

Does Reliable Pirated Product Lead to More Piracy?*

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Abstract

Conventional wisdom would suggest if a pirated product, which is cheaper than the original product, becomes more reliable then the relative demand of the pirated product or the rate of piracy will increase when consumers have different willingness to pay. However, is this always true? We address this question in a framework where the original product developer makes costly investment to deter pirate(s) in a given regime of IPR protection. We show that under commercial piracy, when the original firm and the pirate compete in quantities, the conventional wisdom holds i.e. the more reliable the pirated product, the higher is the rate of piracy. However, the relationship is non-monotonic, hence the wisdom does not hold when they compete in prices or pirates are the end-users. We also compare the survival possibilities of a pirate and optimal deterrence efforts of copyright holder under different scenarios of piracy considered in the analysis.

Keywords: IPR protections, Copyright holder, price competition, quantity competition, product quality

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1. Introduction

When the consumers are heterogeneous in their willingness to pay for a product then a pirated product is likely to sell in the market when piracy is accommodated and it is cheaper than the original product. Now suppose the quality or the reliability of the pirated product (which is usually of lower quality than the original product) gets better; does that mean that the relative demand of the pirated product or the rate of piracy will increase? We ask this question in two different frameworks, namely, in the environment of commercial piracy and the end-users piracy. Under commercial piracy, there are one original product developer, a commercial pirate and a group of heterogeneous consumers; and under end-users piracy there are one original product developer and a group of heterogeneous consumers who are also the pirates.¹

The issue of piracy or copyright violations and intellectual property rights (IPR) protection is presently receiving a great deal of attention in various economic analyses. Copyright violations take place when there is illegal copying or counterfeiting of the original product. These products can be digital products (like software, music CDs, movie DVDs, video games etc.) or non-digital products i.e. regular items (like cloth, shoes, books, bags, medicines etc.).² In recent years, there is a renewed interest to study the implications of piracy, and mostly those of digital goods piracy because of the rapid advancement of digital copying technology. Conventional copying or counterfeiting of non-digital products (e.g. the fake brands of original goods), was always there in several markets and would continue to be there in future as well. But the growth of digital piracy is now posing an additional threat. Since digital piracy is a relatively new phenomenon compared to the conventional counterfeiting, a lot of recent studies have focused their attention on digital piracy. To study the implications of digital piracy, most of these studies considered a scenario where the pirates are mainly the end-users (see Conner and

¹ So far studies on piracy or illegal copying are broadly divided in two categories in the literature, commercial piracy and end-users piracy. Under commercial piracy, a pirate sells the pirated product for profit, whereas under end-users piracy, individual user pirates the product for his/her own use.

² Globally counterfeiting activities have risen to 5-7% of world trade, or about \$200 billion to \$300 billion in lost revenue, according to some estimates for the European Union some years back (see Time Magazine 2001). We believe that the figure has increased in recent years due to the significant increase in digital piracy.

Rumelt (1991), Takeyama (1994), Shy and Thisse (1999), Chen and Png (2003), Bae and Choi (2006), Belleflame and Picard (2007) among many others). Except few studies (see Slive and Bernhardt (1998), Banerjee (2003), Poddar (2005), Kiema (2008)) the issue of commercial piracy has not been addressed adequately so far in the literature. Even if those few studies addressed commercial piracy, the explicit influence of exogenous IPR protection on piracy is never incorporated in the models. Recently, a study by Lu and Poddar (2012) deals with this issue in a model where there is one original product developer (the incumbent) and a commercial pirate (the potential entrant). The original product developer makes costly investment to deter the commercial pirate in a given regime of IPR protection anticipating the entry of the pirate. The IPR protection can be weak or strong and is exogenous to the model. Its impact on the existence (or non-existence) of piracy and its relationship to the original producer's optimal deterrence effort to limit piracy are discussed in detail in that framework.³

Now it is generally observed that the pirated product may also vary widely in terms of quality or reliability as the perceived quality (and hence the price as well) also depends largely on the heterogeneity of the consumer demand and their willingness to pay for it. Our main focus of this paper is to first study whether a more reliable pirated product increases or decreases the rate of piracy (i.e. the relative demand for the pirated good) in the market. Conventional wisdom would suggest that more reliable pirated products would mean higher relative demand of the pirated good or higher piracy rate when consumers are heterogeneous in their willingness to pay. However, we find that the actual relationship between the rate of piracy and the reliability of the pirated product is far more complicated and it depends on the nature of the pirate as well as on the nature of the product market competition if the pirate is commercial and competes with the original firm. Namely, under commercial piracy when the original firm and the pirate compete in quantities in the product market, the conventional wisdom holds i.e. the more reliable the pirated product, the higher is the rate of piracy, thus the relationship is monotonic. However, the same wisdom does not hold when they compete in prices. There we find that the relationship is non-monotonic. When the pirated good is of relatively lower in

³ It is fairly well documented that different countries have different levels of IPR protections. Usually developed nations have stronger IPR laws (and enforcements) than most developing nations.

quality, piracy rate increases with the quality of the pirated good, but it decreases with quality when the pirated good is of relatively higher in quality. Moreover, in the intermediate range of quality of the pirated good, the relationship between the rate of piracy and the quality of the pirated good also depends on the effectiveness of the IPR protections.

