Dynamic Monetary and Fiscal Policy Games under Adaptive Learning *

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Abstract

Monetary and fiscal policy games have often been modelled with the assumption of rational agents in spite of growing criticism for it in the literature. In this paper we relax this assumption and analyse different monetary and fiscal policy games (Nash, Stackelberg & Cooperation) under the assumption of adaptive learning (AL) agents. These agents update their beliefs as new data become available, and are bounded rationally. On calibrating the model, AL expectations is found not to converge to rational expectations (RE) even in the long run (150 periods). Rather it stays around the vicinity of the RE equilibrium. Stackelberg game in which monetary policy leads, adds least to the losses accruing to both the monetary and fiscal authorities. It is found to be the best performing interaction game in terms of anchoring AL inflation expectations to RE.

JEL-Classification: E52, E62

Keywords: Monetary policy, Fiscal policy, Strategic games, Adaptive learning

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1 Introduction

Expectations play a key role in macroeconomic modeling. Macro-policies are formulated considering economic agents' perception about the future, under the assumption that they are rational. About 30 years ago policies were formulated under the assumption that agents have adaptive expectations, which implied that they made systematic errors. However, Muth (1961) proposed Rational Expectations (RE) which undoubtedly revolutionized the way economists modeled expectations. Seminal work of Robert E. Lucas Jr., Stanley Fischer and others made RE more popular in the literature.

Policy formulation has become quite complex over the years owing to the lack of consensus in the literature. Though fiscal and monetary authorities conduct policies differently based on their experience and ideology, they unanimously agree that expectations play a key role in the evolution of macroeconomic variables. Sargent and Wallace (1981) highlighted the inability of monetary authorities to fool the public. If monetary authority tries to achieve objectives by fooling the public then economy would end up in a sub-optimal equilibrium with higher level of inflation. This only serves to highlight the importance of understanding public perception. The importance of inflationary expectations is such that these days monetary authorities conduct their own surveys to get some forward looking information about inflationary expectations.

The assumption of rational agents is in-built in most of the macro-economic models. Such an agent forms expectations using all the available information at a time and does not make systematic errors. In brief, RE theorizes that individual expectations of specific events in the future may be erroneous but on an average they are correct. It assumes that individual expectations are not systematically biased and that individuals use all the relevant information in reaching a decision on the best course for their economic future without bias. However, the assumption of rational agents is not foolproof. In the real world, people make decisions under uncertainty whereas RE demands agents to be extremely knowledgeable. It also assumes that a market or the economy as a whole has only one equilibrium point but a complex system can have many equilibrium points, several of which can be small points within highly unstable regions. As economists we tend to choose those points which are stable and unique, and ignore the rest.

Because of these shortcomings of RE, it is now being looked at with growing skepticism. Scrutinizing this assumption gives rise to a few questions - If agents are not rational, then what? How do economic agents form their beliefs then? Do we have any alternative?

One alternative is to assume limited knowledge, which implies that as time goes by and new data become available the agent changes its forecasts accordingly. This type of expectations is called adaptive learning (AL). Under AL, agents are assumed to be very close to having rational expectations i.e. the agents know the reduced form equations of the model but they do not know the parameters of these equations, which they must learn over the period of time. In its core, AL is a small step away from assuming RE. Agents form their expectations by running regressions every period as new data become available. In the limit these agents converge to rational agents.

AL is quite different from Adaptive Expectations (AE). In crude terms AL is a convex combination of AE and RE. Under AE, agents make systematic errors and do not update their forecasts. Whereas, under AL agents do not make systematic errors, update their forecasts regularly and are close to rationality.

Figure 1 illustrates where agents with different expectations stand on a unit line, with respect to each other.

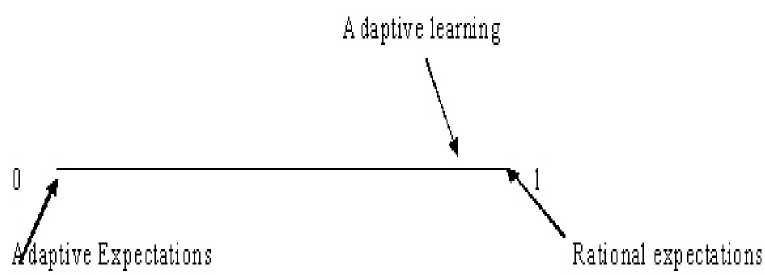

Figure 1: Comparison between AE, AL and RE

Why is AL important?

- In theory we assume that agents have RE but how did they come to possess such expectations?
- As discussed above, by assuming RE we might end up with multiple equilibria. AL offers a device of selecting stable candidate out of these multiple equilibria. Such equilibria are

called 'Expectationally stable'. We should have more confidence in such equilibria then those Rational expectations equilibrium (REE) which are unstable under AL dynamics.

- AL dynamics are found to be more empirically robust. Orphanides and Williams (2004) and Milani (2005) find results in support of AL expectations.

Most of the policy games modelled between monetary and fiscal authorities are based on the assumption that economic agents have RE; despite existing criticism of the RE literature.

Behavioural economics has very often reported the irrationality of consumer behaviour and nature of expectation formulation. In this paper, we study the interaction between monetary and fiscal policy in a game theoretic framework (Cooperation, Nash, Stackelberg (monetary/ fiscal leadership)) explicitly allowing for different assumptions regarding expectations formulation. The model is calibrated to see the key differences in the evolution of important macroeconomic variables and also to understand under which policy game, the AL expectations rapidly converges to RE equilibrium. This paper is a step towards understanding the complex procedure of expectations formation and its effect on monetary and fiscal policy games.

This paper seeks to answer the following:

1. Do policy prescriptions change when agents are assumed to be adaptive learners?
2. Which M-F game performs better in terms of convergence of AL to RE?
3. How do other macro-economic series behave in each kind of game? Whether their volatility changes with the assumption of AL agents? Which policy game performs the best in terms of least volatility of the macro-series?

The paper is structured as follows: Section II reviews the literature on monetary-fiscal policy games, section III discusses the model and M-F games, section IV presents parameter values for calibration, section V analyses the results, section VI compares the results to RE literature, section VII lists out caveats as well as scope for future work and section VIII concludes.

2 Literature review

Games between monetary and fiscal policy have traditionally been generalised as a game of chicken. Wherein it depends on who gives in first and accommodates the other. In the past, monetary authority has been the one that has most often given in to the fiscal pressures. But with greater importance granted to independent monetary policy and stress on rule-based policies, the policy game has changed. In literature, different games between these policies have been extensively studied and debated. Since most of the papers use different models, below we just concentrate on their policy prescriptions rather than the models used, for brevity.

