C_g^2)$, it follows that $(1 - \theta_g)(R_g^2 - R_g^1) = \theta_g(C_g^1 - C_g^2)$. Therefore, $\pi_g(R_g^2, C_g^2) - \pi_g(R_g^1, C_g^1) = \theta_g(1 - \beta)(C_g^1 - C_g^2) > 0$. As long as $C_g^1 > 0$, the informed lender can reduce the collateral requirement to C_g^2 , with offer (R_g^2, C_g^2) which provides borrower g with the same payoff as (R_g^1, C_g^1) .

A monopolist lender's full information offer is the first-best (zero-collateral) maximum, obtained by setting $V_g(R_g, 0) = V_g^0$, so that $R_g^0(\lambda) = \frac{\lambda}{1-\theta_g}$. Notably, $R_g^0 = \bar{R}_g$ if $V_g^0 = V^0$, g = h, l. Under perfect competition, fully informed lenders offer the first-best (zero-collateral) minimum repayment, $\underline{\mathbf{R}}_g(\rho)$, by setting $\pi_g = 0, g = h, l$.

1.2 Contract offers under Incomplete Information

The *revelation principle* ensures that, without loss of generality, an uninformed principal may restrict offers to the cardinality of the type space. This implies that the lender makes at most three offers, one for each borrower type. Accordingly, there would be 14 possible permutations as shown in the Table A.1 below. The offers are denoted by the borrower types that would accept the lender's offers in equilibrium (offers in parentheses denotes pooling or bunching):

Table A.1:	Lender's	menu	comprises of	of

1 b; h; l 2 (b, h); l 8 b	
$\begin{vmatrix} 3 & (b,l); h \end{vmatrix} 9 h$	
4 (l,h); b 10 l	
5 $b; h$ 11 (b, h, l))
$\begin{vmatrix} 6 & b; l & 12 & (b, h) \end{vmatrix}$	
$\begin{bmatrix} 7 & h; l \end{bmatrix} \begin{bmatrix} 13 & (b, l) \end{bmatrix}$	
14 (l,h)	

With $\pi_b < 0$ for any offer, an uninformed lender can easily replace a menu that sorts the *b*-type with a menu that does not include offers to *b*-types. Therefore, replacing menu 1 with menu 7 unambiguously increases profits. Likewise, menus 4, 5 and 6 can be replaced with menus 14, 9 and 10 respectively. Also, offer 8 is redundant. Moreover, as will be demonstrated below, any offer that is accepted by the *b*-type and the *l*-type is also always accepted by the *b*-type. Stated differently, an uninformed lender is unable to sort the *h*-type while continuing to make offers to the *b*- and *l*-types. Therefore, menus 3 and 13 are never offered in equilibrium. Lastly, as shown below, there is no equilibrium wherein the uninformed lender attracts only *l*-types away (10). This reduces the number of menus offered in equilibrium to six as shown in Table A. 2.

Table 1: Uninformed lender's menu offers under different candidate equilibria

Candidate	Profit	Customer types	Menu of contracts	Breakeven
equilibria		accepting offers	offered	cutoff
Screen-1	Π_1^S	h;l	$(R_h^S, C_h^S); (R_l^S, C_l^S)$	$\hat{\lambda}_{b,h}^{m{S}}(ho),\hat{\lambda}_{h,l}^{m{S}}(ho)$
Screen-2	Π_2^S	h	$\left(R_{h}^{S},C_{h}^{S}\right)$	$\hat{\lambda}_{b,h}^{S}(ho)$
Pool-1	Π_1^P	(b,h,l)	$(\mathbf{\bar{R}}_{l}^{P},0)$	$\hat{\lambda}_{1}^{P}(ho, u_{b}, u_{l})$
Pool-2	Π_2^P	(b,h)	$(\mathbf{R}_{h}^{P},0)$	$\hat{\lambda}_2^P(ho, u_b, u_h)$
Hybrid-1	Π_1^Y	(b,h);l	$(\bar{\mathbf{R}}_h^P,0); (R_l^S,C_l^S)$	$\hat{\lambda}_1^P(\rho,\nu_b,\nu_h),\hat{\lambda}_{h,l}^S(\rho)$
Hybrid-2	Π_2^Y	(h,l)	(R_g^Y, C_g^Y)	$\hat{\lambda}^{Y}(ho, u_{h}, u_{l})$

Three categories of equilibria are characterized in terms of the lender's offers:

- 1. Screening equilibria: the lender offers successfully sorts borrower types.¹
- 2. Pooling equilibria: lender's offer of a single contract is accepted by two or more borrower types.
- 3. <u>Hybrid equilibria</u>: involves the bunching (or pooling) of adjacent borrower types while screening the third type. This occurs, if, for example, the lender bunches the creditworthy borrowers (the *l*-type and the *h*-type) while screening the uncreditworthy borrower (*b*-type). We characterize this category of equilibria as *hybrid* because the equilibria offers involve both pooling and screening.

The candidate equilibria (summarized in Table A.2) emerge as the final equilibria of the model for different values of the model parameters (as shown below). Within each category, candidate-1 has a larger number of borrower types accepting offers than candidate-2. For example, in candidate equilibrium Hybrid-2, the lender screens out the *b*-type, but in Hybrid-1 it pools them with *h*-types. If the lender can screen the *b*-type from the *h*-type, but not sort between the *h*-type and the *l*-type, then its offers in Screen-2 would be accepted by the *h*-types. However, if the lender can sort between all borrower types, it can offer Screen-1 whose profits dominate those of Screen-2. Similarly, for a given distribution of borrower types, the lender's offers in Hybrid-1 dominate those in Pool-2. Finally, there is no equilibrium

 $^{^{1}}$ The terms "sorting", "screening" and "separating" are used interchangeably. Also, the terms "bunching" and "pooling" are used interchangeably.

in which the monopolist lender bunches the non-adjacent, b- and l-types or is able to attract only the l-types away.²

1.2.1The lender's optimization problem under incomplete information

Under incomplete information, the optimization problem can be written as follows:³

$$\max \ \Pi \equiv \nu_b \pi_b + \nu_h \pi_h + \nu_l \pi_l, \tag{1}$$

where $\pi_k = (1 - \theta_k)R_k + \beta \theta_k C_k - \rho$, k = b, h, l subject to the following participation constraints

$$V_b(R_b, C_b) \leq V^0 \tag{2}$$

$$V_h(R_h, C_h) \geq V_h^0 \tag{3}$$

$$V_l(R_l, C_l) \geq V_l^0 \tag{4}$$

and the following incentive compatibility constraints

$$V_b(R_b, C_b) \geq V_b(R_h, C_h) \tag{5}$$

$$V_b(R_b, C_b) \geq V_b(R_l, C_l) \tag{6}$$

$$V_{b}(R_{b}, C_{b}) \geq V_{b}(R_{l}, C_{l})$$

$$V_{h}(R_{h}, C_{h}) \geq V_{h}(R_{b}, C_{b})$$

$$V_{h}(R_{h}, C_{h}) \geq V_{h}(R_{l}, C_{l})$$

$$V_{l}(R_{l}, C_{l}) > V_{l}(R_{b}, C_{b})$$

$$(9)$$

$$V_h(R_h, C_h) \geq V_h(R_l, C_l) \tag{8}$$

$$V_l(R_l, C_l) \geq V_l(R_b, C_b) \tag{9}$$

$$V_l(R_l, C_l) \geq V_l(R_h, C_h). \tag{10}$$

where $V_g^0 = (1 - \theta_g)x - \lambda$, g = h, l and $\lambda \in [\rho, (1 - \theta_h)x - V^0]$. Also $V_b = V^0$ throughout. Note that since $\pi_b < 0$, the lender does not offer contract (R_b, C_b) in equilibrium. Therefore, (2), (7) and (9) are redundant. Moreover, one may replace $V_b(R_b, C_b)$ with the b-type's reservation utility V^0 on the left-hand side of (5) and (6). Consequently, the above maximization problem in (1)-(10) reduces to

$$\max \Pi \equiv \nu_b \pi_b + \nu_h \pi_h + \nu_l \pi_l, \tag{11}$$

where $\pi_k = (1 - \theta_k)R_k + \beta \theta_k C_k - \rho$, k = b, h, l subject to the following participation constraints

$$V_h(R_h, C_h) \geq V_h^0 \tag{12}$$

$$V_l(R_l, C_l) \geq V_l^0 \tag{13}$$

and the following incentive compatibility constraints

$$V^0 \geq V_b(R_h, C_h) \tag{14}$$

$$V^0 \geq V_b(R_l, C_l) \tag{15}$$

$$V_h(R_h, C_h) \geq V_h(R_l, C_l) \tag{16}$$

$$V_l(R_l, C_l) \geq V_l(R_h, C_h). \tag{17}$$

Lemma 2 Lender's expected profits from loans to l-types are always non-negative, $\pi_l(R_l, C_l) \geq 0$.

Proof. Suppose not, that is there exist equilibria in which $\pi_l(R_l, C_l) < 0$. With $R \ge C$, it follows that for any (R_l, C_l) we have $\pi_b < \pi_h < \pi_l < 0$. The lender can always drop this contract and increase profits. Therefore, in any equilibrium, it is always the case that $\pi_l(R_l, C_l) \ge 0$.

 $^{^{2}}$ However, such equilibria does exist in situations where an uninformed lender competes with an informed lender (with complete information on borrower types). See Proposition 14 below for details.

 $^{^{3}}$ In this section, for the most part, we drop the superscript denoting offers by borrower U. Therefore, unless otherwise mentioned, we are considering profits and offers of Lender-U only. We will re-introduce superscripts below.

Lemma 3 The IR constraint of the l-type, (13), must bind.

Proof. Case 1: Lender does not sort *h*-type from *l*-type

We prove by contradiction. Suppose not, that is, there exists a solution to (11)- $(15)^4$ characterized by (R_g^1, C_g^1) such that $V_l(R_g^1, C_g^1) > V_l^0$. We will show that there exists an offer (R_g^2, C_g^2) such that it satisfies (13) but yields higher profit. We begin by characterizing this contract. Consider contract (R_g^2, C_g^2) , where $R_g^2 > R_g^1$, $C_g^2 < C_g^1$ such that

$$V_b(R_g^1, C_g^1) = V_b(R_g^2, C_g^2)$$
(18)

It follows that

$$V_h(R_g^2, C_g^2) < V_h(R_g^1, C_g^1)$$

$$V_l(R_a^2, C_g^2) < V_l(R_a^1, C_g^1)$$
(19)

Following the last inequality, we focus our attention to (R_q^2, C_q^2) such that

$$V_l(R_g^1, C_g^1) > V_l(R_g^2, C_g^2) \ge V_l^0$$
(20)

Since (R_g^1, C_g^1) is a solution, it satisfies all the constraints. Consequently, we can show that offer (R_g^2, C_g^2) in (18) and (20) satisfies all other constraints as well. Constraints (13), (14) and (15) are satisfied by construction.⁵

It remains to be shown that (12) is satisfied. We prove by contradiction. Suppose not, then $V_h(R_g^1, C_g^1) \geq V_h^0 > V_h(R_g^2, C_g^2)$. It follows that $\lambda < (1 - \theta_h)R_g^2 + \theta_h C_g^2$. From (20), we obtain $\lambda \geq (1 - \theta_l)R_g^2 + \theta_l C_g^2$. Combining the two, we have $(1 - \theta_h)R_g^2 + \theta_h C_g^2 > (1 - \theta_l)R_g^2 + \theta_l C_g^2$ or $(\theta_h - \theta_l)(C_g^2 - R_g^2) > 0$. Since $\theta_h > \theta_l$, it must be the case that $C_g^2 > R_g^2$. This is impossible given our initial condition $R \geq C$. We have a contradiction. It must be the case that (12) is satisfied.