We then extend our analysis to the case of end-user piracy as it is also quite prevalent in various markets, particularly in the markets for digital products. Here instead of assuming any commercial pirate, we assume there are numerous pirates who are basically the end-user consumers and the market structure is monopoly with the original producer as the only firm. End-users pirate the product for their own benefit only and are not involved in any profit making commercial activity. The IPR protection and the deterrence effort of the original producer now target the end-users. There we find the relationship between the rate of piracy and the reliability of the pirated product is again non-monotonic and it also depends on the effectiveness of the IPR protections.⁴

Finally, we make an overall comparison of the main results from three different scenarios of piracy (namely, commercial piracy under quantity and price competition; and the end-user piracy under monopoly) we considered in the analysis. We find that a pirate is most likely to survive under commercial piracy and when it competes with the original firm in quantities and least likely to survive under end-user piracy. Thus, in terms of optimal deterrence effort of the copyright holder we find that to completely deter piracy, the original producer has to give more effort under quantity competition as opposed to other two situations, which interestingly require similar levels of effort. However, when the pirate is accommodated, the original producer gives least effort for deterrence under quantity competition and the maximum effort is given under end-user piracy. This result is a consequence of the fact that the original firm faces the softest competition from the pirate under quantity competition and the toughest competition from the end-user pirates. Thus, from our analysis, we conclude that it is the nature as

⁴ An alternative scenario which is also consistent with end-user piracy would be when there is a competitive fringe of commercial pirates (i.e. a large number of identical commercial pirates instead of just one) but each pirate makes zero profit due to perfect competition among them. This case is non-strategic. Although the working for this case would be little different from the end-user piracy case, however, it can be easily verified that the final results with regard to rate of piracy and the quality of the pirated good largely remain unchanged (working is available upon request).

well as the modes of competition between the original producer (i.e. the copyright holder) and the pirate(s) play a major role for all the outcomes in different scenarios.

The rest of the paper is organized as follows. In the next section, we set up the model of commercial piracy. The basic framework is borrowed from Lu and Poddar (2012). In section 3, we do our main analysis of commercial piracy under both quantity and price competition. In section 4, we do our analysis of end-user piracy. Section 5 makes overall comparison of outcomes across different piracy scenarios. Section 6 concludes.

2. The Model of Commercial Piracy

2.1 The Original Firm and the Pirate

Consider an original firm and a commercial pirate. The pirate has the know-how or the technology to copy/counterfeit the original product. We assume the pirate produces copies, which are of lower quality than the original. The product quality of the pirated good (compared to original) is captured by the parameter q , $q \in (0,1)$. In the case of digital product, although the pirated copies are almost like original, they do not come with any guarantee or supporting services, thus making them inferior compared to the original.

We consider a two-period model, where in the first period ($t=1$), the original product developer undertakes costly investment in order to deter piracy. It adopts the following entry deterring strategy. It tries to deter the pirate by increasing the cost of copying, in particular, raising the marginal cost of producing a pirated copy. The potential pirate appears in the market of the original product in the second time period ($t=2$). We assume the higher the entry deterring investment made by the original product developer in the first period (the higher the deterrence level), the higher would be the marginal cost of copying by the pirate. The pirate if survives, competes with the original developer in prices or quantities by possibly producing a lower quality.

We assume at $t=1$, the cost of investment of the original product developer to choose the level of deterrence, x , is given by $c_o(x) = x^2/2$. Thus, if the profit of the product developer at $t=2$ is denoted by π_o^2 then the net profit of the developer at the

end of the game is $\pi_o = \pi_o^2 - c_o(x) = \pi_o^2 - x^2/2$. When the level of deterrence is x , the marginal cost of production for the pirate will be $c+x$, where c is a parameter ($c > 0$) *exogenously* given. We would like to interpret c in the following way: it is the degree or the strength of IPR protection in our model. It essentially captures the strength of legal protection and enforcement to stop piracy and it is beyond the control of the original firm (i.e. the copyright holder).⁵ It is generally understood that the government or the regulatory authority can influence c .⁶ In our model, we interpret c as the public effort from the government and x as the private effort from the product developer to stop/limit piracy.

In this part of our study, we first focus on what would be the best entry-detering strategy x (hence, the optimal entry deterring private investment in response to potential piracy) for the original product developer given an enforcement environment of IPR protection (i.e. given c). Secondly, we analyze the main focus of our study, namely, the relationship between the rate of piracy and the quality/reliability of the pirated product and how it depends on the nature of product market competition which can be in prices or in quantities.

2.2 Consumers' Preferences

Consider a continuum of consumers indexed by $X \in [0, \theta]$. X measures the taste or the consumer's willingness to pay for the original product. A high value of X means higher valuation for the product and low value of X means lower valuation for the product. Therefore, one consumer differs from another on the basis of his/her valuation

⁵ It needs to be noted here that without proper enforcements, legal protection may not be effective.

⁶ According to a recent study by Andres (2006) (also see Park and Ginarte (1997)), the strength of IPR protection of a country mainly consists of two categories: *membership in the international copyright treaties* and *enforcement provisions*.

We assumed an additive form between c and the level of deterrence x that is chosen by the original firm. The reason is as follows. We view pirate's copying cost has two components. One is due to original producer's private effort to deter piracy, which may include technological adoption to protect copying; and/or it could be private monitoring, identifying and suing the pirate and all of these efforts can be reflected in x . The other component is due to the IPR regime i.e. the strength of IPR legislations and enforcements which is reflected in c . Both the original firm's private effort (investment) and the legal protection and enforcement of copyright legislations contribute to the deterrence of piracy.

or the taste for the particular product. Valuations are uniformly with density $1/\theta$ distributed over the interval $[0, \theta]$.⁷ Each consumer purchases at most one unit of the good. A consumer's utility function is given as:

$$U = \begin{cases} X - p_o & \text{if buys original product,} \\ qX - p_p & \text{if buys pirated product,} \\ 0 & \text{if buys none,} \end{cases}^8$$

where p_o and p_p are the prices of the original and pirated products respectively.⁹

3. Analysis and Main Results: Commercial Piracy

We look for subgame perfect equilibrium of the two-period game and solve the game using the usual method of backward induction. We start by deriving demands of the product developer and the pirate.

3.1 Deriving Demands of the Product Developer and the Pirate

The demand for the original product and for the pirated product, D_o and D_p , can be derived from the distribution of buyers as follows.

Recall that consumers are heterogeneous with respect to their values towards the product. Thus, the marginal consumer, X who is indifferent between buying the original product and the pirated version, is given by $X - p_o = qX - p_p$, or $X = \frac{(p_o - p_p)}{(1 - q)}$. The marginal consumer, Y who is indifferent between buying the pirated product and not buying any product, is given by $qY - p_p = 0$; $Y = \frac{p_p}{q}$. Thus, the demand for original

⁷ So the number of consumers is normalized to one.