Nordhaus et. al. (1994) find that concentrating on one authority while taking other authority's as given, seriously undermines the policy outcomes. Thus strongly putting the case forward to consider both the policies together. Dixit and Lambertini (2003) (DL hereafter) study fiscal-monetary policy interactions when the monetary authority is more conservative than the fiscal authority. They find Nash equilibrium to be suboptimal and fiscal leadership to be generally better. Lambertini and Rovelli (2004) argue that both the authorities would want to be the second mover in a stackelberg situation where one policy maker pre-commits to a policy rule. They conclude that fiscal authority should adopt a fiscal policy rule based on minimization of a loss function which internalises the objective of price stability. Although the results of these two papers are similar, both had different underlying models. Hallett et. al. (2009) re-examine Rogoff (1985) by introducing growth rate of central government debt in the output equation and conclude that response of conservative central bank may be quite different in such a case. Bartolomeo et. al. (2009) extend the well known model of DL (2003) by including multiplicative uncertainty into the model, which arises because of various coefficients in the model. They argue that under multiplicative uncertainty, achievement of common target by both the authorities may not be feasible because of the time in-consistency problem. Both the authorities can overcome this problem only if they choose their target levels equal to natural levels. Bilbiie (2003) discusses the possible solution to control the ever growing fiscal deficit and debt. He argues for fiscal rules designed on structural deficit rather than actual deficit, as rules on actual deficit may run into credibility trap. Gersl and Zapal (2009) investigate the possibility where both fiscal and monetary authorities are independent of the government. They found this set up to be welfare inducing only when the level of uncertainty between fiscal and monetary authority remains unaltered. Bohn (2009) deals with the problem of expropriation inherent in most of the fiscal systems. According to him, presence of fiscal policy alone does not tackle the time inconsistency problem of monetary policy, as envisaged by DL(2003). It is overcome only when fiscal policy is modelled to expropriate, monetary policy is constrained. Ciccarone and Guili (2009) find that transparency leads to improvement of social welfare only when the ratio between the weight attached to output and that attached to the instrument cost by the fiscal authority falls below a threshold value depending on multiplicative uncertainty and on the weight attached to output by the monetary authority.

All the papers discussed above have two things in common:

1. They find monetary and fiscal policy cooperation to be extremely important.
2. They inherently assume that agents are rational. In this paper we relax this assumption and assume that agents are learning adaptively and bounded rationally.

A few papers on adaptive learning are discussed below. But none of them tackle the central issue of this paper.

It was not until very recently that adaptive learning gained popularity in applied macroeconomics, especially in dynamic general equilibrium setting. Recent literature has used applied adaptive learning to study inter-alia the evolution of US inflation and the importance of expectations for its determination, the effects of monetary policy on macroeconomic variables, hyperinflation, business cycle fluctuations, asset prices, structural changes and policy reforms (see for example, Cho et al., 2002; Bullard and Cho, 2005; Marcet and Nicolini, 2003; Orphanides and Williams, 2005; Bullard and Eusepi, 2005). Orphanides and Williams (2004) and Milani (2005) find that adaptive learning models manage to reproduce important features of empirically observed expectations. Bullard and Mitra (2005) use adaptive learning to examine learnability of monetary policy rules. Similar exercise was done by Kulthanavit and Chen (2006) for Japan. Both the papers find support for adaptive learning expectations. Carceles-Poveda and Giannitsarou (2005) lay down comprehensive framework for computational implementation of adaptive learning algorithms. They find initial values to be highly important for adaptive learning dynamics; and that though in theory the effect of adaptive learning should disappear and expectations should converge to RE, in practice such effects linger for quite some time.

Given that AL is gaining momentum in policy as well as academic circles, and interest in the field of monetary and fiscal policy interaction has renewed. It provides a strong case for the study of monetary and fiscal policy games under the assumption of AL agents. Both these concepts have been studied separately to a great extent but no paper deals with both of them together. This paper steps in here and fills the void in the literature.

3 Methodology

3.1 Model

We discuss, below, the equations of our model. We do not explicitly go into DSGE modelling to derive the model equations. Rather, we borrow them from the existing literature.

1. Monetary policy loss function : $L_m^t = \gamma_1 \; \pi_t^2 + \gamma_2 \; x_t^2$

   γ_1, γ_2 >0, $\gamma_1 + \gamma_2 = 1$, γ_1 >γ_2

   Monetary authority loss function consists of squared deviations of inflation and output. Monetary authorities care about inflation as well as the level of output gap. We considder monetary authority to be conservative implying γ_1 to be greater than γ_2. This implies that MA cares more about inflation than output gap. γ_1 and γ_2 measure the weight attached to inflation (π_t) and output gap ($x_t = Y_t - Y_n$) respectively. Higher the value of γ_1, more concerned is the central bank about inflation. In both the USA and India, central bank act mandates them to balance price stability and growth.

2. Fiscal policy loss function : $L_f^t = \rho_1 \; \pi_t^2 + \rho_2 \; x_t^2 + \rho_3 \; g_t^2$

   ρ_1, ρ_2, ρ_3 >0, $\rho_1 + \rho_2 + \rho_3 = 1$

   Fiscal authority loss function consists of squared deviations of inflation, output and government expenditure. We assume that the governments apart from being concerned about inflation and output gap, also care about the government expenditure. This kind of loss function has been studied by Kirsanova et. al. (2005). For our analysis, we consider fiscal authority to be more concerned about output gap compared to inflation and government expenditure i.e. $\rho_2 > \rho_1 \& \rho_3$. We include government expenditure in the loss function mainly because of the following two reasons:

   - Increasing attention to curb government spending has led to acts inhibiting the government to spend carelessly. For example: Stability and Growth Pact (SGP) in the Euro, Fiscal Responsibility and Budget Management (FRBM) Act in India.

- Moreover, as per Kirsanova et. al. (2005) inclusion of debt in the fiscal authority's loss equation may lead to instability in some of the macroeconomic variables.

- In recent times, as debt to GDP ratio has increased significantly for most of the countries, government expenditure has become quite a sensitive area for fiscal authorities, and also a key variable signalling the economic health of the country globally.

3. IS/AD equation :$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + \psi g_t + \tau_t, \tau_t = \zeta \tau_{t-1} + e_{x,t}$

   $x_t = (Y_t - Y_n)$ is the output gap (difference between actual and potential output), i_t is the nominal interest rate, τ_t is a demand shock, E_t stands for the expectation operator for both inflation rate π_{t+1} and output gap x_{t+1} , the parameter σ >0 represents the inter-temporal elasticity of substitution in private spending and finally, ψ measures the effect of government consumption on output.

4. Phillips/AS equation : $\pi_t = kx_t + \beta E_t \pi_{t+1} + \lambda g_t + v_t, v_t = \vartheta v_{t-1} + e_{\pi,t}$

   k >0 measures the sensitivity of inflation rate to output gap, and is determined by the frequency of price adjustment and the marginal cost elasticity in relation to the real level of economic activity. The discount factor of the private sector and policymakers is represented by β , where 0 <β <1. This parameter measures the sensitivity of the agents to inflation rate. λ measures the proportion of government expenditure that is spent as subsidy to increase supply of goods and services in order to bring down inflation. Government expenditure, g_t, is added to the AS equation following DL (2003) to study the direct impact of government expenditure on inflation and indirect impact on output gap.

5. Debt equation : $b_t = (1 + i^*)b_{t-1} + \bar{b}(i_t - \pi_t) + g_t - \omega x_t$

   Similar to Kirsanova et al. (2005), the real stock of debt at the beginning of period t, b_t, depends on the stock of debt at the last period, b_{t1}, added to the flows of interest payments, government spending, and revenues. i^* is the equilibrium interest rate, $\bar{b}$ accounts for the steady state value of debt, i_t is the nominal interest rate, g_t the government spending, ω the tax rate, x_t the output gap. Tax revenues vary with output through the term x_t. Note that debt does not have an error term of its own. However, debt gets affected by output and inflation shock indirectly.