Finally, since $R_g^2 > R_g^1, C_g^2 < C_g^1$, we have using (18) that $\pi_h(R_g^2, C_g^2) > \pi_h(R_g^1, C_g^1)$. Also, $\pi_l(R_g^2, C_g^2) > \pi_l(R_g^1, C_g^1)$. It follows that $\Pi(R_g^2, C_g^2) > \Pi(R_g^1, C_g^1)$. Therefore, (R_g^1, C_g^1) cannot be an equilibrium because lender can offer an alternative contract of the form (R_g^2, C_g^2) and increase profits. Such profitable deviations are not possible only if $V_l(R_g, C_g) = V_l^0$. Therefore (13) must bind in equilibrium.

Case 2: Lender sorts creditworthy types.

We prove by contradiction. Suppose not, that is, there exists a solution to (11)-(17) characterized by $\{(R_h, C_h), (R_l^1, C_l^1)\}$ such that $V_l(R_l^1, C_l^1) > V_l^0$. We will show that there exist a menu $\{(R_h, C_h), (R_l^2, C_l^2)\}$ such that it satisfies (13) but yields higher profit. We begin by characterizing this contract. Consider contract (R_l^2, C_l^2) , where $R_l^2 > R_l^1, C_l^2 < C_l^1$ such that

$$V_b(R_l^1, C_l^1) = V_b(R_l^2, C_l^2)$$
(21)

This implies

$$V_h(R_l^2, C_l^2) < V_h(R_l^1, C_l^1)$$

$$V_l(R_l^2, C_l^2) < V_l(R_l^1, C_l^1)$$
(22)

Following this, we can choose (R_l^2, C_l^2) such that either of the following are true

$$V_{l}(R_{l}^{1}, C_{l}^{1}) > V_{l}(R_{l}^{2}, C_{l}^{2}) \ge V_{l}(R_{h}, C_{h}) \ge V_{l}^{0}$$

$$(23)$$

$$V_{l}(R_{l}^{1}, C_{l}^{1}) \ge V_{l}(R_{l}^{2}, C_{l}^{2}) \ge V_{l}^{0} \ge V_{l}(R_{h}, C_{h})$$

$$(24)$$

$$V_l(R_l^1, C_l^1) > V_l(R_l^2, C_l^2) \ge V_l^0 > V_l(R_h, C_h)$$
(24)

⁴Since Lender-U does not sort, h-type and l-types are bunched with a single offer. Consequently, (16) and (17) are trivially satisfied.

⁵Since Lender-U has only one offer for all creditworthy types, (14) and (15) are the same constraint.

First, we show that the menu $\{(R_h, C_h), (R_l^2, C_l^2)\}$ in (21) and either (23) or (24) satisfies all other constraints as well. Constraints (12) and (14) are trivially satisfied. Constraint (13), (15) and (17) are satisfied by construction. Lastly, (16) is satisfied because $V_h(R_h, C_h) \ge V_h(R_l^1, C_l^1) > V_h(R_l^2, C_l^2)$.

Next, we show that by offering (R_l^2, C_l^2) instead of (R_l^1, C_l^1) , lender increases profits. Since $R_l^2 > R_l^1$, $C_l^2 < C_l^1$, we have using $(21)^6$, $\pi_l(R_l^2, C_l^2) > \pi_l(R_l^1, C_l^1)$. Therefore, menu $\{(R_h, C_h), (R_l^1, C_l^1)\}$ cannot be an equilibrium because lender can offer an alternative contract of the form $\{(R_h, C_h), (R_l^2, C_l^2)\}$ and increase profits.

Note that this holds for either (23) or (24). For (23), all deviations from (R_l^1, C_l^1) to (R_l^2, C_l^2) yield higher profits, unless $V_l(R_l, C_l) = V_l(R_h, C_h)$; in which case, lender bunches *h*-types and *l*-types and we follow the proof as given in Case 1 above. Alternatively, for (24) such profitable deviations are not possible only if $V_l(R_l, C_l) = V_l^0$. Therefore (13) must bind in equilibrium.

Lemma 4 In any equilibrium that screens out the b-type, the IC constraint of the b-type w.r.t the h-type, (14), must bind.

Proof. Case 1: lender sorts creditworthy types.

We prove by contradiction. Suppose not, that is, there exists a solution to (11)-(17) characterized by $\{(R_h^1, C_h^1), (R_l, C_l)\}$ such that $V^0 > V_b(R_h^1, C_h^1)$. We will show that there exist menus such as $\{(R_h^2, C_h^2), (R_l, C_l)\}$ that satisfy (14) all the other constraints but yield higher profits. We begin by characterizing such contracts. Consider contract (R_h^2, C_h^2) , where $R_h^2 > R_h^1, C_h^2 < C_h^1$ such that

$$V_h(R_h^1, C_h^1) = V_h(R_h^2, C_h^2).$$
(25)

It follows that

$$V_{l}(R_{h}^{2}, C_{h}^{2}) < V_{l}(R_{h}^{1}, C_{h}^{1})$$

$$V_{b}(R_{h}^{2}, C_{h}^{2}) > V_{b}(R_{h}^{1}, C_{h}^{1})$$
(26)

Therefore, if $V^0 > V_b(R_h^1, C_h^1)$, we can choose (R_h^2, C_h^2) so that $V^0 \ge V_b(R_h^2, C_h^2) > V_b(R_h^1, C_h^1)$. Since $\{(R_h^1, C_h^1), (R_l, C_l)\}$ is a solution, it satisfies all the constraints. Consequently, we can show that menus $\{(R_h^2, C_h^2), (R_l, C_l)\}$ satisfy all the constraints as well. Constraints (13) and (15) are trivially satisfied. Constraint (12), (14) and (16) are satisfied by construction. Lastly, (17) is satisfied because $V_l(R_l, C_l) \ge V_l(R_h^1, C_h^1) > V_l(R_h^2, C_h^2)$.

But, since $R_h^2 > R_h^1$ and $C_h^2 < C_h^1$, we use (25) to obtain $\pi_h(R_h^2, C_h^2) > \pi_h(R_h^1, C_h^1)$. Therefore, $\{(R_h^1, C_h^1), (R_l, C_l)\}$ cannot be an equilibrium because lender can offer an alternative contract of the form $\{(R_h^2, C_h^2), (R_l, C_l)\}$ and increase profits. Such profitable deviations are not possible only if $V^0 = V_b(R_h, C_h)$. Therefore (14) must bind in equilibrium.

Case 2: lender does not sort h-type from l-type

This holds for either creditworthy type, yielding two sets of equilibria where lender screens out just the *b*-type. The first occurs when g = h, and lender captures only the *h*-type by screening them from the *b*-type, as described in the candidate equilibria Screen-2. The second occurs when g = l and lender captures both *h*- and *l*-types by bunching them and screening them from the b-type, as described in the candidate equilibria Hybrid-2.

We prove by contradiction for Hybrid-2. Suppose not, that is, there exists a solution to (11)- $(15)^7$ characterized by (R_q^1, C_q^1) , where (R_q^1, C_q^1) is the offer to both the *h*-type and the *l*-type, and $V^0 >$

 $[\]frac{1}{\left[6 \text{ This follows from } \pi_l(R_l^2, C_l^2) - \pi_l(R_l^1, C_l^1) = \frac{\theta_b(1-\theta_l) - \beta\theta_l(1-\theta_b)}{(1-\theta_b)}(C_l^1 - C_l^2) \text{ and } \theta_b(1-\theta_l) - \beta\theta_l(1-\theta_b) > (1-\beta)\theta_l(1-\theta_b) - \beta\theta_l(1-\theta_b) = 0.$

⁷Since Lender-U does not sort, h-type and l-types are bunched with a single offer. Consequently, (16) and (17) are trivially satisfied.

 $V_b(R_g^1, C_g^1)$. Note that one can find an alternative contract (R_g^2, C_g^2) with $R_g^2 > R_g^1$ and $C_g^2 < C_g^1$, such that

$$V_l(R_g^1, C_g^1) = V_l(R_g^2, C_g^2).$$
(27)

It follows that

$$V_h(R_g^2, C_g^2) > V_h(R_g^1, C_g^1)$$
and $V_b(R_g^2, C_g^2) > V_b(R_g^1, C_g^1).$
(28)

Following this, we can restrict our attention to contracts such that

$$V^{0} \ge V_{b}(R_{g}^{2}, C_{g}^{2}) > V_{b}(R_{g}^{1}, C_{g}^{1}).$$
⁽²⁹⁾

Since (R_g^1, C_g^1) is a solution, it satisfies all the constraints. Consequently, we can show that offer (R_g^2, C_g^2) in (27) and (29) satisfies all other constraints as well. Constraints (13), (14) and (15) are satisfied by construction.⁸ Lastly, (12) is satisfied from (28).

In offering (R_g^2, C_g^2) instead of (R_g^1, C_g^1) , lender's change in profits from the *l*-type and *h*-type are given by

$$\begin{aligned} \Delta \pi_l &= (1 - \beta) \theta_l (C_g^1 - C_g^2) > 0 \\ \Delta \pi_h &= \frac{\theta_l (1 - \theta_h) - \beta \theta_h (1 - \theta_l)}{(1 - \theta_l)} (C_g^1 - C_g^2) \end{aligned}$$

Since bunching is only feasible for $\nu_l \geq \nu_h$ and we have $(1-\beta)\theta_l > \frac{(1-\beta)\theta_l(1-\theta_h)}{(1-\theta_l)} > \frac{\theta_l(1-\theta_h)-\beta\theta_h(1-\theta_l)}{(1-\theta_l)}$. Therefore lender's offer of (R_g^2, C_g^2) to both *h*-type and *l*-types yields higher profits than (R_g^1, C_g^1) . That is, $\Pi(R_g^2, C_g^2) > \Pi(R_g^1, C_g^1)$. This implies that (R_g^1, C_g^1) cannot be an equilibrium. Such deviations are no longer possible if $V_b(R_g, C_g) = V^0$.

Proceeding exactly as above, we can prove the same for candidate equilibria Screen-2, where lender's offers are accepted by the h-types only.

Lemma 5 In any equilibrium that sorts the h-types from the l-type, the IC constraint of the h-type w.r.t the l-type,(16), must bind.

Proof. We prove by contradiction. Suppose not, that is, there exists a solution to (11)-(17) characterized by $\{(R_h, C_h), (R_l^1, C_l^1)\}$ such that $V_h(R_h, C_h) > V_h(R_l^1, C_l^1)$. We will show that there exist a menus of contracts $\{(R_h, C_h), (R_l^2, C_l^2)\}$ such that it satisfies (16) and all the other constraints but yields higher profit. We begin by characterizing this contract. Consider contract (R_l^2, C_l^2) , where $R_l^2 > R_l^1$, $C_l^2 < C_l^1$ such that

$$V_l(R_l^1, C_l^1) = V_l(R_l^2, C_l^2) = V_l^0$$
(30)

It follows that $V_h(R_l^2, C_l^2) > V_h(R_l^1, C_l^1)$ and $V_b(R_l^2, C_l^2) > V_b(R_l^1, C_l^1)$. Therefore, if $V_h(R_l^2, C_l^2) > V_h(R_l^1, C_l^1)$, we can choose (R_l^2, C_l^2) so that

$$V_h(R_h, C_h) \ge V_h(R_l^2, C_l^2) > V_h(R_l^1, C_l^1)$$
(31)

Since $\{(R_h, C_h), (R_l^1, C_l^1)\}$ is a solution, it satisfies all the constraints. Consequently, we can show that the menu $\{(R_h, C_h), (R_l^2, C_l^2)\}$ in (30) and (31) satisfies all the constraints as well. Constraints (12) and (14) are trivially satisfied. Constraint (13), (16) and (17) are satisfied by construction.

For (15), we prove by contradiction. Suppose not, then $V_b(R_l^2, C_l^2) > V^0$. In addition, Lemma 4 implies $V_b(R_l^2, C_l^2) > V^0 = V_b(R_h, C_h)$. That is, $(1 - \theta_b)(R_h - R_l^2) > \theta_b(C_l^2 - C_h)$. Also, because (31) holds, it follows that $\theta_h(C_l^2 - C_h) \ge (1 - \theta_h)(R_h - R_l^2)$. Combining both, $(\frac{1 - \theta_b}{\theta_b})(R_h - R_l^2) > (C_l^2 - C_h)$. $\ge (\frac{1 - \theta_h}{\theta_h})(R_h - R_l^2) \text{ or } (\frac{1 - \theta_b}{\theta_b} - \frac{1 - \theta_h}{\theta_h})(R_h - R_l^2) > 0.$

⁸Since Lender-U has only one offer for all creditworthy types, (14) and (15) are the same constraint.