⁸ Note that $q=0$ will eliminate the pirated product, while $q=1$ will make the two products identical. In our model $q=1$ is never possible as we have assumed that the pirated good is of lower quality. Also technically, $q \in (0,1)$ is needed so that demands, prices and profits are not indeterminate.

⁹ The utility representation is borrowed from the standard model of vertical product differentiation in the literature (see Shaked and Sutton (1982), Tirole (1988)).

product is $D_o = \frac{1}{\theta} \int_x^\theta dx = [(1-q)\theta - (p_o - p_p)] / (1-q)\theta$ and the demand for pirated

product is $D_p = \frac{1}{\theta} \int_y^x dx = (qp_o - p_p) / q(1-q)\theta$.

Note that we have implicitly assumed that $qp_o \geq p_p$ when we derive the demand functions as above. When this assumption does not hold true, the demand for pirated product becomes zero while the demand for original producer is $D_o = (\theta - p_o) / \theta$. Thus, we write the demand functions as the following:

$$D_o = \begin{cases} [(1-q)\theta - (p_o - p_p)] / (1-q)\theta & \text{if } qp_o \geq p_p \\ (\theta - p_o) / \theta & \text{otherwise} \end{cases}, \quad (1)$$

and

$$D_p = \begin{cases} (qp_o - p_p) / q(1-q)\theta & \text{if } qp_o \geq p_p \\ 0 & \text{otherwise} \end{cases}. \quad (2)$$

In the second period, the product market competition can be in prices or in quantities. We will analyze both cases in turn.

3.2 Quantity Competition ¹⁰

Using backward induction, one can first obtain equilibrium quantities in the quantity competition stage and then work out the choice of optimal level of deterrence by the original firm in the first period. Note that the original producer can decide to accommodate or deter entry of the pirate completely.

3.2.1 The Entry Accommodation Equilibrium and Entry Deterrence Equilibrium

Assume both original developer and the pirate have positive demand. Then from (1) and (2), one can obtain the following inverse demand functions:

¹⁰ Counterfeit hotel or restaurant chains in tourist places could be an example of this kind of piracy where the competition between the original and the counterfeits are mainly over the number of tourists/visitors, resembling a quantity competition.

$$p_o = \theta(1 - D_o - qD_p), \quad (3)$$

$$p_p = q\theta(1 - D_o - D_p). \quad (4)$$

In the quantity competition stage, the original developer chooses D_o to maximize $\pi_o^2(D_o, D_p) = \theta(1 - D_o - qD_p)D_o$, while the pirate chooses D_p to maximize $\pi_p(D_o, D_p) = [q\theta(1 - D_o - D_p) - c - x]D_p$. From the first-order conditions for profit maximization, we can obtain both firms' reaction functions:

$$D_o = \frac{1}{2}(1 - qD_p),$$

$$D_p = \frac{1}{2}\left(1 - D_o - \frac{c+x}{q\theta}\right).$$

The equilibrium quantities are then

$$D_o = \frac{1}{(4-q)\theta}(c+x+(2-q)\theta),$$

$$D_p = \frac{1}{q(4-q)\theta}(q\theta - 2(c+x)).$$

Note that only when $2(c+x) < q\theta$, $D_p > 0$. So if the original producer chooses x such that such that $2(c+x) \geq q\theta$, i.e., $x \geq q\theta/2 - c$, then $D_p = 0$. It is also clear that if $c \geq q\theta/2$, there is no need to deter piracy.

When $2(c+x) < q\theta$, one can then obtain the following equilibrium prices and profits for both firms:

$$p_o = \frac{1}{4-q}(c+x+(2-q)\theta),$$

$$p_p = \frac{(2-q)(c+x)+q\theta}{4-q},$$

$$\pi_o^2 = \frac{1}{(4-q)^2\theta}(c+x+(2-q)\theta)^2,$$

$$\pi_p = \frac{1}{q(4-q)^2\theta}(q\theta - 2(c+x))^2.$$

Note that $\pi_o^2 = \theta/4$ when $x = q\theta/2 - c$, which is the same as when the firm chooses a deterrence level higher than $q\theta/2 - c$.¹¹ Thus, when the deterrence cost is taken into account, $x > q\theta/2 - c$ is strictly dominated by $x = q\theta/2 - c$.

In stage 1, the original developer chooses the deterrence level x to maximize

$$\pi_o = \pi_o^2 - \frac{x^2}{2} = \frac{1}{(4-q)^2 \theta} (c+x+(2-q)\theta)^2 - \frac{x^2}{2}. \text{ To find the optimal deterrence level } x,$$

we first find $\frac{d\pi_o}{dx} = \frac{2(c+x+(2-q)\theta)}{(4-q)^2 \theta} - x$ and $\frac{d^2\pi_o}{dx^2} = \frac{2}{(4-q)^2 \theta} - 1$. Note that when

evaluated at $x=0$, $\frac{d\pi_o}{dx} = \frac{2(c+(2-q)\theta)}{(4-q)^2 \theta} > 0$. We then distinguish two cases depending

on whether $d^2\pi_o/dx^2$ is positive or negative.

When $(4-q)^2 \theta \leq 2$, $\frac{d^2\pi_o}{dx^2} \geq 0$. Since we also have $\frac{d\pi_o}{dx} > 0$, the profit function is strictly increasing in x . The original producer will choose a deterrence level x as big as possible subject to the constraint $2(c+x) \leq q\theta$. Thus, the optimal deterrence level is $x^* = q\theta/2 - c$.