The model makes it apparent that the behavior of the variables depends on the expectations regarding the evolution of a few key variables. RE literature assumes that the expectations formation process is common knowledge. The same is assumed here in the case of AL. All agents in the economy, be it central bank, government or individuals, know that agents are adaptive learners. Both the central bank and government would design policies keeping in view the expectations formation process of the individual agents.

Woodford (2003) assumptions apply here. The structural model underlying our analysis has monopolistic competition and staggered prices. Output is sub-optimally low because of the monopolistic power of firms. This gives the authorities the incentive to push output closer to the optimal level. Policies considered in this paper are discretionary in nature. As far as the timing of the game is concerned, the private sector forms its expectations first, then shocks are realized and later M-F policy games begin in response to these shocks. Also the economy under consideration is a cashless economy. Thus making interest rate the the policy instrument opposed to monetary aggregates.

We study the M-F policy interaction under the following games:

1. Cooperation: Both the authorities coordinate to minimize fiscal authorities' loss function, which coincides with the society's loss function

2. Nash: Both the authorities act simultaneously and non-cooperatively, minimizing their own loss functions.

3. Stackelberg: One authority moves first and the other follows, non-cooperatively. Both the possibilities, one in which fiscal policy leads (Fiscal leadership, thereafter FM) and the other in which monetary policy leads (Monetary leadership, thereafter MF), are considered.

All these games differ in the timing of their policy decisions. Under Nash and cooperation, both the authorities move simultaneously, whereas under stackelberg they move sequentially.

The model laid out above is solved under different M-F policy games with different assumptions about expectations formation. First step is to understand the nature of these games, and second, to compress the model equations to a set of reduced form equations. Once these reduced form equations are derived, the role of expectations sets in.

To estimate the AL algorithm, different methods have been proposed in the literature. Carceles-Poveda and Giannitsarou (2007) do an extensive analysis of AL procedures. They discuss three learning algorithms namely: Recursive Least Squares (RLS), Stohastic Gain (SG) and last, Constant Gain (CG). These algorithms can be initialized using one of the three conditions: Randomly generated data, ad hoc initial conditions and initial conditions drawn from a distribution. They discuss in detail the pros and cons of each algorithm and leave the choice of selection to the situation the researcher wishes to study. SG algorithm is simple but inefficient compared to RLS. CG is an extension to both these algorithms, and the recent literature has focussed on this method for modelling AL. Under CG-RLS, agents attach higher weight to the recent observations and smaller weight to the past observations. More details in the Appendix.

We assume that agents have reached an equilibrium, but then in the next period the economic regime changes completely and agents have no information whatsoever about the evolution of the variables, i.e. they start collecting information afresh all over again. As discussed, ad hoc initial conditions are best suited to such kind of problems. To estimate models with rationally expecting agents, we use the method of undetermined coefficients.

3.2 Calibration

Table 1 below reports values used for calibration of the model parameters[1].

Table 1: Parameter values for calibration

Parameter	Definition	Calibration	Reference
σ	Intertemporal elasticity of substitution in private spending	5.00	Nunes and Portugal (2009)
k	Sensitivity of inflation to output gap	0.50	Gouvea (2007), Walsh (2003)
β	Agents sensitivity to inflation rate	0.99	Cavallari (2008), Pires (2003)
i*	Natural interest rate	0.07	Barcelos Neto and Portugal (2009)
$\bar{b}$	steady state debt value	0.20	Kirsanova et. al (2005), Portugal (2009)
ω	Tax rate	0.26	Kirsanova et. al. (2005) & Portugal (2009)
γ_1	Weight attached to inflation by monetary authority	0.70	Conservative central bank
γ_2	Weight attached to output gap by monetary authority	0.30	
ρ_1	Weight attached to inflation by fiscal authority	0.30	
ρ_2	Weight attached to output gap by fiscal authority	0.50	FP more concerned about output gap
ρ_3	Weight attached to government expenditure by fiscal authority	0.20	
ψ	Effect of government consumption on output	0.50	
λ	Effect of government expenditure on inflation	0.50	
γ	persistence of demand shock	0.80	
f	Persistence of supply shock	0.80	

Parameter values accorded to the coefficients in the loss functions imply the following:

- Monetary authority is more conservative about inflation compared to fiscal authority.

[1] As a part of our simulation exercise, the standard deviation of shocks are set to 0.007 and 0.008 for demand and supply shock respectively. The seed of random numbers has been fixed to to 73 in Matlab to ensure that results are reproducible.

- Fiscal auhtority cares more about output gap than monetary authority
- Shocks are persistent.

4 Results and analysis

4.1 Reduced form equations

On solving the model, we get the following reduced form equations in all the games:

$$g_t = f(E_t\pi_{t+1}, v_t) \tag{1}$$

$$i_t = f(E_t\pi_{t+1}, E_t x_{t+1}, v_t, \tau_t) \tag{2}$$

$$\pi_t = f(E_t\pi_{t+1}, v_t) \tag{3}$$

$$x_t = f(E_t\pi_{t+1}, v_t) \tag{4}$$

$$b_t = f(E_t\pi_{t+1}, E_t x_{t+1}, v_t, \tau_t) \tag{5}$$

Though reduced form equations are same for all the games, their coefficients differ. Game-wise reduced form equations are derived in appendix 2. A few observations about the reduced form equations of the different policy games are discussed below:

1. Cooperation:

   Inflationary expectations and supply shocks affect inflation very strongly. Since both the authorities are concerned about output gap, it increases the sensitivity of inflation to shocks.

2. Stackelberg :

   - When monetary policy moves first, the output gap becomes more sensitive to inflationary expectations.
   - Government expenditure reacts greatly to inflationary expectations under Stackelberg monetary leadership regime.

- As expected, interest rates strongly react to inflation expectation under Stackelberg monetary leadership regime.
- Debt reacts strongly to inflationary expectations under Stackelberg monetary leadership regime.

3. Expected output gap and demand shock affect variables in the same way in all the games.

4.2 Calibration results and analysis

A comparison of the policy games entails three criteria. First, convergence of AL expectations to RE, second, differences in the evolution of macro series under RE and AL, and lastly, contribution to the authorities' loss functions by AL agents over and above those contributed by RE agents.

4.2.1 Convergence

AL expectations are expected to converge to RE in the long run (Evans and Hankopojha, 2003; Carceles-Poveda and Giannitsarou, 2007). Figure 2 reports the evolution of expectations under AL (dark continuous line) as well as under RE (dotted line). Dotted line plots rational expectations and the dark continuous line plots the evolution of AL.