Since the first expression is negative, this would imply $R_l^2 > R_h$. Similarly we can show that $C_l^2 < C_h$. However, since (17) is satisfied, we get $(1 - \theta_l)(R_h - R_l^2) > \theta_l(C_l^2 - C_h)$. Again, using (31) it follows that $\theta_h(C_l^2 - C_h) \ge (1 - \theta_h)(R_h - R_l^2)$. Combining both, $(\frac{1 - \theta_l}{\theta_l})(R_h - R_l^2) > (C_l^2 - C_h) \ge (\frac{1 - \theta_h}{\theta_h})(R_h - R_l^2)$ or $(\frac{1 - \theta_l}{\theta_l} - \frac{1 - \theta_h}{\theta_h})(R_h - R_l^2) > 0$. Now, since the first expression is positive, this would imply $R_l^2 < R_h$. Similarly we can show that $C_l^2 > C_h$. We have a contradiction. Therefore, it cannot be the case that $V_b(R_l^2, C_l^2) > V^0$ and (15) is satisfied.

Since $R_l^2 > R_l^1, C_l^2 < C_l^1$, using (30), we get $\pi_l(R_l^2, C_l^2) > \pi_l(R_l^1, C_l^1)$. Therefore, menu $\{(R_h, C_h), (R_l^1, C_l^1)\}$ cannot be an equilibrium because the lender can offer an alternative contract of the form $\{(R_h, C_h), (R_l^1, C_l^2)\}$ and increase profits. Such profitable deviations are not possible only if $V_h(R_h, C_h) = V_h(R_l, C_l)$. Therefore (16) must bind in equilibrium.

1.2.2 Lender's offers in screening equilibria

Consequently, the above maximization problem in (1)-(10) reduces to

$$\max \Pi \equiv \nu_h \pi_h + \nu_l \pi_l, \tag{32}$$

where $\pi_k = (1 - \theta_k)R_k + \beta \theta_k C_k - \rho$, subject to the following participation constraints

$$V_h(R_h, C_h) \geq V_h^0 \tag{33}$$

$$V_l(R_l, C_l) = V_l^0 \tag{34}$$

and the following incentive compatibility constraints

$$V^0 = V_b(R_h, C_h) \tag{35}$$

$$V^0 \geq V_b(R_l, C_l) \tag{36}$$

$$V_h(R_h, C_h) = V_h(R_l, C_l) \tag{37}$$

$$V_l(R_l, C_l) \geq V_l(R_h, C_h). \tag{38}$$

Using the equations (33)-(38), we obtain expressions for (R_h, C_h) and (R_l, C_l) as follows:

$$\begin{array}{rcl} R_g^Y & \geq & R_l \geq R_l^S, \\ R_h^S & \geq & R_h \geq R_g^Y \\ C_g^Y & \leq & C_l \leq C_l^S \\ C_h^S & \leq & C_h \leq C_g^Y \end{array}$$

where

$$R_l^S = C_l^S = \lambda, \tag{39}$$

$$R_g^Y = \frac{\theta_b}{\theta_b - \theta_l} \lambda - \frac{\theta_l}{\theta_b - \theta_l} [(1 - \theta_b)x - V^0]$$
(40)

$$C_g^Y = -\frac{1-\theta_b}{\theta_b-\theta_l}\lambda + \frac{1-\theta_l}{\theta_b-\theta_l}[(1-\theta_b)x - V^0]$$
(41)

$$R_{h}^{S} = \frac{\theta_{b}}{\theta_{b} - \theta_{h}} \lambda - \frac{\theta_{h}}{\theta_{b} - \theta_{h}} [(1 - \theta_{b})x - V^{0}]$$

$$\tag{42}$$

$$C_h^S = -\frac{1-\theta_b}{\theta_b-\theta_h}\lambda + \frac{1-\theta_h}{\theta_b-\theta_h}[(1-\theta_b)x - V^0].$$
(43)

Using conditions 1, 2 and $\lambda > \rho > 0$, all of the offers given by (39)-(43) are strictly positive.

From (33)-(38), it follows that the screening offers to *h*-types and *l*-types are (R_h^S, C_h^S) and (R_l^S, C_l^S) respectively. Likewise, (R_g^Y, C_g^Y) is an offer satisfying (33)-(38) in which the lender bunches both *h*-types and *l*-types while screening out the *b*-types.

Lemma 6 In any equilibrium that sorts the h-types from the l-type, the IR constraint of the h-type, (33) must bind.

Proof. First note that $V_h(R_h^S, C_h^S) = V_h^0$. Also lemmas 3-5 hold.

We prove by contradiction. Suppose not. That is, there exists a menu $\{(R_h^2, C_h^2), (R_l^2, C_l^2)\}$ which satisfies (33)-(38) where $R_h^2 < R_h^S$ and $C_h^2 > C_h^S$ such that $V_b(R_h^2, C_h^2) = V_b(R_h^S, C_h^S)$. It follows that $V_h(R_h^2, C_h^2) > V_h(R_h^S, C_h^S) = V_h^0$. or (33) is slack. Also, from (34) and (37) it follows that $R_l^2 > R_l^S$, $C_l^2 < C_l^S$. Therefore, replacing $\{(R_h^S, C_h^S), (R_l^S, C_l^S)\}$ by $\{(R_h^2, C_h^2), (R_l^2, C_l^2)\}$ would increase profits from the *l*-types but decrease profits from the *h*-types as follows:

$$\Delta \pi_{l} = (1 - \beta)\theta_{l} \frac{(1 - \theta_{l})}{(1 - \theta_{b})} (\frac{\theta_{b} - \theta_{h}}{\theta_{h} - \theta_{l}}) (C_{h}^{2} - C_{h}^{S}) > 0$$

$$\Delta \pi_{h} = -\frac{\theta_{b}(1 - \theta_{h}) - \beta\theta_{h}(1 - \theta_{b})}{(1 - \theta_{b})} (C_{h}^{2} - C_{h}^{S}) < 0$$

For fixed (non-zero) ν_h and ν_l , the effect on total profits Π is either monotonically increasing or monotonically decreasing with the magnitude of the deviation, $(C_h^2 - C_h^S)$.⁹ If the total effect is positive, all deviations with menus of the type $\{(R_h^2, C_h^2), (R_l^2, C_l^2)\}$ yield higher profits than $\{(R_h^S, C_h^S), (R_l^S, C_l^S)\}$. That is, profits are maximized by offering (R_g^Y, C_g^Y) which does not induce separation. Therefore, in an equilibrium that induces separation of the *h*-types and *l*-types, (33) must bind.

We can now list the set of equilibria in term of the propositions given below. Lenders need to recover the cost of screening to break-even on screening offers. To break-even, borrower's (exogenous) reservation payoff has to be lower than a threshold or alternatively, $\lambda \geq \hat{\lambda}$. Stated differently, this candidate screening equilibrium are feasible only if the borrower's (exogenous) reservation payoff is significantly low- that is one of two screening cutoffs (one for each pair of adjacent types) are satisfied. The first cutoff is $\hat{\lambda}_{h,l}^{S}$ for screening the *h*-types from the *l*-types and the second is $\hat{\lambda}_{b,h}^{S}$ for screening the *h*-types from the *b*-types.

$$\hat{\lambda}_{h,l}^{S}(\rho) = \frac{1}{1 - (1 - \beta)\theta_l}\rho \tag{44}$$

$$\hat{\lambda}_{b,h}^{S}(\rho) = \frac{\theta_b - \theta_h}{\theta_b(1 - \theta_h) - \beta\theta_h(1 - \theta_b)}\rho + \frac{(1 - \beta)\theta_h(1 - \theta_h)}{\theta_b(1 - \theta_h) - \beta\theta_h(1 - \theta_b)}[(1 - \theta_b)x - V^0].$$
(45)

Clearly for both cutoffs, $\hat{\lambda}'(\rho) > 0$ and $\hat{\lambda}''(\rho) = 0$. Note that there is a one-to-one correspondence between λ and V_g^0 , g = h, l.

1.3 Candidate Equilibria Under Incomplete Information

The candidate equilibria are given below in Propositions 7-13. For a given set of parameter values, the candidate that provides maximum payoff to the lender emerges as the final equilibrium. In the paper, we provide numerical exercises to determine how the final equilibrium changes for different parameter values.

1.3.1 Screening Equilibria

Candidate equilibrium: Screen-1

Proposition 7 If $\lambda \geq \max(\hat{\lambda}_{h,l}^{S}(\rho), \hat{\lambda}_{b,h}^{S}(\rho))$, where $\hat{\lambda}_{h,l}^{S}(\rho)$ and $\hat{\lambda}_{b,h}^{S}(\rho)$ are given in (44) and (45), a pure strategy equilibrium wherein the lender sorts all borrower types is characterized as follows:

⁹Except for the case when $\Delta \Pi^U \equiv \nu_h \Delta \pi_l^U + \nu_l \Delta \pi_l^U = 0$. Then, all points satisfying (33)-(38) can be supported as equilbira. However, we restrict our attention to the screening offer given in lemma 6.

- (a) lender offers menu $\{(R_h^S, C_h^S), (R_l^S, C_l^S)\}$ given by (39), (42) and (43).

(b) both creditworthy types accept their respective offers but b-types types reject both offers (c) lender expected profits are $\Pi_1^S = \nu_h [\frac{\theta_b(1-\theta_h)-\beta\theta_h(1-\theta_b)}{\theta_b-\theta_h}\lambda - \frac{(1-\beta)\theta_h(1-\theta_h)}{\theta_b-\theta_h}\{(1-\theta_b)x - V^0\}] - \nu_l [1-\theta_b)x - V^0\}$ $(1-\beta)\theta_l \lambda - (\nu_h + \nu_l)\rho$

Proof. In any screening equilibrium wherein the lender separates all borrower types, Lemmas 1-6 must hold. This implies that the lender offers menu $\{(R_h^S, C_h^S), (R_l^S, C_l^S)\}$. For the lender to screen the *l*-type from the *h*-type, profits $\pi_l(R_l^S, C_l^S) \ge 0$. This occurs when $\lambda \ge \hat{\lambda}_{h,l}^S(\rho) = \frac{1}{1-(1-\beta)\theta_l}\rho$. Similarly, the lender can screen the *h*-type from the *b*-type, if $\pi_h(R_h^S, C_h^S) \ge 0$. That is if $\lambda \ge \hat{\lambda}_{b,h}^S(\rho)$, which, using (42) and (43), gives $\hat{\lambda}_{b,h}^{S}(\rho)$ as in (45).

In order to screen all borrower types, we must have $\lambda > \max(\hat{\lambda}_{h,l}^S(\rho), \hat{\lambda}_{b,h}^S(\rho))$. However, if $\hat{\lambda}_{h,l}^S > \lambda \ge \hat{\lambda}_{b,h}^S$ or $\hat{\lambda}_{b,h}^S > \lambda \ge \hat{\lambda}_{h,l}^S$, the lender still has the option to just screen one creditworthy type, as given by the following proposition.