When $(4-q)^2 \theta > 2$, $\frac{d^2\pi_o}{dx^2} < 0$. The profit function is concave in x . When evaluated at $x = \frac{q\theta}{2} - c$, $\frac{d\pi_o}{dx} = c - \frac{q(4-q)\theta - 2}{2(4-q)}$, which is positive when $c > \frac{q(4-q)\theta - 2}{2(4-q)}$ and negative when $c < \frac{q(4-q)\theta - 2}{2(4-q)}$. Therefore, when $c > \frac{q(4-q)\theta - 2}{2(4-q)}$, the optimal deterrence level is $x^* = q\theta/2 - c$, while when $c < \frac{q(4-q)\theta - 2}{2(4-q)}$, the optimal deterrence

¹¹ When $x > q\theta/2 - c$, the original producer, as a monopolist, will choose $p_o = \theta/2$ and obtain profits of $\theta/4$.

level is determined by $\frac{d\pi_o}{dx} = \frac{2(c+x+(2-q)\theta)}{(4-q)^2\theta} - x = 0$ and therefore,

$$x^* = \frac{2(c+(2-q)\theta)}{(4-q)^2\theta - 2}.$$

We thus have the following proposition characterizing the entry accommodation equilibrium and entry deterrence equilibrium.

Define $\phi(q, \theta) \equiv \frac{q(4-q)\theta - 2}{2(4-q)}$.

Proposition 1

- (i) When $(4-q)^2\theta \leq 2$ and $c < q\theta/2$, the original producer's optimal level of deterrence is $x^* = q\theta/2 - c$. In this case, it deters the pirate and the pirate has no demand.
- (ii) When $(4-q)^2\theta > 2$ and $c < q\theta/2$,
 - (a) when $c \leq \phi(q, \theta)$, the original producer's optimal level of deterrence is $x^* = 2(c+(2-q)\theta)/((4-q)^2\theta - 2)$. In this case, it accommodates the pirate and shares the market with the pirate.
 - (b) When $\phi(q, \theta) \leq c < q\theta/2$, the original producer's optimal level of deterrence is $x^* = q\theta/2 - c$. In this case, it deters the pirate and the pirate has no demand.
- (iii) When $c \geq q\theta/2$, there is no need to deter the pirate strategically. Piracy is blockaded anyway due to exogenous high level of IPR protection.

The condition $(4-q)^2\theta \leq 2$ in Proposition 1(i) can be interpreted as when the consumers' tastes are not sufficiently diverse, i.e., for any given q , θ is not sufficiently big ($\theta \leq 2/(4-q)^2$). In such a case, the original producer necessarily deters the pirate as long as the degree of intellectual property right is not sufficiently high (i.e. $c < q\theta/2$).

On the contrary, when the consumer taste is sufficiently diverse (i.e. $\theta > 2/(4-q)^2$) the original producer deters the pirate only if the degree of intellectual property right is relatively high (i.e. $\phi(q, \theta) \leq c < q\theta/2$). On the other hand, deterrence is too costly if the degree of intellectual property right is low (i.e. $c \leq \phi(q, \theta)$), there the original producer accommodates. Note that the numerator of $\phi(q, \theta)$ has to be positive, i.e. $q(4-q)\theta - 2 > 0$, for entry accommodation to arise in equilibrium. Since $q(4-q)$ is maximized on the interval $[0,1]$ at $q=1$ and the maximum is 3, a necessary condition for entry accommodation to be optimal is $\theta > 2/3$. One can also note that as long as $\theta > 2/3$, the condition $(4-q)^2 \theta > 2$ is satisfied for all $q \in (0,1)$.

3.2.2 Rate of Piracy and Quality of the Pirated Product

We define the ratio of $D_p/(D_o + D_p)$ to measure the rate of piracy. Thus the higher the ratio, the higher will be the rate of piracy. When $(4-q)^2 \theta > 2$ and $c \leq \phi(q, \theta)$, i.e. when the original firm accommodates the pirate, it is straightforward to get $\frac{D_p}{D_o + D_p} = \frac{(4-q)(q\theta - 2c) - 2}{(4-q)(q(3-q)\theta - (2-q)c) - 2}$. In all the other cases, entry is either deterred or blockaded; thus, the rate of piracy is zero.

When $(4-q)^2 \theta > 2$ and $c \leq \phi(q, \theta)$, simple computation yields

$$\frac{\partial}{\partial q} \left(\frac{D_p}{D_o + D_p} \right) = \frac{2(4-q)^2 c^2 + 4 \left((4-q)^2 (1-q)\theta + 2-q \right) c + \left(q^2 (4-q)^2 \theta + 2(8-12q+3q^2) \right) \theta}{\left((4-q)(q(3-q)\theta - (2-q)c) - 2 \right)^2}. \quad (5)$$

Since $(4-q)^2 \theta > 2$ and thus

$q^2 (4-q)^2 \theta + 2(8-12q+3q^2) > 2q^2 + 2(8-12q+3q^2) = 8(1-q)(2-q)$, the last term in

the numerator is positive. Therefore, $\frac{\partial}{\partial q} \left(\frac{D_p}{D_o + D_p} \right) > 0$. This result is summarized in the

following lemma.

Lemma 1

When firms compete in quantities, the relationship between the rate of piracy and the quality of the pirated product is monotonic i.e. the more reliable the pirated product, the higher is the rate of piracy.

The intuition for above result is as follows. When a consumer chooses between a pirated copy and an original one, she cares about both the reliability/quality of the chosen product and the price difference between the two. Since the price difference effect when firms compete in quantities remain small compared to the reliability effect as the quality of the pirated good improves, the relative demand of the pirated product increases. Hence, we get a monotonic relationship between the rate of piracy and the quality of the pirated product obtains.

3.3 Price Competition

As in section 3.2, we first obtain equilibrium prices in the price competition stage and then work out the choice of optimal level of deterrence by the original firm in the first period. Since this problem has been analyzed in detail by Lu and Poddar (2012), here we just summarize the main findings from that paper.

3.3.1 The Entry Accommodation Equilibrium and Entry Deterrence Equilibrium

The entry accommodation equilibrium and entry deterrence equilibrium in the whole parameter space of c, q and θ is characterized by Proposition 1 in Lu and Poddar (2011) which we replicate below.