ϕ's in figure 2 represent evolution of expectations for different state variables. In our system we have two state variables, demand shock(τ_t) and supply shock(v_t).

$$\begin{pmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{pmatrix} = \begin{bmatrix} \phi_1 & \phi_2 \\ \phi_3 & \phi_4 \end{bmatrix} \begin{bmatrix} \tau_t \\ v_t \end{bmatrix}$$

Figures obtained for all the games are similar in nature. It is difficult to bring out the differences through naked inspection. Therefore to make results more comprehensible, average distances between the two expectations and variation (standard deviation) in AL expectations are compared. (Table 2)

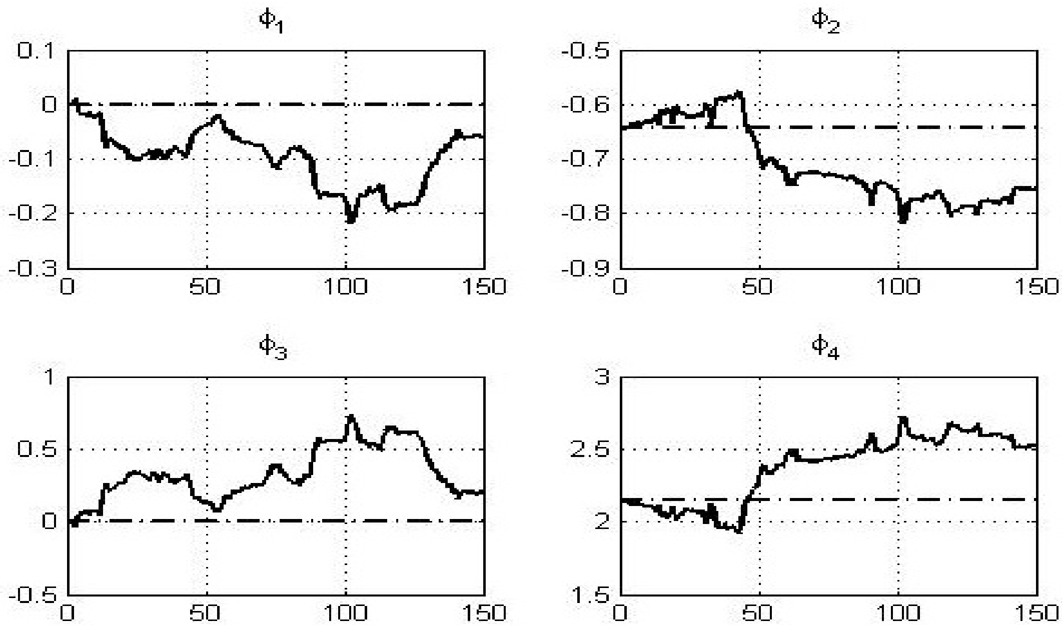

Figure 2: Convergence under cooperation

Table 2: Average distance between RE and AL (RE-AL)

ϕ	Cooperation	Nash	FM	MF
ϕ_1	-0.096	-0.130	-0.144	-0.136
ϕ_2	-0.071	-0.090	-0.096	-0.090
ϕ_3	0.322	0.115	0.124	0.101
ϕ_4	0.237	0.077	0.083	0.067

Note: Author's calculations

Table 2 reports average distance between RE and AL. ϕ_1 and ϕ_2 represent the evolution of the output expectations in response to demand and supply shocks. On an average, output expectations for AL are below rational expecting agents. On the other hand, evolution of inflationary expectations is above rational expectations on an average. In aggregate, evolution of AL expectations remain in the same region across the policy games. Incidentally, AL expectations lend downward bias to output expectations and upward bias to inflationary expectations. They only differ in terms of magnitude, which is the next agenda for discussion below.

Key findings:

1. Adaptive learning expectations do not converge even in 150 periods. In theory, the difference must become zero. Carceles-Poveda and Giannitsarou (2007) report similar results

in their paper especially for CG-RLS method.

2. When both the policies cooodinate, expectations about future output are better managed. This is obvious because both the authorities minimize a loss function in which more weight is attached to output. However, at the same time since lower weight is attached to inflation by both the authorities, AL inflationary expectations is way off from RE.

3. Under MF game, AL inflationary expectations remain close to RE. This implies that when monetary policy takes the lead then expectations related to inflation are much better anchored owing to monetary policy's ability to commit to a lower inflation because of more weight attached to inflation stabilization.

Comparing convergence only on one criterion may be misleading if lower average is associated with higher variance. Therefore, we compare variations in AL expectations across regimes. Table 3 reports standard deviation of adaptive learning expectations [2].

Table 3: Standard deviation of adaptive learning expectations

ϕ	Cooperation	Nash	FM	MF
ϕ_1	0.0548	0.0753	0.0807	0.0762
ϕ_2	0.0678	0.0964	0.1032	0.0974
ϕ_3	0.1827	0.0645	0.0691	0.0563
ϕ_4	0.2262	0.0826	0.0885	0.0720

Note: Author's calculations

Standard deviations help concretize the results presented above. AL output expectations are most contained under cooperation. Whereas, AL inflationary expectations are more contained under the stackelberg regime of monetary leadership.

4.2.2 Adaptive learning vs. Rational expectations

This section compares the evolution of various macro series generated out of two different assumptions about expectations formation.

1. Difference between evolution of variables under AL and RE

[2] Note: Standard deviation of RE is zero.

Table 4: Difference in the evolution of series under different games, normalised on cooperation

Variables	Cooperation	Nash	FM	MF
Output	1	1.04	1.16	0.98
Inflation	1	0.27	0.30	0.22
Interest rate	1	0.40	0.36	0.28
Government expenditure	1	1.43	0.62	1.35
Debt	1	1.18	0.77	1.13