Candidate Equilibrium: Screen-2

Proposition 8 If $\hat{\lambda}_{h,l}^S > \lambda \geq \hat{\lambda}_{b,h}^S$, where $\hat{\lambda}_{h,l}^S(\rho)$ and $\hat{\lambda}_{b,h}^S(\rho)$ are given in (44) and (45), a pure strategy equilibrium wherein the lender screeens out the b-type and lends to the h-types only, is characterized as follows:

(a) lender offers contract (R_h^S, C_h^S) , given by (42) and (43)

(a) binder δ_{jj} in the equation (1, n) is (1, n) if (1, n) if (1, n) is (1, n) if (1, n) if (1, n) is (1, n) if (1, n) if (1, n) is (1, n) if (1, n) if (1, n) is (1, n) if (1, n) if (1, n) is (1, n) if (1, n) if (1, n) if (1, n) is (1, n) if (1, n) if (1, n) if (1, n) is (1, n) if (1,

The maximization problem for Screen-2 is the same as that of Screen-1, except that $\hat{\lambda}_{h,l}^S > \lambda \ge \hat{\lambda}_{b,h}^S$. Therefore, lender's screening offer (R_l^S, C_l^S) to *l*-types does not satisfy (34). Stated differently, *l*-types would reject offer (R_l^S, C_l^S) because $V_l(R_l^S, C_l^S) < V_l^0$. Accordingly, all the results in Screen-1 hold for Screen-2 except for the lenders' offers to the l-type.

No equilibrium where lender gets the *l*-types only. Importantly, a separating equilibrium, wherein the uninformed lender can screen out both b-types and h-types, and only lend to the l-types does not exist. Even for $\hat{\lambda}_{b,h}^S > \lambda \ge \hat{\lambda}_{h,l}^S$. If the lender offers menu $\{(R_h^S, C_h^S), (R_l^S, C_l^S)\}$, h-types would reject this offer because it doesn't satisfy their participation constraint $V_h(R_h^S, C_h^S) < V_h^0$. Instead, they would accept the offer intended for *l*-types, (R_l^S, C_l^S) essentially rendering such an offer unprofitable. However, such equilibria are possible in the presence of an informed lender, as shown below. If the h-types accept offers made by the informed lender, then the uninformed lender's offers remain attractive only to *l*-types.

1.3.2Pooling Equilibria (pooling the *b*-type)

When ν_b is sufficiently small, the lender may choose to pool them with creditworthy types. There are two such pooling equilibria; one, where b-types are pooled with h-types only and the other where all types are pooled together.

Lemma 9 In any equilibrium where the lender pools b-types, it offers a zero collateral contract.

Proof. The proof is by contradiction. Suppose not, that is, there exists a pooling equilibrium where the lender's offers (R_P^1, C_P^1) and pools all borrowers. Since *l*-types accept this contract, we must have from Lemma 3 that $V_l(R_P^1, C_P^1) = V_l^0$. Consider alternative offer (R_P^2, C_P^2) with $R_P^2 > R_P^1$ and $C_P^2 < C_P^1$, such that $V_l(R_P^2, C_P^2) = V_l(R_P^1, C_P^1)$. It follows that $V_h(R_P^2, C_P^2) > V_h(R_P^1, C_P^1)$ and $V_b(R_P^2, C_P^2) > V_b(R_P^1, C_P^1)$. Therefore, both b-types and h-types accept this new contract as it yields them higher payoff. But (R_P^2, C_P^2) yields the lender higher profits than $(R_P^1, C_P^1)^{10}$ Accordingly, in equilibria where the lender pools all borrowers, $C_P = 0$. Similarly, we can show that this result holds in equilibria where the lender pools just the h- and b-types, but not the l-types.

Candidate Equilibrium: Pool-1

Proposition 10 If $\lambda \geq \hat{\lambda}_1^P \equiv (\frac{1-\theta_l}{1-\mathbb{E}(\theta)})\rho$, a pure strategy equilibrium wherein the lender pools all borrowers is characterized as follows:

(a) lender offers contract $(R_l^0, 0)$, such that $R_l^0 = x - \frac{V_l^0}{1 - \theta_l} = \frac{\lambda}{1 - \theta_l}$.

(b) all borrowers accept this offer. (c) lender's profits are $\Pi_1^P = \frac{1 - \mathbb{E}(\theta)}{1 - \theta_l} \lambda - \rho$ where $\mathbb{E}(\theta) \equiv \nu_b \theta_b + \nu_h \theta_h + \nu_l \theta_l$.

Proof. From Lemma 10, it follows that the pooling offer is of the form $(R_P, 0)$. Since the pooling offer must yield non-negative profits, it must satisfy $[1 - \mathbb{E}(\theta)]R_P \ge \rho$, where $E(\theta) \equiv \nu_b \theta_b + \nu_h \theta_h + \nu_l \theta_l$ is the expected value of θ . This implies that pooling is feasible for contracts of the form $(R_P, 0)$ such that $R_P \ge \rho/[1 - E(\theta)]$. For pooling equilibria, the lender has to ensure that *l*-types accept its offer. Therefore, it must be true that $R_P \leq R_l^0$, that is $\lambda \geq (\frac{1-\theta_l}{1-\mathbb{E}(\theta)})\rho \equiv \hat{\lambda}_1^P(\rho)$. Since increasing R_P increases profits, the lender offers $(R_l^0, 0)$ such that $R_l^0 = \frac{\lambda}{1-\theta_l}$ and the participation constraint of *l*-types just bind.

Candidate Equilibrium: Pool-2

Proposition 11 If $\lambda \geq \hat{\lambda}_2^P \equiv (\frac{(\nu_b + \nu_h)(1 - \theta_h)}{\nu_b(1 - \theta_b) + \nu_h(1 - \theta_h)})\rho$, a pure strategy equilibrium wherein the lender pools *h*-types and *b*-types only, is characterized as follows:

(a) lender offers contract $(R_h^0, 0)$, where $R_h^0 = x - \frac{V_h^0}{1-\theta_h} = \frac{\lambda}{1-\theta_h}$. (b) b-types and h-types accept this offer but l-types reject this offer (c) lender's expected profits are $\Pi_2^P = \frac{\nu_b(1-\theta_b)+\nu_h(1-\theta_h)}{1-\theta_h}\lambda - (\nu_b + \nu_h)\rho$

Proof. Following the same procedure as above, we know that the pooling offer must yield nonnegative profits. So it must satisfy $[\nu_b(1-\theta_b)+\nu_b(1-\theta_b)]R_P \geq (\nu_b+\nu_b)\rho$. Clearly, pooling is feasible for contracts of the form $(R_P, 0)$ such that $R_P \ge (\nu_b + \nu_h)\rho/[\nu_b(1-\theta_b) + \nu_h(1-\theta_h)]$. For this pooling contract $(R_P, 0)$ to hold, the entrant has to ensure that *h*-types accept its offer. Therefore, it must be true that $R_P \leq R_h^0$, where $R_h^0 = \frac{\lambda}{1-\theta_h}$. That is, $\lambda \geq (\frac{(\nu_b + \nu_h)(1-\theta_h)}{\nu_b(1-\theta_b)+\nu_h(1-\theta_h)})\rho \equiv \hat{\lambda}_2^P$. Since increasing R_P increases profits, the lender offers $(R_h^0, 0)$ such that the participation constraint of the *h*-types just bind.

1.3.3Hybrid Equilibria

Hybrid equilibria has elements of both pooling and screening. This occurs when the lender pools or bunches adjacent types and screens the third type.

Candidate Equilibrium: Hybrid-1

 $^{^{10}}$ One can show this proceeding in a similar way as in Case 2 for Lemma 4. Also note that pooling is feasible only if $\nu_l \ge \nu_b + \nu_h.$

Proposition 12 If $\hat{\lambda}_{b,h}^S > \lambda \ge \hat{\lambda}_{h,l}^S$ and $\lambda \ge \hat{\lambda}_2^P$, a pure strategy equilibrium wherein the lender separates only the l-types and bunches (pools) b-types and h-types is characterized as follows:

(a) lender offers menu $\{(R_h^0, 0); (R_l^S, C_l^S)\}$ given by (39).and $R_h^0 = x - \frac{V_h^0}{1-\theta_h} = \frac{\lambda}{1-\theta_h}$ (b) l-types accepts the offer (R_l^S, C_l^S) , while b-types and h-types accept the offer $(R_h^0, 0)$. (c) lender expected profits are $\Pi_1^Y = \frac{\nu_b(1-\theta_b)+\nu_h(1-\theta_h)}{1-\theta_h}\lambda + \nu_l[1-(1-\beta)\theta_l]\lambda - \rho$

Proof. Since $\hat{\lambda}_{b,h}^S > \lambda \ge \hat{\lambda}_{h,l}^S$, the lender cannot sort the *h*-types from *b*-types but can sort *l*-types from *h*-types. The lender offers (R_l^S, C_l^S) as given in (39). Just as in Screen-1, this is accepted by *l*-types and rejected by b-types and the h-types. Also, as was the case for Pool-2, for a sufficiently low ν_b and $\lambda \geq \hat{\lambda}_2^P$, the lender offers $(R_h^0, 0)$, where $R_h^0 = \frac{\lambda}{1-\theta_h}$. This offer is accepted by the *b*-type and the *h*-type and yields non-negative profits overall.¹¹

Candidate Equilibrium: Hybrid-2

Proposition 13 A pure strategy equilibrium wherein the lender screens out the b-types and bunches the *l-type is characterized as follows:*

(a) the lender offers contract (R_q^Y, C_q^Y) given by (40) and (41)

(b) h-types and l-types accept the offer (R_g^Y, C_g^Y) , but b-types reject this offer. (c) lender expected profits are $\Pi_2^Y = [\nu_h \{\theta_b(1-\theta_h) - \beta\theta_h(1-\theta_b)\} + \nu_l \{\theta_b(1-\theta_l) - \beta\theta_l(1-\theta_h)\}] \frac{\lambda}{\theta_b - \theta_l}$ $-[\nu_h \{\theta_l(1-\theta_h) - \beta\theta_h(1-\theta_h)\} + \nu_l \{\theta_b(1-\beta)\theta_l(1-\theta_l)\}] \frac{\{(1-\theta_b)x-V^0\}}{\theta_b - \theta_l} - (\nu_h + \nu_l)\rho$

Proof. First, as *l*-types accept the lender's offer Lemma 3 must hold. Also, because the lender screens out the *b*-types, Lemma 4 must hold. Therefore, $V_l(R, C) = V_l^0$ and $V^0 = V_b(R, C)$. Solving these two equations for (R, C), we get the lender's offers to be (R_g^Y, C_g^Y) , given by (40) and (41). Note that $V_h(R_g^Y, C_g^Y) > V_h^0$, and therefore, *h*-types accept this offer as well.

2 Competition between Informed and Uninformed Lender

We have solved the model using differences in exogenous payoff for h-type and l-type borrowers by varying the parameter λ . In this section, we show that the optimization problem of the previous section is similar to that of an uninformed lender (Lender-U) facing this adverse selection problem in competing with an informed lender (Lender-I). Lender-I has the information advantage, in that it has complete information over borrower types. As will be demostrated below, the parameter λ in the previous exercise is identical to the cost of funds for the informed lender, ρ^{I} .

In what follows, we solve the problem of competition between asymmetrically informed lenders by fixing the uninformed lender's cost of funds, ρ^U , and varying the informed lender's cost of funds, ρ^I (similar to variations in λ). Needless to say, borrowers accept offers from the uninformed lender only in situations where $\rho^{I} \geq \rho^{U}$. The list of equilibria are provided in Table A.3 (note the additional equilibria in Screen-3).

¹¹Note that, since $\lambda > \rho$, the lender can make offers with $R_h < \mathbb{R}_h$, but $(R_h^0, 0)$ maximizes lender profits by bunching.