Define

$$\eta(q, \theta) \equiv \frac{q(1-q)(16-12q+q^2)\theta + 6q - 8 - q\sqrt{(1-q)(2+q^2\theta)((4-q)^2(1-q)\theta - 2)}}{2(2-q)(8-8q+q^2)}.$$

Proposition 2

- (i) When $q(4-q)(1-q)\theta \leq 2$ and $c < q\theta/2$, the original producer's optimal level of deterrence is $x^* = (q\theta - 2c)/(q^2\theta + 2)$. In this case, it deters the pirate and the pirate has no demand.
- (ii) When $q(4-q)(1-q)\theta > 2$ and $c < q\theta/2$,
- (a) When $c \leq \eta(q, \theta)$, the original producer's optimal level of deterrence is $x^* = 2(c + 2(1-q)\theta)/((4-q)^2(1-q)\theta - 2)$. In this case, it accommodates the pirate and shares the market with the pirate.
- (b) When $\eta(q, \theta) \leq c < q\theta/2$, the original producer's optimal level of deterrence is $x^* = (q\theta - 2c)/(q^2\theta + 2)$. In this case, it deters the pirate and the pirate has no demand.
- (iii) When $c \geq q\theta/2$, there is no need to deter the pirate strategically. Piracy is blockaded anyway due to exogenous high level of IPR protection.

Similar to the case of quantity competition, in Proposition 2(i), the condition $q(4-q)(1-q)\theta \leq 2$ can be interpreted as when the consumers' tastes are not sufficiently diverse, i.e., for any given q , θ is not sufficiently big ($\theta \leq 2/q(4-q)(1-q)$). In such a case, the original producer necessarily deters the pirate as long as the degree of intellectual property right is not sufficiently high (i.e. $c < q\theta/2$).

On the contrary, when the consumer taste is sufficiently diverse (i.e. $\theta > 2/q(4-q)(1-q)$), the original producer deters the pirate only if the degree of intellectual property right is relatively high (i.e. $\eta(q, \theta) \leq c < q\theta/2$). On the other hand, deterrence is too costly if the degree of intellectual property right is low (i.e. $c \leq \eta(q, \theta)$), there the original producer accommodates. Note that $2/q(4-q)(1-q)$ is minimized at $q=0.465$ and the minimum is 2.274 and thus, a necessary condition for entry accommodation to be optimal is $\theta > 2.274$.

Furthermore, if the degree of intellectual property right is sufficiently high ($c \geq q\theta/2$), deterrence is blockaded.

3.3.2 Rate of Piracy and Quality of the Pirated Product

As before, we define the ratio of $D_p/(D_o + D_p)$ to measure the rate of piracy. When $c \leq \eta(q, \theta)$ and $q(4-q)(1-q)\theta > 2$, i.e. when the original firm accommodates the pirate, it is straightforward to get $\frac{D_p}{D_o + D_p} = \frac{q(1-q)(4-q)\theta - (2-q)(4-q)c - 2}{3q(1-q)(4-q)\theta - 2(1-q)(4-q)c - 2}$. In all the other cases, entry is either deterred or blockaded; thus, the rate of piracy is zero.

When $c \leq \eta(q, \theta)$ and $q(4-q)(1-q)\theta > 2$, simple computation yields

$$\frac{\partial}{\partial q} \left(\frac{D_p}{D_o + D_p} \right) = \frac{2(4-q)^2 c^2 + ((4-q)^2(4-8q+q^2)\theta + 8-4q)c + 4(4-10q+3q^2)\theta}{(3q(1-q)(4-q)\theta - 2(1-q)(4-q)c - 2)^2}. \quad (6)$$

As illustrated by numerical examples in Lu and Poddar (2012) (see appendix 1), the pattern of the change of the rate of piracy as the quality of pirated products increases is as follows: When $q < 0.465$ (i.e. when q is small), the rate of piracy is increasing in q ; when q is sufficiently large, it is decreasing in q ; when q is intermediate, it is decreasing in q when c is small and increasing in q when c is large. Thus, we have the following.

Lemma 2 (same as Proposition 4 in Lu and Poddar (2012))

When firms compete in prices, the relationship between the rate of piracy and the quality of the pirated product is non-monotonic.

The intuition for above result is as follows. When a consumer chooses between a pirated copy and original one, she cares about the reliability/quality of the chosen product and the price differential between the two. When the reliability of the pirated product is far from the original product, price competition is less intense and thus the price differential is large. However, as the pirated product becomes more reliable, the price competition between the pirated product and the original one becomes more intense; as a result, the price differential becomes smaller. Now it is the interaction between the price

and quality differential of the original and the pirated product leads to a non-monotonic relationship. When q is small, the price difference effect dominates (as the price of the pirated product is low) and a larger fraction of consumers choose to buy pirated product as q increases from its relatively low value. When q is sufficiently large, the reliability effect dominates (as both the pirated good and the original product are close in quality) and thus a smaller fraction of consumers choose to buy the pirated product as q increases further. Finally, When q is intermediate, which effect dominates depends on the degree of IPR protection since the price difference effect is larger when c is big than when c is small. In other words, the price competition is softer (i.e. prices are far apart) when c is big than when c is small. Moreover, we can also show the optimal level of deterrence also increases in c , which makes the price competition even softer for a big c . As a result, the rate of piracy increases in q when c is big while decreases in q when c is small.

3.4 Comparison between Price and Quantity Competition

It is useful to point out how the competition in prices or in quantities in the product market affects the strategic responses of the original product developer and the pirate. To do that we first explore how the accommodation/deterrence possibilities of the pirate are affected due to the nature of market competition. Comparing the condition in Proposition 1(*ii*a) and the one in Proposition 2(*ii*a), we have the following finding.

Proposition 3

Entry accommodation of the pirate is more likely to be observed under quantity competition than under price competition.

Proof: Observing the fact that $(4-q)^2 \theta > q(4-q)(1-q)\theta$ and $\eta(q, \theta) < \phi(q, \theta)$.