Note: Ratios normalized to results under cooperation

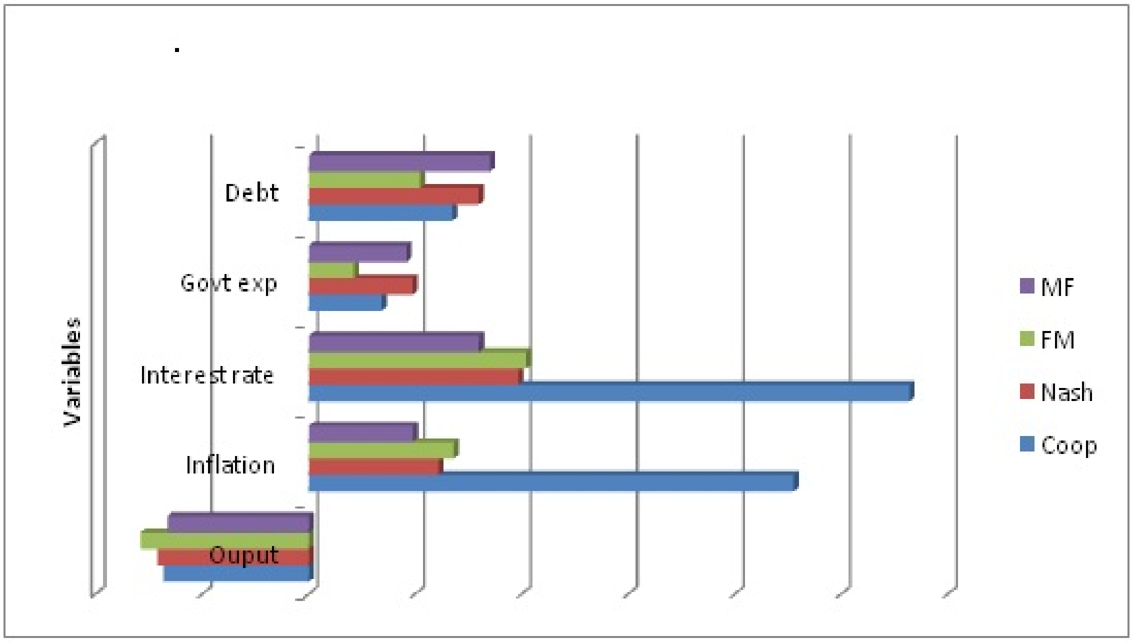

Figure 3: Convergence under cooperation

Monetary leadership performs best in the case of three variables, namely, output, inflation and interest rate. On the other hand, fiscal leadership performs better in case of government expenditure and debt. Hence a choice between MF and FM, would entail significant trade-offs. MF would provide lower variation in output, inflation and interest rate but higher variation in government expenditure and debt. FM, on the other hand, would contain debt and government expenditure closer to RE levels, but lead to higher variation in output, inflation and interest rate[3].

2. Absolute deviations

[3]Comparison of standard deviations across regimes throws similar results. Therefore, for brevity, we report only mean results.

Table 5: Absolute deviation across M-F games

Type of expectations	Variables	Cooperation	Nash	FM	MF
Adaptive learning	Output	0.945	1.513	1.610	1.55
	Inflation	3.150	1.297	1.380	1.15
	Interest rate	2.834	1.211	1.248	1.091
	Government expenditure	0.472	1.037	0.427	1.080
	Debt	0.688	1.489	0.85	1.545
Rational expectations	Output	0.864	1.439	1.525	1.486
	Inflation	2.883	1.234	1.307	1.099
	Interest rate	2.472	1.092	1.119	0.991
	Government expenditure	0.432	0.987	0.404	1.032
	Debt	0.604	1.398	0.800	1.465

Based on 500 simulations using Hodrick Prescott filtered series. Results similar for non-filtered series as well. Available upon request

- AL variables are more volatile than RE, as documented in the literature.
- When both policies cooperate, then output volatility is least. Under cooperation, since both the policies attach higher weight to output (0.7), output gets managed well. Interest rate and inflation have least variability in case of monetary leadership. On the other hand, government debt and expenditure volatilities are least under fiscal leadership. Since the leader gets the first mover advantage, whichever authority gets to move first tries to minimize its own loss function at the expense of the other authority.

3. Relative deviations

Table 6: Relative deviation across M-F games

Types of expectations	Variables	Cooperation	Nash	FM	MF
Adaptive learning	Output	1	1	1	1
	Inflation	3.334	0.857	0.857	0.739
	Interest rate	2.999	0.800	0.775	0.702
	Government expenditure	0.499	0.685	0.265	0.694
	Debt	0.728	0.979	0.533	0.992
Rational expectations	Output	1	1	1	1
	Inflation	3.334	0.857	0.857	0.739
	Interest rate	2.859	0.759	0.734	0.666
	Government expenditure	0.499	0.685	0.265	0.694
	Debt	0.698	0.971	0.524	0.985

Based on 500 simulations using Hodrick Prescott filtered series. Results similar for non-filtered series as well. Available upon request

It is important to look into relative deviations (w.r.t to output) to see cyclicality of macro series (Table 6). From the reduced form equations, it is clear that output, inflation and government expenditure are dependent only on inflationary expectations and supply shock. Therefore, under both type of expectations, these variables move in the same fashion as output and with same magnitude. However differences arise in interest rate and debt. Under AL, both debt and government expenditure move with output, albeit with a higher magnitude compared to RE results. A few observations:

- Under cooperation, inflation and interest rate are highly correlated with output. Monetary leadership brings down this high correlation due to higher weight attached to inflation in its loss function.
- On the other hand, fiscal leadership brings down correlation of government expenditure and debt with output.

4.3 Loss functions

The analysis would be incomplete if the losses attributed to policy authorities due to AL agents is overlooked. This provides a sense of how much losses are being under/overestimated under the assumption of RE agents (Table 7).

Table 7: Relative deviation across M-F games

	Cooperation	Nash	FM	MF
Monetary policy	2.971	1	1.20	0.82
Fiscal policy	2.971	1	1.09	0.91

Ratios are calculated taking Nash as the base

Observations:

- Net value of losses added to both the authorities' loss function is positive under all the games i.e loss under adaptive learning is always greater than rational expectations game. Thus implying that whenever we assume rational agents we underestimate losses owing to monetary and fiscal policies.
- Under cooperation losses incurred by both the authorities are highest out of all the alternatives. This is because both the authorities concentrate only on output, ignoring other variables especially inflation. Stackelberg MF performs the best.

Results indicate that the monetary leadership under stackelberg regime dominates the other regimes.

5 An aggregate perspective

The performance of various M-F games vary across the applied criterion. In order to employ a selection criterion, the M-F games are compared and ranked on the basis of their performance on a scale of one to four, one being the best (Table 8).

Table 8: Comparison across games

	Cooperation	Nash	FM	MF
Convergence	2.50	2.14	3.50	1.87
Evolution difference	3.20	3.00	2.20	1.60
Absolute deviations	2.60	2.40	2.40	2.60
Relative deviations	3.00	2.87	1.62	2.50
Loss functions	4.00	2.00	3.00	1.00
Total	15.30	12.40	12.72	9.57
Rank	4	2	3	1

Source : Author's calculations

Cooperation:

It performs well in anchoring AL output expectations but performs badly on AL inflationary expectations. It performs worse than all the other games throughout. Since monetary authority is forced to minimize the loss function contradicting with its objectives, we see cooperation performing worse than others in all aspects.

Fiscal leadership:

This game reduces the correlation of debt and interest rate with output, and contains volatility slightly better than other regimes. But in terms of the overall performance, it is ranked above cooperation. Fiscal authority by moving first is able to make sure that AL expectations of government expenditure and debt are better contained, but it fails to do the same for other variables. It does badly in containing the AL inflationary and output expectations.

Nash:

Surprisingly, Nash doesn't outperform other games in any of the aspects. But it performs well on an average, and is ranked above cooperation and fiscal leadership.

Monetary leadership:

This strategy outperforms other games under the assumption of AL agents. It performs consistently well. Though it lags behind in containing the volatility of variables.

6 Conclusion

We find that assuming rational agents leads to underestimation of volatility of variables as well as the welfare losses owing to monetary and fiscal authorities. Literature has largely focused on rational agents. But this study shows that such an assumption may lead to underestimation of key aspects of macro-economic series. Policy responses based on such underestimations in an RE framework are bound to be weak. Stronger policy initiatives are required to tackle greater volatility (due to AL agents) which may increase the variability in the policy variables itself. Thus providing a strong case for the use of AL agents in the models.

On comparing different M-F games, we find monetary leadership (MF) performs better in terms of anchoring AL inflationary expectations. Though cooperation performs better in anchoring AL output expectations, it increases the volatility of the macro-series and adds substantially to the losses of the authorities. Nash, in spite of not outperforming in any criteria, manages to perform better than cooperation and fiscal leadership.