Candidate	Profit	Customer types	Menu of contracts	Breakeven
equilibria		$accepting \ Lender-U$'s offer	offered by Lender-U	$cuto\!f\!f$
Screen-1	Π_1^S	h;l	$(R_h^U, C_h^U); (R_l^U, C_l^U)$	$\tilde{\rho}_{b,h}^S(\rho^U), \tilde{\rho}_{h,l}^S(\rho^U)$
Screen-2	Π_2^S	h	$(R_{h}^{U}, C_{h}^{U}); (R_{l}^{0}, C_{l}^{0})$	$\widetilde{ ho}^{S}_{b,h}(ho^{U})$
Screen-3	Π_S^3	l	(R_l^U, C_l^U)	$\widetilde{ ho}_{h,l}^S(ho^U)$
Pool-1	Π_1^P	(b,h,l)	$(\mathbb{R}_l^I, 0)$	$\widetilde{ ho}_1^P(ho^U, u_b, u_l)$
Pool-2	Π_2^P	(b,h)	$(\underline{R}_{h}^{I},0)$	$\widetilde{ ho}_2^P(ho^U, u_b, u_h)$
Hybrid-1	Π_1^Y	(b,h);l	$(R_{h}^{I}, 0); (R_{l}^{U}, C_{l}^{U})$	$\tilde{\rho}_1^P(\rho^U, \nu_b, \nu_h), \tilde{\rho}_{h,l}^S(\rho^U)$
Hybrid-2	Π_2^Y	(h,l)	(R_g^U, C_g^U)	$\tilde{\rho}^{Y}(\rho^{U},\nu_{h},\nu_{l}),$

TABLE 2. Lender-U's offers under different candidate equilibria

Lemma 14 For borrowers of type-b, the Lender-I denies credit. For borrowers of types g = h, l the Lender-I offers a contract from the set $Z_g^I(\rho^I) = \{(R_g^I, 0) : R_g^I \in [R_g^I(\rho^I), \bar{R}_g]\}$, where $R_g^I(\rho^I) = \frac{\rho^I}{1-\theta_g}$ and $\bar{R}_g = x - \frac{V^0}{1-\theta_g}$, g = h, l are the first-best (zero-collateral) minimum and maximum repayments, respectively.

Proof. Since $\pi_b^I < 0$, the Lender-*I* denies credit to the *b*-type. Lender-*I* knows borrower type, so we can focus our attention, without loss of generality, to either creditworthy type g = h, l. We first show that Lender-*I* will always offer zero-collateral contracts. We prove by contradiction. Suppose not, that is, there exists an offer with a positive collateral requirement from Lender-*U*, (R_g^1, C_g^1) , that yields non-negative profits, $\pi_g^I(R_g^1, C_g^1) \ge 0$. Consider offer (R_g^2, C_g^2) with $R_g^2 > R_g^1, C_g^2 < C_g^1$ such that $V_g(R_g^1, C_g^1) = V_g(R_g^2, C_g^2), g = h, l$. From $V_g(R_g^1, C_g^1) = V_g(R_g^2, C_g^2)$, it follows that $(1 - \theta_g)(R_g^2 - R_g^1) = \theta_g(C_g^1 - C_g^2)$. Therefore, $\pi_g(R_g^2, C_g^2) - \pi_g(R_g^1, C_g^1) = \theta_g(1 - \beta)(C_g^1 - C_g^2) > 0$. As long as $C_g^1 > 0$, Lender-*I* can reduce the collateral requirement to C_g^2 , with offer (R_g^2, C_g^2) which provides borrower *g* with the same payoff as (R_g^1, C_g^1) . All such deviations yield higher profits for Lender-*I*. Therefore, Lender-*I* will always choose a contract that sets its collateral requirement to zero. The first-best (zero-collateral) minimum repayment is obtained by setting $\pi_g^I = 0, g = h, l$. The first-best (zero-collateral) maximum is obtained by setting $V_g(R_g^I, 0) = V^0, g = h, l$.

2.1 Lender-U's offer in equilibria

Under competition, Lender-U has to compete with the best offers from its informed rival, Lender-I. These offers are \bar{V}_h^I and \bar{V}_l^I for the *h*-types and *l*-types respectively. Since, Lender-I denies *b*-types credit, it reservation payoff continues to be V^0 . Lender-U's optimization problem can be written as follows:¹²

$$\max \ \Pi^U \equiv \nu_b \pi^U_b + \nu_h \pi^U_h + \nu_l \pi^U_l, \tag{46}$$

where $\pi_k^U = (1 - \theta_k)R_k^U + \beta \theta_k C_k^U - \rho^U$, k = b, h, l subject to the following participation constraints

$$V_b(R_b, C_b) \leq V^0 \tag{47}$$

$$V_h(R_h, C_h) \geq \bar{V}_h^I \tag{48}$$

$$V_l(R_l, C_l) \geq \bar{V}_l^I \tag{49}$$

 $^{1^{2}}$ In this section, for the most part, we drop the superscript denoting offers by borrower U. Therefore, unless otherwise mentioned, we are considering profits and offers of Lender-U only. We will re-introduce superscripts below.

and the following incentive compatibility constraints

$$V_b(R_b, C_b) \geq V_b(R_h, C_h) \tag{50}$$

$$V_b(R_b, C_b) \geq V_b(R_l, C_l)$$

$$V_b(R_b, C_b) \geq V_b(R_l, C_l)$$
(51)

$$V_h(R_h, C_h) \geq V_h(R_b, C_b) \tag{52}$$

$$V_h(R_h, C_h) \geq V_h(R_h, C_h) \tag{53}$$

$$V_h(R_h, C_h) \geq V_h(R_l, C_l)$$

$$V_l(R_l, C_l) \geq V_l(R_k, C_k)$$
(53)

$$V_l(R_l, C_l) \geq V_l(R_b, C_b)$$

$$V_l(R_l, C_l) \geq V_l(R_b, C_b).$$

$$(54)$$

$$V_l(R_l, C_l) \geq V_l(R_h, C_h).$$
(55)

With $\pi_b^U < 0$, Lender-U does not offer (R_b, C_b) in equilibrium. Therefore, (47), (53) and (54) are redundant. Moreover, we replace $V_b(R_b, C_b)$ with the b-type's reservation utility V^0 in (50) and (51). Consequently, the above maximization problem in (46)-(55) reduces to

$$\max \ \Pi^U \equiv \nu_b \pi_b^U + \nu_h \pi_h^U + \nu_l \pi_l^U, \tag{56}$$

where $\pi_k^U = (1 - \theta_k)R_k^U + \beta \theta_k C_k^U - \rho^U$, k = b, h, l subject to the following participation constraints

$$V_h(R_h, C_h) \geq \bar{V}_h^I \tag{57}$$

$$V_l(R_l, C_l) \geq V_l^{I} \tag{58}$$

and the following incentive compatibility constraints

$$V^0 \geq V_b(R_h, C_h) \tag{59}$$

$$V^0 \geq V_b(R_l, C_l) \tag{60}$$

$$V_h(R_h, C_h) \geq V_h(R_l, C_l) \tag{61}$$

$$V_l(R_l, C_l) \geq V_l(R_h, C_h). \tag{62}$$

Lemma 15 Lender-U's expected profits from loans to l-types are always non-negative, $\pi_l^U(R_l, C_l) \ge 0$.

Proof. Suppose not, that is there exist equilibria in which $\pi_l^U(R_l, C_l) < 0$. With $R \ge C$, it follows that for any (R_l, C_l) we have $\pi_b^U(R_l, C_l) < \pi_h^U(R_l, C_l) < \pi_l^U(R_l, C_l) < 0$. Lender-U can always drop this contract and increase profits. Therefore, in any equilibrium, it is always the case that $\pi_l^U(R_l, C_l) \ge 0$.

Lemma 16 In any equilibirum offer by Lender-U, the IR constraint of the l-type, (58), must bind.

Proof. Case 1: Lender-U does not sort h-type from l-type

We prove by contradiction. Suppose not, that is, there exists a solution to (56)- $(60)^{13}$ characterized by (R_g^1, C_g^1) such that $V_l(R_g^1, C_g^1) > \bar{V}_l^I$. We will show that there exists an offer (R_g^2, C_g^2) such that it satisfies (58) but yields higher profit. We begin by characterizing this contract. Consider contract (R_g^2, C_g^2) , where $R_g^2 > R_g^1$, $C_g^2 < C_g^1$ such that

$$V_b(R_g^1, C_g^1) = V_b(R_g^2, C_g^2)$$
(63)

It follows that

$$V_h(R_g^2, C_g^2) < V_h(R_g^1, C_g^1)$$

$$V_l(R_g^2, C_g^2) < V_l(R_g^1, C_g^1)$$
(64)

 $^{^{13}}$ Since Lender-U does not sort, h-type and l-types are bunched with a single offer. Consequently, (16) and (17) are trivially satisfied.

Following the last inequality, we focus our attention to (R_q^2, C_q^2) such that

$$V_l(R_g^1, C_g^1) > V_l(R_g^2, C_g^2) \ge \bar{V}_l^I$$
(65)

Since (R_q^1, C_q^1) is a solution, it satisfies all the constraints. Consequently, we can show that offer (R_q^2, C_q^2) in (63) and (65) satisfies all other constraints as well. Constraints (58), (59) and (60) are satisfied by construction.¹⁴

It remains to be shown that (57) is satisfied. We prove by contradiction. Suppose not, then $V_h(R_g^1, C_g^1) \geq \bar{V}_h^I > V_h(R_g^2, C_g^2)$. It follows that $\rho^I < (1 - \theta_h)R_g^2 + \theta_h C_g^2$. From (65), we obtain $\rho^I \geq (1 - \theta_l)R_g^2 + \theta_l C_g^2$. Combining the two, we have $(1 - \theta_h)R_g^2 + \theta_h C_g^2 > (1 - \theta_l)R_g^2 + \theta_l C_g^2$ or $(\theta_h - \theta_l)(C_g^2 - R_g^2) > 0$.

Since $\theta_h > \theta_l$, it must be the case that $C_g^2 > R_g^2$. This is impossible given our initial condition 3, so that all lender offers to creditworthy types have the property $R \geq C$. We have a contradiction. It must be the case that (57) is satisfied.

Finally, since $R_g^2 > R_g^1$, $C_g^2 < C_g^1$, we have using (63) that $\pi_h(R_g^2, C_g^2) > \pi_h(R_g^1, C_g^1)$. Also, $\pi_l(R_g^2, C_g^2) > \pi_l(R_g^1, C_g^1)$. It follows that $\Pi(R_g^2, C_g^2) > \Pi(R_g^1, C_g^1)$. Therefore, (R_g^1, C_g^1) cannot be an equilibrium because Lender-U can offer an alternative contract of the form (R_g^2, C_g^2) and increase profits. Such profitable deviations are not possible only if $V_l(R_q, C_q) = \bar{V}_l^I$. Therefore (58) must bind in equilibrium.

Case 2: when Lender-U sorts creditworthy types.

We prove by contradiction. Suppose not, that is, there exists a solution to (56)-(62) characterized by $\{(R_h, C_h), (R_l^1, C_l^1)\}$ such that $V_l(R_l^1, C_l^1) > \overline{V}_l^I$. We will show that there exist a menu $\{(R_h, C_h), (R_l^2, C_l^2)\}$ such that it satisfies (58) but yields higher profit. We begin by characterizing this contract. Consider contract (R_l^2, C_l^2) , where $R_l^2 > R_l^1$, $C_l^2 < C_l^1$ such that

$$V_b(R_l^1, C_l^1) = V_b(R_l^2, C_l^2)$$
(66)

This implies

$$V_h(R_l^2, C_l^2) < V_h(R_l^1, C_l^1)
 V_l(R_l^2, C_l^2) < V_l(R_l^1, C_l^1)$$
(67)

Following this, we can choose (R_l^2, C_l^2) such that either of the following are true

$$V_l(R_l^1, C_l^1) > V_l(R_l^2, C_l^2) \ge V_l(R_h, C_h) \ge \bar{V}_l^I$$
 (68)

$$V_l(R_l^1, C_l^1) > V_l(R_l^2, C_l^2) \ge \bar{V}_l^I > V_l(R_h, C_h)$$
(69)

First, we show that the menu $\{(R_h, C_h), (R_l^2, C_l^2)\}$ in (66) and either (68) or (69) satisfies all other constraints as well. Constraints (57) and (59) are trivially satisfied. Constraint (58), (60) and (62) are satisfied by construction. Lastly, (61) is satisfied because $V_h(R_h, C_h) \geq V_h(R_l^1, C_l^1) > V_h(R_l^2, C_l^2)$.