The above result reflects the fact that quantity competition is less stiff than price competition. Thus when firms compete in quantities, the original developer can more readily accommodate the pirate (i.e. less incentive to deter) compared to price competition scenario.

As for the relationship between the reliability of pirated products and the rate of piracy, combining Lemmas 1 and 2, we have our main result.

Proposition 4

In an environment of commercial piracy, when the original producer and the pirate compete in quantities, the relationship between the rate of piracy and the quality of the pirated product is monotonic (i.e. the more reliable the pirated product, the higher is the rate of piracy); whereas when they compete in prices, the relationship is non-monotonic.

Proof: Follows directly from Lemma 1 and Lemma 2.

In a broader sense, the difference in the results on the rate of piracy is also due to the very nature of price and quantity competition. For any given level of quality q of the pirated product, quantity competition is always less stiff (i.e. softer) than price competition. Thus a competing firm is less sensitive and hence less reactive in its strategic response due to change in q in the case of quantity competition compared to price competition. Moreover, the degree of sensitivity and reaction under price competition gets more pronounced compared to quantity competition as q becomes significantly high (i.e. the products become close). This feature is reflected in the following way. When pirated good is significantly low in quality, rate of piracy is increasing in quality for both quantity and price competition i.e. the qualitative behavior in the change of the piracy rate across the two types of competition matches. However, when the pirated good is significantly high in quality, the intensity of competition with prices gets much higher than with quantities resulting in very sharp reactions from competitors under price competition compared to quantity competition. Hence we get a divergence in the behavior on the piracy rate. In other words, the difference in the intensity or the degree of competition between price and quantity does not seem to matter much when the pirated good is relatively low in quality, but it matters when the pirated good is relatively high in quality.

4. End-User Piracy

Now we extend our analysis to the case of end-user piracy. End-user piracy is quite prevalent, in particular, in the market for digital goods as it is relatively easy to copy a digital product.¹² Here, we assume there is no commercial pirate in the economy, and the consumers (i.e. all potential product users) are the potential pirates. As before, there is one original product developer and consumers' valuations are uniformly distributed over the interval $[0, \theta]$ with density $1/\theta$. Consumers have the choice to buy the original product from the product developer or they can pirate themselves. The activity of the original product firm remains exactly the same as before, except that now it targets the end user pirates to stop or limit piracy as opposed to commercial pirate that we have analyzed before. However, unlike before, here the original firm does not face any direct competition from anybody in the market; instead, it stands to lose its potential market because of end user pirates. Under this circumstance to limit/stop piracy, it invests to raise the cost of piracy to the end users.¹³

Thus a consumer's utility function is given as:

$$U = \begin{cases} X - p & \text{if buys original product} \\ qX - (c + x) & \text{if pirates original product} \\ 0 & \text{otherwise,} \end{cases}$$

where x is the level of deterrence for piracy from the original producer and $c > 0$ is the exogenous cost parameter as before measuring the degree of IPR protection and this time it is targeted to stop/limit end-user piracy.

4.1 Deriving Demand of the Original and Pirated Product

The demand for the original product and for the pirated product, D_o and D_p , can be derived from the distribution of buyers as follows.

The marginal consumer, \hat{X} , who is indifferent between buying the original product and pirating is given by $\hat{X} = \frac{p - (c + x)}{1 - q}$. The marginal consumer, \hat{Y} , who is indifferent

¹² Most common digital products are computer software, music, movies and games.

¹³ Here, we do not need the two period time structure as before, everything can be formulated within a single period without loss of generality. There is no strategic game here; it's a monopoly analysis.

between pirating the product and not buying any product is given by $\hat{Y} = \frac{c+x}{q}$. Thus,

the demand for the original firm is $D_o = \frac{1}{\theta} \int_{\hat{x}}^{\theta} dx = \frac{(1-q)\theta - p + (c+x)}{(1-q)\theta}$ and the demand

for the pirated product is $D_p = \frac{1}{\theta} \int_{\hat{Y}}^{\hat{x}} dx = \frac{qp - (c+x)}{q(1-q)\theta}$. Here we have implicitly assumed

$qp \geq c+x$ so that the demand for the pirate product is nonnegative. When instead

$qp \leq c+x$, the developer's demand is $D_o = \frac{\theta - p}{\theta}$.

4.2 Choice of Optimal Price and Level of Deterrence by the Product Developer

When the developer chooses p and x such that $qp \geq c+x$, the firm's profit maximization problem is

$$\max_{p \geq 0, x \geq 0} \pi_o = pD_o - c_o(x) = p \left(\frac{(1-q)\theta - p + (c+x)}{(1-q)\theta} \right) - \frac{1}{2}x^2,$$

$$s.t. \quad qp \geq c+x$$

which is labeled Problem I.

When the developer chooses p and x such that $qp \leq c+x$, the firm's profit maximization problem is

$$\max_{p \geq 0, x \geq 0} \pi_o = pD_o - c_o(x) = p \left(\frac{\theta - p}{\theta} \right) - \frac{1}{2}x^2,$$

$$s.t. \quad qp \leq c+x$$

which is labeled Problem II.

4.2.1 The Optimum

We summarize the optimum in the following proposition after solving Problems I and II (see appendix 2 for all the details).

$$\text{Define } \delta(q, \theta) \equiv \frac{q(1-q)\theta - 1}{2-q}.$$

Proposition 5

(i) When $c \leq \delta(q, \theta)$ (this implicitly requires $q(1-q)\theta > 1$), the original developer

accommodates piracy, the optimal price is $p^* = \frac{(1-q)\theta((1-q)\theta+c)}{2(1-q)\theta-1}$ and the

optimal level of deterrence is $x^* = \frac{(1-q)\theta+c}{2(1-q)\theta-1}$.

(ii) When $\delta(q, \theta) \leq c \leq \frac{q\theta}{2}$, the original developer deters piracy, the optimal price is

$p^* = \frac{\theta(1+qc)}{2+q^2\theta}$ and the optimal level of deterrence is $x^* = \frac{q\theta-2c}{2+q^2\theta}$.