7 Future work

1. There are a number of alternative simulation techniques available (discussed in methodology section; Carceles-Poveda and Giannitsarou, 2007). The CG-RLS technique is applied in this paper to address the issue of AL. Future work could concentrate on other techniques to enhance the robustness of the analysis.

2. The present model could further be extended to include other variables like exchange rate. The assumption here is that the economy under consideration is relatively small and closed to the world. Inclusion of exchange rate variable extends the analysis to a whole new area of international economics.

Appendix

On solving the games, we get the following reduced form equations:

$$g_t = f(E_t\pi_{t+1}, v_t)$$
$$i_t = f(E_t\pi_{t+1}, E_t x_{t+1}, v_t, \tau_t)$$
$$\pi_t = f(E_t\pi_{t+1}, v_t)$$
$$x_t = f(E_t\pi_{t+1}, v_t)$$
$$b_t = f(E_t\pi_{t+1}, E_t x_{t+1}, v_t, \tau_t)$$

Cooperation

In this game both the authorities solve the same loss function simultaneosuly, that of fiscal authoritys. The game is set up as follows:

Loss function: $\mathrm{L}_t^{m,f} = \rho_1\pi_t^2 + \rho_2 x_t^2 +_3 g_t^2$

Subject to:

(1) $\mathrm{x}_t = E_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1}) + \psi g_t + \tau_t, \tau_t = \zeta\tau_{t-1} + e_{x,t}$

(2)$\pi_t = kx_t + \beta E_t\pi_{t+1} + \lambda g_t + v_t, v_t = \vartheta v_{t-1} + e_{\pi,t}$

F.O.C

Monetary authority (w.r.t i_t)

(3)$\pi_t = \frac{-\rho_2 x_t}{\rho_1 k}$

Fiscal authority (w.r.t g_t)

(4)$\pi_t = \frac{-\rho_2 x_t\psi - \rho_3 g_t}{\rho_1(k\psi+\lambda)}$

Substituting (3) and (4) in (1) and (2) respectively, we get:

(5) $\mathrm{g}_t = \frac{-\lambda(E_t\pi_{t+1}+v_t)\rho_1\rho_2}{\Theta}$

$where, \Theta = \lambda^2\rho_1\rho_2 + k^2\rho_1\rho_3 + \rho_2\rho_3$

(6) $\mathrm{i}_t = \frac{-kg_t\rho_3+\lambda\rho_2(E_tx_{t+1}+\sigma E_t\pi_{t+1}+\psi g_t+\tau_t)}{\lambda\sigma\rho_2}$

From (5) in (6) we get equilibrium i_t

(7) $\mathrm{i}_t = \frac{E_tx_{t+1}}{\sigma} - \frac{v_t\rho_1(\lambda\rho_2-k\rho_3)}{\sigma\Theta} + \frac{E_t\pi_{t+1}(\sigma\Theta-\beta\rho_1(\lambda\psi\rho_2-k\rho_3)}{\sigma\Theta} + \frac{\tau_t}{\sigma}$

Substituting (5) and (7) in (1) and (2) we get equilibrium $\mathrm{x}_t and \pi_t$,

(8) $\mathrm{x}_t = \frac{-k(E_t\pi_{t+1}+v_t)\rho_1\rho_3}{\Theta}$

(9) $\mathrm{g}_t = \frac{(E_t\pi_{t+1}+v_t)\rho_2\rho_3}{\Theta}$

Putting (8) and (9) in debt equation we get,

(10) $\mathrm{b}_t = \frac{(E_tx_{t+1}+\tau_t)\bar{b}}{\sigma} + \frac{v_t(\sigma\rho_1\Upsilon-\bar{b}\Delta)}{\sigma\Theta} + \frac{E_t\pi_{t+1}(\beta\sigma\rho_1\Upsilon+(\sigma\Theta-\beta\Delta)\bar{b})}{\sigma\Theta} + (1+i^*)b_{t-1}$

where

$\Upsilon = -\lambda\rho_2 + k\omega\rho_3$

$\Delta = -\lambda\psi\rho_1\rho_2 + k\rho_1\rho_3 + \sigma\rho_2\rho_3$

Substituting parameter values for calibration, we get the following equations:

$\mathrm{g}_t = 0.127E_t\pi_{t+1} + 0.128v_t$

$i_t = 1.058E_t\pi_{t+1} + 0.2E_tx_{t+1} + 0.592v_t + 0.2\tau_t$

$\pi_t = 0.849E_t\pi_{t+1} + 0.858v_t$

$x_t = -0.254E_t\pi_{t+1} - 0.2575v_t$

$b_t = 1.07b_{t-1} + 0.235E_t\pi_{t+1} + 0.04E_tx_{t+1} + 0.358v_t + 0.04\tau_t$

Note: For the purpose of analysis, we report results for debt growth rate i.e. $b_t - b_{t-1}$.

Nash

It is a non-cooperative game where both the authorities move simultaneously. They solve for their response based on the other authoritys best response reaction function.

Fiscal authority's loss function: $L_t^f = \rho_1 \pi_t^2 + \rho_2 x_t^2 +_3 g_t^2$

Monetary authority's loss function: $L_t^m = \gamma_1 \pi_t^2 + \gamma_2 x_t^2$

Subject to:

(1) $x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + \psi g_t + \tau_t, \tau_t = \zeta \tau_{t-1} + e_{x,t}$

(2)$\pi_t = k x_t + \beta E_t \pi_{t+1} + \lambda g_t + v_t, v_t = \vartheta v_{t-1} + e_{\pi,t}$

F.O.Cs

Monetary authority (w.r.t i_t)

(3)$\pi_t = \frac{-\gamma_2 x_t}{\gamma_1 k}$

Fiscal authority (w.r.t g_t)

(4)$\pi_t = \frac{-\rho_2 x_t \psi - \rho_3 g_t}{\rho_1 (k\psi + \lambda)}$

Solving (3) and (4) to get x_t and π_t:

(5) $x_t = \frac{k g_t \gamma_1 \rho_3}{\Xi}$

(6)$\pi_t = \frac{g_t \gamma_2 \rho_3}{\Xi}$

where $\Xi = -(\lambda + k\psi)\gamma_2 \rho_1 + k\psi \gamma_1 \rho_2$

Substituting (5) and (6) in (1) and (2) to get i_t and g_t:

(7) $g_t = \frac{(\beta E_t \pi_{t+1} + v_t)\Xi}{\Omega}$

Where $\Omega = \gamma_2(\lambda(\lambda + k\psi)\rho_1 + \rho_3) + k\gamma_1(-\lambda\psi\rho_1 + k\rho_3)$

(8) $i_t = \frac{E_t x_{t+1} + \sigma E_t \pi_{t+1} + g_t(\psi + (k/\Xi)) + \tau_t}{\sigma}$

Solving for optimal i_t and g_t from (7) and (8),

(9) $g_t = \frac{(\beta E_t \pi_{t+1} + v_t)\Xi}{\Omega}$

(10) $i_t = \frac{E_t x_{t+1} + \tau_t}{\sigma} + \frac{v_t \Lambda}{\sigma\Omega} + \frac{E_t \pi_{t+1}(\sigma + (\beta\Lambda/\Omega))}{\sigma}$