Next, we show that by offering (R_l^2, C_l^2) instead of (R_l^1, C_l^1) , Lender-U increases profits. Since $R_l^2 > R_l^1, C_l^2 < C_l^1$, we have using $(66)^{15}, \pi_l(R_l^2, C_l^2) > \pi_l(R_l^1, C_l^1)$. Therefore, menu $\{(R_h, C_h), (R_l^1, C_l^1)\}$ cannot be an equilibrium because Lender-U can offer an alternative contract of the form $\{(R_h, C_h), (R_l^2, C_l^2)\}$ and increase profits.

Note that this holds for either (68) or (69). For (68), all deviations from (R_l^1, C_l^1) to (R_l^2, C_l^2) yield higher profits, unless $V_l(R_l, C_l) = V_l(R_h, C_h)$; in which case, Lender-U bunches h-types and l-types and we follow the proof as given in Case 1 above. Alternatively, for (69) such profitable deviations are not possible only if $V_l(R_l, C_l) = \overline{V}_l^I$. Therefore (58) must bind in equilibrium.

¹⁴Since Lender-U has only one offer for all creditworthy types, (14) and (15) are the same constraint. ¹⁵This follows from $\pi_l(R_l^2, C_l^2) - \pi_l(R_l^1, C_l^1) = \frac{\theta_b(1-\theta_l) - \beta\theta_l(1-\theta_b)}{(1-\theta_b)}(C_l^1 - C_l^2)$ and $\theta_b(1-\theta_l) - \beta\theta_l(1-\theta_b) > (1-\beta)\theta_l(1-\theta_b)$ θ_b) > 0.

Lemma 17 If Lender-U screens out the b-type, the IC constraint of the b-type w.r.t the h-type, (59), must bind.

Proof. Case 1: when Lender-U sorts creditworthy types.

We prove by contradiction. Suppose not, that is, there exists a solution to (56)-(62) characterized by $\{(R_h^1, C_h^1), (R_l, C_l)\}$ such that $V^0 > V_b(R_h^1, C_h^1)$. We will show that there exist menus such as $\{(R_h^2, C_h^2), (R_l, C_l)\}$ that satisfy (59) all the other constraints but yield higher profits. We begin by characterizing such contracts. Consider contract (R_h^2, C_h^2) , where $R_h^2 > R_h^1, C_h^2 < C_h^1$ such that

$$V_h(R_h^1, C_h^1) = V_h(R_h^2, C_h^2).$$
(70)

It follows that

$$V_{l}(R_{h}^{2}, C_{h}^{2}) < V_{l}(R_{h}^{1}, C_{h}^{1})$$

$$V_{b}(R_{h}^{2}, C_{h}^{2}) > V_{b}(R_{h}^{1}, C_{h}^{1})$$
(71)

Therefore, if $V^0 > V_b(R_h^1, C_h^1)$, we can choose (R_h^2, C_h^2) so that $V^0 \ge V_b(R_h^2, C_h^2) > V_b(R_h^1, C_h^1)$. Since $\{(R_h^1, C_h^1), (R_l, C_l)\}$ is a solution, it satisfies all the constraints. Consequently, we can show that menus $\{(R_h^2, C_h^2), (R_l, C_l)\}$ satisfy all the constraints as well. Constraints (58) and (60) are trivially satisfied. Constraint (57), (60) and (61) are satisfied by construction. Lastly, (62) is satisfied because $V_l(R_l, C_l) \ge V_l(R_h^1, C_h^1) > V_l(R_h^2, C_h^2)$.

But, since $R_h^2 > R_h^1$ and $C_h^2 < C_h^1$, we use (70) to obtain $\pi_h(R_h^2, C_h^2) > \pi_h(R_h^1, C_h^1)$. Therefore, $\{(R_h^1, C_h^1), (R_l, C_l)\}$ cannot be an equilibrium because Lender-U can offer an alternative contract of the form $\{(R_h^2, C_h^2), (R_l, C_l)\}$ and increase profits. Such profitable deviations are not possible only if $V^0 = V_b(R_h, C_h)$. Therefore (59) must bind in equilibrium.

Case 2: Lender-U does not sort h-type from l-type

This holds for either creditworthy type, yielding two sets of equilibria where Lender-U screens out just the b-type. The first occurs when g = h, and Lender-U captures only the h-type by screening them from the b-type, as described in the candidate equilibria Screen-2. The second occurs when g = l and Lender-U captures both h- and l-types by bunching them and screening them from the b-type, as described in the candidate equilibria them and screening them from the b-type, as described in the candidate equilibria them and screening them from the b-type, as described in the candidate equilibria them and screening them from the b-type, as described in the candidate equilibria them and screening them from the b-type, as described in the candidate equilibria them and screening them from the b-type, as described in the candidate equilibria them and screening them from the b-type, as described in the candidate equilibria them and screening them from the b-type, as described in the candidate equilibria them and screening them from the b-type, as described in the candidate equilibria them and screening them from the b-type, as described in the candidate equilibria them and screening them from the b-type.

We prove by contradiction for Hybrid-2. Suppose not, that is, there exists a solution to (56)- $(60)^{16}$ characterized by (R_g^1, C_g^1) , where (R_g^1, C_g^1) is the offer to both the *h*-type and the *l*-type, and $V^0 > V_b(R_g^1, C_g^1)$. Note that one can find an alternative contract (R_g^2, C_g^2) with $R_g^2 > R_g^1$ and $C_g^2 < C_g^1$, such that

$$V_l(R_g^1, C_g^1) = V_l(R_g^2, C_g^2).$$
(72)

It follows that

$$V_h(R_g^2, C_g^2) > V_h(R_g^1, C_g^1)$$
and $V_b(R_g^2, C_g^2) > V_b(R_g^1, C_g^1).$
(73)

Following this, we can restrict our attention to contracts such that

$$V^{0} \ge V_{b}(R_{g}^{2}, C_{g}^{2}) > V_{b}(R_{g}^{1}, C_{g}^{1}).$$
(74)

Since (R_g^1, C_g^1) is a solution, it satisfies all the constraints. Consequently, we can show that offer (R_g^2, C_g^2) in (72) and (74) satisfies all other constraints as well. Constraints (58), (59) and (60) are satisfied by construction.¹⁷ Lastly, (57) is satisfied from (73).

 $^{^{16}}$ Since Lender-U does not sort, h-type and l-types are bunched with a single offer. Consequently, (16) and (17) are trivially satisfied.

 $^{^{17}}$ Since Lender-U has only one offer for all creditworthy types, (16) and (17) are the same constraint.

In offering (R_g^2, C_g^2) instead of (R_g^1, C_g^1) , Lender-U's change in profits from the *l*-type and *h*-type are given by

$$\begin{aligned} \Delta \pi_l^U &= (1-\beta)\theta_l (C_g^1 - C_g^2) > 0\\ \Delta \pi_h^U &= \frac{\theta_l (1-\theta_h) - \beta \theta_h (1-\theta_l)}{(1-\theta_l)} (C_g^1 - C_g^2) \end{aligned}$$

Since bunching is only feasible for $\nu_l \geq \nu_h$ and we have $(1-\beta)\theta_l > \frac{(1-\beta)\theta_l(1-\theta_h)}{(1-\theta_l)} > \frac{\theta_l(1-\theta_h)-\beta\theta_h(1-\theta_l)}{(1-\theta_l)}$. Therefore Lender-U's offer of (R_g^2, C_g^2) to both h-type and l-types yields higher profits than (R_g^1, C_g^1) . That is, $\Pi(R_g^2, C_g^2) > \Pi(R_g^1, C_g^1)$. This implies that (R_g^1, C_g^1) cannot be an equilibrium. Such deviations are no longer possible if $V_b(R_g, C_g) = V^0$. Proceeding exactly as above, we can prove the same for candidate equilibria Screen-2, where Lender-U's offers are accepted by the h-types only.

Lemma 18 In any equilibrium wherein Lender-U sorts the h-types from the l-type, the IC constraint of the h-type w.r.t the l-type, (61), must bind.

Proof. We prove by contradiction. Suppose not, that is, there exists a solution to (56)-(62) characterized by $\{(R_h, C_h), (R_l^1, C_l^1)\}$ such that $V_h(R_h, C_h) > V_h(R_l^1, C_l^1)$. We will show that there exist a menus of contracts $\{(R_h, C_h), (R_l^2, C_l^2)\}$ such that it satisfies (61) and all the other constraints but yields higher profit. We begin by characterizing this contract. Consider contract (R_l^2, C_l^2) , where $R_l^2 > R_l^1$, $C_l^2 < C_l^1$ such that

$$V_l(R_l^1, C_l^1) = V_l(R_l^2, C_l^2) = \bar{V}_l^I$$
(75)

It follows that $V_h(R_l^2, C_l^2) > V_h(R_l^1, C_l^1)$ and $V_b(R_l^2, C_l^2) > V_b(R_l^1, C_l^1)$. Therefore, if $V_h(R_l^2, C_l^2) > V_h(R_l^1, C_l^1)$, we can choose (R_l^2, C_l^2) so that

$$V_h(R_h, C_h) \ge V_h(R_l^2, C_l^2) > V_h(R_l^1, C_l^1)$$
(76)

Since $\{(R_h, C_h), (R_l^1, C_l^1)\}$ is a solution, it satisfies all the constraints. Consequently, we can show that the menu $\{(R_h, C_h), (R_l^2, C_l^2)\}$ in (75) and (76) satisfies all the constraints as well. Constraints (57) and (59) are trivially satisfied. Constraint (58), (61) and (62) are satisfied by construction.

For (60), we prove by contradiction. Suppose not, then $V_b(R_l^2, C_l^2) > V^0$. In addition, Lemma 4 implies $V_b(R_l^2, C_l^2) > V^0 = V_b(R_h, C_h)$. That is, $(1 - \theta_b)(R_h - R_l^2) > \theta_b(C_l^2 - C_h)$. Also, because (76) holds, it follows that $\theta_h(C_l^2 - C_h) \ge (1 - \theta_h)(R_h - R_l^2)$. Combining both, $(\frac{1 - \theta_b}{\theta_b})(R_h - R_l^2) > (C_l^2 - C_h) \ge (\frac{1 - \theta_h}{\theta_b})(R_h - R_l^2) = 0$. Since the first expression is negative, this would imply $R_l^2 > R_h$. Similarly we can show that $C_l^2 < C_h$. However, since (62) is satisfied, we get $(1 - \theta_l)(R_h - R_l^2) > \theta_l(C_l^2 - C_h)$. Again, using (76) it follows that $\theta_h(C_l^2 - C_h) \ge (1 - \theta_h)(R_h - R_l^2)$. Combining both, $(\frac{1 - \theta_l}{\theta_l})(R_h - R_l^2) > (C_l^2 - C_h) \ge (C_l^2 - C_h) \ge (\frac{1 - \theta_h}{\theta_h})(R_h - R_l^2)$ or $(\frac{1 - \theta_l}{\theta_l} - \frac{1 - \theta_h}{\theta_h})(R_h - R_l^2) > 0$. Now, since the first expression is positive, this would imply $R_l^2 < R_h$. Similarly we can show that $C_l^2 < C_h$. Similarly we can show that $C_l^2 - C_h \ge (1 - \theta_h)(R_h - R_l^2) > 0$. Now, since the first expression is positive, this would imply $R_l^2 < R_h$. Similarly we can show that $C_l^2 < C_h$. However, $(1 - \theta_h)(R_h - R_l^2) > 0$. Now, since the first expression is positive, this would imply $R_l^2 < R_h$. Similarly we can show that $C_l^2 > C_h$.

Since $R_l^2 > R_l^1, C_l^2 < C_l^1$, using (75), we get $\pi_l(R_l^2, C_l^2) > \pi_l(R_l^1, C_l^1)$. Therefore, menu $\{(R_h, C_h), (R_l^1, C_l^1)\}$ cannot be an equilibrium because Lender-U can offer an alternative contract of the form $\{(R_h, C_h), (R_l^2, C_l^2)\}$ and increase profits. Such profitable deviations are not possible only if $V_h(R_h, C_h) = V_h(R_l, C_l)$. Therefore (61) must bind in equilibrium.