(iii) When $c \geq \frac{q\theta}{2}$, the piracy is blockaded and the original developer's optimal price

is the monopoly price $p^* = \frac{\theta}{2}$.

4.3 Rate of Piracy and Quality of the Pirated Product

As before, we define the ratio of $D_p / (D_o + D_p)$ to measure the rate of piracy.

When $q(1-q)\theta > 1$ and $c \leq \delta(q, \theta)$, i.e. when the original firm accommodates the pirate,

it is straightforward to get $\frac{D_p}{D_o + D_p} = \frac{q(1-q)\theta - (2-q)c - 1}{2q(1-q)\theta - 2(1-q)c - 1}$. In all the other cases,

entry is either deterred or blockaded; thus, the rate of piracy is zero.

When $q(1-q)\theta > 1$ and $c \leq \delta(q, \theta)$, simple computation yields

$$\frac{\partial}{\partial q} \left(\frac{D_p}{D_o + D_p} \right) = \frac{(2c+1)(c+(1-2q)\theta)}{(2q(1-q)\theta - 2(1-q)c - 1)^2}. \quad (7)$$

Clearly, when $q \leq \frac{1}{2}$, the sign of the partial derivative is positive; when $q > \frac{1}{2}$, it is

positive if c is relatively large, i.e. $(2q-1)\theta < c \leq \delta(q, \theta)$ while negative if c is relatively

small, i.e. $0 \leq c < (2q-1)\theta$. Thus, we have a similar conclusion as in Lemma 2.

Proposition 6

The relationship between the rate of piracy and the quality of the pirated product under end user piracy is non-monotonic.

To illustrate this result, we present some numerical examples. Fix $\theta = 5$. Then the condition $q(1-q)\theta > 1$ is satisfied when $0.2764 < q < 0.7236$. $\delta(q, \theta)$ is maximized at $q=0.5168$ and the maximum is 0.1676. Thus, a necessary condition for piracy accommodation to be optimal is $c \leq 0.1676$. When $c=0.05$, then

$$\frac{d}{dq} \left(\frac{D_p}{D_o + D_p} \right) = \frac{11(101 - 200q)}{2(11 - 101q + 100q^2)^2},$$

which is positive when $0.2764 < q < 0.505$ and

negative when $0.505 < q < 0.7236$. When $c=0.1$, then
$$\frac{d}{dq} \left(\frac{D_p}{D_o + D_p} \right) = \frac{3(51 - 100q)}{(6 - 51q + 50q^2)^2},$$

which is positive when $0.2764 < q < 0.51$ and negative when $0.51 < q < 0.7236$. When

$c=0.15$, then
$$\frac{d}{dq} \left(\frac{D_p}{D_o + D_p} \right) = \frac{13(103 - 200q)}{2(13 - 103q + 100q^2)^2},$$
 which is positive when

$0.2764 < q < 0.515$ and negative when $0.515 < q < 0.7236$.

The intuition for this result is similar to the one for Lemma 2.

4.4 Comparison between End-user piracy and Commercial Piracy under Price Competition

Comparing the condition in Proposition 2(ii) and the one in Proposition 5(i), we have the following result.

Lemma 3

Entry accommodation of pirates is less likely to be observed under end-user piracy than under commercial piracy under price competition.

Proof: Follows from the fact that $q(4-q)(1-q)\theta > 2q(1-q)\theta$ and $\delta(q, \theta) < \eta(q, \theta)$.

Under accommodation, the original firm faces more severe competition from the end-user pirates compared to the commercial pirate under price competition. Note that the cost of piracy to the end-user pirates is $(c + x)$; whereas the price of the commercial pirate satisfies $p_p \geq c + x$ and more often it is more than $(c + x)$. Thus keeping other things constant, the original firm will have a greater incentive to avoid the situation of accommodation with end-user pirates compared to the commercial pirate under price competition.

5. Comparison Across all Three Scenarios

5.1 Accommodation/Deterrence Possibilities of the Pirate

Now we consider all the three alternative scenarios (i.e. (i) commercial piracy under quantity competition, (ii) commercial piracy under price competition, (iii) end-user piracy under monopoly) we discussed so far to make an overall comparison of accommodation and deterrence possibilities of the pirate(s).

Given $\delta(q, \theta) < \eta(q, \theta) < \phi(q, \theta)$ and $(4 - q)^2 \theta > q(4 - q)(1 - q)\theta > 2q(1 - q)\theta$, we have a clear ordering on the accommodation/deterrence possibilities of the pirate(s) which we summarize below in the following proposition.

Proposition 7

A pirate is most likely to survive under commercial piracy and when it competes with the original firm in quantities and least likely to survive under end-user piracy.

This result is also a consequence of the fact that the original firm faces the softest competition from the pirate under quantity competition and the toughest competition from the end-user pirates. The survival possibility of the pirate under price competition is in between these two cases.

5.2 Piracy Deterrence Effort by the Copyright holder across three Scenarios

The comparison of the optimal deterrence efforts x^* of the original producer to stop/limit piracy across the three scenarios can be summarized as follows. Since $x^* = 0$ in the blockaded entry case, we exclude it in the following discussion.

Proposition 8

Under deterrence: $x^*(quantity) > x^*(price) = x^*(end - user)$

Under accommodation: $x^*(end - user) > x^*(price) > x^*(quantity)$

Proof: Follows directly from comparing the relevant expressions we derived before.

To deter completely under quantity competition, the original firm has to incur higher effort level as the pirate is most easily accommodated in this case compared to other two. Also note that in this situation the optimal deterrence effort of the copyright holder is exactly same under price competition and under the case of end user piracy.

On the other hand, for accommodation, since the competition is most relaxed under quantity, deterrence effort is the least for the original firm in that situation, whereas it is highest under the end-users piracy case where the competition is toughest. The optimal deterrence effort under price competition is in between these two cases as the intensity of the price competition lies in between these two cases as well.