Where $\Lambda = -\psi(\lambda + k\psi)\gamma_2\rho_1 + k\gamma_1(\psi^2\rho_2 + \rho_3)$

Substituting (9) and (10) in (1) and (2) we get x_t and π_t,

(11) $x_t = \frac{-k(\beta E_t \pi_{t+1} + v_t)\gamma_1\rho_3}{\Omega}$

(12) $\pi_t = \frac{(\beta E_t \pi_{t+1} + v_t)\gamma_2\rho_3}{\Omega}$

Substituting (9)-(12) in debt equation to get;

(13) $b_t = \frac{(E_t x_{t+1} + \tau_t)\bar{b}}{\sigma} + \frac{v_t((\Xi + k\omega\gamma_1\rho_3) + \Lambda)}{\sigma\Omega} + \frac{E_t \pi_{t+1}(\beta\sigma(\Xi + k\omega\gamma_1\rho_3) + \sigma\Omega - \beta(\Lambda + \sigma_1\rho_3))}{\sigma\Omega} + b_{t-1}(1 + i^*)$

Substituting parameter values for calibration, we get the following equations:

$g_t = 0.476 E_t \pi_{t+1} + 0.480 v_t$

$i_t = 1.16 E_t \pi_{t+1} + 0.2 E_t x_{t+1} + 0.169 v_t + 0.2 \tau_t$

$\pi_t = 0.595 E_t \pi_{t+1} + 0.60 v_t$

$x_t = -0.69 E_t \pi_{t+1} - 0.70 v_t$

$b_t = 1.07 b_{t-1} + 0.77 E_t \pi_{t+1} + 0.04 E_t x_{t+1} + 0.576 v_t + 0.04 \tau_t$

Note: For the purpose of analysis, we report results for debt growth rate i.e. $b_t - b_{t-1}$.

Monetary leadership

In this type of game one authority leads and the other one follows. It is solved using backward induction. Authority which leads solves for its best response keeping in mind followers best response.We solve the game via backward induction, solving for fiscal authority's reaction function first and then solving for monetary authority keeping fiscal authority's reaction function in mind.

Stage 1: Monetary policy takes a decision

Stage 2: Fiscal policy follows

Stage 2:

Fiscal authority's loss function: $L_t^f = \rho_1\pi_t^2 + \rho_2 x_t^2 +_3 g_t^2$

Subject to:

(1) $x_t = E_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1}) + \psi g_t + \tau_t, \tau_t = \zeta\tau_{t-1} + e_{x,t}$

(2)$\pi_t = kx_t + \beta E_t\pi_{t+1} + \lambda g_t + v_t, v_t = \vartheta v_{t-1} + e_{\pi,t}$

F.O.C

(3) $g_t = \frac{-\psi\rho_2 x_t - \rho_1(k\psi+\lambda)\pi_t}{\rho_3}$

Substitute (1) and (2) in (3) to get optimal g_t

(4) $g_t = \frac{-(\lambda+k\psi)\rho_1 v_t}{\Im} - \frac{(E_t x_{t+1}+\tau_t)\Gamma}{\Im} + \frac{\sigma i_t\Gamma}{\Im} - \frac{E_t\pi_{t+1}(\beta(\lambda+k\psi)\rho_1)+\Gamma}{\Im}$

Where

$\Im = (\lambda + k\psi)^2\rho_1 + \psi^2\rho_2 + \rho_3$

$\Gamma = k(\lambda + k\psi)\rho_1 + \psi\rho_2$

Putting back (4) in (1) and (2) to get new constraints for stage 1,

(5) $\pi_t = \frac{v_t(\psi^2\rho_2+\rho_3)}{\Im} + \frac{(E_t x_{t+1}+\tau_t)(-\lambda\psi\rho_2+k\rho_3)}{\Im} + \frac{i_t(\lambda\sigma\psi\rho_2-k\sigma\rho_3)}{\Im} + \frac{E_t\pi_{t+1}(-\lambda\sigma\psi\rho_2+\beta\psi^2\rho_2+\beta\rho_3+k\sigma\rho_3)}{\Im}$

(6) $\mathrm{x}_t = \frac{-\psi\rho_1 v_t(\lambda+k\psi)}{\Im} + \frac{(E_t x_{t+1}+\tau_t)(\lambda(\lambda+k\psi)\rho_1+\rho_3)}{\Im} + \frac{i_t(-\lambda\sigma\rho_1(\lambda+k\psi)-\sigma\rho_3)}{\Im} + \frac{E_t\pi_{t+1}(\lambda\sigma\rho_1(\lambda+k\psi)-k\psi\rho_1(\lambda+k\psi)+\sigma\rho_3}{\Im}$

Stage 1: Monetary authority's loss function: $\mathrm{L}_t^m = \gamma_1\pi_t^2 + \gamma_2 x_t^2$

subject to (5) and (6).

F.O.C

(7) $\pi_t = \frac{-\gamma_2(-\lambda(\lambda+k\psi)\rho_1-\rho_3)x_t}{\gamma_1(\lambda\psi\rho_2-k\rho_3)}$

Substituting (5) and (6) in (7) we get optimal i_t

(8) $\mathrm{i}_t = \frac{v_t\nabla+(E_t x_{t+1}+\tau_t)\aleph+E_t\pi_{t+1}(\sigma\aleph+\beta\nabla)}{\sigma\wp}$

Where

$\nabla = \psi\gamma_2\rho_1(\lambda+k\psi)(\lambda\rho_1(\lambda+k\psi)+\rho_3) - (\psi^2\gamma_1\rho_2-\gamma_1\rho_3)(\lambda\psi\rho_2-k\rho_3)$

$\aleph = (\lambda\rho_1(\lambda+k\psi)+\rho_3)(\lambda\gamma_2\rho_1(\lambda+k\psi)+\gamma_2\rho_3) + (\lambda\psi\rho_2-k\rho_3)(\lambda\psi\gamma_1\rho_2-k\gamma_1\rho_3)$

$\wp = (\gamma_2(\lambda\rho_1(\lambda+k\psi)+\rho_3)^2 + \gamma_1(\lambda\psi\rho_2-k\rho_3)^2)$

Putting it back in (5) and (6) we get optimal path of π_t, x_t and i_t,

(9) $\mathrm{x}_t = \frac{\gamma_1\rho_3(\lambda\psi\rho_2-k\rho_3)(\beta E_t\pi_{t+1}+v_t)}{\wp}$

(10) $\pi_t = \frac{\gamma_2\rho_3(\lambda(\lambda+k\psi)\rho_1+\rho_3)(\beta E_t\pi_{t+1}+v_t)}{\wp}$

(11) $g_t = \frac{((\lambda+k\psi)\gamma_2\rho_1(\lambda(\lambda+k\psi)\rho_1+\rho_3)+\psi\gamma_1\rho_2(\lambda\psi\rho_2-k\rho_3))(\beta E_t\pi_{t+1}+v_t)}{\wp}$

Substituting (8)-(11) in debt equation we get,

(12) $b_t = \frac{v_tA+(E_tx_{t+1}+\tau_t)(\bar{b}/\sigma)+E_t\pi_{t+1}B+(1+i^*)b_{t-1})}{\sigma\wp}$