2.2 Lender-U's offers in screening equilibria

Consequently, the above maximization problem in (46)-(55) reduces to

$$\max \Pi^U \equiv \nu_h \pi_h^U + \nu_l \pi_l^U, \tag{77}$$

where $\pi_k^U = (1 - \theta_k) R_k^U + \beta \theta_k C_k^U - \rho^U$, subject to the following participation constraints

$$V_h(R_h, C_h) \geq \bar{V}_h^I \tag{78}$$

$$V_l(R_l, C_l) = \overline{V}_l^I \tag{79}$$

and the following incentive compatibility constraints

$$V^0 = V_b(R_h, C_h) \tag{80}$$

$$V^0 \geq V_b(R_l, C_l) \tag{81}$$

$$V_h(R_h, C_h) = V_h(R_l, C_l)$$
(82)

$$V_l(R_l, C_l) \geq V_l(R_h, C_h). \tag{83}$$

Using the equations (78)-(83), we obtain expressions for (R_h, C_h) and (R_l, C_l) as follows:

$$\begin{array}{rcl} R_g^U & \geq & R_l \geq R_l^U, \\ R_h^U & \geq & R_h \geq R_g^U \\ C_g^U & \leq & C_l \leq C_l^U \\ C_h^U & \leq & C_h \leq C_g^U \end{array}$$

where

$$R_l^U = C_l^U = \rho^I, \tag{84}$$

$$R_g^U = \frac{\theta_b}{\theta_b - \theta_l} \rho^I - \frac{\theta_l}{\theta_b - \theta_l} [(1 - \theta_b)x - V^0]$$
(85)

$$C_g^U = -\frac{1-\theta_b}{\theta_b-\theta_l}\rho^I + \frac{1-\theta_l}{\theta_b-\theta_l}[(1-\theta_b)x - V^0]$$
(86)

$$R_{h}^{U} = \frac{\theta_{b}}{\theta_{b} - \theta_{h}} \rho^{I} - \frac{\theta_{h}}{\theta_{b} - \theta_{h}} [(1 - \theta_{b})x - V^{0}]$$
(87)

$$C_h^U = -\frac{1-\theta_b}{\theta_b-\theta_h}\rho^I + \frac{1-\theta_h}{\theta_b-\theta_h}[(1-\theta_b)x - V^0].$$
(88)

Using conditions 1, 2 we can show that all of the offers given by (84)-(88) are strictly positive. From (78)-(83), note that if the offer to *h*-types is $(R_h, C_h) = (R_h^U, C_h^U)$, then the offer to *l*-types is $(R_l, C_l) = (R_l^U, C_l^U)$ and vice-versa. Likewise, $(R_h, C_h) = (R_l, C_l) = (R_g^U, C_g^U)$ is an offer satisfying (78)-(83) in which Lender-U bunches both *h*-types and *l*-types.

Lemma 19 In any equilibrium wherein Lender-U sorts the h-types from the l-type, the IR constraint of the h-type, (78) must bind.

Proof. First note that $V_h(R_h^U, C_h^U) = \overline{V}_h^I$. Also lemmas 16-18 hold.

We prove by contradiction. Suppose not. That is, there exists a menu $\{(R_h^2, C_h^2), (R_l^2, C_l^2)\}$ which satifies (78)-(83) where $R_h^2 < R_h^U$ and $C_h^2 > C_h^U$ such that $V_b(R_h^2, C_h^2) = V_b(R_h^U, C_h^U)$. It follows that $V_h(R_h^2, C_h^2) > V_h(R_h^U, C_h^U) = \bar{V}_h^I$ or (78) is slack. Also, from (79) and (82) it follows that $R_l^2 > R_l^U$, $C_l^2 < C_l^U$. Therefore, replacing $\{(R_h^U, C_h^U), (R_l^U, C_l^U)\}$ by $\{(R_h^2, C_h^2), (R_l^2, C_l^2)\}$ would increase profits from the *l*-types but decrease profits from the *h*-types as follows:

$$\begin{aligned} \Delta \pi_l^U &= (1-\beta)\theta_l \frac{(1-\theta_l)}{(1-\theta_b)} (\frac{\theta_b - \theta_h}{\theta_h - \theta_l}) (C_h^2 - C_h^U) > 0 \\ \Delta \pi_h^U &= -\frac{\theta_b (1-\theta_h) - \beta \theta_h (1-\theta_b)}{(1-\theta_b)} (C_h^2 - C_h^U) < 0 \end{aligned}$$

For given values of ν_h and ν_l , the effect on total profits Π^U is either monotonically increasing or monotonically decreasing with the magnitude of the deviation $(C_h^2 - C_h^U)$.¹⁸ In particular, if the effect on Π^U is positive, all deviations with menus of the type $\{(R_h^2, C_h^2), (R_l^2, C_l^2)\}$ yield higher profits than $\{(R_h^U, C_h^U), (R_l^U, C_l^U)\}$. That is, profits are maximized by offering (R_g^U, C_g^U) which does not induce separation. Therefore, in an equilibrium that induces separation of the h-types and l-types, (78) must bind.

2.3Candidate Equilibria with Competing Lenders

We list the candidate equilibria of the model below in Propositions 20-27. In what follows, we hold the value of Lender-U's lending costs fixed at ρ^U and vary the value of Lender-I's lending costs, ρ^I . Depending on the distribution of borrower types ν_h and ν_h , and the value of ρ^I , we can determine which of the following candidate equilibria will emerge as the final equilibrium of the model. Hereafter, we re-introduce the superscripts I and U for lenders' offers and profits. Also, we introduce the two threshold values of ρ^I

$$\tilde{\rho}_{h,l}^S(\rho^U) = \frac{1}{1 - (1 - \beta)\theta_l}\rho^U \tag{89}$$

$$\tilde{\rho}_{b,h}^{S}(\rho^{U}) = \frac{\theta_{b} - \theta_{h}}{\theta_{b}(1 - \theta_{h}) - \beta\theta_{h}(1 - \theta_{b})}\rho^{U} + \frac{(1 - \beta)\theta_{h}(1 - \theta_{h})}{\theta_{b}(1 - \theta_{h}) - \beta\theta_{h}(1 - \theta_{b})}[(1 - \theta_{b})x - V^{0}].$$
(90)

2.3.1Screening Equilibria

Candidate equilibrium: Screen-1 We can now state Lender-U's offers under Screen-1 in terms of the following proposition.

Proposition 20 If $\rho^I \ge \max(\tilde{\rho}_{h,l}^S, \tilde{\rho}_{b,h}^S)$, where $\tilde{\rho}_{h,l}^S$ and $\tilde{\rho}_{b,h}^S$ are given in (89) and (90), a pure strategy equilibrium wherein Lender-U sorts all borrower types is characterized as follows: (a) Lender-U offers menu $\{(R_h^U, C_h^U), (R_l^U, C_l^U)\}$ given by (84), (87) and (88). (b) Lender-I offers $(\underline{R}_h^I, 0)$ to h-types and $(\underline{R}_l^I, 0)$ to l-types.

(c) Both creditworthy types accept offers from Lender-U but b-types types reject offers from both lenders (d) Lender-U's expected profits are $\Pi_1^S = \nu_h [\frac{\theta_b(1-\theta_h)-\beta\theta_h(1-\theta_b)}{\theta_b-\theta_h}\rho^I - \frac{(1-\beta)\theta_h(1-\theta_h)}{\theta_b-\theta_h}\{(1-\theta_b)x - V^0\}] -$ $\nu_l [1 - (1 - \beta)\theta_l]\rho^I - (\nu_h + \nu_l)\rho^U$

Proof. In any screening equilibrium wherein Lender-U separates all borrower types, Lemmas 1-6 must hold. This implies that Lender-U offers menu $\{(R_h^U, C_h^U), (R_l^U, C_l^U)\}$. Moreover, in equilibrium, Lender-I gives the break-even contract offers to each type, that is $(\underline{\mathbf{R}}_{h}^{I}, 0)$ to h-types and $(\underline{\mathbf{R}}_{l}^{I}, 0)$ to *l*-types, which the borrowers reject. Finally, for Lender-*U* to screen the *l*-type from the *h*-type, profits $\pi_l^U(R_l^U, C_l^U) \ge 0$. This occurs when $\rho^I \ge \tilde{\rho}_{h,l}^S = \frac{1}{1-(1-\beta)\theta_l}\rho^U$. Similarly, Lender-*U* can screen the *h*-type from the *b*-type, if $\pi_h^U(R_h^U, C_h^U) \ge 0$. That is if $\rho^I \ge \tilde{\rho}_{b,h}^S$, where $\tilde{\rho}_{b,h}^S$ is given in (90).

In order to screen all borrower types, we must have $\rho^I > \max(\tilde{\rho}_{b,h}^S, \tilde{\rho}_{h,l}^S)$. However, if $\tilde{\rho}_{h,l}^S > \rho^I \ge \tilde{\rho}_{b,h}^S$ or $\tilde{\rho}_{b,h}^S > \rho^I \ge \tilde{\rho}_{h,l}^S$, Lender-U still has the option to just screen one creditworthy type, as given by the following propositions.

Candidate Equilibrium: Screen-2

Proposition 21 If $\tilde{\rho}_{h,l}^S > \rho^I \geq \tilde{\rho}_{b,h}^S$, where $\tilde{\rho}_{h,l}^S$ and $\tilde{\rho}_{b,h}^S$ are given in (89) and (90), a pure strategy equilibrium wherein Lender-U screens out the b-type and lends to the h-types only, is characterized as

¹⁸Strictly speaking, if $\Delta \Pi^U \equiv \nu_h \Delta \pi_l^U + \nu_l \Delta \pi_l^U = 0$, then all points satisfying (33)-(38) can be supported as equilbira. However, we restrict our attention to the screening offer given in lemma 6.

follows:

(a) Lender-U offers menu $\{(R_h^U, C_h^U); (R_l^0, C_l^0)\}$, with (R_h^U, C_h^U) given by (87) and (88) (b) Lender-I offers $(R_h^I, 0)$ to h-types and $(R_l^I, 0)$ to l-types so that $\pi_l^U(R_l^0, C_l^0) = 0$, $V_h(R_h^I, 0) =$ $V_h(R_l^0, C_l^0)$, and $V_l(R_l^0, C_l^0) = V_l(R_l^I, 0)$.

(c) The l-types accept offers from Lender-I, but h-types accepts offers from Lender-U. The b-types reject all offers.

(d) Lender-U's expected profits are $\Pi_2^S = \nu_h \left[\frac{\theta_b(1-\theta_h)-\beta\theta_h(1-\theta_b)}{\theta_b-\theta_h}\rho^I - \frac{(1-\beta)\theta_h(1-\theta_h)}{\theta_b-\theta_h}\{(1-\theta_b)x-V^0\}\right] - \nu_h \rho^U$

Proof. The maximization problem for Screen-2 is the same as that of Screen-1, except for the fact that now $\tilde{\rho}_{h,l}^S > \rho^I \ge \tilde{\rho}_{b,h}^S$. Therefore, in Screen-2, all the results of Screen-1 hold except for the lenders' offers to the *l*-type. This implies that (R_l^U, C_l^U) as given in (84), if offered, would yield negative profits for Lender-U. Also, so that h-types do not find the offer to the l-types attractive, any such offer would have to satisfy (82). Therefore, the best contract (highest borrower payoff) Lender-U can offer to the *l*-type is given by (R_l^0, C_l^0) , where $\pi_l^U(R_l^0, C_l^0) = 0$. For any such offer, (79) is not satisfied: $V_l(R_l^0, C_l^0) < \bar{V}_l^I$. This implies that Lender-I does not need to offer $(\mathbf{R}_l^I, 0)$ to retain l-types. Lender-I can offer $(\mathbf{R}_l^I, 0)$ instead, so that $V_l(R_l^0, C_l^0) = V_l(R_l^I, 0)$ and still retain the *l*-type with positive profits.