6. Conclusion

In this paper, we study whether reliable pirated products lead to higher rate of piracy. We address this question in a framework where the original product developer i.e. the copyright holder makes costly investment to deter the pirate(s) in a given regime of IPR protection. The pirate can be commercial or end-users. The IPR protection can be weak or strong and is exogenous to the model. The pirated product may vary widely in terms of quality or reliability. In this set-up, we show that the relationship between the rate of piracy and the reliability of the pirated product depends very much on the nature of the pirate as well as on the nature of the market competition if the pirate is commercial and competes with the original producer. Under commercial piracy, when the original

firm and the pirate compete in quantities, the conventional wisdom holds i.e. the more reliable the pirated product, the higher is the rate of piracy. However, the relationship is non-monotonic, hence the wisdom does not hold when they compete in prices or the pirates are the end-users.

We also find that a pirate is most likely to survive under commercial piracy and when it competes with the original firm in quantities and least likely to survive under end-user piracy. The survival possibility of the pirate under price competition is in between these two cases. The main results of this analysis are consequences of the fact that the original firm faces the softest competition from the pirate under quantity competition and the toughest competition from the end-user pirates. The optimal levels of deterrence of the original firm also reflect that fact under the various scenarios of piracy considered in the analysis.

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Appendix 1

Numerical examples:

q	θ	$\eta(q, \theta)$	$\text{sign}\left[\partial\left(D_p/(D_o + D_p)\right)/\partial q\right]$
0.5	5	0.345	< 0 when $c < 0.192$ > 0 when $0.192 < c < 0.345$
0.47	5	0.340	< 0 when $c < 0.021$ > 0 when $0.021 < c < 0.340$
0.6	5	0.320	< 0 when $c < 0.320$
0.9	10*	0.089	< 0 when $c < 0.089$

*Note: When $q=0.9$, $2/q(4-q)(1-q)=7.169$.

Appendix 2

Problem I

Now we solve Problem I first. Define Lagrangian

$$L_1(p, x, \lambda) = p \left(\frac{(1-q)\theta - p + (c+x)}{(1-q)\theta} \right) - \frac{1}{2}x^2 + \lambda(qp - c - x) . \quad \text{The sufficient and}$$

necessary conditions for the optimum are the following:

$$\frac{\partial L_1(p, x, \lambda)}{\partial p} = \frac{(1-q)\theta - 2p + (c+x)}{(1-q)\theta} + \lambda q = 0, \quad (\text{A1})$$

$$\frac{\partial L_1(p, x, \lambda)}{\partial x} = \frac{p}{(1-q)\theta} - x - \lambda = 0, \quad (\text{A2})$$

$$\lambda(qp - c - x) = 0, \quad \lambda \geq 0, \quad qp \geq c + x. \quad (\text{A3})$$

If $\lambda = 0$, then we can solve for p and x from (A1) and (A2) after plugging $\lambda = 0$

into these equations and get $p = \frac{(1-q)\theta((1-q)\theta + c)}{2(1-q)\theta - 1}$ and $x = \frac{(1-q)\theta + c}{2(1-q)\theta - 1}$. We also

need to check whether $qp \geq c + x$ is satisfied and we find that this condition is satisfied

when $c \leq \frac{q(1-q)\theta-1}{2-q}$. In this case, the developer's profit is $\pi_o^A = \frac{((1-q)\theta+c)^2}{2(2(1-q)\theta-1)}$,

where the superscript A indicates this is an accommodation case.

If instead $qp = c + x$, then we can solve for p and x from (A1), (A2) and $qp = c + x$, and get $p = \frac{\theta(1+qc)}{2+q^2\theta}$, and $x = \frac{q\theta-2c}{2+q^2\theta}$. Note that $x \geq 0$ when $c \leq \frac{q\theta}{2}$. We also need to

check whether $\lambda \geq 0$ is satisfied and we find that this condition is satisfied when

$c \geq \frac{q(1-q)\theta-1}{2-q}$. In this case, the developer's profit is $\pi_o^D = \frac{\theta+2c(q\theta-c)}{2(2+q^2\theta)}$, where the

superscript D indicates this is a deterrence case.

Problem II

Next we turn to Problem II. Define Lagrangian

$L_2(p, x, \kappa) = p\left(\frac{\theta-p}{\theta}\right) - \frac{1}{2}x^2 - \kappa(qp - c - x)$. The sufficient and necessary conditions

for the optimum are the following:

$$\frac{\partial L_2(p, x, \kappa)}{\partial p} = \frac{\theta-2p}{\theta} - \kappa q = 0, \quad (\text{A4})$$

$$\frac{\partial L_2(p, x, \kappa)}{\partial x} = -x + \kappa = 0, \quad (\text{A5})$$

$$\kappa(qp - c - x) = 0, \quad \kappa \geq 0, \quad qp \leq c + x. \quad (\text{A6})$$

If $\kappa = 0$, then we can solve for p and x from (A4) and (A5) after plugging $\kappa = 0$ into these equations and get $p = \frac{\theta}{2}$ and $x = 0$. We also need to check whether $qp \leq c + x$

is satisfied and we find that this condition is satisfied when $c \geq \frac{q\theta}{2}$. This is clearly the

blockade case since the condition $qp \leq c + x$ is satisfied when the original developer

chooses the monopoly price, $p = \frac{\theta}{2}$, and zero deterrence level, $x=0$. In this case, the

developer's profit is $\pi_o^B = \frac{\theta}{4}$, where the superscript B indicates this is a blockade case.

If instead $qp = c + x$, then we can solve for p and x from (A4), (A5) and $qp = c + x$, and get $p = \frac{\theta(1+qc)}{2+q^2\theta}$, and $x = \frac{q\theta - 2c}{2+q^2\theta}$. We also need to check whether $\kappa \geq 0$ is satisfied and we find that this condition is satisfied when $c \leq \frac{q\theta}{2}$. This is clearly the deterrence case and the developer's profit is $\pi_o^D = \frac{\theta + 2c(q\theta - c)}{2(2+q^2\theta)}$.