Where,

A = -($\sigma(\lambda + k\psi)\gamma_2\rho_1(\lambda(\lambda + k\psi)\rho_1 + \rho_3) + (\sigma\psi\gamma_1\rho_2 + \omega\sigma\gamma_1\rho_3)(\lambda\psi\rho_2 - k\rho_3) + \bar{b}((\lambda + k\psi)\rho_1 + \rho_3)(\psi(\lambda + k\psi)\gamma_2\rho_1 + \sigma\gamma_2\rho_3) + (\lambda\psi\rho_2 - k\rho_3)(\psi^2\gamma_1\rho_2 +_1 \rho_3)))$

B =-($\beta\sigma(\lambda+k\psi)\gamma_2\rho_1(\lambda(\lambda+k\psi)\rho_1+\rho_3)+(\lambda\psi\rho_2-k\rho_2)(\psi\rho_2+\omega\rho_3)\beta\sigma\gamma_1)+\bar{b}(\gamma_2(\lambda(\lambda+k\psi)\rho_1+\rho_3)(\rho_1(\lambda+k\psi)(\lambda\sigma-\beta\psi)+(1-\beta)\sigma\rho_3)+\gamma_1(\lambda\psi\rho_2-k\rho_3)(\psi\rho_2(\lambda\sigma-\beta\psi)-(\beta+k\sigma)\rho_3))$

Substituting parameter values for calibration, we get the following equations:

$g_t = 0.525E_t\pi_{t+1} + 0.530v_t$

$i_t = 1.118E_t\pi_{t+1} + 0.2E_tx_{t+1} + 0.184v_t + 0.2\tau_t$

$\pi_t = 0.559E_t\pi_{t+1} + 0.563v_t$

$x_t = -0.756E_t\pi_{t+1} - 0.763v_t$

$b_t = 1.07b_{t-1} + 0.846E_t\pi_{t+1} + 0.04E_tx_{t+1} + 0.652v_t + 0.04\tau_t$

Note: For the purpose of analysis, we report results for debt growth rate i.e. $b_t - b_{t-1}$.

Fiscal leadership

We solve the game via backward induction, solving for fiscal authority's reaction function first and then solving for monetary authority keeping fiscal authority's reaction function in mind.

Stage 1: Fiscal policy takes a decision

Stage 2: Monetary policy follows

Stage 2:

Monetary authority's loss function: $L_t^f = \gamma_1\pi_t^2 + \gamma_2 x_t^2$

Subject to:

(1) $x_t = E_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1}) + \psi g_t + \tau_t, \tau_t = \zeta\tau_{t-1} + e_{x,t}$

(2)$\pi_t = kx_t + \beta E_t\pi_{t+1} + \lambda g_t + v_t, v_t = \vartheta v_{t-1} + e_{\pi,t}$

F.O.C

Monetary authority (w.r.t i_t)

(3)$\pi_t = \frac{-\gamma_2 x_t}{\gamma_1 k}$

Put equation (1) and (2) and (3) to get i_t,

(4) $i_t = \frac{E_t x_{t+1}+\tau_t}{\sigma} + \frac{k\gamma_1 v_t + E_t\pi_{t+1}(k\beta\gamma_1+\sigma U)+(k(\lambda+k\psi)\gamma_1+\psi\gamma_2)g_t)}{\sigma U}$

Where

$U = (k^2\gamma_1 + \gamma_2)$

Substitute (4) in (1) and (2) to get x_t and π_t in terms of g_t,

(5) $\pi_t = \frac{(\beta E_t\pi_{t+1}+\lambda g_t+v_t)\gamma_2}{U}$

(6) $x_t = \frac{-k\gamma_1(\beta E_t\pi_{t+1}+\lambda g_t+v_t)}{U}$

Stage 1: Fiscal authority's loss function: $L_t^f = \rho_1\pi_t^2 + \rho_2 x_t^2 +_3 g_t^2$

subject to (5) and (6).

F.O.C

(7) $\pi_t = \frac{k\lambda\gamma_1\rho_2 x_t - \rho_3 U g_t}{\rho_1\lambda\gamma_2}$

Substituting (5) and (6) in (7) we get optimal g_t,

(8) $g_t = \frac{-\lambda(E_t\pi_{t+1}+v_t)Z}{\lambda^2 Z+U^2\rho_3}$

Where

Z= $\gamma_2^2\rho_1 + k^2\gamma_1^2\rho_2$

Substituting (8) in in (4), (6) and (7), we get optimal paths of i_t, π_t and x_t,

(9) $\pi_t = \frac{\gamma_2\rho_3 U(E_t\pi_{t+1}+v_t)}{\lambda^2 Z+U^2\rho_3}$

(10) $x_t = \frac{-k\gamma_1\rho_3 U(E_t\pi_{t+1}+v_t)}{\lambda^2 Z+U^2\rho_3}$

(11) $i_t = \frac{(\lambda^2 Z+U^2\rho_3)(E_t x_{t+1}+\tau_t)+v_t(-\lambda\psi Z+k\gamma_1\rho_3 U)+E_t\pi_{t+1}((\lambda^2\sigma-\beta\lambda\psi)Z+k\beta\gamma_1\rho_3 U+\sigma U^2\rho_3)}{\sigma(\lambda^2 Z+U^2\rho_3)}$

Substituting (8)-(11) in debt equation to get:

(12) $b_t = \frac{(\tau_t+E_t x_{t+1}\bar{b})(\lambda^2 Z+U^2\rho_3)+v_t T+E_t\pi_{t+1}Q}{\sigma(\lambda^2 Z+U^2\rho_3)} + (1+i^*)b_{t-1}$

Where

T = $\lambda\sigma Z + k\omega\sigma\gamma_1\rho_3 U + (-\lambda\psi Z + k\gamma_1\rho_3 U)\bar{b} - \sigma\gamma_2\rho_3 U\bar{b}$

Q = $-\beta\lambda\sigma Z + k\omega\beta\sigma\gamma_1\rho_3 U + \bar{b}((\lambda^2\sigma - \beta\lambda\psi)Z + k\beta\gamma_1\rho_3 U + U^2\sigma\rho_3 - \beta\sigma\gamma_2\rho_3 U)$

Substituting parameter values for calibration, we get the following equations:

$g_t = 0.189E_t\pi_{t+1} + 0.192v_t$

$i_t = 1.15E_t\pi_{t+1} + 0.2E_t x_{t+1} + 0.156v_t + 0.2\tau_t$

$\pi_t = 0.613E_t\pi_{t+1} + 0.619v_t$

$x_t = -0.715 E_t \pi_{t+1} - 0.722 v_t$

$b_t = 1.07 b_{t-1} + 0.484 E_t \pi_{t+1} + 0.04 E_t x_{t+1} + 0.287 v_t + 0.04 \tau_t$

Note: For the purpose of analysis, we report results for debt growth rate i.e. $\mathrm{b}_t - b_{t-1}$.

Once the reduced form equations are derived, we use the code provided by Carceles-Poveda and Giannitsarou (2007), adapt it as per our model and estimate the calibration results.

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