Candidate equilibrium: Screen-3

Proposition 22 If $\tilde{\rho}_{b,h}^S > \rho^I \ge \tilde{\rho}_{h,l}^S$ and $\tilde{\rho}^h \ge \rho^I \ge \tilde{\rho}_{h,l}^S$, where $\tilde{\rho}^h = \frac{1}{1-(1-\beta)\theta_h}\rho^U$, a pure strategy equilibrium wherein Lender-U separates only l-types is characterized as follows:

(a) Lender-U offers (R_l^U, C_l^U) given by (84). (b) Lender-I offers $(R_h^I, 0)$ to h-types and $(R_l^I, 0)$ to l-types, (c) The h-types accept offers from Lender-I, but l-types accepts offers from Lender-U. The b-types reject all offers.

(d) Lender-U's expected profits are $\Pi_3^S = \nu_l [1 - (1 - \beta)\theta_l] \rho^I - \nu_l \rho^U$

Proof. The maximization problem for Screen-3 is the same as that of Screen-1, except for the fact that now $\tilde{\rho}_{b,h}^S > \rho^I \geq \tilde{\rho}_{h,l}^S$. Lender-U can simply make and offer to (R_l^U, C_l^U) as given in (84). This implies that since $\tilde{\rho}_{b,h}^S > \rho^I \ge \tilde{\rho}_{h,l}^S$, (R_h^U, C_h^U) as given in (87)-(88), if offered, would yield negative profits for Lender-U. Therefore, Lender-U cannot screen out the b-types. Lender-I continues to offer $(\mathbb{R}_{h}^{I}, 0)$ as before. Note that while Lender-U cannot match this offer, Lender-I cannot raise R_h^I because the *h*-types are just indifferent between its offer of $(\mathbb{R}_{h}^{I}, 0)$ and Lender-U's offer of (R_{l}^{U}, C_{l}^{U}) . However, with $\tilde{\rho}_{b,h}^{S} > \rho^{I}$, $\pi_{h}(R_{l}^{U}, C_{l}^{U}) < \pi_{h}(R_{h}^{U}, C_{h}^{U}) < 0$ so that Lender-I's offers $(\mathbb{R}_{h}^{I}, 0)$ yield higher profits from h-types.

Equilibria with Lender-U pooling the b-type 2.3.2

When the proportion of the *b*-type is sufficiently small, Lender-U can choose to pool them with creditworthy types. There are two such types of equilibria; one, where b-types are pooled with the h-type and the other where all types are pooled together.

Lemma 23 In any equibrium where the Lender-U pools the bad-risk type, it offers a zero collateral contract.

Proof. The proof is by contradiction. Suppose not, that is, there exists a pooling equilibrium where Lender-U's offers (R_P^1, C_P^1) and pools all borrowers. Since *l*-types accept this contract, we must have from Lemma 3 that $V_l(R_P^1, C_P^1) = \bar{V}_l^I$. Consider alternative offer (R_P^2, C_P^2) with $R_P^2 > R_P^1$ and $C_P^2 < C_P^1$, such that $V_l(R_P^2, C_P^2) = V_l(R_P^1, C_P^1)$. It follows that $V_h(R_P^2, C_P^2) > V_h(R_P^1, C_P^1)$ and $V_b(R_P^2, C_P^2) >$ $V_b(R_P^1, C_P^1)$. Therefore, both b-types and h-types accept this new contract as it yields them higher

payoff. But (R_P^2, C_P^2) yields Lender-U higher profits than $(R_P^1, C_P^1)^{19}$ Proceeding just as in the cases above, this implies that in equilibrium where Lender-U pools all borrowers, $C_P = 0$. Note that in the same way, we can show that this result holds in an equilibrium where Lender-U pools just the h- and *b*-types, but not the *l*-types. \blacksquare

Candidate Equilibrium: Pool-1

Proposition 24 If $\rho^I \geq \tilde{\rho}_1^P \equiv (\frac{1-\theta}{1-\mathbb{E}(\theta)})\rho^U$, where $E(\theta) \equiv \nu_b \theta_b + \nu_h \theta_h + \nu_l \theta_l$, a pure strategy equilibrium wherein Lender-U pools all borrowers is characterized as follows:

- (a) Lender-U offers $(\underline{R}_{I}^{I}, 0)$.
- (b) Lender-I offers $(R_h^I, 0)$ to h-types and $(R_l^I, 0)$ to l-types. (c) All borrowers go to the Lender-U for loans.
- (d) Lender-U 's profits are $\Pi_1^P = (\frac{1-\mathbb{E}(\theta)}{1-\theta_i})\rho^I \rho^U$.

Proof. First, from lemma 10, it follows that the pooling offer is of the form $(R_P, 0)$. Since the pooling offer must yield non-negative profits, it must satisfy $[1 - \mathbb{E}(\theta)]R_P \geq \rho^U$, where $E(\theta)$ is the expected value of θ . This implies that pooling is feasible for contracts of the form $(R_P, 0)$ such that $R_P \ge \rho^U / [1 - E(\theta)]$. Third, for a pooling contract $(R_P, 0)$ to hold, the entrant has to ensure that *l*-types accept its offer. Therefore, it must be true that $R_P \le \mathbb{R}_l^I$, that is $\rho^I \ge (\frac{1 - \theta_l}{1 - \mathbb{E}(\theta)})\rho^U \equiv \tilde{\rho}_1^P$. Since increasing R_P increases profits, Lender-U offers $(\mathbf{R}_l^I, 0)$ such that the participation constraint of *l*-types just bind.

Candidate Equilibrium: Pool-2

Proposition 25 If $\rho^I \geq \tilde{\rho}_2^P \equiv (\frac{(\nu_b + \nu_h)(1 - \theta_h)}{\nu_b(1 - \theta_b) + \nu_h(1 - \theta_h)})\rho^U$, a pure strategy equilibrium wherein the lender pools h-types and b-types only, is characterized as follows: (a) Lender-U offers $(\underline{R}_{h}^{I}, 0)$.

(b) Lender-I offers $(R_{h}^{I}, 0)$ to h-types and $(R_{l}^{I}, 0)$ to l-types where $\pi_{l}^{U}(R_{l}^{0}, C_{l}^{0}) = 0$, $V_{l}(R_{l}^{0}, C_{l}^{0}) = V_{l}(R_{l}^{I}, 0)$ and $V_l(\underline{R}_h^I, 0) = V_l(R_l^0, C_l^0).$

(c) Both b-type and h-type borrowers accept Lender-U's offer, while l-types accept Lender-I's offer. (d) Lender-U's expected profits are $\Pi_2^P = \frac{\nu_b(1-\theta_b)+\nu_h(1-\theta_h)}{1-\theta_h}\rho^I - (\nu_b + \nu_h)\rho^U$

Proof. Following the same procedure as above, we know that the pooling offer must yield nonnegative profits. So it must satisfy $[\nu_b(1-\theta_b)+\nu_h(1-\theta_h)]R_P \geq (\nu_b+\nu_h)\rho$. Clearly, pooling is feasible for contracts of the form $(R_P, 0)$ such that $R_P \ge (\nu_b + \nu_h)\rho/[\nu_b(1-\theta_b) + \nu_h(1-\theta_h)]$. For this pooling contract $(R_P, 0)$ to hold, the entrant has to ensure that h-types accept its offer. Therefore, it must be true that $R_P \leq \mathbb{R}_h^I$, where $\mathbb{R}_h^I = \frac{\rho^I}{1-\theta_h}$. That is, $\rho^I \geq (\frac{(\nu_b+\nu_h)(1-\theta_h)}{\nu_b(1-\theta_b)+\nu_h(1-\theta_h)})\rho \equiv \hat{\lambda}_2^P$. Since increasing R_P increases profits, Lender-U offers $(\mathbb{R}_h^I, 0)$ such that the participation constraint of the h-types just bind. Likewise, Lender-I's offers $(R_l^I, 0)$ where $R_l^I \geq \bar{R}_l^I$ to l-types is exactly as given in Screen-2, and yields the best payoff that Lender-U could offer them.

2.3.3Hybrid Equilibria

Hybrid equilibria has elements of pooling and screening. This occurs when Lender-U pools or bunches adjacent types but screens the non-adjacent types. The hybrid equilibria can be described as follows.

¹⁹One can show this proceeding in a similar way as in Case 2 for Lemma 4. Also note that pooling is feasible only if $\nu_l \ge \nu_b + \nu_h.$

Candidate Equilibrium: Hybrid-1

Proposition 26 If $\tilde{\rho}_{b,h}^S > \rho^I \ge \tilde{\rho}_{h,l}^S$ and $\rho^I \ge \tilde{\rho}_2^P$, a pure strategy equilibrium wherein Lender-U separates only the l-types and bunches (pools) b-types and h-types is characterized as follows:

(a) Lender-U offers menu $\{(\underline{R}_{h}^{I}, 0); (R_{l}^{U}, C_{l}^{U})\}$ given by (84).

(b) Lender-I offers $(\underline{R}_{h}^{I}, 0)$ to h-types and $(\underline{R}_{l}^{I}, 0)$ to l-types. (c) The l-types accept offer $(\underline{R}_{l}^{U}, C_{l}^{U})$ from Lender-U. The b-types and h-types accept Lender-U's offer $(R_{h}^{I}, 0).$

(d) Lender-U's expected profits are $\Pi_1^Y = \frac{\nu_b(1-\theta_b)+\nu_h(1-\theta_h)}{1-\theta_h}\rho^I + \nu_l[1-(1-\beta)\theta_l]\rho^I - \rho^U$

Proof. Since $\tilde{\rho}_{b,h}^S > \rho^I \ge \tilde{\rho}_{h,l}^S$, Lender-U cannot sort the h-types from b-types but can sort l-types from h-types. Lender-U offers (R_l^U, C_l^U) as given in (84). Just as in Screen-1, this is accepted by l-types and rejected by b-types and the h-types. Also, as was the case for Pool-2, for a sufficiently low ν_b and $\rho^{I} \geq \tilde{\rho}_{2}^{P}$, Lender-U offers ($\underline{\mathbf{R}}_{h}^{I}, 0$), which is accepted by the b-type and the h-type and yields non-negative profits.²⁰ \blacksquare

Candidate Equilibrium: Hybrid-2

Proposition 27 A pure strategy equilibrium wherein Lender-U screens out the b-types and bunches the *l-types and h-types is characterized as follows:*

(a) Lender-U offers menu (R_g^U, C_g^U) given by (85) and (86) (b) Lender-I offers $(\underline{R}_h^I, 0)$ to h-types and $(\underline{R}_l^I, 0)$ to l-types. (c) The h-type and the l-type accept the offer (R_g^U, C_g^U) . The b-types reject this offer.

(d) Lender-U's expected profits are $\Pi_2^Y = \left[\nu_h \{\theta_b(1-\theta_h) - \beta \theta_h(1-\theta_b)\} + \nu_l \{\theta_b(1-\theta_l) - \beta \theta_l(1-\theta_h)\}\right] \frac{\rho^I}{\theta_b - \theta_l} - \left[\nu_h \{\theta_l(1-\theta_h) - \beta \theta_h(1-\theta_h)\} + \nu_l \{\theta_b(1-\beta) \theta_l(1-\theta_l)\}\right] \frac{\{(1-\theta_b)x - V^0\}}{\theta_b - \theta_l} - (\nu_h + \nu_l)\rho^U$

Proof. First, as *l*-types accept Lender-U's offer Lemma 3 must hold. Also, because Lender-U screens out the *b*-types, Lemma 4 must hold. Therefore, $V_l(R, C) = \bar{V}_l^I$ and $V^0 = V_b(R, C)$. Solving these two equations for (R, C), we get Lender-U's offers to be (R_g^U, C_g^U) . Note that $V_h(R_g^U, C_g^U) > \bar{V}_h^I$, and therefore, h-types borrow from Lender-U.

²⁰Note that, since $\rho^I > \rho^U$, Lender-U can make offers with $R_h^I < \mathbb{R}_h^I$, but $(\mathbb{R}_h^I, 0)$ maximizes the Lender-U's profits from this bunching.